Computer algebra independent integration tests

7-Inverse-hyperbolic-functions/7.1-Inverse-hyperbolic-sine/7.1.5-Inverse-hyperbolic-sine-functions

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [371]. This is test number [188].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 99.73 (370)	% 0.27 (1)
Mathematica	% 99.73 (370)	% 0.27 (1)
Maple	% 62.80 (233)	% 37.20 (138)
Maxima	% 31.54 (117)	% 68.46 (254)
Fricas	% 39.62 (147)	% 60.38 (224)
Sympy	% 25.07 (93)	% 74.93 (278)
Giac	% 26.15 (97)	% 73.85 (274)
Mupad	% 21.02 (78)	% 78.98 (293)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

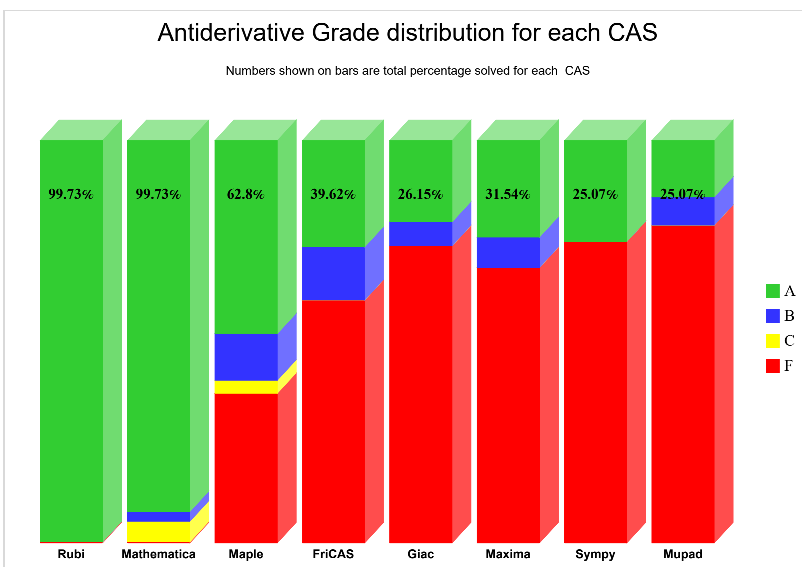
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

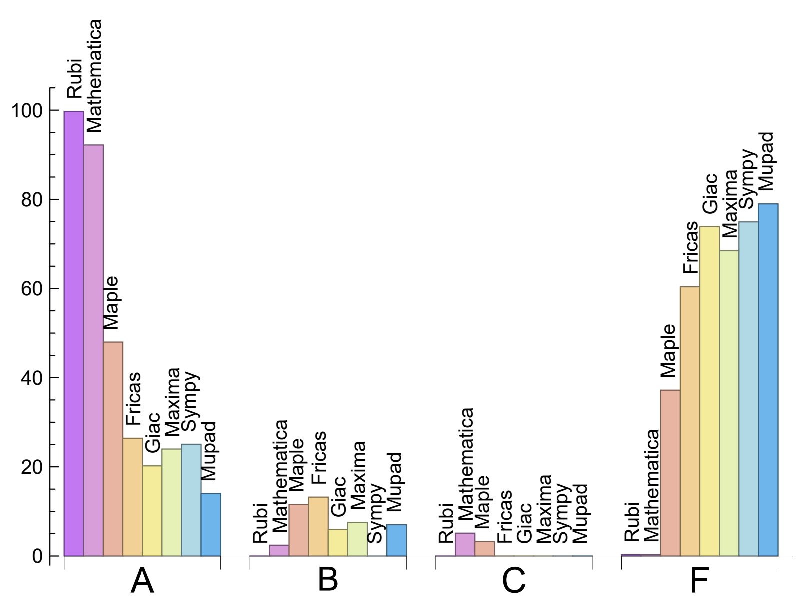
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.73	0.00	0.00	0.27
Mathematica	92.18	2.43	5.12	0.27
Maple	47.98	11.59	3.23	37.20
Maxima	23.99	7.55	0.00	68.46
Fricas	26.42	13.21	0.00	60.38
Sympy	25.07	0.00	0.00	74.93
Giac	20.22	5.93	0.00	73.85
Mupad	14.02	7.01	0.00	78.98

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	1	100.00 %	0.00 %	0.00 %
Mathematica	1	100.00 %	0.00 %	0.00 %
Maple	138	63.04 %	0.00 %	36.96 %
Maxima	254	92.52 %	2.36 %	5.12 %
Fricas	224	61.61 %	0.00 %	38.39 %
Sympy	278	79.50 %	11.51 %	8.99 %
Giac	274	79.93 %	2.92 %	17.15 %
Mupad	293	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

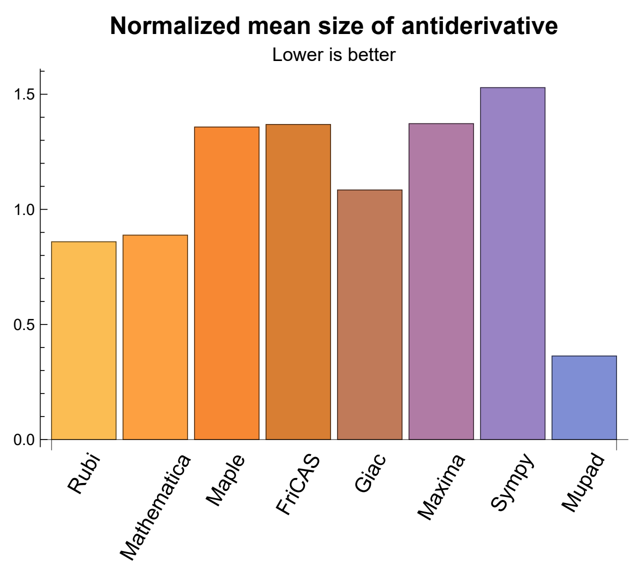
1.3 Performance

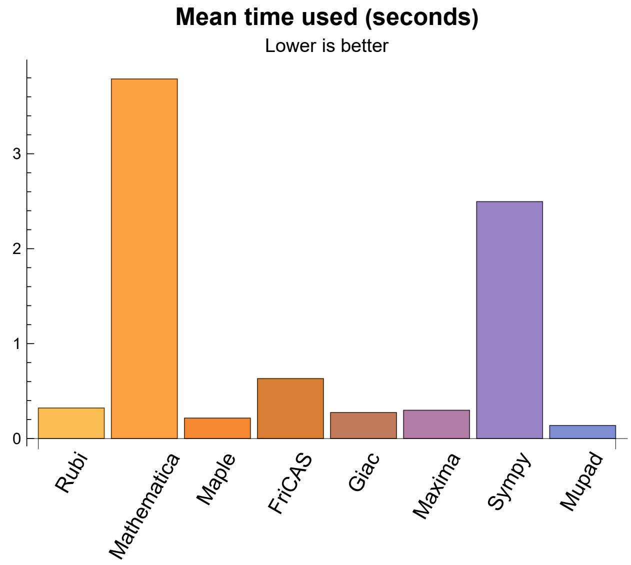
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.32	178.03	0.86	132.50	1.00
Mathematica	3.79	202.25	0.89	113.00	0.92
Maple	0.22	279.21	1.36	131.00	1.22
Maxima	0.30	102.36	1.37	30.00	0.66
Fricas	0.63	149.95	1.37	87.00	1.31
Sympy	2.50	254.38	1.53	26.00	1.10
Giac	0.27	93.24	1.08	0.00	0.00
Mupad	0.14	16.62	0.36	-1.00	-0.01

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{23, 24, 28, 29, 30, 32, 33, 53, 57, 84, 88, 92, 93, 97, 136, 146, 155, 161, 167, 173, 179, 185, 191, 197, 203, 209, 215, 221, 227, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 281, 282, 342, 346, 347, 362, 363}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {120, 132, 142, 144, 151, 153, 343, 344, 345}

Mathematica {37, 38, 42, 46, 51, 52, 74, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 112, 113, 120, 132, 133, 135, 142, 143, 144, 145, 151, 152, 153, 154, 180, 181, 182, 183, 184, 186, 187, 188, 189, 190, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 204, 205, 206, 207, 208, 210, 211, 212, 213, 214, 216, 217, 218, 219, 220, 222, 223, 224, 225, 226, 278, 279, 325, 326, 327, 343, 344, 345, 349, 350, 351, 352, 353, 354, 355, 356, 357}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

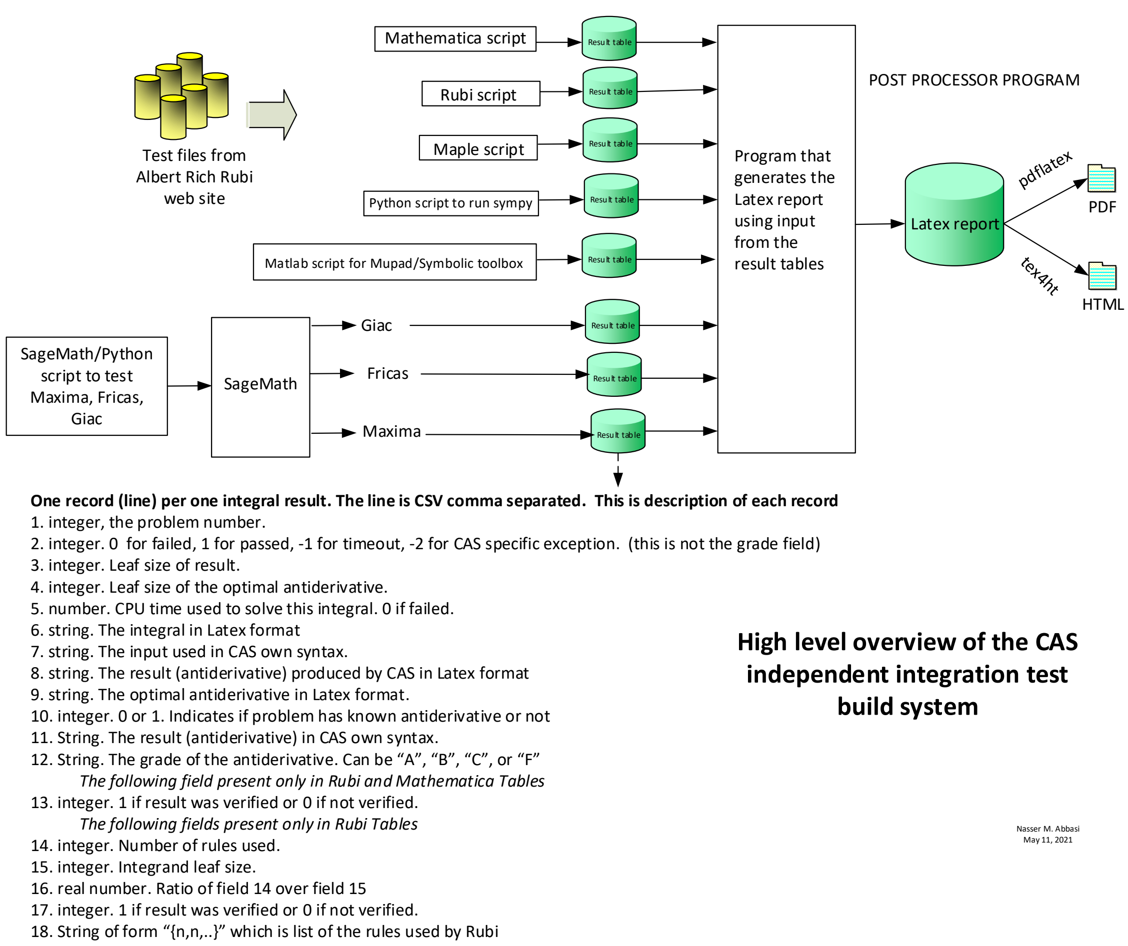
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 370, 371 }

B grade: { }

C grade: { }

F grade: { 369 }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 39, 40, 41, 43, 44, 45, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 146, 147, 148, 149, 150, 151, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 197, 198, 199, 200, 201, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 285, 287, 289, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 369, 370, 371 }

B grade: { 103, 104, 145, 152, 154, 196, 202, 302, 368 }

C grade: { 37, 38, 42, 46, 74, 125, 153, 228, 229, 230, 231, 232, 233, 234, 235, 284, 286, 288, 290 }

F grade: { 31 }

2.1.3 Maple

A grade: { 1, 4, 5, 6, 7, 8, 12, 13, 14, 15, 17, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 37, 39, 40, 41, 42, 43, 44, 45, 47, 49, 50, 53, 57, 58, 59, 60, 61, 63, 64, 65, 66, 67, 68, 69, 70, 72, 73, 74, 75, 76, 77, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 97, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 127, 128, 129, 130, 131, 133, 135, 136, 137, 138, 139, 140, 141, 146, 147, 148, 155, 156, 157, 158, 159, 160, 161, 164, 165, 166, 167, 171, 172, 173, 177, 178, 179, 185, 191, 197, 203, 209, 215, 221, 227, 260, 261, 262, 263, 264, 265, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 281, 282, 283, 285, 289, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 311, 317, 324, 342, 345, 346, 347, 348, 349, 350, 351, 353, 362, 363, 364, 365, 366, 368 }

B grade: { 9, 10, 11, 18, 35, 36, 38, 46, 48, 51, 52, 62, 132, 134, 142, 143, 144, 145, 149, 150, 151, 152, 153, 154, 162, 163, 168, 169, 170, 174, 175, 176, 266, 267, 268, 280, 343, 344, 352, 354, 355, 356, 357 }

C grade: { 228, 229, 230, 231, 232, 233, 234, 235, 284, 286, 288, 290 }

F grade: { 2, 3, 16, 31, 54, 55, 56, 71, 78, 79, 80, 94, 95, 96, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 126, 180, 181, 182, 183, 184, 186, 187, 188, 189, 190, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 204, 205, 206, 207, 208, 210, 211, 212, 213, 214, 216, 217, 218, 219, 220, 222, 223, 224, 225, 226, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 287, 291, 307, 308, 309, 310, 312, 313, 314, 315, 316, 318, 319, 320, 321, 322, 323, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 358, 359, 360, 361, 367, 369, 370, 371 }

2.1.4 Maxima

A grade: { 4, 5, 6, 7, 9, 10, 12, 13, 14, 15, 23, 24, 28, 29, 30, 32, 33, 49, 50, 53, 57, 61, 64, 84, 88, 92, 93, 97, 119, 121, 136, 146, 155, 161, 167, 173, 185, 191, 197, 203, 209, 215, 221, 227, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 281, 282, 285, 289, 292, 293, 294, 296, 297, 298, 299, 300, 301, 302, 304, 306, 317, 324, 342, 346, 347, 350, 353, 354, 362, 363, 366, 367, 369 }

B grade: { 58, 59, 60, 63, 65, 66, 115, 116, 117, 118, 122, 124, 134, 262, 268, 272, 273, 274, 276, 280, 283, 305, 349, 351, 352, 355, 356, 357 }

C grade: { }

F grade: { 1, 2, 3, 8, 11, 16, 17, 18, 19, 20, 21, 22, 25, 26, 27, 31, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 51, 52, 54, 55, 56, 62, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 85, 86, 87, 89, 90, 91, 94, 95, 96, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 120, 123, 125, 126, 127, 128, 129, 130, 131, 132, 133, 135, 137, 138, 139, 140, 141, 142, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 154, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 168, 169, 170, 171, 172, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 186, 187, 188, 189, 190, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 204, 205, 206, 207, 208, 210, 211, 212, 213, 214, 216, 217, 218, 219, 220, 222, 223, 224, 225, 226, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 260, 261, 263, 264, 265, 266, 267, 269, 270, 271, 275, 277, 278, 279, 284, 286, 287, 288, 290, 291, 295, 303, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 318, 319, 320, 321, 322, 323, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 343, 344, 345, 348, 358, 359, 360, 361, 364, 365, 368, 370, 371 }

2.1.5 FriCAS

A grade: { 4, 5, 6, 7, 12, 13, 14, 23, 24, 28, 29, 30, 32, 33, 53, 57, 58, 59, 60, 61, 67, 68, 69, 75, 76, 77, 84, 88, 92, 93, 97, 118, 119, 136, 146, 155, 161, 167, 173, 179, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 266, 267, 268, 281, 282, 283, 285, 292, 293, 294, 296, 297, 298, 299, 301, 304, 305, 306, 315, 316, 317, 322, 323, 324, 342, 346, 347, 349, 350, 351, 353, 355, 356, 357, 362, 363, 366, 369, 371 }

B grade: { 9, 10, 11, 15, 63, 64, 65, 66, 70, 115, 116, 117, 121, 122, 123, 124, 125, 127, 128, 129, 130, 131, 134, 137, 138, 139, 140, 141, 147, 148, 149, 150, 272, 273, 274, 275, 276, 277, 280, 289, 300, 302, 314, 321, 352, 354, 367, 368, 370 }

C grade: { }

F grade: { 1, 2, 3, 8, 16, 17, 18, 19, 20, 21, 22, 25, 26, 27, 31, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 55, 56, 62, 71, 72, 73, 74, 78, 79, 80, 81, 82, 83, 85, 86, 87, 89, 90, 91, 94, 95, 96, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 120, 126, 132, 133, 135, 142, 143, 144, 145, 151, 152, 153, 154, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 168, 169, 170, 171, 172, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 263, 264, 265, 269, 270, 271, 278, 279, 284, 286, 287, 288, 290, 291, 295, 303, 307, 308, 309, 310, 311, 312, 313, 318, 319, 320, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 343, 344, 345, 348, 358, 359, 360, 361, 364, 365 }

2.1.6 Sympy

A grade: { 4, 5, 6, 7, 12, 13, 14, 15, 23, 24, 28, 29, 30, 32, 33, 57, 58, 59, 60, 61, 67, 68, 69, 70, 75, 76, 77, 84, 88, 92, 93, 97, 115, 116, 117, 118, 119, 127, 128, 129, 130, 131, 136, 137, 138, 139, 140, 141, 146, 147, 148, 149, 150, 155, 161, 167, 173, 179, 185, 191, 197, 209, 215, 221, 246, 247, 248, 249, 250, 254, 255, 256, 257, 258, 266, 267, 268, 272, 273, 274, 275, 276, 277, 281, 282, 283, 285, 294, 304, 362, 363, 366, 367 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 8, 9, 10, 11, 16, 17, 18, 19, 20, 21, 22, 25, 26, 27, 31, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 62, 63, 64, 65, 66, 71, 72, 73, 74, 78, 79, 80, 81, 82, 83, 85, 86, 87, 89, 90, 91, 94, 95, 96, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 120, 121, 122, 123, 124, 125, 126, 132, 133, 134, 135, 142, 143, 144, 145, 151, 152, 153, 154, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 168, 169, 170, 171, 172, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 186, 187, 188, 189, 190, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 210, 211, 212, 213, 214, 216, 217, 218, 219, 220, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 251, 252, 253, 259, 260, 261, 262, 263, 264, 265, 269, 270, 271, 278, 279, 280, 284, 286, 287, 288, 289, 290, 291, 292, 293, 295, 296, 297, 298, 299, 300, 301, 302, 303, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 364, 365, 368, 369, 370, 371 }

2.1.7 Giac

A grade: { 6, 7, 23, 24, 28, 29, 30, 32, 33, 57, 58, 59, 60, 84, 88, 92, 97, 136, 146, 155, 161, 167, 173, 179, 185, 191, 197, 203, 209, 215, 221, 227, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 280, 281, 282, 283, 285, 292, 293, 294, 296, 297, 298, 299, 300, 301, 304, 305, 306, 342, 346, 347, 349, 350, 351, 353, 362, 363, 369 }

B grade: { 9, 15, 61, 63, 64, 65, 66, 115, 116, 117, 118, 119, 121, 289, 302, 352, 354, 355, 356, 357, 366, 367 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 8, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 21, 22, 25, 26, 27, 31, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 62, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 85, 86, 87, 89, 90, 91, 93, 94, 95, 96, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 120, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 137, 138, 139, 140, 141, 142, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 154, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 168, 169, 170, 171, 172, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 186, 187, 188, 189, 190, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 204, 205, 206, 207, 208, 210, 211, 212, 213, 214, 216, 217, 218, 219, 220, 222, 223, 224, 225, 226, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 284, 286, 287, 288, 290, 291, 295, 303, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 343, 344, 345, 348, 358, 359, 360, 361, 364, 365, 368, 370, 371 }

2.1.8 Mupad

A grade: { 23, 24, 28, 29, 30, 32, 33, 53, 57, 84, 88, 92, 93, 97, 136, 146, 155, 161, 167, 173, 179, 185, 191, 197, 203, 209, 215, 221, 227, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 281, 282, 342, 346, 347, 362, 363 }

B grade: { 6, 7, 61, 119, 272, 273, 274, 275, 276, 277, 283, 285, 294, 301, 302, 304, 305, 317, 324, 353, 354, 366, 367, 368, 369, 371 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 25, 26, 27, 31, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 55, 56, 58, 59, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 85, 86, 87, 89, 90, 91, 94, 95, 96, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 137, 138, 139, 140, 141, 142, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 154, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 168, 169, 170, 171, 172, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 186, 187, 188, 189, 190, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 204, 205, 206, 207, 208, 210, 211, 212, 213, 214, 216, 217, 218, 219, 220, 222, 223, 224, 225, 226, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 278, 279, 280, 284, 286, 287, 288, 289, 290, 291, 292, 293, 295, 296, 297, 298, 299, 300, 303, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 318, 319, 320, 321, 322, 323, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 343, 344, 345, 348, 349, 350, 351, 352, 355, 356, 357, 358, 359, 360, 361, 364, 365, 370 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	168	263	0	0	0	0	-1
normalized size	1	1.00	0.99	1.55	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.264	0.010	0.129	0.000	0.592	0.000	0.000	0.000

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	240	0	0	0	0	0	-1
normalized size	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.405	0.136	0.203	0.000	0.533	0.000	0.000	0.000

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	348	348	322	0	0	0	0	0	-1
normalized size	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.433	0.043	0.183	0.000	0.540	0.000	0.000	0.000

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	166	259	230	214	316	0	-1
normalized size	1	1.00	0.94	1.47	1.31	1.22	1.80	0.00	-0.01
time (sec)	N/A	0.170	0.145	0.009	0.396	0.769	1.479	0.000	0.000

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	121	189	150	147	190	0	-1
normalized size	1	1.00	0.98	1.52	1.21	1.19	1.53	0.00	-0.01
time (sec)	N/A	0.096	0.090	0.006	0.404	0.574	0.661	0.000	0.000

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	91	96	82	87	99	124	78
normalized size	1	1.00	0.94	0.99	0.85	0.90	1.02	1.28	0.80
time (sec)	N/A	0.052	0.041	0.007	0.370	0.477	0.303	0.823	0.289

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	31	30	43	26	41	28
normalized size	1	1.00	1.00	1.03	1.00	1.43	0.87	1.37	0.93
time (sec)	N/A	0.013	0.009	0.006	0.402	0.560	0.133	0.232	0.002

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	175	282	0	0	0	0	-1
normalized size	1	1.00	0.94	1.51	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.259	0.049	0.044	0.000	0.573	0.000	0.000	0.000

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	79	178	94	253	0	214	-1
normalized size	1	1.00	0.96	2.17	1.15	3.09	0.00	2.61	-0.01
time (sec)	N/A	0.055	0.101	0.017	0.457	0.663	0.000	0.861	0.000

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	166	279	158	566	0	0	-1
normalized size	1	1.00	1.30	2.18	1.23	4.42	0.00	0.00	-0.01
time (sec)	N/A	0.084	0.360	0.062	0.390	0.764	0.000	0.000	0.000

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	205	516	0	977	0	0	-1
normalized size	1	1.00	1.12	2.82	0.00	5.34	0.00	0.00	-0.01
time (sec)	N/A	0.138	0.432	0.013	0.000	1.673	0.000	0.000	0.000

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	368	368	354	641	590	475	743	0	-1
normalized size	1	1.00	0.96	1.74	1.60	1.29	2.02	0.00	-0.00
time (sec)	N/A	0.759	0.557	0.122	0.503	0.686	3.945	0.000	0.000

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	248	410	378	319	454	0	-1
normalized size	1	1.00	1.04	1.72	1.58	1.33	1.90	0.00	-0.00
time (sec)	N/A	0.512	0.382	0.094	0.430	0.615	1.721	0.000	0.000

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	142	193	219	183	233	0	-1
normalized size	1	1.00	1.01	1.38	1.56	1.31	1.66	0.00	-0.01
time (sec)	N/A	0.322	0.342	0.070	0.369	0.633	0.855	0.000	0.000

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	74	72	72	96	82	111	-1
normalized size	1	1.00	1.61	1.57	1.57	2.09	1.78	2.41	-0.02
time (sec)	N/A	0.063	0.062	0.000	0.334	0.454	0.278	0.785	0.000

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	273	0	0	0	0	0	-1
normalized size	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.469	0.224	0.030	0.000	0.674	0.000	0.000	0.000

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	191	529	0	0	0	0	-1
normalized size	1	1.00	0.73	2.01	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.472	0.218	0.344	0.000	0.836	0.000	0.000	0.000

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	270	1013	0	0	0	0	-1
normalized size	1	1.00	0.77	2.90	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.609	0.708	0.667	0.000	0.663	0.000	0.000	0.000

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	394	394	305	394	0	0	0	0	-1
normalized size	1	1.00	0.77	1.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.172	0.694	0.480	0.000	0.592	0.000	0.000	0.000

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	188	254	0	0	0	0	-1
normalized size	1	1.00	0.77	1.04	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.702	0.413	0.323	0.000	0.752	0.000	0.000	0.000

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	98	120	0	0	0	0	-1
normalized size	1	1.00	0.84	1.03	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.318	0.166	0.225	0.000	0.524	0.000	0.000	0.000

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	45	56	0	0	0	0	-1
normalized size	1	1.00	0.83	1.04	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.074	0.022	0.000	0.000	0.584	0.000	0.000	0.000

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.042	0.205	0.205	0.000	0.606	0.000	0.000	0.000

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.039	0.390	0.408	0.000	0.559	0.000	0.000	0.000

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	359	351	288	616	0	0	0	0	-1
normalized size	1	0.98	0.80	1.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.688	1.610	0.377	0.000	0.484	0.000	0.000	0.000

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	176	150	272	0	0	0	0	-1
normalized size	1	0.98	0.83	1.51	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.337	0.749	0.270	0.000	0.572	0.000	0.000	0.000

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	81	71	118	0	0	0	0	-1
normalized size	1	0.95	0.84	1.39	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.179	0.188	0.000	0.000	0.450	0.000	0.000	0.000

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.031	3.218	0.228	0.000	0.531	0.000	0.000	0.000

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.029	5.209	0.414	0.000	0.646	0.000	0.000	0.000

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	75	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.261	4.173	3.603	0.000	0.560	0.000	0.000	0.000

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.101	0.046	3.604	0.000	0.565	0.000	0.000	0.000

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.029	0.367	1.197	0.000	0.469	0.000	0.000	0.000

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.031	0.781	1.260	0.000	0.568	0.000	0.000	0.000

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	640	640	413	1119	0	0	0	0	-1
normalized size	1	1.00	0.65	1.75	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.687	1.493	0.803	0.000	0.565	0.000	0.000	0.000

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	431	431	301	791	0	0	0	0	-1
normalized size	1	1.00	0.70	1.84	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.532	0.958	0.618	0.000	0.546	0.000	0.000	0.000

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	208	423	0	0	0	0	-1
normalized size	1	1.00	0.92	1.86	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.250	1.212	0.423	0.000	0.629	0.000	0.000	0.000

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	664	664	1353	992	0	0	0	0	-1
normalized size	1	1.00	2.04	1.49	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.650	6.368	0.350	0.000	0.466	0.000	0.000	0.000

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	781	781	1384	1814	0	0	0	0	-1
normalized size	1	1.00	1.77	2.32	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.547	9.653	0.714	0.000	0.529	0.000	0.000	0.000

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	918	918	779	1510	0	0	0	0	-1
normalized size	1	1.00	0.85	1.64	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.932	3.709	0.964	0.000	0.524	0.000	0.000	0.000

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	651	651	546	1087	0	0	0	0	-1
normalized size	1	1.00	0.84	1.67	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.731	2.218	0.845	0.000	0.579	0.000	0.000	0.000

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	353	392	601	0	0	0	0	-1
normalized size	1	1.00	1.11	1.70	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.335	1.232	0.499	0.000	0.533	0.000	0.000	0.000

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	984	984	2889	1838	0	0	0	0	-1
normalized size	1	1.00	2.94	1.87	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.889	13.619	0.368	0.000	0.542	0.000	0.000	0.000

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1228	1228	1899	1954	0	0	0	0	-1
normalized size	1	1.00	1.55	1.59	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.139	7.079	1.010	0.000	0.594	0.000	0.000	0.000

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	901	901	1047	1424	0	0	0	0	-1
normalized size	1	1.00	1.16	1.58	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.926	2.783	0.887	0.000	0.587	0.000	0.000	0.000

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	494	494	656	805	0	0	0	0	-1
normalized size	1	1.00	1.33	1.63	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.397	1.344	0.571	0.000	0.563	0.000	0.000	0.000

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1536	1536	7163	3928	0	0	0	0	-1
normalized size	1	1.00	4.66	2.56	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.448	25.937	0.470	0.000	0.693	0.000	0.000	0.000

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	430	430	304	751	0	0	0	0	-1
normalized size	1	1.00	0.71	1.75	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.575	0.951	0.810	0.000	0.594	0.000	0.000	0.000

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	233	486	0	0	0	0	-1
normalized size	1	1.00	0.90	1.88	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.430	0.608	0.614	0.000	0.620	0.000	0.000	0.000

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	158	209	87	0	0	0	-1
normalized size	1	1.00	1.32	1.74	0.72	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.217	0.264	0.406	0.419	0.547	0.000	0.000	0.000

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	48	77	28	0	0	0	-1
normalized size	1	1.00	1.02	1.64	0.60	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.057	0.036	0.006	0.452	0.705	0.000	0.000	0.000

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	256	678	0	0	0	0	-1
normalized size	1	1.00	0.79	2.09	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.548	0.625	0.272	0.000	0.444	0.000	0.000	0.000

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	444	444	448	1770	0	0	0	0	-1
normalized size	1	1.00	1.01	3.99	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.661	2.172	0.686	0.000	0.550	0.000	0.000	0.000

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.186	0.138	2.384	0.000	0.527	0.000	0.000	0.000

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	438	438	397	0	0	0	0	0	-1
normalized size	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.727	0.234	2.638	0.000	0.572	0.000	0.000	0.000

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	332	332	304	0	0	0	0	0	-1
normalized size	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.559	0.229	2.128	0.000	0.549	0.000	0.000	0.000

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	206	0	0	0	0	0	-1
normalized size	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.300	0.018	0.176	0.000	0.574	0.000	0.000	0.000

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.195	0.273	1.279	0.000	0.528	0.000	0.000	0.000

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	95	200	318	110	255	162	-1
normalized size	1	1.00	0.73	1.53	2.43	0.84	1.95	1.24	-0.01
time (sec)	N/A	0.172	0.088	0.016	0.336	0.463	1.438	1.004	0.000

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	74	130	210	91	170	131	-1
normalized size	1	1.00	0.82	1.44	2.33	1.01	1.89	1.46	-0.01
time (sec)	N/A	0.113	0.059	0.004	0.365	0.552	0.666	0.329	0.000

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	60	74	149	75	104	111	-1
normalized size	1	1.00	0.79	0.97	1.96	0.99	1.37	1.46	-0.01
time (sec)	N/A	0.066	0.042	0.004	0.369	0.531	0.302	0.364	0.000

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	40	31	30	57	46	92	76
normalized size	1	1.00	1.18	0.91	0.88	1.68	1.35	2.71	2.24
time (sec)	N/A	0.014	0.028	0.002	0.388	0.536	0.164	0.359	0.446

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	153	388	0	0	0	0	-1
normalized size	1	1.00	1.17	2.96	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.243	0.012	0.171	0.000	0.537	0.000	0.000	0.000

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	57	71	111	167	0	110	-1
normalized size	1	1.00	1.00	1.25	1.95	2.93	0.00	1.93	-0.02
time (sec)	N/A	0.070	0.041	0.006	0.325	0.578	0.000	0.449	0.000

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	110	106	146	236	0	199	-1
normalized size	1	1.00	1.20	1.15	1.59	2.57	0.00	2.16	-0.01
time (sec)	N/A	0.102	0.178	0.008	0.409	0.663	0.000	0.608	0.000

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	149	203	284	285	0	381	-1
normalized size	1	1.00	1.16	1.57	2.20	2.21	0.00	2.95	-0.01
time (sec)	N/A	0.154	0.235	0.008	0.359	0.717	0.000	0.470	0.000

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	179	275	357	343	0	709	-1
normalized size	1	1.00	1.07	1.65	2.14	2.05	0.00	4.25	-0.01
time (sec)	N/A	0.228	0.214	0.009	0.364	0.707	0.000	0.570	0.000

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	145	377	0	182	366	0	-1
normalized size	1	1.00	0.44	1.14	0.00	0.55	1.11	0.00	-0.00
time (sec)	N/A	0.547	0.191	0.109	0.000	0.535	3.107	0.000	0.000

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	107	212	0	146	243	0	-1
normalized size	1	1.00	0.51	1.00	0.00	0.69	1.15	0.00	-0.00
time (sec)	N/A	0.375	0.143	0.086	0.000	0.527	1.385	0.000	0.000

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	79	113	0	114	138	0	-1
normalized size	1	1.00	0.63	0.90	0.00	0.90	1.10	0.00	-0.01
time (sec)	N/A	0.236	0.083	0.077	0.000	0.572	0.676	0.000	0.000

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	47	46	0	88	63	0	-1
normalized size	1	1.00	1.04	1.02	0.00	1.96	1.40	0.00	-0.02
time (sec)	N/A	0.051	0.023	0.053	0.000	0.553	0.262	0.000	0.000

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	251	0	0	0	0	0	-1
normalized size	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.350	0.032	0.217	0.000	0.498	0.000	0.000	0.000

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	178	217	0	0	0	0	-1
normalized size	1	1.00	1.00	1.22	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.383	0.120	0.394	0.000	0.515	0.000	0.000	0.000

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	279	384	0	0	0	0	-1
normalized size	1	1.00	1.19	1.63	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.486	0.130	0.662	0.000	0.582	0.000	0.000	0.000

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	478	478	1830	730	0	0	0	0	-1
normalized size	1	1.00	3.83	1.53	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.573	10.523	0.887	0.000	0.672	0.000	0.000	0.000

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	355	355	175	311	0	225	432	0	-1
normalized size	1	1.00	0.49	0.88	0.00	0.63	1.22	0.00	-0.00
time (sec)	N/A	0.450	0.207	0.099	0.000	0.487	3.036	0.000	0.000

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	129	169	0	180	248	0	-1
normalized size	1	1.00	0.64	0.83	0.00	0.89	1.22	0.00	-0.00
time (sec)	N/A	0.303	0.137	0.076	0.000	0.499	1.295	0.000	0.000

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	70	67	0	139	109	0	-1
normalized size	1	1.00	0.90	0.86	0.00	1.78	1.40	0.00	-0.01
time (sec)	N/A	0.074	0.031	0.052	0.000	0.501	0.598	0.000	0.000

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	346	0	0	0	0	0	-1
normalized size	1	1.00	1.26	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.398	0.033	0.191	0.000	0.610	0.000	0.000	0.000

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	259	0	0	0	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.580	0.117	0.446	0.000	0.746	0.000	0.000	0.000

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	514	514	524	0	0	0	0	0	-1
normalized size	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.883	0.239	0.765	0.000	0.647	0.000	0.000	0.000

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	44	49	0	0	0	0	-1
normalized size	1	1.00	0.73	0.82	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.527	0.147	0.122	0.000	0.478	0.000	0.000	0.000

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	27	0	0	0	0	-1
normalized size	1	1.00	1.00	0.90	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.211	0.053	0.058	0.000	0.652	0.000	0.000	0.000

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	0	0	0	0	-1
normalized size	1	1.00	1.00	1.09	0.00	0.00	0.00	0.00	-0.09
time (sec)	N/A	0.023	0.007	0.049	0.000	0.580	0.000	0.000	0.000

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.044	0.203	0.171	0.000	0.535	0.000	0.000	0.000

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	83	146	0	0	0	0	-1
normalized size	1	1.00	0.54	0.95	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.216	0.463	0.131	0.000	0.467	0.000	0.000	0.000

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	62	73	0	0	0	0	-1
normalized size	1	1.00	0.74	0.87	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.132	0.167	0.081	0.000	0.581	0.000	0.000	0.000

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	35	34	0	0	0	0	-1
normalized size	1	1.00	0.92	0.89	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.073	0.023	0.053	0.000	0.586	0.000	0.000	0.000

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.039	2.140	0.181	0.000	0.557	0.000	0.000	0.000

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	110	215	0	0	0	0	-1
normalized size	1	1.00	0.43	0.84	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.497	0.403	0.158	0.000	0.509	0.000	0.000	0.000

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	117	107	0	0	0	0	-1
normalized size	1	1.00	0.80	0.73	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.250	0.109	0.083	0.000	0.616	0.000	0.000	0.000

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	53	51	0	0	0	0	-1
normalized size	1	1.00	0.84	0.81	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.079	0.042	0.028	0.000	0.544	0.000	0.000	0.000

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.040	2.142	0.181	0.000	0.523	0.000	0.000	0.000

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.053	0.445	0.188	0.000	0.568	0.000	0.000	0.000

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	545	545	345	0	0	0	0	0	-1
normalized size	1	1.00	0.63	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.152	1.028	180.000	0.000	0.526	0.000	0.000	0.000

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	228	0	0	0	0	0	-1
normalized size	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.483	0.189	0.317	0.000	0.570	0.000	0.000	0.000

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	109	0	0	0	0	0	-1
normalized size	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.124	0.119	0.104	0.000	0.740	0.000	0.000	0.000

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.060	0.172	0.185	0.000	0.533	0.000	0.000	0.000

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	496	496	656	0	0	0	0	0	-1
normalized size	1	1.00	1.32	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.847	1.775	180.000	0.000	0.000	0.000	0.000	0.000

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	251	0	0	0	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.700	1.822	0.520	0.000	0.000	0.000	0.000	0.000

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	111	0	0	0	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.249	0.079	0.070	0.000	0.000	0.000	0.000	0.000

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	582	0	0	0	0	0	-1
normalized size	1	1.00	1.79	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.965	5.193	0.504	0.000	0.000	0.000	0.000	0.000

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	272	0	0	0	0	0	-1
normalized size	1	1.00	1.81	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.249	0.148	0.120	0.000	0.000	0.000	0.000	0.000

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	389	389	939	0	0	0	0	0	-1
normalized size	1	1.00	2.41	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.126	10.191	0.507	0.000	0.000	0.000	0.000	0.000

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	458	0	0	0	0	0	-1
normalized size	1	1.00	2.56	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.388	2.446	0.128	0.000	0.000	0.000	0.000	0.000

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	411	411	471	0	0	0	0	0	-1
normalized size	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.857	1.099	180.000	0.000	0.000	0.000	0.000	0.000

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	217	0	0	0	0	0	-1
normalized size	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.392	1.020	0.486	0.000	0.000	0.000	0.000	0.000

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	111	0	0	0	0	0	-1
normalized size	1	1.00	1.21	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.129	0.113	0.067	0.000	0.000	0.000	0.000	0.000

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	301	0	0	0	0	0	-1
normalized size	1	1.00	1.12	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.542	2.694	0.472	0.000	0.000	0.000	0.000	0.000

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	155	0	0	0	0	0	-1
normalized size	1	1.00	1.27	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.239	0.097	0.124	0.000	0.000	0.000	0.000	0.000

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	365	365	375	0	0	0	0	0	-1
normalized size	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.883	2.731	0.483	0.000	0.000	0.000	0.000	0.000

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	207	0	0	0	0	0	-1
normalized size	1	1.00	1.31	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.276	0.557	0.125	0.000	0.000	0.000	0.000	0.000

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	445	445	429	0	0	0	0	0	-1
normalized size	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.047	2.592	0.476	0.000	0.000	0.000	0.000	0.000

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	238	0	0	0	0	0	-1
normalized size	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.411	0.544	0.126	0.000	0.000	0.000	0.000	0.000

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	79	0	0	0	0	0	-1
normalized size	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.048	2.146	0.000	0.697	0.000	0.000	0.000

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	71	93	1231	279	527	817	-1
normalized size	1	1.00	0.71	0.93	12.31	2.79	5.27	8.17	-0.01
time (sec)	N/A	0.079	0.103	0.014	0.483	0.558	3.504	1.931	0.000

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	83	86	790	228	394	594	-1
normalized size	1	1.00	0.79	0.82	7.52	2.17	3.75	5.66	-0.01
time (sec)	N/A	0.070	0.076	0.006	0.388	0.491	1.692	1.633	0.000

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	64	73	445	168	258	403	-1
normalized size	1	1.00	0.84	0.96	5.86	2.21	3.39	5.30	-0.01
time (sec)	N/A	0.066	0.046	0.006	0.357	0.619	0.769	1.309	0.000

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	57	62	201	108	148	243	-1
normalized size	1	1.00	0.84	0.91	2.96	1.59	2.18	3.57	-0.01
time (sec)	N/A	0.039	0.064	0.006	0.444	0.548	0.335	0.956	0.000

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	50	36	35	65	51	99	85
normalized size	1	1.00	1.28	0.92	0.90	1.67	1.31	2.54	2.18
time (sec)	N/A	0.023	0.031	0.003	0.340	0.505	0.158	0.356	0.484

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	70	159	0	0	0	0	-1
normalized size	1	1.00	0.86	1.96	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.112	0.026	0.100	0.000	0.659	0.000	0.000	0.000

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	43	54	80	175	0	131	-1
normalized size	1	1.00	0.88	1.10	1.63	3.57	0.00	2.67	-0.02
time (sec)	N/A	0.054	0.039	0.005	0.367	0.576	0.000	0.491	0.000

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	57	60	117	118	0	0	-1
normalized size	1	1.00	0.97	1.02	1.98	2.00	0.00	0.00	-0.02
time (sec)	N/A	0.051	0.058	0.006	0.406	0.549	0.000	0.000	0.000

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	82	74	0	343	0	0	-1
normalized size	1	1.00	0.98	0.88	0.00	4.08	0.00	0.00	-0.01
time (sec)	N/A	0.071	0.086	0.006	0.000	0.557	0.000	0.000	0.000

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	61	80	258	210	0	0	-1
normalized size	1	1.00	0.68	0.89	2.87	2.33	0.00	0.00	-0.01
time (sec)	N/A	0.067	0.075	0.006	0.356	0.571	0.000	0.000	0.000

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	61	94	0	509	0	0	-1
normalized size	1	1.00	0.53	0.82	0.00	4.43	0.00	0.00	-0.01
time (sec)	N/A	0.086	0.082	0.008	0.000	0.718	0.000	0.000	0.000

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	155	0	0	0	0	0	-1
normalized size	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.207	0.120	1.951	0.000	0.665	0.000	0.000	0.000

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	192	218	0	618	1268	0	-1
normalized size	1	1.00	0.97	1.11	0.00	3.14	6.44	0.00	-0.01
time (sec)	N/A	0.307	0.260	0.052	0.000	0.756	7.462	0.000	0.000

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	170	194	0	486	916	0	-1
normalized size	1	1.00	0.99	1.13	0.00	2.83	5.33	0.00	-0.01
time (sec)	N/A	0.258	0.203	0.038	0.000	0.556	4.593	0.000	0.000

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	147	163	0	358	610	0	-1
normalized size	1	1.00	1.08	1.20	0.00	2.63	4.49	0.00	-0.01
time (sec)	N/A	0.205	0.176	0.041	0.000	0.575	1.864	0.000	0.000

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	120	135	0	230	335	0	-1
normalized size	1	1.00	1.17	1.31	0.00	2.23	3.25	0.00	-0.01
time (sec)	N/A	0.145	0.185	0.033	0.000	0.554	0.857	0.000	0.000

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	87	90	0	141	143	0	-1
normalized size	1	1.00	1.53	1.58	0.00	2.47	2.51	0.00	-0.02
time (sec)	N/A	0.070	0.101	0.052	0.000	0.562	0.325	0.000	0.000

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	116	115	100	404	0	0	0	0	-1
normalized size	1	0.99	0.86	3.48	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.198	0.049	0.082	0.000	0.719	0.000	0.000	0.000

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	154	229	0	0	0	0	-1
normalized size	1	1.00	1.54	2.29	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.176	0.605	0.138	0.000	0.552	0.000	0.000	0.000

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	120	180	230	319	0	0	-1
normalized size	1	1.00	1.41	2.12	2.71	3.75	0.00	0.00	-0.01
time (sec)	N/A	0.137	0.251	0.205	0.416	0.658	0.000	0.000	0.000

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	212	310	0	0	0	0	-1
normalized size	1	1.00	1.25	1.83	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.247	1.720	0.278	0.000	0.578	0.000	0.000	0.000

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.173	3.749	1.957	0.000	0.534	0.000	0.000	0.000

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	355	420	0	1077	2518	0	-1
normalized size	1	1.00	1.09	1.29	0.00	3.30	7.72	0.00	-0.00
time (sec)	N/A	0.474	0.476	0.052	0.000	0.608	17.075	0.000	0.000

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	303	365	0	832	1828	0	-1
normalized size	1	1.00	1.09	1.31	0.00	2.98	6.55	0.00	-0.00
time (sec)	N/A	0.386	0.402	0.065	0.000	0.625	10.072	0.000	0.000

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	258	302	0	613	1173	0	-1
normalized size	1	1.00	1.14	1.33	0.00	2.70	5.17	0.00	-0.00
time (sec)	N/A	0.300	0.328	0.041	0.000	0.629	4.994	0.000	0.000

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	200	243	0	391	685	0	-1
normalized size	1	1.00	1.24	1.51	0.00	2.43	4.25	0.00	-0.01
time (sec)	N/A	0.213	0.211	0.056	0.000	0.538	2.082	0.000	0.000

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	147	160	0	239	282	0	-1
normalized size	1	1.00	1.47	1.60	0.00	2.39	2.82	0.00	-0.01
time (sec)	N/A	0.108	0.167	0.071	0.000	0.663	0.804	0.000	0.000

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	128	736	0	0	0	0	-1
normalized size	1	1.00	0.83	4.75	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.224	0.049	0.089	0.000	0.524	0.000	0.000	0.000

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	315	481	0	0	0	0	-1
normalized size	1	1.00	1.90	2.90	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.255	0.723	0.149	0.000	0.618	0.000	0.000	0.000

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	229	409	0	0	0	0	-1
normalized size	1	1.00	1.46	2.61	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.261	0.812	0.217	0.000	0.592	0.000	0.000	0.000

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	694	651	0	0	0	0	-1
normalized size	1	1.00	2.66	2.49	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.410	7.141	0.311	0.000	0.725	0.000	0.000	0.000

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.179	1.920	2.039	0.000	0.548	0.000	0.000	0.000

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	475	573	0	1241	2876	0	-1
normalized size	1	1.00	1.36	1.64	0.00	3.56	8.24	0.00	-0.00
time (sec)	N/A	0.673	0.638	0.048	0.000	0.588	20.597	0.000	0.000

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	412	473	0	900	1889	0	-1
normalized size	1	1.00	1.47	1.68	0.00	3.20	6.72	0.00	-0.00
time (sec)	N/A	0.485	0.452	0.046	0.000	0.751	9.646	0.000	0.000

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	300	371	0	574	1027	0	-1
normalized size	1	1.00	1.54	1.90	0.00	2.94	5.27	0.00	-0.01
time (sec)	N/A	0.321	0.323	0.038	0.000	0.598	4.852	0.000	0.000

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	226	245	0	344	444	0	-1
normalized size	1	1.00	1.97	2.13	0.00	2.99	3.86	0.00	-0.01
time (sec)	N/A	0.160	0.264	0.060	0.000	0.540	1.695	0.000	0.000

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	157	1153	0	0	0	0	-1
normalized size	1	1.00	0.84	6.20	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.259	0.065	0.116	0.000	0.615	0.000	0.000	0.000

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	501	820	0	0	0	0	-1
normalized size	1	1.00	2.14	3.50	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.317	1.596	0.199	0.000	0.649	0.000	0.000	0.000

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	360	723	0	0	0	0	-1
normalized size	1	1.00	1.94	3.89	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.328	1.270	0.223	0.000	0.649	0.000	0.000	0.000

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	385	385	1182	1202	0	0	0	0	-1
normalized size	1	1.00	3.07	3.12	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.565	8.593	0.312	0.000	0.916	0.000	0.000	0.000

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.060	1.150	0.787	0.000	0.639	0.000	0.000	0.000

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	209	151	194	0	0	0	0	-1
normalized size	1	0.98	0.71	0.91	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.469	0.307	0.318	0.000	0.482	0.000	0.000	0.000

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	109	134	0	0	0	0	-1
normalized size	1	1.00	0.75	0.92	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.340	0.216	0.248	0.000	0.487	0.000	0.000	0.000

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	137	102	130	0	0	0	0	-1
normalized size	1	0.97	0.72	0.92	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.293	0.184	0.155	0.000	0.494	0.000	0.000	0.000

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	61	66	0	0	0	0	-1
normalized size	1	1.00	0.88	0.96	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.161	0.090	0.039	0.000	0.549	0.000	0.000	0.000

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	49	60	0	0	0	0	-1
normalized size	1	1.00	0.84	1.03	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.089	0.022	0.031	0.000	0.475	0.000	0.000	0.000

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.066	0.840	0.169	0.000	0.517	0.000	0.000	0.000

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	252	281	602	0	0	0	0	-1
normalized size	1	0.98	1.10	2.35	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.409	1.096	0.377	0.000	0.582	0.000	0.000	0.000

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	193	388	0	0	0	0	-1
normalized size	1	1.00	1.03	2.06	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.311	0.880	0.302	0.000	0.656	0.000	0.000	0.000

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	180	138	342	0	0	0	0	-1
normalized size	1	0.98	0.75	1.86	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.285	0.738	0.235	0.000	0.584	0.000	0.000	0.000

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	97	160	0	0	0	0	-1
normalized size	1	1.00	0.94	1.55	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.148	0.299	0.056	0.000	0.621	0.000	0.000	0.000

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	87	77	128	0	0	0	0	-1
normalized size	1	0.96	0.85	1.41	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.174	0.089	0.050	0.000	0.452	0.000	0.000	0.000

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.064	1.443	0.167	0.000	0.589	0.000	0.000	0.000

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	316	316	896	0	0	0	0	-1
normalized size	1	0.99	0.99	2.80	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.892	1.250	0.388	0.000	0.611	0.000	0.000	0.000

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	179	579	0	0	0	0	-1
normalized size	1	1.00	0.72	2.34	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.682	0.658	0.310	0.000	0.520	0.000	0.000	0.000

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	305	216	507	0	0	0	0	-1
normalized size	1	1.24	0.88	2.06	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.612	0.725	0.241	0.000	0.637	0.000	0.000	0.000

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	120	239	0	0	0	0	-1
normalized size	1	1.00	0.77	1.53	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.331	0.315	0.066	0.000	0.769	0.000	0.000	0.000

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	100	190	0	0	0	0	-1
normalized size	1	1.00	0.80	1.52	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.177	0.281	0.059	0.000	0.755	0.000	0.000	0.000

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.062	1.058	0.192	0.000	0.616	0.000	0.000	0.000

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	410	406	410	1244	0	0	0	0	-1
normalized size	1	0.99	1.00	3.03	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.878	1.904	0.431	0.000	0.991	0.000	0.000	0.000

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	340	340	318	800	0	0	0	0	-1
normalized size	1	1.00	0.94	2.35	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.696	1.112	0.352	0.000	0.953	0.000	0.000	0.000

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	327	258	709	0	0	0	0	-1
normalized size	1	0.99	0.78	2.14	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.675	0.843	0.281	0.000	0.726	0.000	0.000	0.000

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	181	333	0	0	0	0	-1
normalized size	1	1.00	0.89	1.63	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.341	0.808	0.076	0.000	1.288	0.000	0.000	0.000

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	156	130	272	0	0	0	0	-1
normalized size	1	0.98	0.81	1.70	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.267	0.439	0.068	0.000	0.633	0.000	0.000	0.000

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.063	3.630	0.208	0.000	1.075	0.000	0.000	0.000

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	361	361	342	0	0	0	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.875	0.653	180.000	0.000	0.000	0.000	0.000	0.000

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	272	272	223	0	0	0	0	0	-1
normalized size	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.665	0.290	180.000	0.000	0.000	0.000	0.000	0.000

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	238	0	0	0	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.650	0.408	180.000	0.000	0.000	0.000	0.000	0.000

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	140	0	0	0	0	0	-1
normalized size	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.429	0.101	0.111	0.000	0.000	0.000	0.000	0.000

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	111	0	0	0	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.246	0.122	0.000	0.000	0.000	0.000	0.000	0.000

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.088	2.164	0.218	0.000	0.000	0.000	0.000	0.000

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	601	601	343	0	0	0	0	0	-1
normalized size	1	1.00	0.57	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.662	0.477	180.000	0.000	0.000	0.000	0.000	0.000

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	360	360	225	0	0	0	0	0	-1
normalized size	1	1.00	0.62	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.059	0.288	180.000	0.000	0.000	0.000	0.000	0.000

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	328	328	238	0	0	0	0	0	-1
normalized size	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.882	0.299	180.000	0.000	0.000	0.000	0.000	0.000

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	142	0	0	0	0	0	-1
normalized size	1	1.00	0.69	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.483	0.108	0.115	0.000	0.000	0.000	0.000	0.000

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	272	0	0	0	0	0	-1
normalized size	1	1.00	1.81	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.254	0.227	0.009	0.000	0.000	0.000	0.000	0.000

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.103	1.495	0.247	0.000	0.000	0.000	0.000	0.000

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	701	701	342	0	0	0	0	0	-1
normalized size	1	1.00	0.49	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.203	0.700	180.000	0.000	0.000	0.000	0.000	0.000

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	455	455	223	0	0	0	0	0	-1
normalized size	1	1.00	0.49	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.520	0.321	180.000	0.000	0.000	0.000	0.000	0.000

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	394	394	238	0	0	0	0	0	-1
normalized size	1	1.00	0.60	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.244	0.425	180.000	0.000	0.000	0.000	0.000	0.000

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	126	0	0	0	0	0	-1
normalized size	1	1.00	0.48	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.731	0.077	0.109	0.000	0.000	0.000	0.000	0.000

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	458	0	0	0	0	0	-1
normalized size	1	1.00	2.56	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.402	1.498	0.003	0.000	0.000	0.000	0.000	0.000

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.105	1.131	0.243	0.000	0.000	0.000	0.000	0.000

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	835	835	343	0	0	0	0	0	-1
normalized size	1	1.00	0.41	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.289	0.517	180.000	0.000	0.000	0.000	0.000	0.000

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	547	547	225	0	0	0	0	0	-1
normalized size	1	1.00	0.41	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.143	0.325	180.000	0.000	0.000	0.000	0.000	0.000

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	481	481	238	0	0	0	0	0	-1
normalized size	1	1.00	0.49	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.680	0.336	180.000	0.000	0.000	0.000	0.000	0.000

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	305	305	125	0	0	0	0	0	-1
normalized size	1	1.00	0.41	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.796	0.075	0.108	0.000	0.000	0.000	0.000	0.000

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	698	0	0	0	0	0	-1
normalized size	1	1.00	3.23	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.419	4.682	0.075	0.000	0.000	0.000	0.000	0.000

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.102	1.138	0.253	0.000	0.000	0.000	0.000	0.000

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	320	0	0	0	0	0	-1
normalized size	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.637	0.370	180.000	0.000	0.000	0.000	0.000	0.000

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	205	0	0	0	0	0	-1
normalized size	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.455	0.229	180.000	0.000	0.000	0.000	0.000	0.000

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	217	0	0	0	0	0	-1
normalized size	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.455	0.240	180.000	0.000	0.000	0.000	0.000	0.000

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	119	0	0	0	0	0	-1
normalized size	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.243	0.067	0.118	0.000	0.000	0.000	0.000	0.000

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	111	0	0	0	0	0	-1
normalized size	1	1.00	1.21	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.126	0.066	0.000	0.000	0.000	0.000	0.000	0.000

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.098	0.070	0.204	0.000	0.000	0.000	0.000	0.000

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	367	367	490	0	0	0	0	0	-1
normalized size	1	1.00	1.34	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.651	0.668	180.000	0.000	0.000	0.000	0.000	0.000

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	253	0	0	0	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.453	0.452	180.000	0.000	0.000	0.000	0.000	0.000

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	327	0	0	0	0	0	-1
normalized size	1	1.00	1.28	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.465	0.372	180.000	0.000	0.000	0.000	0.000	0.000

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	147	0	0	0	0	0	-1
normalized size	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.231	0.117	0.095	0.000	0.000	0.000	0.000	0.000

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	155	0	0	0	0	0	-1
normalized size	1	1.00	1.27	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.255	0.113	0.000	0.000	0.000	0.000	0.000	0.000

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.108	0.076	0.219	0.000	0.000	0.000	0.000	0.000

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	437	437	551	0	0	0	0	0	-1
normalized size	1	1.00	1.26	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.555	3.454	180.000	0.000	0.000	0.000	0.000	0.000

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	390	0	0	0	0	0	-1
normalized size	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.096	1.777	180.000	0.000	0.000	0.000	0.000	0.000

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	321	321	389	0	0	0	0	0	-1
normalized size	1	1.00	1.21	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.960	1.548	180.000	0.000	0.000	0.000	0.000	0.000

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	227	0	0	0	0	0	-1
normalized size	1	1.00	1.09	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.542	0.696	0.099	0.000	0.000	0.000	0.000	0.000

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	207	0	0	0	0	0	-1
normalized size	1	1.00	1.31	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.285	0.294	0.000	0.000	0.000	0.000	0.000	0.000

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.109	0.080	0.225	0.000	0.000	0.000	0.000	0.000

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	531	531	701	0	0	0	0	0	-1
normalized size	1	1.00	1.32	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.467	2.717	180.000	0.000	0.000	0.000	0.000	0.000

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	420	420	429	0	0	0	0	0	-1
normalized size	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.098	2.106	180.000	0.000	0.000	0.000	0.000	0.000

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	410	410	474	0	0	0	0	0	-1
normalized size	1	1.00	1.16	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.110	1.494	180.000	0.000	0.000	0.000	0.000	0.000

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	235	0	0	0	0	0	-1
normalized size	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.562	0.881	0.095	0.000	0.000	0.000	0.000	0.000

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	238	0	0	0	0	0	-1
normalized size	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.465	0.186	0.000	0.000	0.000	0.000	0.000	0.000

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.117	0.082	0.221	0.000	0.000	0.000	0.000	0.000

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	113	238	0	0	0	0	-1
normalized size	1	1.00	0.38	0.80	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.332	0.213	0.026	0.000	0.612	0.000	0.000	0.000

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	113	212	0	0	0	0	-1
normalized size	1	1.00	0.64	1.20	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.159	0.186	0.012	0.000	0.540	0.000	0.000	0.000

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	87	205	0	0	0	0	-1
normalized size	1	1.00	0.33	0.79	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.243	0.047	0.010	0.000	0.535	0.000	0.000	0.000

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	87	179	0	0	0	0	-1
normalized size	1	1.00	0.61	1.26	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.126	0.032	0.011	0.000	0.671	0.000	0.000	0.000

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	61	161	0	0	0	0	-1
normalized size	1	1.00	0.27	0.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.212	0.032	0.014	0.000	0.576	0.000	0.000	0.000

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	56	140	0	0	0	0	-1
normalized size	1	1.00	0.53	1.32	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.113	0.026	0.013	0.000	0.552	0.000	0.000	0.000

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	58	202	0	0	0	0	-1
normalized size	1	1.00	0.22	0.76	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.238	0.031	0.013	0.000	0.636	0.000	0.000	0.000

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	61	176	0	0	0	0	-1
normalized size	1	1.00	0.42	1.21	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.136	0.036	0.016	0.000	0.591	0.000	0.000	0.000

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	110	0	0	0	0	0	-1
normalized size	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.212	0.133	180.000	0.000	0.580	0.000	0.000	0.000

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	110	0	0	0	0	0	-1
normalized size	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.221	0.119	180.000	0.000	0.588	0.000	0.000	0.000

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	110	0	0	0	0	0	-1
normalized size	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.217	0.102	180.000	0.000	0.547	0.000	0.000	0.000

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	110	0	0	0	0	0	-1
normalized size	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.209	0.084	180.000	0.000	0.569	0.000	0.000	0.000

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	110	0	0	0	0	0	-1
normalized size	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.203	0.063	180.000	0.000	1.124	0.000	0.000	0.000

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	109	0	0	0	0	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.222	0.099	180.000	0.000	0.727	0.000	0.000	0.000

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	106	0	0	0	0	0	-1
normalized size	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.226	0.077	180.000	0.000	0.769	0.000	0.000	0.000

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	110	0	0	0	0	0	-1
normalized size	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.230	0.087	180.000	0.000	0.725	0.000	0.000	0.000

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-2)	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.208	89.803	180.000	0.000	2.031	0.000	0.000	0.000

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-2)	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.203	108.231	180.000	0.000	0.721	0.000	0.000	0.000

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-2)	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.209	73.209	180.000	0.000	1.063	0.000	0.000	0.000

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-2)	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.202	93.831	180.000	0.000	0.887	0.000	0.000	0.000

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-2)	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	78	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.183	9.439	180.000	0.000	0.689	0.000	0.000	0.000

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-2)	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	78	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.193	19.934	180.000	0.000	0.715	0.000	0.000	0.000

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-2)	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.209	23.070	180.000	0.000	0.756	0.000	0.000	0.000

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-2)	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.217	73.711	180.000	0.000	0.520	0.000	0.000	0.000

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-2)	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.206	98.228	180.000	0.000	0.903	0.000	0.000	0.000

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-2)	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.207	128.470	180.000	0.000	1.009	0.000	0.000	0.000

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-2)	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.206	83.669	180.000	0.000	0.577	0.000	0.000	0.000

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-2)	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.194	120.915	180.000	0.000	0.890	0.000	0.000	0.000

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-2)	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	78	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.182	9.331	180.000	0.000	0.617	0.000	0.000	0.000

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-2)	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	78	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.194	37.756	180.000	0.000	0.861	0.000	0.000	0.000

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-2)	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.206	43.218	180.000	0.000	0.472	0.000	0.000	0.000

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-2)	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.207	112.483	180.000	0.000	0.700	0.000	0.000	0.000

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	127	204	0	199	0	0	-1
normalized size	1	1.00	0.97	1.56	0.00	1.52	0.00	0.00	-0.01
time (sec)	N/A	0.189	0.122	0.133	0.000	0.521	0.000	0.000	0.000

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	110	167	0	161	0	0	-1
normalized size	1	1.00	1.03	1.56	0.00	1.50	0.00	0.00	-0.01
time (sec)	N/A	0.119	0.094	0.112	0.000	0.711	0.000	0.000	0.000

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	91	238	98	0	0	-1
normalized size	1	1.00	1.00	1.49	3.90	1.61	0.00	0.00	-0.02
time (sec)	N/A	0.068	0.059	0.098	0.710	0.600	0.000	0.000	0.000

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	24	28	0	0	0	0	-1
normalized size	1	1.00	0.77	0.90	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.119	0.066	0.126	0.000	0.459	0.000	0.000	0.000

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	47	44	0	0	0	0	-1
normalized size	1	1.00	1.31	1.22	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.115	0.064	0.122	0.000	0.488	0.000	0.000	0.000

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	69	85	63	0	0	0	0	-1
normalized size	1	0.97	1.20	0.89	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.106	0.294	0.126	0.000	0.615	0.000	0.000	0.000

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	266	592	0	332	694	0	-1
normalized size	1	1.00	1.13	2.52	0.00	1.41	2.95	0.00	-0.00
time (sec)	N/A	0.309	0.244	0.139	0.000	0.462	17.366	0.000	0.000

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	211	479	0	259	568	0	-1
normalized size	1	1.00	1.12	2.53	0.00	1.37	3.01	0.00	-0.01
time (sec)	N/A	0.190	0.172	0.126	0.000	0.462	10.347	0.000	0.000

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	124	262	394	160	298	0	-1
normalized size	1	1.00	1.17	2.47	3.72	1.51	2.81	0.00	-0.01
time (sec)	N/A	0.104	0.100	0.110	0.793	0.434	5.279	0.000	0.000

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	37	42	0	0	0	0	-1
normalized size	1	1.00	0.79	0.89	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.144	0.320	0.128	0.000	0.416	0.000	0.000	0.000

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	70	72	0	0	0	0	-1
normalized size	1	1.00	1.30	1.33	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.157	0.289	0.123	0.000	0.489	0.000	0.000	0.000

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	108	110	0	0	0	0	-1
normalized size	1	1.00	1.29	1.31	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.281	0.404	0.120	0.000	0.656	0.000	0.000	0.000

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	179	32	26	0	13
normalized size	1	1.00	1.00	0.93	11.93	2.13	1.73	0.00	0.87
time (sec)	N/A	0.069	0.022	0.074	0.769	0.490	1.198	0.000	0.225

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	132	32	26	0	13
normalized size	1	1.00	1.00	0.93	8.80	2.13	1.73	0.00	0.87
time (sec)	N/A	0.068	0.020	0.067	0.779	0.833	0.836	0.000	0.195

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	84	32	24	0	13
normalized size	1	1.00	1.00	0.93	5.60	2.13	1.60	0.00	0.87
time (sec)	N/A	0.042	0.016	0.074	0.521	0.548	0.736	0.000	0.199

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	0	30	22	0	11
normalized size	1	1.00	1.00	1.09	0.00	2.73	2.00	0.00	1.00
time (sec)	N/A	0.075	0.030	0.077	0.000	0.413	1.222	0.000	0.221

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	150	32	26	0	13
normalized size	1	1.00	1.00	1.08	11.54	2.46	2.00	0.00	1.00
time (sec)	N/A	0.075	0.015	0.077	0.771	0.469	1.880	0.000	0.210

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	0	32	29	0	13
normalized size	1	1.00	1.00	0.93	0.00	2.13	1.93	0.00	0.87
time (sec)	N/A	0.070	0.015	0.081	0.000	0.886	2.342	0.000	0.203

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	128	203	0	0	0	0	-1
normalized size	1	1.00	1.11	1.77	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.206	0.629	0.259	0.000	0.410	0.000	0.000	0.000

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	98	168	0	0	0	0	-1
normalized size	1	1.00	1.14	1.95	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.160	0.400	0.248	0.000	0.562	0.000	0.000	0.000

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	62	131	119	115	0	76	-1
normalized size	1	1.00	1.35	2.85	2.59	2.50	0.00	1.65	-0.02
time (sec)	N/A	0.058	0.098	0.231	0.600	0.629	0.000	0.549	0.000

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.082	0.601	0.294	0.000	0.699	0.000	0.000	0.000

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	55	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.117	2.918	0.250	0.000	0.625	0.000	0.000	0.000

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	44	67	102	52	42	74	45
normalized size	1	1.00	0.88	1.34	2.04	1.04	0.84	1.48	0.90
time (sec)	N/A	0.037	0.019	0.033	0.653	0.758	0.848	0.459	0.247

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	75	89	0	0	0	0	-1
normalized size	1	1.00	0.74	0.88	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.129	0.009	0.000	0.721	0.000	0.000	0.000

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	31	30	42	27	40	28
normalized size	1	1.00	1.00	0.91	0.88	1.24	0.79	1.18	0.82
time (sec)	N/A	0.022	0.015	0.002	0.310	0.483	0.223	0.326	0.250

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	35	77	0	0	0	0	-1
normalized size	1	1.00	0.22	0.48	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.054	0.005	0.007	0.000	0.652	0.000	0.000	0.000

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.062	0.008	0.069	0.000	0.544	0.000	0.000	0.000

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	42	66	0	0	0	0	-1
normalized size	1	1.00	0.56	0.88	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.023	0.042	0.008	0.000	0.607	0.000	0.000	0.000

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	46	106	0	58	-1
normalized size	1	1.00	1.00	0.85	1.39	3.21	0.00	1.76	-0.03
time (sec)	N/A	0.026	0.007	0.011	0.366	1.048	0.000	0.302	0.000

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	88	101	0	0	0	0	-1
normalized size	1	1.00	0.45	0.51	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.090	0.169	0.010	0.000	0.591	0.000	0.000	0.000

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.062	0.008	0.054	0.000	0.548	0.000	0.000	0.000

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	43	47	46	40	0	50	-1
normalized size	1	1.00	0.60	0.65	0.64	0.56	0.00	0.69	-0.01
time (sec)	N/A	0.025	0.024	0.013	0.864	0.783	0.000	0.339	0.000

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	37	37	36	35	0	48	-1
normalized size	1	1.00	0.66	0.66	0.64	0.62	0.00	0.86	-0.02
time (sec)	N/A	0.017	0.018	0.003	0.786	0.718	0.000	0.477	0.000

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	33	24	23	28	29	40	31
normalized size	1	1.00	0.94	0.69	0.66	0.80	0.83	1.14	0.89
time (sec)	N/A	0.011	0.041	0.003	0.850	0.772	0.303	0.519	0.920

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	78	0	0	0	0	-1
normalized size	1	1.00	1.00	1.70	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.064	0.009	0.068	0.000	0.717	0.000	0.000	0.000

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	21	20	25	0	35	-1
normalized size	1	1.00	1.00	0.81	0.77	0.96	0.00	1.35	-0.04
time (sec)	N/A	0.013	0.011	0.003	0.860	0.551	0.000	0.367	0.000

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	34	31	30	32	0	52	-1
normalized size	1	1.00	0.74	0.67	0.65	0.70	0.00	1.13	-0.02
time (sec)	N/A	0.017	0.015	0.003	0.841	0.569	0.000	0.316	0.000

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	39	41	40	37	0	67	-1
normalized size	1	1.00	0.63	0.66	0.65	0.60	0.00	1.08	-0.02
time (sec)	N/A	0.022	0.017	0.004	0.889	0.607	0.000	0.375	0.000

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	44	51	50	42	0	82	-1
normalized size	1	1.00	0.56	0.65	0.64	0.54	0.00	1.05	-0.01
time (sec)	N/A	0.027	0.019	0.007	0.858	0.594	0.000	0.765	0.000

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	57	54	69	122	0	74	-1
normalized size	1	1.00	1.02	0.96	1.23	2.18	0.00	1.32	-0.02
time (sec)	N/A	0.040	0.044	0.027	0.460	0.676	0.000	0.549	0.000

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	29	38	27	45	0	47	27
normalized size	1	1.00	0.88	1.15	0.82	1.36	0.00	1.42	0.82
time (sec)	N/A	0.017	0.024	0.005	0.318	0.875	0.000	0.492	0.034

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	77	31	43	96	0	49	25
normalized size	1	1.00	3.08	1.24	1.72	3.84	0.00	1.96	1.00
time (sec)	N/A	0.017	0.096	0.006	0.631	0.503	0.000	0.323	0.549

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	114	0	0	0	0	-1
normalized size	1	1.00	1.00	2.19	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.062	0.008	0.007	0.000	0.471	0.000	0.000	0.000

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	31	30	49	20	39	27
normalized size	1	1.00	1.00	1.07	1.03	1.69	0.69	1.34	0.93
time (sec)	N/A	0.024	0.017	0.005	0.667	0.684	1.927	0.418	0.227

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	44	46	97	58	0	84	43
normalized size	1	1.00	0.88	0.92	1.94	1.16	0.00	1.68	0.86
time (sec)	N/A	0.034	0.024	0.005	0.633	0.700	0.000	0.425	0.236

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	48	53	47	62	0	75	-1
normalized size	1	1.00	0.89	0.98	0.87	1.15	0.00	1.39	-0.02
time (sec)	N/A	0.040	0.031	0.007	0.462	0.809	0.000	0.668	0.000

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	74	0	0	0	0	0	-1
normalized size	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.081	0.040	0.000	0.000	0.000	0.000	0.000

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	66	0	0	0	0	0	-1
normalized size	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.035	0.053	0.023	0.000	0.000	0.000	0.000	0.000

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	58	0	0	0	0	0	-1
normalized size	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.030	0.053	0.012	0.000	0.000	0.000	0.000	0.000

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.023	0.028	0.012	0.000	0.000	0.000	0.000	0.000

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	133	0	0	0	0	-1
normalized size	1	1.00	1.00	2.22	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.066	0.008	0.004	0.000	0.000	0.000	0.000	0.000

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	61	0	0	0	0	0	-1
normalized size	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.033	0.057	0.013	0.000	0.000	0.000	0.000	0.000

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	62	0	0	0	0	0	-1
normalized size	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.039	0.047	0.015	0.000	0.000	0.000	0.000	0.000

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	149	0	0	269	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	1.76	0.00	0.00	-0.01
time (sec)	N/A	0.036	0.125	0.234	0.000	0.523	0.000	0.000	0.000

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	180	0	0	188	0	0	-1
normalized size	1	1.00	1.40	0.00	0.00	1.46	0.00	0.00	-0.01
time (sec)	N/A	0.064	0.147	0.212	0.000	0.481	0.000	0.000	0.000

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	76	0	0	114	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	1.50	0.00	0.00	-0.01
time (sec)	N/A	0.015	0.028	0.203	0.000	0.540	0.000	0.000	0.000

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	48	47	44	52	0	0	39
normalized size	1	1.00	0.96	0.94	0.88	1.04	0.00	0.00	0.78
time (sec)	N/A	0.040	0.026	0.023	0.729	0.454	0.000	0.000	0.534

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	150	0	0	0	0	0	-1
normalized size	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.048	0.791	0.155	0.000	0.565	0.000	0.000	0.000

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	197	0	0	0	0	0	-1
normalized size	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.043	1.447	0.108	0.000	0.844	0.000	0.000	0.000

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	229	0	0	0	0	0	-1
normalized size	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.054	0.581	0.107	0.000	0.512	0.000	0.000	0.000

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	149	0	0	269	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	1.76	0.00	0.00	-0.01
time (sec)	N/A	0.032	0.125	0.191	0.000	0.570	0.000	0.000	0.000

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	180	0	0	188	0	0	-1
normalized size	1	1.00	1.40	0.00	0.00	1.46	0.00	0.00	-0.01
time (sec)	N/A	0.059	0.146	0.186	0.000	0.587	0.000	0.000	0.000

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	76	0	0	114	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	1.50	0.00	0.00	-0.01
time (sec)	N/A	0.013	0.020	0.177	0.000	0.514	0.000	0.000	0.000

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	48	48	44	52	0	0	39
normalized size	1	1.00	0.96	0.96	0.88	1.04	0.00	0.00	0.78
time (sec)	N/A	0.039	0.027	0.017	0.704	0.484	0.000	0.000	0.452

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	146	0	0	0	0	0	-1
normalized size	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.023	0.691	0.105	0.000	0.427	0.000	0.000	0.000

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	196	0	0	0	0	0	-1
normalized size	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.029	1.491	0.102	0.000	0.677	0.000	0.000	0.000

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	272	272	227	0	0	0	0	0	-1
normalized size	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.051	0.771	0.104	0.000	0.471	0.000	0.000	0.000

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	348	348	337	0	0	0	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.112	0.288	0.108	0.000	0.000	0.000	0.000	0.000

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	312	312	258	0	0	0	0	0	-1
normalized size	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.105	0.228	0.103	0.000	0.000	0.000	0.000	0.000

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	259	0	0	0	0	0	-1
normalized size	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.027	0.055	0.115	0.000	0.000	0.000	0.000	0.000

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	180	0	0	0	0	0	-1
normalized size	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.030	0.005	0.112	0.000	0.000	0.000	0.000	0.000

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	291	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.052	0.388	0.125	0.000	0.000	0.000	0.000	0.000

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	308	0	0	0	0	0	-1
normalized size	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.072	0.832	0.108	0.000	0.000	0.000	0.000	0.000

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	389	389	365	0	0	0	0	0	-1
normalized size	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.088	0.995	0.133	0.000	0.000	0.000	0.000	0.000

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	348	348	337	0	0	0	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.097	0.288	0.109	0.000	0.000	0.000	0.000	0.000

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	255	0	0	0	0	0	-1
normalized size	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.096	0.263	0.106	0.000	0.000	0.000	0.000	0.000

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	259	0	0	0	0	0	-1
normalized size	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.028	0.055	0.105	0.000	0.000	0.000	0.000	0.000

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	180	0	0	0	0	0	-1
normalized size	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.028	0.004	0.107	0.000	0.000	0.000	0.000	0.000

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	291	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.049	0.375	0.104	0.000	0.000	0.000	0.000	0.000

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	308	0	0	0	0	0	-1
normalized size	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.073	0.849	0.103	0.000	0.000	0.000	0.000	0.000

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	389	389	370	0	0	0	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.081	0.981	0.111	0.000	0.000	0.000	0.000	0.000

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	43	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.044	0.093	0.470	0.000	0.619	0.000	0.000	0.000

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	244	1175	0	0	0	0	-1
normalized size	1	1.00	0.93	4.50	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.226	0.064	0.706	0.000	0.496	0.000	0.000	0.000

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	194	195	187	649	0	0	0	0	-1
normalized size	1	1.01	0.96	3.35	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.216	0.064	0.009	0.000	0.474	0.000	0.000	0.000

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	127	263	0	0	0	0	-1
normalized size	1	1.00	0.95	1.98	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.122	0.030	0.008	0.000	0.528	0.000	0.000	0.000

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	43	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.049	0.100	0.283	0.000	0.534	0.000	0.000	0.000

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	43	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.045	0.848	0.267	0.000	0.478	0.000	0.000	0.000

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	76	166	0	0	0	0	-1
normalized size	1	1.00	1.00	2.18	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.076	0.528	0.006	0.000	0.000	0.000	0.000	0.000

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	119	322	491	138	0	173	-1
normalized size	1	1.00	0.72	1.95	2.98	0.84	0.00	1.05	-0.01
time (sec)	N/A	0.172	0.089	0.012	0.698	0.559	0.000	0.414	0.000

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	102	264	273	117	0	140	-1
normalized size	1	1.00	0.89	2.30	2.37	1.02	0.00	1.22	-0.01
time (sec)	N/A	0.125	0.118	0.005	0.692	0.633	0.000	0.605	0.000

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	73	138	175	93	0	106	-1
normalized size	1	1.00	1.09	2.06	2.61	1.39	0.00	1.58	-0.01
time (sec)	N/A	0.071	0.108	0.004	0.682	0.558	0.000	0.322	0.000

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	46	89	141	73	0	80	-1
normalized size	1	1.00	1.48	2.87	4.55	2.35	0.00	2.58	-0.03
time (sec)	N/A	0.017	0.036	0.003	0.463	0.597	0.000	0.295	0.000

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	99	126	160	136	0	158	180
normalized size	1	1.00	1.11	1.42	1.80	1.53	0.00	1.78	2.02
time (sec)	N/A	0.121	0.079	0.005	0.344	0.494	0.000	0.423	1.038

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	110	267	170	183	0	234	269
normalized size	1	1.00	1.11	2.70	1.72	1.85	0.00	2.36	2.72
time (sec)	N/A	0.105	0.171	0.010	0.357	0.687	0.000	0.938	1.261

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	129	457	313	181	0	384	-1
normalized size	1	1.00	1.11	3.94	2.70	1.56	0.00	3.31	-0.01
time (sec)	N/A	0.086	0.217	0.008	0.963	0.712	0.000	0.382	0.000

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	162	501	352	230	0	715	-1
normalized size	1	1.00	1.04	3.21	2.26	1.47	0.00	4.58	-0.01
time (sec)	N/A	0.108	0.122	0.010	0.563	0.513	0.000	0.529	0.000

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	192	841	594	295	0	1173	-1
normalized size	1	1.00	0.93	4.06	2.87	1.43	0.00	5.67	-0.00
time (sec)	N/A	0.171	0.809	0.010	0.453	0.685	0.000	0.492	0.000

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	359	359	198	0	0	0	0	0	-1
normalized size	1	1.00	0.55	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.743	0.333	0.011	0.000	0.629	0.000	0.000	0.000

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	138	0	0	0	0	0	-1
normalized size	1	1.00	0.55	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.516	0.205	0.010	0.000	0.567	0.000	0.000	0.000

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	76	0	0	0	0	0	-1
normalized size	1	1.00	0.65	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.277	0.108	0.009	0.000	0.497	0.000	0.000	0.000

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	44	0	0	0	0	0	-1
normalized size	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.055	0.039	0.004	0.000	0.577	0.000	0.000	0.000

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.038	0.142	0.009	0.000	0.609	0.000	0.000	0.000

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.037	0.474	0.009	0.000	0.477	0.000	0.000	0.000

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	52	125	0	0	0	0	-1
normalized size	1	1.00	0.87	2.08	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.096	0.018	0.057	0.000	0.718	0.000	0.000	0.000

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	0	0	0	0	-1
normalized size	1	1.00	1.00	1.33	0.00	0.00	0.00	0.00	-0.33
time (sec)	N/A	0.060	0.064	0.126	0.000	0.608	0.000	0.000	0.000

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	41	38	37	66	61	105	88
normalized size	1	1.00	0.91	0.84	0.82	1.47	1.36	2.33	1.96
time (sec)	N/A	0.049	0.027	0.003	0.314	0.694	0.847	0.321	0.569

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	41	0	39	152	76	113	99
normalized size	1	1.00	0.89	0.00	0.85	3.30	1.65	2.46	2.15
time (sec)	N/A	0.050	0.042	0.031	0.345	0.637	71.048	0.364	0.361

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	131	46	0	242	0	0	41
normalized size	1	1.00	2.67	0.94	0.00	4.94	0.00	0.00	0.84
time (sec)	N/A	0.032	0.128	0.044	0.000	0.898	0.000	0.000	1.086

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	F	A	A	F	A	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	28	0	1	1	0	1	1
normalized size	1	0.00	1.04	0.00	0.04	0.04	0.00	0.04	0.04
time (sec)	N/A	0.039	0.558	0.095	0.780	0.484	0.000	0.278	0.224

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	0	0	108	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	2.92	0.00	0.00	-0.03
time (sec)	N/A	0.067	0.044	0.252	0.000	0.572	0.000	0.000	0.000

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	24	0	0	33	0	0	23
normalized size	1	1.00	0.83	0.00	0.00	1.14	0.00	0.00	0.79
time (sec)	N/A	0.062	0.024	0.214	0.000	0.477	0.000	0.000	0.245

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [74] had the largest ratio of [1.333]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	8	5	1.00	12	0.417
2	A	10	6	1.00	14	0.429
3	A	12	7	1.00	14	0.500
4	A	5	5	1.00	16	0.312
5	A	4	4	1.00	16	0.250
6	A	4	4	1.00	14	0.286
7	A	3	2	1.00	8	0.250
8	A	8	5	1.00	16	0.312
9	A	3	3	1.00	16	0.188
10	A	4	4	1.00	16	0.250
11	A	5	5	1.00	16	0.312
12	A	18	7	1.00	18	0.389
13	A	13	7	1.00	18	0.389
14	A	9	7	1.00	16	0.438
15	A	3	3	1.00	10	0.300
16	A	10	6	1.00	18	0.333
17	A	10	7	1.00	18	0.389
18	A	13	10	1.00	18	0.556
19	A	27	7	1.00	18	0.389
20	A	17	6	1.00	18	0.333
21	A	11	7	1.00	16	0.438
22	A	4	4	1.00	10	0.400
23	A	0	0	0.00	0	0.000
24	A	0	0	0.00	0	0.000
25	A	19	7	0.98	18	0.389
26	A	11	7	0.98	16	0.438
27	A	5	5	0.95	10	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
28	A	0	0	0.00	0	0.000
29	A	0	0	0.00	0	0.000
30	A	0	0	0.00	0	0.000
31	A	3	3	1.00	16	0.188
32	A	0	0	0.00	0	0.000
33	A	0	0	0.00	0	0.000
34	A	16	12	1.00	30	0.400
35	A	13	8	1.00	30	0.267
36	A	8	6	1.00	28	0.214
37	A	22	20	1.00	30	0.667
38	A	35	22	1.00	30	0.733
39	A	24	17	1.00	30	0.567
40	A	20	12	1.00	30	0.400
41	A	12	9	1.00	28	0.321
42	A	29	24	1.00	30	0.800
43	A	30	18	1.00	30	0.600
44	A	26	15	1.00	30	0.500
45	A	14	10	1.00	28	0.357
46	A	37	29	1.00	30	0.967
47	A	13	7	1.00	30	0.233
48	A	9	7	1.00	30	0.233
49	A	6	5	1.00	28	0.179
50	A	2	2	1.00	23	0.087
51	A	10	7	1.00	30	0.233
52	A	13	10	1.00	30	0.333
53	A	0	0	0.00	0	0.000
54	A	13	9	1.00	34	0.265
55	A	11	8	1.00	32	0.250
56	A	9	7	1.00	24	0.292
57	A	0	0	0.00	0	0.000
58	A	6	6	1.00	10	0.600
59	A	5	5	1.00	10	0.500
60	A	5	5	1.00	8	0.625
61	A	3	3	1.00	6	0.500
62	A	9	6	1.00	10	0.600
63	A	4	4	1.00	10	0.400

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
64	A	5	5	1.00	10	0.500
65	A	6	6	1.00	10	0.600
66	A	7	7	1.00	10	0.700
67	A	19	8	1.00	12	0.667
68	A	14	8	1.00	12	0.667
69	A	10	8	1.00	10	0.800
70	A	4	4	1.00	8	0.500
71	A	11	7	1.00	12	0.583
72	A	11	8	1.00	12	0.667
73	A	14	11	1.00	12	0.917
74	A	40	16	1.00	12	1.333
75	A	18	11	1.00	12	0.917
76	A	12	10	1.00	10	1.000
77	A	5	4	1.00	8	0.500
78	A	13	8	1.00	12	0.667
79	A	13	9	1.00	12	0.750
80	A	21	13	1.00	12	1.083
81	A	14	8	1.00	12	0.667
82	A	10	8	1.00	10	0.800
83	A	3	3	1.00	8	0.375
84	A	0	0	0.00	0	0.000
85	A	12	7	1.00	12	0.583
86	A	8	7	1.00	10	0.700
87	A	4	4	1.00	8	0.500
88	A	0	0	0.00	0	0.000
89	A	24	12	1.00	12	1.000
90	A	14	12	1.00	10	1.200
91	A	5	5	1.00	8	0.625
92	A	0	0	0.00	0	0.000
93	A	0	0	0.00	0	0.000
94	A	22	9	1.00	16	0.562
95	A	14	9	1.00	14	0.643
96	A	5	4	1.00	12	0.333
97	A	0	0	0.00	0	0.000
98	A	23	12	1.00	18	0.667
99	A	14	10	1.00	16	0.625

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
100	A	8	7	1.00	14	0.500
101	A	16	10	1.00	16	0.625
102	A	9	8	1.00	14	0.571
103	A	18	10	1.00	16	0.625
104	A	10	8	1.00	14	0.571
105	A	20	9	1.00	18	0.500
106	A	12	8	1.00	16	0.500
107	A	7	6	1.00	14	0.429
108	A	16	10	1.00	16	0.625
109	A	8	7	1.00	14	0.500
110	A	22	15	1.00	16	0.938
111	A	9	8	1.00	14	0.571
112	A	21	13	1.00	16	0.812
113	A	10	8	1.00	14	0.571
114	A	3	3	1.00	21	0.143
115	A	6	5	1.00	21	0.238
116	A	6	5	1.00	21	0.238
117	A	6	5	1.00	21	0.238
118	A	5	5	1.00	19	0.263
119	A	4	3	1.00	10	0.300
120	A	7	7	1.00	21	0.333
121	A	6	6	1.00	21	0.286
122	A	4	4	1.00	21	0.190
123	A	7	7	1.00	21	0.333
124	A	5	5	1.00	21	0.238
125	A	8	7	1.00	21	0.333
126	A	3	3	1.00	23	0.130
127	A	9	7	1.00	23	0.304
128	A	8	6	1.00	23	0.261
129	A	7	7	1.00	23	0.304
130	A	6	6	1.00	21	0.286
131	A	4	4	1.00	12	0.333
132	A	8	8	0.99	23	0.348
133	A	9	7	1.00	23	0.304
134	A	5	5	1.00	23	0.217
135	A	11	9	1.00	23	0.391

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
136	A	0	0	0.00	0	0.000
137	A	17	9	1.00	23	0.391
138	A	13	7	1.00	23	0.304
139	A	12	9	1.00	23	0.391
140	A	8	7	1.00	21	0.333
141	A	6	4	1.00	12	0.333
142	A	9	9	1.00	23	0.391
143	A	11	8	1.00	23	0.348
144	A	9	9	1.00	23	0.391
145	A	16	12	1.00	23	0.522
146	A	0	0	0.00	0	0.000
147	A	16	6	1.00	23	0.261
148	A	13	8	1.00	23	0.348
149	A	9	6	1.00	21	0.286
150	A	6	4	1.00	12	0.333
151	A	10	9	1.00	23	0.391
152	A	13	9	1.00	23	0.391
153	A	10	10	1.00	23	0.435
154	A	21	12	1.00	23	0.522
155	A	0	0	0.00	0	0.000
156	A	14	7	0.98	23	0.304
157	A	11	7	1.00	23	0.304
158	A	11	7	0.97	23	0.304
159	A	8	7	1.00	21	0.333
160	A	5	5	1.00	12	0.417
161	A	0	0	0.00	0	0.000
162	A	13	6	0.98	23	0.261
163	A	10	6	1.00	23	0.261
164	A	10	6	0.98	23	0.261
165	A	6	6	1.00	21	0.286
166	A	6	6	0.96	12	0.500
167	A	0	0	0.00	0	0.000
168	A	26	9	0.99	23	0.391
169	A	20	9	1.00	23	0.391
170	A	18	10	1.24	23	0.435
171	A	11	10	1.00	21	0.476

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
172	A	7	7	1.00	12	0.583
173	A	0	0	0.00	0	0.000
174	A	24	8	0.99	23	0.348
175	A	17	8	1.00	23	0.348
176	A	18	10	0.99	23	0.435
177	A	9	9	1.00	21	0.429
178	A	8	7	0.98	12	0.583
179	A	0	0	0.00	0	0.000
180	A	21	9	1.00	25	0.360
181	A	16	9	1.00	25	0.360
182	A	16	9	1.00	25	0.360
183	A	11	9	1.00	23	0.391
184	A	8	7	1.00	14	0.500
185	A	0	0	0.00	0	0.000
186	A	43	12	1.00	25	0.480
187	A	27	11	1.00	25	0.440
188	A	24	12	1.00	25	0.480
189	A	13	11	1.00	23	0.478
190	A	9	8	1.00	14	0.571
191	A	0	0	0.00	0	0.000
192	A	46	12	1.00	25	0.480
193	A	29	11	1.00	25	0.440
194	A	26	12	1.00	25	0.480
195	A	14	11	1.00	23	0.478
196	A	10	8	1.00	14	0.571
197	A	0	0	0.00	0	0.000
198	A	77	13	1.00	25	0.520
199	A	42	11	1.00	25	0.440
200	A	35	13	1.00	25	0.520
201	A	16	11	1.00	23	0.478
202	A	11	8	1.00	14	0.571
203	A	0	0	0.00	0	0.000
204	A	20	8	1.00	25	0.320
205	A	15	8	1.00	25	0.320
206	A	15	8	1.00	25	0.320
207	A	10	8	1.00	23	0.348

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
208	A	7	6	1.00	14	0.429
209	A	0	0	0.00	0	0.000
210	A	19	7	1.00	25	0.280
211	A	14	7	1.00	25	0.280
212	A	14	7	1.00	25	0.280
213	A	8	7	1.00	23	0.304
214	A	8	7	1.00	14	0.500
215	A	0	0	0.00	0	0.000
216	A	36	10	1.00	25	0.400
217	A	26	10	1.00	25	0.400
218	A	24	11	1.00	25	0.440
219	A	13	11	1.00	23	0.478
220	A	9	8	1.00	14	0.571
221	A	0	0	0.00	0	0.000
222	A	34	9	1.00	25	0.360
223	A	23	9	1.00	25	0.360
224	A	24	11	1.00	25	0.440
225	A	11	10	1.00	23	0.435
226	A	10	8	1.00	14	0.571
227	A	0	0	0.00	0	0.000
228	A	8	7	1.00	23	0.304
229	A	6	5	1.00	23	0.217
230	A	7	7	1.00	23	0.304
231	A	5	5	1.00	23	0.217
232	A	6	6	1.00	23	0.261
233	A	4	4	1.00	23	0.174
234	A	7	7	1.00	23	0.304
235	A	5	5	1.00	23	0.217
236	A	3	3	1.00	25	0.120
237	A	3	3	1.00	25	0.120
238	A	3	3	1.00	25	0.120
239	A	3	3	1.00	25	0.120
240	A	3	3	1.00	25	0.120
241	A	3	3	1.00	25	0.120
242	A	3	3	1.00	25	0.120
243	A	3	3	1.00	25	0.120

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
244	A	0	0	0.00	0	0.000
245	A	0	0	0.00	0	0.000
246	A	0	0	0.00	0	0.000
247	A	0	0	0.00	0	0.000
248	A	0	0	0.00	0	0.000
249	A	0	0	0.00	0	0.000
250	A	0	0	0.00	0	0.000
251	A	0	0	0.00	0	0.000
252	A	0	0	0.00	0	0.000
253	A	0	0	0.00	0	0.000
254	A	0	0	0.00	0	0.000
255	A	0	0	0.00	0	0.000
256	A	0	0	0.00	0	0.000
257	A	0	0	0.00	0	0.000
258	A	0	0	0.00	0	0.000
259	A	0	0	0.00	0	0.000
260	A	7	6	1.00	30	0.200
261	A	6	6	1.00	30	0.200
262	A	4	4	1.00	28	0.143
263	A	5	4	1.00	30	0.133
264	A	6	6	1.00	30	0.200
265	A	4	4	0.97	30	0.133
266	A	15	9	1.00	30	0.300
267	A	11	9	1.00	30	0.300
268	A	7	6	1.00	28	0.214
269	A	6	4	1.00	30	0.133
270	A	7	5	1.00	30	0.167
271	A	11	8	1.00	30	0.267
272	A	2	2	1.00	30	0.067
273	A	2	2	1.00	30	0.067
274	A	2	2	1.00	28	0.071
275	A	2	2	1.00	30	0.067
276	A	2	2	1.00	30	0.067
277	A	2	2	1.00	30	0.067
278	A	8	8	1.00	30	0.267
279	A	7	7	1.00	30	0.233

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
280	A	3	3	1.00	28	0.107
281	A	0	0	0.00	0	0.000
282	A	0	0	0.00	0	0.000
283	A	5	5	1.00	10	0.500
284	A	4	4	1.00	10	0.400
285	A	3	3	1.00	8	0.375
286	A	5	5	1.00	6	0.833
287	A	5	5	1.00	10	0.500
288	A	3	3	1.00	10	0.300
289	A	5	5	1.00	10	0.500
290	A	6	6	1.00	10	0.600
291	A	5	5	1.00	10	0.500
292	A	7	5	1.00	10	0.500
293	A	6	5	1.00	8	0.625
294	A	6	6	1.00	6	1.000
295	A	5	5	1.00	10	0.500
296	A	3	3	1.00	10	0.300
297	A	4	4	1.00	10	0.400
298	A	5	4	1.00	10	0.400
299	A	6	4	1.00	10	0.400
300	A	6	6	1.00	10	0.600
301	A	3	3	1.00	8	0.375
302	A	5	5	1.00	6	0.833
303	A	5	5	1.00	10	0.500
304	A	3	3	1.00	10	0.300
305	A	5	5	1.00	10	0.500
306	A	5	4	1.00	10	0.400
307	A	3	3	1.00	10	0.300
308	A	3	3	1.00	10	0.300
309	A	3	3	1.00	8	0.375
310	A	3	3	1.00	6	0.500
311	A	5	5	1.00	10	0.500
312	A	3	3	1.00	10	0.300
313	A	3	3	1.00	10	0.300
314	A	3	2	1.00	20	0.100
315	A	5	4	1.00	20	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
316	A	2	2	1.00	20	0.100
317	A	4	3	1.00	18	0.167
318	A	1	1	1.00	20	0.050
319	A	1	1	1.00	20	0.050
320	A	2	2	1.00	20	0.100
321	A	3	2	1.00	20	0.100
322	A	5	4	1.00	20	0.200
323	A	2	2	1.00	20	0.100
324	A	4	3	1.00	18	0.167
325	A	1	1	1.00	20	0.050
326	A	1	1	1.00	20	0.050
327	A	2	2	1.00	20	0.100
328	A	2	2	1.00	22	0.091
329	A	2	2	1.00	22	0.091
330	A	1	1	1.00	22	0.045
331	A	1	1	1.00	22	0.045
332	A	1	1	1.00	22	0.045
333	A	2	2	1.00	22	0.091
334	A	2	2	1.00	22	0.091
335	A	2	2	1.00	22	0.091
336	A	2	2	1.00	22	0.091
337	A	1	1	1.00	22	0.045
338	A	1	1	1.00	22	0.045
339	A	1	1	1.00	22	0.045
340	A	2	2	1.00	22	0.091
341	A	2	2	1.00	22	0.091
342	A	0	0	0.00	0	0.000
343	A	8	8	1.00	40	0.200
344	A	7	7	1.01	40	0.175
345	A	6	7	1.00	38	0.184
346	A	0	0	0.00	0	0.000
347	A	0	0	0.00	0	0.000
348	A	6	6	1.00	10	0.600
349	A	5	4	1.00	12	0.333
350	A	5	4	1.00	12	0.333
351	A	5	4	1.00	10	0.400

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
352	A	5	4	1.00	8	0.500
353	A	9	8	1.00	12	0.667
354	A	9	8	1.00	12	0.667
355	A	6	5	1.00	12	0.417
356	A	7	6	1.00	12	0.500
357	A	8	7	1.00	12	0.583
358	A	37	8	1.00	14	0.571
359	A	27	8	1.00	14	0.571
360	A	17	8	1.00	12	0.667
361	A	7	4	1.00	10	0.400
362	A	0	0	0.00	0	0.000
363	A	0	0	0.00	0	0.000
364	A	7	7	1.00	19	0.368
365	A	2	2	1.00	15	0.133
366	A	4	4	1.00	12	0.333
367	A	4	4	1.00	14	0.286
368	A	6	6	1.00	10	0.600
369	F	0	0	N/A	0	N/A
370	A	2	2	1.00	26	0.077
371	A	2	2	1.00	26	0.077

Chapter 3

Listing of integrals

3.1 $\int \frac{\sinh^{-1}(cx)}{d+ex} \, dx$

Optimal. Leaf size=170

$$\frac{\text{Li}_2\left(-\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{\text{Li}_2\left(-\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{\sinh^{-1}(cx)\log\left(\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e}+\frac{\sinh^{-1}(cx)\log\left(\frac{ee^{\sinh^{-1}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)}{e}-$$

```
[Out] -1/2*arcsinh(c*x)^2/e+arcsinh(c*x)*ln(1+e*(c*x+(c^2*x^2+1)^(1/2))/(c*d-(c^2
*d^2+e^2)^(1/2)))/e+arcsinh(c*x)*ln(1+e*(c*x+(c^2*x^2+1)^(1/2))/(c*d+(c^2*d
^2+e^2)^(1/2)))/e+polylog(2,-e*(c*x+(c^2*x^2+1)^(1/2))/(c*d-(c^2*d^2+e^2)^(
1/2)))/e+polylog(2,-e*(c*x+(c^2*x^2+1)^(1/2))/(c*d+(c^2*d^2+e^2)^(1/2)))/e
```

Rubi [A] time = 0.26, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5799, 5561, 2190, 2279, 2391}

$$\frac{\text{PolyLog}\left(2,-\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{\text{PolyLog}\left(2,-\frac{ee^{\sinh^{-1}(cx)}}{\sqrt{c^2d^2+e^2}+cd}\right)}{e}+\frac{\sinh^{-1}(cx)\log\left(\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e}+\frac{\sinh^{-1}(cx)\log\left(-\right.}{e}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[c*x]/(d + e*x),x]
```

```
[Out] -ArcSinh[c*x]^2/(2*e) + (ArcSinh[c*x]*Log[1 + (e*E^ArcSinh[c*x])/(c*d - Sqr
t[c^2*d^2 + e^2])])/e + (ArcSinh[c*x]*Log[1 + (e*E^ArcSinh[c*x])/(c*d + Sqr
t[c^2*d^2 + e^2])])/e + PolyLog[2, -((e*E^ArcSinh[c*x])/(c*d - Sqrt[c^2*d^2
 + e^2]))]/e + PolyLog[2, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))
]/e
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Cosh[x])/(c*d + e*Sinh[x]), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sinh^{-1}(cx)}{d+ex}\,dx = \text{Subst}\left(\int \frac{x\cosh(x)}{cd+e\sinh(x)}\,dx, x, \sinh^{-1}(cx)\right)$$
$$= -\frac{\sinh^{-1}(cx)^2}{2e} + \text{Subst}\left(\int \frac{e^x x}{cd-\sqrt{c^2d^2+e^2}+ee^x}\,dx, x, \sinh^{-1}(cx)\right) + \text{Subst}\left(\int \frac{e^x}{cd+\sqrt{c^2d^2}}\right.$$
$$= -\frac{\sinh^{-1}(cx)^2}{2e} + \frac{\sinh^{-1}(cx)\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\sinh^{-1}(cx)\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e} - \frac{\text{Subs}}{}$$
$$= -\frac{\sinh^{-1}(cx)^2}{2e} + \frac{\sinh^{-1}(cx)\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\sinh^{-1}(cx)\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e} - \frac{\text{Subs}}{}$$
$$= -\frac{\sinh^{-1}(cx)^2}{2e} + \frac{\sinh^{-1}(cx)\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\sinh^{-1}(cx)\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\text{Li}_2\left(\right.}{}$$

Mathematica [A] time = 0.01, size = 168, normalized size = 0.99

$$\frac{\text{Li}_2\left(\frac{ee^{\sinh^{-1}(cx)}}{\sqrt{c^2d^2+e^2}-cd}\right)}{e} + \frac{\text{Li}_2\left(-\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\sinh^{-1}(cx)\log\left(\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e} + \frac{\sinh^{-1}(cx)\log\left(\frac{ee^{\sinh^{-1}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)}{e} - \frac{\text{sinh}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[c*x]/(d + e*x),x]
```

```
[Out] -1/2*ArcSinh[c*x]^2/e + (ArcSinh[c*x]*Log[1 + (e*E^ArcSinh[c*x])/(c*d - Sqr
t[c^2*d^2 + e^2])])/e + (ArcSinh[c*x]*Log[1 + (e*E^ArcSinh[c*x])/(c*d + Sqr
t[c^2*d^2 + e^2])])/e + PolyLog[2, (e*E^ArcSinh[c*x])/(-(c*d) + Sqrt[c^2*d^
2 + e^2])]/e + PolyLog[2, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))
]/e
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{arsinh}\,(cx)}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(c*x)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(arcsinh(c*x)/(e*x + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}(cx)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(c*x)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(c*x)/(e*x + d), x)
```

maple [A] time = 0.13, size = 263, normalized size = 1.55

$$-\frac{\operatorname{arcsinh}(cx)^2}{2e}+\frac{\operatorname{arcsinh}(cx)\ln\left(\frac{-\left(cx+\sqrt{c^2x^2+1}\right)e-cd+\sqrt{c^2d^2+e^2}}{-cd+\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{\operatorname{arcsinh}(cx)\ln\left(\frac{\left(cx+\sqrt{c^2x^2+1}\right)e+cd+\sqrt{c^2d^2+e^2}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}+\text{dil}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(c*x)/(e*x+d),x)
```

```
[Out] -1/2*arcsinh(c*x)^2/e+1/e*arcsinh(c*x)*ln((-(c*x+(c^2*x^2+1)^(1/2))*e-c*d+(
c^2*d^2+e^2)^(1/2))/(-c*d+(c^2*d^2+e^2)^(1/2)))+1/e*arcsinh(c*x)*ln(((c*x+(
c^2*x^2+1)^(1/2))*e+c*d+(c^2*d^2+e^2)^(1/2))/(c*d+(c^2*d^2+e^2)^(1/2)))+1/e
*dilog(((c*x+(c^2*x^2+1)^(1/2))*e+c*d+(c^2*d^2+e^2)^(1/2))/(c*d+(c^2*d^2+e^
2)^(1/2)))+1/e*dilog((-(c*x+(c^2*x^2+1)^(1/2))*e-c*d+(c^2*d^2+e^2)^(1/2))/(
-c*d+(c^2*d^2+e^2)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}(cx)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(c*x)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(c*x)/(e*x + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{asinh}(c\,x)}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(c*x)/(d + e*x),x)
```

```
[Out] int(asinh(c*x)/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}(cx)}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(c*x)/(e*x+d),x)
```

```
[Out] Integral(asinh(c*x)/(d + e*x), x)
```

3.2 $\int \frac{\sinh^{-1}(cx)^2}{d+ex}\, dx$

Optimal. Leaf size=260

$$\frac{2\sinh^{-1}(cx)\text{Li}_2\left(-\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{2\sinh^{-1}(cx)\text{Li}_2\left(-\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}-\frac{2\text{Li}_3\left(-\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}-\frac{2\text{Li}_3\left(-\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}+\text{sir}$$

```
[Out] -1/3*arcsinh(c*x)^3/e+arcsinh(c*x)^2*ln(1+e*(c*x+(c^2*x^2+1)^(1/2))/(c*d-(c
^2*d^2+e^2)^(1/2)))/e+arcsinh(c*x)^2*ln(1+e*(c*x+(c^2*x^2+1)^(1/2))/(c*d+(c
^2*d^2+e^2)^(1/2)))/e+2*arcsinh(c*x)*polylog(2,-e*(c*x+(c^2*x^2+1)^(1/2))/(
c*d-(c^2*d^2+e^2)^(1/2)))/e+2*arcsinh(c*x)*polylog(2,-e*(c*x+(c^2*x^2+1)^(1
/2))/(c*d+(c^2*d^2+e^2)^(1/2)))/e-2*polylog(3,-e*(c*x+(c^2*x^2+1)^(1/2))/(c
*d-(c^2*d^2+e^2)^(1/2)))/e-2*polylog(3,-e*(c*x+(c^2*x^2+1)^(1/2))/(c*d+(c^2
*d^2+e^2)^(1/2)))/e
```

Rubi [A] time = 0.40, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5799, 5561, 2190, 2531, 2282, 6589}

$$\frac{2\sinh^{-1}(cx)\text{PolyLog}\left(2,-\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{2\sinh^{-1}(cx)\text{PolyLog}\left(2,-\frac{ee^{\sinh^{-1}(cx)}}{\sqrt{c^2d^2+e^2}+cd}\right)}{e}-\frac{2\text{PolyLog}\left(3,-\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}-$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[c*x]^2/(d + e*x),x]
```

```
[Out] -ArcSinh[c*x]^3/(3*e) + (ArcSinh[c*x]^2*Log[1 + (e*E^ArcSinh[c*x])/(c*d - S
qrt[c^2*d^2 + e^2])])/e + (ArcSinh[c*x]^2*Log[1 + (e*E^ArcSinh[c*x])/(c*d +
 Sqrt[c^2*d^2 + e^2])])/e + (2*ArcSinh[c*x]*PolyLog[2, -((e*E^ArcSinh[c*x])
/(c*d - Sqrt[c^2*d^2 + e^2]))])/e + (2*ArcSinh[c*x]*PolyLog[2, -((e*E^ArcSi
nh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))])/e - (2*PolyLog[3, -((e*E^ArcSinh[c*
x])/(c*d - Sqrt[c^2*d^2 + e^2]))])/e - (2*PolyLog[3, -((e*E^ArcSinh[c*x])/(
c*d + Sqrt[c^2*d^2 + e^2]))])/e
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Cosh[x])/(c*d + e*Sinh[x]), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\sinh^{-1}(cx)^2}{d+ex}\,dx = \text{Subst}\left(\int \frac{x^2\cosh(x)}{cd+e\sinh(x)}\,dx, x, \sinh^{-1}(cx)\right)$$

$$= -\frac{\sinh^{-1}(cx)^3}{3e} + \text{Subst}\left(\int \frac{e^x x^2}{cd-\sqrt{c^2d^2+e^2}+ee^x}\,dx, x, \sinh^{-1}(cx)\right) + \text{Subst}\left(\int \frac{}{cd+\sqrt{c}}\right.$$

$$= -\frac{\sinh^{-1}(cx)^3}{3e} + \frac{\sinh^{-1}(cx)^2\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\sinh^{-1}(cx)^2\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e} -$$

$$= -\frac{\sinh^{-1}(cx)^3}{3e} + \frac{\sinh^{-1}(cx)^2\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\sinh^{-1}(cx)^2\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e} +$$

$$= -\frac{\sinh^{-1}(cx)^3}{3e} + \frac{\sinh^{-1}(cx)^2\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\sinh^{-1}(cx)^2\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e} +$$

$$= -\frac{\sinh^{-1}(cx)^3}{3e} + \frac{\sinh^{-1}(cx)^2\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\sinh^{-1}(cx)^2\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e} +$$

Mathematica [A] time = 0.14, size = 240, normalized size = 0.92

$$-\frac{-6\sinh^{-1}(cx)\text{Li}_2\left(\frac{ee^{\sinh^{-1}(cx)}}{\sqrt{c^2d^2+e^2}-cd}\right) - 6\sinh^{-1}(cx)\text{Li}_2\left(-\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right) + 6\text{Li}_3\left(\frac{ee^{\sinh^{-1}(cx)}}{\sqrt{c^2d^2+e^2}-cd}\right) + 6\text{Li}_3\left(-\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[c*x]^2/(d + e*x),x]
```

```
[Out] -1/3*(ArcSinh[c*x]^3 - 3*ArcSinh[c*x]^2*Log[1 + (e*E^ArcSinh[c*x])/(c*d - S
qrt[c^2*d^2 + e^2])] - 3*ArcSinh[c*x]^2*Log[1 + (e*E^ArcSinh[c*x])/(c*d + S
qrt[c^2*d^2 + e^2])] - 6*ArcSinh[c*x]*PolyLog[2, (e*E^ArcSinh[c*x])/(-(c*d)
 + Sqrt[c^2*d^2 + e^2])] - 6*ArcSinh[c*x]*PolyLog[2, -((e*E^ArcSinh[c*x])/(
c*d + Sqrt[c^2*d^2 + e^2]))] + 6*PolyLog[3, (e*E^ArcSinh[c*x])/(-(c*d) + Sq
```

```
rt[c^2*d^2 + e^2])] + 6*PolyLog[3, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2
 + e^2]))])/e
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\operatorname{arsinh}(cx)^2}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(c*x)^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(arcsinh(c*x)^2/(e*x + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}(cx)^2}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(c*x)^2/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(c*x)^2/(e*x + d), x)
```

maple [F] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsinh}(cx)^2}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(c*x)^2/(e*x+d),x)
```

```
[Out] int(arcsinh(c*x)^2/(e*x+d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}(cx)^2}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(c*x)^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(c*x)^2/(e*x + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{asinh}(c\,x)^2}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(c*x)^2/(d + e*x),x)
```

```
[Out] int(asinh(c*x)^2/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}^2(cx)}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(c*x)**2/(e*x+d),x)
```

```
[Out] Integral(asinh(c*x)**2/(d + e*x), x)
```

3.3 $\int \frac{\sinh^{-1}(cx)^3}{d+ex}\,dx$

Optimal. Leaf size=348

$$\frac{3\sinh^{-1}(cx)^2\text{Li}_2\left(-\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{3\sinh^{-1}(cx)^2\text{Li}_2\left(-\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}-\frac{6\sinh^{-1}(cx)\text{Li}_3\left(-\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}-\frac{6\sinh^{-1}}{}$$

```
[Out] -1/4*arcsinh(c*x)^4/e+arcsinh(c*x)^3*ln(1+e*(c*x+(c^2*x^2+1)^(1/2))/(c*d-(c
^2*d^2+e^2)^(1/2)))/e+arcsinh(c*x)^3*ln(1+e*(c*x+(c^2*x^2+1)^(1/2))/(c*d+(c
^2*d^2+e^2)^(1/2)))/e+3*arcsinh(c*x)^2*polylog(2,-e*(c*x+(c^2*x^2+1)^(1/2))
/(c*d-(c^2*d^2+e^2)^(1/2)))/e+3*arcsinh(c*x)^2*polylog(2,-e*(c*x+(c^2*x^2+1
)^(1/2))/(c*d+(c^2*d^2+e^2)^(1/2)))/e-6*arcsinh(c*x)*polylog(3,-e*(c*x+(c^2
*x^2+1)^(1/2))/(c*d-(c^2*d^2+e^2)^(1/2)))/e-6*arcsinh(c*x)*polylog(3,-e*(c*
x+(c^2*x^2+1)^(1/2))/(c*d+(c^2*d^2+e^2)^(1/2)))/e+6*polylog(4,-e*(c*x+(c^2*
x^2+1)^(1/2))/(c*d-(c^2*d^2+e^2)^(1/2)))/e+6*polylog(4,-e*(c*x+(c^2*x^2+1)^
(1/2))/(c*d+(c^2*d^2+e^2)^(1/2)))/e
```

Rubi [A] time = 0.43, antiderivative size = 348, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5799, 5561, 2190, 2531, 6609, 2282, 6589}

$$\frac{3\sinh^{-1}(cx)^2\text{PolyLog}\left(2,-\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{3\sinh^{-1}(cx)^2\text{PolyLog}\left(2,-\frac{ee^{\sinh^{-1}(cx)}}{\sqrt{c^2d^2+e^2}+cd}\right)}{e}-\frac{6\sinh^{-1}(cx)\text{PolyLog}\left(3,\right.}{e}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[c*x]^3/(d + e*x),x]
```

```
[Out] -ArcSinh[c*x]^4/(4*e) + (ArcSinh[c*x]^3*Log[1 + (e*E^ArcSinh[c*x])/(c*d - S
qrt[c^2*d^2 + e^2])])/e + (ArcSinh[c*x]^3*Log[1 + (e*E^ArcSinh[c*x])/(c*d +
 Sqrt[c^2*d^2 + e^2])])/e + (3*ArcSinh[c*x]^2*PolyLog[2, -((e*E^ArcSinh[c*x
])/(c*d - Sqrt[c^2*d^2 + e^2]))])/e + (3*ArcSinh[c*x]^2*PolyLog[2, -((e*E^A
rcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))])/e - (6*ArcSinh[c*x]*PolyLog[3,
-((e*E^ArcSinh[c*x])/(c*d - Sqrt[c^2*d^2 + e^2]))])/e - (6*ArcSinh[c*x]*Pol
yLog[3, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))])/e + (6*PolyLog[
4, -((e*E^ArcSinh[c*x])/(c*d - Sqrt[c^2*d^2 + e^2]))])/e + (6*PolyLog[4, -(
(e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))])/e
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
```

```
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Cosh[x])/(c*d + e*Sinh[x]), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{\sinh^{-1}(cx)^3}{d+ex}\,dx = \text{Subst}\left(\int \frac{x^3\cosh(x)}{cd+e\sinh(x)}\,dx, x, \sinh^{-1}(cx)\right)$$

$$= -\frac{\sinh^{-1}(cx)^4}{4e} + \text{Subst}\left(\int \frac{e^x x^3}{cd-\sqrt{c^2d^2+e^2}+ee^x}\,dx, x, \sinh^{-1}(cx)\right) + \text{Subst}\left(\int \frac{e^x}{cd+\sqrt{c^2d^2}} \cdots\right.$$

$$= -\frac{\sinh^{-1}(cx)^4}{4e} + \frac{\sinh^{-1}(cx)^3\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\sinh^{-1}(cx)^3\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e} - \frac{3}{} \cdots$$

$$= -\frac{\sinh^{-1}(cx)^4}{4e} + \frac{\sinh^{-1}(cx)^3\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\sinh^{-1}(cx)^3\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{3}{} \cdots$$

$$= -\frac{\sinh^{-1}(cx)^4}{4e} + \frac{\sinh^{-1}(cx)^3\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\sinh^{-1}(cx)^3\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{3}{} \cdots$$

$$= -\frac{\sinh^{-1}(cx)^4}{4e} + \frac{\sinh^{-1}(cx)^3\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\sinh^{-1}(cx)^3\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{3}{} \cdots$$

$$= -\frac{\sinh^{-1}(cx)^4}{4e} + \frac{\sinh^{-1}(cx)^3\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\sinh^{-1}(cx)^3\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{3}{} \cdots$$

Mathematica [A] time = 0.04, size = 322, normalized size = 0.93

$$\frac{12\sinh^{-1}(cx)^2\text{Li}_2\left(\frac{ee^{\sinh^{-1}(cx)}}{\sqrt{c^2d^2+e^2}-cd}\right)+12\sinh^{-1}(cx)^2\text{Li}_2\left(-\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)-24\sinh^{-1}(cx)\text{Li}_3\left(\frac{ee^{\sinh^{-1}(cx)}}{\sqrt{c^2d^2+e^2}-cd}\right)-24\sin}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[c*x]^3/(d + e*x),x]
```

```
[Out] (-ArcSinh[c*x]^4 + 4*ArcSinh[c*x]^3*Log[1 + (e*E^ArcSinh[c*x])/(c*d - Sqrt[
c^2*d^2 + e^2])] + 4*ArcSinh[c*x]^3*Log[1 + (e*E^ArcSinh[c*x])/(c*d + Sqrt[
c^2*d^2 + e^2])] + 12*ArcSinh[c*x]^2*PolyLog[2, (e*E^ArcSinh[c*x])/(-(c*d)
+ Sqrt[c^2*d^2 + e^2])] + 12*ArcSinh[c*x]^2*PolyLog[2, -((e*E^ArcSinh[c*x])
/(c*d + Sqrt[c^2*d^2 + e^2]))] - 24*ArcSinh[c*x]*PolyLog[3, (e*E^ArcSinh[c*
x])/(-(c*d) + Sqrt[c^2*d^2 + e^2])] - 24*ArcSinh[c*x]*PolyLog[3, -((e*E^Arc
Sinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))] + 24*PolyLog[4, (e*E^ArcSinh[c*x])
/(-(c*d) + Sqrt[c^2*d^2 + e^2])] + 24*PolyLog[4, -((e*E^ArcSinh[c*x])/(c*d
+ Sqrt[c^2*d^2 + e^2]))])/(4*e)
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\operatorname{arsinh}(cx)^3}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(c*x)^3/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(arcsinh(c*x)^3/(e*x + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}(cx)^3}{ex+d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(c*x)^3/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(c*x)^3/(e*x + d), x)
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsinh}(cx)^3}{ex+d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(c*x)^3/(e*x+d),x)
```

```
[Out] int(arcsinh(c*x)^3/(e*x+d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}(cx)^3}{ex+d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(c*x)^3/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(c*x)^3/(e*x + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{asinh}(c\,x)^3}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(c*x)^3/(d + e*x),x)
```

```
[Out] int(asinh(c*x)^3/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}^3(cx)}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(c*x)**3/(e*x+d),x)
```

```
[Out] Integral(asinh(c*x)**3/(d + e*x), x)
```

3.4 $\int (d+ex)^3 \left(a+b\sinh^{-1}(cx)\right) dx$

Optimal. Leaf size=176

$$\frac{(d+ex)^4\left(a+b\sinh^{-1}(cx)\right)}{4e}-\frac{b\sqrt{c^2x^2+1}\,(d+ex)^3}{16c}-\frac{7bd\sqrt{c^2x^2+1}\,(d+ex)^2}{48c}-\frac{b\left(8c^4d^4-24c^2d^2e^2+3e^4\right)\sinh^{-}}{32c^4e}$$

```
[Out] -1/32*b*(8*c^4*d^4-24*c^2*d^2*e^2+3*e^4)*arcsinh(c*x)/c^4/e+1/4*(e*x+d)^4*(
a+b*arcsinh(c*x))/e-7/48*b*d*(e*x+d)^2*(c^2*x^2+1)^(1/2)/c-1/16*b*(e*x+d)^3
*(c^2*x^2+1)^(1/2)/c-1/96*b*(4*d*(19*c^2*d^2-16*e^2)+e*(26*c^2*d^2-9*e^2)*x
)*(c^2*x^2+1)^(1/2)/c^3
```

Rubi [A] time = 0.17, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = $\{5801, 743, 833, 780, 215\}$

$$\frac{(d+ex)^4\left(a+b\sinh^{-1}(cx)\right)}{4e}-\frac{b\sqrt{c^2x^2+1}\,\left(ex\left(26c^2d^2-9e^2\right)+4d\left(19c^2d^2-16e^2\right)\right)}{96c^3}-\frac{b\left(-24c^2d^2e^2+8c^4d^4+3\right.}{32c^4e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-7*b*d*(d + e*x)^2*Sqrt[1 + c^2*x^2])/(48*c) - (b*(d + e*x)^3*Sqrt[1 + c^2
*x^2])/(16*c) - (b*(4*d*(19*c^2*d^2 - 16*e^2) + e*(26*c^2*d^2 - 9*e^2)*x)*S
qrt[1 + c^2*x^2])/(96*c^3) - (b*(8*c^4*d^4 - 24*c^2*d^2*e^2 + 3*e^4)*ArcSin
h[c*x])/(32*c^4*e) + ((d + e*x)^4*(a + b*ArcSinh[c*x]))/(4*e)
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 5801

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n
 - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rubi steps

$$\int (d+ex)^3\left(a+b\sinh^{-1}(cx)\right)dx = \frac{(d+ex)^4\left(a+b\sinh^{-1}(cx)\right)}{4e} - \frac{(bc)\int \frac{(d+ex)^4}{\sqrt{1+c^2x^2}}dx}{4e}$$

$$= -\frac{b(d+ex)^3\sqrt{1+c^2x^2}}{16c} + \frac{(d+ex)^4\left(a+b\sinh^{-1}(cx)\right)}{4e} - \frac{b\int \frac{(d+ex)^2\left(4c^2d^2-3e^2+7\right.}{\sqrt{1+c^2x^2}}}{16ce}$$

$$= -\frac{7bd(d+ex)^2\sqrt{1+c^2x^2}}{48c} - \frac{b(d+ex)^3\sqrt{1+c^2x^2}}{16c} + \frac{(d+ex)^4\left(a+b\sinh^{-1}(c\right.}{4e}$$

$$= -\frac{7bd(d+ex)^2\sqrt{1+c^2x^2}}{48c} - \frac{b(d+ex)^3\sqrt{1+c^2x^2}}{16c} - \frac{b\left(4d\left(19c^2d^2-16e^2\right)+e\right.}{9}$$

$$= -\frac{7bd(d+ex)^2\sqrt{1+c^2x^2}}{48c} - \frac{b(d+ex)^3\sqrt{1+c^2x^2}}{16c} - \frac{b\left(4d\left(19c^2d^2-16e^2\right)+e\right.}{9}$$

Mathematica [A] time = 0.14, size = 166, normalized size = 0.94

$$\frac{24ac^4x\left(4d^3+6d^2ex+4de^2x^2+e^3x^3\right)-bc\sqrt{c^2x^2+1}\left(c^2\left(96d^3+72d^2ex+32de^2x^2+6e^3x^3\right)-e^2(64d+9ex)\right)+}{96c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (24*a*c^4*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3) - b*c*Sqrt[1 + c^2*
x^2]*(-(e^2*(64*d + 9*e*x)) + c^2*(96*d^3 + 72*d^2*e*x + 32*d*e^2*x^2 + 6*e
^3*x^3)) + 3*b*(24*c^2*d^2*e - 3*e^3 + 8*c^4*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2
*x^2 + e^3*x^3))*ArcSinh[c*x])/(96*c^4)
```

fricas [A] time = 0.77, size = 214, normalized size = 1.22

$$24\,ac^4e^3x^4+96\,ac^4de^2x^3+144\,ac^4d^2ex^2+96\,ac^4d^3x+3\left(8\,bc^4e^3x^4+32\,bc^4de^2x^3+48\,bc^4d^2ex^2+32\,bc^4d^3x+2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/96*(24*a*c^4*e^3*x^4 + 96*a*c^4*d*e^2*x^3 + 144*a*c^4*d^2*e*x^2 + 96*a*c^
4*d^3*x + 3*(8*b*c^4*e^3*x^4 + 32*b*c^4*d*e^2*x^3 + 48*b*c^4*d^2*e*x^2 + 32
*b*c^4*d^3*x + 24*b*c^2*d^2*e - 3*b*e^3)*log(c*x + sqrt(c^2*x^2 + 1)) - (6*
b*c^3*e^3*x^3 + 32*b*c^3*d*e^2*x^2 + 96*b*c^3*d^3 - 64*b*c*d*e^2 + 9*(8*b*c
^3*d^2*e - b*c*e^3)*x)*sqrt(c^2*x^2 + 1))/c^4
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [A] time = 0.01, size = 259, normalized size = 1.47

$$\frac{\frac{(cex+cd)^4 a}{4c^3e} + \frac{b\left(\frac{e^3 \operatorname{arcsinh}(cx)c^4x^4}{4} + e^2 \operatorname{arcsinh}(cx)c^4x^3d + \frac{3e \operatorname{arcsinh}(cx)c^4x^2d^2}{2} + \operatorname{arcsinh}(cx)c^4x\, d^3 + \frac{\operatorname{arcsinh}(cx)c^4d^4}{4e} - \frac{e^4\left(\frac{c^3x^3\sqrt{c^2x^2+1}}{4} - \frac{3cx\sqrt{c^2x^2+1}}{8} + \frac{3\operatorname{ar}}{}\right.}{}\right.}{c^3}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/c*(1/4*(c*e*x+c*d)^4*a/c^3/e+b/c^3*(1/4*e^3*arcsinh(c*x)*c^4*x^4+e^2*arcs
inh(c*x)*c^4*x^3*d+3/2*e*arcsinh(c*x)*c^4*x^2*d^2+arcsinh(c*x)*c^4*x*d^3+1/
4/e*arcsinh(c*x)*c^4*d^4-1/4/e*(e^4*(1/4*c^3*x^3*(c^2*x^2+1)^(1/2)-3/8*c*x*
(c^2*x^2+1)^(1/2)+3/8*arcsinh(c*x))+4*c*d*e^3*(1/3*c^2*x^2*(c^2*x^2+1)^(1/2
)-2/3*(c^2*x^2+1)^(1/2))+6*c^2*d^2*e^2*(1/2*c*x*(c^2*x^2+1)^(1/2)-1/2*arcsi
nh(c*x))+4*c^3*d^3*e*(c^2*x^2+1)^(1/2)+c^4*d^4*arcsinh(c*x))))
```

maxima [A] time = 0.40, size = 230, normalized size = 1.31

$$\frac{1}{4}\, ae^3x^4 + ade^2x^3 + \frac{3}{2}\, ad^2ex^2 + \frac{3}{4}\left(2\, x^2 \operatorname{arsinh}(cx) - c\left(\frac{\sqrt{c^2x^2+1}\, x}{c^2} - \frac{\operatorname{arsinh}(cx)}{c^3}\right)\right)bd^2e + \frac{1}{3}\left(3\, x^3 \operatorname{arsinh}(cx) - c\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/4*a*e^3*x^4 + a*d*e^2*x^3 + 3/2*a*d^2*e*x^2 + 3/4*(2*x^2*arcsinh(c*x) - c
*(sqrt(c^2*x^2 + 1)*x/c^2 - arcsinh(c*x)/c^3))*b*d^2*e + 1/3*(3*x^3*arcsinh
(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*b*d*e^2 +
1/32*(8*x^4*arcsinh(c*x) - (2*sqrt(c^2*x^2 + 1)*x^3/c^2 - 3*sqrt(c^2*x^2 +
1)*x/c^4 + 3*arcsinh(c*x)/c^5)*c)*b*e^3 + a*d^3*x + (c*x*arcsinh(c*x) - sqr
t(c^2*x^2 + 1))*b*d^3/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b\operatorname{asinh}(c\, x))\ (d + e\, x)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))*(d + e*x)^3,x)
```

```
[Out] int((a + b*asinh(c*x))*(d + e*x)^3, x)
```

sympy [A] time = 1.48, size = 316, normalized size = 1.80

$$\begin{cases} ad^3x + \frac{3ad^2ex^2}{2} + ade^2x^3 + \frac{ae^3x^4}{4} + bd^3x\operatorname{asinh}(cx) + \frac{3bd^2ex^2\operatorname{asinh}(cx)}{2} + bde^2x^3\operatorname{asinh}(cx) + \frac{be^3x^4\operatorname{asinh}(cx)}{4} - \frac{bd^3\sqrt{}}{} \\ a\left(d^3x + \frac{3d^2ex^2}{2} + de^2x^3 + \frac{e^3x^4}{4}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*d**3*x + 3*a*d**2*e*x**2/2 + a*d*e**2*x**3 + a*e**3*x**4/4 + b
*d**3*x*asinh(c*x) + 3*b*d**2*e*x**2*asinh(c*x)/2 + b*d*e**2*x**3*asinh(c*x
) + b*e**3*x**4*asinh(c*x)/4 - b*d**3*sqrt(c**2*x**2 + 1)/c - 3*b*d**2*e*x*
sqrt(c**2*x**2 + 1)/(4*c) - b*d*e**2*x**2*sqrt(c**2*x**2 + 1)/(3*c) - b*e**
3*x**3*sqrt(c**2*x**2 + 1)/(16*c) + 3*b*d**2*e*asinh(c*x)/(4*c**2) + 2*b*d*
e**2*sqrt(c**2*x**2 + 1)/(3*c**3) + 3*b*e**3*x*sqrt(c**2*x**2 + 1)/(32*c**3
) - 3*b*e**3*asinh(c*x)/(32*c**4), Ne(c, 0)), (a*(d**3*x + 3*d**2*e*x**2/2
+ d*e**2*x**3 + e**3*x**4/4), True))
```

3.5 $\int (d+ex)^2 \left(a+b\sinh^{-1}(cx)\right) dx$

Optimal. Leaf size=124

$$\frac{(d+ex)^3\left(a+b\sinh^{-1}(cx)\right)}{3e}-\frac{bd\left(2d^2-\frac{3e^2}{c^2}\right)\sinh^{-1}(cx)}{6e}-\frac{b\sqrt{c^2x^2+1}\,(d+ex)^2}{9c}-\frac{b\sqrt{c^2x^2+1}\,\left(4\left(4c^2d^2-e^2\right)+5\right.}{18c^3}$$

```
[Out] -1/6*b*d*(2*d^2-3*e^2/c^2)*arcsinh(c*x)/e+1/3*(e*x+d)^3*(a+b*arcsinh(c*x))/
e-1/9*b*(e*x+d)^2*(c^2*x^2+1)^(1/2)/c-1/18*b*(5*c^2*d*e*x+16*c^2*d^2-4*e^2)
*(c^2*x^2+1)^(1/2)/c^3
```

Rubi [A] time = 0.10, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5801, 743, 780, 215}

$$\frac{(d+ex)^3\left(a+b\sinh^{-1}(cx)\right)}{3e}-\frac{b\sqrt{c^2x^2+1}\,\left(4\left(4c^2d^2-e^2\right)+5c^2dex\right)}{18c^3}-\frac{bd\left(2d^2-\frac{3e^2}{c^2}\right)\sinh^{-1}(cx)}{6e}-\frac{b\sqrt{c^2x^2+1}\,(d}{9c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -(b*(d + e*x)^2*Sqrt[1 + c^2*x^2])/(9*c) - (b*(4*(4*c^2*d^2 - e^2) + 5*c^2*
d*e*x)*Sqrt[1 + c^2*x^2])/(18*c^3) - (b*d*(2*d^2 - (3*e^2)/c^2)*ArcSinh[c*x
])/(6*e) + ((d + e*x)^3*(a + b*ArcSinh[c*x]))/(3*e)
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 5801

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n
 - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rubi steps

$$\int (d+ex)^2 \left(a+b\sinh^{-1}(cx)\right) dx = \frac{(d+ex)^3 \left(a+b\sinh^{-1}(cx)\right)}{3e} - \frac{(bc)\int \frac{(d+ex)^3}{\sqrt{1+c^2x^2}} dx}{3e}$$

$$= -\frac{b(d+ex)^2\sqrt{1+c^2x^2}}{9c} + \frac{(d+ex)^3 \left(a+b\sinh^{-1}(cx)\right)}{3e} - \frac{b\int \frac{(d+ex)\left(3c^2d^2-2e^2+5c\right.}{\sqrt{1+c^2x^2}}}{9ce}$$

$$= -\frac{b(d+ex)^2\sqrt{1+c^2x^2}}{9c} - \frac{b\left(4\left(4c^2d^2-e^2\right)+5c^2dex\right)\sqrt{1+c^2x^2}}{18c^3} + \frac{(d+ex)^3}{}$$

$$= -\frac{b(d+ex)^2\sqrt{1+c^2x^2}}{9c} - \frac{b\left(4\left(4c^2d^2-e^2\right)+5c^2dex\right)\sqrt{1+c^2x^2}}{18c^3} - \frac{bd\left(2d^2-\right.}{}$$

Mathematica [A] time = 0.09, size = 121, normalized size = 0.98

$$\frac{6ac^3x\left(3d^2+3dex+e^2x^2\right)-b\sqrt{c^2x^2+1}\left(c^2\left(18d^2+9dex+2e^2x^2\right)-4e^2\right)+3bc\sinh^{-1}(cx)\left(6c^2d^2x+3d\left(2c^2ex^2\right.\right.}{18c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (6*a*c^3*x*(3*d^2 + 3*d*e*x + e^2*x^2) - b*Sqrt[1 + c^2*x^2]*(-4*e^2 + c^2*
(18*d^2 + 9*d*e*x + 2*e^2*x^2)) + 3*b*c*(6*c^2*d^2*x + 2*c^2*e^2*x^3 + 3*d*
(e + 2*c^2*e*x^2))*ArcSinh[c*x])/(18*c^3)
```

fricas [A] time = 0.57, size = 147, normalized size = 1.19

$$\frac{6\,ac^3e^2x^3+18\,ac^3dex^2+18\,ac^3d^2x+3\left(2\,bc^3e^2x^3+6\,bc^3dex^2+6\,bc^3d^2x+3\,bcde\right)\log\left(cx+\sqrt{c^2x^2+1}\right)-\left(2\,bc^2\right.}{18\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/18*(6*a*c^3*e^2*x^3 + 18*a*c^3*d*e*x^2 + 18*a*c^3*d^2*x + 3*(2*b*c^3*e^2*
x^3 + 6*b*c^3*d*e*x^2 + 6*b*c^3*d^2*x + 3*b*c*d*e)*log(c*x + sqrt(c^2*x^2 +
 1)) - (2*b*c^2*e^2*x^2 + 9*b*c^2*d*e*x + 18*b*c^2*d^2 - 4*b*e^2)*sqrt(c^2*
x^2 + 1))/c^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [A] time = 0.01, size = 189, normalized size = 1.52

$$\frac{\frac{(cex+cd)^3a}{3c^2e}+\frac{b\left(\frac{e^2\operatorname{arcsinh}(cx)c^3x^3}{3}+e\operatorname{arcsinh}(cx)c^3x^2d+\operatorname{arcsinh}(cx)c^3x\,d^2+\frac{\operatorname{arcsinh}(cx)c^3d^3}{3e}-\frac{e^3\left(\frac{c^2x^2\sqrt{c^2x^2+1}}{3}-\frac{2\sqrt{c^2x^2+1}}{3}\right)+3cd\,e^2\left(\frac{cx\sqrt{c^2x^2+1}}{2}-\frac{\operatorname{arcsinh}(cx)}{2}\right)+}{3e}\right.}{c^2}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/c*(1/3*(c*e*x+c*d)^3*a/c^2/e+b/c^2*(1/3*e^2*arcsinh(c*x)*c^3*x^3+e*arcsin
h(c*x)*c^3*x^2*d+arcsinh(c*x)*c^3*x*d^2+1/3/e*arcsinh(c*x)*c^3*d^3-1/3/e*(e
^3*(1/3*c^2*x^2*(c^2*x^2+1)^(1/2)-2/3*(c^2*x^2+1)^(1/2))+3*c*d*e^2*(1/2*c*x
*(c^2*x^2+1)^(1/2)-1/2*arcsinh(c*x))+3*c^2*d^2*e*(c^2*x^2+1)^(1/2)+c^3*d^3*
arcsinh(c*x))))
```

maxima [A] time = 0.40, size = 150, normalized size = 1.21

$$\frac{1}{3}\,ae^2x^3+adex^2+\frac{1}{2}\left(2\,x^2\,\operatorname{arsinh}\left(cx\right)-c\left(\frac{\sqrt{c^2x^2+1}\,x}{c^2}-\frac{\operatorname{arsinh}\left(cx\right)}{c^3}\right)\right)bde+\frac{1}{9}\left(3\,x^3\,\operatorname{arsinh}\left(cx\right)-c\left(\frac{\sqrt{c^2x^2+1}\,x^2}{c^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*e^2*x^3 + a*d*e*x^2 + 1/2*(2*x^2*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*
x/c^2 - arcsinh(c*x)/c^3))*b*d*e + 1/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^
2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*b*e^2 + a*d^2*x + (c*x*arcsinh(c
*x) - sqrt(c^2*x^2 + 1))*b*d^2/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,{\left(d+e\,x\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))*(d + e*x)^2,x)
```

```
[Out] int((a + b*asinh(c*x))*(d + e*x)^2, x)
```

sympy [A] time = 0.66, size = 190, normalized size = 1.53

$$\begin{cases} ad^2x + adex^2 + \frac{ae^2x^3}{3} + bd^2x\operatorname{asinh}\left(cx\right) + bdex^2\operatorname{asinh}\left(cx\right) + \frac{be^2x^3\operatorname{asinh}\left(cx\right)}{3} - \frac{bd^2\sqrt{c^2x^2+1}}{c} - \frac{bdex\sqrt{c^2x^2+1}}{2c} - \frac{be^2x^2\sqrt{c}}{9c} \\ a\left(d^2x + dex^2 + \frac{e^2x^3}{3}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*d**2*x + a*d*e*x**2 + a*e**2*x**3/3 + b*d**2*x*asinh(c*x) + b*
d*e*x**2*asinh(c*x) + b*e**2*x**3*asinh(c*x)/3 - b*d**2*sqrt(c**2*x**2 + 1)
/c - b*d*e*x*sqrt(c**2*x**2 + 1)/(2*c) - b*e**2*x**2*sqrt(c**2*x**2 + 1)/(9
*c) + b*d*e*asinh(c*x)/(2*c**2) + 2*b*e**2*sqrt(c**2*x**2 + 1)/(9*c**3), Ne
(c, 0)), (a*(d**2*x + d*e*x**2 + e**2*x**3/3), True))
```

3.6 $\int (d+ex)\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=97

$$\frac{(d+ex)^2\left(a+b\sinh^{-1}(cx)\right)}{2e}-\frac{b\left(2d^2-\frac{e^2}{c^2}\right)\sinh^{-1}(cx)}{4e}-\frac{b\sqrt{c^2x^2+1}\,(d+ex)}{4c}-\frac{3bd\sqrt{c^2x^2+1}}{4c}$$

```
[Out] -1/4*b*(2*d^2-e^2/c^2)*arcsinh(c*x)/e+1/2*(e*x+d)^2*(a+b*arcsinh(c*x))/e-3/
4*b*d*(c^2*x^2+1)^(1/2)/c-1/4*b*(e*x+d)*(c^2*x^2+1)^(1/2)/c
```

Rubi [A] time = 0.05, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5801, 743, 641, 215}

$$\frac{(d+ex)^2\left(a+b\sinh^{-1}(cx)\right)}{2e}-\frac{b\left(2d^2-\frac{e^2}{c^2}\right)\sinh^{-1}(cx)}{4e}-\frac{b\sqrt{c^2x^2+1}\,(d+ex)}{4c}-\frac{3bd\sqrt{c^2x^2+1}}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-3*b*d*Sqrt[1 + c^2*x^2])/(4*c) - (b*(d + e*x)*Sqrt[1 + c^2*x^2])/(4*c) -
(b*(2*d^2 - e^2/c^2)*ArcSinh[c*x])/(4*e) + ((d + e*x)^2*(a + b*ArcSinh[c*x]
))/(2*e)
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 5801

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n
 - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rubi steps

$$\int (d+ex)\left(a+b\sinh^{-1}(cx)\right)dx = \frac{(d+ex)^2\left(a+b\sinh^{-1}(cx)\right)}{2e} - \frac{(bc)\int \frac{(d+ex)^2}{\sqrt{1+c^2x^2}}dx}{2e}$$

$$= -\frac{b(d+ex)\sqrt{1+c^2x^2}}{4c} + \frac{(d+ex)^2\left(a+b\sinh^{-1}(cx)\right)}{2e} - \frac{b\int \frac{2c^2d^2-e^2+3c^2dex}{\sqrt{1+c^2x^2}}d}{4ce}$$

$$= -\frac{3bd\sqrt{1+c^2x^2}}{4c} - \frac{b(d+ex)\sqrt{1+c^2x^2}}{4c} + \frac{(d+ex)^2\left(a+b\sinh^{-1}(cx)\right)}{2e} - \frac{1}{4}$$

$$= -\frac{3bd\sqrt{1+c^2x^2}}{4c} - \frac{b(d+ex)\sqrt{1+c^2x^2}}{4c} - \frac{b\left(2d^2-\frac{e^2}{c^2}\right)\sinh^{-1}(cx)}{4e} + \frac{(d+e}{}$$

Mathematica [A] time = 0.04, size = 91, normalized size = 0.94

$$adx+\frac{1}{2}aex^2-\frac{bd\sqrt{c^2x^2+1}}{c}-\frac{bex\sqrt{c^2x^2+1}}{4c}+\frac{be\sinh^{-1}(cx)}{4c^2}+bdx\sinh^{-1}(cx)+\frac{1}{2}bex^2\sinh^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] a*d*x + (a*e*x^2)/2 - (b*d*Sqrt[1 + c^2*x^2])/c - (b*e*x*Sqrt[1 + c^2*x^2])
/(4*c) + (b*e*ArcSinh[c*x])/(4*c^2) + b*d*x*ArcSinh[c*x] + (b*e*x^2*ArcSinh
[c*x])/2
```

fricas [A] time = 0.48, size = 87, normalized size = 0.90

$$\frac{2\,ac^2ex^2+4\,ac^2dx+\left(2\,bc^2ex^2+4\,bc^2dx+be\right)\log\left(cx+\sqrt{c^2x^2+1}\right)-(bcex+4\,bcd)\sqrt{c^2x^2+1}}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*c^2*e*x^2 + 4*a*c^2*d*x + (2*b*c^2*e*x^2 + 4*b*c^2*d*x + b*e)*log(
c*x + sqrt(c^2*x^2 + 1)) - (b*c*e*x + 4*b*c*d)*sqrt(c^2*x^2 + 1))/c^2
```

giac [A] time = 0.82, size = 124, normalized size = 1.28

$$\left(x\log\left(cx+\sqrt{c^2x^2+1}\right)-\frac{\sqrt{c^2x^2+1}}{c}\right)bd+adx+\frac{1}{4}\left(2\,ax^2+\left(2\,x^2\log\left(cx+\sqrt{c^2x^2+1}\right)-c\left(\frac{\sqrt{c^2x^2+1}\,x}{c^2}+\frac{\log}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] (x*log(c*x + sqrt(c^2*x^2 + 1)) - sqrt(c^2*x^2 + 1)/c)*b*d + a*d*x + 1/4*(2
*a*x^2 + (2*x^2*log(c*x + sqrt(c^2*x^2 + 1)) - c*(sqrt(c^2*x^2 + 1)*x/c^2 +
 log(-x*abs(c) + sqrt(c^2*x^2 + 1))/(c^2*abs(c))))*b)*e
```

maple [A] time = 0.01, size = 96, normalized size = 0.99

$$\frac{\frac{a\left(\frac{1}{2}c^2x^2e+c^2dx\right)}{c}+\frac{b\left(\frac{\operatorname{arcsinh}(cx)c^2x^2e}{2}+\operatorname{arcsinh}(cx)c^2xd-\frac{e\left(\frac{cx\sqrt{c^2x^2+1}}{2}-\frac{\operatorname{arcsinh}(cx)}{2}\right)}{2}-cd\sqrt{c^2x^2+1}\right)}{c}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/c*(a/c*(1/2*c^2*x^2*e+c^2*d*x)+b/c*(1/2*arcsinh(c*x)*c^2*x^2*e+arcsinh(c*
x)*c^2*x*d-1/2*e*(1/2*c*x*(c^2*x^2+1)^(1/2)-1/2*arcsinh(c*x))-c*d*(c^2*x^2+
1)^(1/2)))
```

maxima [A] time = 0.37, size = 82, normalized size = 0.85

$$\frac{1}{2}aex^2+\frac{1}{4}\left(2x^2\operatorname{arsinh}(cx)-c\left(\frac{\sqrt{c^2x^2+1}\,x}{c^2}-\frac{\operatorname{arsinh}(cx)}{c^3}\right)\right)be+adx+\frac{\left(cx\operatorname{arsinh}(cx)-\sqrt{c^2x^2+1}\right)bd}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/2*a*e*x^2 + 1/4*(2*x^2*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x/c^2 - arcsin
h(c*x)/c^3))*b*e + a*d*x + (c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*b*d/c
```

mupad [B] time = 0.29, size = 78, normalized size = 0.80

$$\frac{a\,x\,(2\,d+e\,x)}{2}-\frac{b\,d\,\left(\sqrt{c^2\,x^2+1}-c\,x\,\operatorname{asinh}(c\,x)\right)}{c}-\frac{b\,e\,x\,\sqrt{c^2\,x^2+1}}{4\,c}+b\,e\,x\,\operatorname{asinh}(c\,x)\,\left(\frac{x}{2}+\frac{1}{4\,c^2\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))*(d + e*x),x)
```

```
[Out] (a*x*(2*d + e*x))/2 - (b*d*((c^2*x^2 + 1)^(1/2) - c*x*asinh(c*x)))/c - (b*e
*x*(c^2*x^2 + 1)^(1/2))/(4*c) + b*e*x*asinh(c*x)*(x/2 + 1/(4*c^2*x))
```

sympy [A] time = 0.30, size = 99, normalized size = 1.02

$$\begin{cases} adx+\frac{aex^2}{2}+bdx\operatorname{asinh}(cx)+\frac{bex^2\operatorname{asinh}(cx)}{2}-\frac{bd\sqrt{c^2x^2+1}}{c}-\frac{bex\sqrt{c^2x^2+1}}{4c}+\frac{be\operatorname{asinh}(cx)}{4c^2} & \text{for } c\neq 0\\ a\left(dx+\frac{ex^2}{2}\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*d*x + a*e*x**2/2 + b*d*x*asinh(c*x) + b*e*x**2*asinh(c*x)/2 -
b*d*sqrt(c**2*x**2 + 1)/c - b*e*x*sqrt(c**2*x**2 + 1)/(4*c) + b*e*asinh(c*x
)/(4*c**2), Ne(c, 0)), (a*(d*x + e*x**2/2), True))
```

3.7 $\int \left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=30

$$ax-\frac{b\sqrt{c^2x^2+1}}{c}+bx\sinh^{-1}(cx)$$

```
[Out] a*x+b*x*arcsinh(c*x)-b*(c^2*x^2+1)^(1/2)/c
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5653, 261}

$$ax-\frac{b\sqrt{c^2x^2+1}}{c}+bx\sinh^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcSinh[c*x],x]
[Out] a*x - (b*Sqrt[1 + c^2*x^2])/c + b*x*ArcSinh[c*x]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sinh^{-1}(cx)\right)\,dx &= ax+b\int \sinh^{-1}(cx)\,dx\\ &= ax+bx\sinh^{-1}(cx)-(bc)\int \frac{x}{\sqrt{1+c^2x^2}}\,dx\\ &= ax-\frac{b\sqrt{1+c^2x^2}}{c}+bx\sinh^{-1}(cx)\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 1.00

$$ax-\frac{b\sqrt{c^2x^2+1}}{c}+bx\sinh^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcSinh[c*x],x]
[Out] a*x - (b*Sqrt[1 + c^2*x^2])/c + b*x*ArcSinh[c*x]
```

fricas [A] time = 0.56, size = 43, normalized size = 1.43

$$\frac{bcx\log\left(cx+\sqrt{c^2x^2+1}\right)+acx-\sqrt{c^2x^2+1}\,b}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arcsinh(c*x),x, algorithm="fricas")
```

```
[Out] (b*c*x*log(c*x + sqrt(c^2*x^2 + 1)) + a*c*x - sqrt(c^2*x^2 + 1)*b)/c
```

giac [A] time = 0.23, size = 41, normalized size = 1.37

$$\left(x \log\left(cx + \sqrt{c^2x^2 + 1}\right) - \frac{\sqrt{c^2x^2 + 1}}{c}\right)b + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arcsinh(c*x),x, algorithm="giac")
```

```
[Out] (x*log(c*x + sqrt(c^2*x^2 + 1)) - sqrt(c^2*x^2 + 1)/c)*b + a*x
```

maple [A] time = 0.01, size = 31, normalized size = 1.03

$$ax + \frac{b\left(\operatorname{arcsinh}(cx)\,cx - \sqrt{c^2x^2 + 1}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arcsinh(c*x),x)
```

```
[Out] a*x+b/c*(arcsinh(c*x)*c*x-(c^2*x^2+1)^(1/2))
```

maxima [A] time = 0.40, size = 30, normalized size = 1.00

$$ax + \frac{\left(cx\,\operatorname{arsinh}(cx) - \sqrt{c^2x^2 + 1}\right)b}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arcsinh(c*x),x, algorithm="maxima")
```

```
[Out] a*x + (c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*b/c
```

mupad [B] time = 0.00, size = 28, normalized size = 0.93

$$a\,x - \frac{b\,\sqrt{c^2\,x^2 + 1}}{c} + b\,x\,\operatorname{asinh}(c\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*asinh(c*x),x)
```

```
[Out] a*x - (b*(c^2*x^2 + 1)^(1/2))/c + b*x*asinh(c*x)
```

sympy [A] time = 0.13, size = 26, normalized size = 0.87

$$ax + b\left(\begin{cases} x \operatorname{asinh}(cx) - \frac{\sqrt{c^2x^2+1}}{c} & \text{for}\ c \neq 0 \\ 0 & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*asinh(c*x),x)
```

```
[Out] a*x + b*Piecewise((x*asinh(c*x) - sqrt(c**2*x**2 + 1)/c, Ne(c, 0)), (0, Tru
e))
```

3.8 $\int \frac{a+b\sinh^{-1}(cx)}{d+ex}\,dx$

Optimal. Leaf size=187

$$\frac{\left(a+b\sinh^{-1}(cx)\right)\log\left(\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e}+\frac{\left(a+b\sinh^{-1}(cx)\right)\log\left(\frac{ee^{\sinh^{-1}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)}{e}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2be}+\frac{b\mathrm{Li}_2}{}$$

```
[Out] -1/2*(a+b*arcsinh(c*x))^2/b/e+(a+b*arcsinh(c*x))*ln(1+e*(c*x+(c^2*x^2+1)^(1
/2))/(c*d-(c^2*d^2+e^2)^(1/2)))/e+(a+b*arcsinh(c*x))*ln(1+e*(c*x+(c^2*x^2+1
)^(1/2))/(c*d+(c^2*d^2+e^2)^(1/2)))/e+b*polylog(2,-e*(c*x+(c^2*x^2+1)^(1/2)
)/(c*d-(c^2*d^2+e^2)^(1/2)))/e+b*polylog(2,-e*(c*x+(c^2*x^2+1)^(1/2))/(c*d+
(c^2*d^2+e^2)^(1/2)))/e
```

Rubi [A] time = 0.26, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {5799, 5561, 2190, 2279, 2391}

$$\frac{b\text{PolyLog}\left(2,-\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{b\text{PolyLog}\left(2,-\frac{ee^{\sinh^{-1}(cx)}}{\sqrt{c^2d^2+e^2}+cd}\right)}{e}+\frac{\left(a+b\sinh^{-1}(cx)\right)\log\left(\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e}+\frac{\left(a+b\mathrm{s}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(d + e*x),x]
```

```
[Out] -(a + b*ArcSinh[c*x])^2/(2*b*e) + ((a + b*ArcSinh[c*x])*Log[1 + (e*E^ArcSin
h[c*x])/(c*d - Sqrt[c^2*d^2 + e^2])])/e + ((a + b*ArcSinh[c*x])*Log[1 + (e*
E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2])])/e + (b*PolyLog[2, -((e*E^ArcS
inh[c*x])/(c*d - Sqrt[c^2*d^2 + e^2]))])/e + (b*PolyLog[2, -((e*E^ArcSinh[c
*x])/(c*d + Sqrt[c^2*d^2 + e^2]))])/e
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Cosh[x])/(c*d + e*Sinh[x]), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{d+ex}\,dx = \text{Subst}\left(\int \frac{(a+bx)\cosh(x)}{cd+e\sinh(x)}\,dx, x, \sinh^{-1}(cx)\right)$$
$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2be} + \text{Subst}\left(\int \frac{e^x(a+bx)}{cd-\sqrt{c^2d^2+e^2}+ee^x}\,dx, x, \sinh^{-1}(cx)\right) + \text{Subst}\Bigg(\int$$
$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2be} + \frac{\left(a+b\sinh^{-1}(cx)\right)\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\left(a+b\sinh^{-1}(cx)\right)\text{lo}}{e}$$
$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2be} + \frac{\left(a+b\sinh^{-1}(cx)\right)\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\left(a+b\sinh^{-1}(cx)\right)\text{lo}}{e}$$
$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2be} + \frac{\left(a+b\sinh^{-1}(cx)\right)\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\left(a+b\sinh^{-1}(cx)\right)\text{lo}}{e}$$

Mathematica [A] time = 0.05, size = 175, normalized size = 0.94

$$\frac{-\left(a+b\sinh^{-1}(cx)\right)\left(a-2b\log\left(\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)-2b\log\left(\frac{ee^{\sinh^{-1}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)+b\sinh^{-1}(cx)\right)+2b^2\text{Li}_2\left(\frac{ee^{\sinh^{-1}(cx)}}{\sqrt{c^2d^2+e^2}-}\right.}{2be}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(d + e*x),x]
```

```
[Out] (-((a + b*ArcSinh[c*x])*(a + b*ArcSinh[c*x] - 2*b*Log[1 + (e*E^ArcSinh[c*x]
)/(c*d - Sqrt[c^2*d^2 + e^2])] - 2*b*Log[1 + (e*E^ArcSinh[c*x])/(c*d + Sqrt
[c^2*d^2 + e^2])])) + 2*b^2*PolyLog[2, (e*E^ArcSinh[c*x])/(-(c*d) + Sqrt[c^
2*d^2 + e^2])] + 2*b^2*PolyLog[2, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2
+ e^2]))])/(2*b*e)
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\,\text{arsinh}\,(cx)+a}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/(e*x + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{arsinh}\,(cx)+a}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(e*x + d), x)
```

maple [A] time = 0.04, size = 282, normalized size = 1.51

$$\frac{a\ln(cex+cd)}{e}-\frac{b\,\mathrm{arcsinh}(cx)^2}{2e}+\frac{b\,\mathrm{arcsinh}(cx)\ln\left(\frac{-\left(cx+\sqrt{c^2x^2+1}\right)e-cd+\sqrt{c^2d^2+e^2}}{-cd+\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{b\,\mathrm{arcsinh}(cx)\ln\left(\frac{\left(cx+\sqrt{c^2x^2+1}\right.}{cd+\sqrt{}}\right.}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(e*x+d),x)
```

```
[Out] a*ln(c*e*x+c*d)/e-1/2*b*arcsinh(c*x)^2/e+b/e*arcsinh(c*x)*ln((-(c*x+(c^2*x^
2+1)^(1/2))*e-c*d+(c^2*d^2+e^2)^(1/2))/(-c*d+(c^2*d^2+e^2)^(1/2)))+b/e*arcs
inh(c*x)*ln(((c*x+(c^2*x^2+1)^(1/2))*e+c*d+(c^2*d^2+e^2)^(1/2))/(c*d+(c^2*d
^2+e^2)^(1/2)))+b/e*dilog(((c*x+(c^2*x^2+1)^(1/2))*e+c*d+(c^2*d^2+e^2)^(1/2
))/(c*d+(c^2*d^2+e^2)^(1/2)))+b/e*dilog((-(c*x+(c^2*x^2+1)^(1/2))*e-c*d+(c^
2*d^2+e^2)^(1/2))/(-c*d+(c^2*d^2+e^2)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b\int\frac{\log\left(cx+\sqrt{c^2x^2+1}\right)}{ex+d}\,dx+\frac{a\log(ex+d)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x+d),x, algorithm="maxima")
```

```
[Out] b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/(e*x + d), x) + a*log(e*x + d)/e
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{a+b\,\mathrm{asinh}(c\,x)}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(d + e*x),x)
```

```
[Out] int((a + b*asinh(c*x))/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a+b\,\mathrm{asinh}(cx)}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(e*x+d),x)
```

```
[Out] Integral((a + b*asinh(c*x))/(d + e*x), x)
```

3.9 $\int \frac{a+b\sinh^{-1}(cx)}{(d+ex)^2}\,dx$

Optimal. Leaf size=82

$$-\frac{a+b\sinh^{-1}(cx)}{e(d+ex)}-\frac{bc\tanh^{-1}\left(\frac{e-c^2dx}{\sqrt{c^2x^2+1}\sqrt{c^2d^2+e^2}}\right)}{e\sqrt{c^2d^2+e^2}}$$

```
[Out] (-a-b*arcsinh(c*x))/e/(e*x+d)-b*c*arctanh((-c^2*d*x+e)/(c^2*d^2+e^2)^(1/2)/
(c^2*x^2+1)^(1/2))/e/(c^2*d^2+e^2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {5801, 725, 206}

$$-\frac{a+b\sinh^{-1}(cx)}{e(d+ex)}-\frac{bc\tanh^{-1}\left(\frac{e-c^2dx}{\sqrt{c^2x^2+1}\sqrt{c^2d^2+e^2}}\right)}{e\sqrt{c^2d^2+e^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(d + e*x)^2,x]
```

```
[Out] -((a + b*ArcSinh[c*x])/(e*(d + e*x))) - (b*c*ArcTanh[(e - c^2*d*x)/(Sqrt[c^
2*d^2 + e^2]*Sqrt[1 + c^2*x^2])])/(e*Sqrt[c^2*d^2 + e^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 5801

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n
 - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\sinh^{-1}(cx)}{(d+ex)^2}\,dx &= -\frac{a+b\sinh^{-1}(cx)}{e(d+ex)}+\frac{(bc)\int \frac{1}{(d+ex)\sqrt{1+c^2x^2}}\,dx}{e}\\
&= -\frac{a+b\sinh^{-1}(cx)}{e(d+ex)}-\frac{(bc)\,\text{Subst}\left(\int \frac{1}{c^2d^2+e^2-x^2}\,dx,x,\frac{e-c^2dx}{\sqrt{1+c^2x^2}}\right)}{e}\\
&= -\frac{a+b\sinh^{-1}(cx)}{e(d+ex)}-\frac{bc\tanh^{-1}\left(\frac{e-c^2dx}{\sqrt{c^2d^2+e^2}\sqrt{1+c^2x^2}}\right)}{e\sqrt{c^2d^2+e^2}}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 79, normalized size = 0.96

$$-\frac{\frac{a+b\sinh^{-1}(cx)}{d+ex}+\frac{bc\tanh^{-1}\left(\frac{e-c^2dx}{\sqrt{c^2x^2+1}\sqrt{c^2d^2+e^2}}\right)}{\sqrt{c^2d^2+e^2}}}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(d + e*x)^2,x]
```

```
[Out] -(((a + b*ArcSinh[c*x])/(d + e*x) + (b*c*ArcTanh[(e - c^2*d*x)/(Sqrt[c^2*d^
2 + e^2]*Sqrt[1 + c^2*x^2])])/Sqrt[c^2*d^2 + e^2])/e)
```

fricas [B] time = 0.66, size = 253, normalized size = 3.09

$$-\frac{ac^2d^3+ade^2-\left(bc^2d^2e+be^3\right)x\log\left(cx+\sqrt{c^2x^2+1}\right)-\left(bcdex+bcd^2\right)\sqrt{c^2d^2+e^2}\log\left(-\frac{c^3d^2x-cde+\sqrt{c^2d^2+e^2}\left(c^2\right.}{}\right.}{c^2d^4e+d^2e^3+\left(c^2d^3e^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -(a*c^2*d^3 + a*d*e^2 - (b*c^2*d^2*e + b*e^3)*x*log(c*x + sqrt(c^2*x^2 + 1)
) - (b*c*d*e*x + b*c*d^2)*sqrt(c^2*d^2 + e^2)*log(-(c^3*d^2*x - c*d*e + sqr
t(c^2*d^2 + e^2)*(c^2*d*x - e) + (c^2*d^2 + sqrt(c^2*d^2 + e^2)*c*d + e^2)*
sqrt(c^2*x^2 + 1))/(e*x + d)) - (b*c^2*d^3 + b*d*e^2 + (b*c^2*d^2*e + b*e^3
)*x)*log(-c*x + sqrt(c^2*x^2 + 1)))/(c^2*d^4*e + d^2*e^3 + (c^2*d^3*e^2 + d
*e^4)*x)
```

giac [B] time = 0.86, size = 214, normalized size = 2.61

$$\left(\frac{ce^{(-1)}\log\left(-c^2d+\sqrt{c^2d^2+e^2}\,|c|\right)\operatorname{sgn}\left(\frac{1}{xe+d}\right)}{\sqrt{c^2d^2+e^2}}-\frac{ce^{(-1)}\log\left(-c^2d+\sqrt{c^2d^2+e^2}\left(\sqrt{c^2-\frac{2c^2d}{xe+d}+\frac{c^2d^2}{(xe+d)^2}+\frac{e^2}{(xe+d)^2}}\right.\right.}{\sqrt{c^2d^2+e^2}\operatorname{sgn}\left(\frac{1}{xe+d}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] (c*e^(-1)*log(-c^2*d + sqrt(c^2*d^2 + e^2)*abs(c))*sgn(1/(x*e + d))/sqrt(c^
2*d^2 + e^2) - c*e^(-1)*log(-c^2*d + sqrt(c^2*d^2 + e^2)*(sqrt(c^2 - 2*c^2*
d/(x*e + d) + c^2*d^2/(x*e + d)^2 + e^2/(x*e + d)^2) + sqrt(c^2*d^2*e^2 + e
^4)*e^(-1)/(x*e + d)))/(sqrt(c^2*d^2 + e^2)*sgn(1/(x*e + d))) - e^(-1)*log(
c*x + sqrt(c^2*x^2 + 1))/(x*e + d))*b - a*e^(-1)/(x*e + d)
```

maple [B] time = 0.02, size = 178, normalized size = 2.17

$$-\frac{ca}{(cex+cd)\,e}-\frac{cb\operatorname{arcsinh}(cx)}{(cex+cd)\,e}-\frac{cb\ln\left(\frac{\frac{2c^2d^2+2e^2}{e^2}-\frac{2cd\left(cx+\frac{cd}{e}\right)}{e}+2\sqrt{\frac{c^2d^2+e^2}{e^2}}\sqrt{\left(cx+\frac{cd}{e}\right)^2-\frac{2cd\left(cx+\frac{cd}{e}\right)}{e}+\frac{c^2d^2+e^2}{e^2}}}{cx+\frac{cd}{e}}\right)}{e^2\sqrt{\frac{c^2d^2+e^2}{e^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(e*x+d)^2,x)
```

```
[Out] -c*a/(c*e*x+c*d)/e-c*b/(c*e*x+c*d)/e*arcsinh(c*x)-c*b/e^2/((c^2*d^2+e^2)/e^
2)^(1/2)*ln((2*(c^2*d^2+e^2)/e^2-2*c*d/e*(c*x+c*d/e)+2*((c^2*d^2+e^2)/e^2)^
(1/2)*((c*x+c*d/e)^2-2*c*d/e*(c*x+c*d/e)+(c^2*d^2+e^2)/e^2)^(1/2))/(c*x+c*d
/e))
```

maxima [A] time = 0.46, size = 94, normalized size = 1.15

$$-b\left(\frac{\operatorname{arsinh}(cx)}{e^2x+de}-\frac{c\operatorname{arsinh}\left(\frac{cdex}{|e^2x+de|}-\frac{e^2}{c|e^2x+de|}\right)}{\sqrt{\frac{c^2d^2}{e^2}+1}\,e^2}\right)-\frac{a}{e^2x+de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -b*(arcsinh(c*x)/(e^2*x + d*e) - c*arcsinh(c*d*e*x/abs(e^2*x + d*e) - e^2/(
c*abs(e^2*x + d*e)))/(sqrt(c^2*d^2/e^2 + 1)*e^2)) - a/(e^2*x + d*e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\operatorname{asinh}(c\,x)}{(d+e\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(d + e*x)^2,x)
```

```
[Out] int((a + b*asinh(c*x))/(d + e*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\operatorname{asinh}(cx)}{(d+ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(e*x+d)**2,x)
```

```
[Out] Integral((a + b*asinh(c*x))/(d + e*x)**2, x)
```

3.10 $\int \frac{a+b\sinh^{-1}(cx)}{(d+ex)^3}\,dx$

Optimal. Leaf size=128

$$-\frac{a+b\sinh^{-1}(cx)}{2e(d+ex)^2}-\frac{bc\sqrt{c^2x^2+1}}{2\left(c^2d^2+e^2\right)(d+ex)}-\frac{bc^3d\tanh^{-1}\left(\frac{e-c^2dx}{\sqrt{c^2x^2+1}\sqrt{c^2d^2+e^2}}\right)}{2e\left(c^2d^2+e^2\right)^{3/2}}$$

```
[Out] 1/2*(-a-b*arcsinh(c*x))/e/(e*x+d)^2-1/2*b*c^3*d*arctanh((-c^2*d*x+e)/(c^2*d
^2+e^2)^(1/2)/(c^2*x^2+1)^(1/2))/e/(c^2*d^2+e^2)^(3/2)-1/2*b*c*(c^2*x^2+1)^
(1/2)/(c^2*d^2+e^2)/(e*x+d)
```

Rubi [A] time = 0.08, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5801, 731, 725, 206}

$$-\frac{a+b\sinh^{-1}(cx)}{2e(d+ex)^2}-\frac{bc\sqrt{c^2x^2+1}}{2\left(c^2d^2+e^2\right)(d+ex)}-\frac{bc^3d\tanh^{-1}\left(\frac{e-c^2dx}{\sqrt{c^2x^2+1}\sqrt{c^2d^2+e^2}}\right)}{2e\left(c^2d^2+e^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(d + e*x)^3,x]
```

```
[Out] -(b*c*Sqrt[1 + c^2*x^2])/(2*(c^2*d^2 + e^2)*(d + e*x)) - (a + b*ArcSinh[c*x
])/(2*e*(d + e*x)^2) - (b*c^3*d*ArcTanh[(e - c^2*d*x)/(Sqrt[c^2*d^2 + e^2]*
Sqrt[1 + c^2*x^2])])/(2*e*(c^2*d^2 + e^2)^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 731

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[(c*d)/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 5801

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n
 - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{(d+ex)^3}\,dx = -\frac{a+b\sinh^{-1}(cx)}{2e(d+ex)^2}+\frac{(bc)\int \frac{1}{(d+ex)^2\sqrt{1+c^2x^2}}\,dx}{2e}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2\left(c^2d^2+e^2\right)(d+ex)}-\frac{a+b\sinh^{-1}(cx)}{2e(d+ex)^2}+\frac{\left(bc^3d\right)\int \frac{1}{(d+ex)\sqrt{1+c^2x^2}}\,dx}{2e\left(c^2d^2+e^2\right)}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2\left(c^2d^2+e^2\right)(d+ex)}-\frac{a+b\sinh^{-1}(cx)}{2e(d+ex)^2}-\frac{\left(bc^3d\right)\text{Subst}\left(\int \frac{1}{c^2d^2+e^2-x^2}\,dx, x, \frac{e-c^2dx}{\sqrt{1+c^2x^2}}\right)}{2e\left(c^2d^2+e^2\right)}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2\left(c^2d^2+e^2\right)(d+ex)}-\frac{a+b\sinh^{-1}(cx)}{2e(d+ex)^2}-\frac{bc^3d\tanh^{-1}\left(\frac{e-c^2dx}{\sqrt{c^2d^2+e^2}\sqrt{1+c^2x^2}}\right)}{2e\left(c^2d^2+e^2\right)^{3/2}}$$

Mathematica [A] time = 0.36, size = 166, normalized size = 1.30

$$\frac{1}{2}\left(-\frac{a}{e(d+ex)^2}-\frac{bc\sqrt{c^2x^2+1}}{\left(c^2d^2+e^2\right)(d+ex)}-\frac{bc^3d\log\left(\sqrt{c^2x^2+1}\sqrt{c^2d^2+e^2}+c^2(-d)x+e\right)}{e\left(c^2d^2+e^2\right)^{3/2}}+\frac{bc^3d\log(d+ex)}{e\left(c^2d^2+e^2\right)^{3/2}}-\frac{b\,s}{e(}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(d + e*x)^3,x]
```

```
[Out] (-(a/(e*(d + e*x)^2)) - (b*c*Sqrt[1 + c^2*x^2])/((c^2*d^2 + e^2)*(d + e*x))
 - (b*ArcSinh[c*x])/(e*(d + e*x)^2) + (b*c^3*d*Log[d + e*x])/(e*(c^2*d^2 +
e^2)^(3/2)) - (b*c^3*d*Log[e - c^2*d*x + Sqrt[c^2*d^2 + e^2]*Sqrt[1 + c^2*x
^2]])/(e*(c^2*d^2 + e^2)^(3/2)))/2
```

fricas [B] time = 0.76, size = 566, normalized size = 4.42

$$-\frac{(a+b)c^4d^6+(2\,a+b)c^2d^4e^2+ad^2e^4+\left(bc^4d^4e^2+bc^2d^2e^4\right)x^2-\left(bc^3d^3e^2x^2+2\,bc^3d^4ex+bc^3d^5\right)\sqrt{c^2d^2+e^2}\,\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] -1/2*((a + b)*c^4*d^6 + (2*a + b)*c^2*d^4*e^2 + a*d^2*e^4 + (b*c^4*d^4*e^2
+ b*c^2*d^2*e^4)*x^2 - (b*c^3*d^3*e^2*x^2 + 2*b*c^3*d^4*e*x + b*c^3*d^5)*sq
rt(c^2*d^2 + e^2)*log(-(c^3*d^2*x - c*d*e + sqrt(c^2*d^2 + e^2)*(c^2*d*x -
e) + (c^2*d^2 + sqrt(c^2*d^2 + e^2)*c*d + e^2)*sqrt(c^2*x^2 + 1))/(e*x + d)
) + 2*(b*c^4*d^5*e + b*c^2*d^3*e^3)*x - ((b*c^4*d^4*e^2 + 2*b*c^2*d^2*e^4 +
 b*e^6)*x^2 + 2*(b*c^4*d^5*e + 2*b*c^2*d^3*e^3 + b*d*e^5)*x)*log(c*x + sqrt
(c^2*x^2 + 1)) - (b*c^4*d^6 + 2*b*c^2*d^4*e^2 + b*d^2*e^4 + (b*c^4*d^4*e^2
+ 2*b*c^2*d^2*e^4 + b*e^6)*x^2 + 2*(b*c^4*d^5*e + 2*b*c^2*d^3*e^3 + b*d*e^5
)*x)*log(-c*x + sqrt(c^2*x^2 + 1)) + (b*c^3*d^5*e + b*c*d^3*e^3 + (b*c^3*d^
4*e^2 + b*c*d^2*e^4)*x)*sqrt(c^2*x^2 + 1))/(c^4*d^8*e + 2*c^2*d^6*e^3 + d^4
*e^5 + (c^4*d^6*e^3 + 2*c^2*d^4*e^5 + d^2*e^7)*x^2 + 2*(c^4*d^7*e^2 + 2*c^2
*d^5*e^4 + d^3*e^6)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\mathrm{arsinh}\left(cx\right)+a}{\left(ex+d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(e*x + d)^3, x)
```

maple [B] time = 0.06, size = 279, normalized size = 2.18

$$-\frac{c^2 a}{2\left(cex+cd\right)^2 e}-\frac{c^2 b \operatorname{arcsinh}\left(cx\right)}{2\left(cex+cd\right)^2 e}-\frac{c^2 b\sqrt{\left(cx+\frac{cd}{e}\right)^2-\frac{2cd\left(cx+\frac{cd}{e}\right)}{e}+\frac{c^2d^2+e^2}{e^2}}}{2e\left(c^2d^2+e^2\right)\left(cx+\frac{cd}{e}\right)}-\frac{c^3 bd\ln\left(\frac{\frac{2c^2d^2+2e^2}{e^2}-\frac{2cd\left(cx+\frac{cd}{e}\right)}{e}+2\sqrt{\frac{c^2d^2+e^2}{e^2}}}{cx}\right.}{2e^2\left(c^2d^2+e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(e*x+d)^3,x)
```

```
[Out] -1/2*c^2*a/(c*e*x+c*d)^2/e-1/2*c^2*b/(c*e*x+c*d)^2/e*arcsinh(c*x)-1/2*c^2*b
/e/(c^2*d^2+e^2)/(c*x+c*d/e)*((c*x+c*d/e)^2-2*c*d/e*(c*x+c*d/e)+(c^2*d^2+e^
2)/e^2)^(1/2)-1/2*c^3*b/e^2*d/(c^2*d^2+e^2)/((c^2*d^2+e^2)/e^2)^(1/2)*ln((2
*(c^2*d^2+e^2)/e^2-2*c*d/e*(c*x+c*d/e)+2*((c^2*d^2+e^2)/e^2)^(1/2)*((c*x+c*
d/e)^2-2*c*d/e*(c*x+c*d/e)+(c^2*d^2+e^2)/e^2)^(1/2))/(c*x+c*d/e))
```

maxima [A] time = 0.39, size = 158, normalized size = 1.23

$$-\frac{1}{2}\left(c\left(\frac{\sqrt{c^2x^2+1}}{c^2d^2ex+c^2d^3+e^3x+de^2}-\frac{c^2 d\operatorname{arsinh}\left(\frac{cdx}{e\left|x+\frac{d}{e}\right|}-\frac{1}{c\left|x+\frac{d}{e}\right|}\right)}{\left(\frac{c^2d^2}{e^2}+1\right)^{\frac{3}{2}}e^4}\right)+\frac{\operatorname{arsinh}\left(cx\right)}{e^3x^2+2\,de^2x+d^2e}\right)b-\frac{a}{2\left(e^3x^2+2\,de^2x+d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(c*(sqrt(c^2*x^2 + 1)/(c^2*d^2*e*x + c^2*d^3 + e^3*x + d*e^2) - c^2*d*
arcsinh(c*d*x/(e*abs(x + d/e)) - 1/(c*abs(x + d/e)))/((c^2*d^2/e^2 + 1)^(3/
2)*e^4)) + arcsinh(c*x)/(e^3*x^2 + 2*d*e^2*x + d^2*e))*b - 1/2*a/(e^3*x^2 +
 2*d*e^2*x + d^2*e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\operatorname{asinh}\left(c\,x\right)}{\left(d+e\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(d + e*x)^3,x)
```

```
[Out] int((a + b*asinh(c*x))/(d + e*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\operatorname{asinh}\left(cx\right)}{\left(d+ex\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(e*x+d)**3,x)
```

```
[Out] Integral((a + b*asinh(c*x))/(d + e*x)**3, x)
```

3.11 $\int \frac{a+b\sinh^{-1}(cx)}{(d+ex)^4}\,dx$

Optimal. Leaf size=183

$$-\frac{a+b\sinh^{-1}(cx)}{3e(d+ex)^3}-\frac{bc\sqrt{c^2x^2+1}}{6\left(c^2d^2+e^2\right)(d+ex)^2}-\frac{bc^3d\sqrt{c^2x^2+1}}{2\left(c^2d^2+e^2\right)^2(d+ex)}-\frac{bc^3\left(2c^2d^2-e^2\right)\tanh^{-1}\left(\frac{e-c^2dx}{\sqrt{c^2x^2+1}\,\sqrt{c^2d^2+e^2}}\right)}{6e\left(c^2d^2+e^2\right)^{5/2}}$$

```
[Out] 1/3*(-a-b*arcsinh(c*x))/e/(e*x+d)^3-1/6*b*c^3*(2*c^2*d^2-e^2)*arctanh((-c^2
*d*x+e)/(c^2*d^2+e^2)^(1/2)/(c^2*x^2+1)^(1/2))/e/(c^2*d^2+e^2)^(5/2)-1/6*b*
c*(c^2*x^2+1)^(1/2)/(c^2*d^2+e^2)/(e*x+d)^2-1/2*b*c^3*d*(c^2*x^2+1)^(1/2)/(
c^2*d^2+e^2)^2/(e*x+d)
```

Rubi [A] time = 0.14, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5801, 745, 807, 725, 206}

$$-\frac{a+b\sinh^{-1}(cx)}{3e(d+ex)^3}-\frac{bc^3d\sqrt{c^2x^2+1}}{2\left(c^2d^2+e^2\right)^2(d+ex)}-\frac{bc\sqrt{c^2x^2+1}}{6\left(c^2d^2+e^2\right)(d+ex)^2}-\frac{bc^3\left(2c^2d^2-e^2\right)\tanh^{-1}\left(\frac{e-c^2dx}{\sqrt{c^2x^2+1}\,\sqrt{c^2d^2+e^2}}\right)}{6e\left(c^2d^2+e^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(d + e*x)^4,x]
```

```
[Out] -(b*c*Sqrt[1 + c^2*x^2])/(6*(c^2*d^2 + e^2)*(d + e*x)^2) - (b*c^3*d*Sqrt[1
+ c^2*x^2])/(2*(c^2*d^2 + e^2)^2*(d + e*x)) - (a + b*ArcSinh[c*x])/(3*e*(d
+ e*x)^3) - (b*c^3*(2*c^2*d^2 - e^2)*ArcTanh[(e - c^2*d*x)/(Sqrt[c^2*d^2 +
e^2]*Sqrt[1 + c^2*x^2])])/(6*e*(c^2*d^2 + e^2)^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
```

```
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 5801

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n
 - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{(d+ex)^4}\,dx = -\frac{a+b\sinh^{-1}(cx)}{3e(d+ex)^3} + \frac{(bc)\int \frac{1}{(d+ex)^3\sqrt{1+c^2x^2}}\,dx}{3e}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{6\left(c^2d^2+e^2\right)(d+ex)^2} - \frac{a+b\sinh^{-1}(cx)}{3e(d+ex)^3} - \frac{\left(bc^3\right)\int \frac{-2d+ex}{(d+ex)^2\sqrt{1+c^2x^2}}\,dx}{6e\left(c^2d^2+e^2\right)}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{6\left(c^2d^2+e^2\right)(d+ex)^2} - \frac{bc^3d\sqrt{1+c^2x^2}}{2\left(c^2d^2+e^2\right)^2(d+ex)} - \frac{a+b\sinh^{-1}(cx)}{3e(d+ex)^3} + \frac{\left(bc^3\left(2c^2d^2-\right.\right.}{6e}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{6\left(c^2d^2+e^2\right)(d+ex)^2} - \frac{bc^3d\sqrt{1+c^2x^2}}{2\left(c^2d^2+e^2\right)^2(d+ex)} - \frac{a+b\sinh^{-1}(cx)}{3e(d+ex)^3} - \frac{\left(bc^3\left(2c^2d^2-\right.\right.}{}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{6\left(c^2d^2+e^2\right)(d+ex)^2} - \frac{bc^3d\sqrt{1+c^2x^2}}{2\left(c^2d^2+e^2\right)^2(d+ex)} - \frac{a+b\sinh^{-1}(cx)}{3e(d+ex)^3} - \frac{bc^3\left(2c^2d^2-\right.}{}$$

Mathematica [A] time = 0.43, size = 205, normalized size = 1.12

$$\frac{1}{6}\left(-\frac{2a}{e(d+ex)^3} - \frac{bc\sqrt{c^2x^2+1}\left(c^2d(4d+3ex)+e^2\right)}{\left(c^2d^2+e^2\right)^2(d+ex)^2} + \frac{bc^3\left(e^2-2c^2d^2\right)\log\left(\sqrt{c^2x^2+1}\sqrt{c^2d^2+e^2}+c^2(-d)x+\right.}{e\left(c^2d^2+e^2\right)^{5/2}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(d + e*x)^4,x]
```

```
[Out] ((-2*a)/(e*(d + e*x)^3) - (b*c*Sqrt[1 + c^2*x^2]*(e^2 + c^2*d*(4*d + 3*e*x)
))/((c^2*d^2 + e^2)^2*(d + e*x)^2) - (2*b*ArcSinh[c*x])/(e*(d + e*x)^3) - (
b*c^3*(-2*c^2*d^2 + e^2)*Log[d + e*x])/(e*(c^2*d^2 + e^2)^(5/2)) + (b*c^3*(
-2*c^2*d^2 + e^2)*Log[e - c^2*d*x + Sqrt[c^2*d^2 + e^2]*Sqrt[1 + c^2*x^2]])
/(e*(c^2*d^2 + e^2)^(5/2)))/6
```

fricas [B] time = 1.67, size = 977, normalized size = 5.34

$$-\frac{(2a+3b)c^6d^9+3(2a+b)c^4d^7e^2+6ac^2d^5e^4+2ad^3e^6+3\left(bc^6d^6e^3+bc^4d^4e^5\right)x^3+9\left(bc^6d^7e^2+bc^4d^5e^4\right)x^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x+d)^4,x, algorithm="fricas")
```

[Out] -1/6*((2*a + 3*b)*c^6*d^9 + 3*(2*a + b)*c^4*d^7*e^2 + 6*a*c^2*d^5*e^4 + 2*a *d^3*e^6 + 3*(b*c^6*d^6*e^3 + b*c^4*d^4*e^5)*x^3 + 9*(b*c^6*d^7*e^2 + b*c^4 *d^5*e^4)*x^2 + (2*b*c^5*d^8 - b*c^3*d^6*e^2 + (2*b*c^5*d^5*e^3 - b*c^3*d^3 *e^5)*x^3 + 3*(2*b*c^5*d^6*e^2 - b*c^3*d^4*e^4)*x^2 + 3*(2*b*c^5*d^7*e - b* c^3*d^5*e^3)*x)*sqrt(c^2*d^2 + e^2)*log(-(c^3*d^2*x - c*d*e - sqrt(c^2*d^2 + e^2)*(c^2*d*x - e) + (c^2*d^2 - sqrt(c^2*d^2 + e^2)*c*d + e^2)*sqrt(c^2*x ^2 + 1))/(e*x + d)) + 9*(b*c^6*d^8*e + b*c^4*d^6*e^3)*x - 2*((b*c^6*d^6*e^3 + 3*b*c^4*d^4*e^5 + 3*b*c^2*d^2*e^7 + b*e^9)*x^3 + 3*(b*c^6*d^7*e^2 + 3*b* c^4*d^5*e^4 + 3*b*c^2*d^3*e^6 + b*d*e^8)*x^2 + 3*(b*c^6*d^8*e + 3*b*c^4*d^6 *e^3 + 3*b*c^2*d^4*e^5 + b*d^2*e^7)*x)*log(c*x + sqrt(c^2*x^2 + 1)) - 2*(b* c^6*d^9 + 3*b*c^4*d^7*e^2 + 3*b*c^2*d^5*e^4 + b*d^3*e^6 + (b*c^6*d^6*e^3 + 3*b*c^4*d^4*e^5 + 3*b*c^2*d^2*e^7 + b*e^9)*x^3 + 3*(b*c^6*d^7*e^2 + 3*b*c^4 *d^5*e^4 + 3*b*c^2*d^3*e^6 + b*d*e^8)*x^2 + 3*(b*c^6*d^8*e + 3*b*c^4*d^6*e^ 3 + 3*b*c^2*d^4*e^5 + b*d^2*e^7)*x)*log(-c*x + sqrt(c^2*x^2 + 1)) + (4*b*c^ 5*d^8*e + 5*b*c^3*d^6*e^3 + b*c*d^4*e^5 + 3*(b*c^5*d^6*e^3 + b*c^3*d^4*e^5) *x^2 + (7*b*c^5*d^7*e^2 + 8*b*c^3*d^5*e^4 + b*c*d^3*e^6)*x)*sqrt(c^2*x^2 + 1))/(c^6*d^12*e + 3*c^4*d^10*e^3 + 3*c^2*d^8*e^5 + d^6*e^7 + (c^6*d^9*e^4 + 3*c^4*d^7*e^6 + 3*c^2*d^5*e^8 + d^3*e^10)*x^3 + 3*(c^6*d^10*e^3 + 3*c^4*d^ 8*e^5 + 3*c^2*d^6*e^7 + d^4*e^9)*x^2 + 3*(c^6*d^11*e^2 + 3*c^4*d^9*e^4 + 3* c^2*d^7*e^6 + d^5*e^8)*x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arsinh}(cx) + a}{(ex + d)^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsinh(c*x))/(e*x+d)^4,x, algorithm="giac")

[Out] integrate((b*arcsinh(c*x) + a)/(e*x + d)^4, x)

maple [B] time = 0.01, size = 516, normalized size = 2.82

$$-\frac{c^3 a}{3(cex+cd)^3 e} - \frac{c^3 b \operatorname{arcsinh}(cx)}{3(cex+cd)^3 e} - \frac{c^3 b \sqrt{\left(cx+\frac{cd}{e}\right)^2 - \frac{2cd\left(cx+\frac{cd}{e}\right)}{e} + \frac{c^2d^2+e^2}{e^2}}}{6e^2\left(c^2d^2+e^2\right)\left(cx+\frac{cd}{e}\right)^2} - \frac{c^4 bd \sqrt{\left(cx+\frac{cd}{e}\right)^2 - \frac{2cd\left(cx+\frac{cd}{e}\right)}{e} + \frac{c^2d^2+e^2}{e^2}}}{2e\left(c^2d^2+e^2\right)^2\left(cx+\frac{cd}{e}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arcsinh(c*x))/(e*x+d)^4,x)

[Out] -1/3*c^3*a/(c*e*x+c*d)^3/e-1/3*c^3*b/(c*e*x+c*d)^3/e*arcsinh(c*x)-1/6*c^3*b /e^2/(c^2*d^2+e^2)/(c*x+c*d/e)^2*((c*x+c*d/e)^2-2*c*d/e*(c*x+c*d/e)+(c^2*d^ 2+e^2)/e^2)^(1/2)-1/2*c^4*b/e*d/(c^2*d^2+e^2)^2/(c*x+c*d/e)*((c*x+c*d/e)^2- 2*c*d/e*(c*x+c*d/e)+(c^2*d^2+e^2)/e^2)^(1/2)-1/2*c^5*b/e^2*d^2/(c^2*d^2+e^2)^2/((c^2*d^2+e^2)/e^2)^(1/2)*ln((2*(c^2*d^2+e^2)/e^2-2*c*d/e*(c*x+c*d/e)+2 *((c^2*d^2+e^2)/e^2)^(1/2)*((c*x+c*d/e)^2-2*c*d/e*(c*x+c*d/e)+(c^2*d^2+e^2) /e^2)^(1/2))/(c*x+c*d/e))+1/6*c^3*b/e^2/(c^2*d^2+e^2)/((c^2*d^2+e^2)/e^2)^(1/2)*ln((2*(c^2*d^2+e^2)/e^2-2*c*d/e*(c*x+c*d/e)+2*((c^2*d^2+e^2)/e^2)^(1/2)*((c*x+c*d/e)^2-2*c*d/e*(c*x+c*d/e)+(c^2*d^2+e^2)/e^2)^(1/2))/(c*x+c*d/e))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{6}\left(6c\int \frac{1}{3\left(c^3e^4x^6 + 3c^3de^3x^5 + 3cd^2e^2x^2 + cd^3ex + \left(3c^3d^2e^2 + ce^4\right)x^4 + \left(c^3d^3e + 3cde^3\right)x^3 + \left(c^2e^4x^5 + 3c^2de^3x\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(6*c*integrate(1/3/(c^3*e^4*x^6 + 3*c^3*d*e^3*x^5 + 3*c*d^2*e^2*x^2 + c
*d^3*e*x + (3*c^3*d^2*e^2 + c*e^4)*x^4 + (c^3*d^3*e + 3*c*d*e^3)*x^3 + (c^2
*e^4*x^5 + 3*c^2*d*e^3*x^4 + 3*d^2*e^2*x + d^3*e + (3*c^2*d^2*e^2 + e^4)*x^
3 + (c^2*d^3*e + 3*d*e^3)*x^2)*sqrt(c^2*x^2 + 1)), x) - 2*(c^6*d^3 - 3*c^4*
d*e^2)*log(e*x + d)/(c^6*d^6*e + 3*c^4*d^4*e^3 + 3*c^2*d^2*e^5 + e^7) + (3*
c^6*d^6 + 2*c^4*d^4*e^2 - c^2*d^2*e^4 + 2*(c^6*d^4*e^2 - c^2*e^6)*x^2 + (5*
c^6*d^5*e + 2*c^4*d^3*e^3 - 3*c^2*d*e^5)*x + (c^6*d^6 - 3*c^4*d^4*e^2 + (c^
6*d^3*e^3 - 3*c^4*d*e^5)*x^3 + 3*(c^6*d^4*e^2 - 3*c^4*d^2*e^4)*x^2 + 3*(c^6
*d^5*e - 3*c^4*d^3*e^3)*x)*log(c^2*x^2 + 1) - 2*(c^6*d^6 + 3*c^4*d^4*e^2 +
3*c^2*d^2*e^4 + e^6)*log(c*x + sqrt(c^2*x^2 + 1)))/(c^6*d^9*e + 3*c^4*d^7*e
^3 + 3*c^2*d^5*e^5 + d^3*e^7 + (c^6*d^6*e^4 + 3*c^4*d^4*e^6 + 3*c^2*d^2*e^8
 + e^10)*x^3 + 3*(c^6*d^7*e^3 + 3*c^4*d^5*e^5 + 3*c^2*d^3*e^7 + d*e^9)*x^2
+ 3*(c^6*d^8*e^2 + 3*c^4*d^6*e^4 + 3*c^2*d^4*e^6 + d^2*e^8)*x) - I*(3*c^6*d
^2 - c^4*e^2)*(log(I*c*x + 1) - log(-I*c*x + 1))/((c^6*d^6 + 3*c^4*d^4*e^2
+ 3*c^2*d^2*e^4 + e^6)*c))*b - 1/3*a/(e^4*x^3 + 3*d*e^3*x^2 + 3*d^2*e^2*x +
 d^3*e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b \operatorname{asinh}(c\, x)}{(d + e\, x)^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(d + e*x)^4,x)
```

```
[Out] int((a + b*asinh(c*x))/(d + e*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{asinh}(cx)}{(d + ex)^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(e*x+d)**4,x)
```

```
[Out] Integral((a + b*asinh(c*x))/(d + e*x)**4, x)
```

3.12 $\int (d+ex)^3 \left(a+b\sinh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=368

$$-\frac{3e^3\left(a+b\sinh^{-1}(cx)\right)^2}{32c^4}-\frac{2bd^3\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{c}-\frac{3bd^2ex\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{2c}+\frac{3d^2e\left(a+b\sinh^{-1}(cx)\right)^2}{4c^2}\ldots$$

```
[Out] 2*b^2*d^3*x-4/3*b^2*d*e^2*x/c^2+3/4*b^2*d^2*e*x^2-3/32*b^2*e^3*x^2/c^2+2/9*
b^2*d*e^2*x^3+1/32*b^2*e^3*x^4-1/4*d^4*(a+b*arcsinh(c*x))^2/e+3/4*d^2*e*(a+
b*arcsinh(c*x))^2/c^2-3/32*e^3*(a+b*arcsinh(c*x))^2/c^4+1/4*(e*x+d)^4*(a+b*
arcsinh(c*x))^2/e-2*b*d^3*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c+4/3*b*d*e^
2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3-3/2*b*d^2*e*x*(a+b*arcsinh(c*x))
*(c^2*x^2+1)^(1/2)/c+3/16*b*e^3*x*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3-
2/3*b*d*e^2*x^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c-1/8*b*e^3*x^3*(a+b*a
rcsinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

Rubi [A] time = 0.76, antiderivative size = 368, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {5801, 5821, 5675, 5717, 8, 5758, 30}

$$-\frac{3bd^2ex\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{2c}+\frac{3d^2e\left(a+b\sinh^{-1}(cx)\right)^2}{4c^2}-\frac{2bd^3\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{c}+\frac{4bde^2\sqrt{c^2x\ldots}}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] 2*b^2*d^3*x - (4*b^2*d*e^2*x)/(3*c^2) + (3*b^2*d^2*e*x^2)/4 - (3*b^2*e^3*x^
2)/(32*c^2) + (2*b^2*d*e^2*x^3)/9 + (b^2*e^3*x^4)/32 - (2*b*d^3*Sqrt[1 + c^
2*x^2]*(a + b*ArcSinh[c*x]))/c + (4*b*d*e^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSin
h[c*x]))/(3*c^3) - (3*b*d^2*e*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(2*
c) + (3*b*e^3*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(16*c^3) - (2*b*d*e
^2*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(3*c) - (b*e^3*x^3*Sqrt[1 +
c^2*x^2]*(a + b*ArcSinh[c*x]))/(8*c) - (d^4*(a + b*ArcSinh[c*x])^2)/(4*e) +
 (3*d^2*e*(a + b*ArcSinh[c*x])^2)/(4*c^2) - (3*e^3*(a + b*ArcSinh[c*x])^2)/
(32*c^4) + ((d + e*x)^4*(a + b*ArcSinh[c*x])^2)/(4*e)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
```

```
0] && NeQ[p, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5801

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n
 - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\int (d+ex)^3\left(a+b\sinh^{-1}(cx)\right)^2\,dx = \frac{(d+ex)^4\left(a+b\sinh^{-1}(cx)\right)^2}{4e} - \frac{(bc)\int \frac{(d+ex)^4\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\,dx}{2e}$$

$$= \frac{(d+ex)^4\left(a+b\sinh^{-1}(cx)\right)^2}{4e} - \frac{(bc)\int \left(\frac{d^4\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}} + \frac{4d^3ex\left(a+b\sinh^{-1}(c\right.}{\sqrt{1+c^2x^2}}\right.}{}$$

$$= \frac{(d+ex)^4\left(a+b\sinh^{-1}(cx)\right)^2}{4e} - \left(2bcd^3\right)\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\,dx - \frac{(bc}{}$$

$$= -\frac{2bd^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c} - \frac{3bd^2ex\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx\right.}{2c}$$

$$= 2b^2d^3x + \frac{3}{4}b^2d^2ex^2 + \frac{2}{9}b^2de^2x^3 + \frac{1}{32}b^2e^3x^4 - \frac{2bd^3\sqrt{1+c^2x^2}\left(a+b\sinh\right.}{c}$$

$$= 2b^2d^3x - \frac{4b^2de^2x}{3c^2} + \frac{3}{4}b^2d^2ex^2 - \frac{3b^2e^3x^2}{32c^2} + \frac{2}{9}b^2de^2x^3 + \frac{1}{32}b^2e^3x^4 - \frac{2bd^3}{}$$

Mathematica [A] time = 0.56, size = 354, normalized size = 0.96

$$\frac{c\left(72a^2c^3x\left(4d^3+6d^2ex+4de^2x^2+e^3x^3\right) - 6ab\sqrt{c^2x^2+1}\left(c^2\left(96d^3+72d^2ex+32de^2x^2+6e^3x^3\right) - e^2(64d+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (c*(72*a^2*c^3*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3) - 6*a*b*Sqrt[1
 + c^2*x^2]*(-(e^2*(64*d + 9*e*x)) + c^2*(96*d^3 + 72*d^2*e*x + 32*d*e^2*x^
2 + 6*e^3*x^3)) + b^2*c*x*(-3*e^2*(128*d + 9*e*x) + c^2*(576*d^3 + 216*d^2*
e*x + 64*d*e^2*x^2 + 9*e^3*x^3))) - 6*b*(-3*a*(24*c^2*d^2*e - 3*e^3 + 8*c^4
*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3)) + b*c*Sqrt[1 + c^2*x^2]*(-(
e^2*(64*d + 9*e*x)) + c^2*(96*d^3 + 72*d^2*e*x + 32*d*e^2*x^2 + 6*e^3*x^3))
)*ArcSinh[c*x] + 9*b^2*(24*c^2*d^2*e - 3*e^3 + 8*c^4*x*(4*d^3 + 6*d^2*e*x +
 4*d*e^2*x^2 + e^3*x^3))*ArcSinh[c*x]^2)/(288*c^4)
```

fricas [A] time = 0.69, size = 475, normalized size = 1.29

$$\frac{9\left(8a^2+b^2\right)c^4e^3x^4+32\left(9a^2+2b^2\right)c^4de^2x^3+27\left(8\left(2a^2+b^2\right)c^4d^2e-b^2c^2e^3\right)x^2+9\left(8b^2c^4e^3x^4+32b^2c^4de^2x^3\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/288*(9*(8*a^2 + b^2)*c^4*e^3*x^4 + 32*(9*a^2 + 2*b^2)*c^4*d*e^2*x^3 + 27*
(8*(2*a^2 + b^2)*c^4*d^2*e - b^2*c^2*e^3)*x^2 + 9*(8*b^2*c^4*e^3*x^4 + 32*b
^2*c^4*d*e^2*x^3 + 48*b^2*c^4*d^2*e*x^2 + 32*b^2*c^4*d^3*x + 24*b^2*c^2*d^2
*e - 3*b^2*e^3)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 96*(3*(a^2 + 2*b^2)*c^4*d^
3 - 4*b^2*c^2*d*e^2)*x + 6*(24*a*b*c^4*e^3*x^4 + 96*a*b*c^4*d*e^2*x^3 + 144
*a*b*c^4*d^2*e*x^2 + 96*a*b*c^4*d^3*x + 72*a*b*c^2*d^2*e - 9*a*b*e^3 - (6*b
^2*c^3*e^3*x^3 + 32*b^2*c^3*d*e^2*x^2 + 96*b^2*c^3*d^3 - 64*b^2*c*d*e^2 + 9
*(8*b^2*c^3*d^2*e - b^2*c*e^3)*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2
 + 1)) - 6*(6*a*b*c^3*e^3*x^3 + 32*a*b*c^3*d*e^2*x^2 + 96*a*b*c^3*d^3 - 64*
a*b*c*d*e^2 + 9*(8*a*b*c^3*d^2*e - a*b*c*e^3)*x)*sqrt(c^2*x^2 + 1))/c^4
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [A] time = 0.12, size = 641, normalized size = 1.74

$$\frac{\frac{(cex+cd)^4a^2}{4c^3e}+\frac{b^2\left(c^3d^3\left(\operatorname{arcsinh}(cx)^2cx-2\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}+2cx\right)+\frac{3c^2d^2e\left(2\operatorname{arcsinh}(cx)^2c^2x^2-2\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}\,cx+\operatorname{arcsinh}(cx)^2+c^2x^2+1\right)}{4}+\frac{cde^2\left(9\operatorname{arcs}\cdots\right.}{\cdots}\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/c*(1/4*(c*e*x+c*d)^4*a^2/c^3/e+b^2/c^3*(c^3*d^3*(arcsinh(c*x)^2*c*x-2*arc
sinh(c*x)*(c^2*x^2+1)^(1/2)+2*c*x)+3/4*c^2*d^2*e*(2*arcsinh(c*x)^2*c^2*x^2-
2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c*x+arcsinh(c*x)^2+c^2*x^2+1)+1/9*c*d*e^2*
(9*arcsinh(c*x)^2*c^3*x^3-6*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^2*x^2+27*arcsi
nh(c*x)^2*c*x+2*c^3*x^3-42*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+42*c*x)+1/32*e^3*
(8*arcsinh(c*x)^2*c^4*x^4-4*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^3*x^3+16*arcsi
nh(c*x)^2*c^2*x^2+c^4*x^4-10*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c*x+5*arcsinh(c
```

```
*x)^2+5*c^2*x^2+4)-3*c*d*e^2*(arcsinh(c*x)^2*c*x-2*arcsinh(c*x)*(c^2*x^2+1)
^(1/2)+2*c*x)-1/4*e^3*(2*arcsinh(c*x)^2*c^2*x^2-2*arcsinh(c*x)*(c^2*x^2+1)^
(1/2)*c*x+arcsinh(c*x)^2+c^2*x^2+1))+2*a*b/c^3*(1/4*e^3*arcsinh(c*x)*c^4*x^
4+e^2*arcsinh(c*x)*c^4*x^3*d+3/2*e*arcsinh(c*x)*c^4*x^2*d^2+arcsinh(c*x)*c^
4*x*d^3+1/4/e*arcsinh(c*x)*c^4*d^4-1/4/e*(e^4*(1/4*c^3*x^3*(c^2*x^2+1)^(1/2
)-3/8*c*x*(c^2*x^2+1)^(1/2)+3/8*arcsinh(c*x))+4*c*d*e^3*(1/3*c^2*x^2*(c^2*x
^2+1)^(1/2)-2/3*(c^2*x^2+1)^(1/2))+6*c^2*d^2*e^2*(1/2*c*x*(c^2*x^2+1)^(1/2)
-1/2*arcsinh(c*x))+4*c^3*d^3*e*(c^2*x^2+1)^(1/2)+c^4*d^4*arcsinh(c*x))))
```

maxima [A] time = 0.50, size = 590, normalized size = 1.60

$$\frac{1}{4}b^2e^3x^4\operatorname{arsinh}(cx)^2+b^2de^2x^3\operatorname{arsinh}(cx)^2+\frac{1}{4}a^2e^3x^4+\frac{3}{2}b^2d^2ex^2\operatorname{arsinh}(cx)^2+a^2de^2x^3+b^2d^3x\operatorname{arsinh}(cx)^2+\frac{3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*e^3*x^4*arcsinh(c*x)^2 + b^2*d*e^2*x^3*arcsinh(c*x)^2 + 1/4*a^2*e^3
*x^4 + 3/2*b^2*d^2*e*x^2*arcsinh(c*x)^2 + a^2*d*e^2*x^3 + b^2*d^3*x*arcsinh
(c*x)^2 + 3/2*a^2*d^2*e*x^2 + 3/2*(2*x^2*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1
)*x/c^2 - arcsinh(c*x)/c^3))*a*b*d^2*e + 3/4*(c^2*(x^2/c^2 - log(c*x + sqrt
(c^2*x^2 + 1))^2/c^4) - 2*c*(sqrt(c^2*x^2 + 1)*x/c^2 - arcsinh(c*x)/c^3)*ar
csinh(c*x))*b^2*d^2*e + 2/3*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/
c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*a*b*d*e^2 - 2/9*(3*c*(sqrt(c^2*x^2 + 1)*x^2
/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4)*arcsinh(c*x) - (c^2*x^3 - 6*x)/c^2)*b^2*d*e
^2 + 1/16*(8*x^4*arcsinh(c*x) - (2*sqrt(c^2*x^2 + 1)*x^3/c^2 - 3*sqrt(c^2*x
^2 + 1)*x/c^4 + 3*arcsinh(c*x)/c^5)*c)*a*b*e^3 + 1/32*((x^4/c^2 - 3*x^2/c^4
 + 3*log(c*x + sqrt(c^2*x^2 + 1))^2/c^6)*c^2 - 2*(2*sqrt(c^2*x^2 + 1)*x^3/c
^2 - 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*arcsinh(c*x)/c^5)*c*arcsinh(c*x))*b^2*e^
3 + 2*b^2*d^3*(x - sqrt(c^2*x^2 + 1)*arcsinh(c*x)/c) + a^2*d^3*x + 2*(c*x*a
rcsinh(c*x) - sqrt(c^2*x^2 + 1))*a*b*d^3/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\operatorname{asinh}(c\,x))^2\,(d+e\,x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2*(d + e*x)^3,x)
```

```
[Out] int((a + b*asinh(c*x))^2*(d + e*x)^3, x)
```

sympy [A] time = 3.94, size = 743, normalized size = 2.02

$$\begin{cases} a^2d^3x+\frac{3a^2d^2ex^2}{2}+a^2de^2x^3+\frac{a^2e^3x^4}{4}+2abd^3x\operatorname{asinh}(cx)+3abd^2ex^2\operatorname{asinh}(cx)+2abde^2x^3\operatorname{asinh}(cx)+\frac{abe^3x^4}{} \\ a^2\left(d^3x+\frac{3d^2ex^2}{2}+de^2x^3+\frac{e^3x^4}{4}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*d**3*x + 3*a**2*d**2*e*x**2/2 + a**2*d*e**2*x**3 + a**2*e**
3*x**4/4 + 2*a*b*d**3*x*asinh(c*x) + 3*a*b*d**2*e*x**2*asinh(c*x) + 2*a*b*d
*e**2*x**3*asinh(c*x) + a*b*e**3*x**4*asinh(c*x)/2 - 2*a*b*d**3*sqrt(c**2*x
**2 + 1)/c - 3*a*b*d**2*e*x*sqrt(c**2*x**2 + 1)/(2*c) - 2*a*b*d*e**2*x**2*s
qrt(c**2*x**2 + 1)/(3*c) - a*b*e**3*x**3*sqrt(c**2*x**2 + 1)/(8*c) + 3*a*b*
```

```
d**2*e*asinh(c*x)/(2*c**2) + 4*a*b*d*e**2*sqrt(c**2*x**2 + 1)/(3*c**3) + 3*
a*b*e**3*x*sqrt(c**2*x**2 + 1)/(16*c**3) - 3*a*b*e**3*asinh(c*x)/(16*c**4)
+ b**2*d**3*x*asinh(c*x)**2 + 2*b**2*d**3*x + 3*b**2*d**2*e*x**2*asinh(c*x)
**2/2 + 3*b**2*d**2*e*x**2/4 + b**2*d*e**2*x**3*asinh(c*x)**2 + 2*b**2*d*e*
*2*x**3/9 + b**2*e**3*x**4*asinh(c*x)**2/4 + b**2*e**3*x**4/32 - 2*b**2*d**
3*sqrt(c**2*x**2 + 1)*asinh(c*x)/c - 3*b**2*d**2*e*x*sqrt(c**2*x**2 + 1)*as
inh(c*x)/(2*c) - 2*b**2*d*e**2*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(3*c) -
b**2*e**3*x**3*sqrt(c**2*x**2 + 1)*asinh(c*x)/(8*c) + 3*b**2*d**2*e*asinh(c
*x)**2/(4*c**2) - 4*b**2*d*e**2*x/(3*c**2) - 3*b**2*e**3*x**2/(32*c**2) + 4
*b**2*d*e**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(3*c**3) + 3*b**2*e**3*x*sqrt(c
**2*x**2 + 1)*asinh(c*x)/(16*c**3) - 3*b**2*e**3*asinh(c*x)**2/(32*c**4), N
e(c, 0)), (a**2*(d**3*x + 3*d**2*e*x**2/2 + d*e**2*x**3 + e**3*x**4/4), Tru
e))
```

3.13 $\int (d+ex)^2 \left(a+b\sinh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=239

$$-\frac{2bd^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{c}-\frac{bdex\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{c}+\frac{de\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2}-\frac{2be^2x^2\sqrt{c^2x^2}}{}$$

```
[Out] 2*b^2*d^2*x-4/9*b^2*e^2*x/c^2+1/2*b^2*d*e*x^2+2/27*b^2*e^2*x^3-1/3*d^3*(a+b
*arcsinh(c*x))^2/e+1/2*d*e*(a+b*arcsinh(c*x))^2/c^2+1/3*(e*x+d)^3*(a+b*arcs
inh(c*x))^2/e-2*b*d^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c+4/9*b*e^2*(a+b
*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3-b*d*e*x*(a+b*arcsinh(c*x))*(c^2*x^2+1)
^(1/2)/c-2/9*b*e^2*x^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

Rubi [A] time = 0.51, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {5801, 5821, 5675, 5717, 8, 5758, 30}

$$-\frac{2bd^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{c}-\frac{bdex\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{c}+\frac{de\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2}+\frac{4be^2\sqrt{c^2x^2+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] 2*b^2*d^2*x - (4*b^2*e^2*x)/(9*c^2) + (b^2*d*e*x^2)/2 + (2*b^2*e^2*x^3)/27
- (2*b*d^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/c + (4*b*e^2*Sqrt[1 + c^
2*x^2]*(a + b*ArcSinh[c*x]))/(9*c^3) - (b*d*e*x*Sqrt[1 + c^2*x^2]*(a + b*Ar
cSinh[c*x]))/c - (2*b*e^2*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(9*c)
 - (d^3*(a + b*ArcSinh[c*x])^2)/(3*e) + (d*e*(a + b*ArcSinh[c*x])^2)/(2*c^2
) + ((d + e*x)^3*(a + b*ArcSinh[c*x])^2)/(3*e)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
```

```
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5801

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n
 - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\int (d+ex)^2\left(a+b\sinh^{-1}(cx)\right)^2\,dx=\frac{(d+ex)^3\left(a+b\sinh^{-1}(cx)\right)^2}{3e}-\frac{(2bc)\int\frac{(d+ex)^3\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\,dx}{3e}$$

$$=\frac{(d+ex)^3\left(a+b\sinh^{-1}(cx)\right)^2}{3e}-\frac{(2bc)\int\left(\frac{d^3\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}+\frac{3d^2ex\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\right.}{}$$

$$=\frac{(d+ex)^3\left(a+b\sinh^{-1}(cx)\right)^2}{3e}-\left(2bcd^2\right)\int\frac{x\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\,dx-\left(2bcd^3\right.$$

$$=-\frac{2bd^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c}-\frac{bdex\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c}-$$

$$=2b^2d^2x+\frac{1}{2}b^2dex^2+\frac{2}{27}b^2e^2x^3-\frac{2bd^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c}+\frac{4be^2}{}$$

$$=2b^2d^2x-\frac{4b^2e^2x}{9c^2}+\frac{1}{2}b^2dex^2+\frac{2}{27}b^2e^2x^3-\frac{2bd^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx\right.}{c}$$

Mathematica [A] time = 0.38, size = 248, normalized size = 1.04

$$18a^2c^3x\left(3d^2+3dex+e^2x^2\right)-6ab\sqrt{c^2x^2+1}\left(c^2\left(18d^2+9dex+2e^2x^2\right)-4e^2\right)-6b\sinh^{-1}(cx)\left(b\sqrt{c^2x^2+1}\left(c^2\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (18*a^2*c^3*x*(3*d^2 + 3*d*e*x + e^2*x^2) - 6*a*b*Sqrt[1 + c^2*x^2]*(-4*e^2
 + c^2*(18*d^2 + 9*d*e*x + 2*e^2*x^2)) + b^2*c*x*(-24*e^2 + c^2*(108*d^2 +
27*d*e*x + 4*e^2*x^2)) - 6*b*(-3*a*(3*c*d*e + 2*c^3*x*(3*d^2 + 3*d*e*x + e^
```

```
2*x^2)) + b*Sqrt[1 + c^2*x^2]*(-4*e^2 + c^2*(18*d^2 + 9*d*e*x + 2*e^2*x^2))
)*ArcSinh[c*x] + 9*b^2*c*(6*c^2*d^2*x + 2*c^2*e^2*x^3 + 3*d*(e + 2*c^2*e*x^
2))*ArcSinh[c*x]^2)/(54*c^3)
```

fricas [A] time = 0.61, size = 319, normalized size = 1.33

$$\frac{2\left(9a^2+2b^2\right)c^3e^2x^3+27\left(2a^2+b^2\right)c^3dex^2+9\left(2b^2c^3e^2x^3+6b^2c^3dex^2+6b^2c^3d^2x+3b^2cde\right)\log\left(cx+\sqrt{c^2\cdots}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/54*(2*(9*a^2 + 2*b^2)*c^3*e^2*x^3 + 27*(2*a^2 + b^2)*c^3*d*e*x^2 + 9*(2*b
^2*c^3*e^2*x^3 + 6*b^2*c^3*d*e*x^2 + 6*b^2*c^3*d^2*x + 3*b^2*c*d*e)*log(c*x
 + sqrt(c^2*x^2 + 1))^2 + 6*(9*(a^2 + 2*b^2)*c^3*d^2 - 4*b^2*c*e^2)*x + 6*(
6*a*b*c^3*e^2*x^3 + 18*a*b*c^3*d*e*x^2 + 18*a*b*c^3*d^2*x + 9*a*b*c*d*e - (
2*b^2*c^2*e^2*x^2 + 9*b^2*c^2*d*e*x + 18*b^2*c^2*d^2 - 4*b^2*e^2)*sqrt(c^2*
x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) - 6*(2*a*b*c^2*e^2*x^2 + 9*a*b*c^2*d
*e*x + 18*a*b*c^2*d^2 - 4*a*b*e^2)*sqrt(c^2*x^2 + 1))/c^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [A] time = 0.09, size = 410, normalized size = 1.72

$$\frac{\frac{(cex+cd)^3a^2}{3c^2e}+\frac{b^2\left(c^2d^2\left(\operatorname{arcsinh}(cx)^2cx-2\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}+2cx\right)+\frac{cde\left(2\operatorname{arcsinh}(cx)^2c^2x^2-2\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}\,cx+\operatorname{arcsinh}(cx)^2+c^2x^2+1\right)}{2}+\frac{e^2\left(9\operatorname{arcsi}\cdots\right.}{\cdots}\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/c*(1/3*(c*e*x+c*d)^3*a^2/c^2/e+b^2/c^2*(c^2*d^2*(arcsinh(c*x)^2*c*x-2*arc
sinh(c*x)*(c^2*x^2+1)^(1/2)+2*c*x)+1/2*c*d*e*(2*arcsinh(c*x)^2*c^2*x^2-2*ar
csinh(c*x)*(c^2*x^2+1)^(1/2)*c*x+arcsinh(c*x)^2+c^2*x^2+1)+1/27*e^2*(9*arcs
inh(c*x)^2*c^3*x^3-6*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^2*x^2+27*arcsinh(c*x)
^2*c*x+2*c^3*x^3-42*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+42*c*x)-e^2*(arcsinh(c*x
)^2*c*x-2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+2*c*x))+2*a*b/c^2*(1/3*e^2*arcsinh
(c*x)*c^3*x^3+e*arcsinh(c*x)*c^3*x^2*d+arcsinh(c*x)*c^3*x*d^2+1/3/e*arcsinh
(c*x)*c^3*d^3-1/3/e*(e^3*(1/3*c^2*x^2*(c^2*x^2+1)^(1/2)-2/3*(c^2*x^2+1)^(1/
2))+3*c*d*e^2*(1/2*c*x*(c^2*x^2+1)^(1/2)-1/2*arcsinh(c*x))+3*c^2*d^2*e*(c^2
*x^2+1)^(1/2)+c^3*d^3*arcsinh(c*x))))
```

maxima [A] time = 0.43, size = 378, normalized size = 1.58

$$\frac{1}{3}b^2e^2x^3\operatorname{arsinh}(cx)^2+b^2dex^2\operatorname{arsinh}(cx)^2+\frac{1}{3}a^2e^2x^3+b^2d^2x\operatorname{arsinh}(cx)^2+a^2dex^2+\left(2x^2\operatorname{arsinh}(cx)-c\left(\frac{\sqrt{c^2x^2\cdots}}{c\cdots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*e^2*x^3*arcsinh(c*x)^2 + b^2*d*e*x^2*arcsinh(c*x)^2 + 1/3*a^2*e^2*x
^3 + b^2*d^2*x*arcsinh(c*x)^2 + a^2*d*e*x^2 + (2*x^2*arcsinh(c*x) - c*(sqrt
(c^2*x^2 + 1)*x/c^2 - arcsinh(c*x)/c^3))*a*b*d*e + 1/2*(c^2*(x^2/c^2 - log(
c*x + sqrt(c^2*x^2 + 1))^2/c^4) - 2*c*(sqrt(c^2*x^2 + 1)*x/c^2 - arcsinh(c*
x)/c^3)*arcsinh(c*x))*b^2*d*e + 2/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 +
 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*a*b*e^2 - 2/27*(3*c*(sqrt(c^2*x^2 +
 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4)*arcsinh(c*x) - (c^2*x^3 - 6*x)/c^2)*
b^2*e^2 + 2*b^2*d^2*(x - sqrt(c^2*x^2 + 1)*arcsinh(c*x)/c) + a^2*d^2*x + 2*
(c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*a*b*d^2/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b\,\mathrm{asinh}\,(c\,x))^2\,(d + e\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2*(d + e*x)^2,x)
```

```
[Out] int((a + b*asinh(c*x))^2*(d + e*x)^2, x)
```

sympy [A] time = 1.72, size = 454, normalized size = 1.90

$$\begin{cases} a^2d^2x + a^2dex^2 + \frac{a^2e^2x^3}{3} + 2abd^2x\,\mathrm{asinh}\,(cx) + 2abdex^2\,\mathrm{asinh}\,(cx) + \frac{2abe^2x^3\,\mathrm{asinh}\,(cx)}{3} - \frac{2abd^2\sqrt{c^2x^2+1}}{c} - \frac{abdex\sqrt{c^2x^2+1}}{c} \\ a^2\left(d^2x + dex^2 + \frac{e^2x^3}{3}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*d**2*x + a**2*d*e*x**2 + a**2*e**2*x**3/3 + 2*a*b*d**2*x*as
inh(c*x) + 2*a*b*d*e*x**2*asinh(c*x) + 2*a*b*e**2*x**3*asinh(c*x)/3 - 2*a*b
*d**2*sqrt(c**2*x**2 + 1)/c - a*b*d*e*x*sqrt(c**2*x**2 + 1)/c - 2*a*b*e**2*
x**2*sqrt(c**2*x**2 + 1)/(9*c) + a*b*d*e*asinh(c*x)/c**2 + 4*a*b*e**2*sqrt(
c**2*x**2 + 1)/(9*c**3) + b**2*d**2*x*asinh(c*x)**2 + 2*b**2*d**2*x + b**2*
d*e*x**2*asinh(c*x)**2 + b**2*d*e*x**2/2 + b**2*e**2*x**3*asinh(c*x)**2/3 +
 2*b**2*e**2*x**3/27 - 2*b**2*d**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/c - b**2*
d*e*x*sqrt(c**2*x**2 + 1)*asinh(c*x)/c - 2*b**2*e**2*x**2*sqrt(c**2*x**2 +
1)*asinh(c*x)/(9*c) + b**2*d*e*asinh(c*x)**2/(2*c**2) - 4*b**2*e**2*x/(9*c*
*2) + 4*b**2*e**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(9*c**3), Ne(c, 0)), (a**2
*(d**2*x + d*e*x**2 + e**2*x**3/3), True))
```

3.14 $\int (d+ex)\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=140

$$-\frac{2bd\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{c}-\frac{bex\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{2c}+\frac{e\left(a+b\sinh^{-1}(cx)\right)^2}{4c^2}-\frac{d^2\left(a+b\sinh^{-1}\right.}{2e}$$

```
[Out] 2*b^2*d*x+1/4*b^2*e*x^2-1/2*d^2*(a+b*arcsinh(c*x))^2/e+1/4*e*(a+b*arcsinh(c
*x))^2/c^2+1/2*(e*x+d)^2*(a+b*arcsinh(c*x))^2/e-2*b*d*(a+b*arcsinh(c*x))*(c
^2*x^2+1)^(1/2)/c-1/2*b*e*x*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

Rubi [A] time = 0.32, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5801, 5821, 5675, 5717, 8, 5758, 30}

$$-\frac{2bd\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{c}-\frac{bex\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{2c}+\frac{e\left(a+b\sinh^{-1}(cx)\right)^2}{4c^2}-\frac{d^2\left(a+b\sinh^{-1}\right.}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] 2*b^2*d*x + (b^2*e*x^2)/4 - (2*b*d*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/
c - (b*e*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(2*c) - (d^2*(a + b*ArcS
inh[c*x])^2)/(2*e) + (e*(a + b*ArcSinh[c*x])^2)/(4*c^2) + ((d + e*x)^2*(a +
 b*ArcSinh[c*x])^2)/(2*e)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

```
&& GtQ[m, 1] && IntegerQ[m]
```

Rule 5801

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n
 - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\int (d+ex)\left(a+b\sinh^{-1}(cx)\right)^2\,dx = \frac{(d+ex)^2\left(a+b\sinh^{-1}(cx)\right)^2}{2e} - \frac{(bc)\int \frac{(d+ex)^2\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\,dx}{e}$$

$$= \frac{(d+ex)^2\left(a+b\sinh^{-1}(cx)\right)^2}{2e} - \frac{(bc)\int \left(\frac{d^2\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}} + \frac{2dex\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}} + \right.}{e}$$

$$= \frac{(d+ex)^2\left(a+b\sinh^{-1}(cx)\right)^2}{2e} - (2bcd)\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\,dx - \frac{\left(bcd^2\right)\int}{}$$

$$= -\frac{2bd\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c} - \frac{bex\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{2c} - \frac{d^2(}{}$$

$$= 2b^2dx + \frac{1}{4}b^2ex^2 - \frac{2bd\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c} - \frac{bex\sqrt{1+c^2x^2}\left(a+b\,\mathrm{s}\right.}{2c}$$

Mathematica [A] time = 0.34, size = 142, normalized size = 1.01

$$\frac{c\left(2a^2cx(2d+ex) - 2ab\sqrt{c^2x^2+1}\,(4d+ex) + b^2cx(8d+ex)\right) + 2b\sinh^{-1}(cx)\left(a\left(4c^2dx+2c^2ex^2+e\right) - bc\sqrt{c^2x^2}\right.}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (c*(2*a^2*c*x*(2*d + e*x) + b^2*c*x*(8*d + e*x) - 2*a*b*(4*d + e*x)*Sqrt[1
+ c^2*x^2]) + 2*b*(-(b*c*(4*d + e*x)*Sqrt[1 + c^2*x^2]) + a*(e + 4*c^2*d*x
+ 2*c^2*e*x^2))*ArcSinh[c*x] + b^2*(e + 4*c^2*d*x + 2*c^2*e*x^2)*ArcSinh[c*
x]^2)/(4*c^2)
```

fricas [A] time = 0.63, size = 183, normalized size = 1.31

$$\frac{\left(2\,a^2+b^2\right)c^2ex^2 + 4\left(a^2+2\,b^2\right)c^2dx + \left(2\,b^2c^2ex^2 + 4\,b^2c^2dx + b^2e\right)\log\left(cx+\sqrt{c^2x^2+1}\right)^2 + 2\left(2\,abc^2ex^2 + 4\,abc\right.}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/4*((2*a^2 + b^2)*c^2*e*x^2 + 4*(a^2 + 2*b^2)*c^2*d*x + (2*b^2*c^2*e*x^2 +
 4*b^2*c^2*d*x + b^2*e)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 2*(2*a*b*c^2*e*x^2
 + 4*a*b*c^2*d*x + a*b*e - (b^2*c*e*x + 4*b^2*c*d)*sqrt(c^2*x^2 + 1))*log(c
*x + sqrt(c^2*x^2 + 1)) - 2*(a*b*c*e*x + 4*a*b*c*d)*sqrt(c^2*x^2 + 1))/c^2
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [A] time = 0.07, size = 193, normalized size = 1.38

$$\frac{\frac{a^2\left(\frac{1}{2}c^2x^2e+c^2dx\right)}{c}+\frac{b^2\left(\frac{e\left(2\operatorname{arcsinh}(cx)^2c^2x^2-2\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}\,cx+\operatorname{arcsinh}(cx)^2+c^2x^2+1\right)}{4}+cd\left(\operatorname{arcsinh}(cx)^2cx-2\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}+2cx\right)\right)}{c}+\frac{2ab}{}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/c*(a^2/c*(1/2*c^2*x^2*e+c^2*d*x)+b^2/c*(1/4*e*(2*arcsinh(c*x)^2*c^2*x^2-2
*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c*x+arcsinh(c*x)^2+c^2*x^2+1)+c*d*(arcsinh(
c*x)^2*c*x-2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+2*c*x))+2*a*b/c*(1/2*arcsinh(c*
x)*c^2*x^2*e+arcsinh(c*x)*c^2*x*d-1/2*e*(1/2*c*x*(c^2*x^2+1)^(1/2)-1/2*arcs
inh(c*x))-c*d*(c^2*x^2+1)^(1/2)))
```

maxima [A] time = 0.37, size = 219, normalized size = 1.56

$$\frac{1}{2}b^2ex^2\operatorname{arsinh}(cx)^2+b^2dx\operatorname{arsinh}(cx)^2+\frac{1}{2}a^2ex^2+\frac{1}{2}\left(2x^2\operatorname{arsinh}(cx)-c\left(\frac{\sqrt{c^2x^2+1}\,x}{c^2}-\frac{\operatorname{arsinh}(cx)}{c^3}\right)\right)abe+\frac{1}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*e*x^2*arcsinh(c*x)^2 + b^2*d*x*arcsinh(c*x)^2 + 1/2*a^2*e*x^2 + 1/2
*(2*x^2*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x/c^2 - arcsinh(c*x)/c^3))*a*b*
e + 1/4*(c^2*(x^2/c^2 - log(c*x + sqrt(c^2*x^2 + 1))^2/c^4) - 2*c*(sqrt(c^2
*x^2 + 1)*x/c^2 - arcsinh(c*x)/c^3)*arcsinh(c*x))*b^2*e + 2*b^2*d*(x - sqrt
(c^2*x^2 + 1)*arcsinh(c*x)/c) + a^2*d*x + 2*(c*x*arcsinh(c*x) - sqrt(c^2*x^
2 + 1))*a*b*d/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\operatorname{asinh}(c\,x))^2\,(d+e\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2*(d + e*x),x)
```

```
[Out] int((a + b*asinh(c*x))^2*(d + e*x), x)
```

sympy [A] time = 0.85, size = 233, normalized size = 1.66

$$\begin{cases} a^2dx + \frac{a^2ex^2}{2} + 2abdx\operatorname{asinh}(cx) + abex^2\operatorname{asinh}(cx) - \frac{2abd\sqrt{c^2x^2+1}}{c} - \frac{abex\sqrt{c^2x^2+1}}{2c} + \frac{abe\operatorname{asinh}(cx)}{2c^2} + b^2dx\operatorname{asinh}^2(cx) \\ a^2\left(dx + \frac{ex^2}{2}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*d*x + a**2*e*x**2/2 + 2*a*b*d*x*asinh(c*x) + a*b*e*x**2*asi
nh(c*x) - 2*a*b*d*sqrt(c**2*x**2 + 1)/c - a*b*e*x*sqrt(c**2*x**2 + 1)/(2*c)
 + a*b*e*asinh(c*x)/(2*c**2) + b**2*d*x*asinh(c*x)**2 + 2*b**2*d*x + b**2*e
*x**2*asinh(c*x)**2/2 + b**2*e*x**2/4 - 2*b**2*d*sqrt(c**2*x**2 + 1)*asinh(
c*x)/c - b**2*e*x*sqrt(c**2*x**2 + 1)*asinh(c*x)/(2*c) + b**2*e*asinh(c*x)*
*2/(4*c**2), Ne(c, 0)), (a**2*(d*x + e*x**2/2), True))
```

3.15 $\int \left(a+b \sinh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=46

$$-\frac{2b\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{c}+x\left(a+b\sinh^{-1}(cx)\right)^2+2b^2x$$

```
[Out] 2*b^2*x+x*(a+b*arcsinh(c*x))^2-2*b*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

Rubi [A] time = 0.06, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5653, 5717, 8}

$$-\frac{2b\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{c}+x\left(a+b\sinh^{-1}(cx)\right)^2+2b^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] 2*b^2*x - (2*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/c + x*(a + b*ArcSinh
[c*x])^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sinh^{-1}(cx)\right)^2 dx &= x\left(a+b\sinh^{-1}(cx)\right)^2-(2bc)\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\,dx\\ &= -\frac{2b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c}+x\left(a+b\sinh^{-1}(cx)\right)^2+\left(2b^2\right)\int 1\,dx\\ &= 2b^2x-\frac{2b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c}+x\left(a+b\sinh^{-1}(cx)\right)^2\end{aligned}$$

Mathematica [A] time = 0.06, size = 74, normalized size = 1.61

$$x\left(a^2+2b^2\right)-\frac{2ab\sqrt{c^2x^2+1}}{c}+\frac{2b\sinh^{-1}(cx)\left(acx-b\sqrt{c^2x^2+1}\right)}{c}+b^2x\sinh^{-1}(cx)^2$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (a^2 + 2*b^2)*x - (2*a*b*Sqrt[1 + c^2*x^2])/c + (2*b*(a*c*x - b*Sqrt[1 + c^
2*x^2])*ArcSinh[c*x])/c + b^2*x*ArcSinh[c*x]^2
```

fricas [B] time = 0.45, size = 96, normalized size = 2.09

$$\frac{b^2cx\log\left(cx+\sqrt{c^2x^2+1}\right)^2+\left(a^2+2\,b^2\right)cx-2\,\sqrt{c^2x^2+1}\,ab+2\left(abcx-\sqrt{c^2x^2+1}\,b^2\right)\log\left(cx+\sqrt{c^2x^2+1}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] (b^2*c*x*log(c*x + sqrt(c^2*x^2 + 1))^2 + (a^2 + 2*b^2)*c*x - 2*sqrt(c^2*x^
2 + 1)*a*b + 2*(a*b*c*x - sqrt(c^2*x^2 + 1)*b^2)*log(c*x + sqrt(c^2*x^2 + 1
)))/c
```

giac [B] time = 0.78, size = 111, normalized size = 2.41

$$2\left(x\log\left(cx+\sqrt{c^2x^2+1}\right)-\frac{\sqrt{c^2x^2+1}}{c}\right)ab+\left(x\log\left(cx+\sqrt{c^2x^2+1}\right)^2+2\,c\left(\frac{x}{c}-\frac{\sqrt{c^2x^2+1}\,\log\left(cx+\sqrt{c^2x^2+1}\right)}{c^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] 2*(x*log(c*x + sqrt(c^2*x^2 + 1)) - sqrt(c^2*x^2 + 1)/c)*a*b + (x*log(c*x +
 sqrt(c^2*x^2 + 1))^2 + 2*c*(x/c - sqrt(c^2*x^2 + 1)*log(c*x + sqrt(c^2*x^2
 + 1))/c^2))*b^2 + a^2*x
```

maple [A] time = 0.00, size = 72, normalized size = 1.57

$$\frac{a^2cx+b^2\left(\operatorname{arcsinh}\left(cx\right)^2cx-2\operatorname{arcsinh}\left(cx\right)\sqrt{c^2x^2+1}\,+2cx\right)+2ab\left(\operatorname{arcsinh}\left(cx\right)cx-\sqrt{c^2x^2+1}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/c*(a^2*c*x+b^2*(arcsinh(c*x)^2*c*x-2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+2*c*x
)+2*a*b*(arcsinh(c*x)*c*x-(c^2*x^2+1)^(1/2)))
```

maxima [A] time = 0.33, size = 72, normalized size = 1.57

$$b^2x\operatorname{arsinh}\left(cx\right)^2+2\,b^2\left(x-\frac{\sqrt{c^2x^2+1}\,\operatorname{arsinh}\left(cx\right)}{c}\right)+a^2x+\frac{2\left(cx\operatorname{arsinh}\left(cx\right)-\sqrt{c^2x^2+1}\right)ab}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] b^2*x*arcsinh(c*x)^2 + 2*b^2*(x - sqrt(c^2*x^2 + 1)*arcsinh(c*x)/c) + a^2*x
 + 2*(c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*a*b/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2,x)
```

```
[Out] int((a + b*asinh(c*x))^2, x)
```

sympy [A] time = 0.28, size = 82, normalized size = 1.78

$$\begin{cases} a^2x + 2abx\operatorname{asinh}(cx) - \frac{2ab\sqrt{c^2x^2+1}}{c} + b^2x\operatorname{asinh}^2(cx) + 2b^2x - \frac{2b^2\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{c} & \text{for } c \neq 0 \\ a^2x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*x + 2*a*b*x*asinh(c*x) - 2*a*b*sqrt(c**2*x**2 + 1)/c + b**2
*x*asinh(c*x)**2 + 2*b**2*x - 2*b**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/c, Ne(c
, 0)), (a**2*x, True))
```

3.16 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d+ex}\,dx$

Optimal. Leaf size=291

$$\frac{2b\left(a+b\sinh^{-1}(cx)\right)\text{Li}_2\left(-\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{2b\left(a+b\sinh^{-1}(cx)\right)\text{Li}_2\left(-\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(\frac{ee^{s}}{cd-}\right.}{e}$$

```
[Out] -1/3*(a+b*arcsinh(c*x))^3/b/e+(a+b*arcsinh(c*x))^2*ln(1+e*(c*x+(c^2*x^2+1)^
(1/2))/(c*d-(c^2*d^2+e^2)^(1/2)))/e+(a+b*arcsinh(c*x))^2*ln(1+e*(c*x+(c^2*x
^2+1)^(1/2))/(c*d+(c^2*d^2+e^2)^(1/2)))/e+2*b*(a+b*arcsinh(c*x))*polylog(2,
-e*(c*x+(c^2*x^2+1)^(1/2))/(c*d-(c^2*d^2+e^2)^(1/2)))/e+2*b*(a+b*arcsinh(c*
x))*polylog(2,-e*(c*x+(c^2*x^2+1)^(1/2))/(c*d+(c^2*d^2+e^2)^(1/2)))/e-2*b^2
*polylog(3,-e*(c*x+(c^2*x^2+1)^(1/2))/(c*d-(c^2*d^2+e^2)^(1/2)))/e-2*b^2*po
lylog(3,-e*(c*x+(c^2*x^2+1)^(1/2))/(c*d+(c^2*d^2+e^2)^(1/2)))/e
```

Rubi [A] time = 0.47, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5799, 5561, 2190, 2531, 2282, 6589}

$$\frac{2b\left(a+b\sinh^{-1}(cx)\right)\text{PolyLog}\left(2,-\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{2b\left(a+b\sinh^{-1}(cx)\right)\text{PolyLog}\left(2,-\frac{ee^{\sinh^{-1}(cx)}}{\sqrt{c^2d^2+e^2}+cd}\right)}{e}-\frac{2b^2\text{PolyLog}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(d + e*x),x]
```

```
[Out] -(a + b*ArcSinh[c*x])^3/(3*b*e) + ((a + b*ArcSinh[c*x])^2*Log[1 + (e*E^ArcS
inh[c*x])/(c*d - Sqrt[c^2*d^2 + e^2])])/e + ((a + b*ArcSinh[c*x])^2*Log[1 +
 (e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2])])/e + (2*b*(a + b*ArcSinh[c
*x])*PolyLog[2, -((e*E^ArcSinh[c*x])/(c*d - Sqrt[c^2*d^2 + e^2]))])/e + (2*
b*(a + b*ArcSinh[c*x])*PolyLog[2, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2
+ e^2]))])/e - (2*b^2*PolyLog[3, -((e*E^ArcSinh[c*x])/(c*d - Sqrt[c^2*d^2 +
 e^2]))])/e - (2*b^2*PolyLog[3, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 +
e^2]))])/e
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Cosh[x])/(c*d + e*Sinh[x]), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d+ex}\,dx = \text{Subst}\left(\int \frac{(a+bx)^2\cosh(x)}{cd+e\sinh(x)}\,dx, x, \sinh^{-1}(cx)\right)$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^3}{3be} + \text{Subst}\left(\int \frac{e^x(a+bx)^2}{cd-\sqrt{c^2d^2+e^2}+ee^x}\,dx, x, \sinh^{-1}(cx)\right) + \text{Sul}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^3}{3be} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\left(a+b\sinh^{-1}\right.}{}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^3}{3be} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\left(a+b\sinh^{-1}\right.}{}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^3}{3be} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\left(a+b\sinh^{-1}\right.}{}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^3}{3be} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\left(a+b\sinh^{-1}\right.}{}$$

Mathematica [A] time = 0.22, size = 273, normalized size = 0.94

$$\frac{6b\left(a+b\sinh^{-1}(cx)\right)\text{Li}_2\left(\frac{ee^{\sinh^{-1}(cx)}}{\sqrt{c^2d^2+e^2}-cd}\right)+6b\left(a+b\sinh^{-1}(cx)\right)\text{Li}_2\left(-\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)+3\left(a+b\sinh^{-1}(cx)\right)^2\log}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(d + e*x),x]
```

```
[Out] (-((a + b*ArcSinh[c*x])^3/b) + 3*(a + b*ArcSinh[c*x])^2*Log[1 + (e*E^ArcSin
h[c*x])/(c*d - Sqrt[c^2*d^2 + e^2])] + 3*(a + b*ArcSinh[c*x])^2*Log[1 + (e*
```

```
E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2])] + 6*b*(a + b*ArcSinh[c*x])*Pol
yLog[2, (e*E^ArcSinh[c*x])/(-(c*d) + Sqrt[c^2*d^2 + e^2])] + 6*b*(a + b*Arc
Sinh[c*x])*PolyLog[2, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))] -
6*b^2*PolyLog[3, (e*E^ArcSinh[c*x])/(-(c*d) + Sqrt[c^2*d^2 + e^2])] - 6*b^2
*PolyLog[3, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))])/(3*e)
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{b^2 \operatorname{arsinh}(cx)^2 + 2\,ab \operatorname{arsinh}(cx) + a^2}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(e*x + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arsinh}(cx) + a)^2}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/(e*x + d), x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{arcsinh}(cx))^2}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(e*x+d),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/(e*x+d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^2 \log(ex+d)}{e} + \int \frac{b^2 \log\left(cx + \sqrt{c^2x^2+1}\right)^2}{ex+d} + \frac{2\,ab \log\left(cx + \sqrt{c^2x^2+1}\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] a^2*log(e*x + d)/e + integrate(b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(e*x + d)
 + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(e*x + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b \operatorname{asinh}(c\,x))^2}{d + e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(d + e*x),x)
```

```
[Out] int((a + b*asinh(c*x))^2/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{asinh}(cx))^2}{d + ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(e*x+d),x)
```

```
[Out] Integral((a + b*asinh(c*x))**2/(d + e*x), x)
```

3.17 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{(d+ex)^2}\,dx$

Optimal. Leaf size=263

$$\frac{2bc\left(a+b\sinh^{-1}(cx)\right)\log\left(\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e\sqrt{c^2d^2+e^2}}-\frac{2bc\left(a+b\sinh^{-1}(cx)\right)\log\left(\frac{ee^{\sinh^{-1}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)}{e\sqrt{c^2d^2+e^2}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{e(d+ex)}+$$

```
[Out] -(a+b*arcsinh(c*x))^2/e/(e*x+d)+2*b*c*(a+b*arcsinh(c*x))*ln(1+e*(c*x+(c^2*x
^2+1)^(1/2))/(c*d-(c^2*d^2+e^2)^(1/2)))/e/(c^2*d^2+e^2)^(1/2)-2*b*c*(a+b*ar
csinh(c*x))*ln(1+e*(c*x+(c^2*x^2+1)^(1/2))/(c*d+(c^2*d^2+e^2)^(1/2)))/e/(c^
2*d^2+e^2)^(1/2)+2*b^2*c*polylog(2,-e*(c*x+(c^2*x^2+1)^(1/2))/(c*d-(c^2*d^2
+e^2)^(1/2)))/e/(c^2*d^2+e^2)^(1/2)-2*b^2*c*polylog(2,-e*(c*x+(c^2*x^2+1)^(
1/2))/(c*d+(c^2*d^2+e^2)^(1/2)))/e/(c^2*d^2+e^2)^(1/2)
```

Rubi [A] time = 0.47, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {5801, 5831, 3322, 2264, 2190, 2279, 2391}

$$\frac{2b^2c\text{PolyLog}\left(2,-\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e\sqrt{c^2d^2+e^2}}-\frac{2b^2c\text{PolyLog}\left(2,-\frac{ee^{\sinh^{-1}(cx)}}{\sqrt{c^2d^2+e^2}+cd}\right)}{e\sqrt{c^2d^2+e^2}}+\frac{2bc\left(a+b\sinh^{-1}(cx)\right)\log\left(\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e\sqrt{c^2d^2+e^2}}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(d + e*x)^2,x]
```

```
[Out] -((a + b*ArcSinh[c*x])^2/(e*(d + e*x))) + (2*b*c*(a + b*ArcSinh[c*x])*Log[1
 + (e*E^ArcSinh[c*x])/(c*d - Sqrt[c^2*d^2 + e^2])])/(e*Sqrt[c^2*d^2 + e^2])
 - (2*b*c*(a + b*ArcSinh[c*x])*Log[1 + (e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d
^2 + e^2])])/(e*Sqrt[c^2*d^2 + e^2]) + (2*b^2*c*PolyLog[2, -((e*E^ArcSinh[c
*x])/(c*d - Sqrt[c^2*d^2 + e^2]))])/(e*Sqrt[c^2*d^2 + e^2]) - (2*b^2*c*Poly
Log[2, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))])/(e*Sqrt[c^2*d^2
+ e^2])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5801

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n
 - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5831

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{(d+ex)^2}\,dx = -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{e(d+ex)} + \frac{(2bc)\int \frac{a+b\sinh^{-1}(cx)}{(d+ex)\sqrt{1+c^2x^2}}\,dx}{e}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{e(d+ex)} + \frac{(2bc)\,\mathrm{Subst}\left(\int \frac{a+bx}{cd+e\sinh(x)}\,dx, x, \sinh^{-1}(cx)\right)}{e}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{e(d+ex)} + \frac{(4bc)\,\mathrm{Subst}\left(\int \frac{e^x(a+bx)}{-e+2cde^x+ee^{2x}}\,dx, x, \sinh^{-1}(cx)\right)}{e}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{e(d+ex)} + \frac{(4bc)\,\mathrm{Subst}\left(\int \frac{e^x(a+bx)}{2cd-2\sqrt{c^2d^2+e^2}+2ee^x}\,dx, x, \sinh^{-1}(cx)\right)}{\sqrt{c^2d^2+e^2}} - (4b$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{e(d+ex)} + \frac{2bc\left(a+b\sinh^{-1}(cx)\right)\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e\sqrt{c^2d^2+e^2}} - \frac{2bc\left(a+b\right.}{}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{e(d+ex)} + \frac{2bc\left(a+b\sinh^{-1}(cx)\right)\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e\sqrt{c^2d^2+e^2}} - \frac{2bc\left(a+b\right.}{}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{e(d+ex)} + \frac{2bc\left(a+b\sinh^{-1}(cx)\right)\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e\sqrt{c^2d^2+e^2}} - \frac{2bc\left(a+b\right.}{}$$

Mathematica [A] time = 0.22, size = 191, normalized size = 0.73

$$\frac{\frac{2bc\left(\left(a+b\sinh^{-1}(cx)\right)\left(\log\left(\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)-\log\left(\frac{ee^{\sinh^{-1}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)\right)+b\text{Li}_2\left(\frac{ee^{\sinh^{-1}(cx)}}{\sqrt{c^2d^2+e^2}-cd}\right)-b\text{Li}_2\left(-\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)\right)}{\sqrt{c^2d^2+e^2}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d+ex}}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(d + e*x)^2,x]
```

```
[Out] (-((a + b*ArcSinh[c*x])^2/(d + e*x)) + (2*b*c*((a + b*ArcSinh[c*x])*(Log[1
+ (e*E^ArcSinh[c*x])/(c*d - Sqrt[c^2*d^2 + e^2])] - Log[1 + (e*E^ArcSinh[c*
x])/(c*d + Sqrt[c^2*d^2 + e^2])]) + b*PolyLog[2, (e*E^ArcSinh[c*x])/(-(c*d)
 + Sqrt[c^2*d^2 + e^2])] - b*PolyLog[2, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^
2*d^2 + e^2]))]))/Sqrt[c^2*d^2 + e^2])/e
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2 \operatorname{arsinh}(cx)^2 + 2\,ab \operatorname{arsinh}(cx) + a^2}{e^2x^2 + 2\,dex + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(e^2*x^2 + 2*d*e*x
 + d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arsinh}(cx) + a)^2}{(ex + d)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/(e*x + d)^2, x)
```

maple [A] time = 0.34, size = 529, normalized size = 2.01

$$-\frac{c\,a^2}{(cex+cd)\,e}-\frac{c\,b^2 \operatorname{arcsinh}(cx)^2}{e\,(cex+cd)}+\frac{2c\,b^2 \operatorname{arcsinh}(cx)\ln\left(\frac{-\left(cx+\sqrt{c^2x^2+1}\right)e-cd+\sqrt{c^2d^2+e^2}}{-cd+\sqrt{c^2d^2+e^2}}\right)}{e\sqrt{c^2d^2+e^2}}-\frac{2c\,b^2 \operatorname{arcsinh}(cx)\ln\left(\frac{\left(cx+\sqrt{c\right.}}{}\right.}{e\sqrt{c^2d^2+\,}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(e*x+d)^2,x)
```

```
[Out] -c*a^2/(c*e*x+c*d)/e-c*b^2*arcsinh(c*x)^2/e/(c*e*x+c*d)+2*c*b^2/e*arcsinh(c
*x)/(c^2*d^2+e^2)^(1/2)*ln((-(c*x+(c^2*x^2+1)^(1/2))*e-c*d+(c^2*d^2+e^2)^(1
/2))/(-c*d+(c^2*d^2+e^2)^(1/2)))-2*c*b^2/e*arcsinh(c*x)/(c^2*d^2+e^2)^(1/2)
*ln(((c*x+(c^2*x^2+1)^(1/2))*e+c*d+(c^2*d^2+e^2)^(1/2))/(c*d+(c^2*d^2+e^2)^
(1/2)))+2*c*b^2/e/(c^2*d^2+e^2)^(1/2)*dilog((-(c*x+(c^2*x^2+1)^(1/2))*e-c*d
+(c^2*d^2+e^2)^(1/2))/(-c*d+(c^2*d^2+e^2)^(1/2)))-2*c*b^2/e/(c^2*d^2+e^2)^(
1/2)*dilog(((c*x+(c^2*x^2+1)^(1/2))*e+c*d+(c^2*d^2+e^2)^(1/2))/(c*d+(c^2*d^
2+e^2)^(1/2)))-2*c*a*b/(c*e*x+c*d)/e*arcsinh(c*x)-2*c*a*b/e^2/((c^2*d^2+e^2
)/e^2)^(1/2)*ln((2*(c^2*d^2+e^2)/e^2-2*c*d/e*(c*x+c*d/e)+2*((c^2*d^2+e^2)/e
```

```
^2)^(1/2)*((c*x+c*d/e)^2-2*c*d/e*(c*x+c*d/e)+(c^2*d^2+e^2)/e^2)^(1/2))/(c*x
+c*d/e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-b^2\left(\frac{\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{e^2x+de}-\int\frac{2\left(c^3x^2+\sqrt{c^2x^2+1}\,c^2x+c\right)\log\left(cx+\sqrt{c^2x^2+1}\right)}{c^3e^2x^4+c^3dex^3+ce^2x^2+cdex+\left(c^2e^2x^3+c^2dex^2+e^2x+de\right)\sqrt{c^2x^2+1}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -b^2*(log(c*x + sqrt(c^2*x^2 + 1))^2/(e^2*x + d*e) - integrate(2*(c^3*x^2 +
 sqrt(c^2*x^2 + 1)*c^2*x + c)*log(c*x + sqrt(c^2*x^2 + 1))/(c^3*e^2*x^4 + c
^3*d*e*x^3 + c*e^2*x^2 + c*d*e*x + (c^2*e^2*x^3 + c^2*d*e*x^2 + e^2*x + d*e
)*sqrt(c^2*x^2 + 1)), x)) - 2*a*b*(arcsinh(c*x)/(e^2*x + d*e) - c*arcsinh(c
*d*e*x/abs(e^2*x + d*e) - e^2/(c*abs(e^2*x + d*e)))/(sqrt(c^2*d^2/e^2 + 1)*
e^2)) - a^2/(e^2*x + d*e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{(a+b\,\mathrm{asinh}\,(c\,x))^2}{(d+e\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(d + e*x)^2,x)
```

```
[Out] int((a + b*asinh(c*x))^2/(d + e*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(a+b\,\mathrm{asinh}\,(cx))^2}{(d+ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(e*x+d)**2,x)
```

```
[Out] Integral((a + b*asinh(c*x))**2/(d + e*x)**2, x)
```

3.18 $\int \frac{(a+b\sinh^{-1}(cx))^2}{(d+ex)^3}\,dx$

Optimal. Leaf size=349

$$-\frac{bc\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{\left(c^2d^2+e^2\right)(d+ex)}+\frac{bc^3d\left(a+b\sinh^{-1}(cx)\right)\log\left(\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e\left(c^2d^2+e^2\right)^{3/2}}-\frac{bc^3d\left(a+b\sinh^{-1}(cx)\right)\log\left(\frac{e e}{\sqrt{c}}\right.}{e\left(c^2d^2+e^2\right)^{3/2}}$$

```
[Out] -1/2*(a+b*arcsinh(c*x))^2/e/(e*x+d)^2+b^2*c^2*ln(e*x+d)/e/(c^2*d^2+e^2)+b*c
^3*d*(a+b*arcsinh(c*x))*ln(1+e*(c*x+(c^2*x^2+1)^(1/2))/(c*d-(c^2*d^2+e^2)^(
1/2)))/e/(c^2*d^2+e^2)^(3/2)-b*c^3*d*(a+b*arcsinh(c*x))*ln(1+e*(c*x+(c^2*x^
2+1)^(1/2))/(c*d+(c^2*d^2+e^2)^(1/2)))/e/(c^2*d^2+e^2)^(3/2)+b^2*c^3*d*poly
log(2,-e*(c*x+(c^2*x^2+1)^(1/2))/(c*d-(c^2*d^2+e^2)^(1/2)))/e/(c^2*d^2+e^2)
^(3/2)-b^2*c^3*d*polylog(2,-e*(c*x+(c^2*x^2+1)^(1/2))/(c*d+(c^2*d^2+e^2)^(1
/2)))/e/(c^2*d^2+e^2)^(3/2)-b*c*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/(c^2*d
^2+e^2)/(e*x+d)
```

Rubi [A] time = 0.61, antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {5801, 5831, 3324, 3322, 2264, 2190, 2279, 2391, 2668, 31}

$$\frac{b^2c^3d\,\text{PolyLog}\left(2,-\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e\left(c^2d^2+e^2\right)^{3/2}}-\frac{b^2c^3d\,\text{PolyLog}\left(2,-\frac{ee^{\sinh^{-1}(cx)}}{\sqrt{c^2d^2+e^2}+cd}\right)}{e\left(c^2d^2+e^2\right)^{3/2}}-\frac{bc\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{\left(c^2d^2+e^2\right)(d+ex)}+\frac{bc^3d\left(a+b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(d + e*x)^3,x]
```

```
[Out] -((b*c*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/((c^2*d^2 + e^2)*(d + e*x)))
 - (a + b*ArcSinh[c*x])^2/(2*e*(d + e*x)^2) + (b*c^3*d*(a + b*ArcSinh[c*x])
*Log[1 + (e*E^ArcSinh[c*x])/(c*d - Sqrt[c^2*d^2 + e^2])])/(e*(c^2*d^2 + e^2
)^(3/2)) - (b*c^3*d*(a + b*ArcSinh[c*x])*Log[1 + (e*E^ArcSinh[c*x])/(c*d +
Sqrt[c^2*d^2 + e^2])])/(e*(c^2*d^2 + e^2)^(3/2)) + (b^2*c^2*Log[d + e*x])/(
e*(c^2*d^2 + e^2)) + (b^2*c^3*d*PolyLog[2, -((e*E^ArcSinh[c*x])/(c*d - Sqrt
[c^2*d^2 + e^2]))])/(e*(c^2*d^2 + e^2)^(3/2)) - (b^2*c^3*d*PolyLog[2, -((e*
E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))])/(e*(c^2*d^2 + e^2)^(3/2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
```

```
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3324

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[(b*(c + d*x)^m*Cos[e + f*x])/(f*(a^2 - b^2)*(a + b*Sin[e +
f*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[(b*d*m)/(f*(a^2 - b^2)), Int[((c + d*x)^(m - 1)*Cos[e + f*x])/(a
 + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 5801

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n
 - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5831

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{(d+ex)^3}\,dx = -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2e(d+ex)^2}+\frac{(bc)\int \frac{a+b\sinh^{-1}(cx)}{(d+ex)^2\sqrt{1+c^2x^2}}\,dx}{e}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2e(d+ex)^2}+\frac{\left(bc^2\right)\text{Subst}\left(\int \frac{a+bx}{(cd+e\sinh(x))^2}\,dx,x,\sinh^{-1}(cx)\right)}{e}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{\left(c^2d^2+e^2\right)(d+ex)}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2e(d+ex)^2}+\frac{\left(b^2c^2\right)\text{Subst}\left(\int \frac{\cosh(x)}{cd+e\sinh(x}\right.}{c^2d^2+e}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{\left(c^2d^2+e^2\right)(d+ex)}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2e(d+ex)^2}+\frac{\left(b^2c^2\right)\text{Subst}\left(\int \frac{1}{cd+x}\,dx,x\right.}{e\left(c^2d^2+e^2\right)}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{\left(c^2d^2+e^2\right)(d+ex)}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2e(d+ex)^2}+\frac{b^2c^2\log(d+ex)}{e\left(c^2d^2+e^2\right)}+\frac{\left(2bc^3d\right.}{}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{\left(c^2d^2+e^2\right)(d+ex)}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2e(d+ex)^2}+\frac{bc^3d\left(a+b\sinh^{-1}(cx)\right)\text{lo}}{e\left(c^2d^2+e\right.}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{\left(c^2d^2+e^2\right)(d+ex)}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2e(d+ex)^2}+\frac{bc^3d\left(a+b\sinh^{-1}(cx)\right)\text{lo}}{e\left(c^2d^2+e\right.}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{\left(c^2d^2+e^2\right)(d+ex)}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2e(d+ex)^2}+\frac{bc^3d\left(a+b\sinh^{-1}(cx)\right)\text{lo}}{e\left(c^2d^2+e\right.}$$

Mathematica [A] time = 0.71, size = 270, normalized size = 0.77

$$\frac{-\frac{2bce\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{\left(c^2d^2+e^2\right)(d+ex)}+\frac{2bc^3d\left(\left(a+b\sinh^{-1}(cx)\right)\left(\log\left(\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)-\log\left(\frac{ee^{\sinh^{-1}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)\right)+b\text{Li}_2\left(\frac{ee^{\sinh^{-1}(cx)}}{\sqrt{c^2d^2+e^2}-cd}\right)-b\text{Li}_2\left(-\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right.\right.}{\left(c^2d^2+e^2\right)^{3/2}}}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(d + e*x)^3,x]
```

```
[Out] ((-2*b*c*e*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/((c^2*d^2 + e^2)*(d + e*
x)) - (a + b*ArcSinh[c*x])^2/(d + e*x)^2 + (2*b^2*c^2*Log[d + e*x])/(c^2*d^
2 + e^2) + (2*b*c^3*d*((a + b*ArcSinh[c*x])*(Log[1 + (e*E^ArcSinh[c*x])/(c*
d - Sqrt[c^2*d^2 + e^2])] - Log[1 + (e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2
+ e^2])]) + b*PolyLog[2, (e*E^ArcSinh[c*x])/(-(c*d) + Sqrt[c^2*d^2 + e^2])]
 - b*PolyLog[2, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))]))/(c^2*d
^2 + e^2)^(3/2))/(2*e)
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\,\text{arsinh}\,(cx)^2+2\,ab\,\text{arsinh}\,(cx)+a^2}{e^3x^3+3\,de^2x^2+3\,d^2ex+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(e^3*x^3 + 3*d*e^2
*x^2 + 3*d^2*e*x + d^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arsinh}(cx) + a)^2}{(ex + d)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/(e*x + d)^3, x)
```

maple [B] time = 0.67, size = 1013, normalized size = 2.90

$$-\frac{c^2 a^2}{2 (cex + cd)^2 e} - \frac{c^4 b^2 \operatorname{arcsinh}(cx)^2 d^2}{2e (cex + cd)^2 \left(c^2 d^2 + e^2\right)} - \frac{c^3 b^2 \operatorname{arcsinh}(cx) e \sqrt{c^2 x^2 + 1}\, x}{(cex + cd)^2 \left(c^2 d^2 + e^2\right)} - \frac{c^3 b^2 \operatorname{arcsinh}(cx) d \sqrt{c^2 x^2 + 1}}{(cex + cd)^2 \left(c^2 d^2 + e^2\right)} + \frac{c}{(c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(e*x+d)^3,x)
```

```
[Out] -1/2*c^2*a^2/(c*e*x+c*d)^2/e-1/2*c^4*b^2*arcsinh(c*x)^2/e/(c*e*x+c*d)^2/(c^
2*d^2+e^2)*d^2-c^3*b^2*arcsinh(c*x)*e/(c*e*x+c*d)^2/(c^2*d^2+e^2)*(c^2*x^2+
1)^(1/2)*x-c^3*b^2*arcsinh(c*x)/(c*e*x+c*d)^2/(c^2*d^2+e^2)*d*(c^2*x^2+1)^(
1/2)+c^4*b^2*arcsinh(c*x)*e/(c*e*x+c*d)^2/(c^2*d^2+e^2)*x^2+2*c^4*b^2*arcsi
nh(c*x)/(c*e*x+c*d)^2/(c^2*d^2+e^2)*d*x+c^4*b^2*arcsinh(c*x)/e/(c*e*x+c*d)^
2/(c^2*d^2+e^2)*d^2-1/2*c^2*b^2*arcsinh(c*x)^2*e/(c*e*x+c*d)^2/(c^2*d^2+e^2
)-2*c^2*b^2/e/(c^2*d^2+e^2)*ln(c*x+(c^2*x^2+1)^(1/2))+c^2*b^2/e/(c^2*d^2+e^
2)*ln(2*c*d*(c*x+(c^2*x^2+1)^(1/2))+(c*x+(c^2*x^2+1)^(1/2))^2*e-e)+c^3*b^2/
e/(c^2*d^2+e^2)^(3/2)*d*arcsinh(c*x)*ln((-(c*x+(c^2*x^2+1)^(1/2))*e-c*d+(c^
2*d^2+e^2)^(1/2))/(-c*d+(c^2*d^2+e^2)^(1/2)))-c^3*b^2/e/(c^2*d^2+e^2)^(3/2)
*d*arcsinh(c*x)*ln(((c*x+(c^2*x^2+1)^(1/2))*e+c*d+(c^2*d^2+e^2)^(1/2))/(c*d
+(c^2*d^2+e^2)^(1/2)))+c^3*b^2/e/(c^2*d^2+e^2)^(3/2)*d*dilog((-(c*x+(c^2*x^
2+1)^(1/2))*e-c*d+(c^2*d^2+e^2)^(1/2))/(-c*d+(c^2*d^2+e^2)^(1/2)))-c^3*b^2/
e/(c^2*d^2+e^2)^(3/2)*d*dilog(((c*x+(c^2*x^2+1)^(1/2))*e+c*d+(c^2*d^2+e^2)^
(1/2))/(c*d+(c^2*d^2+e^2)^(1/2)))-c^2*a*b/(c*e*x+c*d)^2/e*arcsinh(c*x)-c^2*
a*b/e/(c^2*d^2+e^2)/(c*x+c*d/e)*((c*x+c*d/e)^2-2*c*d/e*(c*x+c*d/e)+(c^2*d^2
+e^2)/e^2)^(1/2)-c^3*a*b/e^2*d/(c^2*d^2+e^2)/((c^2*d^2+e^2)/e^2)^(1/2)*ln((
2*(c^2*d^2+e^2)/e^2-2*c*d/e*(c*x+c*d/e)+2*((c^2*d^2+e^2)/e^2)^(1/2)*((c*x+c
*d/e)^2-2*c*d/e*(c*x+c*d/e)+(c^2*d^2+e^2)/e^2)^(1/2))/(c*x+c*d/e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\left(\left(c\left(\frac{\sqrt{c^2x^2+1}}{c^2d^2ex + c^2d^3 + e^3x + de^2} - \frac{c^2 d \operatorname{arsinh}\left(\frac{cdx}{e\left|x+\frac{d}{e}\right|} - \frac{1}{c\left|x+\frac{d}{e}\right|}\right)}{\left(\frac{c^2d^2}{e^2}+1\right)^{\frac{3}{2}} e^4}\right)\right) + \frac{\operatorname{arsinh}(cx)}{e^3x^2 + 2\,de^2x + d^2e}\right)ab - \frac{1}{2}b^2\left(\frac{\log\left(cx + \sqrt{c^2x}\right.}{e^3x^2 + 2\,de^2x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] -(c*(sqrt(c^2*x^2 + 1)/(c^2*d^2*e*x + c^2*d^3 + e^3*x + d*e^2) - c^2*d*arcs
inh(c*d*x/(e*abs(x + d/e)) - 1/(c*abs(x + d/e)))/((c^2*d^2/e^2 + 1)^(3/2)*e
^4)) + arcsinh(c*x)/(e^3*x^2 + 2*d*e^2*x + d^2*e))*a*b - 1/2*b^2*(log(c*x +
```

```
 sqrt(c^2*x^2 + 1))^2/(e^3*x^2 + 2*d*e^2*x + d^2*e) - 2*integrate((c^3*x^2
+ sqrt(c^2*x^2 + 1)*c^2*x + c)*log(c*x + sqrt(c^2*x^2 + 1))/(c^3*e^3*x^5 +
2*c^3*d*e^2*x^4 + 2*c*d*e^2*x^2 + c*d^2*e*x + (c^3*d^2*e + c*e^3)*x^3 + (c^
2*e^3*x^4 + 2*c^2*d*e^2*x^3 + 2*d*e^2*x + d^2*e + (c^2*d^2*e + e^3)*x^2)*sq
rt(c^2*x^2 + 1)), x)) - 1/2*a^2/(e^3*x^2 + 2*d*e^2*x + d^2*e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\mathrm{asinh}\,(c\,x))^2}{(d + e\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(d + e*x)^3,x)
```

```
[Out] int((a + b*asinh(c*x))^2/(d + e*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{asinh}\,(cx))^2}{(d + ex)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(e*x+d)**3,x)
```

```
[Out] Integral((a + b*asinh(c*x))**2/(d + e*x)**3, x)
```

3.19 $\int \frac{(d+ex)^3}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=394

$$\frac{e^3\sinh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{4bc^4}-\frac{e^3\sinh\left(\frac{4a}{b}\right)\mathrm{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(cx)\right)}{8bc^4}-\frac{e^3\cosh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(\frac{2a}{b}+2\sinh^{-1}(c\right.}{4bc^4}$$

```
[Out] d^3*Chi(a/b+arcsinh(c*x))*cosh(a/b)/b/c-3/4*d*e^2*Chi(a/b+arcsinh(c*x))*cos
h(a/b)/b/c^3+3/4*d*e^2*Chi(3*a/b+3*arcsinh(c*x))*cosh(3*a/b)/b/c^3+3/2*d^2*
e*cosh(2*a/b)*Shi(2*a/b+2*arcsinh(c*x))/b/c^2-1/4*e^3*cosh(2*a/b)*Shi(2*a/b
+2*arcsinh(c*x))/b/c^4+1/8*e^3*cosh(4*a/b)*Shi(4*a/b+4*arcsinh(c*x))/b/c^4-
d^3*Shi(a/b+arcsinh(c*x))*sinh(a/b)/b/c+3/4*d*e^2*Shi(a/b+arcsinh(c*x))*sin
h(a/b)/b/c^3-3/2*d^2*e*Chi(2*a/b+2*arcsinh(c*x))*sinh(2*a/b)/b/c^2+1/4*e^3*
Chi(2*a/b+2*arcsinh(c*x))*sinh(2*a/b)/b/c^4-3/4*d*e^2*Shi(3*a/b+3*arcsinh(c
*x))*sinh(3*a/b)/b/c^3-1/8*e^3*Chi(4*a/b+4*arcsinh(c*x))*sinh(4*a/b)/b/c^4
```

Rubi [A] time = 1.17, antiderivative size = 394, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {5805, 6742, 3303, 3298, 3301, 5448, 12}

$$-\frac{3d^2e\sinh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{2bc^2}-\frac{3de^2\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{4bc^3}+\frac{3de^2\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3a}{b}+3\sin\right.}{4bc^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^3*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]])/(b*c) - (3*d*e^2*Cosh[a/b]
*CoshIntegral[a/b + ArcSinh[c*x]])/(4*b*c^3) + (3*d*e^2*Cosh[(3*a)/b]*CoshI
ntegral[(3*a)/b + 3*ArcSinh[c*x]])/(4*b*c^3) - (3*d^2*e*CoshIntegral[(2*a)/
b + 2*ArcSinh[c*x]]*Sinh[(2*a)/b])/(2*b*c^2) + (e^3*CoshIntegral[(2*a)/b +
2*ArcSinh[c*x]]*Sinh[(2*a)/b])/(4*b*c^4) - (e^3*CoshIntegral[(4*a)/b + 4*Ar
cSinh[c*x]]*Sinh[(4*a)/b])/(8*b*c^4) - (d^3*Sinh[a/b]*SinhIntegral[a/b + Ar
cSinh[c*x]])/(b*c) + (3*d*e^2*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]])/(
4*b*c^3) + (3*d^2*e*Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSinh[c*x]])/(
2*b*c^2) - (e^3*Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSinh[c*x]])/(4*b*
c^4) - (3*d*e^2*Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSinh[c*x]])/(4*b*
c^3) + (e^3*Cosh[(4*a)/b]*SinhIntegral[(4*a)/b + 4*ArcSinh[c*x]])/(8*b*c^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5805

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]*(c*d + e*Sinh[x
])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m, 0
]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(d+ex)^3}{a+b\sinh^{-1}(cx)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh(x)(cd+e\sinh(x))^3}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{c^4}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{c^3d^3\cosh(x)}{a+bx} + \frac{3c^2d^2e\cosh(x)\sinh(x)}{a+bx} + \frac{3cde^2\cosh(x)\sinh^2(x)}{a+bx} + \frac{e^3\cosh(x)\sinh^3(x)}{a+bx}\right) dx, x, \text{s}\right.}{c^4}$$

$$= \frac{d^3\,\text{Subst}\left(\int \frac{\cosh(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{c} + \frac{\left(3d^2e\right)\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{c^2}$$

$$= \frac{\left(3d^2e\right)\text{Subst}\left(\int \frac{\sinh(2x)}{2(a+bx)}\,dx, x, \sinh^{-1}(cx)\right)}{c^2} + \frac{\left(3de^2\right)\text{Subst}\left(\int \left(-\frac{\cosh(x)}{4(a+bx)} + \frac{\cosh(3x)}{4(a+bx)}\right) dx, x\right.}{c^3}$$

$$= \frac{d^3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{bc} - \frac{d^3\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{bc} + \frac{\left(3d^2e\right)\text{Subst}\left(\right.}{}$$

$$= \frac{d^3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{bc} - \frac{d^3\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{bc} - \frac{\left(3de^2\cosh\left(\frac{a}{b}\right)\right)}{}$$

$$= \frac{d^3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{bc} - \frac{3de^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{4bc^3} + \frac{3de^2\cosh\left(\right.}{}$$

Mathematica [A] time = 0.69, size = 305, normalized size = 0.77

$$\frac{e^3\left(2\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right) - \sinh\left(\frac{4a}{b}\right)\text{Chi}\left(4\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right) - 2\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\sinh^{-1}(c\right.\right.\right.}{8bc^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^3*(Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]] - Sinh[a/b]*SinhIntegral[a
/b + ArcSinh[c*x]]))/(b*c) + (3*d*e^2*(-(Cosh[a/b]*CoshIntegral[a/b + ArcSi
nh[c*x]]) + Cosh[(3*a)/b]*CoshIntegral[3*(a/b + ArcSinh[c*x])] + Sinh[a/b]*
SinhIntegral[a/b + ArcSinh[c*x]] - Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcS
inh[c*x])]))/(4*b*c^3) + (e^3*(2*CoshIntegral[2*(a/b + ArcSinh[c*x])]*Sinh[
(2*a)/b] - CoshIntegral[4*(a/b + ArcSinh[c*x])]*Sinh[(4*a)/b] - 2*Cosh[(2*a
)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])] + Cosh[(4*a)/b]*SinhIntegral[4*(a
/b + ArcSinh[c*x])]))/(8*b*c^4) - (3*d^2*e*(CoshIntegral[(2*a)/b + 2*ArcSin
h[c*x]]*Sinh[(2*a)/b] - Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSinh[c*x]
]))/(2*b*c^2)
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{e^3x^3 + 3\,de^2x^2 + 3\,d^2ex + d^3}{b\,\text{arsinh}\,(cx) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)/(b*arcsinh(c*x) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex + d)^3}{b\,\text{arsinh}\,(cx) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3/(b*arcsinh(c*x) + a), x)
```

maple [A] time = 0.48, size = 394, normalized size = 1.00

$$\frac{-\frac{e^3 e^{-\frac{4a}{b}}\,\text{Ei}\left(1,-4\,\text{arcsinh}(cx)-\frac{4a}{b}\right)}{16c^3b} + \frac{e^3 e^{\frac{4a}{b}}\,\text{Ei}\left(1,4\,\text{arcsinh}(cx)+\frac{4a}{b}\right)}{16c^3b} + \frac{3e\,e^{\frac{2a}{b}}\,\text{Ei}\left(1,2\,\text{arcsinh}(cx)+\frac{2a}{b}\right)d^2}{4cb} - \frac{e^3 e^{\frac{2a}{b}}\,\text{Ei}\left(1,2\,\text{arcsinh}(cx)+\frac{2a}{b}\right)}{8c^3b} - \frac{3e}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/c*(-1/16/c^3*e^3/b*exp(-4*a/b)*Ei(1,-4*arcsinh(c*x)-4*a/b)+1/16/c^3*e^3/b
*exp(4*a/b)*Ei(1,4*arcsinh(c*x)+4*a/b)+3/4/c*e/b*exp(2*a/b)*Ei(1,2*arcsinh(
c*x)+2*a/b)*d^2-1/8/c^3*e^3/b*exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b)-3/4/c*e
/b*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b)*d^2+1/8/c^3*e^3/b*exp(-2*a/b)*Ei
(1,-2*arcsinh(c*x)-2*a/b)-3/8/c^2*d*e^2/b*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-
3*a/b)-1/2/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)*d^3+3/8/c^2*d/b*exp(a/b)*Ei(1,
arcsinh(c*x)+a/b)*e^2-1/2/b*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)*d^3+3/8/c^2*d
/b*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)*e^2-3/8/c^2*d*e^2/b*exp(3*a/b)*Ei(1,3*
arcsinh(c*x)+3*a/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex + d)^3}{b\,\text{arsinh}\,(cx) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^3/(b*arcsinh(c*x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^3}{a+b\,\mathrm{asinh}\left(c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3/(a + b*asinh(c*x)),x)
[Out] int((d + e*x)^3/(a + b*asinh(c*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^3}{a+b\,\mathrm{asinh}\left(cx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(a+b*asinh(c*x)),x)
[Out] Integral((d + e*x)**3/(a + b*asinh(c*x)), x)
```

3.20 $\int \frac{(d+ex)^2}{a+b\sinh^{-1}(cx)} dx$

Optimal. Leaf size=245

$$-\frac{e^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{4bc^3}+\frac{e^2\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)}{4bc^3}+\frac{e^2\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{4bc^3}-$$

```
[Out] d^2*Chi(a/b+arcsinh(c*x))*cosh(a/b)/b/c-1/4*e^2*Chi(a/b+arcsinh(c*x))*cosh(
a/b)/b/c^3+1/4*e^2*Chi(3*a/b+3*arcsinh(c*x))*cosh(3*a/b)/b/c^3+d*e*cosh(2*a
/b)*Shi(2*a/b+2*arcsinh(c*x))/b/c^2-d^2*Shi(a/b+arcsinh(c*x))*sinh(a/b)/b/c
+1/4*e^2*Shi(a/b+arcsinh(c*x))*sinh(a/b)/b/c^3-d*e*Chi(2*a/b+2*arcsinh(c*x)
)*sinh(2*a/b)/b/c^2-1/4*e^2*Shi(3*a/b+3*arcsinh(c*x))*sinh(3*a/b)/b/c^3
```

Rubi [A] time = 0.70, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5805, 6742, 3303, 3298, 3301, 5448}

$$-\frac{de\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{bc^2}-\frac{e^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{4bc^3}+\frac{e^2\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx\right.}{4bc^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^2*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]])/(b*c) - (e^2*Cosh[a/b]*Cos
hIntegral[a/b + ArcSinh[c*x]])/(4*b*c^3) + (e^2*Cosh[(3*a)/b]*CoshIntegral[
(3*a)/b + 3*ArcSinh[c*x]])/(4*b*c^3) - (d*e*CoshIntegral[(2*a)/b + 2*ArcSin
h[c*x]]*Sinh[(2*a)/b])/(b*c^2) - (d^2*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[
c*x]])/(b*c) + (e^2*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]])/(4*b*c^3) +
 (d*e*Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSinh[c*x]])/(b*c^2) - (e^2*
Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSinh[c*x]])/(4*b*c^3)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5805

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]*(c*d + e*Sinh[x
])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m, 0
]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(d+ex)^2}{a+b\sinh^{-1}(cx)}\,dx = \frac{\mathrm{Subst}\left(\int \frac{\cosh(x)(cd+e\sinh(x))^2}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{c^3}$$

$$= \frac{\mathrm{Subst}\left(\int \left(\frac{c^2d^2\cosh(x)}{a+bx}+\frac{e^2\cosh(x)\sinh^2(x)}{a+bx}+\frac{cde\sinh(2x)}{a+bx}\right)dx, x, \sinh^{-1}(cx)\right)}{c^3}$$

$$= \frac{d^2\,\mathrm{Subst}\left(\int \frac{\cosh(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{c} + \frac{(de)\,\mathrm{Subst}\left(\int \frac{\sinh(2x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{c^2} + \frac{e^2\,\mathrm{Su}}{}$$

$$= \frac{e^2\,\mathrm{Subst}\left(\int \left(-\frac{\cosh(x)}{4(a+bx)}+\frac{\cosh(3x)}{4(a+bx)}\right)dx, x, \sinh^{-1}(cx)\right)}{c^3} + \frac{\left(d^2\cosh\left(\frac{a}{b}\right)\right)\mathrm{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\right.}{c}$$

$$= \frac{d^2\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{bc} - \frac{de\mathrm{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{bc^2} - \frac{d^2\sinh\left(\frac{a}{b}\right)}{}$$

$$= \frac{d^2\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{bc} - \frac{de\mathrm{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{bc^2} - \frac{d^2\sinh\left(\frac{a}{b}\right)}{}$$

$$= \frac{d^2\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{bc} - \frac{e^2\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{4bc^3} + \frac{e^2\cosh\left(\frac{3a}{b}\right)\mathrm{C}}{}$$

Mathematica [A] time = 0.41, size = 188, normalized size = 0.77

$$\frac{\cosh\left(\frac{a}{b}\right)\left(4c^2d^2-e^2\right)\mathrm{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)-4c^2d^2\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)-4cde\sinh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(2\left(\frac{a}{b}+\mathrm{si}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(a + b*ArcSinh[c*x]),x]
```

```
[Out] ((4*c^2*d^2 - e^2)*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]] + e^2*Cosh[(3
*a)/b]*CoshIntegral[3*(a/b + ArcSinh[c*x])] - 4*c*d*e*CoshIntegral[2*(a/b +
 ArcSinh[c*x])]*Sinh[(2*a)/b] - 4*c^2*d^2*Sinh[a/b]*SinhIntegral[a/b + ArcS
inh[c*x]] + e^2*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + 4*c*d*e*Cosh[(
2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])] - e^2*Sinh[(3*a)/b]*SinhIntegr
al[3*(a/b + ArcSinh[c*x])])/(4*b*c^3)
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{e^2x^2+2\,dex+d^2}{b\,\mathrm{arsinh}\,(cx)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)/(b*arcsinh(c*x) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^2}{b\operatorname{arsinh}(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2/(b*arcsinh(c*x) + a), x)
```

maple [A] time = 0.32, size = 254, normalized size = 1.04

$$\frac{-\frac{e^2 \mathrm{e}^{-\frac{3a}{b}} \operatorname{Ei}\left(1,-3\operatorname{arcsinh}(cx)-\frac{3a}{b}\right)}{8c^2b}-\frac{e^2 \mathrm{e}^{\frac{3a}{b}} \operatorname{Ei}\left(1,3\operatorname{arcsinh}(cx)+\frac{3a}{b}\right)}{8c^2b}-\frac{\mathrm{e}^{\frac{a}{b}} \operatorname{Ei}\left(1,\operatorname{arcsinh}(cx)+\frac{a}{b}\right)d^2}{2b}+\frac{\mathrm{e}^{\frac{a}{b}} \operatorname{Ei}\left(1,\operatorname{arcsinh}(cx)+\frac{a}{b}\right)e^2}{8c^2b}-\frac{\mathrm{e}^{-\frac{a}{b}} \operatorname{Ei}\left(1,-a\right.}{}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/c*(-1/8/c^2*e^2/b*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b)-1/8/c^2*e^2/b*e
xp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)-1/2/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)*
d^2+1/8/c^2/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)*e^2-1/2/b*exp(-a/b)*Ei(1,-arc
sinh(c*x)-a/b)*d^2+1/8/c^2/b*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)*e^2-1/2/c*d*
e/b*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b)+1/2/c*d*e/b*exp(2*a/b)*Ei(1,2*a
rcsinh(c*x)+2*a/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^2}{b\operatorname{arsinh}(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^2/(b*arcsinh(c*x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^2}{a+b\operatorname{asinh}(c\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2/(a + b*asinh(c*x)),x)
```

```
[Out] int((d + e*x)^2/(a + b*asinh(c*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^2}{a+b\operatorname{asinh}(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(a+b*asinh(c*x)),x)
```

```
[Out] Integral((d + e*x)**2/(a + b*asinh(c*x)), x)
```

3.21 $\int \frac{d+ex}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=116

$$-\frac{e\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{2bc^2}+\frac{e\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{2bc^2}+\frac{d\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{bc}-\frac{d\text{ s}}{}$$

```
[Out] d*Chi(a/b+arcsinh(c*x))*cosh(a/b)/b/c+1/2*e*cosh(2*a/b)*Shi(2*a/b+2*arcsinh
(c*x))/b/c^2-d*Shi(a/b+arcsinh(c*x))*sinh(a/b)/b/c-1/2*e*Chi(2*a/b+2*arcsin
h(c*x))*sinh(2*a/b)/b/c^2
```

Rubi [A] time = 0.32, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5805, 6742, 3303, 3298, 3301, 5448, 12}

$$-\frac{e\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{2bc^2}+\frac{e\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{2bc^2}+\frac{d\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{bc}-\frac{d\text{ s}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]])/(b*c) - (e*CoshIntegral[(2*a
)/b + 2*ArcSinh[c*x]]*Sinh[(2*a)/b])/(2*b*c^2) - (d*Sinh[a/b]*SinhIntegral[
a/b + ArcSinh[c*x]])/(b*c) + (e*Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcS
inh[c*x]])/(2*b*c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5805

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]*(c*d + e*Sinh[x
])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m, 0
]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{d+ex}{a+b\sinh^{-1}(cx)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh(x)(cd+e\sinh(x))}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{c^2}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{cd\cosh(x)}{a+bx}+\frac{e\cosh(x)\sinh(x)}{a+bx}\right)dx, x, \sinh^{-1}(cx)\right)}{c^2}$$

$$= \frac{d\,\text{Subst}\left(\int \frac{\cosh(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{c} + \frac{e\,\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{c^2}$$

$$= \frac{e\,\text{Subst}\left(\int \frac{\sinh(2x)}{2(a+bx)}\,dx, x, \sinh^{-1}(cx)\right)}{c^2} + \frac{\left(d\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \sinh^{-1}\right.}{c}$$

$$= \frac{d\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{bc} - \frac{d\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{bc} + \frac{e\,\text{Subst}\left(\int \frac{\text{sinh}}{a+}\right.}{}$$

$$= \frac{d\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{bc} - \frac{d\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{bc} + \frac{\left(e\cosh\left(\frac{2a}{b}\right)\right)}{}$$

$$= \frac{d\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{bc} - \frac{e\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{2bc^2} - \frac{d\sinh\left(\frac{a}{b}\right)}{}$$

Mathematica [A] time = 0.17, size = 98, normalized size = 0.84

$$\frac{2cd\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right) - e\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right) - 2cd\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{2bc^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (2*c*d*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]] - e*CoshIntegral[2*(a/b +
 ArcSinh[c*x])]*Sinh[(2*a)/b] - 2*c*d*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[
c*x]] + e*Cosh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])])/(2*b*c^2)
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{ex+d}{b\,\text{arsinh}\,(cx)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

[Out] integral((e*x + d)/(b*arcsinh(c*x) + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex+d}{b\operatorname{arsinh}(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)/(a+b*arcsinh(c*x)),x, algorithm="giac")

[Out] integrate((e*x + d)/(b*arcsinh(c*x) + a), x)

maple [A] time = 0.22, size = 120, normalized size = 1.03

$$\frac{-\frac{e^{\frac{a}{b}}\operatorname{Ei}\left(1,\operatorname{arcsinh}(cx)+\frac{a}{b}\right)d}{2b}-\frac{e^{-\frac{a}{b}}\operatorname{Ei}\left(1,-\operatorname{arcsinh}(cx)-\frac{a}{b}\right)d}{2b}-\frac{e\,e^{-\frac{2a}{b}}\operatorname{Ei}\left(1,-2\operatorname{arcsinh}(cx)-\frac{2a}{b}\right)}{4cb}+\frac{e\,e^{\frac{2a}{b}}\operatorname{Ei}\left(1,2\operatorname{arcsinh}(cx)+\frac{2a}{b}\right)}{4cb}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d)/(a+b*arcsinh(c*x)),x)

[Out] 1/c*(-1/2/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)*d-1/2/b*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)*d-1/4/c*e/b*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b)+1/4/c*e/b*exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex+d}{b\operatorname{arsinh}(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)/(a+b*arcsinh(c*x)),x, algorithm="maxima")

[Out] integrate((e*x + d)/(b*arcsinh(c*x) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{d+e\,x}{a+b\,\mathrm{asinh}\left(c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x)/(a + b*asinh(c*x)),x)

[Out] int((d + e*x)/(a + b*asinh(c*x)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex}{a+b\operatorname{asinh}(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)/(a+b*asinh(c*x)),x)

[Out] Integral((d + e*x)/(a + b*asinh(c*x)), x)

3.22 $\int \frac{1}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=54

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc}-\frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc}$$

```
[Out] Chi((a+b*arcsinh(c*x))/b)*cosh(a/b)/b/c-Shi((a+b*arcsinh(c*x))/b)*sinh(a/b)
/b/c
```

Rubi [A] time = 0.07, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5657, 3303, 3298, 3301}

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc}-\frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^(-1),x]
```

```
[Out] (Cosh[a/b]*CoshIntegral[(a + b*ArcSinh[c*x])/b])/(b*c) - (Sinh[a/b]*SinhInt
egral[(a + b*ArcSinh[c*x])/b])/(b*c)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rubi steps

$$\int \frac{1}{a+b\sinh^{-1}(cx)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\sinh^{-1}(cx)\right)}{bc}$$
$$= \frac{\cosh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\sinh^{-1}(cx)\right)}{bc} - \frac{\sinh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx, x, a+\right.}{bc}$$
$$= \frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc} - \frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc}$$

Mathematica [A] time = 0.02, size = 45, normalized size = 0.83

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)-\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])^(-1),x]
```

```
[Out] (Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]] - Sinh[a/b]*SinhIntegral[a/b +
ArcSinh[c*x]])/(b*c)
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{b\,\text{arsinh}\,(cx)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(1/(b*arcsinh(c*x) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{b\,\text{arsinh}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(1/(b*arcsinh(c*x) + a), x)
```

maple [A] time = 0.00, size = 56, normalized size = 1.04

$$\frac{-\frac{e^{\frac{a}{b}}\,\text{Ei}\left(1,\text{arcsinh}(cx)+\frac{a}{b}\right)}{2b}-\frac{e^{-\frac{a}{b}}\,\text{Ei}\left(1,-\text{arcsinh}(cx)-\frac{a}{b}\right)}{2b}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/c*(-1/2/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)-1/2/b*exp(-a/b)*Ei(1,-arcsinh(c
*x)-a/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{b\,\text{arsinh}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*arcsinh(c*x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{a + b \operatorname{asinh}(c\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*asinh(c*x)),x)
```

```
[Out] int(1/(a + b*asinh(c*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + b \operatorname{asinh}(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(c*x)),x)
```

```
[Out] Integral(1/(a + b*asinh(c*x)), x)
```

3.23 $\int \frac{1}{(d+ex)\left(a+b\sinh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=21

$$\operatorname{Int}\left(\frac{1}{(d+ex)\left(a+b\sinh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable(1/(e*x+d)/(a+b*arcsinh(c*x)),x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(d+ex)\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((d + e*x)*(a + b*ArcSinh[c*x])),x]
[Out] Defer[Int][1/((d + e*x)*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(a+b\sinh^{-1}(cx)\right)}\,dx = \int \frac{1}{(d+ex)\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Mathematica [A] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex)\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((d + e*x)*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[1/((d + e*x)*(a + b*ArcSinh[c*x])), x]
```

fricas [A] time = 0.61, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{1}{aex+ad+(bex+bd)\operatorname{arsinh}(cx)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral(1/(a*e*x + a*d + (b*e*x + b*d)*arcsinh(c*x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ex+d)(b\operatorname{arsinh}(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] integrate(1/((e*x + d)*(b*arcsinh(c*x) + a)), x)
```

maple [A] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{1}{(ex+d)\,(a+b\,\mathrm{arcsinh}\,(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(a+b*arcsinh(c*x)),x)
[Out] int(1/(e*x+d)/(a+b*arcsinh(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ex+d)(b\,\mathrm{arsinh}\,(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate(1/((e*x + d)*(b*arcsinh(c*x) + a)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{(a+b\,\mathrm{asinh}\,(c\,x))\,\,(d+e\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*asinh(c*x))*(d + e*x)),x)
[Out] int(1/((a + b*asinh(c*x))*(d + e*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\,\mathrm{asinh}\,(cx))\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*asinh(c*x)),x)
[Out] Integral(1/((a + b*asinh(c*x))*(d + e*x)), x)
```

3.24 $\int \frac{1}{(d+ex)^2\left(a+b\sinh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{1}{(d+ex)^2\left(a+b\sinh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable(1/(e*x+d)^2/(a+b*arcsinh(c*x)),x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(d+ex)^2\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((d + e*x)^2*(a + b*ArcSinh[c*x])),x]
[Out] Defer[Int][1/((d + e*x)^2*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\left(a+b\sinh^{-1}(cx)\right)}\,dx=\int \frac{1}{(d+ex)^2\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Mathematica [A] time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex)^2\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((d + e*x)^2*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[1/((d + e*x)^2*(a + b*ArcSinh[c*x])), x]
```

fricas [A] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{ae^2x^2+2\,adex+ad^2+\left(be^2x^2+2\,bdex+bd^2\right)\text{arsinh}\,(cx)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral(1/(a*e^2*x^2 + 2*a*d*e*x + a*d^2 + (b*e^2*x^2 + 2*b*d*e*x + b*d^2)
*arcsinh(c*x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ex+d)^2(b\,\text{arsinh}\,(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(1/((e*x + d)^2*(b*arcsinh(c*x) + a)), x)
```

maple [A] time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{1}{(ex+d)^2 (a+b\,\mathrm{arcsinh}\,(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(a+b*arcsinh(c*x)),x)
[Out] int(1/(e*x+d)^2/(a+b*arcsinh(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ex+d)^2 (b\,\mathrm{arsinh}\,(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate(1/((e*x + d)^2*(b*arcsinh(c*x) + a)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{(a+b\,\mathrm{asinh}\,(c\,x))\,(d+e\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*asinh(c*x))*(d + e*x)^2),x)
[Out] int(1/((a + b*asinh(c*x))*(d + e*x)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\,\mathrm{asinh}\,(cx))\,(d+ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(a+b*asinh(c*x)),x)
[Out] Integral(1/((a + b*asinh(c*x))*(d + e*x)**2), x)
```

3.25 $\int \frac{(d+ex)^2}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=359

$$\frac{e^2\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{4b^2c^3}-\frac{3e^2\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{4b^2c^3}-\frac{e^2\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{4b^2c^3}+\frac{3e^2\cosh\left(\frac{3a}{b}\right)}{}$$

```
[Out] 2*d*e*Chi(2*(a+b*arcsinh(c*x))/b)*cosh(2*a/b)/b^2/c^2+d^2*cosh(a/b)*Shi((a+
b*arcsinh(c*x))/b)/b^2/c-1/4*e^2*cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b^2/c^
3+3/4*e^2*cosh(3*a/b)*Shi(3*(a+b*arcsinh(c*x))/b)/b^2/c^3-d^2*Chi((a+b*arcs
inh(c*x))/b)*sinh(a/b)/b^2/c+1/4*e^2*Chi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b^
2/c^3-2*d*e*Shi(2*(a+b*arcsinh(c*x))/b)*sinh(2*a/b)/b^2/c^2-3/4*e^2*Chi(3*(
a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b^2/c^3-d^2*(c^2*x^2+1)^(1/2)/b/c/(a+b*arc
sinh(c*x))-2*d*e*x*(c^2*x^2+1)^(1/2)/b/c/(a+b*arcsinh(c*x))-e^2*x^2*(c^2*x^
2+1)^(1/2)/b/c/(a+b*arcsinh(c*x))
```

Rubi [A] time = 0.69, antiderivative size = 351, normalized size of antiderivative = 0.98, number of steps used = 19, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {5803, 5655, 5779, 3303, 3298, 3301, 5665}

$$\frac{2de\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{b^2c^2}+\frac{e^2\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{4b^2c^3}-\frac{3e^2\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)}{4b^2c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -((d^2*Sqrt[1 + c^2*x^2])/(b*c*(a + b*ArcSinh[c*x]))) - (2*d*e*x*Sqrt[1 + c
^2*x^2])/(b*c*(a + b*ArcSinh[c*x])) - (e^2*x^2*Sqrt[1 + c^2*x^2])/(b*c*(a +
 b*ArcSinh[c*x])) + (2*d*e*Cosh[(2*a)/b]*CoshIntegral[(2*a)/b + 2*ArcSinh[c
*x]])/(b^2*c^2) - (d^2*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b])/(b^2*c)
+ (e^2*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b])/(4*b^2*c^3) - (3*e^2*Cos
hIntegral[(3*a)/b + 3*ArcSinh[c*x]]*Sinh[(3*a)/b])/(4*b^2*c^3) + (d^2*Cosh[
a/b]*SinhIntegral[a/b + ArcSinh[c*x]])/(b^2*c) - (e^2*Cosh[a/b]*SinhIntegra
l[a/b + ArcSinh[c*x]])/(4*b^2*c^3) - (2*d*e*Sinh[(2*a)/b]*SinhIntegral[(2*a
)/b + 2*ArcSinh[c*x]])/(b^2*c^2) + (3*e^2*Cosh[(3*a)/b]*SinhIntegral[(3*a)/
b + 3*ArcSinh[c*x]])/(4*b^2*c^3)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 5803

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*ArcSinh[c*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[m, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{(d+ex)^2}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = \int\left(\frac{d^2}{\left(a+b\sinh^{-1}(cx)\right)^2}+\frac{2dex}{\left(a+b\sinh^{-1}(cx)\right)^2}+\frac{e^2x^2}{\left(a+b\sinh^{-1}(cx)\right)^2}\right)dx$$

$$= d^2\int\frac{1}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx+(2de)\int\frac{x}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx+e^2\int\frac{x^2}{\left(a+b\sinh\right.}$$

$$= -\frac{d^2\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{2dex\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{e^2x^2\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{\left(cd^2\right)\int}{}$$

$$= -\frac{d^2\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{2dex\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{e^2x^2\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{d^2\,\mathrm{Subs}}{}$$

$$= -\frac{d^2\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{2dex\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{e^2x^2\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{2de\cos}{}$$

$$= -\frac{d^2\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{2dex\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{e^2x^2\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{2de\cos}{}$$

Mathematica [A] time = 1.61, size = 288, normalized size = 0.80

$$-\frac{\sinh\left(\frac{a}{b}\right)\left(4c^2d^2-e^2\right)\mathrm{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)-4c^2d^2\cosh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)+\frac{4bc^2d^2\sqrt{c^2x^2+1}}{a+b\sinh^{-1}(cx)}+\frac{8bc^2dex\sqrt{c}}{a+b\sinh^{-}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -1/4*((4*b*c^2*d^2*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]) + (8*b*c^2*d*e*x
*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]) + (4*b*c^2*e^2*x^2*Sqrt[1 + c^2*x^
2])/(a + b*ArcSinh[c*x]) - 8*c*d*e*Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcS
inh[c*x])] + (4*c^2*d^2 - e^2)*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b] +
 3*e^2*CoshIntegral[3*(a/b + ArcSinh[c*x])]*Sinh[(3*a)/b] - 4*c^2*d^2*Cosh[
a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + e^2*Cosh[a/b]*SinhIntegral[a/b + Ar
cSinh[c*x]] + 8*c*d*e*Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])] -
3*e^2*Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c*x])])/(b^2*c^3)
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{e^2x^2+2\,dex+d^2}{b^2\,\mathrm{arsinh}\,(cx)^2+2\,ab\,\mathrm{arsinh}\,(cx)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x)
 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(ex+d)^2}{(b\,\mathrm{arsinh}\,(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2/(b*arcsinh(c*x) + a)^2, x)
```

maple [A] time = 0.38, size = 616, normalized size = 1.72

$$\frac{\frac{\left(4c^3x^3-4c^2x^2\sqrt{c^2x^2+1}+3cx-\sqrt{c^2x^2+1}\right)e^2}{8c^2b(a+b\,\mathrm{arcsinh}(cx))}+\frac{3e^2\mathrm{e}^{\frac{3a}{b}}\,\mathrm{Ei}\left(1,3\,\mathrm{arcsinh}(cx)+\frac{3a}{b}\right)}{8c^2b^2}-\frac{e^2\left(4c^3x^3+3cx+4c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}\right)}{8b\,c^2(a+b\,\mathrm{arcsinh}(cx))}-\frac{3e^2\mathrm{e}^{-\frac{3a}{b}}\,\mathrm{Ei}\left(1,-3\,\mathrm{arcs}\right.}{8b^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/c*(1/8*(4*c^3*x^3-4*c^2*x^2*(c^2*x^2+1)^(1/2)+3*c*x-(c^2*x^2+1)^(1/2))*e^
2/c^2/b/(a+b*arcsinh(c*x))+3/8*e^2/c^2/b^2*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3
*a/b)-1/8/b*e^2/c^2*(4*c^3*x^3+3*c*x+4*c^2*x^2*(c^2*x^2+1)^(1/2)+(c^2*x^2+1
)^(1/2))/(a+b*arcsinh(c*x))-3/8/b^2*e^2/c^2*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x
)-3*a/b)+1/2*(c*x-(c^2*x^2+1)^(1/2))*d^2/b/(a+b*arcsinh(c*x))+1/2*d^2/b^2*e
xp(a/b)*Ei(1,arcsinh(c*x)+a/b)-1/8*(c*x-(c^2*x^2+1)^(1/2))*e^2/c^2/b/(a+b*a
rcsinh(c*x))-1/8/c^2*e^2/b^2*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)-1/2/b*d^2*(c*x
+(c^2*x^2+1)^(1/2))/(a+b*arcsinh(c*x))-1/2/b^2*d^2*exp(-a/b)*Ei(1,-arcsinh(
c*x)-a/b)+1/8/c^2/b*e^2*(c*x+(c^2*x^2+1)^(1/2))/(a+b*arcsinh(c*x))+1/8/c^2/
b^2*e^2*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)+1/2*(2*c^2*x^2-2*c*x*(c^2*x^2+1)^
(1/2)+1)*d*e/c/(a+b*arcsinh(c*x))/b-e*d/c/b^2*exp(2*a/b)*Ei(1,2*arcsinh(c*x
)+2*a/b)-1/2/b*e*d/c*(2*c^2*x^2+1+2*c*x*(c^2*x^2+1)^(1/2))/(a+b*arcsinh(c*x
))-1/b^2*e*d/c*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c^3e^2x^5+2\,c^3dex^4+2\,cdex^2+cd^2x+\left(c^3d^2+ce^2\right)x^3+\left(c^2e^2x^4+2\,c^2dex^3+2\,dex+\left(c^2d^2+e^2\right)x^2+d^2\right)\sqrt{c^2x^2+}}{abc^3x^2+\sqrt{c^2x^2+1}\,abc^2x+abc+\left(b^2c^3x^2+\sqrt{c^2x^2+1}\,b^2c^2x+b^2c\right)\log\left(cx+\sqrt{c^2x^2+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*e^2*x^5 + 2*c^3*d*e*x^4 + 2*c*d*e*x^2 + c*d^2*x + (c^3*d^2 + c*e^2)*x
^3 + (c^2*e^2*x^4 + 2*c^2*d*e*x^3 + 2*d*e*x + (c^2*d^2 + e^2)*x^2 + d^2)*sq
rt(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b^2*
c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2 + 1))
) + integrate((3*c^5*e^2*x^6 + 4*c^5*d*e*x^5 + 8*c^3*d*e*x^3 + (c^5*d^2 + 6
*c^3*e^2)*x^4 + 4*c*d*e*x + c*d^2 + (2*c^3*d^2 + 3*c*e^2)*x^2 + (3*c^3*e^2*
x^4 + 4*c^3*d*e*x^3 - c*d^2 + (c^3*d^2 + c*e^2)*x^2)*(c^2*x^2 + 1) + (6*c^4
*e^2*x^5 + 8*c^4*d*e*x^4 + 8*c^2*d*e*x^2 + (2*c^4*d^2 + 7*c^2*e^2)*x^3 + 2*
d*e + (c^2*d^2 + 2*e^2)*x)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^4 + (c^2*x^2 + 1)*
a*b*c^3*x^2 + 2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 + (c^2*x^2 + 1)*b^2*c^3*
x^2 + 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 + b^2*c^2*x)*sqrt(c^2*x^2 + 1)
)*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^3 + a*b*c^2*x)*sqrt(c^2*x^2 +
 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^2}{(a+b\,\mathrm{asinh}\,(c\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2/(a + b*asinh(c*x))^2,x)
```

```
[Out] int((d + e*x)^2/(a + b*asinh(c*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^2}{(a+b\,\mathrm{asinh}\,(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral((d + e*x)**2/(a + b*asinh(c*x))**2, x)
```

3.26 $\int \frac{d+ex}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=180

$$\frac{e\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{b^2c^2}-\frac{e\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{b^2c^2}-\frac{d\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{b^2c}+\frac{d\cosh\left(\frac{a}{b}\right)\text{Sh}}{b}$$

```
[Out] e*Chi(2*(a+b*arcsinh(c*x))/b)*cosh(2*a/b)/b^2/c^2+d*cosh(a/b)*Shi((a+b*arcs
inh(c*x))/b)/b^2/c-d*Chi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b^2/c-e*Shi(2*(a+b
*arcsinh(c*x))/b)*sinh(2*a/b)/b^2/c^2-d*(c^2*x^2+1)^(1/2)/b/c/(a+b*arcsinh(
c*x))-e*x*(c^2*x^2+1)^(1/2)/b/c/(a+b*arcsinh(c*x))
```

Rubi [A] time = 0.34, antiderivative size = 176, normalized size of antiderivative = 0.98, number of steps used = 11, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5803, 5655, 5779, 3303, 3298, 3301, 5665}

$$\frac{e\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{b^2c^2}-\frac{e\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{b^2c^2}-\frac{d\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{b^2c}+\frac{d\cos}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -((d*Sqrt[1 + c^2*x^2])/(b*c*(a + b*ArcSinh[c*x]))) - (e*x*Sqrt[1 + c^2*x^2
])/(b*c*(a + b*ArcSinh[c*x])) + (e*Cosh[(2*a)/b]*CoshIntegral[(2*a)/b + 2*A
rcSinh[c*x]])/(b^2*c^2) - (d*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b])/(b
^2*c) + (d*Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]])/(b^2*c) - (e*Sinh[(2
*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSinh[c*x]])/(b^2*c^2)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 5803

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*ArcSinh[c*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[m, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{d+ex}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = \int \left(\frac{d}{\left(a+b\sinh^{-1}(cx)\right)^2}+\frac{ex}{\left(a+b\sinh^{-1}(cx)\right)^2}\right)dx$$

$$= d\int \frac{1}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx + e\int \frac{x}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

$$= -\frac{d\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)} - \frac{ex\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{(cd)\int \frac{x}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx}{b}$$

$$= -\frac{d\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)} - \frac{ex\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{d\,\text{Subst}\left(\int \frac{\sinh(x)}{a+bx}\,dx, x, \sinh^{-1}(\right.}{bc}$$

$$= -\frac{d\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)} - \frac{ex\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{e\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(\right.}{b^2c^2}$$

$$= -\frac{d\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)} - \frac{ex\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{e\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(\right.}{b^2c^2}$$

Mathematica [A] time = 0.75, size = 150, normalized size = 0.83

$$-\frac{\frac{bcd\sqrt{c^2x^2+1}}{a+b\sinh^{-1}(cx)}+\frac{bcex\sqrt{c^2x^2+1}}{a+b\sinh^{-1}(cx)}+cd\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)-e\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-cd\,\text{cc}}{b^2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -(((b*c*d*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]) + (b*c*e*x*Sqrt[1 + c^2*x
^2])/(a + b*ArcSinh[c*x]) - e*Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcSinh[c
*x])] + c*d*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b] - c*d*Cosh[a/b]*Sinh
```

```
Integral[a/b + ArcSinh[c*x]] + e*Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSin
h[c*x]])])/(b^2*c^2))
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{ex+d}{b^2\,\text{arsinh}\,(cx)^2+2\,ab\,\text{arsinh}\,(cx)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((e*x + d)/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{ex+d}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)/(b*arcsinh(c*x) + a)^2, x)
```

maple [A] time = 0.27, size = 272, normalized size = 1.51

$$\frac{\frac{\left(cx-\sqrt{c^2x^2+1}\right)d}{2b(a+b\,\text{arcsinh}(cx))}+\frac{d\,e^{\frac{a}{b}}\,\text{Ei}\left(1,\text{arcsinh}(cx)+\frac{a}{b}\right)}{2b^2}-\frac{d\left(cx+\sqrt{c^2x^2+1}\right)}{2b(a+b\,\text{arcsinh}(cx))}-\frac{d\,e^{-\frac{a}{b}}\,\text{Ei}\left(1,-\text{arcsinh}(cx)-\frac{a}{b}\right)}{2b^2}+\frac{\left(2c^2x^2-2cx\sqrt{c^2x^2+1}+1\right)e}{4c(a+b\,\text{arcsinh}(cx))b}-\frac{e\,e^{\frac{2a}{b}}\,\text{Ei}\left(\right.}{}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/c*(1/2*(c*x-(c^2*x^2+1)^(1/2))*d/b/(a+b*arcsinh(c*x))+1/2*d/b^2*exp(a/b)*
Ei(1,arcsinh(c*x)+a/b)-1/2/b*d*(c*x+(c^2*x^2+1)^(1/2))/(a+b*arcsinh(c*x))-1
/2/b^2*d*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)+1/4*(2*c^2*x^2-2*c*x*(c^2*x^2+1)
^(1/2)+1)*e/c/(a+b*arcsinh(c*x))/b-1/2/c*e/b^2*exp(2*a/b)*Ei(1,2*arcsinh(c*
x)+2*a/b)-1/4/c*e/b*(2*c^2*x^2+1+2*c*x*(c^2*x^2+1)^(1/2))/(a+b*arcsinh(c*x)
)-1/2/c*e/b^2*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c^3ex^4+c^3dx^3+cex^2+cdx+\left(c^2ex^3+c^2dx^2+ex+d\right)\sqrt{c^2x^2+1}}{abc^3x^2+\sqrt{c^2x^2+1}\,abc^2x+abc+\left(b^2c^3x^2+\sqrt{c^2x^2+1}\,b^2c^2x+b^2c\right)\log\left(cx+\sqrt{c^2x^2+1}\right)}+\int\frac{}{abc^5x^4+\left(c^2x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*e*x^4 + c^3*d*x^3 + c*e*x^2 + c*d*x + (c^2*e*x^3 + c^2*d*x^2 + e*x +
d)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c +
(b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2
+ 1))) + integrate((2*c^5*e*x^5 + c^5*d*x^4 + 4*c^3*e*x^3 + 2*c^3*d*x^2 + 2
*c*e*x + (2*c^3*e*x^3 + c^3*d*x^2 - c*d)*(c^2*x^2 + 1) + c*d + (4*c^4*e*x^4
 + 2*c^4*d*x^3 + 4*c^2*e*x^2 + c^2*d*x + e)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^4
 + (c^2*x^2 + 1)*a*b*c^3*x^2 + 2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 + (c^2*
x^2 + 1)*b^2*c^3*x^2 + 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 + b^2*c^2*x)*
sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^3 + a*b*c^2*
x)*sqrt(c^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{d+e\,x}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)/(a + b*asinh(c*x))^2,x)
[Out] int((d + e*x)/(a + b*asinh(c*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex}{\left(a+b\operatorname{asinh}\left(cx\right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*asinh(c*x))**2,x)
[Out] Integral((d + e*x)/(a + b*asinh(c*x))**2, x)
```

3.27 $\int \frac{1}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=85

$$-\frac{\sinh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{b^2c}+\frac{\cosh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{b^2c}-\frac{\sqrt{c^2x^2+1}}{bc\left(a+b\sinh^{-1}(cx)\right)}$$

```
[Out] cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b^2/c-Chi((a+b*arcsinh(c*x))/b)*sinh(a/
b)/b^2/c-(c^2*x^2+1)^(1/2)/b/c/(a+b*arcsinh(c*x))
```

Rubi [A] time = 0.18, antiderivative size = 81, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5655, 5779, 3303, 3298, 3301}

$$-\frac{\sinh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{b^2c}+\frac{\cosh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{b^2c}-\frac{\sqrt{c^2x^2+1}}{bc\left(a+b\sinh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^(-2),x]
```

```
[Out] -(Sqrt[1 + c^2*x^2]/(b*c*(a + b*ArcSinh[c*x]))) - (CoshIntegral[a/b + ArcSi
nh[c*x]]*Sinh[a/b])/(b^2*c) + (Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]])/
(b^2*c)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
```

```
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx &= -\frac{\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{c\int \frac{x}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx}{b}\\
&= -\frac{\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{\text{Subst}\left(\int \frac{\sinh(x)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{bc}\\
&= -\frac{\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{\cosh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{bc}-\frac{\sinh\left(\right.}{}\\
&= -\frac{\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{b^2c}+\frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh\right.}{b^2c}
\end{aligned}$$

Mathematica [A] time = 0.19, size = 71, normalized size = 0.84

$$\frac{-\frac{b\sqrt{c^2x^2+1}}{a+b\sinh^{-1}(cx)}-\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)+\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{b^2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])^(-2),x]
[Out] (-((b*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x])) - CoshIntegral[a/b + ArcSinh
[c*x]]*Sinh[a/b] + Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]])/(b^2*c)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{b^2\,\text{arsinh}\,(cx)^2+2\,ab\,\text{arsinh}\,(cx)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral(1/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^(-2), x)
```

maple [A] time = 0.00, size = 118, normalized size = 1.39

$$\frac{\frac{cx-\sqrt{c^2x^2+1}}{2b(a+b\,\text{arcsinh}(cx))}+\frac{e^{\frac{a}{b}}\,\text{Ei}\left(1,\text{arcsinh}(cx)+\frac{a}{b}\right)}{2b^2}-\frac{cx+\sqrt{c^2x^2+1}}{2b(a+b\,\text{arcsinh}(cx))}-\frac{e^{-\frac{a}{b}}\,\text{Ei}\left(1,-\text{arcsinh}(cx)-\frac{a}{b}\right)}{2b^2}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/c*(1/2*(c*x-(c^2*x^2+1)^(1/2))/b/(a+b*arcsinh(c*x))+1/2/b^2*exp(a/b)*Ei(1
,arcsinh(c*x)+a/b)-1/2/b*(c*x+(c^2*x^2+1)^(1/2))/(a+b*arcsinh(c*x))-1/2/b^2
*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c^3x^3 + cx + \left(c^2x^2+1\right)^{\frac{3}{2}}}{abc^3x^2 + \sqrt{c^2x^2+1}\,abc^2x + abc + \left(b^2c^3x^2 + \sqrt{c^2x^2+1}\,b^2c^2x + b^2c\right)\log\left(cx + \sqrt{c^2x^2+1}\right)} + \int \frac{}{abc^4x^4 + \left(c^2x^2 +\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b
*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*
x + sqrt(c^2*x^2 + 1))) + integrate((c^4*x^4 + 2*c^2*x^2 + (c^2*x^2 + 1)*(c
^2*x^2 - 1) + (2*c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1) + 1)/(a*b*c^4*x^4 + (c^2*
x^2 + 1)*a*b*c^2*x^2 + 2*a*b*c^2*x^2 + a*b + (b^2*c^4*x^4 + (c^2*x^2 + 1)*b
^2*c^2*x^2 + 2*b^2*c^2*x^2 + b^2 + 2*(b^2*c^3*x^3 + b^2*c*x)*sqrt(c^2*x^2 +
 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^3*x^3 + a*b*c*x)*sqrt(c^2*x^2
+ 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*asinh(c*x))^2,x)
```

```
[Out] int(1/(a + b*asinh(c*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b \operatorname{asinh}{\left(cx \right)}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral((a + b*asinh(c*x))**(-2), x)
```

3.28 $\int \frac{1}{(d+ex)\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{1}{(d+ex)\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(e*x+d)/(a+b*arcsinh(c*x))^2,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(d+ex)\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((d + e*x)*(a + b*ArcSinh[c*x])^2),x]
[Out] Defer[Int][1/((d + e*x)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=\int \frac{1}{(d+ex)\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A] time = 3.22, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex)\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((d + e*x)*(a + b*ArcSinh[c*x])^2),x]
[Out] Integrate[1/((d + e*x)*(a + b*ArcSinh[c*x])^2), x]
```

fricas [A] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{a^2ex+a^2d+\left(b^2ex+b^2d\right)\text{arsinh}\,(cx)^2+2\,(abex+abd)\,\text{arsinh}\,(cx)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral(1/(a^2*e*x + a^2*d + (b^2*e*x + b^2*d)*arcsinh(c*x)^2 + 2*(a*b*e*x
 + a*b*d)*arcsinh(c*x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ex+d)(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((e*x + d)*(b*arcsinh(c*x) + a)^2), x)
```

maple [A] time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{1}{(ex+d)\,(a+b\,\mathrm{arcsinh}\,(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(1/(e*x+d)/(a+b*arcsinh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c^3x^3+cx+\left(c^2x^2+1\right)^{\frac{3}{2}}}{abc^3ex^3+abc^3dx^2+abcex+abcd+\left(b^2c^3ex^3+b^2c^3dx^2+b^2cex+b^2cd+\left(b^2c^2ex^2+b^2c^2dx\right)\sqrt{c^2x^2+1}\right)\log\left(c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2))/(a*b*c^3*e*x^3 + a*b*c^3*d*x^2 + a*b
*c*e*x + a*b*c*d + (b^2*c^3*e*x^3 + b^2*c^3*d*x^2 + b^2*c*e*x + b^2*c*d + (
b^2*c^2*e*x^2 + b^2*c^2*d*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)
) + (a*b*c^2*e*x^2 + a*b*c^2*d*x)*sqrt(c^2*x^2 + 1)) + integrate((c^5*d*x^4
 + 2*c^3*d*x^2 + (c^3*d*x^2 - 2*c*e*x - c*d)*(c^2*x^2 + 1) + c*d + (2*c^4*d
*x^3 - 2*c^2*e*x^2 + c^2*d*x - e)*sqrt(c^2*x^2 + 1))/(a*b*c^5*e^2*x^6 + 2*a
*b*c^5*d*e*x^5 + 4*a*b*c^3*d*e*x^3 + (c^5*d^2 + 2*c^3*e^2)*a*b*x^4 + 2*a*b*
c*d*e*x + a*b*c*d^2 + (2*c^3*d^2 + c*e^2)*a*b*x^2 + (a*b*c^3*e^2*x^4 + 2*a*
b*c^3*d*e*x^3 + a*b*c^3*d^2*x^2)*(c^2*x^2 + 1) + (b^2*c^5*e^2*x^6 + 2*b^2*c
^5*d*e*x^5 + 4*b^2*c^3*d*e*x^3 + (c^5*d^2 + 2*c^3*e^2)*b^2*x^4 + 2*b^2*c*d*
e*x + b^2*c*d^2 + (2*c^3*d^2 + c*e^2)*b^2*x^2 + (b^2*c^3*e^2*x^4 + 2*b^2*c^
3*d*e*x^3 + b^2*c^3*d^2*x^2)*(c^2*x^2 + 1) + 2*(b^2*c^4*e^2*x^5 + 2*b^2*c^4
*d*e*x^4 + 2*b^2*c^2*d*e*x^2 + b^2*c^2*d^2*x + (c^4*d^2 + c^2*e^2)*b^2*x^3)
*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*e^2*x^5 + 2*a
*b*c^4*d*e*x^4 + 2*a*b*c^2*d*e*x^2 + a*b*c^2*d^2*x + (c^4*d^2 + c^2*e^2)*a*
b*x^3)*sqrt(c^2*x^2 + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*asinh(c*x))^2*(d + e*x)),x)
```

```
[Out] int(1/((a + b*asinh(c*x))^2*(d + e*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\,\mathrm{asinh}\,(cx))^2\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(1/((a + b*asinh(c*x))**2*(d + e*x)), x)
```

3.29 $\int \frac{1}{(d+ex)^2\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{1}{(d+ex)^2\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(e*x+d)^2/(a+b*arcsinh(c*x))^2,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(d+ex)^2\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((d + e*x)^2*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Defer[Int][1/((d + e*x)^2*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=\int \frac{1}{(d+ex)^2\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A] time = 5.21, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex)^2\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((d + e*x)^2*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Integrate[1/((d + e*x)^2*(a + b*ArcSinh[c*x])^2), x]
```

fricas [A] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{a^2e^2x^2+2\,a^2dex+a^2d^2+\left(b^2e^2x^2+2\,b^2dex+b^2d^2\right)\text{arsinh}\,(cx)^2+2\left(abe^2x^2+2\,abdex+abd^2\right)\text{arsi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(a^2*e^2*x^2 + 2*a^2*d*e*x + a^2*d^2 + (b^2*e^2*x^2 + 2*b^2*d*e*
x + b^2*d^2)*arcsinh(c*x)^2 + 2*(a*b*e^2*x^2 + 2*a*b*d*e*x + a*b*d^2)*arcsi
nh(c*x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ex+d)^2(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((e*x + d)^2*(b*arcsinh(c*x) + a)^2), x)
```

maple [A] time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ex+d\right)^{2} \left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(1/(e*x+d)^2/(a+b*arcsinh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c^{3}\cdots}{abc^{3}e^{2}x^{4}+2\,abc^{3}dex^{3}+2\,abcdex+abcd^{2}+\left(c^{3}d^{2}+ce^{2}\right)abx^{2}+\left(b^{2}c^{3}e^{2}x^{4}+2\,b^{2}c^{3}dex^{3}+2\,b^{2}cdex+b^{2}cd^{2}+\left(c^{3}\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2))/(a*b*c^3*e^2*x^4 + 2*a*b*c^3*d*e*x^3
 + 2*a*b*c*d*e*x + a*b*c*d^2 + (c^3*d^2 + c*e^2)*a*b*x^2 + (b^2*c^3*e^2*x^4
 + 2*b^2*c^3*d*e*x^3 + 2*b^2*c*d*e*x + b^2*c*d^2 + (c^3*d^2 + c*e^2)*b^2*x^
2 + (b^2*c^2*e^2*x^3 + 2*b^2*c^2*d*e*x^2 + b^2*c^2*d^2*x)*sqrt(c^2*x^2 + 1)
)*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^2*e^2*x^3 + 2*a*b*c^2*d*e*x^2 + a*b
*c^2*d^2*x)*sqrt(c^2*x^2 + 1)) - integrate((c^5*e*x^5 - c^5*d*x^4 + 2*c^3*e
*x^3 - 2*c^3*d*x^2 + c*e*x + (c^3*e*x^3 - c^3*d*x^2 + 3*c*e*x + c*d)*(c^2*x
^2 + 1) - c*d + (2*c^4*e*x^4 - 2*c^4*d*x^3 + 5*c^2*e*x^2 - c^2*d*x + 2*e)*s
qrt(c^2*x^2 + 1))/(a*b*c^5*e^3*x^7 + 3*a*b*c^5*d*e^2*x^6 + (3*c^5*d^2*e + 2
*c^3*e^3)*a*b*x^5 + 3*a*b*c*d^2*e*x + (c^5*d^3 + 6*c^3*d*e^2)*a*b*x^4 + a*b
*c*d^3 + (6*c^3*d^2*e + c*e^3)*a*b*x^3 + (2*c^3*d^3 + 3*c*d*e^2)*a*b*x^2 +
(a*b*c^3*e^3*x^5 + 3*a*b*c^3*d*e^2*x^4 + 3*a*b*c^3*d^2*e*x^3 + a*b*c^3*d^3*
x^2)*(c^2*x^2 + 1) + (b^2*c^5*e^3*x^7 + 3*b^2*c^5*d*e^2*x^6 + (3*c^5*d^2*e
+ 2*c^3*e^3)*b^2*x^5 + 3*b^2*c*d^2*e*x + (c^5*d^3 + 6*c^3*d*e^2)*b^2*x^4 +
b^2*c*d^3 + (6*c^3*d^2*e + c*e^3)*b^2*x^3 + (2*c^3*d^3 + 3*c*d*e^2)*b^2*x^2
 + (b^2*c^3*e^3*x^5 + 3*b^2*c^3*d*e^2*x^4 + 3*b^2*c^3*d^2*e*x^3 + b^2*c^3*d
^3*x^2)*(c^2*x^2 + 1) + 2*(b^2*c^4*e^3*x^6 + 3*b^2*c^4*d*e^2*x^5 + 3*b^2*c^
2*d^2*e*x^2 + b^2*c^2*d^3*x + (3*c^4*d^2*e + c^2*e^3)*b^2*x^4 + (c^4*d^3 +
3*c^2*d*e^2)*b^2*x^3)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(
a*b*c^4*e^3*x^6 + 3*a*b*c^4*d*e^2*x^5 + 3*a*b*c^2*d^2*e*x^2 + a*b*c^2*d^3*x
 + (3*c^4*d^2*e + c^2*e^3)*a*b*x^4 + (c^4*d^3 + 3*c^2*d*e^2)*a*b*x^3)*sqrt(
c^2*x^2 + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^{2}\,{\left(d+e\,x\right)}^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*asinh(c*x))^2*(d + e*x)^2),x)
```

```
[Out] int(1/((a + b*asinh(c*x))^2*(d + e*x)^2), x)
```

sympy [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\operatorname{asinh}(cx))^2(d+ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(1/((a + b*asinh(c*x))**2*(d + e*x)**2), x)
```

3.30 $\int (d+ex)^m \left(a+b\sinh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=75

$$\frac{(d+ex)^{m+1}\left(a+b\sinh^{-1}(cx)\right)^2}{e(m+1)}-\frac{2bc\text{Int}\left(\frac{(d+ex)^{m+1}\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2x^2+1}},x\right)}{e(m+1)}$$

```
[Out] (e*x+d)^(1+m)*(a+b*arcsinh(c*x))^2/e/(1+m)-2*b*c*Unintegrable((e*x+d)^(1+m)
*(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x)/e/(1+m)
```

Rubi [A] time = 0.26, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (d+ex)^m \left(a+b\sinh^{-1}(cx)\right)^2 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(d + e*x)^m*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] ((d + e*x)^(1 + m)*(a + b*ArcSinh[c*x])^2)/(e*(1 + m)) - (2*b*c*Defer[Int][
((d + e*x)^(1 + m)*(a + b*ArcSinh[c*x]))/Sqrt[1 + c^2*x^2], x])/(e*(1 + m))
```

Rubi steps

$$\int (d+ex)^m \left(a+b\sinh^{-1}(cx)\right)^2 \, dx=\frac{(d+ex)^{1+m}\left(a+b\sinh^{-1}(cx)\right)^2}{e(1+m)}-\frac{(2bc)\int \frac{(d+ex)^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}} \, dx}{e(1+m)}$$

Mathematica [A] time = 4.17, size = 0, normalized size = 0.00

$$\int (d+ex)^m \left(a+b\sinh^{-1}(cx)\right)^2 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d + e*x)^m*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] Integrate[(d + e*x)^m*(a + b*ArcSinh[c*x])^2, x]
```

fricas [A] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2\,\text{arsinh}\,(cx)^2+2\,ab\,\text{arsinh}\,(cx)+a^2\right)(ex+d)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)*(e*x + d)^m, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\,\text{arsinh}\,(cx)+a)^2(ex+d)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*(e*x + d)^m, x)
```

maple [A] time = 3.60, size = 0, normalized size = 0.00

$$\int (ex+d)^{m} (a+b \operatorname{arcsinh}(cx))^{2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int((e*x+d)^m*(a+b*arcsinh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(b^{2}ex+b^{2}d\right)(ex+d)^{m}\log\left(cx+\sqrt{c^{2}x^{2}+1}\right)^{2}}{e(m+1)}+\frac{(ex+d)^{m+1}a^{2}}{e(m+1)}+\int -\frac{2\left(\left(b^{2}c^{2}dx-abe(m+1)-\left(abc^{2}e(m+1)-\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] (b^2*e*x + b^2*d)*(e*x + d)^m*log(c*x + sqrt(c^2*x^2 + 1))^2/(e*(m + 1)) +
(e*x + d)^(m + 1)*a^2/(e*(m + 1)) + integrate(-2*((b^2*c^2*d*x - a*b*e*(m +
 1) - (a*b*c^2*e*(m + 1) - b^2*c^2*e)*x^2)*sqrt(c^2*x^2 + 1)*(e*x + d)^m +
(b^2*c^3*d*x^2 + b^2*c*d - (a*b*c^3*e*(m + 1) - b^2*c^3*e)*x^3 - (a*b*c*e*(
m + 1) - b^2*c*e)*x)*(e*x + d)^m)*log(c*x + sqrt(c^2*x^2 + 1))/(c^3*e*(m +
1)*x^3 + c*e*(m + 1)*x + (c^2*e*(m + 1)*x^2 + e*(m + 1))*sqrt(c^2*x^2 + 1))
, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,\mathrm{asinh}\left(c\,x\right))^2\,(d+e\,x)^m \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2*(d + e*x)^m,x)
```

```
[Out] int((a + b*asinh(c*x))^2*(d + e*x)^m, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b \operatorname{asinh}(cx))^{2} (d+ex)^{m} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral((a + b*asinh(c*x))**2*(d + e*x)**m, x)
```

3.31 $\int (d+ex)^m \left(a+b\sinh^{-1}(cx)\right) dx$

Optimal. Leaf size=179

$$\frac{(d+ex)^{m+1}\left(a+b\sinh^{-1}(cx)\right)}{e(m+1)}-\frac{bc\sqrt{1-\frac{d+ex}{d-\frac{e}{\sqrt{-c^2}}}}\sqrt{1-\frac{d+ex}{\frac{e}{\sqrt{-c^2}}+d}}(d+ex)^{m+2}F_1\left(m+2;\frac{1}{2},\frac{1}{2};m+3;\frac{d+ex}{d-\frac{e}{\sqrt{-c^2}}},\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}\right)}{e^2(m+1)(m+2)\sqrt{c^2x^2+1}}$$

```
[Out] (e*x+d)^(1+m)*(a+b*arcsinh(c*x))/e/(1+m)-b*c*(e*x+d)^(2+m)*AppellF1(2+m,1/2
,1/2,3+m,(e*x+d)/(d-e/(-c^2)^(1/2)),(e*x+d)/(d+e/(-c^2)^(1/2)))*(1+(-e*x-d)
/(d-e/(-c^2)^(1/2)))^(1/2)*(1+(-e*x-d)/(d+e/(-c^2)^(1/2)))^(1/2)/e^2/(1+m)/
(2+m)/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {5801, 760, 133}

$$\frac{(d+ex)^{m+1}\left(a+b\sinh^{-1}(cx)\right)}{e(m+1)}-\frac{bc\sqrt{1-\frac{d+ex}{d-\frac{e}{\sqrt{-c^2}}}}\sqrt{1-\frac{d+ex}{\frac{e}{\sqrt{-c^2}}+d}}(d+ex)^{m+2}F_1\left(m+2;\frac{1}{2},\frac{1}{2};m+3;\frac{d+ex}{d-\frac{e}{\sqrt{-c^2}}},\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}\right)}{e^2(m+1)(m+2)\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -((b*c*(d + e*x)^(2 + m)*Sqrt[1 - (d + e*x)/(d - e/Sqrt[-c^2])]*Sqrt[1 - (d
 + e*x)/(d + e/Sqrt[-c^2])]*AppellF1[2 + m, 1/2, 1/2, 3 + m, (d + e*x)/(d -
 e/Sqrt[-c^2]), (d + e*x)/(d + e/Sqrt[-c^2])])/(e^2*(1 + m)*(2 + m)*Sqrt[1
+ c^2*x^2])) + ((d + e*x)^(1 + m)*(a + b*ArcSinh[c*x]))/(e*(1 + m))
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 760

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[(a + c*x^2)^p/(e*(1 - (d + e*x)/(d + (e*q)/c))^p*
(1 - (d + e*x)/(d - (e*q)/c))^p), Subst[Int[x^m*Simp[1 - x/(d + (e*q)/c), x
]^p*Simp[1 - x/(d - (e*q)/c), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, c, d,
 e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 5801

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n
 - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rubi steps

$$\int (d+ex)^m \left(a+b\sinh^{-1}(cx)\right) dx = \frac{(d+ex)^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{e(1+m)} - \frac{(bc)\int \frac{(d+ex)^{1+m}}{\sqrt{1+c^2x^2}} dx}{e(1+m)}$$

$$= \frac{(d+ex)^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{e(1+m)} - \frac{\left(bc\sqrt{1-\frac{d+ex}{d-\frac{\sqrt{-c^2}e}{c^2}}}\sqrt{1-\frac{d+ex}{d+\frac{\sqrt{-c^2}e}{c^2}}}\right)\text{Subst}}{e^2(1+m)}$$

$$= -\frac{bc(d+ex)^{2+m}\sqrt{1-\frac{d+ex}{d-\frac{e}{\sqrt{-c^2}}}}\sqrt{1-\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}}F_1\left(2+m;\frac{1}{2},\frac{1}{2};3+m;\frac{d+ex}{d-\frac{e}{\sqrt{-c^2}}},\right.}{e^2(1+m)(2+m)\sqrt{1+c^2x^2}}$$

Mathematica [F] time = 0.05, size = 0, normalized size = 0.00

$$\int (d+ex)^m \left(a+b\sinh^{-1}(cx)\right) dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d + e*x)^m*(a + b*ArcSinh[c*x]),x]
[Out] Integrate[(d + e*x)^m*(a + b*ArcSinh[c*x]), x]
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left((b\,\text{arsinh}\,(cx)+a)(ex+d)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral((b*arcsinh(c*x) + a)*(e*x + d)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\,\text{arsinh}\,(cx)+a)(ex+d)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)*(e*x + d)^m, x)
```

maple [F] time = 3.60, size = 0, normalized size = 0.00

$$\int (ex+d)^m\,(a+b\,\text{arcsinh}\,(cx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(a+b*arcsinh(c*x)),x)
[Out] int((e*x+d)^m*(a+b*arcsinh(c*x)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b\left(\frac{(ex+d)(ex+d)^m\log\left(cx+\sqrt{c^2x^2+1}\right)}{e(m+1)} - \int \frac{\left(c^2ex^2+c^2dx\right)(ex+d)^m}{c^2e(m+1)x^2+e(m+1)}\,dx - \int \frac{(cex+}{c^3e(m+1)x^3+ce(m+1)x+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] b*((e*x + d)*(e*x + d)^m*log(c*x + sqrt(c^2*x^2 + 1))/(e*(m + 1)) - integra
te((c^2*e*x^2 + c^2*d*x)*(e*x + d)^m/(c^2*e*(m + 1)*x^2 + e*(m + 1)), x) -
integrate((c*e*x + c*d)*(e*x + d)^m/(c^3*e*(m + 1)*x^3 + c*e*(m + 1)*x + (c
^2*e*(m + 1)*x^2 + e*(m + 1))*sqrt(c^2*x^2 + 1)), x)) + (e*x + d)^(m + 1)*a
/(e*(m + 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b\,\mathrm{asinh}\,(c\,x))\,\,(d + e\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))*(d + e*x)^m,x)
```

```
[Out] int((a + b*asinh(c*x))*(d + e*x)^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\,\mathrm{asinh}\,(cx))\,(d + ex)^m\,\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(a+b*asinh(c*x)),x)
```

```
[Out] Integral((a + b*asinh(c*x))*(d + e*x)**m, x)
```

3.32 $\int \frac{(d+ex)^m}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{(d+ex)^m}{a+b\sinh^{-1}(cx)},x\right)$$

```
[Out] Unintegrable((e*x+d)^m/(a+b*arcsinh(c*x)),x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(d+ex)^m}{a+b\sinh^{-1}(cx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d + e*x)^m/(a + b*ArcSinh[c*x]),x]
[Out] Defer[Int][(d + e*x)^m/(a + b*ArcSinh[c*x]), x]
```

Rubi steps

$$\int \frac{(d+ex)^m}{a+b\sinh^{-1}(cx)}\,dx=\int \frac{(d+ex)^m}{a+b\sinh^{-1}(cx)}\,dx$$

Mathematica [A] time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^m}{a+b\sinh^{-1}(cx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d + e*x)^m/(a + b*ArcSinh[c*x]),x]
[Out] Integrate[(d + e*x)^m/(a + b*ArcSinh[c*x]), x]
```

fricas [A] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(ex+d)^m}{b\,\text{arsinh}\,(cx)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral((e*x + d)^m/(b*arcsinh(c*x) + a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^m}{b\,\text{arsinh}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] integrate((e*x + d)^m/(b*arcsinh(c*x) + a), x)
```

maple [A] time = 1.20, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^m}{a+b\,\mathrm{arcsinh}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(a+b*arcsinh(c*x)),x)
[Out] int((e*x+d)^m/(a+b*arcsinh(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^m}{b\,\mathrm{arsinh}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate((e*x + d)^m/(b*arcsinh(c*x) + a), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(d+e\,x)^m}{a+b\,\mathrm{asinh}\,(c\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^m/(a + b*asinh(c*x)),x)
[Out] int((d + e*x)^m/(a + b*asinh(c*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^m}{a+b\,\mathrm{asinh}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(a+b*asinh(c*x)),x)
[Out] Integral((d + e*x)**m/(a + b*asinh(c*x)), x)
```

3.33 $\int \frac{(d+ex)^m}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=21

$$\operatorname{Int}\left(\frac{(d+ex)^m}{\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable((e*x+d)^m/(a+b*arcsinh(c*x))^2,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(d+ex)^m}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d + e*x)^m/(a + b*ArcSinh[c*x])^2,x]
[Out] Defer[Int][(d + e*x)^m/(a + b*ArcSinh[c*x])^2, x]
```

Rubi steps

$$\int \frac{(d+ex)^m}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = \int \frac{(d+ex)^m}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A] time = 0.78, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^m}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d + e*x)^m/(a + b*ArcSinh[c*x])^2,x]
[Out] Integrate[(d + e*x)^m/(a + b*ArcSinh[c*x])^2, x]
```

fricas [A] time = 0.57, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{(ex+d)^m}{b^2\operatorname{arsinh}(cx)^2+2\,ab\operatorname{arsinh}(cx)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
[Out] integral((e*x + d)^m/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^m}{(b\operatorname{arsinh}(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^m/(b*arcsinh(c*x) + a)^2, x)
```

maple [A] time = 1.26, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^m}{(a+b\,\mathrm{arcsinh}\,(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int((e*x+d)^m/(a+b*arcsinh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(c^2x^2+1\right)^{\frac{3}{2}}(ex+d)^m+\left(c^3x^3+cx\right)(ex+d)^m}{abc^3x^2+\sqrt{c^2x^2+1}\,abc^2x+abc+\left(b^2c^3x^2+\sqrt{c^2x^2+1}\,b^2c^2x+b^2c\right)\log\left(cx+\sqrt{c^2x^2+1}\right)}+\int \frac{}{abc^5ex^5+abc^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^2*x^2 + 1)^(3/2)*(e*x + d)^m + (c^3*x^3 + c*x)*(e*x + d)^m)/(a*b*c^3*x
^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)
*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate(((c^3*e*(m +
1)*x^3 + c^3*d*x^2 + c*e*(m - 1)*x - c*d)*(c^2*x^2 + 1)*(e*x + d)^m + (2*c^
4*e*(m + 1)*x^4 + 2*c^4*d*x^3 + c^2*e*(3*m + 1)*x^2 + c^2*d*x + e*m)*sqrt(c
^2*x^2 + 1)*(e*x + d)^m + (c^5*e*(m + 1)*x^5 + c^5*d*x^4 + 2*c^3*e*(m + 1)*
x^3 + 2*c^3*d*x^2 + c*e*(m + 1)*x + c*d)*(e*x + d)^m)/(a*b*c^5*e*x^5 + a*b*
c^5*d*x^4 + 2*a*b*c^3*e*x^3 + 2*a*b*c^3*d*x^2 + a*b*c*e*x + a*b*c*d + (a*b*
c^3*e*x^3 + a*b*c^3*d*x^2)*(c^2*x^2 + 1) + (b^2*c^5*e*x^5 + b^2*c^5*d*x^4 +
 2*b^2*c^3*e*x^3 + 2*b^2*c^3*d*x^2 + b^2*c*e*x + b^2*c*d + (b^2*c^3*e*x^3 +
 b^2*c^3*d*x^2)*(c^2*x^2 + 1) + 2*(b^2*c^4*e*x^4 + b^2*c^4*d*x^3 + b^2*c^2*
e*x^2 + b^2*c^2*d*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a
*b*c^4*e*x^4 + a*b*c^4*d*x^3 + a*b*c^2*e*x^2 + a*b*c^2*d*x)*sqrt(c^2*x^2 +
1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(d+e\,x)^m}{(a+b\,\mathrm{asinh}\,(c\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^m/(a + b*asinh(c*x))^2,x)
```

```
[Out] int((d + e*x)^m/(a + b*asinh(c*x))^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^m}{(a+b\,\mathrm{asinh}\,(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral((d + e*x)**m/(a + b*asinh(c*x))**2, x)
```

3.34 $\int (f+gx)^3\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)dx$

Optimal. Leaf size=640

$$\frac{1}{2}f^3x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{f^3\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc\sqrt{c^2x^2+1}}+\frac{f^2g\left(c^2x^2+1\right)\sqrt{c^2dx^2+d}\left(a+b\sin\right.}{c^2}$$

```
[Out] 1/2*f^3*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)+3/8*f*g^2*x*(a+b*arcsinh(c
*x))*(c^2*d*x^2+d)^(1/2)/c^2+3/4*f*g^2*x^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)
^(1/2)+f^2*g*(c^2*x^2+1)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^2-1/3*g^3
*(c^2*x^2+1)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^4+1/5*g^3*(c^2*x^2+1)
^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^4-b*f^2*g*x*(c^2*d*x^2+d)^(1/2)
/c/(c^2*x^2+1)^(1/2)+2/15*b*g^3*x*(c^2*d*x^2+d)^(1/2)/c^3/(c^2*x^2+1)^(1/2)
-1/4*b*c*f^3*x^2*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-3/16*b*f*g^2*x^2*(c^
2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-1/3*b*c*f^2*g*x^3*(c^2*d*x^2+d)^(1/2)/
(c^2*x^2+1)^(1/2)-1/45*b*g^3*x^3*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-3/
16*b*c*f*g^2*x^4*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/25*b*c*g^3*x^5*(c^
2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+1/4*f^3*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+
d)^(1/2)/b/c/(c^2*x^2+1)^(1/2)-3/16*f*g^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d
)^(1/2)/b/c^3/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.69, antiderivative size = 640, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5835, 5821, 5682, 5675, 30, 5717, 5742, 5758, 266, 43, 5732, 12}

$$\frac{f^2g\left(c^2x^2+1\right)\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{c^2}+\frac{1}{2}f^3x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{f^3\sqrt{c^2dx^2+d}\left(a+b\sin\right.}{4bc\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -((b*f^2*g*x*Sqrt[d + c^2*d*x^2])/(c*Sqrt[1 + c^2*x^2])) + (2*b*g^3*x*Sqrt[
d + c^2*d*x^2])/(15*c^3*Sqrt[1 + c^2*x^2]) - (b*c*f^3*x^2*Sqrt[d + c^2*d*x^
2])/(4*Sqrt[1 + c^2*x^2]) - (3*b*f*g^2*x^2*Sqrt[d + c^2*d*x^2])/(16*c*Sqrt[
1 + c^2*x^2]) - (b*c*f^2*g*x^3*Sqrt[d + c^2*d*x^2])/(3*Sqrt[1 + c^2*x^2]) -
 (b*g^3*x^3*Sqrt[d + c^2*d*x^2])/(45*c*Sqrt[1 + c^2*x^2]) - (3*b*c*f*g^2*x^
4*Sqrt[d + c^2*d*x^2])/(16*Sqrt[1 + c^2*x^2]) - (b*c*g^3*x^5*Sqrt[d + c^2*d
*x^2])/(25*Sqrt[1 + c^2*x^2]) + (f^3*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c
*x]))/2 + (3*f*g^2*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(8*c^2) + (3
*f*g^2*x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/4 + (f^2*g*(1 + c^2*x^
2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/c^2 - (g^3*(1 + c^2*x^2)*Sqrt[
d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(3*c^4) + (g^3*(1 + c^2*x^2)^2*Sqrt[d
+ c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(5*c^4) + (f^3*Sqrt[d + c^2*d*x^2]*(a +
b*ArcSinh[c*x])^2)/(4*b*c*Sqrt[1 + c^2*x^2]) - (3*f*g^2*Sqrt[d + c^2*d*x^2]
*(a + b*ArcSinh[c*x])^2)/(16*b*c^3*Sqrt[1 + c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5732

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(1 + c^2*x^2)^p, x]}, Dist[d^p*(a + b
*ArcSinh[c*x]), u, x] - Dist[b*c*d^p, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*
x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegerQ
[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0]) && NeQ[p, -2^
(-1)] && GtQ[d, 0]
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
```

```
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5835

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(d^IntPart[p]*(d + e*x^2)^FracPa
rt[p])/(1 + c^2*x^2)^FracPart[p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*Ar
cSinh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d
] && IntegerQ[m] && IntegerQ[p - 1/2] &&  !GtQ[d, 0]
```

Rubi steps

$$\int (f+gx)^3\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)dx=\frac{\sqrt{d+c^2dx^2}\int (f+gx)^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)dx}{\sqrt{1+c^2x^2}}$$

$$=\frac{\sqrt{d+c^2dx^2}\int\left(f^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)+3f^2gx\sqrt{1+$$

$$=\frac{\left(f^3\sqrt{d+c^2dx^2}\right)\int\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)dx}{\sqrt{1+c^2x^2}}+\frac{\left(3f^2g$$

$$=\frac{1}{2}f^3x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{4}fg^2x^3\sqrt{d+c^2dx^2}\left(a$$

$$=-\frac{bf^2gx\sqrt{d+c^2dx^2}}{c\sqrt{1+c^2x^2}}-\frac{bcf^3x^2\sqrt{d+c^2dx^2}}{4\sqrt{1+c^2x^2}}-\frac{bcf^2gx^3\sqrt{d+c^2}$$

$$=-\frac{bf^2gx\sqrt{d+c^2dx^2}}{c\sqrt{1+c^2x^2}}+\frac{2bg^3x\sqrt{d+c^2dx^2}}{15c^3\sqrt{1+c^2x^2}}-\frac{bcf^3x^2\sqrt{d+c^2dx}$$

Mathematica [A] time = 1.49, size = 413, normalized size = 0.65

$$\frac{3600ac\sqrt{d}f\sqrt{c^2x^2+1}\left(4c^2f^2-3g^2\right)\log\left(\sqrt{d}\sqrt{c^2dx^2+d}+cdx\right)+240a\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}\left(6c^4x\left(10f^3+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (240*a*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]*(-16*g^3 + c^2*g*(120*f^2 + 45
*f*g*x + 8*g^2*x^2) + 6*c^4*x*(10*f^3 + 20*f^2*g*x + 15*f*g^2*x^2 + 4*g^3*x
^3)) - 9600*b*c^2*f^2*g*Sqrt[d + c^2*d*x^2]*(3*c*x + c^3*x^3 - 3*(1 + c^2*x
^2)^(3/2)*ArcSinh[c*x]) - 128*b*g^3*Sqrt[d + c^2*d*x^2]*(c*x*(-30 + 5*c^2*x
```

```
^2 + 9*c^4*x^4) - 15*Sqrt[1 + c^2*x^2]*(-2 + c^2*x^2 + 3*c^4*x^4)*ArcSinh[c
*x]) + 3600*a*c*Sqrt[d]*f*(4*c^2*f^2 - 3*g^2)*Sqrt[1 + c^2*x^2]*Log[c*d*x +
 Sqrt[d]*Sqrt[d + c^2*d*x^2]] - 3600*b*c^3*f^3*Sqrt[d + c^2*d*x^2]*(Cosh[2*
ArcSinh[c*x]] - 2*ArcSinh[c*x]*(ArcSinh[c*x] + Sinh[2*ArcSinh[c*x]])) - 675
*b*c*f*g^2*Sqrt[d + c^2*d*x^2]*(8*ArcSinh[c*x]^2 + Cosh[4*ArcSinh[c*x]] - 4
*ArcSinh[c*x]*Sinh[4*ArcSinh[c*x]]))/(28800*c^4*Sqrt[1 + c^2*x^2])
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(ag^3x^3 + 3\,afg^2x^2 + 3\,af^2gx + af^3 + \left(bg^3x^3 + 3\,bfg^2x^2 + 3\,bf^2gx + bf^3\right)\text{arsinh}\,(cx)\right)\sqrt{c^2dx^2 + d}\,, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="fr
icas")
```

```
[Out] integral((a*g^3*x^3 + 3*a*f*g^2*x^2 + 3*a*f^2*g*x + a*f^3 + (b*g^3*x^3 + 3*
b*f*g^2*x^2 + 3*b*f^2*g*x + b*f^3)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="gi
ac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [A] time = 0.80, size = 1119, normalized size = 1.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -2/15*a*g^3/d/c^4*(c^2*d*x^2+d)^(3/2)+1/2*a*f^3*x*(c^2*d*x^2+d)^(1/2)+3/4*b
*(d*(c^2*x^2+1))^(1/2)*f*g^2*c^2/(c^2*x^2+1)*arcsinh(c*x)*x^5+3/8*b*(d*(c^2
*x^2+1))^(1/2)*f*g^2/c^2/(c^2*x^2+1)*arcsinh(c*x)*x+b*(d*(c^2*x^2+1))^(1/2)
*g*c^2/(c^2*x^2+1)*arcsinh(c*x)*x^4*f^2+1/5*a*g^3*x^2*(c^2*d*x^2+d)^(3/2)/c
^2/d-3/8*a*f*g^2/c^2*x*(c^2*d*x^2+d)^(1/2)+a*f^2*g/c^2/d*(c^2*d*x^2+d)^(3/2
)+1/2*b*(d*(c^2*x^2+1))^(1/2)*f^3/(c^2*x^2+1)*arcsinh(c*x)*x+1/4*b*(d*(c^2*
x^2+1))^(1/2)*f^3*arcsinh(c*x)^2/(c^2*x^2+1)^(1/2)/c-2/15*b*(d*(c^2*x^2+1))
^(1/2)*g^3/c^4/(c^2*x^2+1)*arcsinh(c*x)+4/15*b*(d*(c^2*x^2+1))^(1/2)*g^3/(c
^2*x^2+1)*arcsinh(c*x)*x^4-1/25*b*(d*(c^2*x^2+1))^(1/2)*g^3*c/(c^2*x^2+1)^(
1/2)*x^5-1/45*b*(d*(c^2*x^2+1))^(1/2)*g^3/c/(c^2*x^2+1)^(1/2)*x^3+2/15*b*(d
*(c^2*x^2+1))^(1/2)*g^3/c^3/(c^2*x^2+1)^(1/2)*x-3/128*b*(d*(c^2*x^2+1))^(1/
2)*f*g^2/c^3/(c^2*x^2+1)^(1/2)-1/4*b*(d*(c^2*x^2+1))^(1/2)*f^3*c/(c^2*x^2+1
)^(1/2)*x^2-1/8*b*(d*(c^2*x^2+1))^(1/2)*f^3/c/(c^2*x^2+1)^(1/2)+1/2*a*f^3*d
*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)-3/8*a*f*g^2/c^
2*d*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)-3/16*b*(d*(
c^2*x^2+1))^(1/2)*f*g^2*c/(c^2*x^2+1)^(1/2)*x^4-3/16*b*(d*(c^2*x^2+1))^(1/2
)*f*g^2/c/(c^2*x^2+1)^(1/2)*x^2-1/3*b*(d*(c^2*x^2+1))^(1/2)*g*c/(c^2*x^2+1)
^(1/2)*x^3*f^2-b*(d*(c^2*x^2+1))^(1/2)*g/c/(c^2*x^2+1)^(1/2)*x*f^2-3/16*b*(
d*(c^2*x^2+1))^(1/2)*f*arcsinh(c*x)^2/(c^2*x^2+1)^(1/2)/c^3*g^2+1/5*b*(d*(c
^2*x^2+1))^(1/2)*g^3*c^2/(c^2*x^2+1)*arcsinh(c*x)*x^6-1/15*b*(d*(c^2*x^2+1)
)^(1/2)*g^3/c^2/(c^2*x^2+1)*arcsinh(c*x)*x^2+b*(d*(c^2*x^2+1))^(1/2)*g/c^2/
(c^2*x^2+1)*arcsinh(c*x)*f^2+1/2*b*(d*(c^2*x^2+1))^(1/2)*f^3*c^2/(c^2*x^2+1
```

```
)*arcsinh(c*x)*x^3+2*b*(d*(c^2*x^2+1))^(1/2)*g/(c^2*x^2+1)*arcsinh(c*x)*x^2
*f^2+9/8*b*(d*(c^2*x^2+1))^(1/2)*f*g^2/(c^2*x^2+1)*arcsinh(c*x)*x^3+3/4*a*f
*g^2*x*(c^2*d*x^2+d)^(3/2)/c^2/d
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="ma
xima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(f+g\,x\right)^3\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,\sqrt{d\,c^2\,x^2+d} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^3*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2),x)
```

```
[Out] int((f + g*x)^3*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d \left(c^{2} x^{2} + 1\right)} \left(a + b \operatorname{asinh}{\left(c x \right)}\right) \left(f + g x\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(a+b*asinh(c*x))*(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))*(f + g*x)**3, x)
```

3.35 $\int (f+gx)^2 \sqrt{d+c^2dx^2} \left(a+b\sinh^{-1}(cx)\right) dx$

Optimal. Leaf size=431

$$\frac{1}{2}f^2x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{f^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc\sqrt{c^2x^2+1}}+\frac{2fg\left(c^2x^2+1\right)\sqrt{c^2dx^2+d}\left(a+b\sinh\right.}{3c^2}$$

```
[Out] 1/2*f^2*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)+1/8*g^2*x*(a+b*arcsinh(c*x
))*(c^2*d*x^2+d)^(1/2)/c^2+1/4*g^2*x^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/
2)+2/3*f*g*(c^2*x^2+1)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^2-2/3*b*f*g
*x*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-1/4*b*c*f^2*x^2*(c^2*d*x^2+d)^(1
/2)/(c^2*x^2+1)^(1/2)-1/16*b*g^2*x^2*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2
)-2/9*b*c*f*g*x^3*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/16*b*c*g^2*x^4*(c
^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+1/4*f^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2
+d)^(1/2)/b/c/(c^2*x^2+1)^(1/2)-1/16*g^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)
^(1/2)/b/c^3/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.53, antiderivative size = 431, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {5835, 5821, 5682, 5675, 30, 5717, 5742, 5758}

$$\frac{1}{2}f^2x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{f^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc\sqrt{c^2x^2+1}}+\frac{2fg\left(c^2x^2+1\right)\sqrt{c^2dx^2+d}\left(a+b\sinh\right.}{3c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-2*b*f*g*x*Sqrt[d + c^2*d*x^2])/(3*c*Sqrt[1 + c^2*x^2]) - (b*c*f^2*x^2*Sqr
t[d + c^2*d*x^2])/(4*Sqrt[1 + c^2*x^2]) - (b*g^2*x^2*Sqrt[d + c^2*d*x^2])/(
16*c*Sqrt[1 + c^2*x^2]) - (2*b*c*f*g*x^3*Sqrt[d + c^2*d*x^2])/(9*Sqrt[1 + c
^2*x^2]) - (b*c*g^2*x^4*Sqrt[d + c^2*d*x^2])/(16*Sqrt[1 + c^2*x^2]) + (f^2*
x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/2 + (g^2*x*Sqrt[d + c^2*d*x^2]*
(a + b*ArcSinh[c*x]))/(8*c^2) + (g^2*x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh
[c*x]))/4 + (2*f*g*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/
(3*c^2) + (f^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(4*b*c*Sqrt[1 +
c^2*x^2]) - (g^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(16*b*c^3*Sqrt
[1 + c^2*x^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5835

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(d^IntPart[p]*(d + e*x^2)^FracPa
rt[p])/(1 + c^2*x^2)^FracPart[p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*Ar
cSinh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d
] && IntegerQ[m] && IntegerQ[p - 1/2] &&  !GtQ[d, 0]
```

Rubi steps

$$\int (f+gx)^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)dx = \frac{\sqrt{d+c^2dx^2}\int (f+gx)^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)dx}{\sqrt{1+c^2x^2}}$$
$$= \frac{\sqrt{d+c^2dx^2}\int \left(f^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)+2fgx\sqrt{1+c^2x}\right.}{\sqrt{1+c}}$$
$$= \frac{\left(f^2\sqrt{d+c^2dx^2}\right)\int \sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)dx}{\sqrt{1+c^2x^2}} + \frac{\left(2fg\sqrt{d}\right.}{}$$
$$= \frac{1}{2}f^2x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{4}g^2x^3\sqrt{d+c^2dx^2}\left(a+b\right.$$
$$= -\frac{2bfgx\sqrt{d+c^2dx^2}}{3c\sqrt{1+c^2x^2}}-\frac{bcf^2x^2\sqrt{d+c^2dx^2}}{4\sqrt{1+c^2x^2}}-\frac{2bcfgx^3\sqrt{d+c^2dx^2}}{9\sqrt{1+c^2x^2}}$$
$$= -\frac{2bfgx\sqrt{d+c^2dx^2}}{3c\sqrt{1+c^2x^2}}-\frac{bcf^2x^2\sqrt{d+c^2dx^2}}{4\sqrt{1+c^2x^2}}-\frac{bg^2x^2\sqrt{d+c^2dx^2}}{16c\sqrt{1+c^2x^2}}-$$

Mathematica [A] time = 0.96, size = 301, normalized size = 0.70

$$\frac{48ac\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}\left(12c^2f^2x+16f\left(c^2gx^2+g\right)+3g^2x\left(2c^2x^2+1\right)\right)+144a\sqrt{d}\sqrt{c^2x^2+1}(2cf-g)(2cf+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (48*a*c*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]*(12*c^2*f^2*x + 3*g^2*x*(1 +
2*c^2*x^2) + 16*f*(g + c^2*g*x^2)) - 256*b*c*f*g*Sqrt[d + c^2*d*x^2]*(3*c*x
 + c^3*x^3 - 3*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]) + 144*a*Sqrt[d]*(2*c*f - g
)*(2*c*f + g)*Sqrt[1 + c^2*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] -
144*b*c^2*f^2*Sqrt[d + c^2*d*x^2]*(Cosh[2*ArcSinh[c*x]] - 2*ArcSinh[c*x]*(A
rcSinh[c*x] + Sinh[2*ArcSinh[c*x]])) - 9*b*g^2*Sqrt[d + c^2*d*x^2]*(8*ArcSi
nh[c*x]^2 + Cosh[4*ArcSinh[c*x]] - 4*ArcSinh[c*x]*Sinh[4*ArcSinh[c*x]]))/(1
152*c^3*Sqrt[1 + c^2*x^2])
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{c^2dx^2+d}\left(ag^2x^2+2\,afgx+af^2+\left(bg^2x^2+2\,bfgx+bf^2\right)\operatorname{arsinh}(cx)\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="fr
icas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(a*g^2*x^2 + 2*a*f*g*x + a*f^2 + (b*g^2*x^2 +
2*b*f*g*x + b*f^2)*arcsinh(c*x)), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="gi
ac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [B] time = 0.62, size = 791, normalized size = 1.84

$$\frac{a\,g^{2}x\left(c^{2}d\,x^{2}+d\right)^{\frac{3}{2}}}{4c^{2}d}-\frac{a\,g^{2}x\sqrt{c^{2}d\,x^{2}+d}}{8c^{2}}-\frac{a\,g^{2}d\ln\left(\frac{x\,c^{2}d}{\sqrt{c^{2}d}}+\sqrt{c^{2}d\,x^{2}+d}\right)}{8c^{2}\sqrt{c^{2}d}}+\frac{2afg\left(c^{2}d\,x^{2}+d\right)^{\frac{3}{2}}}{3c^{2}d}+\frac{a\,f^{2}x\sqrt{c^{2}d\,x^{2}+}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] 1/4*a*g^2*x*(c^2*d*x^2+d)^(3/2)/c^2/d-1/8*a*g^2/c^2*x*(c^2*d*x^2+d)^(1/2)-1
/8*a*g^2/c^2*d*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+
2/3*a*f*g/c^2/d*(c^2*d*x^2+d)^(3/2)+1/2*a*f^2*x*(c^2*d*x^2+d)^(1/2)+1/2*a*f
^2*d*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+1/4*b*(d*(
c^2*x^2+1))^(1/2)*g^2*c^2/(c^2*x^2+1)*arcsinh(c*x)*x^5-1/16*b*(d*(c^2*x^2+1
))^(1/2)*g^2*c/(c^2*x^2+1)^(1/2)*x^4+3/8*b*(d*(c^2*x^2+1))^(1/2)*g^2/(c^2*x
^2+1)*arcsinh(c*x)*x^3-1/16*b*(d*(c^2*x^2+1))^(1/2)*g^2/c/(c^2*x^2+1)^(1/2)
*x^2+1/8*b*(d*(c^2*x^2+1))^(1/2)*g^2/c^2/(c^2*x^2+1)*arcsinh(c*x)*x+2/3*b*(
d*(c^2*x^2+1))^(1/2)*f*g/c^2/(c^2*x^2+1)*arcsinh(c*x)+1/2*b*(d*(c^2*x^2+1))
^(1/2)*f^2*c^2/(c^2*x^2+1)*arcsinh(c*x)*x^3-1/4*b*(d*(c^2*x^2+1))^(1/2)*f^2
*c/(c^2*x^2+1)^(1/2)*x^2+1/2*b*(d*(c^2*x^2+1))^(1/2)*f^2/(c^2*x^2+1)*arcsin
h(c*x)*x+2/3*b*(d*(c^2*x^2+1))^(1/2)*f*g*c^2/(c^2*x^2+1)*arcsinh(c*x)*x^4-2
/9*b*(d*(c^2*x^2+1))^(1/2)*f*g*c/(c^2*x^2+1)^(1/2)*x^3+4/3*b*(d*(c^2*x^2+1)
)^(1/2)*f*g/(c^2*x^2+1)*arcsinh(c*x)*x^2-2/3*b*(d*(c^2*x^2+1))^(1/2)*f*g/c/
(c^2*x^2+1)^(1/2)*x+1/4*b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)^2/(c^2*x^2+1)^
(1/2)/c*f^2-1/16*b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)^2/(c^2*x^2+1)^(1/2)/c
^3*g^2-1/128*b*(d*(c^2*x^2+1))^(1/2)*g^2/c^3/(c^2*x^2+1)^(1/2)-1/8*b*(d*(c^
2*x^2+1))^(1/2)*f^2/c/(c^2*x^2+1)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="ma
xima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(f+g\,x\right)}^2\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,\sqrt{d\,c^2\,x^2+d}\,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2),x)
```

```
[Out] int((f + g*x)^2*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d\left(c^{2}x^{2}+1\right)}\left(a+b\operatorname{asinh}{\left(cx\right)}\right)\left(f+gx\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(a+b*asinh(c*x))*(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))*(f + g*x)**2, x)
```

3.36 $\int (f+gx)\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=227

$$\frac{1}{2}fx\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)+\frac{f\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)^2}{4bc\sqrt{c^2x^2+1}}+\frac{g\left(c^2x^2+1\right)\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx\right.}{3c^2}$$

```
[Out] 1/2*f*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)+1/3*g*(c^2*x^2+1)*(a+b*arcsi
nh(c*x))*(c^2*d*x^2+d)^(1/2)/c^2-1/3*b*g*x*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1
)^(1/2)-1/4*b*c*f*x^2*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/9*b*c*g*x^3*(
c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+1/4*f*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+
d)^(1/2)/b/c/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5835, 5821, 5682, 5675, 30, 5717}

$$\frac{1}{2}fx\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)+\frac{f\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)^2}{4bc\sqrt{c^2x^2+1}}+\frac{g\left(c^2x^2+1\right)\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx\right.}{3c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -(b*g*x*Sqrt[d + c^2*d*x^2])/(3*c*Sqrt[1 + c^2*x^2]) - (b*c*f*x^2*Sqrt[d +
c^2*d*x^2])/(4*Sqrt[1 + c^2*x^2]) - (b*c*g*x^3*Sqrt[d + c^2*d*x^2])/(9*Sqrt
[1 + c^2*x^2]) + (f*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/2 + (g*(1 +
 c^2*x^2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(3*c^2) + (f*Sqrt[d + c
^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(4*b*c*Sqrt[1 + c^2*x^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5835

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(d^IntPart[p]*(d + e*x^2)^FracPa
rt[p])/(1 + c^2*x^2)^FracPart[p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*Ar
cSinh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d
] && IntegerQ[m] && IntegerQ[p - 1/2] && !GtQ[d, 0]
```

Rubi steps

$$\int (f+gx)\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)dx = \frac{\sqrt{d+c^2dx^2}\int (f+gx)\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)dx}{\sqrt{1+c^2x^2}}$$

$$= \frac{\sqrt{d+c^2dx^2}\int \left(f\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)+gx\sqrt{1+c^2x^2}\right.}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(f\sqrt{d+c^2dx^2}\right)\int \sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)dx}{\sqrt{1+c^2x^2}} + \frac{\left(g\sqrt{d+\right.}{}$$

$$= \frac{1}{2}fx\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right) + \frac{g\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}}{3c^2}$$

$$= -\frac{bgx\sqrt{d+c^2dx^2}}{3c\sqrt{1+c^2x^2}} - \frac{bcfx^2\sqrt{d+c^2dx^2}}{4\sqrt{1+c^2x^2}} - \frac{bcgx^3\sqrt{d+c^2dx^2}}{9\sqrt{1+c^2x^2}} +$$

Mathematica [A] time = 1.21, size = 208, normalized size = 0.92

$$\frac{1}{6}a\sqrt{c^2dx^2+d}\left(\frac{2g}{c^2}+x(3f+2gx)\right)+\frac{a\sqrt{d}f\log\left(\sqrt{d}\sqrt{c^2dx^2+d}+cdx\right)}{2c}+\frac{bf\sqrt{c^2dx^2+d}\left(2\sinh^{-1}(cx)\left(\sinh^{-1}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (a*Sqrt[d + c^2*d*x^2]*((2*g)/c^2 + x*(3*f + 2*g*x)))/6 - (b*g*Sqrt[d + c^2
*d*x^2]*(3*c*x + c^3*x^3 - 3*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]))/(9*c^2*Sqrt
[1 + c^2*x^2]) + (a*Sqrt[d]*f*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]])/(2*
c) + (b*f*Sqrt[d + c^2*d*x^2]*(-Cosh[2*ArcSinh[c*x]] + 2*ArcSinh[c*x]*(ArcS
inh[c*x] + Sinh[2*ArcSinh[c*x]])))/(8*c*Sqrt[1 + c^2*x^2])
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{c^2dx^2+d}\left(agx+af+\left(bgx+bf\right)\text{arsinh}\left(cx\right)\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="fric
as")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(a*g*x + a*f + (b*g*x + b*f)*arcsinh(c*x)), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="giac
")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [B] time = 0.42, size = 423, normalized size = 1.86

$$\frac{ag\left(c^2d\,x^2+d\right)^{\frac{3}{2}}}{3c^2d}+\frac{afx\sqrt{c^2d\,x^2+d}}{2}+\frac{afd\ln\left(\frac{x\,c^2d}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{2\sqrt{c^2d}}+\frac{b\sqrt{d\left(c^2x^2+1\right)}\,g\,\mathrm{arcsinh}\,(cx)}{3c^2\left(c^2x^2+1\right)}-\frac{b\sqrt{d\left(c^2x^2+\right.}}{8c\sqrt{c^2x^2+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] 1/3*a*g/c^2/d*(c^2*d*x^2+d)^(3/2)+1/2*a*f*x*(c^2*d*x^2+d)^(1/2)+1/2*a*f*d*l
n(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+1/3*b*(d*(c^2*x^
2+1))^(1/2)*g/c^2/(c^2*x^2+1)*arcsinh(c*x)-1/8*b*(d*(c^2*x^2+1))^(1/2)*f/c/
(c^2*x^2+1)^(1/2)+1/3*b*(d*(c^2*x^2+1))^(1/2)*g*c^2/(c^2*x^2+1)*arcsinh(c*x
)*x^4-1/9*b*(d*(c^2*x^2+1))^(1/2)*g*c/(c^2*x^2+1)^(1/2)*x^3+2/3*b*(d*(c^2*x
^2+1))^(1/2)*g/(c^2*x^2+1)*arcsinh(c*x)*x^2-1/3*b*(d*(c^2*x^2+1))^(1/2)*g/c
/(c^2*x^2+1)^(1/2)*x+1/2*b*(d*(c^2*x^2+1))^(1/2)*f*c^2/(c^2*x^2+1)*arcsinh(
c*x)*x^3-1/4*b*(d*(c^2*x^2+1))^(1/2)*f*c/(c^2*x^2+1)^(1/2)*x^2+1/2*b*(d*(c^
2*x^2+1))^(1/2)*f/(c^2*x^2+1)*arcsinh(c*x)*x+1/4*b*(d*(c^2*x^2+1))^(1/2)/(c
^2*x^2+1)^(1/2)/c*f*arcsinh(c*x)^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(f+g\,x\right)\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,\sqrt{d\,c^2\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2),x)
```

```
[Out] int((f + g*x)*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d\left(c^2x^2+1\right)}\,\left(a+b\,\mathrm{asinh}\,(cx)\right)\left(f+gx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*asinh(c*x))*(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))*(f + g*x), x)
```

3.37 $\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{f+gx}\,dx$

Optimal. Leaf size=664

$$-\frac{\sqrt{c^2dx^2+d}\left(\frac{c^2f^2}{g^2}+1\right)\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{c^2x^2+1}\,(f+gx)}+\frac{\sqrt{c^2x^2+1}\,\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)^2}{2bc(f+gx)}-\frac{cx\sqrt{c^2dx^2+d}\,\left(a+b\,\mathrm{si}\right.}{2bg\sqrt{c^2x^2+1}}$$

```
[Out] a*(c^2*d*x^2+d)^(1/2)/g+b*arcsinh(c*x)*(c^2*d*x^2+d)^(1/2)/g-b*c*x*(c^2*d*x
^2+d)^(1/2)/g/(c^2*x^2+1)^(1/2)-1/2*c*x*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^
(1/2)/b/g/(c^2*x^2+1)^(1/2)-1/2*(1+c^2*f^2/g^2)*(a+b*arcsinh(c*x))^2*(c^2*d
*x^2+d)^(1/2)/b/c/(g*x+f)/(c^2*x^2+1)^(1/2)-a*arctanh((-c^2*f*x+g)/(c^2*f^2
+g^2)^(1/2)/(c^2*x^2+1)^(1/2))*(c^2*f^2+g^2)^(1/2)*(c^2*d*x^2+d)^(1/2)/g^2/
(c^2*x^2+1)^(1/2)+b*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f
^2+g^2)^(1/2)))*(c^2*f^2+g^2)^(1/2)*(c^2*d*x^2+d)^(1/2)/g^2/(c^2*x^2+1)^(1/
2)-b*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2+g^2)^(1/2)))
*(c^2*f^2+g^2)^(1/2)*(c^2*d*x^2+d)^(1/2)/g^2/(c^2*x^2+1)^(1/2)+b*polylog(2,
-(c*x+(c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2+g^2)^(1/2)))*(c^2*f^2+g^2)^(1/2)*(
c^2*d*x^2+d)^(1/2)/g^2/(c^2*x^2+1)^(1/2)-b*polylog(2,-(c*x+(c^2*x^2+1)^(1/2
))*g/(c*f+(c^2*f^2+g^2)^(1/2)))*(c^2*f^2+g^2)^(1/2)*(c^2*d*x^2+d)^(1/2)/g^2
/(c^2*x^2+1)^(1/2)+1/2*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)*(c^2*d*x^2+d)
^(1/2)/b/c/(g*x+f)
```

Rubi [A] time = 1.65, antiderivative size = 664, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 20, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5835, 5823, 683, 5815, 6742, 261, 725, 206, 5859, 1654, 12, 5857, 5717, 8, 5831, 3322, 2264, 2190, 2279, 2391}

$$\frac{b\sqrt{c^2dx^2+d}\,\sqrt{c^2f^2+g^2}\,\mathrm{PolyLog}\left(2,-\frac{ge^{\sinh^{-1}(cx)}}{cf-\sqrt{c^2f^2+g^2}}\right)}{g^2\sqrt{c^2x^2+1}}-\frac{b\sqrt{c^2dx^2+d}\,\sqrt{c^2f^2+g^2}\,\mathrm{PolyLog}\left(2,-\frac{ge^{\sinh^{-1}(cx)}}{\sqrt{c^2f^2+g^2}+cf}\right)}{g^2\sqrt{c^2x^2+1}}-\frac{\sqrt{c^2d}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(f + g*x),x]
```

```
[Out] (a*Sqrt[d + c^2*d*x^2])/g - (b*c*x*Sqrt[d + c^2*d*x^2])/(g*Sqrt[1 + c^2*x^2
]) + (b*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x])/g - (c*x*Sqrt[d + c^2*d*x^2]*(a +
 b*ArcSinh[c*x])^2)/(2*b*g*Sqrt[1 + c^2*x^2]) - ((1 + (c^2*f^2)/g^2)*Sqrt[d
 + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c*(f + g*x)*Sqrt[1 + c^2*x^2]) +
 (Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c*(f +
 g*x)) - (a*Sqrt[c^2*f^2 + g^2]*Sqrt[d + c^2*d*x^2]*ArcTanh[(g - c^2*f*x)/(
Sqrt[c^2*f^2 + g^2]*Sqrt[1 + c^2*x^2])])/(g^2*Sqrt[1 + c^2*x^2]) + (b*Sqrt[
c^2*f^2 + g^2]*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]*Log[1 + (E^ArcSinh[c*x]*g)/
(c*f - Sqrt[c^2*f^2 + g^2])])/(g^2*Sqrt[1 + c^2*x^2]) - (b*Sqrt[c^2*f^2 + g
^2]*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]*Log[1 + (E^ArcSinh[c*x]*g)/(c*f + Sqrt
[c^2*f^2 + g^2])])/(g^2*Sqrt[1 + c^2*x^2]) + (b*Sqrt[c^2*f^2 + g^2]*Sqrt[d
+ c^2*d*x^2]*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2]))])
/(g^2*Sqrt[1 + c^2*x^2]) - (b*Sqrt[c^2*f^2 + g^2]*Sqrt[d + c^2*d*x^2]*PolyL
og[2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))])/(g^2*Sqrt[1 + c^2
*x^2])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5815

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.) + (g_.)*(x_) + (h_.)*(
x_)^2)^(p_.))/((d_) + (e_.)*(x_))^2, x_Symbol] :> With[{u = IntHide[(f + g*
x + h*x^2)^p/(d + e*x)^2, x]}, Dist[(a + b*ArcSinh[c*x])^n, u, x] - Dist[b*
c*n, Int[SimplifyIntegrand[(u*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^
2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[n, 0] && IGtQ
[p, 0] && EqQ[e*g - 2*d*h, 0]
```

Rule 5823

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.) + (g_.)*(x_))^(m_)*Sqr
t[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[((f + g*x)^m*(d + e*x^2)*(a + b*A
rcSinh[c*x])^(n + 1))/(b*c*Sqrt[d]*(n + 1)), x] - Dist[1/(b*c*Sqrt[d]*(n +
1)), Int[(d*g*m + 2*e*f*x + e*g*(m + 2)*x^2)*(f + g*x)^(m - 1)*(a + b*ArcSi
nh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d
] && ILtQ[m, 0] && GtQ[d, 0] && IGtQ[n, 0]
```

Rule 5831

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 5835

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(d^IntPart[p]*(d + e*x^2)^FracPa
rt[p])/(1 + c^2*x^2)^FracPart[p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*Ar
cSinh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d
] && IntegerQ[m] && IntegerQ[p - 1/2] &&  !GtQ[d, 0]
```

Rule 5857

```
Int[ArcSinh[(c_.)*(x_)]^(n_.)*(RFx_)*((d_) + (e_.)*(x_)^2)^(p_), x_Symbol]
:> With[{u = ExpandIntegrand[(d + e*x^2)^p*ArcSinh[c*x]^n, RFx, x]}, Int[u,
 x] /; SumQ[u]] /; FreeQ[{c, d, e}, x] && RationalFunctionQ[RFx, x] && IGtQ
[n, 0] && EqQ[e, c^2*d] && IntegerQ[p - 1/2]
```

Rule 5859

```
Int[(ArcSinh[(c_.)*(x_)]*(b_.) + (a_))^(n_.)*(RFx_)*((d_) + (e_.)*(x_)^2)^(
p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p, RFx*(a + b*ArcSinh[c*x
])^n, x], x] /; FreeQ[{a, b, c, d, e}, x] && RationalFunctionQ[RFx, x] && I
GtQ[n, 0] && EqQ[e, c^2*d] && IntegerQ[p - 1/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{f+gx}\,dx = \frac{\sqrt{d+c^2dx^2}\int \frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{f+gx}\,dx}{\sqrt{1+c^2x^2}}$$

$$= \frac{\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc(f+gx)} - \frac{\sqrt{d+c^2dx^2}\int \frac{(-g+2c^2fx+c^2gx}{(f}}{2bc\sqrt{1+c^2}}$$

$$= -\frac{cx\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bg\sqrt{1+c^2x^2}} - \frac{\left(1+\frac{c^2f^2}{g^2}\right)\sqrt{d+c^2dx^2}\left(a+b\sinh\right.}{2bc(f+gx)\sqrt{1+c^2x^2}}$$

$$= -\frac{cx\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bg\sqrt{1+c^2x^2}} - \frac{\left(1+\frac{c^2f^2}{g^2}\right)\sqrt{d+c^2dx^2}\left(a+b\sinh\right.}{2bc(f+gx)\sqrt{1+c^2x^2}}$$

$$= -\frac{cx\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bg\sqrt{1+c^2x^2}} - \frac{\left(1+\frac{c^2f^2}{g^2}\right)\sqrt{d+c^2dx^2}\left(a+b\sinh\right.}{2bc(f+gx)\sqrt{1+c^2x^2}}$$

$$= \frac{a\sqrt{d+c^2dx^2}}{g} - \frac{cx\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bg\sqrt{1+c^2x^2}} - \frac{\left(1+\frac{c^2f^2}{g^2}\right)\sqrt{d+c^2d}}{2bc(f+g}$$

$$= \frac{a\sqrt{d+c^2dx^2}}{g} - \frac{cx\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bg\sqrt{1+c^2x^2}} - \frac{\left(1+\frac{c^2f^2}{g^2}\right)\sqrt{d+c^2d}}{2bc(f+g}$$

$$= \frac{a\sqrt{d+c^2dx^2}}{g} + \frac{b\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{g} - \frac{cx\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(\right.}{2bg\sqrt{1+c^2x^2}}$$

$$= \frac{a\sqrt{d+c^2dx^2}}{g} - \frac{bcx\sqrt{d+c^2dx^2}}{g\sqrt{1+c^2x^2}} + \frac{b\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{g} - \frac{cx\sqrt{d+c^2}}{2}$$

$$= \frac{a\sqrt{d+c^2dx^2}}{g} - \frac{bcx\sqrt{d+c^2dx^2}}{g\sqrt{1+c^2x^2}} + \frac{b\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{g} - \frac{cx\sqrt{d+c^2}}{2}$$

$$= \frac{a\sqrt{d+c^2dx^2}}{g} - \frac{bcx\sqrt{d+c^2dx^2}}{g\sqrt{1+c^2x^2}} + \frac{b\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{g} - \frac{cx\sqrt{d+c^2}}{2}$$

$$= \frac{a\sqrt{d+c^2dx^2}}{g} - \frac{bcx\sqrt{d+c^2dx^2}}{g\sqrt{1+c^2x^2}} + \frac{b\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{g} - \frac{cx\sqrt{d+c^2}}{2}$$

$$= \frac{a\sqrt{d+c^2dx^2}}{g} - \frac{bcx\sqrt{d+c^2dx^2}}{g\sqrt{1+c^2x^2}} + \frac{b\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{g} - \frac{cx\sqrt{d+c^2}}{2}$$

Mathematica [C] time = 6.37, size = 1353, normalized size = 2.04

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(f + g*x),x]
```

```
[Out] (2*a*g*Sqrt[d + c^2*d*x^2] + 2*a*Sqrt[d]*Sqrt[c^2*f^2 + g^2]*Log[f + g*x] -
 2*a*c*Sqrt[d]*f*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] - 2*a*Sqrt[d]*Sqr
t[c^2*f^2 + g^2]*Log[d*(g - c^2*f*x) + Sqrt[d]*Sqrt[c^2*f^2 + g^2]*Sqrt[d +
 c^2*d*x^2]] + b*Sqrt[d + c^2*d*x^2]*((-2*c*g*x)/Sqrt[1 + c^2*x^2] + 2*g*Ar
cSinh[c*x] - (c*f*ArcSinh[c*x]^2)/Sqrt[1 + c^2*x^2] + (2*(c^2*f^2 + g^2)*((
(-I)*Pi*ArcTanh[(-g + c*f*Tanh[ArcSinh[c*x]/2])/Sqrt[c^2*f^2 + g^2]])/Sqrt[
c^2*f^2 + g^2] - (2*ArcCos[((-I)*c*f)/g]*ArcTanh[((c*f + I*g)*Cot[(Pi + (2*
I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]] + (Pi - (2*I)*ArcSinh[c*x])*Ar
cTanh[((c*f - I*g)*Tan[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]
] + (ArcCos[((-I)*c*f)/g] - (2*I)*ArcTanh[((c*f + I*g)*Cot[(Pi + (2*I)*ArcS
inh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]] - (2*I)*ArcTanh[((c*f - I*g)*Tan[(Pi
+ (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]])*Log[((1/2 - I/2)*Sqrt[-(
c^2*f^2) - g^2])/(E^(ArcSinh[c*x]/2)*Sqrt[(-I)*g]*Sqrt[c*(f + g*x)])] + (Ar
cCos[((-I)*c*f)/g] + (2*I)*(ArcTanh[((c*f + I*g)*Cot[(Pi + (2*I)*ArcSinh[c*
x])/4])/Sqrt[-(c^2*f^2) - g^2]] + ArcTanh[((c*f - I*g)*Tan[(Pi + (2*I)*ArcS
inh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]]))*Log[((1/2 + I/2)*E^(ArcSinh[c*x]/2)
*Sqrt[-(c^2*f^2) - g^2])/(Sqrt[(-I)*g]*Sqrt[c*(f + g*x)])] - (ArcCos[((-I)*
c*f)/g] + (2*I)*ArcTanh[((c*f + I*g)*Cot[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt
[-(c^2*f^2) - g^2]])*Log[((I*c*f + g)*((-I)*c*f + g + Sqrt[-(c^2*f^2) - g^2
])*(1 + I*Cot[(Pi + (2*I)*ArcSinh[c*x])/4]))/(g*(I*c*f + g + I*Sqrt[-(c^2*f
^2) - g^2]*Cot[(Pi + (2*I)*ArcSinh[c*x])/4]))] - (ArcCos[((-I)*c*f)/g] - (2
*I)*ArcTanh[((c*f + I*g)*Cot[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2)
- g^2]])*Log[((I*c*f + g)*(I*c*f - g + Sqrt[-(c^2*f^2) - g^2])*(I + Cot[(Pi
 + (2*I)*ArcSinh[c*x])/4]))/(g*(c*f - I*g + Sqrt[-(c^2*f^2) - g^2]*Cot[(Pi
+ (2*I)*ArcSinh[c*x])/4]))] + I*(PolyLog[2, ((I*c*f + Sqrt[-(c^2*f^2) - g^2
])*(I*c*f + g - I*Sqrt[-(c^2*f^2) - g^2]*Cot[(Pi + (2*I)*ArcSinh[c*x])/4]))
/(g*(I*c*f + g + I*Sqrt[-(c^2*f^2) - g^2]*Cot[(Pi + (2*I)*ArcSinh[c*x])/4])
)] - PolyLog[2, ((c*f + I*Sqrt[-(c^2*f^2) - g^2])*(-(c*f) + I*g + Sqrt[-(c^
2*f^2) - g^2]*Cot[(Pi + (2*I)*ArcSinh[c*x])/4]))/(g*(I*c*f + g + I*Sqrt[-(c
^2*f^2) - g^2]*Cot[(Pi + (2*I)*ArcSinh[c*x])/4]))])))/Sqrt[-(c^2*f^2) - g^2]
))/Sqrt[1 + c^2*x^2]))/(2*g^2)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2dx^2+d}\,(b\,\text{arsinh}\,(cx)+a)}{gx+f},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(g*x+f),x, algorithm="fric
as")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(g*x + f), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(g*x+f),x, algorithm="giac
")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.35, size = 992, normalized size = 1.49

$$\frac{a\sqrt{\left(x+\frac{f}{g}\right)^{2} c^{2} d-\frac{2 c^{2} d f \left(x+\frac{f}{g}\right)}{g}+\frac{d \left(c^{2} f^{2}+g^{2}\right)}{g^{2}}}}{g}-\frac{a\,c^{2} d f \ln\left(\frac{-\frac{c^{2} d f}{g}+c^{2} d \left(x+\frac{f}{g}\right)}{\sqrt{c^{2} d}}+\sqrt{\left(x+\frac{f}{g}\right)^{2} c^{2} d-\frac{2 c^{2} d f \left(x+\frac{f}{g}\right)}{g}+\frac{d \left(c^{2} f^{2}+g^{2}\right)}{g^{2}}}\right)}{g^{2}\sqrt{c^{2} d}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(g*x+f),x)
```

```
[Out] a/g*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2)-a/g^2*c
^2*d*f*ln((-c^2*d*f/g+c^2*d*(x+f/g))/(c^2*d)^(1/2)+((x+f/g)^2*c^2*d-2*c^2*d
*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2))/(c^2*d)^(1/2)-a/g^3*d/(d*(c^2*f^2+
g^2)/g^2)^(1/2)*ln((2*d*(c^2*f^2+g^2)/g^2-2*c^2*d*f/g*(x+f/g)+2*(d*(c^2*f^2
+g^2)/g^2)^(1/2)*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^
(1/2))/(x+f/g))*c^2*f^2-a/g*d/(d*(c^2*f^2+g^2)/g^2)^(1/2)*ln((2*d*(c^2*f^2+
g^2)/g^2-2*c^2*d*f/g*(x+f/g)+2*(d*(c^2*f^2+g^2)/g^2)^(1/2)*((x+f/g)^2*c^2*d
-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2))/(x+f/g))-1/2*b*(d*(c^2*x^2
+1))^(1/2)/(c^2*x^2+1)^(1/2)*f*arcsinh(c*x)^2*c/g^2+b*(d*(c^2*x^2+1))^(1/2)
/(c^2*x^2+1)/g*arcsinh(c*x)*x^2*c^2-b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/
2)/g*c*x+b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)/g*arcsinh(c*x)+b*(d*(c^2*x^2+1
))^(1/2)*(c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2)/g^2*arcsinh(c*x)*ln((-(c*x+(
c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2+g^2)^(1/2))/(-c*f+(c^2*f^2+g^2)^(1/2)))-b*
(d*(c^2*x^2+1))^(1/2)*(c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2)/g^2*arcsinh(c*x
)*ln(((c*x+(c^2*x^2+1)^(1/2))*g+c*f+(c^2*f^2+g^2)^(1/2))/(c*f+(c^2*f^2+g^2)
^(1/2)))+b*(d*(c^2*x^2+1))^(1/2)*(c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2)/g^2*
dilog((-(c*x+(c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2+g^2)^(1/2))/(-c*f+(c^2*f^2+g
^2)^(1/2)))-b*(d*(c^2*x^2+1))^(1/2)*(c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2)/g
^2*dilog(((c*x+(c^2*x^2+1)^(1/2))*g+c*f+(c^2*f^2+g^2)^(1/2))/(c*f+(c^2*f^2+
g^2)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\left(\frac{c\sqrt{d}\,f\operatorname{arsinh}(cx)}{g^{2}}-\frac{\sqrt{\frac{c^{2}df^{2}}{g^{2}}+d}\,\operatorname{arsinh}\left(\frac{cfx}{|gx+f|}-\frac{g}{c|gx+f|}\right)}{g}-\frac{\sqrt{c^{2}dx^{2}+d}}{g}\right)a+b\int\frac{\sqrt{c^{2}dx^{2}+d}\,\log\left(cx+\sqrt{c^{2}x^{2}\cdots}\right.}{gx+f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(g*x+f),x, algorithm="maxi
ma")
```

```
[Out] -(c*sqrt(d)*f*arcsinh(c*x)/g^2 - sqrt(c^2*d*f^2/g^2 + d)*arcsinh(c*f*x/abs(
g*x + f) - g/(c*abs(g*x + f)))/g - sqrt(c^2*d*x^2 + d)/g)*a + b*integrate(s
qrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1))/(g*x + f), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\operatorname{asinh}(c\,x))\,\sqrt{d\,c^{2}\,x^{2}+d}}{f+g\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2))/(f + g*x),x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2))/(f + g*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d\left(c^{2}x^{2}+1\right)}\,(a+b\operatorname{asinh}(cx))}{f+gx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))*(c**2*d*x**2+d)**(1/2)/(g*x+f),x)
```

```
[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))/(f + g*x), x)
```

3.38 $\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{(f+gx)^2}\,dx$

Optimal. Leaf size=781

$$-\frac{\sqrt{c^2dx^2+d}\left(g-c^2fx\right)^2\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{c^2x^2+1}\left(c^2f^2+g^2\right)(f+gx)^2}+\frac{\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc(f+gx)^2}+\frac{ac^2f\sqrt{c^2dx^2+d}\tanh}{g^2\sqrt{c^2x^2+1}}$$

```
[Out] -a*(c^2*d*x^2+d)^(1/2)/g/(g*x+f)-b*arcsinh(c*x)*(c^2*d*x^2+d)^(1/2)/g/(g*x+
f)+a*c^3*f^2*arcsinh(c*x)*(c^2*d*x^2+d)^(1/2)/g^2/(c^2*f^2+g^2)/(c^2*x^2+1)
^(1/2)+1/2*b*c^3*f^2*arcsinh(c*x)^2*(c^2*d*x^2+d)^(1/2)/g^2/(c^2*f^2+g^2)/(
c^2*x^2+1)^(1/2)-1/2*(-c^2*f*x+g)^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2
)/b/c/(c^2*f^2+g^2)/(g*x+f)^2/(c^2*x^2+1)^(1/2)+b*c*ln(g*x+f)*(c^2*d*x^2+d)
^(1/2)/g^2/(c^2*x^2+1)^(1/2)+a*c^2*f*arctanh((-c^2*f*x+g)/(c^2*f^2+g^2)^(1/
2)/(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/g^2/(c^2*f^2+g^2)^(1/2)/(c^2*x^2+
1)^(1/2)-b*c^2*f*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2+
g^2)^(1/2)))*(c^2*d*x^2+d)^(1/2)/g^2/(c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2)+
b*c^2*f*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2+g^2)^(1/2
)))*(c^2*d*x^2+d)^(1/2)/g^2/(c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2)-b*c^2*f*p
olylog(2,-(c*x+(c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2+g^2)^(1/2)))*(c^2*d*x^2+d
)^(1/2)/g^2/(c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2)+b*c^2*f*polylog(2,-(c*x+(
c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2+g^2)^(1/2)))*(c^2*d*x^2+d)^(1/2)/g^2/(c^2
*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2)+1/2*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2
)*(c^2*d*x^2+d)^(1/2)/b/c/(g*x+f)^2
```

Rubi [A] time = 2.55, antiderivative size = 781, normalized size of antiderivative = 1.00, number of steps used = 35, number of rules used = 22, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.733, Rules used = {5835, 5823, 37, 5813, 12, 1651, 844, 215, 725, 206, 5859, 5857, 5675, 5831, 3324, 3322, 2264, 2190, 2279, 2391, 2668, 31}

$$-\frac{bc^2f\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,-\frac{ge^{\sinh^{-1}(cx)}}{cf-\sqrt{c^2f^2+g^2}}\right)}{g^2\sqrt{c^2x^2+1}\sqrt{c^2f^2+g^2}}+\frac{bc^2f\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,-\frac{ge^{\sinh^{-1}(cx)}}{\sqrt{c^2f^2+g^2}+cf}\right)}{g^2\sqrt{c^2x^2+1}\sqrt{c^2f^2+g^2}}-\frac{\sqrt{c^2dx^2+d}\left(g-c^2\right.}{2bc\sqrt{c^2x^2+1}\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(f + g*x)^2,x]
```

```
[Out] -((a*Sqrt[d + c^2*d*x^2])/(g*(f + g*x))) - (b*Sqrt[d + c^2*d*x^2]*ArcSinh[c
*x])/(g*(f + g*x)) + (a*c^3*f^2*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x])/(g^2*(c^2
*f^2 + g^2)*Sqrt[1 + c^2*x^2]) + (b*c^3*f^2*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x
]^2)/(2*g^2*(c^2*f^2 + g^2)*Sqrt[1 + c^2*x^2]) - ((g - c^2*f*x)^2*Sqrt[d +
c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c*(c^2*f^2 + g^2)*(f + g*x)^2*Sqrt[
1 + c^2*x^2]) + (Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])
^2)/(2*b*c*(f + g*x)^2) + (a*c^2*f*Sqrt[d + c^2*d*x^2]*ArcTanh[(g - c^2*f*x
)/(Sqrt[c^2*f^2 + g^2]*Sqrt[1 + c^2*x^2])])/(g^2*Sqrt[c^2*f^2 + g^2]*Sqrt[1
 + c^2*x^2]) - (b*c^2*f*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]*Log[1 + (E^ArcSinh
[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2])])/(g^2*Sqrt[c^2*f^2 + g^2]*Sqrt[1 + c^
2*x^2]) + (b*c^2*f*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]*Log[1 + (E^ArcSinh[c*x]
*g)/(c*f + Sqrt[c^2*f^2 + g^2])])/(g^2*Sqrt[c^2*f^2 + g^2]*Sqrt[1 + c^2*x^2
]) + (b*c*Sqrt[d + c^2*d*x^2]*Log[f + g*x])/(g^2*Sqrt[1 + c^2*x^2]) - (b*c^
2*f*Sqrt[d + c^2*d*x^2]*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2
 + g^2]))])/(g^2*Sqrt[c^2*f^2 + g^2]*Sqrt[1 + c^2*x^2]) + (b*c^2*f*Sqrt[d +
 c^2*d*x^2]*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))])/
(g^2*Sqrt[c^2*f^2 + g^2]*Sqrt[1 + c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3324

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[(b*(c + d*x)^m*Cos[e + f*x])/(f*(a^2 - b^2)*(a + b*Sin[e +
f*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[(b*d*m)/(f*(a^2 - b^2)), Int[((c + d*x)^(m - 1)*Cos[e + f*x])/(a
 + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5813

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_))^(m_)*((f_.
) + (g_.)*(x_))^(p_.), x_Symbol] :> With[{u = IntHide[(f + g*x)^p*(d + e*x)
^m, x]}, Dist[(a + b*ArcSinh[c*x])^n, u, x] - Dist[b*c*n, Int[SimplifyInteg
rand[(u*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x], x]] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[m, 0] && L
tQ[m + p + 1, 0]
```

Rule 5823

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.) + (g_.)*(x_))^(m_)*Sqr
t[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[((f + g*x)^m*(d + e*x^2)*(a + b*A
```

```
rcSinh[c*x])^(n + 1))/(b*c*Sqrt[d]*(n + 1)), x] - Dist[1/(b*c*Sqrt[d]*(n +
1)), Int[(d*g*m + 2*e*f*x + e*g*(m + 2)*x^2)*(f + g*x)^(m - 1)*(a + b*ArcSi
nh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d
] && ILtQ[m, 0] && GtQ[d, 0] && IGtQ[n, 0]
```

Rule 5831

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 5835

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(d^IntPart[p]*(d + e*x^2)^FracPa
rt[p])/(1 + c^2*x^2)^FracPart[p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*Ar
cSinh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d
] && IntegerQ[m] && IntegerQ[p - 1/2] &&  !GtQ[d, 0]
```

Rule 5857

```
Int[ArcSinh[(c_.)*(x_)]^(n_.)*(RFx_)*((d_) + (e_.)*(x_)^2)^(p_), x_Symbol]
:> With[{u = ExpandIntegrand[(d + e*x^2)^p*ArcSinh[c*x]^n, RFx, x]}, Int[u,
 x] /; SumQ[u]] /; FreeQ[{c, d, e}, x] && RationalFunctionQ[RFx, x] && IGtQ
[n, 0] && EqQ[e, c^2*d] && IntegerQ[p - 1/2]
```

Rule 5859

```
Int[(ArcSinh[(c_.)*(x_)]*(b_.) + (a_))^(n_.)*(RFx_)*((d_) + (e_.)*(x_)^2)^(
p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p, RFx*(a + b*ArcSinh[c*x
])^n, x], x] /; FreeQ[{a, b, c, d, e}, x] && RationalFunctionQ[RFx, x] && I
GtQ[n, 0] && EqQ[e, c^2*d] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{(f+gx)^2}\,dx = \frac{\sqrt{d+c^2dx^2}\int \frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{(f+gx)^2}\,dx}{\sqrt{1+c^2x^2}}$$

$$= \frac{\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc(f+gx)^2} - \frac{\sqrt{d+c^2dx^2}\int \frac{\left(-2g+2c^2fx\right)(a+}{(f+g}}{2bc\sqrt{1+c^2x^2}}$$

$$= -\frac{\left(g-c^2fx\right)^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\left(c^2f^2+g^2\right)(f+gx)^2\sqrt{1+c^2x^2}} + \frac{\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}\,(a}{2bc(f+g}$$

$$= -\frac{\left(g-c^2fx\right)^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\left(c^2f^2+g^2\right)(f+gx)^2\sqrt{1+c^2x^2}} + \frac{\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}\,(a}{2bc(f+g}$$

$$= -\frac{\left(g-c^2fx\right)^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\left(c^2f^2+g^2\right)(f+gx)^2\sqrt{1+c^2x^2}} + \frac{\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}\,(a}{2bc(f+g}$$

$$= -\frac{\left(g-c^2fx\right)^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\left(c^2f^2+g^2\right)(f+gx)^2\sqrt{1+c^2x^2}} + \frac{\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}\,(a}{2bc(f+g}$$

$$= -\frac{a\sqrt{d+c^2dx^2}}{g(f+gx)} - \frac{\left(g-c^2fx\right)^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\left(c^2f^2+g^2\right)(f+gx)^2\sqrt{1+c^2x^2}} + \frac{\sqrt{1+c^2}}{}$$

$$= -\frac{a\sqrt{d+c^2dx^2}}{g(f+gx)} - \frac{\left(g-c^2fx\right)^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\left(c^2f^2+g^2\right)(f+gx)^2\sqrt{1+c^2x^2}} + \frac{\sqrt{1+c^2}}{}$$

$$= -\frac{a\sqrt{d+c^2dx^2}}{g(f+gx)} + \frac{ac^3f^2\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{g^2\left(c^2f^2+g^2\right)\sqrt{1+c^2x^2}} + \frac{bc^3f^2\sqrt{d+c^2dx^2}\,\mathrm{sinh}}{2g^2\left(c^2f^2+g^2\right)\sqrt{1+}}$$

$$= -\frac{a\sqrt{d+c^2dx^2}}{g(f+gx)} - \frac{b\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{g(f+gx)} + \frac{ac^3f^2\sqrt{d+c^2dx^2}\sinh^{-1}(cx}{g^2\left(c^2f^2+g^2\right)\sqrt{1+c^2x^2}}$$

$$= -\frac{a\sqrt{d+c^2dx^2}}{g(f+gx)} - \frac{b\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{g(f+gx)} + \frac{ac^3f^2\sqrt{d+c^2dx^2}\sinh^{-1}(cx}{g^2\left(c^2f^2+g^2\right)\sqrt{1+c^2x^2}}$$

$$= -\frac{a\sqrt{d+c^2dx^2}}{g(f+gx)} - \frac{b\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{g(f+gx)} + \frac{ac^3f^2\sqrt{d+c^2dx^2}\sinh^{-1}(cx}{g^2\left(c^2f^2+g^2\right)\sqrt{1+c^2x^2}}$$

$$= -\frac{a\sqrt{d+c^2dx^2}}{g(f+gx)} - \frac{b\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{g(f+gx)} + \frac{ac^3f^2\sqrt{d+c^2dx^2}\sinh^{-1}(cx}{g^2\left(c^2f^2+g^2\right)\sqrt{1+c^2x^2}}$$

$$= -\frac{a\sqrt{d+c^2dx^2}}{g(f+gx)} - \frac{b\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{g(f+gx)} + \frac{ac^3f^2\sqrt{d+c^2dx^2}\sinh^{-1}(cx}{g^2\left(c^2f^2+g^2\right)\sqrt{1+c^2x^2}}$$

$$= -\frac{a\sqrt{d+c^2dx^2}}{g(f+gx)} - \frac{b\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{g(f+gx)} + \frac{ac^3f^2\sqrt{d+c^2dx^2}\sinh^{-1}(cx}{g^2\left(c^2f^2+g^2\right)\sqrt{1+c^2x^2}}$$

Mathematica [C] time = 9.65, size = 1384, normalized size = 1.77

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(f + g*x)^2,x]
```

```
[Out] ((-2*a*g*Sqrt[d + c^2*d*x^2])/(f + g*x) - (2*a*c^2*Sqrt[d]*f*Log[f + g*x])/
Sqrt[c^2*f^2 + g^2] + 2*a*c*Sqrt[d]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]
] + (2*a*c^2*Sqrt[d]*f*Log[d*(g - c^2*f*x) + Sqrt[d]*Sqrt[c^2*f^2 + g^2]*Sq
rt[d + c^2*d*x^2]])/Sqrt[c^2*f^2 + g^2] + b*c*Sqrt[d + c^2*d*x^2]*((-2*g*Ar
cSinh[c*x])/(c*f + c*g*x) + ArcSinh[c*x]^2/Sqrt[1 + c^2*x^2] + ((2*I)*c*f*P
i*ArcTanh[(-g + c*f*Tanh[ArcSinh[c*x]/2])/Sqrt[c^2*f^2 + g^2]])/(Sqrt[c^2*f
^2 + g^2]*Sqrt[1 + c^2*x^2]) + (2*Log[1 + (g*x)/f])/Sqrt[1 + c^2*x^2] + (2*
c*f*(2*ArcCos[((-I)*c*f)/g]*ArcTanh[((c*f + I*g)*Cot[(Pi + (2*I)*ArcSinh[c*
x])/4])/Sqrt[-(c^2*f^2) - g^2]] + (Pi - (2*I)*ArcSinh[c*x])*ArcTanh[((c*f -
 I*g)*Tan[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]] + (ArcCos[(
(-I)*c*f)/g] - (2*I)*ArcTanh[((c*f + I*g)*Cot[(Pi + (2*I)*ArcSinh[c*x])/4])
/Sqrt[-(c^2*f^2) - g^2]] - (2*I)*ArcTanh[((c*f - I*g)*Tan[(Pi + (2*I)*ArcSi
nh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]])*Log[((1/2 - I/2)*Sqrt[-(c^2*f^2) - g^
2])/(E^(ArcSinh[c*x]/2)*Sqrt[(-I)*g]*Sqrt[c*(f + g*x)])] + (ArcCos[((-I)*c*
f)/g] + (2*I)*(ArcTanh[((c*f + I*g)*Cot[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[
-(c^2*f^2) - g^2]] + ArcTanh[((c*f - I*g)*Tan[(Pi + (2*I)*ArcSinh[c*x])/4])
/Sqrt[-(c^2*f^2) - g^2]]))*Log[((1/2 + I/2)*E^(ArcSinh[c*x]/2)*Sqrt[-(c^2*f
^2) - g^2])/(Sqrt[(-I)*g]*Sqrt[c*(f + g*x)])] - (ArcCos[((-I)*c*f)/g] + (2*
I)*ArcTanh[((c*f + I*g)*Cot[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) -
 g^2]])*Log[((I*c*f + g)*((-I)*c*f + g + Sqrt[-(c^2*f^2) - g^2])*(1 + I*Cot
[(Pi + (2*I)*ArcSinh[c*x])/4]))/(g*(I*c*f + g + I*Sqrt[-(c^2*f^2) - g^2]*Co
t[(Pi + (2*I)*ArcSinh[c*x])/4]))] - (ArcCos[((-I)*c*f)/g] - (2*I)*ArcTanh[(
(c*f + I*g)*Cot[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]])*Log[
((I*c*f + g)*(I*c*f - g + Sqrt[-(c^2*f^2) - g^2])*(I + Cot[(Pi + (2*I)*ArcS
inh[c*x])/4]))/(g*(c*f - I*g + Sqrt[-(c^2*f^2) - g^2]*Cot[(Pi + (2*I)*ArcSi
nh[c*x])/4]))] + I*(PolyLog[2, ((I*c*f + Sqrt[-(c^2*f^2) - g^2])*(I*c*f + g
 - I*Sqrt[-(c^2*f^2) - g^2]*Cot[(Pi + (2*I)*ArcSinh[c*x])/4]))/(g*(I*c*f +
g + I*Sqrt[-(c^2*f^2) - g^2]*Cot[(Pi + (2*I)*ArcSinh[c*x])/4]))] - PolyLog[
2, ((c*f + I*Sqrt[-(c^2*f^2) - g^2])*(-(c*f) + I*g + Sqrt[-(c^2*f^2) - g^2]
*Cot[(Pi + (2*I)*ArcSinh[c*x])/4]))/(g*(I*c*f + g + I*Sqrt[-(c^2*f^2) - g^2
]*Cot[(Pi + (2*I)*ArcSinh[c*x])/4]))])))/(Sqrt[-(c^2*f^2) - g^2]*Sqrt[1 + c
^2*x^2])))/(2*g^2)
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2dx^2 + d}\,(b\,\text{arsinh}\,(cx) + a)}{g^2x^2 + 2\,fgx + f^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(g*x+f)^2,x, algorithm="fr
icas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(g^2*x^2 + 2*f*g*x + f^2)
, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(g*x+f)^2,x, algorithm="gi
ac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.71, size = 1814, normalized size = 2.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(g*x+f)^2,x)
```

```
[Out] -a/d/(c^2*f^2+g^2)/(x+f/g)*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+
g^2)/g^2)^(3/2)-a/g*c^2*f/(c^2*f^2+g^2)*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g
)+d*(c^2*f^2+g^2)/g^2)^(1/2)+a/g^2*c^4*f^2/(c^2*f^2+g^2)*d*ln((-c^2*d*f/g+c
^2*d*(x+f/g))/(c^2*d)^(1/2)+((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2
+g^2)/g^2)^(1/2))/(c^2*d)^(1/2)+a/g^3*c^4*f^3/(c^2*f^2+g^2)*d/(d*(c^2*f^2+g
^2)/g^2)^(1/2)*ln((2*d*(c^2*f^2+g^2)/g^2-2*c^2*d*f/g*(x+f/g)+2*(d*(c^2*f^2+
g^2)/g^2)^(1/2)*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(
1/2))/(x+f/g))+a/g*c^2*f/(c^2*f^2+g^2)*d/(d*(c^2*f^2+g^2)/g^2)^(1/2)*ln((2*
d*(c^2*f^2+g^2)/g^2-2*c^2*d*f/g*(x+f/g)+2*(d*(c^2*f^2+g^2)/g^2)^(1/2)*((x+f
/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2))/(x+f/g))+a*c^2/
(c^2*f^2+g^2)*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/
2)*x+a*c^2/(c^2*f^2+g^2)*d*ln((-c^2*d*f/g+c^2*d*(x+f/g))/(c^2*d)^(1/2)+((x+
f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2))/(c^2*d)^(1/2)+
1/2*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*c/g^2+b*(d*(c^
2*x^2+1))^(1/2)*arcsinh(c*x)/(c^2*x^2+1)/g^2/(g*x+f)*x^3*c^4*f-b*(d*(c^2*x^
2+1))^(1/2)*arcsinh(c*x)/g^2/(g*x+f)*x*c^2*f-b*(d*(c^2*x^2+1))^(1/2)*arcsin
h(c*x)/(c^2*x^2+1)/g/(g*x+f)*x^2*c^2+b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/(
c^2*x^2+1)^(1/2)/g/(g*x+f)*x*c+b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/(c^2*x^
2+1)/g^2/(g*x+f)*x*c^2*f+b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/(c^2*x^2+1)^(
1/2)/g^2/(g*x+f)*c*f-b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/(c^2*x^2+1)/g/(g*
x+f)-b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/g^2/(c^2*f^2+g^2)^(1/2)*c^2*
f*arcsinh(c*x)*ln((-(c*x+(c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2+g^2)^(1/2))/(-c*
f+(c^2*f^2+g^2)^(1/2)))+b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/g^2/(c^2*
f^2+g^2)^(1/2)*c^2*f*arcsinh(c*x)*ln(((c*x+(c^2*x^2+1)^(1/2))*g+c*f+(c^2*f^
2+g^2)^(1/2))/(c*f+(c^2*f^2+g^2)^(1/2)))-2*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2
+1)^(1/2)/g^2/(c^2*f^2+g^2)*c^3*ln(c*x+(c^2*x^2+1)^(1/2))*f^2+b*(d*(c^2*x^2
+1))^(1/2)/(c^2*x^2+1)^(1/2)/g^2/(c^2*f^2+g^2)*c^3*ln((c*x+(c^2*x^2+1)^(1/2
))^2*g+2*c*f*(c*x+(c^2*x^2+1)^(1/2))-g)*f^2-b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^
2+1)^(1/2)/g^2/(c^2*f^2+g^2)^(1/2)*c^2*f*dilog((-(c*x+(c^2*x^2+1)^(1/2))*g-
c*f+(c^2*f^2+g^2)^(1/2))/(-c*f+(c^2*f^2+g^2)^(1/2)))+b*(d*(c^2*x^2+1))^(1/2
)/(c^2*x^2+1)^(1/2)/g^2/(c^2*f^2+g^2)^(1/2)*c^2*f*dilog(((c*x+(c^2*x^2+1)^(
1/2))*g+c*f+(c^2*f^2+g^2)^(1/2))/(c*f+(c^2*f^2+g^2)^(1/2)))-2*b*(d*(c^2*x^2
+1))^(1/2)/(c^2*x^2+1)^(1/2)/(c^2*f^2+g^2)*c*ln(c*x+(c^2*x^2+1)^(1/2))+b*(d
*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/(c^2*f^2+g^2)*c*ln((c*x+(c^2*x^2+1)^(
1/2))^2*g+2*c*f*(c*x+(c^2*x^2+1)^(1/2))-g)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\left(\frac{c^2 d f \operatorname{arsinh}\left(\frac{cfx}{g\left|x+\frac{f}{g}\right|}-\frac{1}{c\left|x+\frac{f}{g}\right|}\right)}{\sqrt{\frac{c^2 d f^2}{g^2}+d}\, g^3}-\frac{c\sqrt{d}\operatorname{arsinh}(cx)}{g^2}+\frac{\sqrt{c^2 d x^2+d}}{g^2 x+fg}\right)a+b\int\frac{\sqrt{c^2 d x^2+d}\log\left(cx+\sqrt{c^2x^2+1}\right)}{g^2x^2+2fgx+f^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(g*x+f)^2,x, algorithm="ma
xima")
```

```
[Out] -(c^2*d*f*arcsinh(c*f*x/(g*abs(x + f/g)) - 1/(c*abs(x + f/g)))/(sqrt(c^2*d*
f^2/g^2 + d)*g^3) - c*sqrt(d)*arcsinh(c*x)/g^2 + sqrt(c^2*d*x^2 + d)/(g^2*x
 + f*g))*a + b*integrate(sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1))/(
g^2*x^2 + 2*f*g*x + f^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}\,(c\,x))\,\sqrt{d\,c^2\,x^2+d}}{\left(f+g\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2))/(f + g*x)^2,x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2))/(f + g*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d\left(c^2x^2+1\right)}\,(a+b\,\mathrm{asinh}\,(cx))}{\left(f+gx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))*(c**2*d*x**2+d)**(1/2)/(g*x+f)**2,x)
```

```
[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))/(f + g*x)**2, x)
```

3.39 $\int (f+gx)^3 \left(d+c^2dx^2\right)^{3/2} \left(a+b\sinh^{-1}(cx)\right) dx$

Optimal. Leaf size=918

$$-\frac{bc^3dg^3\sqrt{c^2dx^2+d}\,x^7}{49\sqrt{c^2x^2+1}}-\frac{bc^3dfg^2\sqrt{c^2dx^2+d}\,x^6}{12\sqrt{c^2x^2+1}}-\frac{8bcdg^3\sqrt{c^2dx^2+d}\,x^5}{175\sqrt{c^2x^2+1}}-\frac{3bc^3df^2g\sqrt{c^2dx^2+d}\,x^5}{25\sqrt{c^2x^2+1}}-\frac{bc^3df^3\sqrt{c^2dx^2+}}{16\sqrt{c^2x^2+}}$$

```
[Out] 3/8*d*f^3*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)+3/16*d*f*g^2*x*(a+b*arcs
inh(c*x))*(c^2*d*x^2+d)^(1/2)/c^2+3/8*d*f*g^2*x^3*(a+b*arcsinh(c*x))*(c^2*d
*x^2+d)^(1/2)+1/4*d*f^3*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2
)+1/2*d*f*g^2*x^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)+3/5*d*
f^2*g*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^2-1/5*d*g^3*(c
^2*x^2+1)^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^4+1/7*d*g^3*(c^2*x^2+1
)^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^4-3/5*b*d*f^2*g*x*(c^2*d*x^2+d
)^(1/2)/c/(c^2*x^2+1)^(1/2)+2/35*b*d*g^3*x*(c^2*d*x^2+d)^(1/2)/c^3/(c^2*x^2
+1)^(1/2)-5/16*b*c*d*f^3*x^2*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-3/32*b*d
*f*g^2*x^2*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-2/5*b*c*d*f^2*g*x^3*(c^2
*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/105*b*d*g^3*x^3*(c^2*d*x^2+d)^(1/2)/c/(
c^2*x^2+1)^(1/2)-1/16*b*c^3*d*f^3*x^4*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
-7/32*b*c*d*f*g^2*x^4*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-3/25*b*c^3*d*f^
2*g*x^5*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-8/175*b*c*d*g^3*x^5*(c^2*d*x^
2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/12*b*c^3*d*f*g^2*x^6*(c^2*d*x^2+d)^(1/2)/(c^
2*x^2+1)^(1/2)-1/49*b*c^3*d*g^3*x^7*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+3
/16*d*f^3*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/b/c/(c^2*x^2+1)^(1/2)-3/
32*d*f*g^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/b/c^3/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.93, antiderivative size = 918, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 17, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.567, Rules used = {5835, 5821, 5684, 5682, 5675, 30, 14, 5717, 194, 5744, 5742, 5758, 266, 43, 5732, 12, 373}

$$-\frac{bc^3dg^3\sqrt{c^2dx^2+d}\,x^7}{49\sqrt{c^2x^2+1}}-\frac{bc^3dfg^2\sqrt{c^2dx^2+d}\,x^6}{12\sqrt{c^2x^2+1}}-\frac{8bcdg^3\sqrt{c^2dx^2+d}\,x^5}{175\sqrt{c^2x^2+1}}-\frac{3bc^3df^2g\sqrt{c^2dx^2+d}\,x^5}{25\sqrt{c^2x^2+1}}-\frac{bc^3df^3\sqrt{c^2dx^2+}}{16\sqrt{c^2x^2+}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^3*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-3*b*d*f^2*g*x*Sqrt[d + c^2*d*x^2])/(5*c*Sqrt[1 + c^2*x^2]) + (2*b*d*g^3*x
*Sqrt[d + c^2*d*x^2])/(35*c^3*Sqrt[1 + c^2*x^2]) - (5*b*c*d*f^3*x^2*Sqrt[d
+ c^2*d*x^2])/(16*Sqrt[1 + c^2*x^2]) - (3*b*d*f*g^2*x^2*Sqrt[d + c^2*d*x^2]
)/(32*c*Sqrt[1 + c^2*x^2]) - (2*b*c*d*f^2*g*x^3*Sqrt[d + c^2*d*x^2])/(5*Sqr
t[1 + c^2*x^2]) - (b*d*g^3*x^3*Sqrt[d + c^2*d*x^2])/(105*c*Sqrt[1 + c^2*x^2
]) - (b*c^3*d*f^3*x^4*Sqrt[d + c^2*d*x^2])/(16*Sqrt[1 + c^2*x^2]) - (7*b*c*
d*f*g^2*x^4*Sqrt[d + c^2*d*x^2])/(32*Sqrt[1 + c^2*x^2]) - (3*b*c^3*d*f^2*g*
x^5*Sqrt[d + c^2*d*x^2])/(25*Sqrt[1 + c^2*x^2]) - (8*b*c*d*g^3*x^5*Sqrt[d +
 c^2*d*x^2])/(175*Sqrt[1 + c^2*x^2]) - (b*c^3*d*f*g^2*x^6*Sqrt[d + c^2*d*x^
2])/(12*Sqrt[1 + c^2*x^2]) - (b*c^3*d*g^3*x^7*Sqrt[d + c^2*d*x^2])/(49*Sqrt
[1 + c^2*x^2]) + (3*d*f^3*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/8 + (
3*d*f*g^2*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(16*c^2) + (3*d*f*g^2
*x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/8 + (d*f^3*x*(1 + c^2*x^2)*S
qrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/4 + (d*f*g^2*x^3*(1 + c^2*x^2)*Sqr
t[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/2 + (3*d*f^2*g*(1 + c^2*x^2)^2*Sqrt[
d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(5*c^2) - (d*g^3*(1 + c^2*x^2)^2*Sqrt[
d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(5*c^4) + (d*g^3*(1 + c^2*x^2)^3*Sqrt[
d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(7*c^4) + (3*d*f^3*Sqrt[d + c^2*d*x^2]
*(a + b*ArcSinh[c*x])^2)/(16*b*c*Sqrt[1 + c^2*x^2]) - (3*d*f*g^2*Sqrt[d + c
^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(32*b*c^3*Sqrt[1 + c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
```

```
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5732

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(1 + c^2*x^2)^p, x]}, Dist[d^p*(a + b
*ArcSinh[c*x]), u, x] - Dist[b*c*d^p, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*
x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegerQ
[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0]) && NeQ[p, -2^
(-1)] && GtQ[d, 0]
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
```

```
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5835

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(d^IntPart[p]*(d + e*x^2)^FracPa
rt[p])/(1 + c^2*x^2)^FracPart[p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*Ar
cSinh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d
] && IntegerQ[m] && IntegerQ[p - 1/2] &&  !GtQ[d, 0]
```

Rubi steps

$$\int (f+gx)^3\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)dx = \frac{\left(d\sqrt{d+c^2dx^2}\right)\int (f+gx)^3\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(d\sqrt{d+c^2dx^2}\right)\int \left(f^3\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+3f\right.}{}$$

$$= \frac{\left(df^3\sqrt{d+c^2dx^2}\right)\int \left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)dx}{\sqrt{1+c^2x^2}} +$$

$$= \frac{1}{4}df^3x\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{2}dfg^2x$$

$$= \frac{3}{8}df^3x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{8}dfg^2x^3\sqrt{d+c^2d}$$

$$= -\frac{3bdf^2gx\sqrt{d+c^2dx^2}}{5c\sqrt{1+c^2x^2}}-\frac{5bcdf^3x^2\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}}-\frac{2bcdf^2g}{5\sqrt{}}$$

$$= -\frac{3bdf^2gx\sqrt{d+c^2dx^2}}{5c\sqrt{1+c^2x^2}}+\frac{2bdg^3x\sqrt{d+c^2dx^2}}{35c^3\sqrt{1+c^2x^2}}-\frac{5bcdf^3x^2\sqrt{}}{16\sqrt{1}}$$

Mathematica [A] time = 3.71, size = 779, normalized size = 0.85

$$\frac{529200acd^{3/2}f\sqrt{c^2x^2+1}\left(2c^2f^2-g^2\right)\log\left(\sqrt{d}\sqrt{c^2dx^2+d}+cdx\right)+5040ad\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}\left(4c^6x^3\left(35\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^3*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (5040*a*d*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]*(-32*g^3 + c^2*g*(336*f^2 +
 105*f*g*x + 16*g^2*x^2) + 4*c^6*x^3*(35*f^3 + 84*f^2*g*x + 70*f*g^2*x^2 +
20*g^3*x^3) + 2*c^4*x*(175*f^3 + 336*f^2*g*x + 245*f*g^2*x^2 + 64*g^3*x^3))
 - 940800*b*c^2*d*f^2*g*Sqrt[d + c^2*d*x^2]*(3*c*x + c^3*x^3 - 3*(1 + c^2*x
^2)^(3/2)*ArcSinh[c*x]) - 37632*b*c^2*d*f^2*g*Sqrt[d + c^2*d*x^2]*(c*x*(-30
 + 5*c^2*x^2 + 9*c^4*x^4) - 15*Sqrt[1 + c^2*x^2]*(-2 + c^2*x^2 + 3*c^4*x^4)
*ArcSinh[c*x]) - 12544*b*d*g^3*Sqrt[d + c^2*d*x^2]*(c*x*(-30 + 5*c^2*x^2 +
9*c^4*x^4) - 15*Sqrt[1 + c^2*x^2]*(-2 + c^2*x^2 + 3*c^4*x^4)*ArcSinh[c*x])
 - 256*b*d*g^3*Sqrt[d + c^2*d*x^2]*(c*x*(840 - 140*c^2*x^2 + 63*c^4*x^4 + 22
5*c^6*x^6) - 105*Sqrt[1 + c^2*x^2]*(8 - 4*c^2*x^2 + 3*c^4*x^4 + 15*c^6*x^6)
```

```
*ArcSinh[c*x]) + 529200*a*c*d^(3/2)*f*(2*c^2*f^2 - g^2)*Sqrt[1 + c^2*x^2]*L
og[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] - 352800*b*c^3*d*f^3*Sqrt[d + c^2*d
*x^2]*(Cosh[2*ArcSinh[c*x]] - 2*ArcSinh[c*x]*(ArcSinh[c*x] + Sinh[2*ArcSinh
[c*x]])) - 22050*b*c^3*d*f^3*Sqrt[d + c^2*d*x^2]*(8*ArcSinh[c*x]^2 + Cosh[4
*ArcSinh[c*x]] - 4*ArcSinh[c*x]*Sinh[4*ArcSinh[c*x]]) - 66150*b*c*d*f*g^2*S
qrt[d + c^2*d*x^2]*(8*ArcSinh[c*x]^2 + Cosh[4*ArcSinh[c*x]] - 4*ArcSinh[c*x
]*Sinh[4*ArcSinh[c*x]]) + 3675*b*c*d*f*g^2*Sqrt[d + c^2*d*x^2]*(72*ArcSinh[
c*x]^2 + 18*Cosh[2*ArcSinh[c*x]] + 9*Cosh[4*ArcSinh[c*x]] - 2*Cosh[6*ArcSin
h[c*x]] + 12*ArcSinh[c*x]*(-3*Sinh[2*ArcSinh[c*x]] - 3*Sinh[4*ArcSinh[c*x]]
 + Sinh[6*ArcSinh[c*x]])))/(2822400*c^4*Sqrt[1 + c^2*x^2])
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(ac^2dg^3x^5 + 3\,ac^2dfg^2x^4 + 3\,adf^2gx + adf^3 + \left(3\,ac^2df^2g + adg^3\right)x^3 + \left(ac^2df^3 + 3\,adfg^2\right)x^2 + \left(bc^2dg\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="fr
icas")
```

```
[Out] integral((a*c^2*d*g^3*x^5 + 3*a*c^2*d*f*g^2*x^4 + 3*a*d*f^2*g*x + a*d*f^3 +
 (3*a*c^2*d*f^2*g + a*d*g^3)*x^3 + (a*c^2*d*f^3 + 3*a*d*f*g^2)*x^2 + (b*c^2
*d*g^3*x^5 + 3*b*c^2*d*f*g^2*x^4 + 3*b*d*f^2*g*x + b*d*f^3 + (3*b*c^2*d*f^2
*g + b*d*g^3)*x^3 + (b*c^2*d*f^3 + 3*b*d*f*g^2)*x^2)*arcsinh(c*x))*sqrt(c^2
*d*x^2 + d), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="gi
ac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [A] time = 0.96, size = 1510, normalized size = 1.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] 17/16*b*(d*(c^2*x^2+1))^(1/2)*f*g^2*d/(c^2*x^2+1)*arcsinh(c*x)*x^3+9/5*b*(d
*(c^2*x^2+1))^(1/2)*g*d/(c^2*x^2+1)*arcsinh(c*x)*x^2*f^2+3/5*b*(d*(c^2*x^2+
1))^(1/2)*g*d/c^2/(c^2*x^2+1)*arcsinh(c*x)*f^2+1/4*b*(d*(c^2*x^2+1))^(1/2)*
f^3*d*c^4/(c^2*x^2+1)*arcsinh(c*x)*x^5+7/8*b*(d*(c^2*x^2+1))^(1/2)*f^3*d*c^
2/(c^2*x^2+1)*arcsinh(c*x)*x^3-3/32*b*(d*(c^2*x^2+1))^(1/2)*f*arcsinh(c*x)^
2*d/(c^2*x^2+1)^(1/2)/c^3*g^2+1/7*b*(d*(c^2*x^2+1))^(1/2)*g^3*d*c^4/(c^2*x^
2+1)*arcsinh(c*x)*x^8+13/35*b*(d*(c^2*x^2+1))^(1/2)*g^3*d*c^2/(c^2*x^2+1)*a
rcsinh(c*x)*x^6-1/35*b*(d*(c^2*x^2+1))^(1/2)*g^3*d/c^2/(c^2*x^2+1)*arcsinh(
c*x)*x^2-1/12*b*(d*(c^2*x^2+1))^(1/2)*f*g^2*d*c^3/(c^2*x^2+1)^(1/2)*x^6-7/3
2*b*(d*(c^2*x^2+1))^(1/2)*f*g^2*d*c/(c^2*x^2+1)^(1/2)*x^4-3/32*b*(d*(c^2*x^
2+1))^(1/2)*f*g^2*d/c/(c^2*x^2+1)^(1/2)*x^2-3/25*b*(d*(c^2*x^2+1))^(1/2)*g*
d*c^3/(c^2*x^2+1)^(1/2)*x^5*f^2-2/5*b*(d*(c^2*x^2+1))^(1/2)*g*d*c/(c^2*x^2+
1)^(1/2)*x^3*f^2-3/5*b*(d*(c^2*x^2+1))^(1/2)*g*d/c/(c^2*x^2+1)^(1/2)*x*f^2-
2/35*a*g^3/d/c^4*(c^2*d*x^2+d)^(5/2)+1/4*a*f^3*x*(c^2*d*x^2+d)^(3/2)+3/8*a*
f^3*d*x*(c^2*d*x^2+d)^(1/2)+3/8*a*f^3*d^2*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x
```

```
^2+d)^(1/2))/(c^2*d)^(1/2)+11/8*b*(d*(c^2*x^2+1))^(1/2)*f*g^2*d*c^2/(c^2*x^
2+1)*arcsinh(c*x)*x^5+3/16*b*(d*(c^2*x^2+1))^(1/2)*f*g^2*d/c^2/(c^2*x^2+1)*
arcsinh(c*x)*x+1/7*a*g^3*x^2*(c^2*d*x^2+d)^(5/2)/c^2/d-1/8*a*f*g^2/c^2*x*(c
^2*d*x^2+d)^(3/2)+3/5*a*f^2*g/c^2/d*(c^2*d*x^2+d)^(5/2)-17/128*b*(d*(c^2*x^
2+1))^(1/2)*f^3*d/c/(c^2*x^2+1)^(1/2)+2/35*b*(d*(c^2*x^2+1))^(1/2)*g^3*d/c^
3/(c^2*x^2+1)^(1/2)*x+7/768*b*(d*(c^2*x^2+1))^(1/2)*f*g^2*d/c^3/(c^2*x^2+1)
^(1/2)-1/16*b*(d*(c^2*x^2+1))^(1/2)*f^3*d*c^3/(c^2*x^2+1)^(1/2)*x^4-5/16*b*
(d*(c^2*x^2+1))^(1/2)*f^3*d*c/(c^2*x^2+1)^(1/2)*x^2-1/49*b*(d*(c^2*x^2+1))^
(1/2)*g^3*d*c^3/(c^2*x^2+1)^(1/2)*x^7-8/175*b*(d*(c^2*x^2+1))^(1/2)*g^3*d*c
/(c^2*x^2+1)^(1/2)*x^5-1/105*b*(d*(c^2*x^2+1))^(1/2)*g^3*d/c/(c^2*x^2+1)^(1
/2)*x^3+1/2*a*f*g^2*x*(c^2*d*x^2+d)^(5/2)/c^2/d-3/16*a*f*g^2/c^2*d*x*(c^2*d
*x^2+d)^(1/2)-3/16*a*f*g^2/c^2*d^2*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(
1/2))/(c^2*d)^(1/2)+3/16*b*(d*(c^2*x^2+1))^(1/2)*f^3*arcsinh(c*x)^2*d/(c^2*
x^2+1)^(1/2)/c-2/35*b*(d*(c^2*x^2+1))^(1/2)*g^3*d/c^4/(c^2*x^2+1)*arcsinh(c
*x)+5/8*b*(d*(c^2*x^2+1))^(1/2)*f^3*d/(c^2*x^2+1)*arcsinh(c*x)*x+9/35*b*(d*
(c^2*x^2+1))^(1/2)*g^3*d/(c^2*x^2+1)*arcsinh(c*x)*x^4+3/5*b*(d*(c^2*x^2+1))
^(1/2)*g*d*c^4/(c^2*x^2+1)*arcsinh(c*x)*x^6*f^2+9/5*b*(d*(c^2*x^2+1))^(1/2)
*g*d*c^2/(c^2*x^2+1)*arcsinh(c*x)*x^4*f^2+1/2*b*(d*(c^2*x^2+1))^(1/2)*f*g^2
*d*c^4/(c^2*x^2+1)*arcsinh(c*x)*x^7
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="ma
xima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(f+g\,x\right)^3\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,{\left(d\,c^2\,x^2+d\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^3*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2),x)
```

```
[Out] int((f + g*x)^3*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d \left(c^{2} x^{2} + 1\right)\right)^{\frac{3}{2}} \left(a + b \operatorname{asinh}{\left(c x \right)}\right) \left(f + g x\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))*(f + g*x)**3, x)
```

3.40 $\int (f+gx)^2 \left(d+c^2dx^2\right)^{3/2} \left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=651

$$\frac{3}{8}df^2x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{4}df^2x\left(c^2x^2+1\right)\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{3df^2\sqrt{c^2dx^2+d}\,\left(a+b\right.}{16bc\sqrt{c^2x^2+}}$$

```
[Out] 3/8*d*f^2*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)+1/16*d*g^2*x*(a+b*arcsin
h(c*x))*(c^2*d*x^2+d)^(1/2)/c^2+1/8*d*g^2*x^3*(a+b*arcsinh(c*x))*(c^2*d*x^2
+d)^(1/2)+1/4*d*f^2*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)+1/
6*d*g^2*x^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)+2/5*d*f*g*(c
^2*x^2+1)^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^2-2/5*b*d*f*g*x*(c^2*d
*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-5/16*b*c*d*f^2*x^2*(c^2*d*x^2+d)^(1/2)/(c
^2*x^2+1)^(1/2)-1/32*b*d*g^2*x^2*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-4/
15*b*c*d*f*g*x^3*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/16*b*c^3*d*f^2*x^4
*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-7/96*b*c*d*g^2*x^4*(c^2*d*x^2+d)^(1/
2)/(c^2*x^2+1)^(1/2)-2/25*b*c^3*d*f*g*x^5*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(
1/2)-1/36*b*c^3*d*g^2*x^6*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+3/16*d*f^2*
(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/b/c/(c^2*x^2+1)^(1/2)-1/32*d*g^2*(
a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/b/c^3/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.73, antiderivative size = 651, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5835, 5821, 5684, 5682, 5675, 30, 14, 5717, 194, 5744, 5742, 5758}

$$\frac{3}{8}df^2x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{4}df^2x\left(c^2x^2+1\right)\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{3df^2\sqrt{c^2dx^2+d}\,\left(a+b\right.}{16bc\sqrt{c^2x^2+}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-2*b*d*f*g*x*Sqrt[d + c^2*d*x^2])/(5*c*Sqrt[1 + c^2*x^2]) - (5*b*c*d*f^2*x
^2*Sqrt[d + c^2*d*x^2])/(16*Sqrt[1 + c^2*x^2]) - (b*d*g^2*x^2*Sqrt[d + c^2*
d*x^2])/(32*c*Sqrt[1 + c^2*x^2]) - (4*b*c*d*f*g*x^3*Sqrt[d + c^2*d*x^2])/(1
5*Sqrt[1 + c^2*x^2]) - (b*c^3*d*f^2*x^4*Sqrt[d + c^2*d*x^2])/(16*Sqrt[1 + c
^2*x^2]) - (7*b*c*d*g^2*x^4*Sqrt[d + c^2*d*x^2])/(96*Sqrt[1 + c^2*x^2]) - (
2*b*c^3*d*f*g*x^5*Sqrt[d + c^2*d*x^2])/(25*Sqrt[1 + c^2*x^2]) - (b*c^3*d*g^
2*x^6*Sqrt[d + c^2*d*x^2])/(36*Sqrt[1 + c^2*x^2]) + (3*d*f^2*x*Sqrt[d + c^2
*d*x^2]*(a + b*ArcSinh[c*x]))/8 + (d*g^2*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSi
nh[c*x]))/(16*c^2) + (d*g^2*x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/8
 + (d*f^2*x*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/4 + (d*
g^2*x^3*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/6 + (2*d*f*
g*(1 + c^2*x^2)^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(5*c^2) + (3*d*
f^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(16*b*c*Sqrt[1 + c^2*x^2])
 - (d*g^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(32*b*c^3*Sqrt[1 + c^2
*x^2])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5835

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(d^IntPart[p]*(d + e*x^2)^FracPa
rt[p])/(1 + c^2*x^2)^FracPart[p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*Ar
cSinh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d
] && IntegerQ[m] && IntegerQ[p - 1/2] && !GtQ[d, 0]
```

Rubi steps

$$\int (f+gx)^2\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)dx = \frac{\left(d\sqrt{d+c^2dx^2}\right)\int (f+gx)^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)dx}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(d\sqrt{d+c^2dx^2}\right)\int\left(f^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+2fgx$$

$$= \frac{\left(df^2\sqrt{d+c^2dx^2}\right)\int\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)dx}{\sqrt{1+c^2x^2}}+\frac{(2d$$

$$= \frac{1}{4}df^2x\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{6}dg^2x^3\left(1$$

$$= \frac{3}{8}df^2x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{8}dg^2x^3\sqrt{d+c^2dx^2}\left(a$$

$$= -\frac{2bdfgx\sqrt{d+c^2dx^2}}{5c\sqrt{1+c^2x^2}}-\frac{5bcdf^2x^2\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}}-\frac{4bcdfgx^3\sqrt{d}}{15\sqrt{1+c}}$$

$$= -\frac{2bdfgx\sqrt{d+c^2dx^2}}{5c\sqrt{1+c^2x^2}}-\frac{5bcdf^2x^2\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}}-\frac{bdg^2x^2\sqrt{d+}}{32c\sqrt{1+c}}$$

Mathematica [A] time = 2.22, size = 546, normalized size = 0.84

$$3600ad^{3/2}\sqrt{c^2x^2+1}\left(6c^2f^2-g^2\right)\log\left(\sqrt{d}\sqrt{c^2dx^2+d}+cdx\right)+240acd\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}\left(30c^2f^2x\left(2c^2x^2+\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (240*a*c*d*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]*(96*f*g*(1 + c^2*x^2)^2 +
30*c^2*f^2*x*(5 + 2*c^2*x^2) + 5*g^2*x*(3 + 14*c^2*x^2 + 8*c^4*x^4)) - 1280
0*b*c*d*f*g*Sqrt[d + c^2*d*x^2]*(3*c*x + c^3*x^3 - 3*(1 + c^2*x^2)^(3/2)*Ar
cSinh[c*x]) - 512*b*c*d*f*g*Sqrt[d + c^2*d*x^2]*(c*x*(-30 + 5*c^2*x^2 + 9*c
^4*x^4) - 15*Sqrt[1 + c^2*x^2]*(-2 + c^2*x^2 + 3*c^4*x^4)*ArcSinh[c*x]) + 3
600*a*d^(3/2)*(6*c^2*f^2 - g^2)*Sqrt[1 + c^2*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[
d + c^2*d*x^2]] - 7200*b*c^2*d*f^2*Sqrt[d + c^2*d*x^2]*(Cosh[2*ArcSinh[c*x]
] - 2*ArcSinh[c*x]*(ArcSinh[c*x] + Sinh[2*ArcSinh[c*x]])) - 450*b*c^2*d*f^2
*Sqrt[d + c^2*d*x^2]*(8*ArcSinh[c*x]^2 + Cosh[4*ArcSinh[c*x]] - 4*ArcSinh[c
*x]*Sinh[4*ArcSinh[c*x]]) - 450*b*d*g^2*Sqrt[d + c^2*d*x^2]*(8*ArcSinh[c*x]
^2 + Cosh[4*ArcSinh[c*x]] - 4*ArcSinh[c*x]*Sinh[4*ArcSinh[c*x]]) + 25*b*d*g
^2*Sqrt[d + c^2*d*x^2]*(72*ArcSinh[c*x]^2 + 18*Cosh[2*ArcSinh[c*x]] + 9*Cos
h[4*ArcSinh[c*x]] - 2*Cosh[6*ArcSinh[c*x]] + 12*ArcSinh[c*x]*(-3*Sinh[2*Arc
Sinh[c*x]] - 3*Sinh[4*ArcSinh[c*x]] + Sinh[6*ArcSinh[c*x]])))/(57600*c^3*Sq
rt[1 + c^2*x^2])
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(ac^2dg^2x^4 + 2\,ac^2dfgx^3 + 2\,adfgx + adf^2 + \left(ac^2df^2 + adg^2\right)x^2 + \left(bc^2dg^2x^4 + 2\,bc^2dfgx^3 + 2\,bdfg\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="fr
icas")
```

```
[Out] integral((a*c^2*d*g^2*x^4 + 2*a*c^2*d*f*g*x^3 + 2*a*d*f*g*x + a*d*f^2 + (a*
c^2*d*f^2 + a*d*g^2)*x^2 + (b*c^2*d*g^2*x^4 + 2*b*c^2*d*f*g*x^3 + 2*b*d*f*g
*x + b*d*f^2 + (b*c^2*d*f^2 + b*d*g^2)*x^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 +
d), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="gi
ac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [A] time = 0.84, size = 1087, normalized size = 1.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] -1/24*a*g^2/c^2*x*(c^2*d*x^2+d)^(3/2)+6/5*b*(d*(c^2*x^2+1))^(1/2)*f*g*d/(c^
2*x^2+1)*arcsinh(c*x)*x^2+1/6*b*(d*(c^2*x^2+1))^(1/2)*g^2*d*c^4/(c^2*x^2+1)
*arcsinh(c*x)*x^7+11/24*b*(d*(c^2*x^2+1))^(1/2)*g^2*d*c^2/(c^2*x^2+1)*arcsi
nh(c*x)*x^5+1/16*b*(d*(c^2*x^2+1))^(1/2)*g^2*d/c^2/(c^2*x^2+1)*arcsinh(c*x)
*x+2/5*b*(d*(c^2*x^2+1))^(1/2)*f*g*d/c^2/(c^2*x^2+1)*arcsinh(c*x)+1/4*b*(d*
(c^2*x^2+1))^(1/2)*d*c^4/(c^2*x^2+1)*arcsinh(c*x)*x^5*f^2+7/8*b*(d*(c^2*x^2
+1))^(1/2)*d*c^2/(c^2*x^2+1)*arcsinh(c*x)*x^3*f^2-2/5*b*(d*(c^2*x^2+1))^(1/
```

```
2)*f*g*d/c/(c^2*x^2+1)^(1/2)*x-2/25*b*(d*(c^2*x^2+1))^(1/2)*f*g*d*c^3/(c^2*
x^2+1)^(1/2)*x^5-4/15*b*(d*(c^2*x^2+1))^(1/2)*f*g*d*c/(c^2*x^2+1)^(1/2)*x^3
+2/5*b*(d*(c^2*x^2+1))^(1/2)*f*g*d*c^4/(c^2*x^2+1)*arcsinh(c*x)*x^6+6/5*b*(
d*(c^2*x^2+1))^(1/2)*f*g*d*c^2/(c^2*x^2+1)*arcsinh(c*x)*x^4+2/5*a*f*g/c^2/d
*(c^2*d*x^2+d)^(5/2)+1/6*a*g^2*x*(c^2*d*x^2+d)^(5/2)/c^2/d-1/16*a*g^2/c^2*d
*x*(c^2*d*x^2+d)^(1/2)-1/16*a*g^2/c^2*d^2*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x
^2+d)^(1/2))/(c^2*d)^(1/2)-17/128*b*(d*(c^2*x^2+1))^(1/2)*d/c/(c^2*x^2+1)^(
1/2)*f^2+7/2304*b*(d*(c^2*x^2+1))^(1/2)*g^2*d/c^3/(c^2*x^2+1)^(1/2)+3/8*a*f
^2*d*x*(c^2*d*x^2+d)^(1/2)+3/8*a*f^2*d^2*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^
2+d)^(1/2))/(c^2*d)^(1/2)+1/4*a*f^2*x*(c^2*d*x^2+d)^(3/2)-1/36*b*(d*(c^2*x^
2+1))^(1/2)*g^2*d*c^3/(c^2*x^2+1)^(1/2)*x^6-7/96*b*(d*(c^2*x^2+1))^(1/2)*g^
2*d*c/(c^2*x^2+1)^(1/2)*x^4-1/32*b*(d*(c^2*x^2+1))^(1/2)*g^2*d/c/(c^2*x^2+1
)^(1/2)*x^2-1/16*b*(d*(c^2*x^2+1))^(1/2)*d*c^3/(c^2*x^2+1)^(1/2)*x^4*f^2-5/
16*b*(d*(c^2*x^2+1))^(1/2)*d*c/(c^2*x^2+1)^(1/2)*x^2*f^2+3/16*b*(d*(c^2*x^2
+1))^(1/2)*arcsinh(c*x)^2*d/(c^2*x^2+1)^(1/2)/c*f^2-1/32*b*(d*(c^2*x^2+1))^
(1/2)*arcsinh(c*x)^2*d/(c^2*x^2+1)^(1/2)/c^3*g^2+17/48*b*(d*(c^2*x^2+1))^(1
/2)*g^2*d/(c^2*x^2+1)*arcsinh(c*x)*x^3+5/8*b*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x
^2+1)*arcsinh(c*x)*x*f^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="ma
xima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(f + g\,x\right)^2\,\left(a + b\,\mathrm{asinh}\left(c\,x\right)\right)\,\left(d\,c^2\,x^2 + d\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2),x)
```

```
[Out] int((f + g*x)^2*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\left(c^2x^2 + 1\right)\right)^{\frac{3}{2}}\left(a + b\,\mathrm{asinh}\left(cx\right)\right)\left(f + gx\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))*(f + g*x)**2, x)
```

3.41 $\int (f+gx)\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=353

$$\frac{3}{8}dfx\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{4}dfx\left(c^2x^2+1\right)\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{3df\sqrt{c^2dx^2+d}\left(a+b\right.}{16bc\sqrt{c^2x^2+}}$$

```
[Out] 3/8*d*f*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)+1/4*d*f*x*(c^2*x^2+1)*(a+b
*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)+1/5*d*g*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))
*(c^2*d*x^2+d)^(1/2)/c^2-1/5*b*d*g*x*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2
)-5/16*b*c*d*f*x^2*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-2/15*b*c*d*g*x^3*(
c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/16*b*c^3*d*f*x^4*(c^2*d*x^2+d)^(1/2)
/(c^2*x^2+1)^(1/2)-1/25*b*c^3*d*g*x^5*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
+3/16*d*f*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/b/c/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {5835, 5821, 5684, 5682, 5675, 30, 14, 5717, 194}

$$\frac{3}{8}dfx\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{4}dfx\left(c^2x^2+1\right)\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{3df\sqrt{c^2dx^2+d}\left(a+b\right.}{16bc\sqrt{c^2x^2+}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -(b*d*g*x*Sqrt[d + c^2*d*x^2])/(5*c*Sqrt[1 + c^2*x^2]) - (5*b*c*d*f*x^2*Sqr
t[d + c^2*d*x^2])/(16*Sqrt[1 + c^2*x^2]) - (2*b*c*d*g*x^3*Sqrt[d + c^2*d*x^
2])/(15*Sqrt[1 + c^2*x^2]) - (b*c^3*d*f*x^4*Sqrt[d + c^2*d*x^2])/(16*Sqrt[1
 + c^2*x^2]) - (b*c^3*d*g*x^5*Sqrt[d + c^2*d*x^2])/(25*Sqrt[1 + c^2*x^2]) +
 (3*d*f*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/8 + (d*f*x*(1 + c^2*x^2
)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/4 + (d*g*(1 + c^2*x^2)^2*Sqrt[d
 + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(5*c^2) + (3*d*f*Sqrt[d + c^2*d*x^2]*(a
 + b*ArcSinh[c*x])^2)/(16*b*c*Sqrt[1 + c^2*x^2])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5835

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(d^IntPart[p]*(d + e*x^2)^FracPa
rt[p])/(1 + c^2*x^2)^FracPart[p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*Ar
cSinh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d
] && IntegerQ[m] && IntegerQ[p - 1/2] &&  !GtQ[d, 0]
```

Rubi steps

$$\int (f+gx)\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)dx = \frac{\left(d\sqrt{d+c^2dx^2}\right)\int (f+gx)\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)d}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(d\sqrt{d+c^2dx^2}\right)\int \left(f\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+gx\left(1\right.\right.}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(df\sqrt{d+c^2dx^2}\right)\int \left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)dx}{\sqrt{1+c^2x^2}} + \frac{(d}{}$$

$$= \frac{1}{4}dfx\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right) + \frac{dg\left(1+c^2\right.}{}$$

$$= \frac{3}{8}dfx\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right) + \frac{1}{4}dfx\left(1+c^2x^2\right)\sqrt{d}$$

$$= -\frac{bdgx\sqrt{d+c^2dx^2}}{5c\sqrt{1+c^2x^2}} - \frac{5bcdfx^2\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}} - \frac{2bcdgx^3\sqrt{d+c}}{15\sqrt{1+c^2x}}$$

Mathematica [A] time = 1.23, size = 392, normalized size = 1.11

$$\frac{3600acd^{3/2}f\sqrt{c^2x^2+1}\log\left(\sqrt{d}\sqrt{c^2dx^2+d}+cdx\right)+240ad\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}\left(5c^2fx\left(2c^2x^2+5\right)+8g\left(c^2\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-640*b*c*d*g*x*(3 + c^2*x^2)*Sqrt[d + c^2*d*x^2] - 128*b*c^3*d*g*x^3*(5 +
3*c^2*x^2)*Sqrt[d + c^2*d*x^2] + 240*a*d*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x
^2]*(8*g*(1 + c^2*x^2)^2 + 5*c^2*f*x*(5 + 2*c^2*x^2)) + 3200*b*d*g*(1 + c^2
*x^2)^(3/2)*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x] + 640*b*d*g*(1 + c^2*x^2)^(3/2
)*(-2 + 3*c^2*x^2)*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x] - 1200*b*c*d*f*Sqrt[d +
 c^2*d*x^2]*Cosh[2*ArcSinh[c*x]] + 3600*a*c*d^(3/2)*f*Sqrt[1 + c^2*x^2]*Log
[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + 2400*b*c*d*f*Sqrt[d + c^2*d*x^2]*Ar
cSinh[c*x]*(ArcSinh[c*x] + Sinh[2*ArcSinh[c*x]]) - 75*b*c*d*f*Sqrt[d + c^2*
d*x^2]*(8*ArcSinh[c*x]^2 + Cosh[4*ArcSinh[c*x]] - 4*ArcSinh[c*x]*Sinh[4*Arc
Sinh[c*x]]))/(9600*c^2*Sqrt[1 + c^2*x^2])
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(ac^2dgx^3+ac^2dfx^2+adgx+adf+\left(bc^2dgx^3+bc^2dfx^2+bdgx+bdf\right)\operatorname{arsinh}(cx)\right)\sqrt{c^2dx^2+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="fric
as")
```

```
[Out] integral((a*c^2*d*g*x^3 + a*c^2*d*f*x^2 + a*d*g*x + a*d*f + (b*c^2*d*g*x^3
+ b*c^2*d*f*x^2 + b*d*g*x + b*d*f)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="giac
")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [A] time = 0.50, size = 601, normalized size = 1.70

$$\frac{ag\left(c^2d\,x^2+d\right)^{\frac{5}{2}}}{5c^2d}+\frac{afx\left(c^2d\,x^2+d\right)^{\frac{3}{2}}}{4}+\frac{3afdx\sqrt{c^2d\,x^2+d}}{8}+\frac{3af\,d^2\ln\left(\frac{x\,c^2d}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{8\sqrt{c^2d}}+\frac{3b\sqrt{d\left(c^2x^2+1\right)}\,f\,a}{16\sqrt{c^2x^2+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/5*a*g/c^2/d*(c^2*d*x^2+d)^(5/2)+1/4*a*f*x*(c^2*d*x^2+d)^(3/2)+3/8*a*f*d*x
*(c^2*d*x^2+d)^(1/2)+3/8*a*f*d^2*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/
2))/(c^2*d)^(1/2)+3/16*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c*f*arcsin
h(c*x)^2*d+1/5*b*(d*(c^2*x^2+1))^(1/2)*g*d*c^4/(c^2*x^2+1)*arcsinh(c*x)*x^6
-1/25*b*(d*(c^2*x^2+1))^(1/2)*g*d*c^3/(c^2*x^2+1)^(1/2)*x^5+3/5*b*(d*(c^2*x
^2+1))^(1/2)*g*d*c^2/(c^2*x^2+1)*arcsinh(c*x)*x^4-2/15*b*(d*(c^2*x^2+1))^(1
/2)*g*d*c/(c^2*x^2+1)^(1/2)*x^3+3/5*b*(d*(c^2*x^2+1))^(1/2)*g*d/(c^2*x^2+1)
*arcsinh(c*x)*x^2-1/5*b*(d*(c^2*x^2+1))^(1/2)*g*d/c/(c^2*x^2+1)^(1/2)*x+1/4
*b*(d*(c^2*x^2+1))^(1/2)*f*d*c^4/(c^2*x^2+1)*arcsinh(c*x)*x^5-1/16*b*(d*(c^
2*x^2+1))^(1/2)*f*d*c^3/(c^2*x^2+1)^(1/2)*x^4+7/8*b*(d*(c^2*x^2+1))^(1/2)*f
*d*c^2/(c^2*x^2+1)*arcsinh(c*x)*x^3-5/16*b*(d*(c^2*x^2+1))^(1/2)*f*d*c/(c^2
*x^2+1)^(1/2)*x^2+5/8*b*(d*(c^2*x^2+1))^(1/2)*f*d/(c^2*x^2+1)*arcsinh(c*x)*
x-17/128*b*(d*(c^2*x^2+1))^(1/2)*f*d/c/(c^2*x^2+1)^(1/2)+1/5*b*(d*(c^2*x^2+
1))^(1/2)*g*d/c^2/(c^2*x^2+1)*arcsinh(c*x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(f+g\,x\right)\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,{\left(d\,c^2\,x^2+d\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2),x)
```

```
[Out] int((f + g*x)*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\left(c^2x^2+1\right)\right)^{\frac{3}{2}}\left(a+b\operatorname{asinh}\left(cx\right)\right)\left(f+gx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))*(f + g*x), x)
```

3.42 $\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{f+gx}\,dx$

Optimal. Leaf size=984

$$-\frac{bdx^3\sqrt{c^2dx^2+d}\,c^3}{9g\sqrt{c^2x^2+1}}+\frac{bdfx^2\sqrt{c^2dx^2+d}\,c^3}{4g^2\sqrt{c^2x^2+1}}-\frac{dfx\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)c^2}{2g^2}-\frac{d\left(c^2f^2+g^2\right)x\sqrt{c^2dx^2+d}}{2bg^3\sqrt{c^2x^2}}$$

```
[Out] a*d*(c^2*f^2+g^2)*(c^2*d*x^2+d)^(1/2)/g^3+b*d*(c^2*f^2+g^2)*arcsinh(c*x)*(c
^2*d*x^2+d)^(1/2)/g^3-1/2*c^2*d*f*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/
g^2+1/3*d*(c^2*x^2+1)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/g-1/3*b*c*d*x*
(c^2*d*x^2+d)^(1/2)/g/(c^2*x^2+1)^(1/2)-b*c*d*(c^2*f^2+g^2)*x*(c^2*d*x^2+d)
^(1/2)/g^3/(c^2*x^2+1)^(1/2)+1/4*b*c^3*d*f*x^2*(c^2*d*x^2+d)^(1/2)/g^2/(c^2
*x^2+1)^(1/2)-1/9*b*c^3*d*x^3*(c^2*d*x^2+d)^(1/2)/g/(c^2*x^2+1)^(1/2)-1/4*c
*d*f*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/b/g^2/(c^2*x^2+1)^(1/2)-1/2*c
*d*(c^2*f^2+g^2)*x*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/b/g^3/(c^2*x^2+
1)^(1/2)-1/2*d*(c^2*f^2+g^2)^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/b/c
/g^4/(g*x+f)/(c^2*x^2+1)^(1/2)-a*d*(c^2*f^2+g^2)^(3/2)*arctanh((-c^2*f*x+g)
/(c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/g^4/(c^2*x^2+1)
^(1/2)+b*d*(c^2*f^2+g^2)^(3/2)*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))*g/
(c*f-(c^2*f^2+g^2)^(1/2)))*(c^2*d*x^2+d)^(1/2)/g^4/(c^2*x^2+1)^(1/2)-b*d*(c
^2*f^2+g^2)^(3/2)*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2
+g^2)^(1/2)))*(c^2*d*x^2+d)^(1/2)/g^4/(c^2*x^2+1)^(1/2)+b*d*(c^2*f^2+g^2)^(
3/2)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2+g^2)^(1/2)))*(c^2*d
*x^2+d)^(1/2)/g^4/(c^2*x^2+1)^(1/2)-b*d*(c^2*f^2+g^2)^(3/2)*polylog(2,-(c*x
+(c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2+g^2)^(1/2)))*(c^2*d*x^2+d)^(1/2)/g^4/(c
^2*x^2+1)^(1/2)+1/2*d*(c^2*f^2+g^2)*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)*
(c^2*d*x^2+d)^(1/2)/b/c/g^2/(g*x+f)
```

Rubi [A] time = 1.89, antiderivative size = 984, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 24, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {5835, 5825, 5682, 5675, 30, 5717, 5823, 683, 5815, 6742, 261, 725, 206, 5859, 1654, 12, 5857, 8, 5831, 3322, 2264, 2190, 2279, 2391}

$$-\frac{bdx^3\sqrt{c^2dx^2+d}\,c^3}{9g\sqrt{c^2x^2+1}}+\frac{bdfx^2\sqrt{c^2dx^2+d}\,c^3}{4g^2\sqrt{c^2x^2+1}}-\frac{dfx\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)c^2}{2g^2}-\frac{d\left(c^2f^2+g^2\right)x\sqrt{c^2dx^2+d}}{2bg^3\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(f + g*x),x]
```

```
[Out] (a*d*(c^2*f^2 + g^2)*Sqrt[d + c^2*d*x^2])/g^3 - (b*c*d*x*Sqrt[d + c^2*d*x^2
])/(3*g*Sqrt[1 + c^2*x^2]) - (b*c*d*(c^2*f^2 + g^2)*x*Sqrt[d + c^2*d*x^2])/
(g^3*Sqrt[1 + c^2*x^2]) + (b*c^3*d*f*x^2*Sqrt[d + c^2*d*x^2])/(4*g^2*Sqrt[1
 + c^2*x^2]) - (b*c^3*d*x^3*Sqrt[d + c^2*d*x^2])/(9*g*Sqrt[1 + c^2*x^2]) +
(b*d*(c^2*f^2 + g^2)*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x])/g^3 - (c^2*d*f*x*Sqr
t[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(2*g^2) + (d*(1 + c^2*x^2)*Sqrt[d +
c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(3*g) - (c*d*f*Sqrt[d + c^2*d*x^2]*(a + b*
ArcSinh[c*x])^2)/(4*b*g^2*Sqrt[1 + c^2*x^2]) - (c*d*(c^2*f^2 + g^2)*x*Sqrt[
d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*g^3*Sqrt[1 + c^2*x^2]) - (d*(c^
2*f^2 + g^2)^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c*g^4*(f +
g*x)*Sqrt[1 + c^2*x^2]) + (d*(c^2*f^2 + g^2)*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2
*d*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c*g^2*(f + g*x)) - (a*d*(c^2*f^2 + g^2
)^(3/2)*Sqrt[d + c^2*d*x^2]*ArcTanh[(g - c^2*f*x)/(Sqrt[c^2*f^2 + g^2]*Sqrt
[1 + c^2*x^2])])/(g^4*Sqrt[1 + c^2*x^2]) + (b*d*(c^2*f^2 + g^2)^(3/2)*Sqrt[
d + c^2*d*x^2]*ArcSinh[c*x]*Log[1 + (E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2
```

```
+ g^2])])/(g^4*Sqrt[1 + c^2*x^2]) - (b*d*(c^2*f^2 + g^2)^(3/2)*Sqrt[d + c^2
*d*x^2]*ArcSinh[c*x]*Log[1 + (E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2])
])/(g^4*Sqrt[1 + c^2*x^2]) + (b*d*(c^2*f^2 + g^2)^(3/2)*Sqrt[d + c^2*d*x^2]
*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2]))])/(g^4*Sqrt[1
 + c^2*x^2]) - (b*d*(c^2*f^2 + g^2)^(3/2)*Sqrt[d + c^2*d*x^2]*PolyLog[2, -(
(E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))])/(g^4*Sqrt[1 + c^2*x^2])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5815

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.) + (g_.)*(x_) + (h_.)*(
```

```
x_)^2)^(p_.))/((d_) + (e_.)*(x_))^2, x_Symbol] :> With[{u = IntHide[(f + g*
x + h*x^2)^p/(d + e*x)^2, x]}, Dist[(a + b*ArcSinh[c*x])^n, u, x] - Dist[b*
c*n, Int[SimplifyIntegrand[(u*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^
2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[n, 0] && IGtQ
[p, 0] && EqQ[e*g - 2*d*h, 0]
```

Rule 5823

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.) + (g_.)*(x_))^(m_)*Sqr
t[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[((f + g*x)^m*(d + e*x^2)*(a + b*A
rcSinh[c*x])^(n + 1))/(b*c*Sqrt[d]*(n + 1)), x] - Dist[1/(b*c*Sqrt[d]*(n +
1)), Int[(d*g*m + 2*e*f*x + e*g*(m + 2)*x^2)*(f + g*x)^(m - 1)*(a + b*ArcSi
nh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d
] && ILtQ[m, 0] && GtQ[d, 0] && IGtQ[n, 0]
```

Rule 5825

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[Sqrt[d + e*x^2]*(
a + b*ArcSinh[c*x])^n, (f + g*x)^m*(d + e*x^2)^(p - 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IGtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Rule 5831

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 5835

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(d^IntPart[p]*(d + e*x^2)^FracPa
rt[p])/(1 + c^2*x^2)^FracPart[p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*Ar
cSinh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d
] && IntegerQ[m] && IntegerQ[p - 1/2] &&  !GtQ[d, 0]
```

Rule 5857

```
Int[ArcSinh[(c_.)*(x_)]^(n_.)*(RFx_)*((d_) + (e_.)*(x_)^2)^(p_), x_Symbol]
:> With[{u = ExpandIntegrand[(d + e*x^2)^p*ArcSinh[c*x]^n, RFx, x]}, Int[u,
 x] /; SumQ[u]] /; FreeQ[{c, d, e}, x] && RationalFunctionQ[RFx, x] && IGtQ
[n, 0] && EqQ[e, c^2*d] && IntegerQ[p - 1/2]
```

Rule 5859

```
Int[(ArcSinh[(c_.)*(x_)]*(b_.) + (a_))^(n_.)*(RFx_)*((d_) + (e_.)*(x_)^2)^(
p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p, RFx*(a + b*ArcSinh[c*x
])^n, x], x] /; FreeQ[{a, b, c, d, e}, x] && RationalFunctionQ[RFx, x] && I
GtQ[n, 0] && EqQ[e, c^2*d] && IntegerQ[p - 1/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{f+gx}\,dx = \frac{\left(d\sqrt{d+c^2dx^2}\right)\int \frac{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{f+gx}\,dx}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(d\sqrt{d+c^2dx^2}\right)\int\left(-\frac{c^2f\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{g^2}+\frac{c^2x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx\right.}{g}\right.}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(d\left(1+\frac{c^2f^2}{g^2}\right)\sqrt{d+c^2dx^2}\right)\int \frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{f+gx}\,dx}{\sqrt{1+c^2x^2}} - \frac{\left(c^2df\sqrt{d+c\right.}{}$$

$$= -\frac{c^2dfx\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2g^2} + \frac{d\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}\left(a\right.}{3g}$$

$$= -\frac{bcdx\sqrt{d+c^2dx^2}}{3g\sqrt{1+c^2x^2}} + \frac{bc^3dfx^2\sqrt{d+c^2dx^2}}{4g^2\sqrt{1+c^2x^2}} - \frac{bc^3dx^3\sqrt{d+c^2dx^2}}{9g\sqrt{1+c^2x^2}} - \frac{c^2}{}$$

$$= -\frac{bcdx\sqrt{d+c^2dx^2}}{3g\sqrt{1+c^2x^2}} + \frac{bc^3dfx^2\sqrt{d+c^2dx^2}}{4g^2\sqrt{1+c^2x^2}} - \frac{bc^3dx^3\sqrt{d+c^2dx^2}}{9g\sqrt{1+c^2x^2}} - \frac{c^2}{}$$

$$= -\frac{bcdx\sqrt{d+c^2dx^2}}{3g\sqrt{1+c^2x^2}} + \frac{bc^3dfx^2\sqrt{d+c^2dx^2}}{4g^2\sqrt{1+c^2x^2}} - \frac{bc^3dx^3\sqrt{d+c^2dx^2}}{9g\sqrt{1+c^2x^2}} - \frac{c^2}{}$$

$$= \frac{ad\left(c^2f^2+g^2\right)\sqrt{d+c^2dx^2}}{g^3} - \frac{bcdx\sqrt{d+c^2dx^2}}{3g\sqrt{1+c^2x^2}} + \frac{bc^3dfx^2\sqrt{d+c^2dx}}{4g^2\sqrt{1+c^2x^2}}$$

$$= \frac{ad\left(c^2f^2+g^2\right)\sqrt{d+c^2dx^2}}{g^3} - \frac{bcdx\sqrt{d+c^2dx^2}}{3g\sqrt{1+c^2x^2}} + \frac{bc^3dfx^2\sqrt{d+c^2dx}}{4g^2\sqrt{1+c^2x^2}}$$

$$= \frac{ad\left(c^2f^2+g^2\right)\sqrt{d+c^2dx^2}}{g^3} - \frac{bcdx\sqrt{d+c^2dx^2}}{3g\sqrt{1+c^2x^2}} + \frac{bc^3dfx^2\sqrt{d+c^2dx}}{4g^2\sqrt{1+c^2x^2}}$$

$$= \frac{ad\left(c^2f^2+g^2\right)\sqrt{d+c^2dx^2}}{g^3} - \frac{bcdx\sqrt{d+c^2dx^2}}{3g\sqrt{1+c^2x^2}} - \frac{bcd\left(c^2f^2+g^2\right)x}{g^3\sqrt{1+c^2}}$$

$$= \frac{ad\left(c^2f^2+g^2\right)\sqrt{d+c^2dx^2}}{g^3} - \frac{bcdx\sqrt{d+c^2dx^2}}{3g\sqrt{1+c^2x^2}} - \frac{bcd\left(c^2f^2+g^2\right)x}{g^3\sqrt{1+c^2}}$$

$$= \frac{ad\left(c^2f^2+g^2\right)\sqrt{d+c^2dx^2}}{g^3} - \frac{bcdx\sqrt{d+c^2dx^2}}{3g\sqrt{1+c^2x^2}} - \frac{bcd\left(c^2f^2+g^2\right)x}{g^3\sqrt{1+c^2}}$$

$$= \frac{ad\left(c^2f^2+g^2\right)\sqrt{d+c^2dx^2}}{g^3} - \frac{bcdx\sqrt{d+c^2dx^2}}{3g\sqrt{1+c^2x^2}} - \frac{bcd\left(c^2f^2+g^2\right)x}{g^3\sqrt{1+c^2}}$$

$$= \frac{ad\left(c^2f^2+g^2\right)\sqrt{d+c^2dx^2}}{g^3} - \frac{bcdx\sqrt{d+c^2dx^2}}{3g\sqrt{1+c^2x^2}} - \frac{bcd\left(c^2f^2+g^2\right)x}{g^3\sqrt{1+c^2}}$$

Mathematica [C] time = 13.62, size = 2889, normalized size = 2.94

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(f + g*x),x]
```

```
[Out] Sqrt[d*(1 + c^2*x^2)]*((a*d*(3*c^2*f^2 + 4*g^2))/(3*g^3) - (a*c^2*d*f*x)/(2
*g^2) + (a*c^2*d*x^2)/(3*g)) + (a*d^(3/2)*(c^2*f^2 + g^2)^(3/2)*Log[f + g*x
])/g^4 - (a*c*d^(3/2)*f*(2*c^2*f^2 + 3*g^2)*Log[c*d*x + Sqrt[d]*Sqrt[d*(1 +
 c^2*x^2)]])/(2*g^4) - (a*d^(3/2)*(c^2*f^2 + g^2)^(3/2)*Log[d*g - c^2*d*f*x
 + Sqrt[d]*Sqrt[c^2*f^2 + g^2]*Sqrt[d*(1 + c^2*x^2)]])/g^4 + (b*d*Sqrt[d*(1
 + c^2*x^2)]*((-2*c*g*x)/Sqrt[1 + c^2*x^2] + 2*g*ArcSinh[c*x] - (c*f*ArcSin
h[c*x]^2)/Sqrt[1 + c^2*x^2] + (2*(c^2*f^2 + g^2)*(((-I)*Pi*ArcTanh[(-g + c*
f*Tanh[ArcSinh[c*x]/2])/Sqrt[c^2*f^2 + g^2]])/Sqrt[c^2*f^2 + g^2] - (2*ArcC
os[((-I)*c*f)/g]*ArcTanh[((c*f + I*g)*Cot[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqr
t[-(c^2*f^2) - g^2]] + (Pi - (2*I)*ArcSinh[c*x])*ArcTanh[((c*f - I*g)*Tan[(
Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]] + (ArcCos[((-I)*c*f)/g
] - (2*I)*ArcTanh[((c*f + I*g)*Cot[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2
*f^2) - g^2]] - (2*I)*ArcTanh[((c*f - I*g)*Tan[(Pi + (2*I)*ArcSinh[c*x])/4]
)/Sqrt[-(c^2*f^2) - g^2]])*Log[((1/2 - I/2)*Sqrt[-(c^2*f^2) - g^2])/(E^(Arc
Sinh[c*x]/2)*Sqrt[(-I)*g]*Sqrt[c*f + c*g*x])] + (ArcCos[((-I)*c*f)/g] + (2*
I)*(ArcTanh[((c*f + I*g)*Cot[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2)
- g^2]] + ArcTanh[((c*f - I*g)*Tan[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2
*f^2) - g^2]]))*Log[((1/2 + I/2)*E^(ArcSinh[c*x]/2)*Sqrt[-(c^2*f^2) - g^2])
/(Sqrt[(-I)*g]*Sqrt[c*f + c*g*x])] - (ArcCos[((-I)*c*f)/g] + (2*I)*ArcTanh[
((c*f + I*g)*Cot[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]])*Log
[((I*c*f + g)*((-I)*c*f + g + Sqrt[-(c^2*f^2) - g^2])*(1 + I*Cot[(Pi + (2*I
)*ArcSinh[c*x])/4]))/(g*(I*c*f + g + I*Sqrt[-(c^2*f^2) - g^2]*Cot[(Pi + (2*
I)*ArcSinh[c*x])/4]))] - (ArcCos[((-I)*c*f)/g] - (2*I)*ArcTanh[((c*f + I*g)
*Cot[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]])*Log[((I*c*f + g
)*(I*c*f - g + Sqrt[-(c^2*f^2) - g^2])*(I + Cot[(Pi + (2*I)*ArcSinh[c*x])/4
]))/(g*(c*f - I*g + Sqrt[-(c^2*f^2) - g^2]*Cot[(Pi + (2*I)*ArcSinh[c*x])/4]
))] + I*(PolyLog[2, ((I*c*f + Sqrt[-(c^2*f^2) - g^2])*(I*c*f + g - I*Sqrt[-
(c^2*f^2) - g^2]*Cot[(Pi + (2*I)*ArcSinh[c*x])/4]))/(g*(I*c*f + g + I*Sqrt[
-(c^2*f^2) - g^2]*Cot[(Pi + (2*I)*ArcSinh[c*x])/4]))] - PolyLog[2, ((c*f +
I*Sqrt[-(c^2*f^2) - g^2])*(-(c*f) + I*g + Sqrt[-(c^2*f^2) - g^2]*Cot[(Pi +
(2*I)*ArcSinh[c*x])/4]))/(g*(I*c*f + g + I*Sqrt[-(c^2*f^2) - g^2]*Cot[(Pi +
 (2*I)*ArcSinh[c*x])/4]))])))/Sqrt[-(c^2*f^2) - g^2]))/Sqrt[1 + c^2*x^2]))/(
2*g^2) + (b*d*Sqrt[d*(1 + c^2*x^2)]*((-9*(ArcSinh[c*x]*(Log[1 + (E^ArcSinh[
c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2])] - Log[1 + (E^ArcSinh[c*x]*g)/(c*f + Sq
rt[c^2*f^2 + g^2])]) + PolyLog[2, (E^ArcSinh[c*x]*g)/(-(c*f) + Sqrt[c^2*f^2
 + g^2])] - PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))]))
/Sqrt[c^2*f^2 + g^2] + (-18*c*g*(4*c^2*f^2 + g^2)*x + 18*g*(4*c^2*f^2 + g^2
)*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] - 18*c*f*(2*c^2*f^2 + g^2)*ArcSinh[c*x]^2
+ 9*c*f*g^2*Cosh[2*ArcSinh[c*x]] + 6*g^3*ArcSinh[c*x]*Cosh[3*ArcSinh[c*x]]
+ 9*(8*c^4*f^4 + 8*c^2*f^2*g^2 + g^4)*(((-I)*Pi*ArcTanh[(-g + c*f*Tanh[ArcS
inh[c*x]/2])/Sqrt[c^2*f^2 + g^2]])/Sqrt[c^2*f^2 + g^2] - (2*ArcCos[((-I)*c*
f)/g]*ArcTanh[((c*f + I*g)*Cot[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2
) - g^2]] + (Pi - (2*I)*ArcSinh[c*x])*ArcTanh[((c*f - I*g)*Tan[(Pi + (2*I)*
ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]] + (ArcCos[((-I)*c*f)/g] - (2*I)*A
rcTanh[((c*f + I*g)*Cot[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2
]] - (2*I)*ArcTanh[((c*f - I*g)*Tan[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^
2*f^2) - g^2]])*Log[((1/2 - I/2)*Sqrt[-(c^2*f^2) - g^2])/(E^(ArcSinh[c*x]/2
)*Sqrt[(-I)*g]*Sqrt[c*f + c*g*x])] + (ArcCos[((-I)*c*f)/g] + (2*I)*(ArcTanh
[((c*f + I*g)*Cot[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]] + A
rcTanh[((c*f - I*g)*Tan[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2
]]))*Log[((1/2 + I/2)*E^(ArcSinh[c*x]/2)*Sqrt[-(c^2*f^2) - g^2])/(Sqrt[(-I)
*g]*Sqrt[c*f + c*g*x])] - (ArcCos[((-I)*c*f)/g] + (2*I)*ArcTanh[((c*f + I*g
```

```
)*Cot[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]])*Log[((I*c*f +
g)*((-I)*c*f + g + Sqrt[-(c^2*f^2) - g^2])*(1 + I*Cot[(Pi + (2*I)*ArcSinh[c
*x])/4]))/(g*(I*c*f + g + I*Sqrt[-(c^2*f^2) - g^2]*Cot[(Pi + (2*I)*ArcSinh[
c*x])/4]))] - (ArcCos[((-I)*c*f)/g] - (2*I)*ArcTanh[((c*f + I*g)*Cot[(Pi +
(2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]])*Log[((I*c*f + g)*(I*c*f -
g + Sqrt[-(c^2*f^2) - g^2])*(I + Cot[(Pi + (2*I)*ArcSinh[c*x])/4]))/(g*(c*f
 - I*g + Sqrt[-(c^2*f^2) - g^2]*Cot[(Pi + (2*I)*ArcSinh[c*x])/4]))] + I*(Po
lyLog[2, ((I*c*f + Sqrt[-(c^2*f^2) - g^2])*(I*c*f + g - I*Sqrt[-(c^2*f^2) -
 g^2]*Cot[(Pi + (2*I)*ArcSinh[c*x])/4]))/(g*(I*c*f + g + I*Sqrt[-(c^2*f^2)
- g^2]*Cot[(Pi + (2*I)*ArcSinh[c*x])/4]))] - PolyLog[2, ((c*f + I*Sqrt[-(c^
2*f^2) - g^2])*(-(c*f) + I*g + Sqrt[-(c^2*f^2) - g^2]*Cot[(Pi + (2*I)*ArcSi
nh[c*x])/4]))/(g*(I*c*f + g + I*Sqrt[-(c^2*f^2) - g^2]*Cot[(Pi + (2*I)*ArcS
inh[c*x])/4]))]))/Sqrt[-(c^2*f^2) - g^2]) - 18*c*f*g^2*ArcSinh[c*x]*Sinh[2*
ArcSinh[c*x]] - 2*g^3*Sinh[3*ArcSinh[c*x]])/g^4))/(72*Sqrt[1 + c^2*x^2])
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(ac^2dx^2 + ad + \left(bc^2dx^2 + bd\right)\operatorname{arsinh}\left(cx\right)\right)\sqrt{c^2dx^2 + d}}{gx + f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/(g*x+f),x, algorithm="fric
as")
```

```
[Out] integral((a*c^2*d*x^2 + a*d + (b*c^2*d*x^2 + b*d)*arcsinh(c*x))*sqrt(c^2*d*
x^2 + d)/(g*x + f), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/(g*x+f),x, algorithm="giac
")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.37, size = 1838, normalized size = 1.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/(g*x+f),x)
```

```
[Out] -a/g*d^2/(d*(c^2*f^2+g^2)/g^2)^(1/2)*ln((2*d*(c^2*f^2+g^2)/g^2-2*c^2*d*f/g*
(x+f/g)+2*(d*(c^2*f^2+g^2)/g^2)^(1/2)*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+
d*(c^2*f^2+g^2)/g^2)^(1/2))/(x+f/g))-a/g^5*d^2/(d*(c^2*f^2+g^2)/g^2)^(1/2)*
ln((2*d*(c^2*f^2+g^2)/g^2-2*c^2*d*f/g*(x+f/g)+2*(d*(c^2*f^2+g^2)/g^2)^(1/2)
*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2))/(x+f/g))*
c^4*f^4-2*a/g^3*d^2/(d*(c^2*f^2+g^2)/g^2)^(1/2)*ln((2*d*(c^2*f^2+g^2)/g^2-2
*c^2*d*f/g*(x+f/g)+2*(d*(c^2*f^2+g^2)/g^2)^(1/2)*((x+f/g)^2*c^2*d-2*c^2*d*f
/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2))/(x+f/g))*c^2*f^2-3/2*a/g^2*c^2*d^2*f
*ln((-c^2*d*f/g+c^2*d*(x+f/g))/(c^2*d)^(1/2)+((x+f/g)^2*c^2*d-2*c^2*d*f/g*(
x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2))/(c^2*d)^(1/2)-a/g^4*d^2*c^4*f^3*ln((-c^2
*d*f/g+c^2*d*(x+f/g))/(c^2*d)^(1/2)+((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*
(c^2*f^2+g^2)/g^2)^(1/2))/(c^2*d)^(1/2)-1/2*a/g^2*c^2*d*f*((x+f/g)^2*c^2*d-
```

```
2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2)*x-4/3*b*(d*(c^2*x^2+1))^(1/2
)*d/(c^2*x^2+1)^(1/2)/g*c*x+1/8*b*(d*(c^2*x^2+1))^(1/2)*f*c*d/(c^2*x^2+1)^(
1/2)/g^2-1/9*b*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)^(1/2)/g*c^3*x^3-b*(c^2*f
^2+g^2)^(3/2)*d*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/g^4*dilog(((c*x+(c^
2*x^2+1)^(1/2))*g+c*f+(c^2*f^2+g^2)^(1/2))/(c*f+(c^2*f^2+g^2)^(1/2)))+b*(c^
2*f^2+g^2)^(3/2)*d*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/g^4*dilog((-(c*x
+(c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2+g^2)^(1/2))/(-c*f+(c^2*f^2+g^2)^(1/2)))+
a/g^3*d*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2)*c^2
*f^2+4/3*b*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)/g*arcsinh(c*x)+a/g*d*((x+f/g
)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2)-1/2*b*(d*(c^2*x^2+
1))^(1/2)*f*c^4*d/(c^2*x^2+1)/g^2*arcsinh(c*x)*x^3-1/2*b*(d*(c^2*x^2+1))^(1
/2)*f*c^2*d/(c^2*x^2+1)/g^2*arcsinh(c*x)*x+b*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x
^2+1)/g^3*arcsinh(c*x)*x^2*c^4*f^2+b*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)/g^
3*arcsinh(c*x)*c^2*f^2+b*(c^2*f^2+g^2)^(3/2)*d*(d*(c^2*x^2+1))^(1/2)/(c^2*x
^2+1)^(1/2)/g^4*arcsinh(c*x)*ln((-(c*x+(c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2+g^
2)^(1/2))/(-c*f+(c^2*f^2+g^2)^(1/2)))-b*(c^2*f^2+g^2)^(3/2)*d*(d*(c^2*x^2+1
))^(1/2)/(c^2*x^2+1)^(1/2)/g^4*arcsinh(c*x)*ln(((c*x+(c^2*x^2+1)^(1/2))*g+c
*f+(c^2*f^2+g^2)^(1/2))/(c*f+(c^2*f^2+g^2)^(1/2)))-1/2*b*(d*(c^2*x^2+1))^(1
/2)/(c^2*x^2+1)^(1/2)*f^3*arcsinh(c*x)^2*c^3*d/g^4-3/4*b*(d*(c^2*x^2+1))^(1
/2)/(c^2*x^2+1)^(1/2)*f*arcsinh(c*x)^2*c*d/g^2+1/3*b*(d*(c^2*x^2+1))^(1/2)*
d/(c^2*x^2+1)/g*arcsinh(c*x)*x^4*c^4+5/3*b*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2
+1)/g*arcsinh(c*x)*x^2*c^2+1/4*b*(d*(c^2*x^2+1))^(1/2)*f*c^3*d/(c^2*x^2+1)^
(1/2)/g^2*x^2-b*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)^(1/2)/g^3*x*c^3*f^2+1/3
*a/g*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(3/2)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/(g*x+f),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\mathrm{asinh}\,(c\,x))\,\left(d\,c^2\,x^2 + d\right)^{3/2}}{f + g\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2))/(f + g*x),x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2))/(f + g*x), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\left(c^2x^2 + 1\right)\right)^{\frac{3}{2}}\,(a + b\,\mathrm{asinh}\,(cx))}{f + gx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))/(g*x+f),x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))/(f + g*x), x)
```

3.43 $\int (f+gx)^3 \left(d+c^2dx^2\right)^{5/2} \left(a+b\sinh^{-1}(cx)\right) dx$

Optimal. Leaf size=1228

$$-\frac{bc^5d^2g^3\sqrt{c^2dx^2+d}\,x^9}{81\sqrt{c^2x^2+1}}-\frac{3bc^5d^2fg^2\sqrt{c^2dx^2+d}\,x^8}{64\sqrt{c^2x^2+1}}-\frac{19bc^3d^2g^3\sqrt{c^2dx^2+d}\,x^7}{441\sqrt{c^2x^2+1}}-\frac{3bc^5d^2f^2g\sqrt{c^2dx^2+d}\,x^7}{49\sqrt{c^2x^2+1}}-\frac{17bc^3}{}$$

```
[Out] 2/63*b*d^2*g^3*x*(c^2*d*x^2+d)^(1/2)/c^3/(c^2*x^2+1)^(1/2)-25/96*b*c*d^2*f^
3*x^2*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/189*b*d^2*g^3*x^3*(c^2*d*x^2+
d)^(1/2)/c/(c^2*x^2+1)^(1/2)-5/96*b*c^3*d^2*f^3*x^4*(c^2*d*x^2+d)^(1/2)/(c^
2*x^2+1)^(1/2)-1/21*b*c*d^2*g^3*x^5*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1
9/441*b*c^3*d^2*g^3*x^7*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/81*b*c^5*d^
2*g^3*x^9*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+5/32*d^2*f^3*(a+b*arcsinh(c
*x))^2*(c^2*d*x^2+d)^(1/2)/b/c/(c^2*x^2+1)^(1/2)+15/128*d^2*f*g^2*x*(a+b*ar
csinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^2+5/16*d^2*f*g^2*x^3*(c^2*x^2+1)*(a+b*arc
sinh(c*x))*(c^2*d*x^2+d)^(1/2)+3/8*d^2*f*g^2*x^3*(c^2*x^2+1)^2*(a+b*arcsinh
(c*x))*(c^2*d*x^2+d)^(1/2)+3/7*d^2*f^2*g*(c^2*x^2+1)^3*(a+b*arcsinh(c*x))*(
c^2*d*x^2+d)^(1/2)/c^2-3/7*b*c*d^2*f^2*g*x^3*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1
)^(1/2)-59/256*b*c*d^2*f*g^2*x^4*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-9/35
*b*c^3*d^2*f^2*g*x^5*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-17/96*b*c^3*d^2*
f*g^2*x^6*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-3/49*b*c^5*d^2*f^2*g*x^7*(c
^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-3/64*b*c^5*d^2*f*g^2*x^8*(c^2*d*x^2+d)^
(1/2)/(c^2*x^2+1)^(1/2)-15/256*d^2*f*g^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)
^(1/2)/b/c^3/(c^2*x^2+1)^(1/2)-3/7*b*d^2*f^2*g*x*(c^2*d*x^2+d)^(1/2)/c/(c^2
*x^2+1)^(1/2)-15/256*b*d^2*f*g^2*x^2*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2
)+5/16*d^2*f^3*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)-1/36*b*d^2*f^3*(c^2
*x^2+1)^(5/2)*(c^2*d*x^2+d)^(1/2)/c+15/64*d^2*f*g^2*x^3*(a+b*arcsinh(c*x))*
(c^2*d*x^2+d)^(1/2)+5/24*d^2*f^3*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))*(c^2*d*x^
2+d)^(1/2)+1/6*d^2*f^3*x*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/
2)-1/7*d^2*g^3*(c^2*x^2+1)^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^4+1/9
*d^2*g^3*(c^2*x^2+1)^4*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^4
```

Rubi [A] time = 1.14, antiderivative size = 1228, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 18, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5835, 5821, 5684, 5682, 5675, 30, 14, 261, 5717, 194, 5744, 5742, 5758, 266, 43, 5732, 12, 373}

$$-\frac{bc^5d^2g^3\sqrt{c^2dx^2+d}\,x^9}{81\sqrt{c^2x^2+1}}-\frac{3bc^5d^2fg^2\sqrt{c^2dx^2+d}\,x^8}{64\sqrt{c^2x^2+1}}-\frac{19bc^3d^2g^3\sqrt{c^2dx^2+d}\,x^7}{441\sqrt{c^2x^2+1}}-\frac{3bc^5d^2f^2g\sqrt{c^2dx^2+d}\,x^7}{49\sqrt{c^2x^2+1}}-\frac{17bc^3}{}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^3*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-3*b*d^2*f^2*g*x*Sqrt[d + c^2*d*x^2])/(7*c*Sqrt[1 + c^2*x^2]) + (2*b*d^2*g
^3*x*Sqrt[d + c^2*d*x^2])/(63*c^3*Sqrt[1 + c^2*x^2]) - (25*b*c*d^2*f^3*x^2*
Sqrt[d + c^2*d*x^2])/(96*Sqrt[1 + c^2*x^2]) - (15*b*d^2*f*g^2*x^2*Sqrt[d +
c^2*d*x^2])/(256*c*Sqrt[1 + c^2*x^2]) - (3*b*c*d^2*f^2*g*x^3*Sqrt[d + c^2*d
*x^2])/(7*Sqrt[1 + c^2*x^2]) - (b*d^2*g^3*x^3*Sqrt[d + c^2*d*x^2])/(189*c*S
qrt[1 + c^2*x^2]) - (5*b*c^3*d^2*f^3*x^4*Sqrt[d + c^2*d*x^2])/(96*Sqrt[1 +
c^2*x^2]) - (59*b*c*d^2*f*g^2*x^4*Sqrt[d + c^2*d*x^2])/(256*Sqrt[1 + c^2*x^
2]) - (9*b*c^3*d^2*f^2*g*x^5*Sqrt[d + c^2*d*x^2])/(35*Sqrt[1 + c^2*x^2]) -
(b*c*d^2*g^3*x^5*Sqrt[d + c^2*d*x^2])/(21*Sqrt[1 + c^2*x^2]) - (17*b*c^3*d^
2*f*g^2*x^6*Sqrt[d + c^2*d*x^2])/(96*Sqrt[1 + c^2*x^2]) - (3*b*c^5*d^2*f^2*
g*x^7*Sqrt[d + c^2*d*x^2])/(49*Sqrt[1 + c^2*x^2]) - (19*b*c^3*d^2*g^3*x^7*S
qrt[d + c^2*d*x^2])/(441*Sqrt[1 + c^2*x^2]) - (3*b*c^5*d^2*f*g^2*x^8*Sqrt[d
 + c^2*d*x^2])/(64*Sqrt[1 + c^2*x^2]) - (b*c^5*d^2*g^3*x^9*Sqrt[d + c^2*d*x
^2])/(81*Sqrt[1 + c^2*x^2]) - (b*d^2*f^3*(1 + c^2*x^2)^(5/2)*Sqrt[d + c^2*d
```

```
*x^2])/(36*c) + (5*d^2*f^3*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/16 +
 (15*d^2*f*g^2*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(128*c^2) + (15*
d^2*f*g^2*x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/64 + (5*d^2*f^3*x*(
1 + c^2*x^2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/24 + (5*d^2*f*g^2*x^
3*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/16 + (d^2*f^3*x*(
1 + c^2*x^2)^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/6 + (3*d^2*f*g^2*x
^3*(1 + c^2*x^2)^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/8 + (3*d^2*f^2
*g*(1 + c^2*x^2)^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(7*c^2) - (d^2
*g^3*(1 + c^2*x^2)^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(7*c^4) + (d
^2*g^3*(1 + c^2*x^2)^4*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(9*c^4) +
(5*d^2*f^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(32*b*c*Sqrt[1 + c^2
*x^2]) - (15*d^2*f*g^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(256*b*c
^3*Sqrt[1 + c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5732

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(1 + c^2*x^2)^p, x]}, Dist[d^p*(a + b
*ArcSinh[c*x]), u, x] - Dist[b*c*d^p, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*
x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegerQ
[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0]) && NeQ[p, -2^
(-1)] && GtQ[d, 0]
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
```

```
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5835

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(d^IntPart[p]*(d + e*x^2)^FracPa
rt[p])/(1 + c^2*x^2)^FracPart[p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*Ar
cSinh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d
] && IntegerQ[m] && IntegerQ[p - 1/2] &&  !GtQ[d, 0]
```

Rubi steps

$$\int (f+gx)^3 \left(d+c^2dx^2\right)^{5/2} \left(a+b\sinh^{-1}(cx)\right) dx = \frac{\left(d^2\sqrt{d+c^2dx^2}\right)\int (f+gx)^3 \left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right.}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(d^2\sqrt{d+c^2dx^2}\right)\int \left(f^3\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)+3\right.}{}$$

$$= \frac{\left(d^2f^3\sqrt{d+c^2dx^2}\right)\int \left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right) dx}{\sqrt{1+c^2x^2}} +$$

$$= \frac{1}{6}d^2f^3x\left(1+c^2x^2\right)^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{8}d^2f$$

$$= -\frac{bd^2f^3\left(1+c^2x^2\right)^{5/2}\sqrt{d+c^2dx^2}}{36c}+\frac{5}{24}d^2f^3x\left(1+c^2x^2\right)\sqrt{c}$$

$$= -\frac{3bd^2f^2gx\sqrt{d+c^2dx^2}}{7c\sqrt{1+c^2x^2}}-\frac{3bcd^2f^2gx^3\sqrt{d+c^2dx^2}}{7\sqrt{1+c^2x^2}}-\frac{9bc^3d}{}$$

$$= -\frac{3bd^2f^2gx\sqrt{d+c^2dx^2}}{7c\sqrt{1+c^2x^2}}+\frac{2bd^2g^3x\sqrt{d+c^2dx^2}}{63c^3\sqrt{1+c^2x^2}}-\frac{25bcd^2f^3}{96}$$

$$= -\frac{3bd^2f^2gx\sqrt{d+c^2dx^2}}{7c\sqrt{1+c^2x^2}}+\frac{2bd^2g^3x\sqrt{d+c^2dx^2}}{63c^3\sqrt{1+c^2x^2}}-\frac{25bcd^2f^3}{96}$$

Mathematica [A] time = 7.08, size = 1899, normalized size = 1.55

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^3*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] Sqrt[d*(1 + c^2*x^2)]*(-1/63*(a*d^2*g*(-27*c^2*f^2 + 2*g^2))/c^4 + (a*d^2*f
*(88*c^2*f^2 + 15*g^2)*x)/(128*c^2) + (a*d^2*g*(81*c^2*f^2 + g^2)*x^2)/(63*
c^2) + (a*d^2*f*(104*c^2*f^2 + 177*g^2)*x^3)/192 + (a*d^2*g*(27*c^2*f^2 + 5
*g^2)*x^4)/21 + (a*c^2*d^2*f*(8*c^2*f^2 + 51*g^2)*x^5)/48 + (a*c^2*d^2*g*(2
7*c^2*f^2 + 19*g^2)*x^6)/63 + (3*a*c^4*d^2*f*g^2*x^7)/8 + (a*c^4*d^2*g^3*x^
8)/9) + (3*b*d^2*f^2*g*(-1/9*(c*x*Sqrt[d*(1 + c^2*x^2)]*(3 + c^2*x^2))/Sqrt
[1 + c^2*x^2] + ((1 + c^2*x^2)*Sqrt[d*(1 + c^2*x^2)]*ArcSinh[c*x])/3))/c^2
+ (6*b*d^2*f^2*g*((2*c*x*Sqrt[d*(1 + c^2*x^2)]*(3 + c^2*x^2))/(45*Sqrt[1 +
c^2*x^2]) - (c^3*x^3*Sqrt[d*(1 + c^2*x^2)]*(5 + 3*c^2*x^2))/(75*Sqrt[1 + c^
2*x^2]) + ((d*(1 + c^2*x^2))^(3/2)*(-2 + 3*c^2*x^2)*ArcSinh[c*x])/(15*d)))/
c^2 + (b*d^2*g^3*((2*c*x*Sqrt[d*(1 + c^2*x^2)]*(3 + c^2*x^2))/(45*Sqrt[1 +
c^2*x^2]) - (c^3*x^3*Sqrt[d*(1 + c^2*x^2)]*(5 + 3*c^2*x^2))/(75*Sqrt[1 + c^
2*x^2]) + ((d*(1 + c^2*x^2))^(3/2)*(-2 + 3*c^2*x^2)*ArcSinh[c*x])/(15*d)))/
c^4 + (3*b*d^2*f^2*g*((-8*c*x*Sqrt[d*(1 + c^2*x^2)]*(3 + c^2*x^2))/(315*Sqr
t[1 + c^2*x^2]) + (4*c^3*x^3*Sqrt[d*(1 + c^2*x^2)]*(5 + 3*c^2*x^2))/(525*Sq
rt[1 + c^2*x^2]) - (c^5*x^5*Sqrt[d*(1 + c^2*x^2)]*(7 + 5*c^2*x^2))/(245*Sqr
t[1 + c^2*x^2]) + ((d*(1 + c^2*x^2))^(3/2)*(8 - 12*c^2*x^2 + 15*c^4*x^4)*Ar
cSinh[c*x])/(105*d)))/c^2 + (2*b*d^2*g^3*((-8*c*x*Sqrt[d*(1 + c^2*x^2)]*(3
+ c^2*x^2))/(315*Sqrt[1 + c^2*x^2]) + (4*c^3*x^3*Sqrt[d*(1 + c^2*x^2)]*(5 +
 3*c^2*x^2))/(525*Sqrt[1 + c^2*x^2]) - (c^5*x^5*Sqrt[d*(1 + c^2*x^2)]*(7 +
5*c^2*x^2))/(245*Sqrt[1 + c^2*x^2]) + ((d*(1 + c^2*x^2))^(3/2)*(8 - 12*c^2*
x^2 + 15*c^4*x^4)*ArcSinh[c*x])/(105*d)))/c^4 + (b*d^2*g^3*((16*c*x*Sqrt[d*
```

```
(1 + c^2*x^2)]*(3 + c^2*x^2))/(945*Sqrt[1 + c^2*x^2]) - (8*c^3*x^3*Sqrt[d*(
1 + c^2*x^2)]*(5 + 3*c^2*x^2))/(1575*Sqrt[1 + c^2*x^2]) + (2*c^5*x^5*Sqrt[d
*(1 + c^2*x^2)]*(7 + 5*c^2*x^2))/(735*Sqrt[1 + c^2*x^2]) - (c^7*x^7*Sqrt[d*
(1 + c^2*x^2)]*(9 + 7*c^2*x^2))/(567*Sqrt[1 + c^2*x^2]) + ((d*(1 + c^2*x^2)
)^(3/2)*(-16 + 24*c^2*x^2 - 30*c^4*x^4 + 35*c^6*x^6)*ArcSinh[c*x])/(315*d))
)/c^4 + (5*a*d^(5/2)*f*(8*c^2*f^2 - 3*g^2)*Log[c*d*x + Sqrt[d]*Sqrt[d*(1 +
c^2*x^2)]])/(128*c^3) + (b*d^2*f^3*Sqrt[d*(1 + c^2*x^2)]*(-Cosh[2*ArcSinh[c
*x]] + 2*ArcSinh[c*x]*(ArcSinh[c*x] + Sinh[2*ArcSinh[c*x]])))/(8*c*Sqrt[1 +
 c^2*x^2]) - (b*d^2*f^3*Sqrt[d*(1 + c^2*x^2)]*(8*ArcSinh[c*x]^2 + Cosh[4*Ar
cSinh[c*x]] - 4*ArcSinh[c*x]*Sinh[4*ArcSinh[c*x]]))/(64*c*Sqrt[1 + c^2*x^2]
) - (3*b*d^2*f*g^2*Sqrt[d*(1 + c^2*x^2)]*(8*ArcSinh[c*x]^2 + Cosh[4*ArcSinh
[c*x]] - 4*ArcSinh[c*x]*Sinh[4*ArcSinh[c*x]]))/(128*c^3*Sqrt[1 + c^2*x^2])
+ (b*d^2*f^3*Sqrt[d*(1 + c^2*x^2)]*(72*ArcSinh[c*x]^2 + 18*Cosh[2*ArcSinh[c
*x]] + 9*Cosh[4*ArcSinh[c*x]] - 2*Cosh[6*ArcSinh[c*x]] - 36*ArcSinh[c*x]*Si
nh[2*ArcSinh[c*x]] - 36*ArcSinh[c*x]*Sinh[4*ArcSinh[c*x]] + 12*ArcSinh[c*x]
*Sinh[6*ArcSinh[c*x]]))/(2304*c*Sqrt[1 + c^2*x^2]) + (b*d^2*f*g^2*Sqrt[d*(1
 + c^2*x^2)]*(72*ArcSinh[c*x]^2 + 18*Cosh[2*ArcSinh[c*x]] + 9*Cosh[4*ArcSin
h[c*x]] - 2*Cosh[6*ArcSinh[c*x]] - 36*ArcSinh[c*x]*Sinh[2*ArcSinh[c*x]] - 3
6*ArcSinh[c*x]*Sinh[4*ArcSinh[c*x]] + 12*ArcSinh[c*x]*Sinh[6*ArcSinh[c*x]])
)/(384*c^3*Sqrt[1 + c^2*x^2]) - (b*d^2*f*g^2*Sqrt[d*(1 + c^2*x^2)]*(1440*Ar
cSinh[c*x]^2 + 576*Cosh[2*ArcSinh[c*x]] + 144*Cosh[4*ArcSinh[c*x]] - 64*Cos
h[6*ArcSinh[c*x]] + 9*Cosh[8*ArcSinh[c*x]] - 1152*ArcSinh[c*x]*Sinh[2*ArcSi
nh[c*x]] - 576*ArcSinh[c*x]*Sinh[4*ArcSinh[c*x]] + 384*ArcSinh[c*x]*Sinh[6*
ArcSinh[c*x]] - 72*ArcSinh[c*x]*Sinh[8*ArcSinh[c*x]]))/(24576*c^3*Sqrt[1 +
c^2*x^2])
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(ac^4d^2g^3x^7 + 3\,ac^4d^2fg^2x^6 + 3\,ad^2f^2gx + ad^2f^3 + \left(3\,ac^4d^2f^2g + 2\,ac^2d^2g^3\right)x^5 + \left(ac^4d^2f^3 + 6\,ac^2d^2fg\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="fr
icas")
```

```
[Out] integral((a*c^4*d^2*g^3*x^7 + 3*a*c^4*d^2*f*g^2*x^6 + 3*a*d^2*f^2*g*x + a*d
^2*f^3 + (3*a*c^4*d^2*f^2*g + 2*a*c^2*d^2*g^3)*x^5 + (a*c^4*d^2*f^3 + 6*a*c
^2*d^2*f*g^2)*x^4 + (6*a*c^2*d^2*f^2*g + a*d^2*g^3)*x^3 + (2*a*c^2*d^2*f^3
+ 3*a*d^2*f*g^2)*x^2 + (b*c^4*d^2*g^3*x^7 + 3*b*c^4*d^2*f*g^2*x^6 + 3*b*d^2
*f^2*g*x + b*d^2*f^3 + (3*b*c^4*d^2*f^2*g + 2*b*c^2*d^2*g^3)*x^5 + (b*c^4*d
^2*f^3 + 6*b*c^2*d^2*f*g^2)*x^4 + (6*b*c^2*d^2*f^2*g + b*d^2*g^3)*x^3 + (2*
b*c^2*d^2*f^3 + 3*b*d^2*f*g^2)*x^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="gi
ac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [A] time = 1.01, size = 1954, normalized size = 1.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] -2/63*a*g^3/d/c^4*(c^2*d*x^2+d)^(7/2)+1/6*a*f^3*x*(c^2*d*x^2+d)^(5/2)+34/63
*b*(d*(c^2*x^2+1))^(1/2)*g^3*d^2*c^2/(c^2*x^2+1)*arcsinh(c*x)*x^6-1/63*b*(d
*(c^2*x^2+1))^(1/2)*g^3*d^2/c^2/(c^2*x^2+1)*arcsinh(c*x)*x^2+12/7*b*(d*(c^2
*x^2+1))^(1/2)*g*d^2/(c^2*x^2+1)*arcsinh(c*x)*x^2*f^2+133/128*b*(d*(c^2*x^2
+1))^(1/2)*f*g^2*d^2/(c^2*x^2+1)*arcsinh(c*x)*x^3+3/7*b*(d*(c^2*x^2+1))^(1/
2)*g*d^2/c^2/(c^2*x^2+1)*arcsinh(c*x)*f^2+1/6*b*(d*(c^2*x^2+1))^(1/2)*f^3*d
^2*c^6/(c^2*x^2+1)*arcsinh(c*x)*x^7+17/24*b*(d*(c^2*x^2+1))^(1/2)*f^3*d^2*c
^4/(c^2*x^2+1)*arcsinh(c*x)*x^5+59/48*b*(d*(c^2*x^2+1))^(1/2)*f^3*d^2*c^2/(
c^2*x^2+1)*arcsinh(c*x)*x^3-15/256*b*(d*(c^2*x^2+1))^(1/2)*f*arcsinh(c*x)^2
*d^2/(c^2*x^2+1)^(1/2)/c^3*g^2+1/9*b*(d*(c^2*x^2+1))^(1/2)*g^3*d^2*c^6/(c^2
*x^2+1)*arcsinh(c*x)*x^10+26/63*b*(d*(c^2*x^2+1))^(1/2)*g^3*d^2*c^4/(c^2*x^
2+1)*arcsinh(c*x)*x^8-59/256*b*(d*(c^2*x^2+1))^(1/2)*f*g^2*d^2*c/(c^2*x^2+1
)^(1/2)*x^4-15/256*b*(d*(c^2*x^2+1))^(1/2)*f*g^2*d^2/c/(c^2*x^2+1)^(1/2)*x^
2-3/49*b*(d*(c^2*x^2+1))^(1/2)*g*d^2*c^5/(c^2*x^2+1)^(1/2)*x^7*f^2-9/35*b*(
d*(c^2*x^2+1))^(1/2)*g*d^2*c^3/(c^2*x^2+1)^(1/2)*x^5*f^2-3/7*b*(d*(c^2*x^2+
1))^(1/2)*g*d^2*c/(c^2*x^2+1)^(1/2)*x^3*f^2-3/7*b*(d*(c^2*x^2+1))^(1/2)*g*d
^2/c/(c^2*x^2+1)^(1/2)*x*f^2-3/64*b*(d*(c^2*x^2+1))^(1/2)*f*g^2*d^2*c^5/(c^
2*x^2+1)^(1/2)*x^8-17/96*b*(d*(c^2*x^2+1))^(1/2)*f*g^2*d^2*c^3/(c^2*x^2+1)^
(1/2)*x^6+15/128*b*(d*(c^2*x^2+1))^(1/2)*f*g^2*d^2/c^2/(c^2*x^2+1)*arcsinh(
c*x)*x+3/7*b*(d*(c^2*x^2+1))^(1/2)*g*d^2*c^6/(c^2*x^2+1)*arcsinh(c*x)*x^8*f
^2+12/7*b*(d*(c^2*x^2+1))^(1/2)*g*d^2*c^4/(c^2*x^2+1)*arcsinh(c*x)*x^6*f^2+
18/7*b*(d*(c^2*x^2+1))^(1/2)*g*d^2*c^2/(c^2*x^2+1)*arcsinh(c*x)*x^4*f^2+3/8
*b*(d*(c^2*x^2+1))^(1/2)*f*g^2*d^2*c^6/(c^2*x^2+1)*arcsinh(c*x)*x^9+23/16*b
*(d*(c^2*x^2+1))^(1/2)*f*g^2*d^2*c^4/(c^2*x^2+1)*arcsinh(c*x)*x^7+127/64*b*
(d*(c^2*x^2+1))^(1/2)*f*g^2*d^2*c^2/(c^2*x^2+1)*arcsinh(c*x)*x^5-1/21*b*(d*
(c^2*x^2+1))^(1/2)*g^3*d^2*c/(c^2*x^2+1)^(1/2)*x^5-1/189*b*(d*(c^2*x^2+1))^
(1/2)*g^3*d^2/c/(c^2*x^2+1)^(1/2)*x^3+2/63*b*(d*(c^2*x^2+1))^(1/2)*g^3*d^2/
c^3/(c^2*x^2+1)^(1/2)*x+359/24576*b*(d*(c^2*x^2+1))^(1/2)*f*g^2*d^2/c^3/(c^
2*x^2+1)^(1/2)-1/36*b*(d*(c^2*x^2+1))^(1/2)*f^3*d^2*c^5/(c^2*x^2+1)^(1/2)*x
^6-13/96*b*(d*(c^2*x^2+1))^(1/2)*f^3*d^2*c^3/(c^2*x^2+1)^(1/2)*x^4-11/32*b*
(d*(c^2*x^2+1))^(1/2)*f^3*d^2*c/(c^2*x^2+1)^(1/2)*x^2-1/81*b*(d*(c^2*x^2+1)
)^(1/2)*g^3*d^2*c^5/(c^2*x^2+1)^(1/2)*x^9+11/16*b*(d*(c^2*x^2+1))^(1/2)*f^3
*d^2/(c^2*x^2+1)*arcsinh(c*x)*x+16/63*b*(d*(c^2*x^2+1))^(1/2)*g^3*d^2/(c^2*
x^2+1)*arcsinh(c*x)*x^4+5/32*b*(d*(c^2*x^2+1))^(1/2)*f^3*arcsinh(c*x)^2*d^2
/(c^2*x^2+1)^(1/2)/c-2/63*b*(d*(c^2*x^2+1))^(1/2)*g^3*d^2/c^4/(c^2*x^2+1)*a
rcsinh(c*x)+3/8*a*f*g^2*x*(c^2*d*x^2+d)^(7/2)/c^2/d-5/64*a*f*g^2/c^2*d*x*(c
^2*d*x^2+d)^(3/2)-15/128*a*f*g^2/c^2*d^2*x*(c^2*d*x^2+d)^(1/2)-15/128*a*f*g
^2/c^2*d^3*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)-19/4
41*b*(d*(c^2*x^2+1))^(1/2)*g^3*d^2*c^3/(c^2*x^2+1)^(1/2)*x^7+1/9*a*g^3*x^2*
(c^2*d*x^2+d)^(7/2)/c^2/d-1/16*a*f*g^2/c^2*x*(c^2*d*x^2+d)^(5/2)+3/7*a*f^2*
g/c^2/d*(c^2*d*x^2+d)^(7/2)-299/2304*b*(d*(c^2*x^2+1))^(1/2)*f^3*d^2/c/(c^2
*x^2+1)^(1/2)+5/24*a*f^3*d*x*(c^2*d*x^2+d)^(3/2)+5/16*a*f^3*d^2*x*(c^2*d*x^
2+d)^(1/2)+5/16*a*f^3*d^3*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^
2*d)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="ma
xima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(f+g\,x\right)}^3\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,{\left(d\,c^2\,x^2+d\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^3*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2),x)
```

```
[Out] int((f + g*x)^3*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Timed out
```

3.44 $\int (f+gx)^2 \left(d+c^2dx^2\right)^{5/2} \left(a+b\sinh^{-1}(cx)\right) dx$

Optimal. Leaf size=901

$$-\frac{bc^5d^2g^2\sqrt{c^2dx^2+d}\,x^8}{64\sqrt{c^2x^2+1}}-\frac{2bc^5d^2fg\sqrt{c^2dx^2+d}\,x^7}{49\sqrt{c^2x^2+1}}-\frac{17bc^3d^2g^2\sqrt{c^2dx^2+d}\,x^6}{288\sqrt{c^2x^2+1}}-\frac{6bc^3d^2fg\sqrt{c^2dx^2+d}\,x^5}{35\sqrt{c^2x^2+1}}-\frac{5bc^3d^2}{96}$$

```
[Out] -1/36*b*d^2*f^2*(c^2*x^2+1)^(5/2)*(c^2*d*x^2+d)^(1/2)/c+5/16*d^2*f^2*x*(a+b
*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)+5/128*d^2*g^2*x*(a+b*arcsinh(c*x))*(c^2*
d*x^2+d)^(1/2)/c^2+5/64*d^2*g^2*x^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)+
5/24*d^2*f^2*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)+5/48*d^2*
g^2*x^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)+1/6*d^2*f^2*x*(c
^2*x^2+1)^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)+1/8*d^2*g^2*x^3*(c^2*x^2
+1)^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)+2/7*d^2*f*g*(c^2*x^2+1)^3*(a+b
*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^2-2/7*b*d^2*f*g*x*(c^2*d*x^2+d)^(1/2)/
c/(c^2*x^2+1)^(1/2)-25/96*b*c*d^2*f^2*x^2*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(
1/2)-5/256*b*d^2*g^2*x^2*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-2/7*b*c*d^
2*f*g*x^3*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-5/96*b*c^3*d^2*f^2*x^4*(c^2
*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-59/768*b*c*d^2*g^2*x^4*(c^2*d*x^2+d)^(1/2
)/(c^2*x^2+1)^(1/2)-6/35*b*c^3*d^2*f*g*x^5*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^
(1/2)-17/288*b*c^3*d^2*g^2*x^6*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-2/49*b
*c^5*d^2*f*g*x^7*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/64*b*c^5*d^2*g^2*x
^8*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+5/32*d^2*f^2*(a+b*arcsinh(c*x))^2*
(c^2*d*x^2+d)^(1/2)/b/c/(c^2*x^2+1)^(1/2)-5/256*d^2*g^2*(a+b*arcsinh(c*x))^
2*(c^2*d*x^2+d)^(1/2)/b/c^3/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.93, antiderivative size = 901, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 15, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5835, 5821, 5684, 5682, 5675, 30, 14, 261, 5717, 194, 5744, 5742, 5758, 266, 43}

$$-\frac{bc^5d^2g^2\sqrt{c^2dx^2+d}\,x^8}{64\sqrt{c^2x^2+1}}-\frac{2bc^5d^2fg\sqrt{c^2dx^2+d}\,x^7}{49\sqrt{c^2x^2+1}}-\frac{17bc^3d^2g^2\sqrt{c^2dx^2+d}\,x^6}{288\sqrt{c^2x^2+1}}-\frac{6bc^3d^2fg\sqrt{c^2dx^2+d}\,x^5}{35\sqrt{c^2x^2+1}}-\frac{5bc^3d^2}{96}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-2*b*d^2*f*g*x*Sqrt[d + c^2*d*x^2])/(7*c*Sqrt[1 + c^2*x^2]) - (25*b*c*d^2*
f^2*x^2*Sqrt[d + c^2*d*x^2])/(96*Sqrt[1 + c^2*x^2]) - (5*b*d^2*g^2*x^2*Sqrt
[d + c^2*d*x^2])/(256*c*Sqrt[1 + c^2*x^2]) - (2*b*c*d^2*f*g*x^3*Sqrt[d + c^
2*d*x^2])/(7*Sqrt[1 + c^2*x^2]) - (5*b*c^3*d^2*f^2*x^4*Sqrt[d + c^2*d*x^2])
/(96*Sqrt[1 + c^2*x^2]) - (59*b*c*d^2*g^2*x^4*Sqrt[d + c^2*d*x^2])/(768*Sqr
t[1 + c^2*x^2]) - (6*b*c^3*d^2*f*g*x^5*Sqrt[d + c^2*d*x^2])/(35*Sqrt[1 + c^
2*x^2]) - (17*b*c^3*d^2*g^2*x^6*Sqrt[d + c^2*d*x^2])/(288*Sqrt[1 + c^2*x^2]
) - (2*b*c^5*d^2*f*g*x^7*Sqrt[d + c^2*d*x^2])/(49*Sqrt[1 + c^2*x^2]) - (b*c
^5*d^2*g^2*x^8*Sqrt[d + c^2*d*x^2])/(64*Sqrt[1 + c^2*x^2]) - (b*d^2*f^2*(1
+ c^2*x^2)^(5/2)*Sqrt[d + c^2*d*x^2])/(36*c) + (5*d^2*f^2*x*Sqrt[d + c^2*d*
x^2]*(a + b*ArcSinh[c*x]))/16 + (5*d^2*g^2*x*Sqrt[d + c^2*d*x^2]*(a + b*Arc
Sinh[c*x]))/(128*c^2) + (5*d^2*g^2*x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c
*x]))/64 + (5*d^2*f^2*x*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*
x]))/24 + (5*d^2*g^2*x^3*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c
*x]))/48 + (d^2*f^2*x*(1 + c^2*x^2)^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*
x]))/6 + (d^2*g^2*x^3*(1 + c^2*x^2)^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*
x]))/8 + (2*d^2*f*g*(1 + c^2*x^2)^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]
))/(7*c^2) + (5*d^2*f^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(32*b*c
*Sqrt[1 + c^2*x^2]) - (5*d^2*g^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2
)/(256*b*c^3*Sqrt[1 + c^2*x^2])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5835

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(d^IntPart[p]*(d + e*x^2)^FracPa
rt[p])/(1 + c^2*x^2)^FracPart[p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*Ar
cSinh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d
] && IntegerQ[m] && IntegerQ[p - 1/2] &&  !GtQ[d, 0]
```

Rubi steps

$$\int (f+gx)^2 \left(d+c^2dx^2\right)^{5/2} \left(a+b\sinh^{-1}(cx)\right) dx = \frac{\left(d^2\sqrt{d+c^2dx^2}\right)\int (f+gx)^2 \left(1+c^2x^2\right)^{5/2} \left(a+b\sinh^{-1}(cx)\right) d}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(d^2\sqrt{d+c^2dx^2}\right)\int \left(f^2 \left(1+c^2x^2\right)^{5/2} \left(a+b\sinh^{-1}(cx)\right)+2fg\right.}{}$$

$$= \frac{\left(d^2f^2\sqrt{d+c^2dx^2}\right)\int \left(1+c^2x^2\right)^{5/2} \left(a+b\sinh^{-1}(cx)\right) dx}{\sqrt{1+c^2x^2}} + \frac{(2}{}$$

$$= \frac{1}{6}d^2f^2x\left(1+c^2x^2\right)^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{8}d^2g^2x^3$$

$$= -\frac{bd^2f^2\left(1+c^2x^2\right)^{5/2}\sqrt{d+c^2dx^2}}{36c}+\frac{5}{24}d^2f^2x\left(1+c^2x^2\right)\sqrt{d+}$$

$$= -\frac{2bd^2fgx\sqrt{d+c^2dx^2}}{7c\sqrt{1+c^2x^2}}-\frac{2bcd^2fgx^3\sqrt{d+c^2dx^2}}{7\sqrt{1+c^2x^2}}-\frac{6bc^3d^2fgx}{35\sqrt{1}}$$

$$= -\frac{2bd^2fgx\sqrt{d+c^2dx^2}}{7c\sqrt{1+c^2x^2}}-\frac{25bcd^2f^2x^2\sqrt{d+c^2dx^2}}{96\sqrt{1+c^2x^2}}-\frac{2bcd^2fgx}{7\sqrt{1}}$$

$$= -\frac{2bd^2fgx\sqrt{d+c^2dx^2}}{7c\sqrt{1+c^2x^2}}-\frac{25bcd^2f^2x^2\sqrt{d+c^2dx^2}}{96\sqrt{1+c^2x^2}}-\frac{5bd^2g^2x^2}{256c\sqrt{}}$$

Mathematica [A] time = 2.78, size = 1047, normalized size = 1.16

$$\frac{d^2\left(-737280bfgx^7\sqrt{c^2dx^2+d}\,c^8+2257920ag^2x^7\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}\,c^7+5160960afgx^6\sqrt{c^2x^2+1}\sqrt{c^2dx^2+}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^2*(-5160960*b*c^2*f*g*x*Sqrt[d + c^2*d*x^2] - 5160960*b*c^4*f*g*x^3*Sqrt
[d + c^2*d*x^2] - 3096576*b*c^6*f*g*x^5*Sqrt[d + c^2*d*x^2] - 737280*b*c^8*
f*g*x^7*Sqrt[d + c^2*d*x^2] + 5160960*a*c*f*g*Sqrt[1 + c^2*x^2]*Sqrt[d + c^
2*d*x^2] + 12418560*a*c^3*f^2*x*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 705
600*a*c*g^2*x*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 15482880*a*c^3*f*g*x^
2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 9784320*a*c^5*f^2*x^3*Sqrt[1 + c^
2*x^2]*Sqrt[d + c^2*d*x^2] + 5550720*a*c^3*g^2*x^3*Sqrt[1 + c^2*x^2]*Sqrt[d
 + c^2*d*x^2] + 15482880*a*c^5*f*g*x^4*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2
] + 3010560*a*c^7*f^2*x^5*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 6397440*a
*c^5*g^2*x^5*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 5160960*a*c^7*f*g*x^6*
Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 2257920*a*c^7*g^2*x^7*Sqrt[1 + c^2*
x^2]*Sqrt[d + c^2*d*x^2] + 352800*b*(8*c^2*f^2 - g^2)*Sqrt[d + c^2*d*x^2]*A
rcSinh[c*x]^2 - 141120*b*(15*c^2*f^2 - g^2)*Sqrt[d + c^2*d*x^2]*Cosh[2*ArcS
inh[c*x]] - 211680*b*c^2*f^2*Sqrt[d + c^2*d*x^2]*Cosh[4*ArcSinh[c*x]] - 352
80*b*g^2*Sqrt[d + c^2*d*x^2]*Cosh[4*ArcSinh[c*x]] - 15680*b*c^2*f^2*Sqrt[d
+ c^2*d*x^2]*Cosh[6*ArcSinh[c*x]] - 15680*b*g^2*Sqrt[d + c^2*d*x^2]*Cosh[6*
ArcSinh[c*x]] - 2205*b*g^2*Sqrt[d + c^2*d*x^2]*Cosh[8*ArcSinh[c*x]] + 56448
00*a*c^2*Sqrt[d]*f^2*Sqrt[1 + c^2*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x
^2]] - 705600*a*Sqrt[d]*g^2*Sqrt[1 + c^2*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d +
c^2*d*x^2]] + 840*b*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]*(6144*c*f*g*Sqrt[1 + c
^2*x^2] + 18432*c^3*f*g*x^2*Sqrt[1 + c^2*x^2] + 18432*c^5*f*g*x^4*Sqrt[1 +
```

```
c^2*x^2] + 6144*c^7*f*g*x^6*Sqrt[1 + c^2*x^2] + 336*(15*c^2*f^2 - g^2)*Sinh
[2*ArcSinh[c*x]] + 168*(6*c^2*f^2 + g^2)*Sinh[4*ArcSinh[c*x]] + 112*c^2*f^2
*Sinh[6*ArcSinh[c*x]] + 112*g^2*Sinh[6*ArcSinh[c*x]] + 21*g^2*Sinh[8*ArcSin
h[c*x]])))/(18063360*c^3*Sqrt[1 + c^2*x^2])
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(ac^4d^2g^2x^6 + 2\,ac^4d^2fgx^5 + 4\,ac^2d^2fgx^3 + 2\,ad^2fgx + ad^2f^2 + \left(ac^4d^2f^2 + 2\,ac^2d^2g^2\right)x^4 + \left(2\,ac^2d^2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="fr
icas")
```

```
[Out] integral((a*c^4*d^2*g^2*x^6 + 2*a*c^4*d^2*f*g*x^5 + 4*a*c^2*d^2*f*g*x^3 + 2
*a*d^2*f*g*x + a*d^2*f^2 + (a*c^4*d^2*f^2 + 2*a*c^2*d^2*g^2)*x^4 + (2*a*c^2
*d^2*f^2 + a*d^2*g^2)*x^2 + (b*c^4*d^2*g^2*x^6 + 2*b*c^4*d^2*f*g*x^5 + 4*b*
c^2*d^2*f*g*x^3 + 2*b*d^2*f*g*x + b*d^2*f^2 + (b*c^4*d^2*f^2 + 2*b*c^2*d^2*
g^2)*x^4 + (2*b*c^2*d^2*f^2 + b*d^2*g^2)*x^2)*arcsinh(c*x))*sqrt(c^2*d*x^2
+ d), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="gi
ac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [A] time = 0.89, size = 1424, normalized size = 1.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] -1/48*a*g^2/c^2*x*(c^2*d*x^2+d)^(5/2)+1/6*a*f^2*x*(c^2*d*x^2+d)^(5/2)+8/7*b
*(d*(c^2*x^2+1))^(1/2)*f*g*d^2/(c^2*x^2+1)*arcsinh(c*x)*x^2+1/8*b*(d*(c^2*x
^2+1))^(1/2)*g^2*d^2*c^6/(c^2*x^2+1)*arcsinh(c*x)*x^9+23/48*b*(d*(c^2*x^2+1
))^(1/2)*g^2*d^2*c^4/(c^2*x^2+1)*arcsinh(c*x)*x^7+127/192*b*(d*(c^2*x^2+1))
^(1/2)*g^2*d^2*c^2/(c^2*x^2+1)*arcsinh(c*x)*x^5+5/128*b*(d*(c^2*x^2+1))^(1/
2)*g^2*d^2/c^2/(c^2*x^2+1)*arcsinh(c*x)*x+2/7*b*(d*(c^2*x^2+1))^(1/2)*f*g*d
^2/c^2/(c^2*x^2+1)*arcsinh(c*x)+1/6*b*(d*(c^2*x^2+1))^(1/2)*d^2*c^6/(c^2*x^
2+1)*arcsinh(c*x)*x^7*f^2-6/35*b*(d*(c^2*x^2+1))^(1/2)*f*g*d^2*c^3/(c^2*x^2
+1)^(1/2)*x^5-2/7*b*(d*(c^2*x^2+1))^(1/2)*f*g*d^2*c/(c^2*x^2+1)^(1/2)*x^3-2
/7*b*(d*(c^2*x^2+1))^(1/2)*f*g*d^2/c/(c^2*x^2+1)^(1/2)*x-2/49*b*(d*(c^2*x^2
+1))^(1/2)*f*g*d^2*c^5/(c^2*x^2+1)^(1/2)*x^7+17/24*b*(d*(c^2*x^2+1))^(1/2)*
d^2*c^4/(c^2*x^2+1)*arcsinh(c*x)*x^5*f^2+59/48*b*(d*(c^2*x^2+1))^(1/2)*d^2*
c^2/(c^2*x^2+1)*arcsinh(c*x)*x^3*f^2+2/7*b*(d*(c^2*x^2+1))^(1/2)*f*g*d^2*c^
6/(c^2*x^2+1)*arcsinh(c*x)*x^8+8/7*b*(d*(c^2*x^2+1))^(1/2)*f*g*d^2*c^4/(c^2
*x^2+1)*arcsinh(c*x)*x^6+12/7*b*(d*(c^2*x^2+1))^(1/2)*f*g*d^2*c^2/(c^2*x^2+
1)*arcsinh(c*x)*x^4-5/128*a*g^2/c^2*d^2*x*(c^2*d*x^2+d)^(1/2)-5/128*a*g^2/c
^2*d^3*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+2/7*a*f*
g/c^2/d*(c^2*d*x^2+d)^(7/2)+1/8*a*g^2*x*(c^2*d*x^2+d)^(7/2)/c^2/d-5/192*a*g
^2/c^2*d*x*(c^2*d*x^2+d)^(3/2)-299/2304*b*(d*(c^2*x^2+1))^(1/2)*d^2/c/(c^2*
x^2+1)^(1/2)*f^2+359/73728*b*(d*(c^2*x^2+1))^(1/2)*g^2*d^2/c^3/(c^2*x^2+1)^
```

```
(1/2)+5/24*a*f^2*d*x*(c^2*d*x^2+d)^(3/2)+5/16*a*f^2*d^2*x*(c^2*d*x^2+d)^(1/
2)+5/16*a*f^2*d^3*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/
2)-13/96*b*(d*(c^2*x^2+1))^(1/2)*d^2*c^3/(c^2*x^2+1)^(1/2)*x^4*f^2-11/32*b*
(d*(c^2*x^2+1))^(1/2)*d^2*c/(c^2*x^2+1)^(1/2)*x^2*f^2-1/64*b*(d*(c^2*x^2+1)
)^(1/2)*g^2*d^2*c^5/(c^2*x^2+1)^(1/2)*x^8-17/288*b*(d*(c^2*x^2+1))^(1/2)*g^
2*d^2*c^3/(c^2*x^2+1)^(1/2)*x^6-59/768*b*(d*(c^2*x^2+1))^(1/2)*g^2*d^2*c/(c
^2*x^2+1)^(1/2)*x^4-5/256*b*(d*(c^2*x^2+1))^(1/2)*g^2*d^2/c/(c^2*x^2+1)^(1/
2)*x^2-1/36*b*(d*(c^2*x^2+1))^(1/2)*d^2*c^5/(c^2*x^2+1)^(1/2)*x^6*f^2+11/16
*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)*arcsinh(c*x)*x*f^2+5/32*b*(d*(c^2*
x^2+1))^(1/2)*arcsinh(c*x)^2*d^2/(c^2*x^2+1)^(1/2)/c*f^2-5/256*b*(d*(c^2*x^
2+1))^(1/2)*arcsinh(c*x)^2*d^2/(c^2*x^2+1)^(1/2)/c^3*g^2+133/384*b*(d*(c^2*
x^2+1))^(1/2)*g^2*d^2/(c^2*x^2+1)*arcsinh(c*x)*x^3
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="ma
xima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(f+g\,x\right)}^2\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,{\left(d\,c^2\,x^2+d\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2),x)
```

```
[Out] int((f + g*x)^2*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Timed out
```

3.45 $\int (f+gx)\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=494

$$\frac{1}{6}d^2fx\left(c^2x^2+1\right)^2\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)+\frac{5}{16}d^2fx\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)+\frac{5}{24}d^2fx\left(c^2x^2+1\right)$$

```
[Out] -1/36*b*d^2*f*(c^2*x^2+1)^(5/2)*(c^2*d*x^2+d)^(1/2)/c+5/16*d^2*f*x*(a+b*arc
sinh(c*x))*(c^2*d*x^2+d)^(1/2)+5/24*d^2*f*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))*
(c^2*d*x^2+d)^(1/2)+1/6*d^2*f*x*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))*(c^2*d*x^2
+d)^(1/2)+1/7*d^2*g*(c^2*x^2+1)^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^
2-1/7*b*d^2*g*x*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-25/96*b*c*d^2*f*x^2
*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/7*b*c*d^2*g*x^3*(c^2*d*x^2+d)^(1/2
)/(c^2*x^2+1)^(1/2)-5/96*b*c^3*d^2*f*x^4*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1
/2)-3/35*b*c^3*d^2*g*x^5*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/49*b*c^5*d
^2*g*x^7*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+5/32*d^2*f*(a+b*arcsinh(c*x)
)^2*(c^2*d*x^2+d)^(1/2)/b/c/(c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.40, antiderivative size = 494, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {5835, 5821, 5684, 5682, 5675, 30, 14, 261, 5717, 194}

$$\frac{1}{6}d^2fx\left(c^2x^2+1\right)^2\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)+\frac{5}{16}d^2fx\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)+\frac{5}{24}d^2fx\left(c^2x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -(b*d^2*g*x*Sqrt[d + c^2*d*x^2])/(7*c*Sqrt[1 + c^2*x^2]) - (25*b*c*d^2*f*x^
2*Sqrt[d + c^2*d*x^2])/(96*Sqrt[1 + c^2*x^2]) - (b*c*d^2*g*x^3*Sqrt[d + c^2
*d*x^2])/(7*Sqrt[1 + c^2*x^2]) - (5*b*c^3*d^2*f*x^4*Sqrt[d + c^2*d*x^2])/(9
6*Sqrt[1 + c^2*x^2]) - (3*b*c^3*d^2*g*x^5*Sqrt[d + c^2*d*x^2])/(35*Sqrt[1 +
 c^2*x^2]) - (b*c^5*d^2*g*x^7*Sqrt[d + c^2*d*x^2])/(49*Sqrt[1 + c^2*x^2]) -
 (b*d^2*f*(1 + c^2*x^2)^(5/2)*Sqrt[d + c^2*d*x^2])/(36*c) + (5*d^2*f*x*Sqrt
[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/16 + (5*d^2*f*x*(1 + c^2*x^2)*Sqrt[d
+ c^2*d*x^2]*(a + b*ArcSinh[c*x]))/24 + (d^2*f*x*(1 + c^2*x^2)^2*Sqrt[d + c
^2*d*x^2]*(a + b*ArcSinh[c*x]))/6 + (d^2*g*(1 + c^2*x^2)^3*Sqrt[d + c^2*d*x
^2]*(a + b*ArcSinh[c*x]))/(7*c^2) + (5*d^2*f*Sqrt[d + c^2*d*x^2]*(a + b*Arc
Sinh[c*x])^2)/(32*b*c*Sqrt[1 + c^2*x^2])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5835

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(d^IntPart[p]*(d + e*x^2)^FracPa
rt[p])/(1 + c^2*x^2)^FracPart[p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*Ar
cSinh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d
] && IntegerQ[m] && IntegerQ[p - 1/2] &&  !GtQ[d, 0]
```

Rubi steps

$$\int (f+gx)\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)dx = \frac{\left(d^2\sqrt{d+c^2dx^2}\right)\int (f+gx)\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(d^2\sqrt{d+c^2dx^2}\right)\int \left(f\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)+gx\left(\right.\right.}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(d^2f\sqrt{d+c^2dx^2}\right)\int \left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)dx}{\sqrt{1+c^2x^2}} + \frac{(}{}$$

$$= \frac{1}{6}d^2fx\left(1+c^2x^2\right)^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right) + \frac{d^2g\left(1-\right.}{}$$

$$= -\frac{bd^2f\left(1+c^2x^2\right)^{5/2}\sqrt{d+c^2dx^2}}{36c} + \frac{5}{24}d^2fx\left(1+c^2x^2\right)\sqrt{d+}$$

$$= -\frac{bd^2gx\sqrt{d+c^2dx^2}}{7c\sqrt{1+c^2x^2}} - \frac{bcd^2gx^3\sqrt{d+c^2dx^2}}{7\sqrt{1+c^2x^2}} - \frac{3bc^3d^2gx^5\sqrt{d}}{35\sqrt{1+c}}$$

$$= -\frac{bd^2gx\sqrt{d+c^2dx^2}}{7c\sqrt{1+c^2x^2}} - \frac{25bcd^2fx^2\sqrt{d+c^2dx^2}}{96\sqrt{1+c^2x^2}} - \frac{bcd^2gx^3\sqrt{d}}{7\sqrt{1+}}$$

Mathematica [A] time = 1.34, size = 656, normalized size = 1.33

$$\frac{d^2\left(388080ac^2fx\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}+176400ac\sqrt{d}f\sqrt{c^2x^2+1}\log\left(\sqrt{d}\sqrt{c^2dx^2+d}+cdx\right)+241920ac^2g\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^2*(-80640*b*c*g*x*Sqrt[d + c^2*d*x^2] - 80640*b*c^3*g*x^3*Sqrt[d + c^2*d
*x^2] - 48384*b*c^5*g*x^5*Sqrt[d + c^2*d*x^2] - 11520*b*c^7*g*x^7*Sqrt[d +
c^2*d*x^2] + 80640*a*g*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 388080*a*c^2
*f*x*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 241920*a*c^2*g*x^2*Sqrt[1 + c^
2*x^2]*Sqrt[d + c^2*d*x^2] + 305760*a*c^4*f*x^3*Sqrt[1 + c^2*x^2]*Sqrt[d +
c^2*d*x^2] + 241920*a*c^4*g*x^4*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 940
80*a*c^6*f*x^5*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 80640*a*c^6*g*x^6*Sq
rt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 88200*b*c*f*Sqrt[d + c^2*d*x^2]*ArcSi
nh[c*x]^2 - 66150*b*c*f*Sqrt[d + c^2*d*x^2]*Cosh[2*ArcSinh[c*x]] - 6615*b*c
*f*Sqrt[d + c^2*d*x^2]*Cosh[4*ArcSinh[c*x]] - 490*b*c*f*Sqrt[d + c^2*d*x^2]
*Cosh[6*ArcSinh[c*x]] + 176400*a*c*Sqrt[d]*f*Sqrt[1 + c^2*x^2]*Log[c*d*x +
Sqrt[d]*Sqrt[d + c^2*d*x^2]] + 420*b*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]*(192*
g*Sqrt[1 + c^2*x^2] + 576*c^2*g*x^2*Sqrt[1 + c^2*x^2] + 576*c^4*g*x^4*Sqrt[
1 + c^2*x^2] + 192*c^6*g*x^6*Sqrt[1 + c^2*x^2] + 315*c*f*Sinh[2*ArcSinh[c*x
]] + 63*c*f*Sinh[4*ArcSinh[c*x]] + 7*c*f*Sinh[6*ArcSinh[c*x]])))/(564480*c^
2*Sqrt[1 + c^2*x^2])
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(ac^4d^2gx^5 + ac^4d^2fx^4 + 2\,ac^2d^2gx^3 + 2\,ac^2d^2fx^2 + ad^2gx + ad^2f + \left(bc^4d^2gx^5 + bc^4d^2fx^4 + 2\,bc^2d^2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="fric
as")
```

```
[Out] integral((a*c^4*d^2*g*x^5 + a*c^4*d^2*f*x^4 + 2*a*c^2*d^2*g*x^3 + 2*a*c^2*d
^2*f*x^2 + a*d^2*g*x + a*d^2*f + (b*c^4*d^2*g*x^5 + b*c^4*d^2*f*x^4 + 2*b*c
^2*d^2*g*x^3 + 2*b*c^2*d^2*f*x^2 + b*d^2*g*x + b*d^2*f)*arcsinh(c*x))*sqrt(
c^2*d*x^2 + d), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="giac
")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [A] time = 0.57, size = 805, normalized size = 1.63

$$\frac{ag\left(c^2d\,x^2+d\right)^{\frac{7}{2}}}{7c^2d}+\frac{afx\left(c^2d\,x^2+d\right)^{\frac{5}{2}}}{6}+\frac{5afdx\left(c^2d\,x^2+d\right)^{\frac{3}{2}}}{24}+\frac{5af\,d^2x\sqrt{c^2d\,x^2+d}}{16}+\frac{5af\,d^3\ln\left(\frac{x\,c^2d}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right.}{16\sqrt{c^2d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/7*a*g/c^2/d*(c^2*d*x^2+d)^(7/2)+1/6*a*f*x*(c^2*d*x^2+d)^(5/2)+5/24*a*f*d*
x*(c^2*d*x^2+d)^(3/2)+5/16*a*f*d^2*x*(c^2*d*x^2+d)^(1/2)+5/16*a*f*d^3*ln(x*
c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+5/32*b*(d*(c^2*x^2+1
))^(1/2)/(c^2*x^2+1)^(1/2)/c*f*arcsinh(c*x)^2*d^2+1/7*b*(d*(c^2*x^2+1))^(1/
2)*g*d^2*c^6/(c^2*x^2+1)*arcsinh(c*x)*x^8-1/49*b*(d*(c^2*x^2+1))^(1/2)*g*d^
2*c^5/(c^2*x^2+1)^(1/2)*x^7+4/7*b*(d*(c^2*x^2+1))^(1/2)*g*d^2*c^4/(c^2*x^2+
1)*arcsinh(c*x)*x^6-3/35*b*(d*(c^2*x^2+1))^(1/2)*g*d^2*c^3/(c^2*x^2+1)^(1/2
)*x^5+6/7*b*(d*(c^2*x^2+1))^(1/2)*g*d^2*c^2/(c^2*x^2+1)*arcsinh(c*x)*x^4-1/
7*b*(d*(c^2*x^2+1))^(1/2)*g*d^2*c/(c^2*x^2+1)^(1/2)*x^3+4/7*b*(d*(c^2*x^2+1
))^(1/2)*g*d^2/(c^2*x^2+1)*arcsinh(c*x)*x^2-1/7*b*(d*(c^2*x^2+1))^(1/2)*g*d
^2/c/(c^2*x^2+1)^(1/2)*x+1/6*b*(d*(c^2*x^2+1))^(1/2)*f*d^2*c^6/(c^2*x^2+1)*
arcsinh(c*x)*x^7-1/36*b*(d*(c^2*x^2+1))^(1/2)*f*d^2*c^5/(c^2*x^2+1)^(1/2)*x
^6+17/24*b*(d*(c^2*x^2+1))^(1/2)*f*d^2*c^4/(c^2*x^2+1)*arcsinh(c*x)*x^5-13/
96*b*(d*(c^2*x^2+1))^(1/2)*f*d^2*c^3/(c^2*x^2+1)^(1/2)*x^4+59/48*b*(d*(c^2*
x^2+1))^(1/2)*f*d^2*c^2/(c^2*x^2+1)*arcsinh(c*x)*x^3-11/32*b*(d*(c^2*x^2+1)
)^(1/2)*f*d^2*c/(c^2*x^2+1)^(1/2)*x^2+11/16*b*(d*(c^2*x^2+1))^(1/2)*f*d^2/(
c^2*x^2+1)*arcsinh(c*x)*x+1/7*b*(d*(c^2*x^2+1))^(1/2)*g*d^2/c^2/(c^2*x^2+1)
*arcsinh(c*x)-299/2304*b*(d*(c^2*x^2+1))^(1/2)*f*d^2/c/(c^2*x^2+1)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(f+g\,x\right)\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,{\left(d\,c^2\,x^2+d\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2),x)
[Out] int((f + g*x)*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x)),x)
[Out] Timed out
```

3.46 $\int \frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{f+gx}\,dx$

Optimal. Leaf size=1536

$$-\frac{bd^2x^5\sqrt{c^2dx^2+d}\,c^5}{25g\sqrt{c^2x^2+1}}+\frac{bd^2fx^4\sqrt{c^2dx^2+d}\,c^5}{16g^2\sqrt{c^2x^2+1}}-\frac{d^2fx^3\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)c^4}{4g^2}-\frac{bd^2\left(c^2f^2+2g^2\right)x^3\sqrt{c^2dx}}{9g^3\sqrt{c^2x^2+1}}$$

```
[Out] -1/8*c^2*d^2*f*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/g^2-1/4*c^4*d^2*f*x
^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/g^2+2/15*b*c*d^2*x*(c^2*d*x^2+d)^
(1/2)/g/(c^2*x^2+1)^(1/2)-1/45*b*c^3*d^2*x^3*(c^2*d*x^2+d)^(1/2)/g/(c^2*x^2
+1)^(1/2)-1/25*b*c^5*d^2*x^5*(c^2*d*x^2+d)^(1/2)/g/(c^2*x^2+1)^(1/2)-a*d^2*
(c^2*f^2+g^2)^(5/2)*arctanh((-c^2*f*x+g)/(c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1
/2))*(c^2*d*x^2+d)^(1/2)/g^6/(c^2*x^2+1)^(1/2)+b*d^2*(c^2*f^2+g^2)^(5/2)*po
lylog(2,-(c*x+(c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2+g^2)^(1/2)))*(c^2*d*x^2+d)
^(1/2)/g^6/(c^2*x^2+1)^(1/2)-b*d^2*(c^2*f^2+g^2)^(5/2)*polylog(2,-(c*x+(c^2
*x^2+1)^(1/2))*g/(c*f+(c^2*f^2+g^2)^(1/2)))*(c^2*d*x^2+d)^(1/2)/g^6/(c^2*x^
2+1)^(1/2)+1/4*b*c^3*d^2*f*(c^2*f^2+2*g^2)*x^2*(c^2*d*x^2+d)^(1/2)/g^4/(c^2
*x^2+1)^(1/2)-1/4*c*d^2*f*(c^2*f^2+2*g^2)*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d
)^(1/2)/b/g^4/(c^2*x^2+1)^(1/2)-1/2*c*d^2*(c^2*f^2+g^2)^2*x*(a+b*arcsinh(c*
x))^2*(c^2*d*x^2+d)^(1/2)/b/g^5/(c^2*x^2+1)^(1/2)-1/2*d^2*(c^2*f^2+g^2)^3*(
a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/b/c/g^6/(g*x+f)/(c^2*x^2+1)^(1/2)+1
/2*d^2*(c^2*f^2+g^2)^2*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)*(c^2*d*x^2+d)
^(1/2)/b/c/g^4/(g*x+f)-1/2*c^2*d^2*f*(c^2*f^2+2*g^2)*x*(a+b*arcsinh(c*x))*(
c^2*d*x^2+d)^(1/2)/g^4-b*c*d^2*(c^2*f^2+g^2)^2*x*(c^2*d*x^2+d)^(1/2)/g^5/(c
^2*x^2+1)^(1/2)-1/3*b*c*d^2*(c^2*f^2+2*g^2)*x*(c^2*d*x^2+d)^(1/2)/g^3/(c^2*
x^2+1)^(1/2)+1/16*b*c^3*d^2*f*x^2*(c^2*d*x^2+d)^(1/2)/g^2/(c^2*x^2+1)^(1/2)
-1/9*b*c^3*d^2*(c^2*f^2+2*g^2)*x^3*(c^2*d*x^2+d)^(1/2)/g^3/(c^2*x^2+1)^(1/2
)+1/16*b*c^5*d^2*f*x^4*(c^2*d*x^2+d)^(1/2)/g^2/(c^2*x^2+1)^(1/2)+1/16*c*d^2
*f*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/b/g^2/(c^2*x^2+1)^(1/2)+b*d^2*(
c^2*f^2+g^2)^(5/2)*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^
2+g^2)^(1/2)))*(c^2*d*x^2+d)^(1/2)/g^6/(c^2*x^2+1)^(1/2)-b*d^2*(c^2*f^2+g^2
)^(5/2)*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2+g^2)^(1/2
)))*(c^2*d*x^2+d)^(1/2)/g^6/(c^2*x^2+1)^(1/2)+a*d^2*(c^2*f^2+g^2)^2*(c^2*d*
x^2+d)^(1/2)/g^5-1/3*d^2*(c^2*x^2+1)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)
/g+1/5*d^2*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/g+b*d^2*(c^
2*f^2+g^2)^2*arcsinh(c*x)*(c^2*d*x^2+d)^(1/2)/g^5+1/3*d^2*(c^2*f^2+2*g^2)*(
c^2*x^2+1)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/g^3
```

Rubi [A] time = 2.45, antiderivative size = 1536, normalized size of antiderivative = 1.00, number of steps used = 37, number of rules used = 29, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.967, Rules used = {5835, 5825, 5682, 5675, 30, 5717, 5742, 5758, 266, 43, 5732, 12, 5823, 683, 5815, 6742, 261, 725, 206, 5859, 1654, 5857, 8, 5831, 3322, 2264, 2190, 2279, 2391}

$$-\frac{bd^2x^5\sqrt{c^2dx^2+d}\,c^5}{25g\sqrt{c^2x^2+1}}+\frac{bd^2fx^4\sqrt{c^2dx^2+d}\,c^5}{16g^2\sqrt{c^2x^2+1}}-\frac{d^2fx^3\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)c^4}{4g^2}-\frac{bd^2\left(c^2f^2+2g^2\right)x^3\sqrt{c^2dx}}{9g^3\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(f + g*x),x]
```

```
[Out] (a*d^2*(c^2*f^2 + g^2)^2*Sqrt[d + c^2*d*x^2])/g^5 + (2*b*c*d^2*x*Sqrt[d + c
^2*d*x^2])/(15*g*Sqrt[1 + c^2*x^2]) - (b*c*d^2*(c^2*f^2 + g^2)^2*x*Sqrt[d +
 c^2*d*x^2])/(g^5*Sqrt[1 + c^2*x^2]) - (b*c*d^2*(c^2*f^2 + 2*g^2)*x*Sqrt[d
+ c^2*d*x^2])/(3*g^3*Sqrt[1 + c^2*x^2]) + (b*c^3*d^2*f*x^2*Sqrt[d + c^2*d*x
```

```
^2])/(16*g^2*Sqrt[1 + c^2*x^2]) + (b*c^3*d^2*f*(c^2*f^2 + 2*g^2)*x^2*Sqrt[d
 + c^2*d*x^2])/(4*g^4*Sqrt[1 + c^2*x^2]) - (b*c^3*d^2*x^3*Sqrt[d + c^2*d*x^
2])/(45*g*Sqrt[1 + c^2*x^2]) - (b*c^3*d^2*(c^2*f^2 + 2*g^2)*x^3*Sqrt[d + c^
2*d*x^2])/(9*g^3*Sqrt[1 + c^2*x^2]) + (b*c^5*d^2*f*x^4*Sqrt[d + c^2*d*x^2])
/(16*g^2*Sqrt[1 + c^2*x^2]) - (b*c^5*d^2*x^5*Sqrt[d + c^2*d*x^2])/(25*g*Sqr
t[1 + c^2*x^2]) + (b*d^2*(c^2*f^2 + g^2)^2*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]
)/g^5 - (c^2*d^2*f*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(8*g^2) - (c
^2*d^2*f*(c^2*f^2 + 2*g^2)*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(2*g
^4) - (c^4*d^2*f*x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(4*g^2) - (d
^2*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(3*g) + (d^2*(c^
2*f^2 + 2*g^2)*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(3*g
^3) + (d^2*(1 + c^2*x^2)^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(5*g)
+ (c*d^2*f*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(16*b*g^2*Sqrt[1 + c
^2*x^2]) - (c*d^2*f*(c^2*f^2 + 2*g^2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*
x])^2)/(4*b*g^4*Sqrt[1 + c^2*x^2]) - (c*d^2*(c^2*f^2 + g^2)^2*x*Sqrt[d + c^
2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*g^5*Sqrt[1 + c^2*x^2]) - (d^2*(c^2*f^
2 + g^2)^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c*g^6*(f + g*x)
*Sqrt[1 + c^2*x^2]) + (d^2*(c^2*f^2 + g^2)^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2
*d*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c*g^4*(f + g*x)) - (a*d^2*(c^2*f^2 + g
^2)^(5/2)*Sqrt[d + c^2*d*x^2]*ArcTanh[(g - c^2*f*x)/(Sqrt[c^2*f^2 + g^2]*Sq
rt[1 + c^2*x^2])])/(g^6*Sqrt[1 + c^2*x^2]) + (b*d^2*(c^2*f^2 + g^2)^(5/2)*S
qrt[d + c^2*d*x^2]*ArcSinh[c*x]*Log[1 + (E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*
f^2 + g^2])])/(g^6*Sqrt[1 + c^2*x^2]) - (b*d^2*(c^2*f^2 + g^2)^(5/2)*Sqrt[d
 + c^2*d*x^2]*ArcSinh[c*x]*Log[1 + (E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 +
 g^2])])/(g^6*Sqrt[1 + c^2*x^2]) + (b*d^2*(c^2*f^2 + g^2)^(5/2)*Sqrt[d + c^
2*d*x^2]*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2]))])/(g^
6*Sqrt[1 + c^2*x^2]) - (b*d^2*(c^2*f^2 + g^2)^(5/2)*Sqrt[d + c^2*d*x^2]*Pol
yLog[2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))])/(g^6*Sqrt[1 + c
^2*x^2])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
```

```
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5732

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(1 + c^2*x^2)^p, x]}, Dist[d^p*(a + b
*ArcSinh[c*x]), u, x] - Dist[b*c*d^p, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*
x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegerQ
[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0]) && NeQ[p, -2^
(-1)] && GtQ[d, 0]
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
```

```
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5815

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.) + (g_.)*(x_) + (h_.)*(
x_)^2)^(p_.))/((d_) + (e_.)*(x_))^2, x_Symbol] :> With[{u = IntHide[(f + g*
x + h*x^2)^p/(d + e*x)^2, x]}, Dist[(a + b*ArcSinh[c*x])^n, u, x] - Dist[b*
c*n, Int[SimplifyIntegrand[(u*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^
2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[n, 0] && IGtQ
[p, 0] && EqQ[e*g - 2*d*h, 0]
```

Rule 5823

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.) + (g_.)*(x_))^(m_)*Sqr
t[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[((f + g*x)^m*(d + e*x^2)*(a + b*A
rcSinh[c*x])^(n + 1))/(b*c*Sqrt[d]*(n + 1)), x] - Dist[1/(b*c*Sqrt[d]*(n +
1)), Int[(d*g*m + 2*e*f*x + e*g*(m + 2)*x^2)*(f + g*x)^(m - 1)*(a + b*ArcSi
nh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d
] && ILtQ[m, 0] && GtQ[d, 0] && IGtQ[n, 0]
```

Rule 5825

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[Sqrt[d + e*x^2]*(
a + b*ArcSinh[c*x])^n, (f + g*x)^m*(d + e*x^2)^(p - 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IGtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Rule 5831

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 5835

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(d^IntPart[p]*(d + e*x^2)^FracPa
rt[p])/(1 + c^2*x^2)^FracPart[p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*Ar
cSinh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d
] && IntegerQ[m] && IntegerQ[p - 1/2] &&  !GtQ[d, 0]
```

Rule 5857

```
Int[ArcSinh[(c_.)*(x_)]^(n_.)*(RFx_)*((d_) + (e_.)*(x_)^2)^(p_), x_Symbol]
:> With[{u = ExpandIntegrand[(d + e*x^2)^p*ArcSinh[c*x]^n, RFx, x]}, Int[u,
 x] /; SumQ[u]] /; FreeQ[{c, d, e}, x] && RationalFunctionQ[RFx, x] && IGtQ
[n, 0] && EqQ[e, c^2*d] && IntegerQ[p - 1/2]
```

Rule 5859

```
Int[(ArcSinh[(c_.)*(x_)]*(b_.) + (a_))^(n_.)*(RFx_)*((d_) + (e_.)*(x_)^2)^(
p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p, RFx*(a + b*ArcSinh[c*x
])^n, x], x] /; FreeQ[{a, b, c, d, e}, x] && RationalFunctionQ[RFx, x] && I
```

```
GtQ[n, 0] && EqQ[e, c^2*d] && IntegerQ[p - 1/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{f+gx}\,dx = \frac{\left(d^2\sqrt{d+c^2dx^2}\right)\int \frac{\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{f+gx}\,dx}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(d^2\sqrt{d+c^2dx^2}\right)\int\Bigg(\frac{\left(-c^4f^3-2c^2fg^2\right)\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{g^4} + \frac{c^2\left(c^2f^2+2g^2\right)x\sqrt{1+\cdots}}{\cdots}}{\cdots}$$

$$= -\frac{\left(c^4d^2f\sqrt{d+c^2dx^2}\right)\int x^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)dx}{g^2\sqrt{1+c^2x^2}} + \frac{\left(c^4d^2\sqrt{d\cdots}\right.}{\cdots}$$

$$= -\frac{c^2d^2f\left(c^2f^2+2g^2\right)x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2g^4} - \frac{c^4d^2fx^3\sqrt{d+c\cdots}}{\cdots}$$

$$= -\frac{bcd^2\left(c^2f^2+2g^2\right)x\sqrt{d+c^2dx^2}}{3g^3\sqrt{1+c^2x^2}} + \frac{bc^3d^2f\left(c^2f^2+2g^2\right)x^2\sqrt{d+c^2dx^2}}{4g^4\sqrt{1+c^2x^2}}\cdots$$

$$= \frac{2bcd^2x\sqrt{d+c^2dx^2}}{15g\sqrt{1+c^2x^2}} - \frac{bcd^2\left(c^2f^2+2g^2\right)x\sqrt{d+c^2dx^2}}{3g^3\sqrt{1+c^2x^2}} + \frac{bc^3d^2fx^2\sqrt{d+\cdots}}{16g^2\sqrt{1+c\cdots}}$$

$$= \frac{2bcd^2x\sqrt{d+c^2dx^2}}{15g\sqrt{1+c^2x^2}} - \frac{bcd^2\left(c^2f^2+2g^2\right)x\sqrt{d+c^2dx^2}}{3g^3\sqrt{1+c^2x^2}} + \frac{bc^3d^2fx^2\sqrt{d+\cdots}}{16g^2\sqrt{1+c\cdots}}$$

$$= \frac{ad^2\left(c^2f^2+g^2\right)^2\sqrt{d+c^2dx^2}}{g^5} + \frac{2bcd^2x\sqrt{d+c^2dx^2}}{15g\sqrt{1+c^2x^2}} - \frac{bcd^2\left(c^2f^2+2g^2\cdots\right.}{3g^3\sqrt{1+\cdots}}$$

$$= \frac{ad^2\left(c^2f^2+g^2\right)^2\sqrt{d+c^2dx^2}}{g^5} + \frac{2bcd^2x\sqrt{d+c^2dx^2}}{15g\sqrt{1+c^2x^2}} - \frac{bcd^2\left(c^2f^2+2g^2\cdots\right.}{3g^3\sqrt{1+\cdots}}$$

$$= \frac{ad^2\left(c^2f^2+g^2\right)^2\sqrt{d+c^2dx^2}}{g^5} + \frac{2bcd^2x\sqrt{d+c^2dx^2}}{15g\sqrt{1+c^2x^2}} - \frac{bcd^2\left(c^2f^2+2g^2\cdots\right.}{3g^3\sqrt{1+\cdots}}$$

$$= \frac{ad^2\left(c^2f^2+g^2\right)^2\sqrt{d+c^2dx^2}}{g^5} + \frac{2bcd^2x\sqrt{d+c^2dx^2}}{15g\sqrt{1+c^2x^2}} - \frac{bcd^2\left(c^2f^2+g^2\right)^{\cdots}}{g^5\sqrt{1+\cdots}}$$

$$= \frac{ad^2\left(c^2f^2+g^2\right)^2\sqrt{d+c^2dx^2}}{g^5} + \frac{2bcd^2x\sqrt{d+c^2dx^2}}{15g\sqrt{1+c^2x^2}} - \frac{bcd^2\left(c^2f^2+g^2\right)^{\cdots}}{g^5\sqrt{1+\cdots}}$$

$$= \frac{ad^2\left(c^2f^2+g^2\right)^2\sqrt{d+c^2dx^2}}{g^5} + \frac{2bcd^2x\sqrt{d+c^2dx^2}}{15g\sqrt{1+c^2x^2}} - \frac{bcd^2\left(c^2f^2+g^2\right)^{\cdots}}{g^5\sqrt{1+\cdots}}$$

$$= \frac{ad^2\left(c^2f^2+g^2\right)^2\sqrt{d+c^2dx^2}}{g^5} + \frac{2bcd^2x\sqrt{d+c^2dx^2}}{15g\sqrt{1+c^2x^2}} - \frac{bcd^2\left(c^2f^2+g^2\right)^{\cdots}}{g^5\sqrt{1+\cdots}}$$

$$= \frac{ad^2\left(c^2f^2+g^2\right)^2\sqrt{d+c^2dx^2}}{g^5} + \frac{2bcd^2x\sqrt{d+c^2dx^2}}{15g\sqrt{1+c^2x^2}} - \frac{bcd^2\left(c^2f^2+g^2\right)^{\cdots}}{g^5\sqrt{1+\cdots}}$$

Mathematica [C] time = 25.94, size = 7163, normalized size = 4.66

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(f + g*x),x]
```

```
[Out] Result too large to show
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(ac^4d^2x^4 + 2\,ac^2d^2x^2 + ad^2 + \left(bc^4d^2x^4 + 2\,bc^2d^2x^2 + bd^2\right)\operatorname{arsinh}(cx)\right)\sqrt{c^2dx^2 + d}}{gx + f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/(g*x+f),x, algorithm="fric
as")
```

```
[Out] integral((a*c^4*d^2*x^4 + 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 + 2*b*c^
2*d^2*x^2 + b*d^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)/(g*x + f), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/(g*x+f),x, algorithm="giac
")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.47, size = 3928, normalized size = 2.56

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/(g*x+f),x)
```

```
[Out] b*d^2*(d*(c^2*x^2+1))^(1/2)*(c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2)/g^6*dilog
((-(c*x+(c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2+g^2)^(1/2))/(-c*f+(c^2*f^2+g^2)^(
1/2)))*c^4*f^4-b*d^2*(d*(c^2*x^2+1))^(1/2)*(c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)^
(1/2)/g^6*dilog(((c*x+(c^2*x^2+1)^(1/2))*g+c*f+(c^2*f^2+g^2)^(1/2))/(c*f+(c
^2*f^2+g^2)^(1/2)))*c^4*f^4+2*b*d^2*(d*(c^2*x^2+1))^(1/2)*(c^2*f^2+g^2)^(1/
2)/(c^2*x^2+1)^(1/2)/g^4*dilog((-(c*x+(c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2+g^2
)^(1/2))/(-c*f+(c^2*f^2+g^2)^(1/2)))*c^2*f^2-2*b*d^2*(d*(c^2*x^2+1))^(1/2)*
(c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2)/g^4*dilog(((c*x+(c^2*x^2+1)^(1/2))*g+
c*f+(c^2*f^2+g^2)^(1/2))/(c*f+(c^2*f^2+g^2)^(1/2)))*c^2*f^2+b*(d*(c^2*x^2+1
))^(1/2)*d^2/(c^2*x^2+1)/g^5*arcsinh(c*x)*x^2*c^6*f^4+1/3*b*(d*(c^2*x^2+1))
^(1/2)*d^2/(c^2*x^2+1)/g^3*arcsinh(c*x)*x^4*c^6*f^2+8/3*b*(d*(c^2*x^2+1))^(
1/2)*d^2/(c^2*x^2+1)/g^3*arcsinh(c*x)*x^2*c^4*f^2-1/4*b*(d*(c^2*x^2+1))^(1/
2)*f*d^2*c^6/(c^2*x^2+1)/g^2*arcsinh(c*x)*x^5-11/8*b*(d*(c^2*x^2+1))^(1/2)*
f*d^2*c^4/(c^2*x^2+1)/g^2*arcsinh(c*x)*x^3-9/8*b*(d*(c^2*x^2+1))^(1/2)*f*d^
2*c^2/(c^2*x^2+1)/g^2*arcsinh(c*x)*x-1/2*b*(d*(c^2*x^2+1))^(1/2)*f^3*d^2*c^
6/(c^2*x^2+1)/g^4*arcsinh(c*x)*x^3-1/2*b*(d*(c^2*x^2+1))^(1/2)*f^3*d^2*c^4/
(c^2*x^2+1)/g^4*arcsinh(c*x)*x-2*b*d^2*(d*(c^2*x^2+1))^(1/2)*(c^2*f^2+g^2)^
(1/2)/(c^2*x^2+1)^(1/2)/g^4*arcsinh(c*x)*ln(((c*x+(c^2*x^2+1)^(1/2))*g+c*f+
```

```
(c^2*f^2+g^2)^(1/2))/(c*f+(c^2*f^2+g^2)^(1/2)))*c^2*f^2+b*d^2*(d*(c^2*x^2+1
))^(1/2)*(c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2)/g^6*arcsinh(c*x)*ln((-(c*x+(
c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2+g^2)^(1/2))/(-c*f+(c^2*f^2+g^2)^(1/2)))*c^
4*f^4-b*d^2*(d*(c^2*x^2+1))^(1/2)*(c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2)/g^6
*arcsinh(c*x)*ln(((c*x+(c^2*x^2+1)^(1/2))*g+c*f+(c^2*f^2+g^2)^(1/2))/(c*f+(
c^2*f^2+g^2)^(1/2)))*c^4*f^4+2*b*d^2*(d*(c^2*x^2+1))^(1/2)*(c^2*f^2+g^2)^(1
/2)/(c^2*x^2+1)^(1/2)/g^4*arcsinh(c*x)*ln((-(c*x+(c^2*x^2+1)^(1/2))*g-c*f+(
c^2*f^2+g^2)^(1/2))/(-c*f+(c^2*f^2+g^2)^(1/2)))*c^2*f^2+1/5*a/g*((x+f/g)^2*
c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(5/2)+2*a/g^3*d^2*((x+f/g)^2
*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2)*c^2*f^2+23/15*b*(d*(c
^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)/g*arcsinh(c*x)+b*(d*(c^2*x^2+1))^(1/2)*d^2
/(c^2*x^2+1)/g^5*arcsinh(c*x)*c^4*f^4-7/3*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*
x^2+1)^(1/2)/g^3*x*c^3*f^2-1/9*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)^(1/2
)/g^3*x^3*c^5*f^2+1/16*b*(d*(c^2*x^2+1))^(1/2)*f*d^2*c^5/(c^2*x^2+1)^(1/2)/
g^2*x^4+9/16*b*(d*(c^2*x^2+1))^(1/2)*f*d^2*c^3/(c^2*x^2+1)^(1/2)/g^2*x^2+1/
4*b*(d*(c^2*x^2+1))^(1/2)*f^3*d^2*c^5/(c^2*x^2+1)^(1/2)/g^4*x^2+34/15*b*(d*
(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)/g*arcsinh(c*x)*x^2*c^2-5/4*b*(d*(c^2*x^2
+1))^(1/2)/(c^2*x^2+1)^(1/2)*f^3*arcsinh(c*x)^2*d^2*c^3/g^4-15/16*b*(d*(c^2
*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*f*arcsinh(c*x)^2*d^2*c/g^2+1/5*b*(d*(c^2*x
^2+1))^(1/2)*d^2/(c^2*x^2+1)/g*arcsinh(c*x)*x^6*c^6+14/15*b*(d*(c^2*x^2+1))
^(1/2)*d^2/(c^2*x^2+1)/g*arcsinh(c*x)*x^4*c^4+b*d^2*(d*(c^2*x^2+1))^(1/2)*(
c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2)/g^2*arcsinh(c*x)*ln((-(c*x+(c^2*x^2+1)
^(1/2))*g-c*f+(c^2*f^2+g^2)^(1/2))/(-c*f+(c^2*f^2+g^2)^(1/2)))-b*d^2*(d*(c^
2*x^2+1))^(1/2)*(c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2)/g^2*arcsinh(c*x)*ln((
(c*x+(c^2*x^2+1)^(1/2))*g+c*f+(c^2*f^2+g^2)^(1/2))/(c*f+(c^2*f^2+g^2)^(1/2)
))-1/2*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*f^5*arcsinh(c*x)^2*d^2*c^5
/g^6-b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)^(1/2)/g^5*x*c^5*f^4+7/3*b*(d*(
c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)/g^3*arcsinh(c*x)*c^2*f^2+1/3*a/g^3*d*((x+
f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(3/2)*c^2*f^2+a/g^5*d
^2*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2)*c^4*f^4+
1/3*a/g*d*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(3/2)+a
/g*d^2*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2)-1/4*
a/g^2*c^2*d*f*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(3/
2)*x-7/8*a/g^2*c^2*d^2*f*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^
2)/g^2)^(1/2)*x-15/8*a/g^2*c^2*d^3*f*ln((-c^2*d*f/g+c^2*d*(x+f/g))/(c^2*d)^
(1/2)+((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2))/(c^2
*d)^(1/2)-1/2*a/g^4*d^2*c^4*f^3*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2
*f^2+g^2)/g^2)^(1/2)*x-5/2*a/g^4*d^3*c^4*f^3*ln((-c^2*d*f/g+c^2*d*(x+f/g))/
(c^2*d)^(1/2)+((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/
2))/(c^2*d)^(1/2)-a/g^6*d^3*c^6*f^5*ln((-c^2*d*f/g+c^2*d*(x+f/g))/(c^2*d)^(
1/2)+((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2))/(c^2*
d)^(1/2)-a/g^7*d^3/(d*(c^2*f^2+g^2)/g^2)^(1/2)*ln((2*d*(c^2*f^2+g^2)/g^2-2*
c^2*d*f/g*(x+f/g)+2*(d*(c^2*f^2+g^2)/g^2)^(1/2)*((x+f/g)^2*c^2*d-2*c^2*d*f/
g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2))/(x+f/g))*c^6*f^6-3*a/g^5*d^3/(d*(c^2*
f^2+g^2)/g^2)^(1/2)*ln((2*d*(c^2*f^2+g^2)/g^2-2*c^2*d*f/g*(x+f/g)+2*(d*(c^2
*f^2+g^2)/g^2)^(1/2)*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g
^2)^(1/2))/(x+f/g))*c^4*f^4-3*a/g^3*d^3/(d*(c^2*f^2+g^2)/g^2)^(1/2)*ln((2*d
*(c^2*f^2+g^2)/g^2-2*c^2*d*f/g*(x+f/g)+2*(d*(c^2*f^2+g^2)/g^2)^(1/2)*((x+f/
g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2))/(x+f/g))*c^2*f^2
-23/15*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)^(1/2)/g*c*x+33/128*b*(d*(c^2
*x^2+1))^(1/2)*f*d^2*c/(c^2*x^2+1)^(1/2)/g^2+1/8*b*(d*(c^2*x^2+1))^(1/2)*f^
3*d^2*c^3/(c^2*x^2+1)^(1/2)/g^4-1/25*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1
)^(1/2)/g*x^5*c^5-11/45*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)^(1/2)/g*c^3
*x^3+b*d^2*(d*(c^2*x^2+1))^(1/2)*(c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2)/g^2*
dilog((-(c*x+(c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2+g^2)^(1/2))/(-c*f+(c^2*f^2+g
^2)^(1/2)))-b*d^2*(d*(c^2*x^2+1))^(1/2)*(c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/
2)/g^2*dilog(((c*x+(c^2*x^2+1)^(1/2))*g+c*f+(c^2*f^2+g^2)^(1/2))/(c*f+(c^2*
f^2+g^2)^(1/2)))-a/g*d^3/(d*(c^2*f^2+g^2)/g^2)^(1/2)*ln((2*d*(c^2*f^2+g^2)/
g^2-2*c^2*d*f/g*(x+f/g)+2*(d*(c^2*f^2+g^2)/g^2)^(1/2)*((x+f/g)^2*c^2*d-2*c^
```

```
2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2))/(x+f/g))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/(g*x+f),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}\,(c\,x))\,\left(d\,c^2\,x^2+d\right)^{5/2}}{f+g\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2))/(f + g*x),x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2))/(f + g*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\left(c^2x^2+1\right)\right)^{\frac{5}{2}}(a+b\,\mathrm{asinh}\,(cx))}{f+gx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))/(g*x+f),x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(5/2)*(a + b*asinh(c*x))/(f + g*x), x)
```

3.47 $\int \frac{(f+gx)^3\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=430

$$\frac{f^3\sqrt{c^2x^2+1}\,\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{c^2dx^2+d}}+\frac{3f^2g\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c^2\sqrt{c^2dx^2+d}}+\frac{3fg^2x\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{2c^2\sqrt{c^2dx^2+d}}+\frac{g^3x^2}{}$$

```
[Out] 3*f^2*g*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c^2/(c^2*d*x^2+d)^(1/2)-2/3*g^3*(c^2
*x^2+1)*(a+b*arcsinh(c*x))/c^4/(c^2*d*x^2+d)^(1/2)+3/2*f*g^2*x*(c^2*x^2+1)*
(a+b*arcsinh(c*x))/c^2/(c^2*d*x^2+d)^(1/2)+1/3*g^3*x^2*(c^2*x^2+1)*(a+b*arc
sinh(c*x))/c^2/(c^2*d*x^2+d)^(1/2)-3*b*f^2*g*x*(c^2*x^2+1)^(1/2)/c/(c^2*d*x
^2+d)^(1/2)+2/3*b*g^3*x*(c^2*x^2+1)^(1/2)/c^3/(c^2*d*x^2+d)^(1/2)-3/4*b*f*g
^2*x^2*(c^2*x^2+1)^(1/2)/c/(c^2*d*x^2+d)^(1/2)-1/9*b*g^3*x^3*(c^2*x^2+1)^(1
/2)/c/(c^2*d*x^2+d)^(1/2)+1/2*f^3*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/b/
c/(c^2*d*x^2+d)^(1/2)-3/4*f*g^2*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/b/c^
3/(c^2*d*x^2+d)^(1/2)
```

Rubi [A] time = 0.58, antiderivative size = 430, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {5835, 5821, 5675, 5717, 8, 5758, 30}

$$\frac{3f^2g\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c^2\sqrt{c^2dx^2+d}}+\frac{f^3\sqrt{c^2x^2+1}\,\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{c^2dx^2+d}}-\frac{3fg^2\sqrt{c^2x^2+1}\,\left(a+b\sinh^{-1}(cx)\right)^2}{4bc^3\sqrt{c^2dx^2+d}}+\frac{3fg^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^3*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (-3*b*f^2*g*x*Sqrt[1 + c^2*x^2])/(c*Sqrt[d + c^2*d*x^2]) + (2*b*g^3*x*Sqrt[
1 + c^2*x^2])/(3*c^3*Sqrt[d + c^2*d*x^2]) - (3*b*f*g^2*x^2*Sqrt[1 + c^2*x^2
])/(4*c*Sqrt[d + c^2*d*x^2]) - (b*g^3*x^3*Sqrt[1 + c^2*x^2])/(9*c*Sqrt[d +
c^2*d*x^2]) + (3*f^2*g*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c^2*Sqrt[d + c^
2*d*x^2]) - (2*g^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(3*c^4*Sqrt[d + c^2*
d*x^2]) + (3*f*g^2*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(2*c^2*Sqrt[d + c^
2*d*x^2]) + (g^3*x^2*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(3*c^2*Sqrt[d + c^
2*d*x^2]) + (f^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c*Sqrt[d +
c^2*d*x^2]) - (3*f*g^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(4*b*c^3*S
qrt[d + c^2*d*x^2])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
```

```
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5835

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(d^IntPart[p]*(d + e*x^2)^FracPa
rt[p])/(1 + c^2*x^2)^FracPart[p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*Ar
cSinh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d
] && IntegerQ[m] && IntegerQ[p - 1/2] &&  !GtQ[d, 0]
```

Rubi steps

$$\int \frac{(f+gx)^3\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}\,dx = \frac{\sqrt{1+c^2x^2}\int \frac{(f+gx)^3\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+c^2dx^2}}$$

$$= \frac{\sqrt{1+c^2x^2}\int \left(\frac{f^3\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}} + \frac{3f^2gx\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}} + \frac{3fg^2x^2\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\right.}{\sqrt{d+c^2dx^2}}$$

$$= \frac{\left(f^3\sqrt{1+c^2x^2}\right)\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+c^2dx^2}} + \frac{\left(3f^2g\sqrt{1+c^2x^2}\right)\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+c^2dx^2}}$$

$$= \frac{3f^2g\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c^2\sqrt{d+c^2dx^2}} + \frac{3fg^2x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{2c^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{3bf^2gx\sqrt{1+c^2x^2}}{c\sqrt{d+c^2dx^2}} - \frac{3bfg^2x^2\sqrt{1+c^2x^2}}{4c\sqrt{d+c^2dx^2}} - \frac{bg^3x^3\sqrt{1+c^2x^2}}{9c\sqrt{d+c^2dx^2}} + \frac{3f^2g\left(1\right.}{}$$

$$= -\frac{3bf^2gx\sqrt{1+c^2x^2}}{c\sqrt{d+c^2dx^2}} + \frac{2bg^3x\sqrt{1+c^2x^2}}{3c^3\sqrt{d+c^2dx^2}} - \frac{3bfg^2x^2\sqrt{1+c^2x^2}}{4c\sqrt{d+c^2dx^2}} - \frac{bg^3x^3\sqrt{1}}{9c\sqrt{d+}}$$

Mathematica [A] time = 0.95, size = 304, normalized size = 0.71

$$\frac{4\sqrt{d}g\left(3a\left(c^2x^2+1\right)\left(c^2\left(18f^2+9fgx+2g^2x^2\right)-4g^2\right)-2bcx\sqrt{c^2x^2+1}\left(c^2\left(27f^2+g^2x^2\right)-6g^2\right)\right)+36acf\sqrt{c^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^3*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (4*Sqrt[d]*g*(-2*b*c*x*Sqrt[1 + c^2*x^2]*(-6*g^2 + c^2*(27*f^2 + g^2*x^2))
+ 3*a*(1 + c^2*x^2)*(-4*g^2 + c^2*(18*f^2 + 9*f*g*x + 2*g^2*x^2))) + 12*b*S
qrt[d]*g*(1 + c^2*x^2)*(-4*g^2 + c^2*(18*f^2 + 9*f*g*x + 2*g^2*x^2))*ArcSin
h[c*x] + 18*b*c*Sqrt[d]*f*(2*c^2*f^2 - 3*g^2)*Sqrt[1 + c^2*x^2]*ArcSinh[c*x
]^2 - 27*b*c*Sqrt[d]*f*g^2*Sqrt[1 + c^2*x^2]*Cosh[2*ArcSinh[c*x]] + 36*a*c*
f*(2*c^2*f^2 - 3*g^2)*Sqrt[d + c^2*d*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*
d*x^2]])/(72*c^4*Sqrt[d]*Sqrt[d + c^2*d*x^2])
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{ag^3x^3+3\,afg^2x^2+3\,af^2gx+af^3+\left(bg^3x^3+3\,bfg^2x^2+3\,bf^2gx+bf^3\right)\text{arsinh}\left(cx\right)}{\sqrt{c^2dx^2+d}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="fr
icas")
```

```
[Out] integral((a*g^3*x^3 + 3*a*f*g^2*x^2 + 3*a*f^2*g*x + a*f^3 + (b*g^3*x^3 + 3*
b*f*g^2*x^2 + 3*b*f^2*g*x + b*f^3)*arcsinh(c*x))/sqrt(c^2*d*x^2 + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(gx+f\right)^3\left(b\,\text{arsinh}\left(cx\right)+a\right)}{\sqrt{c^2dx^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate((g*x + f)^3*(b*arcsinh(c*x) + a)/sqrt(c^2*d*x^2 + d), x)
```

maple [A] time = 0.81, size = 751, normalized size = 1.75

$$\frac{a\,g^3x^2\sqrt{c^2d\,x^2+d}}{3c^2d}-\frac{2a\,g^3\sqrt{c^2d\,x^2+d}}{3d\,c^4}+\frac{3af\,g^2x\sqrt{c^2d\,x^2+d}}{2c^2d}-\frac{3af\,g^2\ln\left(\frac{x\,c^2d}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{2c^2\sqrt{c^2d}}+\frac{3a\,f^2g\sqrt{c^2d\,x^2+}}{c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] 1/3*a*g^3*x^2/c^2/d*(c^2*d*x^2+d)^(1/2)-2/3*a*g^3/d/c^4*(c^2*d*x^2+d)^(1/2)
+3/2*a*f*g^2*x/c^2/d*(c^2*d*x^2+d)^(1/2)-3/2*a*f*g^2/c^2*ln(x*c^2*d/(c^2*d)
^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+3*a*f^2*g/c^2/d*(c^2*d*x^2+d)^(1/
2)+a*f^3*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)-3/4*b*
(d*(c^2*x^2+1))^(1/2)*f*arcsinh(c*x)^2/(c^2*x^2+1)^(1/2)/c^3/d*g^2+1/3*b*(d
*(c^2*x^2+1))^(1/2)*g^3/d/(c^2*x^2+1)*arcsinh(c*x)*x^4-1/9*b*(d*(c^2*x^2+1)
)^(1/2)*g^3/c/d/(c^2*x^2+1)^(1/2)*x^3-1/3*b*(d*(c^2*x^2+1))^(1/2)*g^3/c^2/d
/(c^2*x^2+1)*arcsinh(c*x)*x^2+2/3*b*(d*(c^2*x^2+1))^(1/2)*g^3/c^3/d/(c^2*x^
```

```
2+1)^(1/2)*x-3/8*b*(d*(c^2*x^2+1))^(1/2)*f*g^2/c^3/d/(c^2*x^2+1)^(1/2)+3*b*
(d*(c^2*x^2+1))^(1/2)*g/c^2/d/(c^2*x^2+1)*arcsinh(c*x)*f^2+3/2*b*(d*(c^2*x^
2+1))^(1/2)*f*g^2/c^2/d/(c^2*x^2+1)*arcsinh(c*x)*x+3*b*(d*(c^2*x^2+1))^(1/2
)*g/d/(c^2*x^2+1)*arcsinh(c*x)*x^2*f^2-3*b*(d*(c^2*x^2+1))^(1/2)*g/c/d/(c^2
*x^2+1)^(1/2)*x*f^2+3/2*b*(d*(c^2*x^2+1))^(1/2)*f*g^2/d/(c^2*x^2+1)*arcsinh
(c*x)*x^3-3/4*b*(d*(c^2*x^2+1))^(1/2)*f*g^2/c/d/(c^2*x^2+1)^(1/2)*x^2+1/2*b
*(d*(c^2*x^2+1))^(1/2)*f^3*arcsinh(c*x)^2/(c^2*x^2+1)^(1/2)/c/d-2/3*b*(d*(c
^2*x^2+1))^(1/2)*g^3/c^4/d/(c^2*x^2+1)*arcsinh(c*x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="ma
xima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(f+g\,x\right)^3\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}{\sqrt{d\,c^2\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^3*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2),x)
```

```
[Out] int(((f + g*x)^3*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\operatorname{asinh}\left(cx\right)\right)\left(f+gx\right)^{3}}{\sqrt{d\left(c^{2}x^{2}+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))*(f + g*x)**3/sqrt(d*(c**2*x**2 + 1)), x)
```

3.48 $\int \frac{(f+gx)^2\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=258

$$\frac{f^2\sqrt{c^2x^2+1}\,\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{c^2dx^2+d}}+\frac{2fg\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c^2\sqrt{c^2dx^2+d}}+\frac{g^2x\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{2c^2\sqrt{c^2dx^2+d}}-\frac{g^2\sqrt{c^2x^2}}{}$$

```
[Out] 2*f*g*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c^2/(c^2*d*x^2+d)^(1/2)+1/2*g^2*x*(c^2
*x^2+1)*(a+b*arcsinh(c*x))/c^2/(c^2*d*x^2+d)^(1/2)-2*b*f*g*x*(c^2*x^2+1)^(1
/2)/c/(c^2*d*x^2+d)^(1/2)-1/4*b*g^2*x^2*(c^2*x^2+1)^(1/2)/c/(c^2*d*x^2+d)^(
1/2)+1/2*f^2*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/b/c/(c^2*d*x^2+d)^(1/2)
-1/4*g^2*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/b/c^3/(c^2*d*x^2+d)^(1/2)
```

Rubi [A] time = 0.43, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {5835, 5821, 5675, 5717, 8, 5758, 30}

$$\frac{f^2\sqrt{c^2x^2+1}\,\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{c^2dx^2+d}}+\frac{2fg\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c^2\sqrt{c^2dx^2+d}}-\frac{g^2\sqrt{c^2x^2+1}\,\left(a+b\sinh^{-1}(cx)\right)^2}{4bc^3\sqrt{c^2dx^2+d}}+\frac{g^2x\left(c^2x\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^2*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (-2*b*f*g*x*Sqrt[1 + c^2*x^2])/(c*Sqrt[d + c^2*d*x^2]) - (b*g^2*x^2*Sqrt[1
+ c^2*x^2])/(4*c*Sqrt[d + c^2*d*x^2]) + (2*f*g*(1 + c^2*x^2)*(a + b*ArcSinh
[c*x]))/(c^2*Sqrt[d + c^2*d*x^2]) + (g^2*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x
]))/(2*c^2*Sqrt[d + c^2*d*x^2]) + (f^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x
])^2)/(2*b*c*Sqrt[d + c^2*d*x^2]) - (g^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c
*x])^2)/(4*b*c^3*Sqrt[d + c^2*d*x^2])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
```

```
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5835

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(d^IntPart[p]*(d + e*x^2)^FracPa
rt[p])/(1 + c^2*x^2)^FracPart[p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*Ar
cSinh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d
] && IntegerQ[m] && IntegerQ[p - 1/2] &&  !GtQ[d, 0]
```

Rubi steps

$$\int \frac{(f+gx)^2\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}\,dx = \frac{\sqrt{1+c^2x^2}\int \frac{(f+gx)^2\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+c^2dx^2}}$$

$$= \frac{\sqrt{1+c^2x^2}\int \left(\frac{f^2\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}+\frac{2fgx\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}+\frac{g^2x^2\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\right)dx}{\sqrt{d+c^2dx^2}}$$

$$= \frac{\left(f^2\sqrt{1+c^2x^2}\right)\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+c^2dx^2}}+\frac{\left(2fg\sqrt{1+c^2x^2}\right)\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+c^2dx^2}}$$

$$= \frac{2fg\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c^2\sqrt{d+c^2dx^2}}+\frac{g^2x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{2c^2\sqrt{d+c^2dx^2}}+$$

$$= -\frac{2bfgx\sqrt{1+c^2x^2}}{c\sqrt{d+c^2dx^2}}-\frac{bg^2x^2\sqrt{1+c^2x^2}}{4c\sqrt{d+c^2dx^2}}+\frac{2fg\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx\right.}{c^2\sqrt{d+c^2dx^2}}$$

Mathematica [A] time = 0.61, size = 233, normalized size = 0.90

$$\frac{4c\sqrt{d}\,g\left(a\left(c^2x^2+1\right)(4f+gx)-4bcfx\sqrt{c^2x^2+1}\right)+4a\sqrt{c^2dx^2+d}\,\left(2c^2f^2-g^2\right)\log\left(\sqrt{d}\,\sqrt{c^2dx^2+d}+cdx\right)}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^2*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (4*c*Sqrt[d]*g*(-4*b*c*f*x*Sqrt[1 + c^2*x^2] + a*(4*f + g*x)*(1 + c^2*x^2))
 + 4*b*c*Sqrt[d]*g*(4*f + g*x)*(1 + c^2*x^2)*ArcSinh[c*x] + 2*b*Sqrt[d]*(2*
c^2*f^2 - g^2)*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2 - b*Sqrt[d]*g^2*Sqrt[1 + c^
2*x^2]*Cosh[2*ArcSinh[c*x]] + 4*a*(2*c^2*f^2 - g^2)*Sqrt[d + c^2*d*x^2]*Log
[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]])/(8*c^3*Sqrt[d]*Sqrt[d + c^2*d*x^2])
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{ag^2x^2 + 2\,afgx + af^2 + \left(bg^2x^2 + 2\,bfgx + bf^2\right)\operatorname{arsinh}(cx)}{\sqrt{c^2dx^2 + d}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="fr
icas")
```

```
[Out] integral((a*g^2*x^2 + 2*a*f*g*x + a*f^2 + (b*g^2*x^2 + 2*b*f*g*x + b*f^2)*a
rcsinh(c*x))/sqrt(c^2*d*x^2 + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(gx + f)^2 (b\operatorname{arsinh}(cx) + a)}{\sqrt{c^2dx^2 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate((g*x + f)^2*(b*arcsinh(c*x) + a)/sqrt(c^2*d*x^2 + d), x)
```

maple [B] time = 0.61, size = 486, normalized size = 1.88

$$\frac{a\,g^2x\sqrt{c^2d\,x^2 + d}}{2c^2d} - \frac{a\,g^2\ln\left(\frac{x\,c^2d}{\sqrt{c^2d}} + \sqrt{c^2d\,x^2 + d}\right)}{2c^2\sqrt{c^2d}} + \frac{2afg\sqrt{c^2d\,x^2 + d}}{c^2d} + \frac{a\,f^2\ln\left(\frac{x\,c^2d}{\sqrt{c^2d}} + \sqrt{c^2d\,x^2 + d}\right)}{\sqrt{c^2d}} + \frac{2b\sqrt{d\left(c^2x^2\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] 1/2*a*g^2*x/c^2/d*(c^2*d*x^2+d)^(1/2)-1/2*a*g^2/c^2*ln(x*c^2*d/(c^2*d)^(1/2
)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+2*a*f*g/c^2/d*(c^2*d*x^2+d)^(1/2)+a*f^
2*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+2*b*(d*(c^2*x
^2+1))^(1/2)*f*g/d/(c^2*x^2+1)*arcsinh(c*x)*x^2-2*b*(d*(c^2*x^2+1))^(1/2)*f
*g/c/d/(c^2*x^2+1)^(1/2)*x+1/2*b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)^2/(c^2*
x^2+1)^(1/2)/c/d*f^2-1/4*b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)^2/(c^2*x^2+1)
^(1/2)/c^3/d*g^2-1/8*b*(d*(c^2*x^2+1))^(1/2)*g^2/c^3/d/(c^2*x^2+1)^(1/2)+1/
2*b*(d*(c^2*x^2+1))^(1/2)*g^2/d/(c^2*x^2+1)*arcsinh(c*x)*x^3-1/4*b*(d*(c^2*
x^2+1))^(1/2)*g^2/c/d/(c^2*x^2+1)^(1/2)*x^2+1/2*b*(d*(c^2*x^2+1))^(1/2)*g^2
/c^2/d/(c^2*x^2+1)*arcsinh(c*x)*x+2*b*(d*(c^2*x^2+1))^(1/2)*f*g/c^2/d/(c^2*
x^2+1)*arcsinh(c*x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="ma
xima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f + g\,x)^2\,(a + b\,\mathrm{asinh}(c\,x))}{\sqrt{d\,c^2\,x^2 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2),x)
[Out] int(((f + g*x)^2*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{asinh}(cx)) \left(f + gx\right)^2}{\sqrt{d \left(c^2 x^2 + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(1/2),x)
[Out] Integral((a + b*asinh(c*x))*(f + g*x)**2/sqrt(d*(c**2*x**2 + 1)), x)
```

3.49 $\int \frac{(f+gx)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=120

$$\frac{f\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{c^2dx^2+d}}+\frac{g\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c^2\sqrt{c^2dx^2+d}}-\frac{bgx\sqrt{c^2x^2+1}}{c\sqrt{c^2dx^2+d}}$$

```
[Out] g*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c^2/(c^2*d*x^2+d)^(1/2)-b*g*x*(c^2*x^2+1)^
(1/2)/c/(c^2*d*x^2+d)^(1/2)+1/2*f*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/b/
c/(c^2*d*x^2+d)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {5835, 5821, 5675, 5717, 8}

$$\frac{f\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{c^2dx^2+d}}+\frac{g\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c^2\sqrt{c^2dx^2+d}}-\frac{bgx\sqrt{c^2x^2+1}}{c\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] -((b*g*x*Sqrt[1 + c^2*x^2])/(c*Sqrt[d + c^2*d*x^2])) + (g*(1 + c^2*x^2)*(a
+ b*ArcSinh[c*x]))/(c^2*Sqrt[d + c^2*d*x^2]) + (f*Sqrt[1 + c^2*x^2]*(a + b*
ArcSinh[c*x])^2)/(2*b*c*Sqrt[d + c^2*d*x^2])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5835

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(d^IntPart[p]*(d + e*x^2)^FracPa
```

```
rt[p])/(1 + c^2*x^2)^FracPart[p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*Ar
cSinh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d
] && IntegerQ[m] && IntegerQ[p - 1/2] && !GtQ[d, 0]
```

Rubi steps

$$\int \frac{(f+gx)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}\,dx = \frac{\sqrt{1+c^2x^2}\int \frac{(f+gx)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+c^2dx^2}}$$
$$= \frac{\sqrt{1+c^2x^2}\int \left(\frac{f\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}+\frac{gx\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\right)dx}{\sqrt{d+c^2dx^2}}$$
$$= \frac{\left(f\sqrt{1+c^2x^2}\right)\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+c^2dx^2}} + \frac{\left(g\sqrt{1+c^2x^2}\right)\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+c^2dx^2}}$$
$$= \frac{g\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c^2\sqrt{d+c^2dx^2}} + \frac{f\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{d+c^2dx^2}} - \frac{(bg\sqrt{}}{c}$$
$$= -\frac{bgx\sqrt{1+c^2x^2}}{c\sqrt{d+c^2dx^2}} + \frac{g\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c^2\sqrt{d+c^2dx^2}} + \frac{f\sqrt{1+c^2x^2}\left(a+b\,\mathrm{si}\right.}{2bc\sqrt{d+c^2d}}$$

Mathematica [A] time = 0.26, size = 158, normalized size = 1.32

$$\frac{2\sqrt{d}\,g\left(ac^2x^2+a-bcx\sqrt{c^2x^2+1}\right)+2acf\sqrt{c^2dx^2+d}\,\log\left(\sqrt{d}\sqrt{c^2dx^2+d}+cdx\right)+bc\sqrt{d}\,f\sqrt{c^2x^2+1}\,\sinh^{-1}}{2c^2\sqrt{d}\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (2*Sqrt[d]*g*(a + a*c^2*x^2 - b*c*x*Sqrt[1 + c^2*x^2]) + 2*b*Sqrt[d]*g*(1 +
 c^2*x^2)*ArcSinh[c*x] + b*c*Sqrt[d]*f*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2 + 2
*a*c*f*Sqrt[d + c^2*d*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]])/(2*c^2
*Sqrt[d]*Sqrt[d + c^2*d*x^2])
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{agx+af+\left(bgx+bf\right)\mathrm{arsinh}\left(cx\right)}{\sqrt{c^2dx^2+d}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="fric
as")
```

```
[Out] integral((a*g*x + a*f + (b*g*x + b*f)*arcsinh(c*x))/sqrt(c^2*d*x^2 + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(gx+f\right)\left(b\,\mathrm{arsinh}\left(cx\right)+a\right)}{\sqrt{c^2dx^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate((g*x + f)*(b*arcsinh(c*x) + a)/sqrt(c^2*d*x^2 + d), x)
```

maple [A] time = 0.41, size = 209, normalized size = 1.74

$$\frac{ag\sqrt{c^2d\,x^2+d}}{c^2d}+\frac{af\ln\left(\frac{x\,c^2d}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{\sqrt{c^2d}}+\frac{b\sqrt{d\left(c^2x^2+1\right)}\,f\operatorname{arcsinh}(cx)^2}{2\sqrt{c^2x^2+1}\,cd}+\frac{b\sqrt{d\left(c^2x^2+1\right)}\,g\operatorname{arcsinh}(cx)\,x^2}{d\left(c^2x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] a*g/c^2/d*(c^2*d*x^2+d)^(1/2)+a*f*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1
/2))/(c^2*d)^(1/2)+1/2*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c/d*f*arcs
inh(c*x)^2+b*(d*(c^2*x^2+1))^(1/2)*g/d/(c^2*x^2+1)*arcsinh(c*x)*x^2-b*(d*(c
^2*x^2+1))^(1/2)*g/c/d/(c^2*x^2+1)^(1/2)*x+b*(d*(c^2*x^2+1))^(1/2)*g/c^2/d/
(c^2*x^2+1)*arcsinh(c*x)
```

maxima [A] time = 0.42, size = 87, normalized size = 0.72

$$\frac{bf\operatorname{arsinh}(cx)^2}{2\,c\sqrt{d}}-\frac{bgx}{c\sqrt{d}}+\frac{af\operatorname{arsinh}(cx)}{c\sqrt{d}}+\frac{\sqrt{c^2dx^2+d}\,bg\operatorname{arsinh}(cx)}{c^2d}+\frac{\sqrt{c^2dx^2+d}\,ag}{c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] 1/2*b*f*arcsinh(c*x)^2/(c*sqrt(d)) - b*g*x/(c*sqrt(d)) + a*f*arcsinh(c*x)/(
c*sqrt(d)) + sqrt(c^2*d*x^2 + d)*b*g*arcsinh(c*x)/(c^2*d) + sqrt(c^2*d*x^2
+ d)*a*g/(c^2*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(f+g\,x\right)\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}{\sqrt{d\,c^2\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2),x)
```

```
[Out] int(((f + g*x)*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(a+b\operatorname{asinh}(cx))\left(f+gx\right)}{\sqrt{d\left(c^2x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))*(f + g*x)/sqrt(d*(c**2*x**2 + 1)), x)
```

3.50 $\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=47

$$\frac{\sqrt{c^2x^2+1}\,\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{c^2dx^2+d}}$$

```
[Out] 1/2*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/b/c/(c^2*d*x^2+d)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5677, 5675}

$$\frac{\sqrt{c^2x^2+1}\,\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c*Sqrt[d + c^2*d*x^2])
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5677

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcSinh[c*x])
^n/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*
d] && !GtQ[d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{d+c^2dx^2}}\,dx &= \frac{\sqrt{1+c^2x^2}\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+c^2dx^2}}\\ &= \frac{\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{d+c^2dx^2}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 48, normalized size = 1.02

$$\frac{\sqrt{c^2x^2+1}\,\sinh^{-1}(cx)\left(2a+b\sinh^{-1}(cx)\right)}{2c\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*(2*a + b*ArcSinh[c*x]))/(2*c*Sqrt[d + c^2*d
*x^2])
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b \operatorname{arsinh}(cx) + a}{\sqrt{c^2dx^2 + d}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/sqrt(c^2*d*x^2 + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arsinh}(cx) + a}{\sqrt{c^2dx^2 + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/sqrt(c^2*d*x^2 + d), x)
```

maple [A] time = 0.01, size = 77, normalized size = 1.64

$$\frac{a \ln\left(\frac{x c^2 d}{\sqrt{c^2 d}} + \sqrt{c^2 d\, x^2 + d}\right)}{\sqrt{c^2 d}} + \frac{b \sqrt{d\left(c^2 x^2 + 1\right)}\, \operatorname{arcsinh}(cx)^2}{2\sqrt{c^2 x^2 + 1}\; cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] a*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+1/2*b*(d*(c^2
*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c/d*arcsinh(c*x)^2
```

maxima [A] time = 0.45, size = 28, normalized size = 0.60

$$\frac{b \operatorname{arsinh}(cx)^2}{2\, c\sqrt{d}} + \frac{a \operatorname{arsinh}(cx)}{c\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*b*arcsinh(c*x)^2/(c*sqrt(d)) + a*arcsinh(c*x)/(c*sqrt(d))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a + b \operatorname{asinh}(c\,x)}{\sqrt{d\, c^2\, x^2 + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(d + c^2*d*x^2)^(1/2),x)
```

```
[Out] int((a + b*asinh(c*x))/(d + c^2*d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{asinh}(cx)}{\sqrt{d\left(c^2x^2 + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))/sqrt(d*(c**2*x**2 + 1)), x)
```

3.51 $\int \frac{a+b\sinh^{-1}(cx)}{(f+gx)\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=325

$$\frac{\sqrt{c^2x^2+1}\,\left(a+b\sinh^{-1}(cx)\right)\log\left(\frac{ge^{\sinh^{-1}(cx)}}{cf-\sqrt{c^2f^2+g^2}}+1\right)}{\sqrt{c^2dx^2+d}\,\sqrt{c^2f^2+g^2}}-\frac{\sqrt{c^2x^2+1}\,\left(a+b\sinh^{-1}(cx)\right)\log\left(\frac{ge^{\sinh^{-1}(cx)}}{\sqrt{c^2f^2+g^2}+cf}+1\right)}{\sqrt{c^2dx^2+d}\,\sqrt{c^2f^2+g^2}}+\frac{b\sqrt{\ldots}}{\ldots}$$

```
[Out] (a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2+g^2)^(1/2))
)*(c^2*x^2+1)^(1/2)/(c^2*f^2+g^2)^(1/2)/(c^2*d*x^2+d)^(1/2)-(a+b*arcsinh(c*
x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2+g^2)^(1/2)))*(c^2*x^2+1)^(
1/2)/(c^2*f^2+g^2)^(1/2)/(c^2*d*x^2+d)^(1/2)+b*polylog(2,-(c*x+(c^2*x^2+1)^
(1/2))*g/(c*f-(c^2*f^2+g^2)^(1/2)))*(c^2*x^2+1)^(1/2)/(c^2*f^2+g^2)^(1/2)/(
c^2*d*x^2+d)^(1/2)-b*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2+g^2
)^(1/2)))*(c^2*x^2+1)^(1/2)/(c^2*f^2+g^2)^(1/2)/(c^2*d*x^2+d)^(1/2)
```

Rubi [A] time = 0.55, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {5835, 5831, 3322, 2264, 2190, 2279, 2391}

$$\frac{b\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,-\frac{ge^{\sinh^{-1}(cx)}}{cf-\sqrt{c^2f^2+g^2}}\right)}{\sqrt{c^2dx^2+d}\,\sqrt{c^2f^2+g^2}}-\frac{b\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,-\frac{ge^{\sinh^{-1}(cx)}}{\sqrt{c^2f^2+g^2}+cf}\right)}{\sqrt{c^2dx^2+d}\,\sqrt{c^2f^2+g^2}}+\frac{\sqrt{c^2x^2+1}\,\left(a+b\sinh^{-1}(cx)\ldots\right.}{\sqrt{c^2dx^2+d}\,\sqrt{\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/((f + g*x)*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Log[1 + (E^ArcSinh[c*x]*g)/(c*f - S
qrt[c^2*f^2 + g^2])])/(Sqrt[c^2*f^2 + g^2]*Sqrt[d + c^2*d*x^2]) - (Sqrt[1 +
 c^2*x^2]*(a + b*ArcSinh[c*x])*Log[1 + (E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f
^2 + g^2])])/(Sqrt[c^2*f^2 + g^2]*Sqrt[d + c^2*d*x^2]) + (b*Sqrt[1 + c^2*x^
2]*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2]))])/(Sqrt[c^2
*f^2 + g^2]*Sqrt[d + c^2*d*x^2]) - (b*Sqrt[1 + c^2*x^2]*PolyLog[2, -((E^Arc
Sinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))])/(Sqrt[c^2*f^2 + g^2]*Sqrt[d + c
^2*d*x^2])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5831

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 5835

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(d^IntPart[p]*(d + e*x^2)^FracPa
rt[p])/(1 + c^2*x^2)^FracPart[p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*Ar
cSinh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d
] && IntegerQ[m] && IntegerQ[p - 1/2] &&  !GtQ[d, 0]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{(f+gx)\sqrt{d+c^2dx^2}}\,dx = \frac{\sqrt{1+c^2x^2}\int \frac{a+b\sinh^{-1}(cx)}{(f+gx)\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+c^2dx^2}}$$

$$= \frac{\sqrt{1+c^2x^2}\,\text{Subst}\left(\int \frac{a+bx}{cf+g\sinh(x)}\,dx, x, \sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}$$

$$= \frac{\left(2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \frac{e^x(a+bx)}{2ce^xf-g+e^{2x}g}\,dx, x, \sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}$$

$$= \frac{\left(2g\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \frac{e^x(a+bx)}{2cf+2e^xg-2\sqrt{c^2f^2+g^2}}\,dx, x, \sinh^{-1}(cx)\right)}{\sqrt{c^2f^2+g^2}\,\sqrt{d+c^2dx^2}} - \frac{\left(2g\sqrt{1+c^2x^2}\right)\text{Sub}}{}$$

$$= \frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\log\left(1+\frac{e^{\sinh^{-1}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{\sqrt{c^2f^2+g^2}\,\sqrt{d+c^2dx^2}} - \frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2f^2+g^2}\,\sqrt{d}}$$

$$= \frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\log\left(1+\frac{e^{\sinh^{-1}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{\sqrt{c^2f^2+g^2}\,\sqrt{d+c^2dx^2}} - \frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2f^2+g^2}\,\sqrt{d}}$$

$$= \frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\log\left(1+\frac{e^{\sinh^{-1}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{\sqrt{c^2f^2+g^2}\,\sqrt{d+c^2dx^2}} - \frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2f^2+g^2}\,\sqrt{d}}$$

Mathematica [A] time = 0.63, size = 256, normalized size = 0.79

$$-\frac{-\frac{a\log\left(\sqrt{d}\sqrt{c^2dx^2+d}\sqrt{c^2f^2+g^2}+d\left(g-c^2fx\right)\right)}{\sqrt{d}}+\frac{a\log(f+gx)}{\sqrt{d}}+\frac{b\sqrt{c^2x^2+1}\left(\text{Li}_2\left(\frac{e^{\sinh^{-1}(cx)}g}{\sqrt{c^2f^2+g^2}-cf}\right)-\text{Li}_2\left(-\frac{e^{\sinh^{-1}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)+\sinh^{-1}(cx)\left(\log\left(\frac{ge^{\sinh^{-1}(cx)}}{cf-\sqrt{c^2f^2+g^2}}\right.\right.\right.}{\sqrt{c^2dx^2+d}}}{\sqrt{c^2f^2+g^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])/((f + g*x)*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] ((a*Log[f + g*x])/Sqrt[d] - (a*Log[d*(g - c^2*f*x) + Sqrt[d]*Sqrt[c^2*f^2 +
 g^2]*Sqrt[d + c^2*d*x^2]])/Sqrt[d] + (b*Sqrt[1 + c^2*x^2]*(ArcSinh[c*x]*(L
og[1 + (E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2])] - Log[1 + (E^ArcSinh
[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2])]) + PolyLog[2, (E^ArcSinh[c*x]*g)/(-(c
*f) + Sqrt[c^2*f^2 + g^2])] - PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c
^2*f^2 + g^2]))]))/Sqrt[d + c^2*d*x^2])/Sqrt[c^2*f^2 + g^2]
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2dx^2+d}\,(b\,\text{arsinh}\,(cx)+a)}{c^2dgx^3+c^2dfx^2+dgx+df},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(g*x+f)/(c^2*d*x^2+d)^(1/2),x, algorithm="fric
as")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^2*d*g*x^3 + c^2*d*f*x^
2 + d*g*x + d*f), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{arsinh}\,(cx)+a}{\sqrt{c^2dx^2+d}\,(gx+f)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(g*x+f)/(c^2*d*x^2+d)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(sqrt(c^2*d*x^2 + d)*(g*x + f)), x)
```

maple [B] time = 0.27, size = 678, normalized size = 2.09

$$-\frac{a\ln\left(\frac{\frac{2d\left(c^2f^2+g^2\right)}{g^2}-\frac{2c^2df\left(x+\frac{f}{g}\right)}{g}+2\sqrt{\frac{d\left(c^2f^2+g^2\right)}{g^2}}\sqrt{\left(x+\frac{f}{g}\right)^2c^2d-\frac{2c^2df\left(x+\frac{f}{g}\right)}{g}+\frac{d\left(c^2f^2+g^2\right)}{g^2}}}{x+\frac{f}{g}}\right)}{g\sqrt{\frac{d\left(c^2f^2+g^2\right)}{g^2}}}+\frac{b\sqrt{d\left(c^2x^2+1\right)}\,\sqrt{c^2f^2+g^2}\,\sqrt{c^2x^2+1}}{d\left(c^4f^2x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(g*x+f)/(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -a/g/(d*(c^2*f^2+g^2)/g^2)^(1/2)*ln((2*d*(c^2*f^2+g^2)/g^2-2*c^2*d*f/g*(x+f
/g)+2*(d*(c^2*f^2+g^2)/g^2)^(1/2)*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c
^2*f^2+g^2)/g^2)^(1/2))/(x+f/g))+b*(d*(c^2*x^2+1))^(1/2)*(c^2*f^2+g^2)^(1/2
)*(c^2*x^2+1)^(1/2)/d/(c^4*f^2*x^2+c^2*g^2*x^2+c^2*f^2+g^2)*arcsinh(c*x)*ln
```

```
((-(c*x+(c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2+g^2)^(1/2))/(-c*f+(c^2*f^2+g^2)^(
1/2)))-b*(d*(c^2*x^2+1))^(1/2)*(c^2*f^2+g^2)^(1/2)*(c^2*x^2+1)^(1/2)/d/(c^4
*f^2*x^2+c^2*g^2*x^2+c^2*f^2+g^2)*arcsinh(c*x)*ln(((c*x+(c^2*x^2+1)^(1/2))*
g+c*f+(c^2*f^2+g^2)^(1/2))/(c*f+(c^2*f^2+g^2)^(1/2)))+b*(d*(c^2*x^2+1))^(1/
2)*(c^2*f^2+g^2)^(1/2)*(c^2*x^2+1)^(1/2)/d/(c^4*f^2*x^2+c^2*g^2*x^2+c^2*f^2
+g^2)*dilog((-(c*x+(c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2+g^2)^(1/2))/(-c*f+(c^2
*f^2+g^2)^(1/2)))-b*(d*(c^2*x^2+1))^(1/2)*(c^2*f^2+g^2)^(1/2)*(c^2*x^2+1)^(
1/2)/d/(c^4*f^2*x^2+c^2*g^2*x^2+c^2*f^2+g^2)*dilog(((c*x+(c^2*x^2+1)^(1/2))
*g+c*f+(c^2*f^2+g^2)^(1/2))/(c*f+(c^2*f^2+g^2)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arsinh}(cx) + a}{\sqrt{c^2dx^2 + d}\left(gx + f\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(g*x+f)/(c^2*d*x^2+d)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(sqrt(c^2*d*x^2 + d)*(g*x + f)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \operatorname{asinh}(c\, x)}{\left(f + g\, x\right)\, \sqrt{d\, c^2\, x^2 + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/((f + g*x)*(d + c^2*d*x^2)^(1/2)),x)
```

```
[Out] int((a + b*asinh(c*x))/((f + g*x)*(d + c^2*d*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{asinh}(cx)}{\sqrt{d\left(c^2x^2 + 1\right)}\left(f + gx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(g*x+f)/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))/(sqrt(d*(c**2*x**2 + 1))*(f + g*x)), x)
```

3.52 $\int \frac{a+b\sinh^{-1}(cx)}{(f+gx)^2\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=444

$$-\frac{g\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2dx^2+d}\left(c^2f^2+g^2\right)(f+gx)}+\frac{c^2f\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)\log\left(\frac{ge^{\sinh^{-1}(cx)}}{cf-\sqrt{c^2f^2+g^2}}+1\right)}{\sqrt{c^2dx^2+d}\left(c^2f^2+g^2\right)^{3/2}}-\frac{c^2f\sqrt{c^2x^2+1}\,(a}{\sqrt{c}}$$

```
[Out] -g*(c^2*x^2+1)*(a+b*arcsinh(c*x))/(c^2*f^2+g^2)/(g*x+f)/(c^2*d*x^2+d)^(1/2)
+b*c*ln(g*x+f)*(c^2*x^2+1)^(1/2)/(c^2*f^2+g^2)/(c^2*d*x^2+d)^(1/2)+c^2*f*(a
+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2+g^2)^(1/2)))*
(c^2*x^2+1)^(1/2)/(c^2*f^2+g^2)^(3/2)/(c^2*d*x^2+d)^(1/2)-c^2*f*(a+b*arcsin
h(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2+g^2)^(1/2)))*(c^2*x^2+
1)^(1/2)/(c^2*f^2+g^2)^(3/2)/(c^2*d*x^2+d)^(1/2)+b*c^2*f*polylog(2,-(c*x+(c
^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2+g^2)^(1/2)))*(c^2*x^2+1)^(1/2)/(c^2*f^2+g^
2)^(3/2)/(c^2*d*x^2+d)^(1/2)-b*c^2*f*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))*g/(
c*f+(c^2*f^2+g^2)^(1/2)))*(c^2*x^2+1)^(1/2)/(c^2*f^2+g^2)^(3/2)/(c^2*d*x^2+
d)^(1/2)
```

Rubi [A] time = 0.66, antiderivative size = 444, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5835, 5831, 3324, 3322, 2264, 2190, 2279, 2391, 2668, 31}

$$\frac{bc^2f\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,-\frac{ge^{\sinh^{-1}(cx)}}{cf-\sqrt{c^2f^2+g^2}}\right)}{\sqrt{c^2dx^2+d}\left(c^2f^2+g^2\right)^{3/2}}-\frac{bc^2f\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,-\frac{ge^{\sinh^{-1}(cx)}}{\sqrt{c^2f^2+g^2}+cf}\right)}{\sqrt{c^2dx^2+d}\left(c^2f^2+g^2\right)^{3/2}}-\frac{g\left(c^2x^2+1\right)\left(a+b\,\text{si}\right.}{\sqrt{c^2dx^2+d}\left(c^2f^2+g\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/((f + g*x)^2*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] -((g*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/((c^2*f^2 + g^2)*(f + g*x)*Sqrt[d
+ c^2*d*x^2])) + (c^2*f*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Log[1 + (E^A
rcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2])])/((c^2*f^2 + g^2)^(3/2)*Sqrt[d
+ c^2*d*x^2]) - (c^2*f*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Log[1 + (E^Ar
cSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2])])/((c^2*f^2 + g^2)^(3/2)*Sqrt[d +
 c^2*d*x^2]) + (b*c*Sqrt[1 + c^2*x^2]*Log[f + g*x])/((c^2*f^2 + g^2)*Sqrt[d
 + c^2*d*x^2]) + (b*c^2*f*Sqrt[1 + c^2*x^2]*PolyLog[2, -((E^ArcSinh[c*x]*g)
/(c*f - Sqrt[c^2*f^2 + g^2]))])/((c^2*f^2 + g^2)^(3/2)*Sqrt[d + c^2*d*x^2])
 - (b*c^2*f*Sqrt[1 + c^2*x^2]*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c
^2*f^2 + g^2]))])/((c^2*f^2 + g^2)^(3/2)*Sqrt[d + c^2*d*x^2])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3324

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[(b*(c + d*x)^m*Cos[e + f*x])/(f*(a^2 - b^2)*(a + b*Sin[e +
f*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[(b*d*m)/(f*(a^2 - b^2)), Int[((c + d*x)^(m - 1)*Cos[e + f*x])/(a
 + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 5831

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 5835

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(d^IntPart[p]*(d + e*x^2)^FracPa
rt[p])/(1 + c^2*x^2)^FracPart[p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*Ar
cSinh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d
] && IntegerQ[m] && IntegerQ[p - 1/2] &&  !GtQ[d, 0]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{(f+gx)^2\sqrt{d+c^2dx^2}}\,dx = \frac{\sqrt{1+c^2x^2}\int \frac{a+b\sinh^{-1}(cx)}{(f+gx)^2\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+c^2dx^2}}$$

$$= \frac{\left(c\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \frac{a+bx}{(cf+g\sinh(x))^2}\,dx, x, \sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}$$

$$= -\frac{g\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{\left(c^2f^2+g^2\right)(f+gx)\sqrt{d+c^2dx^2}} + \frac{\left(c^2f\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \frac{a+bx}{cf+g\sinh(x)}\,dx, x,\right.}{\left(c^2f^2+g^2\right)\sqrt{d+c^2dx^2}}$$

$$= -\frac{g\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{\left(c^2f^2+g^2\right)(f+gx)\sqrt{d+c^2dx^2}} + \frac{\left(bc\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \frac{1}{cf+x}\,dx, x, cgx\right)}{\left(c^2f^2+g^2\right)\sqrt{d+c^2dx^2}} +$$

$$= -\frac{g\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{\left(c^2f^2+g^2\right)(f+gx)\sqrt{d+c^2dx^2}} + \frac{bc\sqrt{1+c^2x^2}\log(f+gx)}{\left(c^2f^2+g^2\right)\sqrt{d+c^2dx^2}} + \frac{\left(2c^2fg\sqrt{1+c\right.}{}$$

$$= -\frac{g\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{\left(c^2f^2+g^2\right)(f+gx)\sqrt{d+c^2dx^2}} + \frac{c^2f\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\log\left(1+\right.}{\left(c^2f^2+g^2\right)^{3/2}\sqrt{d+c^2dx^2}}$$

$$= -\frac{g\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{\left(c^2f^2+g^2\right)(f+gx)\sqrt{d+c^2dx^2}} + \frac{c^2f\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\log\left(1+\right.}{\left(c^2f^2+g^2\right)^{3/2}\sqrt{d+c^2dx^2}}$$

$$= -\frac{g\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{\left(c^2f^2+g^2\right)(f+gx)\sqrt{d+c^2dx^2}} + \frac{c^2f\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\log\left(1+\right.}{\left(c^2f^2+g^2\right)^{3/2}\sqrt{d+c^2dx^2}}$$

Mathematica [A] time = 2.17, size = 448, normalized size = 1.01

$$-ag\left(c^2dx^2+d\right)\sqrt{c^2f^2+g^2} - ac^2\sqrt{d}\,f\sqrt{c^2dx^2+d}\,(f+gx)\log\left(\sqrt{d}\sqrt{c^2dx^2+d}\sqrt{c^2f^2+g^2}+d\left(g-c^2fx\right)\right)+$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])/((f + g*x)^2*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] (-(a*g*Sqrt[c^2*f^2 + g^2]*(d + c^2*d*x^2)) + a*c^2*Sqrt[d]*f*(f + g*x)*Sqr
t[d + c^2*d*x^2]*Log[f + g*x] - a*c^2*Sqrt[d]*f*(f + g*x)*Sqrt[d + c^2*d*x^
2]*Log[d*(g - c^2*f*x) + Sqrt[d]*Sqrt[c^2*f^2 + g^2]*Sqrt[d + c^2*d*x^2]] -
 b*d*Sqrt[1 + c^2*x^2]*(g*Sqrt[c^2*f^2 + g^2]*Sqrt[1 + c^2*x^2]*ArcSinh[c*x
] - c^2*f*(f + g*x)*ArcSinh[c*x]*Log[1 + (E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2
*f^2 + g^2])] + c^2*f*(f + g*x)*ArcSinh[c*x]*Log[1 + (E^ArcSinh[c*x]*g)/(c*
f + Sqrt[c^2*f^2 + g^2])] - c*Sqrt[c^2*f^2 + g^2]*(f + g*x)*Log[c*(f + g*x)
] - c^2*f*(f + g*x)*PolyLog[2, (E^ArcSinh[c*x]*g)/(-(c*f) + Sqrt[c^2*f^2 +
g^2])] + c^2*f*(f + g*x)*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^
2 + g^2]))]))/(d*(c^2*f^2 + g^2)^(3/2)*(f + g*x)*Sqrt[d + c^2*d*x^2])
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c^2dx^2+d}\,(b\,\text{arsinh}\,(cx)+a)}{c^2dg^2x^4+2\,c^2dfgx^3+2\,dfgx+df^2+\left(c^2df^2+dg^2\right)x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(g*x+f)^2/(c^2*d*x^2+d)^(1/2),x, algorithm="fr
icas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^2*d*g^2*x^4 + 2*c^2*d*
f*g*x^3 + 2*d*f*g*x + d*f^2 + (c^2*d*f^2 + d*g^2)*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arsinh}(cx) + a}{\sqrt{c^2dx^2 + d}\,(gx + f)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(g*x+f)^2/(c^2*d*x^2+d)^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(sqrt(c^2*d*x^2 + d)*(g*x + f)^2), x)
```

maple [B] time = 0.69, size = 1770, normalized size = 3.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(g*x+f)^2/(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -a/d/(c^2*f^2+g^2)/(x+f/g)*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+
g^2)/g^2)^(1/2)-a/g*c^2*f/(c^2*f^2+g^2)/(d*(c^2*f^2+g^2)/g^2)^(1/2)*ln((2*d
*(c^2*f^2+g^2)/g^2-2*c^2*d*f/g*(x+f/g)+2*(d*(c^2*f^2+g^2)/g^2)^(1/2)*((x+f/
g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2))/(x+f/g))+b*(d*(c
^2*x^2+1))^(1/2)*arcsinh(c*x)/d/(c^2*x^2+1)/(c^2*f^2+g^2)/(g*x+f)*x^3*c^4*f
-b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/d/(c^2*f^2+g^2)/(g*x+f)*x*c^2*f-b*(d*
(c^2*x^2+1))^(1/2)*arcsinh(c*x)/d/(c^2*x^2+1)/(c^2*f^2+g^2)/(g*x+f)*x^2*c^2
*g+b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/d/(c^2*x^2+1)^(1/2)/(c^2*f^2+g^2)/(
g*x+f)*x*c*g+b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/d/(c^2*x^2+1)/(c^2*f^2+g^
2)/(g*x+f)*x*c^2*f+b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/d/(c^2*x^2+1)^(1/2)
/(c^2*f^2+g^2)/(g*x+f)*c*f-b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/d/(c^2*x^2+
1)/(c^2*f^2+g^2)/(g*x+f)*g+b*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+1)^(1/2)/d/(c^6
*f^4*x^2+2*c^4*f^2*g^2*x^2+c^4*f^4+c^2*g^4*x^2+2*c^2*f^2*g^2+g^4)*c^2*f*arc
sinh(c*x)*(c^2*f^2+g^2)^(1/2)*ln((-(c*x+(c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2+g
^2)^(1/2))/(-c*f+(c^2*f^2+g^2)^(1/2)))-b*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+1)^
(1/2)/d/(c^6*f^4*x^2+2*c^4*f^2*g^2*x^2+c^4*f^4+c^2*g^4*x^2+2*c^2*f^2*g^2+g^
4)*c^2*f*arcsinh(c*x)*(c^2*f^2+g^2)^(1/2)*ln(((c*x+(c^2*x^2+1)^(1/2))*g+c*f
+(c^2*f^2+g^2)^(1/2))/(c*f+(c^2*f^2+g^2)^(1/2)))-2*b*(d*(c^2*x^2+1))^(1/2)*
(c^2*x^2+1)^(1/2)/d/(c^6*f^4*x^2+2*c^4*f^2*g^2*x^2+c^4*f^4+c^2*g^4*x^2+2*c^
2*f^2*g^2+g^4)*c^3*ln(c*x+(c^2*x^2+1)^(1/2))*f^2+b*(d*(c^2*x^2+1))^(1/2)*(c
^2*x^2+1)^(1/2)/d/(c^6*f^4*x^2+2*c^4*f^2*g^2*x^2+c^4*f^4+c^2*g^4*x^2+2*c^2*
f^2*g^2+g^4)*c^3*ln((c*x+(c^2*x^2+1)^(1/2))^2*g+2*c*f*(c*x+(c^2*x^2+1)^(1/2
))-g)*f^2+b*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+1)^(1/2)/d/(c^6*f^4*x^2+2*c^4*f^
2*g^2*x^2+c^4*f^4+c^2*g^4*x^2+2*c^2*f^2*g^2+g^4)*c^2*f*(c^2*f^2+g^2)^(1/2)*
dilog((-(c*x+(c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2+g^2)^(1/2))/(-c*f+(c^2*f^2+g
^2)^(1/2)))-b*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+1)^(1/2)/d/(c^6*f^4*x^2+2*c^4*
f^2*g^2*x^2+c^4*f^4+c^2*g^4*x^2+2*c^2*f^2*g^2+g^4)*c^2*f*(c^2*f^2+g^2)^(1/2
)*dilog(((c*x+(c^2*x^2+1)^(1/2))*g+c*f+(c^2*f^2+g^2)^(1/2))/(c*f+(c^2*f^2+g
^2)^(1/2)))-2*b*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+1)^(1/2)/d/(c^6*f^4*x^2+2*c^
4*f^2*g^2*x^2+c^4*f^4+c^2*g^4*x^2+2*c^2*f^2*g^2+g^4)*c*ln(c*x+(c^2*x^2+1)^(
1/2))*g^2+b*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+1)^(1/2)/d/(c^6*f^4*x^2+2*c^4*f^
2*g^2*x^2+c^4*f^4+c^2*g^4*x^2+2*c^2*f^2*g^2+g^4)*c*ln((c*x+(c^2*x^2+1)^(1/2
))^2*g+2*c*f*(c*x+(c^2*x^2+1)^(1/2))-g)*g^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arsinh}(cx) + a}{\sqrt{c^2 dx^2 + d}\,(gx + f)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(g*x+f)^2/(c^2*d*x^2+d)^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(sqrt(c^2*d*x^2 + d)*(g*x + f)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \operatorname{asinh}(c\,x)}{(f + g\,x)^2\,\sqrt{d\,c^2\,x^2 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/((f + g*x)^2*(d + c^2*d*x^2)^(1/2)),x)
```

```
[Out] int((a + b*asinh(c*x))/((f + g*x)^2*(d + c^2*d*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{asinh}(cx)}{\sqrt{d\left(c^2x^2 + 1\right)}\,(f + gx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(g*x+f)**2/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))/(sqrt(d*(c**2*x**2 + 1))*(f + g*x)**2), x)
```

3.53 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^n \log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}} \, dx$

Optimal. Leaf size=37

$$\text{Int}\left(\frac{\left(a+b\sinh^{-1}(cx)\right)^n \log\left(h(f+gx)^m\right)}{\sqrt{c^2x^2+1}},x\right)$$

```
[Out] Unintegrable((a+b*arcsinh(c*x))^n*ln(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x)
```

Rubi [A] time = 0.19, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^n \log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[((a + b*ArcSinh[c*x])^n*Log[h*(f + g*x)^m])/Sqrt[1 + c^2*x^2],x]
```

```
[Out] Defer[Int][((a + b*ArcSinh[c*x])^n*Log[h*(f + g*x)^m])/Sqrt[1 + c^2*x^2], x
]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^n \log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}} \, dx = \int \frac{\left(a+b\sinh^{-1}(cx)\right)^n \log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}} \, dx$$

Mathematica [A] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^n \log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((a + b*ArcSinh[c*x])^n*Log[h*(f + g*x)^m])/Sqrt[1 + c^2*x^2],x]
```

```
[Out] Integrate[((a + b*ArcSinh[c*x])^n*Log[h*(f + g*x)^m])/Sqrt[1 + c^2*x^2], x]
```

fricas [A] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(b \operatorname{arsinh}(cx)+a)^n \log\left(\left(gx+f\right)^m h\right)}{\sqrt{c^2x^2+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^n*log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algori
thm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)^n*log((g*x + f)^m*h)/sqrt(c^2*x^2 + 1), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^n*log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algori
thm="giac")
```

```
[Out] Timed out
```

maple [A] time = 2.38, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(cx))^{n}\ln\left(h\left(gx+f\right)^{m}\right)}{\sqrt{c^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^n*ln(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^n*ln(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\mathrm{arsinh}\,(cx)+a)^{n}\log\left(\left(gx+f\right)^{m}h\right)}{\sqrt{c^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^n*log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^n*log((g*x + f)^m*h)/sqrt(c^2*x^2 + 1), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\ln\left(h\left(f+g\,x\right)^{m}\right)\,(a+b\,\mathrm{asinh}\,(c\,x))^{n}}{\sqrt{c^{2}\,x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(h*(f + g*x)^m)*(a + b*asinh(c*x))^n)/(c^2*x^2 + 1)^(1/2),x)
```

```
[Out] int((log(h*(f + g*x)^m)*(a + b*asinh(c*x))^n)/(c^2*x^2 + 1)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**n*ln(h*(g*x+f)**m)/(c**2*x**2+1)**(1/2),x)
```

```
[Out] Timed out
```

3.54 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx$

Optimal. Leaf size=438

$$\frac{m\left(a+b\sinh^{-1}(cx)\right)^4}{12b^2c}-\frac{m\left(a+b\sinh^{-1}(cx)\right)^2\text{Li}_2\left(-\frac{e^{\sinh^{-1}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{c}-\frac{m\left(a+b\sinh^{-1}(cx)\right)^2\text{Li}_2\left(-\frac{e^{\sinh^{-1}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{c}+\frac{2b}{}$$

```
[Out] 1/12*m*(a+b*arcsinh(c*x))^4/b^2/c+1/3*(a+b*arcsinh(c*x))^3*ln(h*(g*x+f)^m)/
b/c-1/3*m*(a+b*arcsinh(c*x))^3*ln(1+(c*x+(c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2
+g^2)^(1/2)))/b/c-1/3*m*(a+b*arcsinh(c*x))^3*ln(1+(c*x+(c^2*x^2+1)^(1/2))*g
/(c*f+(c^2*f^2+g^2)^(1/2)))/b/c-m*(a+b*arcsinh(c*x))^2*polylog(2,-(c*x+(c^2
*x^2+1)^(1/2))*g/(c*f-(c^2*f^2+g^2)^(1/2)))/c-m*(a+b*arcsinh(c*x))^2*polylo
g(2,-(c*x+(c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2+g^2)^(1/2)))/c+2*b*m*(a+b*arcs
inh(c*x))*polylog(3,-(c*x+(c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2+g^2)^(1/2)))/c
+2*b*m*(a+b*arcsinh(c*x))*polylog(3,-(c*x+(c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^
2+g^2)^(1/2)))/c-2*b^2*m*polylog(4,-(c*x+(c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2
+g^2)^(1/2)))/c-2*b^2*m*polylog(4,-(c*x+(c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2+
g^2)^(1/2)))/c
```

Rubi [A] time = 0.73, antiderivative size = 438, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.265, Rules used = {5675, 5838, 5799, 5561, 2190, 2531, 6609, 2282, 6589}

$$-\frac{m\left(a+b\sinh^{-1}(cx)\right)^2\text{PolyLog}\left(2,-\frac{ge^{\sinh^{-1}(cx)}}{cf-\sqrt{c^2f^2+g^2}}\right)}{c}-\frac{m\left(a+b\sinh^{-1}(cx)\right)^2\text{PolyLog}\left(2,-\frac{ge^{\sinh^{-1}(cx)}}{\sqrt{c^2f^2+g^2}+cf}\right)}{c}+\frac{2bm\left(a+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcSinh[c*x])^2*Log[h*(f + g*x)^m])/Sqrt[1 + c^2*x^2],x]
```

```
[Out] (m*(a + b*ArcSinh[c*x])^4)/(12*b^2*c) - (m*(a + b*ArcSinh[c*x])^3*Log[1 + (
E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2])])/(3*b*c) - (m*(a + b*ArcSinh
[c*x])^3*Log[1 + (E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2])])/(3*b*c) +
 ((a + b*ArcSinh[c*x])^3*Log[h*(f + g*x)^m])/(3*b*c) - (m*(a + b*ArcSinh[c*
x])^2*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2]))])/c - (m
*(a + b*ArcSinh[c*x])^2*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2
 + g^2]))])/c + (2*b*m*(a + b*ArcSinh[c*x])*PolyLog[3, -((E^ArcSinh[c*x]*g)
/(c*f - Sqrt[c^2*f^2 + g^2]))])/c + (2*b*m*(a + b*ArcSinh[c*x])*PolyLog[3,
-((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))])/c - (2*b^2*m*PolyLog[4,
 -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2]))])/c - (2*b^2*m*PolyLog[4
, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))])/c
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Cosh[x])/(c*d + e*Sinh[x]), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5838

```
Int[(Log[(h_.)*((f_.) + (g_.)*(x_))^(m_.)]*((a_.) + ArcSinh[(c_.)*(x_)]*(b_
.))^(n_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[(Log[h*(f + g*x)^m]
*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*Sqrt[d]*(n + 1)), x] - Dist[(g*m)/(b*c*
Sqrt[d]*(n + 1)), Int[(a + b*ArcSinh[c*x])^(n + 1)/(f + g*x), x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, m}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && IGtQ[n,
 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx = \frac{\left(a+b\sinh^{-1}(cx)\right)^3\log\left(h(f+gx)^m\right)}{3bc} - \frac{(gm)\int\frac{\left(a+b\sinh^{-1}(cx)\right)^3}{f+gx}dx}{3bc}$$

$$= \frac{\left(a+b\sinh^{-1}(cx)\right)^3\log\left(h(f+gx)^m\right)}{3bc} - \frac{(gm)\,\text{Subst}\left(\int\frac{(a+bx)^3\cosh(x)}{cf+g\sinh(x)}\right.}{3bc}$$

$$= \frac{m\left(a+b\sinh^{-1}(cx)\right)^4}{12b^2c} + \frac{\left(a+b\sinh^{-1}(cx)\right)^3\log\left(h(f+gx)^m\right)}{3bc} - \cdots$$

$$= \frac{m\left(a+b\sinh^{-1}(cx)\right)^4}{12b^2c} - \frac{m\left(a+b\sinh^{-1}(cx)\right)^3\log\left(1+\frac{e^{\sinh^{-1}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right.}{3bc}$$

$$= \frac{m\left(a+b\sinh^{-1}(cx)\right)^4}{12b^2c} - \frac{m\left(a+b\sinh^{-1}(cx)\right)^3\log\left(1+\frac{e^{\sinh^{-1}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right.}{3bc}$$

$$= \frac{m\left(a+b\sinh^{-1}(cx)\right)^4}{12b^2c} - \frac{m\left(a+b\sinh^{-1}(cx)\right)^3\log\left(1+\frac{e^{\sinh^{-1}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right.}{3bc}$$

$$= \frac{m\left(a+b\sinh^{-1}(cx)\right)^4}{12b^2c} - \frac{m\left(a+b\sinh^{-1}(cx)\right)^3\log\left(1+\frac{e^{\sinh^{-1}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right.}{3bc}$$

$$= \frac{m\left(a+b\sinh^{-1}(cx)\right)^4}{12b^2c} - \frac{m\left(a+b\sinh^{-1}(cx)\right)^3\log\left(1+\frac{e^{\sinh^{-1}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right.}{3bc}$$

Mathematica [A] time = 0.23, size = 397, normalized size = 0.91

$$-\frac{3bm\left(-2b\left(a+b\sinh^{-1}(cx)\right)\text{Li}_3\left(\frac{e^{\sinh^{-1}(cx)}g}{\sqrt{c^2f^2+g^2}-cf}\right)+\left(a+b\sinh^{-1}(cx)\right)^2\text{Li}_2\left(\frac{e^{\sinh^{-1}(cx)}g}{\sqrt{c^2f^2+g^2}-cf}\right)+2b^2\text{Li}_4\left(\frac{e^{\sinh^{-1}(cx)}g}{\sqrt{c^2f^2+g^2}-cf}\right)\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*ArcSinh[c*x])^2*Log[h*(f + g*x)^m])/Sqrt[1 + c^2*x^2],x]
```

```
[Out] -1/3*(-1/4*(m*(a + b*ArcSinh[c*x])^4)/b + m*(a + b*ArcSinh[c*x])^3*Log[1 +
(E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2])] + m*(a + b*ArcSinh[c*x])^3*
Log[1 + (E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2])] - (a + b*ArcSinh[c*
x])^3*Log[h*(f + g*x)^m] + 3*b*m*((a + b*ArcSinh[c*x])^2*PolyLog[2, (E^ArcS
inh[c*x]*g)/(-(c*f) + Sqrt[c^2*f^2 + g^2])] - 2*b*(a + b*ArcSinh[c*x])*Poly
Log[3, (E^ArcSinh[c*x]*g)/(-(c*f) + Sqrt[c^2*f^2 + g^2])] + 2*b^2*PolyLog[4
, (E^ArcSinh[c*x]*g)/(-(c*f) + Sqrt[c^2*f^2 + g^2])]) + 3*b*m*((a + b*ArcSi
nh[c*x])^2*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))] -
2*b*(a + b*ArcSinh[c*x])*PolyLog[3, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^
2 + g^2]))] + 2*b^2*PolyLog[4, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g
^2]))]))/(b*c)
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2\,\text{arsinh}\,(cx)^2+2\,ab\,\text{arsinh}\,(cx)+a^2\right)\log\left(\left(gx+f\right)^m h\right)}{\sqrt{c^2x^2+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algori
thm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)*log((g*x + f)^m*h)
/sqrt(c^2*x^2 + 1), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algori
thm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 7.14Unable to divide, perha
ps due to rounding error%%%{720,[0,4,1,1,1,1,4,0]%%%}+%%%{-1260,[0,4,1,1,1,
1,3,1]%%%}+%%%{360,[0,4,1,1,1,1,2,2]%%%}+%%%{180,[0,4,1,1,1,1,1,3]%%%}+%%%{
-144,[0,4,1,1,1,0,4,0]%%%}+%%%{387,[0,4,1,1,1,0,3,1]%%%}+%%%{-282,[0,4,1,1,
1,0,2,2]%%%}+%%%{-51,[0,4,1,1,1,0,1,3]%%%}+%%%{180,[0,4,1,1,1,0,0,4]%%%}+%%
%{480,[0,2,3,1,1,1,2,0]%%%}+%%%{420,[0,2,3,1,1,1,1,1]%%%}+%%%{-256,[0,2,3,1
,1,0,2,0]%%%}+%%%{1,[0,2,3,1,1,0,1,1]%%%}+%%%{540,[0,2,3,1,1,0,0,2]%%%}+%%%
{-480,[0,0,5,1,1,1,0,0]%%%}+%%%{256,[0,0,5,1,1,0,0,0]%%%} / %%%{1800,[0,1,0
,0,0,0,0,0]%%%} Error: Bad Argument Value
```

maple [F] time = 2.64, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b \operatorname{arcsinh}\left(c x\right)\right)^{2} \ln\left(h \left(g x+f\right)^{m}\right)}{\sqrt{c^{2} x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2*ln(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2*ln(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \operatorname{arsinh}\left(c x\right)+a\right)^{2} \log\left(\left(g x+f\right)^{m} h\right)}{\sqrt{c^{2} x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*log((g*x + f)^m*h)/sqrt(c^2*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(h \left(f+g\,x\right)^{m}\right)\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^{2}}{\sqrt{c^{2}\,x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(h*(f + g*x)^m)*(a + b*asinh(c*x))^2)/(c^2*x^2 + 1)^(1/2),x)
```

```
[Out] int((log(h*(f + g*x)^m)*(a + b*asinh(c*x))^2)/(c^2*x^2 + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{asinh}(cx))^2 \log\left(h\left(f + gx\right)^m\right)}{\sqrt{c^2x^2 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2*ln(h*(g*x+f)**m)/(c**2*x**2+1)**(1/2),x)
[Out] Integral((a + b*asinh(c*x))**2*log(h*(f + g*x)**m)/sqrt(c**2*x**2 + 1), x)
```

3.55 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx$

Optimal. Leaf size=332

$$\frac{m\left(a+b\sinh^{-1}(cx)\right)^3}{6b^2c}-\frac{m\left(a+b\sinh^{-1}(cx)\right)\text{Li}_2\left(-\frac{e^{\sinh^{-1}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{c}-\frac{m\left(a+b\sinh^{-1}(cx)\right)\text{Li}_2\left(-\frac{e^{\sinh^{-1}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{c}-\frac{m}{}$$

```
[Out] 1/6*m*(a+b*arcsinh(c*x))^3/b^2/c+1/2*(a+b*arcsinh(c*x))^2*ln(h*(g*x+f)^m)/b
/c-1/2*m*(a+b*arcsinh(c*x))^2*ln(1+(c*x+(c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2+
g^2)^(1/2)))/b/c-1/2*m*(a+b*arcsinh(c*x))^2*ln(1+(c*x+(c^2*x^2+1)^(1/2))*g/
(c*f+(c^2*f^2+g^2)^(1/2)))/b/c-m*(a+b*arcsinh(c*x))*polylog(2,-(c*x+(c^2*x^
2+1)^(1/2))*g/(c*f-(c^2*f^2+g^2)^(1/2)))/c-m*(a+b*arcsinh(c*x))*polylog(2,-
(c*x+(c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2+g^2)^(1/2)))/c+b*m*polylog(3,-(c*x+
(c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2+g^2)^(1/2)))/c+b*m*polylog(3,-(c*x+(c^2*
x^2+1)^(1/2))*g/(c*f+(c^2*f^2+g^2)^(1/2)))/c
```

Rubi [A] time = 0.56, antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5675, 5838, 5799, 5561, 2190, 2531, 2282, 6589}

$$-\frac{m\left(a+b\sinh^{-1}(cx)\right)\text{PolyLog}\left(2,-\frac{ge^{\sinh^{-1}(cx)}}{cf-\sqrt{c^2f^2+g^2}}\right)}{c}-\frac{m\left(a+b\sinh^{-1}(cx)\right)\text{PolyLog}\left(2,-\frac{ge^{\sinh^{-1}(cx)}}{\sqrt{c^2f^2+g^2}+cf}\right)}{c}+\frac{bm\text{Poly}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcSinh[c*x])*Log[h*(f + g*x)^m])/Sqrt[1 + c^2*x^2],x]
```

```
[Out] (m*(a + b*ArcSinh[c*x])^3)/(6*b^2*c) - (m*(a + b*ArcSinh[c*x])^2*Log[1 + (E
^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2])])/(2*b*c) - (m*(a + b*ArcSinh[
c*x])^2*Log[1 + (E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2])])/(2*b*c) +
((a + b*ArcSinh[c*x])^2*Log[h*(f + g*x)^m])/(2*b*c) - (m*(a + b*ArcSinh[c*x
])*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2]))])/c - (m*(a
 + b*ArcSinh[c*x])*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^
2]))])/c + (b*m*PolyLog[3, -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2])
)])/c + (b*m*PolyLog[3, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))])
/c
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
```

```
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Cosh[x])/(c*d + e*Sinh[x]), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5838

```
Int[(Log[(h_.)*((f_.) + (g_.)*(x_))^(m_.)]*((a_.) + ArcSinh[(c_.)*(x_)]*(b_
.))^(n_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[(Log[h*(f + g*x)^m]
*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*Sqrt[d]*(n + 1)), x] - Dist[(g*m)/(b*c*
Sqrt[d]*(n + 1)), Int[(a + b*ArcSinh[c*x])^(n + 1)/(f + g*x), x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, m}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && IGtQ[n,
 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx = \frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(h(f+gx)^m\right)}{2bc} - \frac{(gm)\int\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{f+gx}\,dx}{2bc}$$

$$= \frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(h(f+gx)^m\right)}{2bc} - \frac{(gm)\,\mathrm{Subst}\left(\int\frac{(a+bx)^2\cosh}{cf+g\sinh(}\right.}{2bc}$$

$$= \frac{m\left(a+b\sinh^{-1}(cx)\right)^3}{6b^2c} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(h(f+gx)^m\right)}{2bc} -$$

$$= \frac{m\left(a+b\sinh^{-1}(cx)\right)^3}{6b^2c} - \frac{m\left(a+b\sinh^{-1}(cx)\right)^2\log\left(1+\frac{e^{\sinh^{-1}(cx)}}{cf-\sqrt{c^2f^2+}}\right.}{2bc}$$

$$= \frac{m\left(a+b\sinh^{-1}(cx)\right)^3}{6b^2c} - \frac{m\left(a+b\sinh^{-1}(cx)\right)^2\log\left(1+\frac{e^{\sinh^{-1}(cx)}}{cf-\sqrt{c^2f^2+}}\right.}{2bc}$$

$$= \frac{m\left(a+b\sinh^{-1}(cx)\right)^3}{6b^2c} - \frac{m\left(a+b\sinh^{-1}(cx)\right)^2\log\left(1+\frac{e^{\sinh^{-1}(cx)}}{cf-\sqrt{c^2f^2+}}\right.}{2bc}$$

$$= \frac{m\left(a+b\sinh^{-1}(cx)\right)^3}{6b^2c} - \frac{m\left(a+b\sinh^{-1}(cx)\right)^2\log\left(1+\frac{e^{\sinh^{-1}(cx)}}{cf-\sqrt{c^2f^2+}}\right.}{2bc}$$

Mathematica [A] time = 0.23, size = 304, normalized size = 0.92

$$2bm\left(b\mathrm{Li}_3\left(\frac{e^{\sinh^{-1}(cx)}g}{\sqrt{c^2f^2+g^2}-cf}\right)-\left(a+b\sinh^{-1}(cx)\right)\mathrm{Li}_2\left(\frac{e^{\sinh^{-1}(cx)}g}{\sqrt{c^2f^2+g^2}-cf}\right)\right)+2bm\left(b\mathrm{Li}_3\left(-\frac{e^{\sinh^{-1}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)-\left(a+b\sinh^{-1}(\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*ArcSinh[c*x])*Log[h*(f + g*x)^m])/Sqrt[1 + c^2*x^2],x]
```

```
[Out] ((m*(a + b*ArcSinh[c*x])^3)/(3*b) - m*(a + b*ArcSinh[c*x])^2*Log[1 + (E^Arc
Sinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2])] - m*(a + b*ArcSinh[c*x])^2*Log[1
+ (E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2])] + (a + b*ArcSinh[c*x])^2*
Log[h*(f + g*x)^m] + 2*b*m*(-((a + b*ArcSinh[c*x])*PolyLog[2, (E^ArcSinh[c*
x]*g)/(-(c*f) + Sqrt[c^2*f^2 + g^2])]) + b*PolyLog[3, (E^ArcSinh[c*x]*g)/(-
(c*f) + Sqrt[c^2*f^2 + g^2])]) + 2*b*m*(-((a + b*ArcSinh[c*x])*PolyLog[2, -
((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))]) + b*PolyLog[3, -((E^ArcS
inh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))]))/(2*b*c)
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{(b\,\mathrm{arsinh}\,(cx)+a)\log\left(\left(gx+f\right)^m h\right)}{\sqrt{c^2x^2+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algorith
m="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)*log((g*x + f)^m*h)/sqrt(c^2*x^2 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arsinh}(cx) + a)\log\left(\left(gx + f\right)^m h\right)}{\sqrt{c^2x^2 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algorith
m="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*log((g*x + f)^m*h)/sqrt(c^2*x^2 + 1), x)
```

maple [F] time = 2.13, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{arcsinh}(cx))\ln\left(h\left(gx + f\right)^m\right)}{\sqrt{c^2x^2 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))*ln(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))*ln(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arsinh}(cx) + a)\log\left(\left(gx + f\right)^m h\right)}{\sqrt{c^2x^2 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algorith
m="maxima")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*log((g*x + f)^m*h)/sqrt(c^2*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(h\left(f + g\,x\right)^m\right)\,(a + b \operatorname{asinh}(c\,x))}{\sqrt{c^2\,x^2 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(h*(f + g*x)^m)*(a + b*asinh(c*x)))/(c^2*x^2 + 1)^(1/2),x)
```

```
[Out] int((log(h*(f + g*x)^m)*(a + b*asinh(c*x)))/(c^2*x^2 + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{asinh}(cx))\log\left(h\left(f + gx\right)^m\right)}{\sqrt{c^2x^2 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))*ln(h*(g*x+f)**m)/(c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))*log(h*(f + g*x)**m)/sqrt(c**2*x**2 + 1), x)
```

3.56 $\int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx$

Optimal. Leaf size=197

$$-\frac{m\text{Li}_2\left(-\frac{e^{\sinh^{-1}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{c}-\frac{m\text{Li}_2\left(-\frac{e^{\sinh^{-1}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{c}-\frac{m\sinh^{-1}(cx)\log\left(\frac{ge^{\sinh^{-1}(cx)}}{cf-\sqrt{c^2f^2+g^2}}+1\right)}{c}-\frac{m\sinh^{-1}(cx)\log\left(\frac{ge^{\sinh^{-1}}}{\sqrt{c^2f^2+}}\right.}{c}$$

```
[Out] 1/2*m*arcsinh(c*x)^2/c+arcsinh(c*x)*ln(h*(g*x+f)^m)/c-m*arcsinh(c*x)*ln(1+(
c*x+(c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2+g^2)^(1/2)))/c-m*arcsinh(c*x)*ln(1+(
c*x+(c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2+g^2)^(1/2)))/c-m*polylog(2,-(c*x+(c^
2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2+g^2)^(1/2)))/c-m*polylog(2,-(c*x+(c^2*x^2+1
)^(1/2))*g/(c*f+(c^2*f^2+g^2)^(1/2)))/c
```

Rubi [A] time = 0.30, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {215, 2404, 5799, 5561, 2190, 2279, 2391}

$$-\frac{m\text{PolyLog}\left(2,-\frac{ge^{\sinh^{-1}(cx)}}{cf-\sqrt{c^2f^2+g^2}}\right)}{c}-\frac{m\text{PolyLog}\left(2,-\frac{ge^{\sinh^{-1}(cx)}}{\sqrt{c^2f^2+g^2}+cf}\right)}{c}-\frac{m\sinh^{-1}(cx)\log\left(\frac{ge^{\sinh^{-1}(cx)}}{cf-\sqrt{c^2f^2+g^2}}+1\right)}{c}-\frac{m\sinh^{-1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Log[h*(f + g*x)^m]/Sqrt[1 + c^2*x^2],x]
```

```
[Out] (m*ArcSinh[c*x]^2)/(2*c) - (m*ArcSinh[c*x]*Log[1 + (E^ArcSinh[c*x]*g)/(c*f
- Sqrt[c^2*f^2 + g^2])])/c - (m*ArcSinh[c*x]*Log[1 + (E^ArcSinh[c*x]*g)/(c*
f + Sqrt[c^2*f^2 + g^2])])/c + (ArcSinh[c*x]*Log[h*(f + g*x)^m])/c - (m*Pol
yLog[2, -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2]))])/c - (m*PolyLog[
2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))])/c
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/Sqrt[(f_) + (g_.)*
(x_)^2], x_Symbol] :> With[{u = IntHide[1/Sqrt[f + g*x^2], x]}, Simp[u*(a +
```

```
b*Log[c*(d + e*x)^n]), x] - Dist[b*e*n, Int[SimplifyIntegrand[u/(d + e*x),
x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && GtQ[f, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Cosh[x])/(c*d + e*Sinh[x]), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx = \frac{\sinh^{-1}(cx)\log\left(h(f+gx)^m\right)}{c} - (gm)\int \frac{\sinh^{-1}(cx)}{cf+cgx}\,dx$$

$$= \frac{\sinh^{-1}(cx)\log\left(h(f+gx)^m\right)}{c} - (gm)\,\text{Subst}\left(\int \frac{x\cosh(x)}{c^2f+cg\sinh(x)}\,dx, x, \sinh^{-1}(cx)\right)$$

$$= \frac{m\sinh^{-1}(cx)^2}{2c} + \frac{\sinh^{-1}(cx)\log\left(h(f+gx)^m\right)}{c} - (gm)\,\text{Subst}\left(\int \frac{e^x x}{c^2f+ce^xg-c\sqrt{c^2f^2+\cdots}}\right.$$

$$= \frac{m\sinh^{-1}(cx)^2}{2c} - \frac{m\sinh^{-1}(cx)\log\left(1+\frac{e^{\sinh^{-1}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{c} - \frac{m\sinh^{-1}(cx)\log\left(1+\frac{e^{\sinh^{-1}(cx)}}{cf+\sqrt{c^2f^2\cdots}}\right.}{c}$$

$$= \frac{m\sinh^{-1}(cx)^2}{2c} - \frac{m\sinh^{-1}(cx)\log\left(1+\frac{e^{\sinh^{-1}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{c} - \frac{m\sinh^{-1}(cx)\log\left(1+\frac{e^{\sinh^{-1}(cx)}}{cf+\sqrt{c^2f^2\cdots}}\right.}{c}$$

$$= \frac{m\sinh^{-1}(cx)^2}{2c} - \frac{m\sinh^{-1}(cx)\log\left(1+\frac{e^{\sinh^{-1}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{c} - \frac{m\sinh^{-1}(cx)\log\left(1+\frac{e^{\sinh^{-1}(cx)}}{cf+\sqrt{c^2f^2\cdots}}\right.}{c}$$

Mathematica [A] time = 0.02, size = 206, normalized size = 1.05

$$-\frac{m\text{Li}_2\left(-\frac{e^{\sinh^{-1}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{c} - \frac{m\text{Li}_2\left(-\frac{e^{\sinh^{-1}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{c} - \frac{m\sinh^{-1}(cx)\log\left(\frac{cge^{\sinh^{-1}(cx)}}{c^2f-c\sqrt{c^2f^2+g^2}}+1\right)}{c} - \frac{m\sinh^{-1}(cx)\log\left(\frac{cge^{\sinh^{-1}\cdots}}{c\sqrt{c^2f^2+g\cdots}}\right.}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[h*(f + g*x)^m]/Sqrt[1 + c^2*x^2],x]
```

```
[Out] (m*ArcSinh[c*x]^2)/(2*c) - (m*ArcSinh[c*x]*Log[1 + (c*E^ArcSinh[c*x]*g)/(c^
2*f - c*Sqrt[c^2*f^2 + g^2])])/c - (m*ArcSinh[c*x]*Log[1 + (c*E^ArcSinh[c*x
]*g)/(c^2*f + c*Sqrt[c^2*f^2 + g^2])])/c + (ArcSinh[c*x]*Log[h*(f + g*x)^m]
)/c - (m*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2]))])/c -
 (m*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))])/c
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(gx+f\right)^{m}h\right)}{\sqrt{c^{2}x^{2}+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(log((g*x + f)^m*h)/sqrt(c^2*x^2 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(gx+f\right)^{m}h\right)}{\sqrt{c^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(log((g*x + f)^m*h)/sqrt(c^2*x^2 + 1), x)
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(h\left(gx+f\right)^{m}\right)}{\sqrt{c^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x)
[Out] int(ln(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(gx+f\right)^{m}h\right)}{\sqrt{c^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(log((g*x + f)^m*h)/sqrt(c^2*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(h\left(f+g\,x\right)^{m}\right)}{\sqrt{c^{2}\,x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(h*(f + g*x)^m)/(c^2*x^2 + 1)^(1/2),x)
[Out] int(log(h*(f + g*x)^m)/(c^2*x^2 + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(h\left(f+gx\right)^{m}\right)}{\sqrt{c^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(h*(g*x+f)**m)/(c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(log(h*(f + g*x)**m)/sqrt(c**2*x**2 + 1), x)
```

3.57 $\int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=37

$$\mathrm{Int}\left(\frac{\log\left(h(f+gx)^m\right)}{\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable(ln(h*(g*x+f)^m)/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x)
```

Rubi [A] time = 0.19, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[h*(f + g*x)^m]/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Defer[Int][Log[h*(f + g*x)^m]/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx=\int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Mathematica [A] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[h*(f + g*x)^m]/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Integrate[Log[h*(f + g*x)^m]/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])), x]
```

fricas [A] time = 0.53, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{c^2x^2+1}\,\log\left(\left(gx+f\right)^m h\right)}{ac^2x^2+\left(bc^2x^2+b\right)\mathrm{arsinh}\,(cx)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(h*(g*x+f)^m)/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorith
m="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*log((g*x + f)^m*h)/(a*c^2*x^2 + (b*c^2*x^2 + b)*
arcsinh(c*x) + a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(gx+f\right)^m h\right)}{\sqrt{c^2x^2+1}\,(b\,\mathrm{arsinh}\,(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(h*(g*x+f)^m)/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorith
m="giac")
```

```
[Out] integrate(log((g*x + f)^m*h)/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)), x)
```

maple [A] time = 1.28, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(h\left(gx+f\right)^{m}\right)}{\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)\sqrt{c^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(h*(g*x+f)^m)/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x)
```

```
[Out] int(ln(h*(g*x+f)^m)/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(gx+f\right)^{m}h\right)}{\sqrt{c^{2}x^{2}+1}\left(b\,\mathrm{arsinh}\left(cx\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(h*(g*x+f)^m)/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorith
m="maxima")
```

```
[Out] integrate(log((g*x + f)^m*h)/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\ln\left(h\left(f+g\,x\right)^{m}\right)}{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,\sqrt{c^{2}\,x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(h*(f + g*x)^m)/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)),x)
```

```
[Out] int(log(h*(f + g*x)^m)/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(h\left(f+gx\right)^{m}\right)}{\left(a+b\,\mathrm{asinh}\left(cx\right)\right)\sqrt{c^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(h*(g*x+f)**m)/(a+b*asinh(c*x))/(c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(log(h*(f + g*x)**m)/((a + b*asinh(c*x))*sqrt(c**2*x**2 + 1)), x)
```

3.58 $\int x^3 \sinh^{-1}(a+bx)\,dx$

Optimal. Leaf size=131

$$-\frac{\left(4a\left(16-19a^2\right)-\left(9-26a^2\right)(a+bx)\right)\sqrt{(a+bx)^2+1}}{96b^4}-\frac{\left(8a^4-24a^2+3\right)\sinh^{-1}(a+bx)}{32b^4}+\frac{7ax^2\sqrt{(a+bx)^2+}}{48b^2}$$

```
[Out] -1/32*(8*a^4-24*a^2+3)*arcsinh(b*x+a)/b^4+1/4*x^4*arcsinh(b*x+a)+7/48*a*x^2
*(1+(b*x+a)^2)^(1/2)/b^2-1/16*x^3*(1+(b*x+a)^2)^(1/2)/b-1/96*(4*a*(-19*a^2+
16)-(-26*a^2+9)*(b*x+a))*(1+(b*x+a)^2)^(1/2)/b^4
```

Rubi [A] time = 0.17, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5865, 5801, 743, 833, 780, 215}

$$-\frac{\left(4a\left(16-19a^2\right)-\left(9-26a^2\right)(a+bx)\right)\sqrt{(a+bx)^2+1}}{96b^4}-\frac{\left(8a^4-24a^2+3\right)\sinh^{-1}(a+bx)}{32b^4}+\frac{7ax^2\sqrt{(a+bx)^2+}}{48b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcSinh[a + b*x],x]
```

```
[Out] (7*a*x^2*Sqrt[1 + (a + b*x)^2])/(48*b^2) - (x^3*Sqrt[1 + (a + b*x)^2])/(16*
b) - ((4*a*(16 - 19*a^2) - (9 - 26*a^2)*(a + b*x))*Sqrt[1 + (a + b*x)^2])/(
96*b^4) - ((3 - 24*a^2 + 8*a^4)*ArcSinh[a + b*x])/(32*b^4) + (x^4*ArcSinh[a
 + b*x])/4
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 5801

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n
 - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int x^3 \sinh^{-1}(a+bx)\,dx = \frac{\text{Subst}\left(\int \left(-\frac{a}{b}+\frac{x}{b}\right)^3 \sinh^{-1}(x)\,dx, x, a+bx\right)}{b}$$

$$= \frac{1}{4}x^4 \sinh^{-1}(a+bx) - \frac{1}{4}\text{Subst}\left(\int \frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^4}{\sqrt{1+x^2}}\,dx, x, a+bx\right)$$

$$= -\frac{x^3\sqrt{1+(a+bx)^2}}{16b} + \frac{1}{4}x^4 \sinh^{-1}(a+bx) - \frac{1}{16}\text{Subst}\left(\int \frac{\left(-\frac{3-4a^2}{b^2}-\frac{7ax}{b^2}\right)\left(-\frac{a}{b}+\frac{x}{b}\right)^2}{\sqrt{1+x^2}}\,dx\right.$$

$$= \frac{7ax^2\sqrt{1+(a+bx)^2}}{48b^2} - \frac{x^3\sqrt{1+(a+bx)^2}}{16b} + \frac{1}{4}x^4 \sinh^{-1}(a+bx) - \frac{1}{48}\text{Subst}\left(\int \frac{\left(\frac{a(23-1}{b^3}\right.}{}\right.$$

$$= \frac{7ax^2\sqrt{1+(a+bx)^2}}{48b^2} - \frac{x^3\sqrt{1+(a+bx)^2}}{16b} - \frac{\left(4a\left(16-19a^2\right)-\left(9-26a^2\right)(a+bx)\right)\sqrt{1}}{96b^4}$$

$$= \frac{7ax^2\sqrt{1+(a+bx)^2}}{48b^2} - \frac{x^3\sqrt{1+(a+bx)^2}}{16b} - \frac{\left(4a\left(16-19a^2\right)-\left(9-26a^2\right)(a+bx)\right)\sqrt{1}}{96b^4}$$

Mathematica [A] time = 0.09, size = 95, normalized size = 0.73

$$\frac{\sqrt{a^2+2abx+b^2x^2+1}\left(50a^3-26a^2bx+a\left(14b^2x^2-55\right)-6b^3x^3+9bx\right)-3\left(8a^4-24a^2-8b^4x^4+3\right)\sinh^{-1}(a}{96b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcSinh[a + b*x],x]
```

```
[Out] (Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(50*a^3 + 9*b*x - 26*a^2*b*x - 6*b^3*x^3
 + a*(-55 + 14*b^2*x^2)) - 3*(3 - 24*a^2 + 8*a^4 - 8*b^4*x^4)*ArcSinh[a + b
*x])/(96*b^4)
```

fricas [A] time = 0.46, size = 110, normalized size = 0.84

$$\frac{3\left(8\,b^4x^4-8\,a^4+24\,a^2-3\right)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)-\left(6\,b^3x^3-14\,ab^2x^2-50\,a^3+\left(26\,a^2-9\right)bx\right.}{96\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(b*x+a),x, algorithm="fricas")
```

[Out] 1/96*(3*(8*b^4*x^4 - 8*a^4 + 24*a^2 - 3)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - (6*b^3*x^3 - 14*a*b^2*x^2 - 50*a^3 + (26*a^2 - 9)*b*x + 55*a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/b^4

giac [A] time = 1.00, size = 162, normalized size = 1.24

$$\frac{1}{4}x^4\log\left(bx+a+\sqrt{(bx+a)^2+1}\right)-\frac{1}{96}\left(\sqrt{b^2x^2+2abx+a^2+1}\left(\left(2x\left(\frac{3x}{b^2}-\frac{7a}{b^3}\right)+\frac{26a^2b^3-9b^3}{b^7}\right)x-\frac{5(10}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*arcsinh(b*x+a),x, algorithm="giac")

[Out] 1/4*x^4*log(b*x + a + sqrt((b*x + a)^2 + 1)) - 1/96*(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*((2*x*(3*x/b^2 - 7*a/b^3) + (26*a^2*b^3 - 9*b^3)/b^7)*x - 5*(10*a^3*b^2 - 11*a*b^2)/b^7) - 3*(8*a^4 - 24*a^2 + 3)*log(-a*b - (x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*abs(b))/(b^4*abs(b)))*b

maple [A] time = 0.02, size = 200, normalized size = 1.53

$$\frac{\frac{\operatorname{arcsinh}(bx+a)(bx+a)^4}{4}-\operatorname{arcsinh}(bx+a)(bx+a)^3a+\frac{3\operatorname{arcsinh}(bx+a)(bx+a)^2a^2}{2}-\operatorname{arcsinh}(bx+a)(bx+a)a^3-\frac{(bx+a)^3}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*arcsinh(b*x+a),x)

[Out] 1/b^4*(1/4*arcsinh(b*x+a)*(b*x+a)^4-arcsinh(b*x+a)*(b*x+a)^3*a+3/2*arcsinh(b*x+a)*(b*x+a)^2*a^2-arcsinh(b*x+a)*(b*x+a)*a^3-1/16*(b*x+a)^3*(1+(b*x+a)^2)^(1/2)+3/32*(b*x+a)*(1+(b*x+a)^2)^(1/2)-3/32*arcsinh(b*x+a)+a*(1/3*(b*x+a)^2*(1+(b*x+a)^2)^(1/2)-2/3*(1+(b*x+a)^2)^(1/2))-3/2*a^2*(1/2*(b*x+a)*(1+(b*x+a)^2)^(1/2)-1/2*arcsinh(b*x+a))+a^3*(1+(b*x+a)^2)^(1/2))

maxima [B] time = 0.34, size = 318, normalized size = 2.43

$$\frac{1}{4}x^4\operatorname{arsinh}(bx+a)-\frac{1}{96}\left(\frac{6\sqrt{b^2x^2+2abx+a^2+1}\,x^3}{b^2}-\frac{14\sqrt{b^2x^2+2abx+a^2+1}\,ax^2}{b^3}+\frac{105a^4\operatorname{arsinh}\left(\frac{}{\sqrt{-4a}}\right.}{b^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*arcsinh(b*x+a),x, algorithm="maxima")

[Out] 1/4*x^4*arcsinh(b*x + a) - 1/96*(6*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*x^3/b^2 - 14*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a*x^2/b^3 + 105*a^4*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^5 + 35*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^2*x/b^4 - 90*(a^2 + 1)*a^2*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^5 - 105*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^3/b^5 - 9*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)*x/b^4 + 9*(a^2 + 1)^2*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^5 + 55*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)*a/b^5)*b

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\operatorname{asinh}(a+b\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*asinh(a + b*x),x)
```

```
[Out] int(x^3*asinh(a + b*x), x)
```

sympy [A] time = 1.44, size = 255, normalized size = 1.95

$$\begin{cases} -\frac{a^4 \operatorname{asinh}(a+bx)}{4b^4} + \frac{25a^3\sqrt{a^2+2abx+b^2x^2+1}}{48b^4} - \frac{13a^2x\sqrt{a^2+2abx+b^2x^2+1}}{48b^3} + \frac{3a^2 \operatorname{asinh}(a+bx)}{4b^4} + \frac{7ax^2\sqrt{a^2+2abx+b^2x^2+1}}{48b^2} - \frac{55a\sqrt{a^2+2abx+b^2x^2+1}}{96b^4} \\ \frac{x^4 \operatorname{asinh}(a)}{4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*asinh(b*x+a),x)
```

```
[Out] Piecewise((-a**4*asinh(a + b*x)/(4*b**4) + 25*a**3*sqrt(a**2 + 2*a*b*x + b*
*2*x**2 + 1)/(48*b**4) - 13*a**2*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(48
*b**3) + 3*a**2*asinh(a + b*x)/(4*b**4) + 7*a*x**2*sqrt(a**2 + 2*a*b*x + b*
*2*x**2 + 1)/(48*b**2) - 55*a*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(96*b**4
) + x**4*asinh(a + b*x)/4 - x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(16*b
) + 3*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(32*b**3) - 3*asinh(a + b*x)/(
32*b**4), Ne(b, 0)), (x**4*asinh(a)/4, True))
```

3.59 $\int x^2 \sinh^{-1}(a + bx)\,dx$

Optimal. Leaf size=90

$$\frac{(-11a^2 + 5abx + 4)\sqrt{(a + bx)^2 + 1}}{18b^3} - \frac{a(3 - 2a^2)\sinh^{-1}(a + bx)}{6b^3} + \frac{1}{3}x^3\sinh^{-1}(a+bx) - \frac{x^2\sqrt{(a + bx)^2 + 1}}{9b}$$

```
[Out] -1/6*a*(-2*a^2+3)*arcsinh(b*x+a)/b^3+1/3*x^3*arcsinh(b*x+a)-1/9*x^2*(1+(b*x
+a)^2)^(1/2)/b+1/18*(5*a*b*x-11*a^2+4)*(1+(b*x+a)^2)^(1/2)/b^3
```

Rubi [A] time = 0.11, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5865, 5801, 743, 780, 215}

$$\frac{(-11a^2 + 5abx + 4)\sqrt{(a + bx)^2 + 1}}{18b^3} - \frac{a(3 - 2a^2)\sinh^{-1}(a + bx)}{6b^3} - \frac{x^2\sqrt{(a + bx)^2 + 1}}{9b} + \frac{1}{3}x^3\sinh^{-1}(a+bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcSinh[a + b*x],x]
```

```
[Out] -(x^2*Sqrt[1 + (a + b*x)^2])/(9*b) + ((4 - 11*a^2 + 5*a*b*x)*Sqrt[1 + (a +
b*x)^2])/(18*b^3) - (a*(3 - 2*a^2)*ArcSinh[a + b*x])/(6*b^3) + (x^3*ArcSinh
[a + b*x])/3
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 5801

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n
 - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int x^2\sinh^{-1}(a+bx)\,dx = \frac{\text{Subst}\left(\int\left(-\frac{a}{b}+\frac{x}{b}\right)^2\sinh^{-1}(x)\,dx,x,a+bx\right)}{b}$$
$$= \frac{1}{3}x^3\sinh^{-1}(a+bx)-\frac{1}{3}\text{Subst}\left(\int\frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^3}{\sqrt{1+x^2}}\,dx,x,a+bx\right)$$
$$= -\frac{x^2\sqrt{1+(a+bx)^2}}{9b}+\frac{1}{3}x^3\sinh^{-1}(a+bx)-\frac{1}{9}\text{Subst}\left(\int\frac{\left(-\frac{2-3a^2}{b^2}-\frac{5ax}{b^2}\right)\left(-\frac{a}{b}+\frac{x}{b}\right)}{\sqrt{1+x^2}}\,dx,x$$
$$= -\frac{x^2\sqrt{1+(a+bx)^2}}{9b}+\frac{\left(4-11a^2+5abx\right)\sqrt{1+(a+bx)^2}}{18b^3}+\frac{1}{3}x^3\sinh^{-1}(a+bx)-\frac{(a(3}{}$$
$$= -\frac{x^2\sqrt{1+(a+bx)^2}}{9b}+\frac{\left(4-11a^2+5abx\right)\sqrt{1+(a+bx)^2}}{18b^3}-\frac{a\left(3-2a^2\right)\sinh^{-1}(a+bx)}{6b^3}$$

Mathematica [A] time = 0.06, size = 74, normalized size = 0.82

$$\frac{\left(6a^3-9a+6b^3x^3\right)\sinh^{-1}(a+bx)+\sqrt{a^2+2abx+b^2x^2+1}\left(-11a^2+5abx-2b^2x^2+4\right)}{18b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcSinh[a + b*x],x]
```

```
[Out] ((4 - 11*a^2 + 5*a*b*x - 2*b^2*x^2)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] + (-9
*a + 6*a^3 + 6*b^3*x^3)*ArcSinh[a + b*x])/(18*b^3)
```

fricas [A] time = 0.55, size = 91, normalized size = 1.01

$$\frac{3\left(2\,b^3x^3+2\,a^3-3\,a\right)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)-\left(2\,b^2x^2-5\,abx+11\,a^2-4\right)\sqrt{b^2x^2+2\,abx+a^2+}}{18\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/18*(3*(2*b^3*x^3 + 2*a^3 - 3*a)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^
2 + 1)) - (2*b^2*x^2 - 5*a*b*x + 11*a^2 - 4)*sqrt(b^2*x^2 + 2*a*b*x + a^2 +
 1))/b^3
```

giac [A] time = 0.33, size = 131, normalized size = 1.46

$$\frac{1}{3}x^3\log\left(bx+a+\sqrt{(bx+a)^2+1}\right)-\frac{1}{18}\left(\sqrt{b^2x^2+2\,abx+a^2+1}\left(x\left(\frac{2\,x}{b^2}-\frac{5\,a}{b^3}\right)+\frac{11\,a^2b-4\,b}{b^5}\right)+\frac{3\left(2\,a^3-3\,a\right)\text{lo}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/3*x^3*log(b*x + a + sqrt((b*x + a)^2 + 1)) - 1/18*(sqrt(b^2*x^2 + 2*a*b*x
 + a^2 + 1)*(x*(2*x/b^2 - 5*a/b^3) + (11*a^2*b - 4*b)/b^5) + 3*(2*a^3 - 3*a
)*log(-a*b - (x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*abs(b))/(b^3*ab
s(b)))*b
```

maple [A] time = 0.00, size = 130, normalized size = 1.44

$$\frac{\frac{\operatorname{arcsinh}(bx+a)(bx+a)^3}{3} - \operatorname{arcsinh}(bx+a)(bx+a)^2 a + \operatorname{arcsinh}(bx+a)(bx+a)a^2 - \frac{(bx+a)^2\sqrt{1+(bx+a)^2}}{9} + \frac{2\sqrt{1+(bx+a)}}{9}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsinh(b*x+a),x)
```

```
[Out] 1/b^3*(1/3*arcsinh(b*x+a)*(b*x+a)^3-arcsinh(b*x+a)*(b*x+a)^2*a+arcsinh(b*x+
a)*(b*x+a)*a^2-1/9*(b*x+a)^2*(1+(b*x+a)^2)^(1/2)+2/9*(1+(b*x+a)^2)^(1/2)+a*
(1/2*(b*x+a)*(1+(b*x+a)^2)^(1/2)-1/2*arcsinh(b*x+a))-a^2*(1+(b*x+a)^2)^(1/2
))
```

maxima [B] time = 0.36, size = 210, normalized size = 2.33

$$\frac{1}{3}x^3\operatorname{arsinh}(bx+a) - \frac{1}{18}b\left(\frac{2\sqrt{b^2x^2+2abx+a^2+1}\,x^2}{b^2} - \frac{15a^3\operatorname{arsinh}\left(\frac{2\left(b^2x+ab\right)}{\sqrt{-4a^2b^2+4\left(a^2+1\right)b^2}}\right)}{b^4} - \frac{5\sqrt{b^2x^2+2abx}}{b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arcsinh(b*x + a) - 1/18*b*(2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*x^2/
b^2 - 15*a^3*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^
4 - 5*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a*x/b^3 + 9*(a^2 + 1)*a*arcsinh(2*(
b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^4 + 15*sqrt(b^2*x^2 + 2*
a*b*x + a^2 + 1)*a^2/b^4 - 4*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)/b^
4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\operatorname{asinh}(a+b\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*asinh(a + b*x),x)
```

```
[Out] int(x^2*asinh(a + b*x), x)
```

sympy [A] time = 0.67, size = 170, normalized size = 1.89

$$\begin{cases} \frac{a^3\operatorname{asinh}(a+bx)}{3b^3} - \frac{11a^2\sqrt{a^2+2abx+b^2x^2+1}}{18b^3} + \frac{5ax\sqrt{a^2+2abx+b^2x^2+1}}{18b^2} - \frac{a\operatorname{asinh}(a+bx)}{2b^3} + \frac{x^3\operatorname{asinh}(a+bx)}{3} - \frac{x^2\sqrt{a^2+2abx+b^2x^2+1}}{9b} + 2 \\ \frac{x^3\operatorname{asinh}(a)}{3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asinh(b*x+a),x)
```

```
[Out] Piecewise((a**3*asinh(a + b*x)/(3*b**3) - 11*a**2*sqrt(a**2 + 2*a*b*x + b**
2*x**2 + 1)/(18*b**3) + 5*a*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(18*b**2
) - a*asinh(a + b*x)/(2*b**3) + x**3*asinh(a + b*x)/3 - x**2*sqrt(a**2 + 2*
a*b*x + b**2*x**2 + 1)/(9*b) + 2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(9*b*
*3), Ne(b, 0)), (x**3*asinh(a)/3, True))
```

3.60 $\int x\sinh^{-1}(a+bx)\,dx$

Optimal. Leaf size=76

$$\frac{(1-2a^2)\sinh^{-1}(a+bx)}{4b^2}+\frac{3a\sqrt{(a+bx)^2+1}}{4b^2}+\frac{1}{2}x^2\sinh^{-1}(a+bx)-\frac{x\sqrt{(a+bx)^2+1}}{4b}$$

```
[Out] 1/4*(-2*a^2+1)*arcsinh(b*x+a)/b^2+1/2*x^2*arcsinh(b*x+a)+3/4*a*(1+(b*x+a)^2
)^(1/2)/b^2-1/4*x*(1+(b*x+a)^2)^(1/2)/b
```

Rubi [A] time = 0.07, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5865, 5801, 743, 641, 215}

$$\frac{(1-2a^2)\sinh^{-1}(a+bx)}{4b^2}+\frac{3a\sqrt{(a+bx)^2+1}}{4b^2}+\frac{1}{2}x^2\sinh^{-1}(a+bx)-\frac{x\sqrt{(a+bx)^2+1}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcSinh[a + b*x],x]
```

```
[Out] (3*a*Sqrt[1 + (a + b*x)^2])/(4*b^2) - (x*Sqrt[1 + (a + b*x)^2])/(4*b) + ((1
 - 2*a^2)*ArcSinh[a + b*x])/(4*b^2) + (x^2*ArcSinh[a + b*x])/2
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 5801

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n
 - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[(((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int x\sinh^{-1}(a+bx)\,dx = \frac{\text{Subst}\left(\int\left(-\frac{a}{b}+\frac{x}{b}\right)\sinh^{-1}(x)\,dx,x,a+bx\right)}{b}$$
$$= \frac{1}{2}x^2\sinh^{-1}(a+bx)-\frac{1}{2}\text{Subst}\left(\int\frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^2}{\sqrt{1+x^2}}dx,x,a+bx\right)$$
$$= -\frac{x\sqrt{1+(a+bx)^2}}{4b}+\frac{1}{2}x^2\sinh^{-1}(a+bx)-\frac{1}{4}\text{Subst}\left(\int\frac{-\frac{1-2a^2}{b^2}-\frac{3ax}{b^2}}{\sqrt{1+x^2}}dx,x,a+bx\right)$$
$$= \frac{3a\sqrt{1+(a+bx)^2}}{4b^2}-\frac{x\sqrt{1+(a+bx)^2}}{4b}+\frac{1}{2}x^2\sinh^{-1}(a+bx)+\frac{\left(1-2a^2\right)\text{Subst}\left(\int\frac{}{\sqrt{1}}\right.}{4b^2}$$
$$= \frac{3a\sqrt{1+(a+bx)^2}}{4b^2}-\frac{x\sqrt{1+(a+bx)^2}}{4b}+\frac{\left(1-2a^2\right)\sinh^{-1}(a+bx)}{4b^2}+\frac{1}{2}x^2\sinh^{-1}(a+$$

Mathematica [A] time = 0.04, size = 60, normalized size = 0.79

$$\frac{\sqrt{a^2+2abx+b^2x^2+1}\,(3a-bx)+\left(-2a^2+2b^2x^2+1\right)\sinh^{-1}(a+bx)}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcSinh[a + b*x],x]
[Out] ((3*a - b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] + (1 - 2*a^2 + 2*b^2*x^2)*Ar
cSinh[a + b*x])/(4*b^2)
```

fricas [A] time = 0.53, size = 75, normalized size = 0.99

$$\frac{\left(2\,b^2x^2-2\,a^2+1\right)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)-\sqrt{b^2x^2+2\,abx+a^2+1}\,(bx-3\,a)}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(b*x+a),x, algorithm="fricas")
[Out] 1/4*((2*b^2*x^2 - 2*a^2 + 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1
)) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b*x - 3*a))/b^2
```

giac [A] time = 0.36, size = 111, normalized size = 1.46

$$\frac{1}{2}\,x^2\log\left(bx+a+\sqrt{(bx+a)^2+1}\right)-\frac{1}{4}\left(\sqrt{b^2x^2+2\,abx+a^2+1}\left(\frac{x}{b^2}-\frac{3\,a}{b^3}\right)-\frac{\left(2\,a^2-1\right)\log\left(-ab-\left(x|b|-\sqrt{b^2}\right.\right.}{b^2|b|}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(b*x+a),x, algorithm="giac")
[Out] 1/2*x^2*log(b*x + a + sqrt((b*x + a)^2 + 1)) - 1/4*(sqrt(b^2*x^2 + 2*a*b*x
+ a^2 + 1)*(x/b^2 - 3*a/b^3) - (2*a^2 - 1)*log(-a*b - (x*abs(b) - sqrt(b^2*
x^2 + 2*a*b*x + a^2 + 1))*abs(b))/(b^2*abs(b)))*b
```

maple [A] time = 0.00, size = 74, normalized size = 0.97

$$\frac{\frac{\text{arcsinh}(bx+a)(bx+a)^2}{2}-\text{arcsinh}\,(bx+a)\,a\,(bx+a)-\frac{(bx+a)\sqrt{1+(bx+a)^2}}{4}+\frac{\text{arcsinh}(bx+a)}{4}+a\sqrt{1+(bx+a)^2}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsinh(b*x+a),x)
```

```
[Out] 1/b^2*(1/2*arcsinh(b*x+a)*(b*x+a)^2-arcsinh(b*x+a)*a*(b*x+a)-1/4*(b*x+a)*(1
+(b*x+a)^2)^(1/2)+1/4*arcsinh(b*x+a)+a*(1+(b*x+a)^2)^(1/2))
```

maxima [B] time = 0.37, size = 149, normalized size = 1.96

$$\frac{1}{2}x^2 \operatorname{arsinh}(bx+a) - \frac{1}{4}b\left(\frac{3a^2 \operatorname{arsinh}\left(\frac{2(b^2x+ab)}{\sqrt{-4a^2b^2+4(a^2+1)b^2}}\right)}{b^3} + \frac{\sqrt{b^2x^2+2abx+a^2+1}\,x}{b^2} - \frac{(a^2+1)\operatorname{arsinh}\left(\frac{2(b^2x+ab)}{\sqrt{-4a^2b^2+4(a^2+1)b^2}}\right)}{b^3} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arcsinh(b*x + a) - 1/4*b*(3*a^2*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2
*b^2 + 4*(a^2 + 1)*b^2))/b^3 + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*x/b^2 - (a
^2 + 1)*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^3 - 3
*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a/b^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x \operatorname{asinh}(a+b\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*asinh(a + b*x),x)
```

```
[Out] int(x*asinh(a + b*x), x)
```

sympy [A] time = 0.30, size = 104, normalized size = 1.37

$$\begin{cases} -\frac{a^2 \operatorname{asinh}(a+bx)}{2b^2} + \frac{3a\sqrt{a^2+2abx+b^2x^2+1}}{4b^2} + \frac{x^2 \operatorname{asinh}(a+bx)}{2} - \frac{x\sqrt{a^2+2abx+b^2x^2+1}}{4b} + \frac{\operatorname{asinh}(a+bx)}{4b^2} & \text{for } b \neq 0 \\ \frac{x^2 \operatorname{asinh}(a)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asinh(b*x+a),x)
```

```
[Out] Piecewise((-a**2*asinh(a + b*x)/(2*b**2) + 3*a*sqrt(a**2 + 2*a*b*x + b**2*x
**2 + 1)/(4*b**2) + x**2*asinh(a + b*x)/2 - x*sqrt(a**2 + 2*a*b*x + b**2*x*
*2 + 1)/(4*b) + asinh(a + b*x)/(4*b**2), Ne(b, 0)), (x**2*asinh(a)/2, True)
)
```

3.61 $\int \sinh^{-1}(a+bx)\,dx$

Optimal. Leaf size=34

$$\frac{(a+bx)\sinh^{-1}(a+bx)}{b}-\frac{\sqrt{(a+bx)^2+1}}{b}$$

```
[Out] (b*x+a)*arcsinh(b*x+a)/b-(1+(b*x+a)^2)^(1/2)/b
```

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5863, 5653, 261}

$$\frac{(a+bx)\sinh^{-1}(a+bx)}{b}-\frac{\sqrt{(a+bx)^2+1}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a + b*x],x]
```

```
[Out] -(Sqrt[1 + (a + b*x)^2]/b) + ((a + b*x)*ArcSinh[a + b*x])/b
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5863

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\begin{aligned}\int \sinh^{-1}(a+bx)\,dx&=\frac{\text{Subst}\left(\int \sinh^{-1}(x)\,dx,x,a+bx\right)}{b}\\&=\frac{(a+bx)\sinh^{-1}(a+bx)}{b}-\frac{\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}}\,dx,x,a+bx\right)}{b}\\&=-\frac{\sqrt{1+(a+bx)^2}}{b}+\frac{(a+bx)\sinh^{-1}(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.03, size = 40, normalized size = 1.18

$$\frac{(a+bx)\sinh^{-1}(a+bx)-\sqrt{a^2+2abx+b^2x^2+1}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a + b*x],x]
```

```
[Out] (-Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] + (a + b*x)*ArcSinh[a + b*x])/b
```

fricas [A] time = 0.54, size = 57, normalized size = 1.68

$$\frac{(bx+a)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)-\sqrt{b^2x^2+2\,abx+a^2+1}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a),x, algorithm="fricas")
```

```
[Out] ((b*x + a)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - sqrt(b^2*x^2
+ 2*a*b*x + a^2 + 1))/b
```

giac [B] time = 0.36, size = 92, normalized size = 2.71

$$-b\left(\frac{a\log\left(-ab-\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)|b|\right)}{b|b|}+\frac{\sqrt{b^2x^2+2\,abx+a^2+1}}{b^2}\right)+x\log\left(bx+a+\sqrt{(bx+a)^2+1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a),x, algorithm="giac")
```

```
[Out] -b*(a*log(-a*b - (x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*abs(b))/(b*
abs(b)) + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/b^2) + x*log(b*x + a + sqrt((b*
x + a)^2 + 1))
```

maple [A] time = 0.00, size = 31, normalized size = 0.91

$$\frac{(bx+a)\operatorname{arcsinh}(bx+a)-\sqrt{1+(bx+a)^2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a),x)
```

```
[Out] 1/b*((b*x+a)*arcsinh(b*x+a)-(1+(b*x+a)^2)^(1/2))
```

maxima [A] time = 0.39, size = 30, normalized size = 0.88

$$\frac{(bx+a)\operatorname{arsinh}(bx+a)-\sqrt{(bx+a)^2+1}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a),x, algorithm="maxima")
```

```
[Out] ((b*x + a)*arcsinh(b*x + a) - sqrt((b*x + a)^2 + 1))/b
```

mupad [B] time = 0.45, size = 76, normalized size = 2.24

$$x\operatorname{asinh}(a+b\,x)-\frac{\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}}{b}+\frac{a\,\ln\left(\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}+\frac{x\,b^2+a\,b}{\sqrt{b^2}}\right)}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a + b*x),x)
```

```
[Out] x*asinh(a + b*x) - (a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2)/b + (a*log((a^2 + b^
2*x^2 + 2*a*b*x + 1)^(1/2) + (a*b + b^2*x)/(b^2)^(1/2)))/(b^2)^(1/2)
```

sympy [A] time = 0.16, size = 46, normalized size = 1.35

$$\begin{cases} \frac{a \operatorname{asinh}(a+bx)}{b} + x \operatorname{asinh}(a+bx) - \frac{\sqrt{a^2+2abx+b^2x^2+1}}{b} & \text{for } b \neq 0 \\ x \operatorname{asinh}(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a),x)
```

```
[Out] Piecewise((a*asinh(a + b*x)/b + x*asinh(a + b*x) - sqrt(a**2 + 2*a*b*x + b*
*2*x**2 + 1)/b, Ne(b, 0)), (x*asinh(a), True))
```

3.62 $\int \frac{\sinh^{-1}(a+bx)}{x}\,dx$

Optimal. Leaf size=131

$$\text{Li}_2\left(\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)+\text{Li}_2\left(\frac{e^{\sinh^{-1}(a+bx)}}{a+\sqrt{a^2+1}}\right)+\sinh^{-1}(a+bx)\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)+\sinh^{-1}(a+bx)\log\left(1-\frac{e^{\sinh^{-1}(a+bx}}{\sqrt{a^2+1}+}\right.$$

```
[Out] -1/2*arcsinh(b*x+a)^2+arcsinh(b*x+a)*ln(1-(b*x+a+(1+(b*x+a)^2)^(1/2))/(a-(a
^2+1)^(1/2)))+arcsinh(b*x+a)*ln(1-(b*x+a+(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1
/2)))+polylog(2,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a-(a^2+1)^(1/2)))+polylog(2,(b
*x+a+(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))
```

Rubi [A] time = 0.24, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5865, 5799, 5561, 2190, 2279, 2391}

$$\text{PolyLog}\left(2,\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)+\text{PolyLog}\left(2,\frac{e^{\sinh^{-1}(a+bx)}}{\sqrt{a^2+1}+a}\right)+\sinh^{-1}(a+bx)\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)+\sinh^{-1}(a+bx)\log$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a + b*x]/x,x]
```

```
[Out] -ArcSinh[a + b*x]^2/2 + ArcSinh[a + b*x]*Log[1 - E^ArcSinh[a + b*x]/(a - Sq
rt[1 + a^2])] + ArcSinh[a + b*x]*Log[1 - E^ArcSinh[a + b*x]/(a + Sqrt[1 + a
^2])] + PolyLog[2, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])] + PolyLog[2, E^A
rcSinh[a + b*x]/(a + Sqrt[1 + a^2])]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Cosh[x])/(c*d + e*Sinh[x]), x], x, ArcSinh[c*x
```

```
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\sinh^{-1}(a+bx)}{x}\,dx = \frac{\text{Subst}\left(\int \frac{\sinh^{-1}(x)}{-\frac{a}{b}+\frac{x}{b}}\,dx, x, a+bx\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \frac{x\cosh(x)}{-\frac{a}{b}+\frac{\sinh(x)}{b}}\,dx, x, \sinh^{-1}(a+bx)\right)}{b}$$

$$= -\frac{1}{2}\sinh^{-1}(a+bx)^2 + \frac{\text{Subst}\left(\int \frac{e^x x}{-\frac{a}{b}-\frac{\sqrt{1+a^2}}{b}+\frac{e^x}{b}}\,dx, x, \sinh^{-1}(a+bx)\right)}{b} + \frac{\text{Subst}\left(\int \frac{e^{x}}{-\frac{a}{b}+\frac{\sqrt{1}}{}}\right.}{}$$

$$= -\frac{1}{2}\sinh^{-1}(a+bx)^2 + \sinh^{-1}(a+bx)\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{1+a^2}}\right) + \sinh^{-1}(a+bx)\log\left(1-\right.$$

$$= -\frac{1}{2}\sinh^{-1}(a+bx)^2 + \sinh^{-1}(a+bx)\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{1+a^2}}\right) + \sinh^{-1}(a+bx)\log\left(1-\right.$$

$$= -\frac{1}{2}\sinh^{-1}(a+bx)^2 + \sinh^{-1}(a+bx)\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{1+a^2}}\right) + \sinh^{-1}(a+bx)\log\left(1-\right.$$

Mathematica [A] time = 0.01, size = 153, normalized size = 1.17

$$\text{Li}_2\left(-\frac{e^{\sinh^{-1}(a+bx)}}{\sqrt{a^2+1}-a}\right)+\text{Li}_2\left(\frac{e^{\sinh^{-1}(a+bx)}}{a+\sqrt{a^2+1}}\right)+\sinh^{-1}(a+bx)\log\left(\frac{e^{\sinh^{-1}(a+bx)}}{b\left(-\frac{\sqrt{a^2+1}}{b}-\frac{a}{b}\right)}+1\right)+\sinh^{-1}(a+bx)\log\left(\frac{e^{\sinh^{-1}}}{b\left(\frac{\sqrt{a^2+}}{b}\right.}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a + b*x]/x,x]
```

```
[Out] -1/2*ArcSinh[a + b*x]^2 + ArcSinh[a + b*x]*Log[1 + E^ArcSinh[a + b*x]/((-(a
/b) - Sqrt[1 + a^2]/b)*b)] + ArcSinh[a + b*x]*Log[1 + E^ArcSinh[a + b*x]/((
-(a/b) + Sqrt[1 + a^2]/b)*b)] + PolyLog[2, -(E^ArcSinh[a + b*x]/(-a + Sqrt[
1 + a^2]))] + PolyLog[2, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])]
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{arsinh}\,(bx+a)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/x,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(b*x + a)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/x,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(b*x + a)/x, x)
```

maple [B] time = 0.17, size = 388, normalized size = 2.96

$$-\frac{\operatorname{arcsinh}(bx+a)^2}{2}+\frac{\left(a^2+1+\sqrt{a^2+1}\,a\right)\operatorname{arcsinh}(bx+a)\left(2\ln\left(\frac{\sqrt{a^2+1}-bx-\sqrt{1+(bx+a)^2}}{a+\sqrt{a^2+1}}\right)a^2+\ln\left(\frac{\sqrt{a^2+1}-bx-\sqrt{1+(bx-}}{a+\sqrt{a^2+1}}\right.\right.}{a^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)/x,x)
```

```
[Out] -1/2*arcsinh(b*x+a)^2+(a^2+1+(a^2+1)^(1/2)*a)/(a^2+1)*arcsinh(b*x+a)*(2*ln(
((a^2+1)^(1/2)-b*x-(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))*a^2+ln(((a^2+1)^
(1/2)-b*x-(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))-2*ln(((a^2+1)^(1/2)-b*x-(
1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))*(a^2+1)^(1/2)*a+ln(((a^2+1)^(1/2)+b*
x+(1+(b*x+a)^2)^(1/2))/(-a+(a^2+1)^(1/2))))+dilog(((a^2+1)^(1/2)-b*x-(1+(b*
x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))+dilog(((a^2+1)^(1/2)+b*x+(1+(b*x+a)^2)^(1
/2))/(-a+(a^2+1)^(1/2)))+a*arcsinh(b*x+a)/(a^2+1)^(1/2)*ln(((a^2+1)^(1/2)-b
*x-(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))-a*arcsinh(b*x+a)/(a^2+1)^(1/2)*l
n(((a^2+1)^(1/2)+b*x+(1+(b*x+a)^2)^(1/2))/(-a+(a^2+1)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/x,x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(b*x + a)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{asinh}(a+b\,x)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a + b*x)/x,x)
```

```
[Out] int(asinh(a + b*x)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)/x,x)
```

```
[Out] Integral(asinh(a + b*x)/x, x)
```

3.63 $\int \frac{\sinh^{-1}(a+bx)}{x^2} dx$

Optimal. Leaf size=57

$$-\frac{b \tanh^{-1}\left(\frac{a(a+bx)+1}{\sqrt{a^2+1}\sqrt{(a+bx)^2+1}}\right)}{\sqrt{a^2+1}} - \frac{\sinh^{-1}(a+bx)}{x}$$

```
[Out] -arcsinh(b*x+a)/x-b*arctanh((1+a*(b*x+a))/(a^2+1)^(1/2)/(1+(b*x+a)^2)^(1/2)
)/(a^2+1)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5865, 5801, 725, 206}

$$-\frac{b \tanh^{-1}\left(\frac{a(a+bx)+1}{\sqrt{a^2+1}\sqrt{(a+bx)^2+1}}\right)}{\sqrt{a^2+1}} - \frac{\sinh^{-1}(a+bx)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a + b*x]/x^2,x]
```

```
[Out] -(ArcSinh[a + b*x]/x) - (b*ArcTanh[(1 + a*(a + b*x))/(Sqrt[1 + a^2]*Sqrt[1
+ (a + b*x)^2])])/Sqrt[1 + a^2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 5801

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n
 - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\sinh^{-1}(a+bx)}{x^2}\,dx = \frac{\text{Subst}\left(\int \frac{\sinh^{-1}(x)}{\left(-\frac{a}{b}+\frac{x}{b}\right)^2}\,dx, x, a+bx\right)}{b}$$
$$= -\frac{\sinh^{-1}(a+bx)}{x} + \text{Subst}\left(\int \frac{1}{\left(-\frac{a}{b}+\frac{x}{b}\right)\sqrt{1+x^2}}\,dx, x, a+bx\right)$$
$$= -\frac{\sinh^{-1}(a+bx)}{x} - \text{Subst}\left(\int \frac{1}{\frac{1}{b^2}+\frac{a^2}{b^2}-x^2}\,dx, x, \frac{\frac{1}{b}+\frac{a(a+bx)}{b}}{\sqrt{1+(a+bx)^2}}\right)$$
$$= -\frac{\sinh^{-1}(a+bx)}{x} - \frac{b\tanh^{-1}\left(\frac{b\left(\frac{1}{b}+\frac{a(a+bx)}{b}\right)}{\sqrt{1+a^2}\sqrt{1+(a+bx)^2}}\right)}{\sqrt{1+a^2}}$$

Mathematica [A] time = 0.04, size = 57, normalized size = 1.00

$$-\frac{b\tanh^{-1}\left(\frac{a^2+abx+1}{\sqrt{a^2+1}\sqrt{(a+bx)^2+1}}\right)}{\sqrt{a^2+1}} - \frac{\sinh^{-1}(a+bx)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a + b*x]/x^2,x]
```

```
[Out] -(ArcSinh[a + b*x]/x) - (b*ArcTanh[(1 + a^2 + a*b*x)/(Sqrt[1 + a^2]*Sqrt[1
+ (a + b*x)^2])])/Sqrt[1 + a^2]
```

fricas [B] time = 0.58, size = 167, normalized size = 2.93

$$\frac{\sqrt{a^2+1}\,bx\log\left(-\frac{a^2bx+a^3+\sqrt{b^2x^2+2\,abx+a^2+1}\left(a^2-\sqrt{a^2+1}\,a+1\right)-\left(abx+a^2+1\right)\sqrt{a^2+1}+a}{x}\right) + \left(a^2+1\right)x\log\left(-bx-a+\sqrt{b^2x^2+2\,a}\right.}{\left(a^2+1\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/x^2,x, algorithm="fricas")
```

```
[Out] (sqrt(a^2 + 1)*b*x*log(-(a^2*b*x + a^3 + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*
(a^2 - sqrt(a^2 + 1)*a + 1) - (a*b*x + a^2 + 1)*sqrt(a^2 + 1) + a)/x) + (a^
2 + 1)*x*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - (a^2 - (a^2 +
1)*x + 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)))/((a^2 + 1)*x)
```

giac [B] time = 0.45, size = 110, normalized size = 1.93

$$\frac{b\log\left(\frac{\left|-2\,x|b|+2\,\sqrt{b^2x^2+2\,abx+a^2+1}-2\,\sqrt{a^2+1}\right|}{\left|-2\,x|b|+2\,\sqrt{b^2x^2+2\,abx+a^2+1}+2\,\sqrt{a^2+1}\right|}\right)}{\sqrt{a^2+1}} - \frac{\log\left(bx+a+\sqrt{(bx+a)^2+1}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/x^2,x, algorithm="giac")
```

```
[Out] b*log(abs(-2*x*abs(b) + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*sqrt(a^2 +
1))/abs(-2*x*abs(b) + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 2*sqrt(a^2 + 1)
))/sqrt(a^2 + 1) - log(b*x + a + sqrt((b*x + a)^2 + 1))/x
```

maple [A] time = 0.01, size = 71, normalized size = 1.25

$$-\frac{\operatorname{arcsinh}(bx+a)}{x}-\frac{b\ln\left(\frac{2a^2+2+2abx+2\sqrt{a^2+1}\,\sqrt{b^2x^2+2abx+a^2+1}}{bx}\right)}{\sqrt{a^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)/x^2,x)
```

```
[Out] -arcsinh(b*x+a)/x-b/(a^2+1)^(1/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*
x^2+2*a*b*x+a^2+1)^(1/2))/b/x)
```

maxima [B] time = 0.32, size = 111, normalized size = 1.95

$$-\frac{b\operatorname{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^2b^2+4\left(a^2+1\right)b^2}\,|x|}+\frac{2\,a^2}{\sqrt{-4\,a^2b^2+4\left(a^2+1\right)b^2}\,|x|}+\frac{2}{\sqrt{-4\,a^2b^2+4\left(a^2+1\right)b^2}\,|x|}\right)}{\sqrt{a^2+1}}-\frac{\operatorname{arsinh}(bx+a)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/x^2,x, algorithm="maxima")
```

```
[Out] -b*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2*a^2/(sqr
t(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*
b^2)*abs(x)))/sqrt(a^2 + 1) - arcsinh(b*x + a)/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{asinh}(a+b\,x)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a + b*x)/x^2,x)
```

```
[Out] int(asinh(a + b*x)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)/x**2,x)
```

```
[Out] Integral(asinh(a + b*x)/x**2, x)
```

3.64 $\int \frac{\sinh^{-1}(a+bx)}{x^3} dx$

Optimal. Leaf size=92

$$\frac{ab^2 \tanh^{-1}\left(\frac{a(a+bx)+1}{\sqrt{a^2+1}\sqrt{(a+bx)^2+1}}\right)}{2\left(a^2+1\right)^{3/2}} - \frac{b\sqrt{(a+bx)^2+1}}{2\left(a^2+1\right)x} - \frac{\sinh^{-1}(a+bx)}{2x^2}$$

```
[Out] -1/2*arcsinh(b*x+a)/x^2+1/2*a*b^2*arctanh((1+a*(b*x+a))/(a^2+1)^(1/2)/(1+(b
*x+a)^2)^(1/2))/(a^2+1)^(3/2)-1/2*b*(1+(b*x+a)^2)^(1/2)/(a^2+1)/x
```

Rubi [A] time = 0.10, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5865, 5801, 731, 725, 206}

$$\frac{ab^2 \tanh^{-1}\left(\frac{a(a+bx)+1}{\sqrt{a^2+1}\sqrt{(a+bx)^2+1}}\right)}{2\left(a^2+1\right)^{3/2}} - \frac{b\sqrt{(a+bx)^2+1}}{2\left(a^2+1\right)x} - \frac{\sinh^{-1}(a+bx)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a + b*x]/x^3,x]
```

```
[Out] -(b*Sqrt[1 + (a + b*x)^2])/(2*(1 + a^2)*x) - ArcSinh[a + b*x]/(2*x^2) + (a*
b^2*ArcTanh[(1 + a*(a + b*x))/(Sqrt[1 + a^2]*Sqrt[1 + (a + b*x)^2])])/(2*(1
 + a^2)^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 731

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[(c*d)/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 5801

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n
 - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
```

```
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh^{-1}(a+bx)}{x^3}\,dx &= \frac{\text{Subst}\left(\int \frac{\sinh^{-1}(x)}{\left(-\frac{a}{b}+\frac{x}{b}\right)^3}\,dx, x, a+bx\right)}{b}\\
&= -\frac{\sinh^{-1}(a+bx)}{2x^2}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{\left(-\frac{a}{b}+\frac{x}{b}\right)^2\sqrt{1+x^2}}\,dx, x, a+bx\right)\\
&= -\frac{b\sqrt{1+(a+bx)^2}}{2\left(1+a^2\right)x}-\frac{\sinh^{-1}(a+bx)}{2x^2}-\frac{(ab)\,\text{Subst}\left(\int \frac{1}{\left(-\frac{a}{b}+\frac{x}{b}\right)\sqrt{1+x^2}}\,dx, x, a+bx\right)}{2\left(1+a^2\right)}\\
&= -\frac{b\sqrt{1+(a+bx)^2}}{2\left(1+a^2\right)x}-\frac{\sinh^{-1}(a+bx)}{2x^2}+\frac{(ab)\,\text{Subst}\left(\int \frac{1}{\frac{1}{b^2}+\frac{a^2}{b^2}-x^2}\,dx, x, \frac{\frac{1}{b}+\frac{a(a+bx)}{b}}{\sqrt{1+(a+bx)^2}}\right)}{2\left(1+a^2\right)}\\
&= -\frac{b\sqrt{1+(a+bx)^2}}{2\left(1+a^2\right)x}-\frac{\sinh^{-1}(a+bx)}{2x^2}+\frac{ab^2\tanh^{-1}\left(\frac{1+a(a+bx)}{\sqrt{1+a^2}\sqrt{1+(a+bx)^2}}\right)}{2\left(1+a^2\right)^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.18, size = 110, normalized size = 1.20

$$
-\frac{\frac{bx\left(\sqrt{a^2+1}\sqrt{a^2+2abx+b^2x^2+1}-abx\log\left(\sqrt{a^2+1}\sqrt{a^2+2abx+b^2x^2+1}+a^2+abx+1\right)+abx\log(x)\right)}{\left(a^2+1\right)^{3/2}}+\sinh^{-1}(a+bx)}{2x^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a + b*x]/x^3,x]
```

```
[Out] -1/2*(ArcSinh[a + b*x] + (b*x*(Sqrt[1 + a^2]*Sqrt[1 + a^2 + 2*a*b*x + b^2*x
^2] + a*b*x*Log[x] - a*b*x*Log[1 + a^2 + a*b*x + Sqrt[1 + a^2]*Sqrt[1 + a^2
 + 2*a*b*x + b^2*x^2]]))/(1 + a^2)^(3/2))/x^2
```

fricas [B] time = 0.66, size = 236, normalized size = 2.57

$$
\sqrt{a^2+1}\,ab^2x^2\log\left(-\frac{a^2bx+a^3+\sqrt{b^2x^2+2\,abx+a^2+1}\left(a^2+\sqrt{a^2+1}\,a+1\right)+\left(abx+a^2+1\right)\sqrt{a^2+1}+a}{x}\right)-\left(a^2+1\right)b^2x^2+\left(a^4+2\,a^2+1\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(a^2 + 1)*a*b^2*x^2*log(-(a^2*b*x + a^3 + sqrt(b^2*x^2 + 2*a*b*x +
 a^2 + 1)*(a^2 + sqrt(a^2 + 1)*a + 1) + (a*b*x + a^2 + 1)*sqrt(a^2 + 1) + a
)/x) - (a^2 + 1)*b^2*x^2 + (a^4 + 2*a^2 + 1)*x^2*log(-b*x - a + sqrt(b^2*x^
2 + 2*a*b*x + a^2 + 1)) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)*b*x -
 (a^4 - (a^4 + 2*a^2 + 1)*x^2 + 2*a^2 + 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a
*b*x + a^2 + 1)))/((a^4 + 2*a^2 + 1)*x^2)
```

giac [B] time = 0.61, size = 199, normalized size = 2.16

$$-\frac{1}{2}\left(\frac{ab\log\left(\frac{\left|-2\,x|b|+2\,\sqrt{b^2x^2+2\,abx+a^2+1}-2\,\sqrt{a^2+1}\right|}{\left|-2\,x|b|+2\,\sqrt{b^2x^2+2\,abx+a^2+1}+2\,\sqrt{a^2+1}\right|}\right)}{\left(a^2+1\right)^{\frac{3}{2}}}-\frac{2\left(\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)ab+a^2|b|+|b|\right)}{\left(\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)^2-a^2-1\right)\left(a^2+1\right)}\right)b-\frac{\log\left(bx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/x^3,x, algorithm="giac")
```

```
[Out] -1/2*(a*b*log(abs(-2*x*abs(b) + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*sqr
t(a^2 + 1))/abs(-2*x*abs(b) + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 2*sqrt(
a^2 + 1)))/(a^2 + 1)^(3/2) - 2*((x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 +
1))*a*b + a^2*abs(b) + abs(b))/(((x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 +
 1))^2 - a^2 - 1)*(a^2 + 1)))*b - 1/2*log(b*x + a + sqrt((b*x + a)^2 + 1))/
x^2
```

maple [A] time = 0.01, size = 106, normalized size = 1.15

$$-\frac{\operatorname{arcsinh}\left(bx+a\right)}{2x^2}-\frac{b\sqrt{b^2x^2+2abx+a^2+1}}{2\left(a^2+1\right)x}+\frac{b^2a\ln\left(\frac{2a^2+2+2abx+2\sqrt{a^2+1}\,\sqrt{b^2x^2+2abx+a^2+1}}{bx}\right)}{2\left(a^2+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)/x^3,x)
```

```
[Out] -1/2*arcsinh(b*x+a)/x^2-1/2*b/(a^2+1)/x*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+1/2*b
^2*a/(a^2+1)^(3/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2
+1)^(1/2))/b/x)
```

maxima [A] time = 0.41, size = 146, normalized size = 1.59

$$\frac{1}{2}\left(\frac{ab\operatorname{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^2b^2+4\left(a^2+1\right)b^2}\,|x|}+\frac{2\,a^2}{\sqrt{-4\,a^2b^2+4\left(a^2+1\right)b^2}\,|x|}+\frac{2}{\sqrt{-4\,a^2b^2+4\left(a^2+1\right)b^2}\,|x|}\right)}{\left(a^2+1\right)^{\frac{3}{2}}}-\frac{\sqrt{b^2x^2+2\,abx+a^2+1}}{\left(a^2+1\right)x}\right)b-\frac{\operatorname{ars}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*(a*b*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2*a^
2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(a^2
 + 1)*b^2)*abs(x)))/(a^2 + 1)^(3/2) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/((a
^2 + 1)*x))*b - 1/2*arcsinh(b*x + a)/x^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{asinh}\left(a+b\,x\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a + b*x)/x^3,x)
```

```
[Out] int(asinh(a + b*x)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}(a + bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)/x**3,x)
```

```
[Out] Integral(asinh(a + b*x)/x**3, x)
```

3.65 $\int \frac{\sinh^{-1}(a+bx)}{x^4}\,dx$

Optimal. Leaf size=129

$$\frac{(1-2a^2)\,b^3\tanh^{-1}\left(\frac{a(a+bx)+1}{\sqrt{a^2+1}\sqrt{(a+bx)^2+1}}\right)}{6\left(a^2+1\right)^{5/2}}+\frac{ab^2\sqrt{(a+bx)^2+1}}{2\left(a^2+1\right)^2x}-\frac{b\sqrt{(a+bx)^2+1}}{6\left(a^2+1\right)x^2}-\frac{\sinh^{-1}(a+bx)}{3x^3}$$

```
[Out] -1/3*arcsinh(b*x+a)/x^3+1/6*(-2*a^2+1)*b^3*arctanh((1+a*(b*x+a))/(a^2+1)^(1
/2)/(1+(b*x+a)^2)^(1/2))/(a^2+1)^(5/2)-1/6*b*(1+(b*x+a)^2)^(1/2)/(a^2+1)/x^
2+1/2*a*b^2*(1+(b*x+a)^2)^(1/2)/(a^2+1)^2/x
```

Rubi [A] time = 0.15, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5865, 5801, 745, 807, 725, 206}

$$\frac{ab^2\sqrt{(a+bx)^2+1}}{2\left(a^2+1\right)^2x}+\frac{(1-2a^2)\,b^3\tanh^{-1}\left(\frac{a(a+bx)+1}{\sqrt{a^2+1}\sqrt{(a+bx)^2+1}}\right)}{6\left(a^2+1\right)^{5/2}}-\frac{b\sqrt{(a+bx)^2+1}}{6\left(a^2+1\right)x^2}-\frac{\sinh^{-1}(a+bx)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a + b*x]/x^4,x]
```

```
[Out] -(b*Sqrt[1 + (a + b*x)^2])/(6*(1 + a^2)*x^2) + (a*b^2*Sqrt[1 + (a + b*x)^2]
)/(2*(1 + a^2)^2*x) - ArcSinh[a + b*x]/(3*x^3) + ((1 - 2*a^2)*b^3*ArcTanh[(
1 + a*(a + b*x))/(Sqrt[1 + a^2]*Sqrt[1 + (a + b*x)^2])])/(6*(1 + a^2)^(5/2)
)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 5801

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n
 - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\sinh^{-1}(a+bx)}{x^4}\,dx = \frac{\text{Subst}\left(\int \frac{\sinh^{-1}(x)}{\left(-\frac{a}{b}+\frac{x}{b}\right)^4}\,dx, x, a+bx\right)}{b}$$

$$= -\frac{\sinh^{-1}(a+bx)}{3x^3} + \frac{1}{3}\text{Subst}\left(\int \frac{1}{\left(-\frac{a}{b}+\frac{x}{b}\right)^3\sqrt{1+x^2}}\,dx, x, a+bx\right)$$

$$= -\frac{b\sqrt{1+(a+bx)^2}}{6\left(1+a^2\right)x^2} - \frac{\sinh^{-1}(a+bx)}{3x^3} - \frac{b^2\,\text{Subst}\left(\int \frac{\frac{2a}{b}+\frac{x}{b}}{\left(-\frac{a}{b}+\frac{x}{b}\right)^2\sqrt{1+x^2}}\,dx, x, a+bx\right)}{6\left(1+a^2\right)}$$

$$= -\frac{b\sqrt{1+(a+bx)^2}}{6\left(1+a^2\right)x^2} + \frac{ab^2\sqrt{1+(a+bx)^2}}{2\left(1+a^2\right)^2x} - \frac{\sinh^{-1}(a+bx)}{3x^3} - \frac{\left(\left(1-2a^2\right)b^2\right)\text{Subst}\left(\int \ldots\right.}{6\left(1\ldots\right.}$$

$$= -\frac{b\sqrt{1+(a+bx)^2}}{6\left(1+a^2\right)x^2} + \frac{ab^2\sqrt{1+(a+bx)^2}}{2\left(1+a^2\right)^2x} - \frac{\sinh^{-1}(a+bx)}{3x^3} + \frac{\left(\left(1-2a^2\right)b^2\right)\text{Subst}\left(\int \ldots\right.}{6\left(1\ldots\right.}$$

$$= -\frac{b\sqrt{1+(a+bx)^2}}{6\left(1+a^2\right)x^2} + \frac{ab^2\sqrt{1+(a+bx)^2}}{2\left(1+a^2\right)^2x} - \frac{\sinh^{-1}(a+bx)}{3x^3} + \frac{\left(1-2a^2\right)b^3\tanh^{-1}\left(\frac{\ldots}{\sqrt{1+\ldots}}\right.}{6\left(1+a^2\right)^{5/2}\ldots}$$

Mathematica [A] time = 0.24, size = 149, normalized size = 1.16

$$\frac{\left(2a^2-1\right)b^3x^3\log(x) - \sqrt{a^2+1}\,bx\left(a^2-3abx+1\right)\sqrt{a^2+2abx+b^2x^2+1} + \left(1-2a^2\right)b^3x^3\log\left(\sqrt{a^2+1}\sqrt{a^2\ldots}\right.}{6\left(a^2+1\right)^{5/2}x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a + b*x]/x^4,x]
```

```
[Out] (-(Sqrt[1 + a^2]*b*x*(1 + a^2 - 3*a*b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2])
 - 2*(1 + a^2)^(5/2)*ArcSinh[a + b*x] + (-1 + 2*a^2)*b^3*x^3*Log[x] + (1 -
2*a^2)*b^3*x^3*Log[1 + a^2 + a*b*x + Sqrt[1 + a^2]*Sqrt[1 + a^2 + 2*a*b*x +
 b^2*x^2]])/(6*(1 + a^2)^(5/2)*x^3)
```

fricas [B] time = 0.72, size = 285, normalized size = 2.21

$$\frac{\left(2\,a^{2}-1\right)\sqrt{a^{2}+1}\,b^{3}x^{3}\log\left(-\frac{a^{2}bx+a^{3}+\sqrt{b^{2}x^{2}+2\,abx+a^{2}+1}\left(a^{2}-\sqrt{a^{2}+1}\,a+1\right)-\left(abx+a^{2}+1\right)\sqrt{a^{2}+1}+a}{x}\right)+3\left(a^{3}+a\right)b^{3}x^{3}+2\left(a^{6}+3\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*((2*a^2 - 1)*sqrt(a^2 + 1)*b^3*x^3*log(-(a^2*b*x + a^3 + sqrt(b^2*x^2 +
 2*a*b*x + a^2 + 1)*(a^2 - sqrt(a^2 + 1)*a + 1) - (a*b*x + a^2 + 1)*sqrt(a^
2 + 1) + a)/x) + 3*(a^3 + a)*b^3*x^3 + 2*(a^6 + 3*a^4 + 3*a^2 + 1)*x^3*log(
-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - 2*(a^6 + 3*a^4 - (a^6 + 3*a
^4 + 3*a^2 + 1)*x^3 + 3*a^2 + 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2
 + 1)) + (3*(a^3 + a)*b^2*x^2 - (a^4 + 2*a^2 + 1)*b*x)*sqrt(b^2*x^2 + 2*a*b
*x + a^2 + 1))/((a^6 + 3*a^4 + 3*a^2 + 1)*x^3)
```

giac [B] time = 0.47, size = 381, normalized size = 2.95

$$\frac{1}{6}\,b\left(\frac{\left(2\,a^{2}b^{2}-b^{2}\right)\log\left(\frac{\left|-2\,x|b|+2\,\sqrt{b^{2}x^{2}+2\,abx+a^{2}+1}-2\,\sqrt{a^{2}+1}\right|}{\left|-2\,x|b|+2\,\sqrt{b^{2}x^{2}+2\,abx+a^{2}+1}+2\,\sqrt{a^{2}+1}\right|}\right)}{\left(a^{4}+2\,a^{2}+1\right)\sqrt{a^{2}+1}}-\frac{2\left(2\left(x|b|-\sqrt{b^{2}x^{2}+2\,abx+a^{2}+1}\right)^{3}a^{2}b^{2}-6\left(x|b|-\cdots\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/x^4,x, algorithm="giac")
```

```
[Out] 1/6*b*((2*a^2*b^2 - b^2)*log(abs(-2*x*abs(b) + 2*sqrt(b^2*x^2 + 2*a*b*x + a
^2 + 1) - 2*sqrt(a^2 + 1))/abs(-2*x*abs(b) + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2
 + 1) + 2*sqrt(a^2 + 1)))/((a^4 + 2*a^2 + 1)*sqrt(a^2 + 1)) - 2*(2*(x*abs(b
) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^3*a^2*b^2 - 6*(x*abs(b) - sqrt(b^2*x
^2 + 2*a*b*x + a^2 + 1))*a^4*b^2 - 4*a^5*b*abs(b) - (x*abs(b) - sqrt(b^2*x^
2 + 2*a*b*x + a^2 + 1))^3*b^2 - 7*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2
+ 1))*a^2*b^2 - 8*a^3*b*abs(b) - (x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 +
 1))*b^2 - 4*a*b*abs(b))/((a^4 + 2*a^2 + 1)*((x*abs(b) - sqrt(b^2*x^2 + 2*a
*b*x + a^2 + 1))^2 - a^2 - 1)^2)) - 1/3*log(b*x + a + sqrt((b*x + a)^2 + 1)
)/x^3
```

maple [A] time = 0.01, size = 203, normalized size = 1.57

$$-\frac{\operatorname{arcsinh}\left(bx+a\right)}{3x^{3}}-\frac{b\sqrt{b^{2}x^{2}+2abx+a^{2}+1}}{6\left(a^{2}+1\right)x^{2}}+\frac{b^{2}a\sqrt{b^{2}x^{2}+2abx+a^{2}+1}}{2\left(a^{2}+1\right)^{2}x}-\frac{b^{3}a^{2}\ln\left(\frac{2a^{2}+2+2abx+2\sqrt{a^{2}+1}\,\sqrt{b^{2}x^{2}+2abx+\cdots}}{bx}\right.}{2\left(a^{2}+1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)/x^4,x)
```

```
[Out] -1/3*arcsinh(b*x+a)/x^3-1/6*b/(a^2+1)/x^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+1/2
*b^2*a/(a^2+1)^2/x*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-1/2*b^3*a^2/(a^2+1)^(5/2)*
ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/b/x)+1/6
*b^3/(a^2+1)^(3/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2
+1)^(1/2))/b/x)
```

maxima [B] time = 0.36, size = 284, normalized size = 2.20

$$-\frac{1}{6}\left(\frac{3\,a^2b^2\,\operatorname{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^2b^2+4\,\left(a^2+1\right)b^2}\,|x|}+\frac{2\,a^2}{\sqrt{-4\,a^2b^2+4\,\left(a^2+1\right)b^2}\,|x|}+\frac{2}{\sqrt{-4\,a^2b^2+4\,\left(a^2+1\right)b^2}\,|x|}\right)}{\left(a^2+1\right)^{\frac{5}{2}}}-\frac{b^2\,\operatorname{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^2b^2+4\,(a}\right.}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/x^4,x, algorithm="maxima")
```

```
[Out] -1/6*(3*a^2*b^2*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x))
 + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 +
 4*(a^2 + 1)*b^2)*abs(x)))/(a^2 + 1)^(5/2) - b^2*arcsinh(2*a*b*x/(sqrt(-4*a
^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^
2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)))/(a^2 + 1)^(3/2)
 - 3*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a*b/((a^2 + 1)^2*x) + sqrt(b^2*x^2 +
 2*a*b*x + a^2 + 1)/((a^2 + 1)*x^2))*b - 1/3*arcsinh(b*x + a)/x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{asinh}\left(a+b\,x\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a + b*x)/x^4,x)
```

```
[Out] int(asinh(a + b*x)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}\left(a+bx\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)/x**4,x)
```

```
[Out] Integral(asinh(a + b*x)/x**4, x)
```

3.66 $\int \frac{\sinh^{-1}(a+bx)}{x^5} dx$

Optimal. Leaf size=167

$$-\frac{a\left(3-2a^2\right)b^4\tanh^{-1}\left(\frac{a(a+bx)+1}{\sqrt{a^2+1}\sqrt{(a+bx)^2+1}}\right)}{8\left(a^2+1\right)^{7/2}}+\frac{\left(4-11a^2\right)b^3\sqrt{(a+bx)^2+1}}{24\left(a^2+1\right)^3x}+\frac{5ab^2\sqrt{(a+bx)^2+1}}{24\left(a^2+1\right)^2x^2}-\frac{b\sqrt{(a+bx)^2+1}}{12\left(a^2+1\right)x^3}$$

```
[Out] -1/4*arcsinh(b*x+a)/x^4-1/8*a*(-2*a^2+3)*b^4*arctanh((1+a*(b*x+a))/(a^2+1)^
(1/2)/(1+(b*x+a)^2)^(1/2))/(a^2+1)^(7/2)-1/12*b*(1+(b*x+a)^2)^(1/2)/(a^2+1)
/x^3+5/24*a*b^2*(1+(b*x+a)^2)^(1/2)/(a^2+1)^2/x^2+1/24*(-11*a^2+4)*b^3*(1+(
b*x+a)^2)^(1/2)/(a^2+1)^3/x
```

Rubi [A] time = 0.23, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {5865, 5801, 745, 835, 807, 725, 206}

$$\frac{5ab^2\sqrt{(a+bx)^2+1}}{24\left(a^2+1\right)^2x^2}+\frac{\left(4-11a^2\right)b^3\sqrt{(a+bx)^2+1}}{24\left(a^2+1\right)^3x}-\frac{a\left(3-2a^2\right)b^4\tanh^{-1}\left(\frac{a(a+bx)+1}{\sqrt{a^2+1}\sqrt{(a+bx)^2+1}}\right)}{8\left(a^2+1\right)^{7/2}}-\frac{b\sqrt{(a+bx)^2+1}}{12\left(a^2+1\right)x^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a + b*x]/x^5,x]
```

```
[Out] -(b*Sqrt[1 + (a + b*x)^2])/(12*(1 + a^2)*x^3) + (5*a*b^2*Sqrt[1 + (a + b*x)
^2])/(24*(1 + a^2)^2*x^2) + ((4 - 11*a^2)*b^3*Sqrt[1 + (a + b*x)^2])/(24*(1
 + a^2)^3*x) - ArcSinh[a + b*x]/(4*x^4) - (a*(3 - 2*a^2)*b^4*ArcTanh[(1 + a
*(a + b*x))/(Sqrt[1 + a^2]*Sqrt[1 + (a + b*x)^2])])/(8*(1 + a^2)^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
```

```
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 5801

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n
 - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\sinh^{-1}(a+bx)}{x^5}\,dx = \frac{\text{Subst}\left(\int \frac{\sinh^{-1}(x)}{\left(-\frac{a}{b}+\frac{x}{b}\right)^5}\,dx, x, a+bx\right)}{b}$$

$$= -\frac{\sinh^{-1}(a+bx)}{4x^4} + \frac{1}{4}\text{Subst}\left(\int \frac{1}{\left(-\frac{a}{b}+\frac{x}{b}\right)^4\sqrt{1+x^2}}\,dx, x, a+bx\right)$$

$$= -\frac{b\sqrt{1+(a+bx)^2}}{12\left(1+a^2\right)x^3} - \frac{\sinh^{-1}(a+bx)}{4x^4} - \frac{b^2\,\text{Subst}\left(\int \frac{\frac{3a}{b}+\frac{2x}{b}}{\left(-\frac{a}{b}+\frac{x}{b}\right)^3\sqrt{1+x^2}}\,dx, x, a+bx\right)}{12\left(1+a^2\right)}$$

$$= -\frac{b\sqrt{1+(a+bx)^2}}{12\left(1+a^2\right)x^3} + \frac{5ab^2\sqrt{1+(a+bx)^2}}{24\left(1+a^2\right)^2x^2} - \frac{\sinh^{-1}(a+bx)}{4x^4} + \frac{b^4\,\text{Subst}\left(\int \frac{-\frac{2\left(2-3a^2\right)}{b^2}+\frac{5a}{b}\cdots}{\left(-\frac{a}{b}+\frac{x}{b}\right)^2\sqrt{1+\cdots}}\cdots\right.}{24\left(1+a^2\cdots\right.}$$

$$= -\frac{b\sqrt{1+(a+bx)^2}}{12\left(1+a^2\right)x^3} + \frac{5ab^2\sqrt{1+(a+bx)^2}}{24\left(1+a^2\right)^2x^2} + \frac{\left(4-11a^2\right)b^3\sqrt{1+(a+bx)^2}}{24\left(1+a^2\right)^3x} - \frac{\sinh^{-1}(a\cdots}{4x^4}$$

$$= -\frac{b\sqrt{1+(a+bx)^2}}{12\left(1+a^2\right)x^3} + \frac{5ab^2\sqrt{1+(a+bx)^2}}{24\left(1+a^2\right)^2x^2} + \frac{\left(4-11a^2\right)b^3\sqrt{1+(a+bx)^2}}{24\left(1+a^2\right)^3x} - \frac{\sinh^{-1}(a\cdots}{4x^4}$$

$$= -\frac{b\sqrt{1+(a+bx)^2}}{12\left(1+a^2\right)x^3} + \frac{5ab^2\sqrt{1+(a+bx)^2}}{24\left(1+a^2\right)^2x^2} + \frac{\left(4-11a^2\right)b^3\sqrt{1+(a+bx)^2}}{24\left(1+a^2\right)^3x} - \frac{\sinh^{-1}(a\cdots}{4x^4}$$

Mathematica [A] time = 0.21, size = 179, normalized size = 1.07

$$\frac{1}{8}\left(-\frac{a\left(2a^2-3\right)b^4\log(x)}{\left(a^2+1\right)^{7/2}}+\frac{a\left(2a^2-3\right)b^4\log\left(\sqrt{a^2+1}\sqrt{a^2+2abx+b^2x^2+1}+a^2+abx+1\right)}{\left(a^2+1\right)^{7/2}}-\frac{b\sqrt{a^2+2abx+\cdots}}{\cdots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a + b*x]/x^5,x]
```

```
[Out] (-1/3*(b*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(2 + 2*a^4 - 5*a*b*x - 5*a^3*b*x
 - 4*b^2*x^2 + a^2*(4 + 11*b^2*x^2)))/((1 + a^2)^3*x^3) - (2*ArcSinh[a + b*
x])/x^4 - (a*(-3 + 2*a^2)*b^4*Log[x])/(1 + a^2)^(7/2) + (a*(-3 + 2*a^2)*b^4
*Log[1 + a^2 + a*b*x + Sqrt[1 + a^2]*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]])/(1
 + a^2)^(7/2))/8
```

fricas [B] time = 0.71, size = 343, normalized size = 2.05

$$\frac{3\left(2\,a^3-3\,a\right)\sqrt{a^2+1}\,b^4x^4\log\left(-\frac{a^2bx+a^3+\sqrt{b^2x^2+2\,abx+a^2+1}\left(a^2+\sqrt{a^2+1}\,a+1\right)+\left(abx+a^2+1\right)\sqrt{a^2+1}+a}{x}\right)-\left(11\,a^4+7\,a^2-4\right)b^4x^4\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/x^5,x, algorithm="fricas")
```

```
[Out] 1/24*(3*(2*a^3 - 3*a)*sqrt(a^2 + 1)*b^4*x^4*log(-(a^2*b*x + a^3 + sqrt(b^2*
x^2 + 2*a*b*x + a^2 + 1)*(a^2 + sqrt(a^2 + 1)*a + 1) + (a*b*x + a^2 + 1)*sq
rt(a^2 + 1) + a)/x) - (11*a^4 + 7*a^2 - 4)*b^4*x^4 + 6*(a^8 + 4*a^6 + 6*a^4
 + 4*a^2 + 1)*x^4*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - 6*(a^
8 + 4*a^6 - (a^8 + 4*a^6 + 6*a^4 + 4*a^2 + 1)*x^4 + 6*a^4 + 4*a^2 + 1)*log(
b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - ((11*a^4 + 7*a^2 - 4)*b^3*x^
3 - 5*(a^5 + 2*a^3 + a)*b^2*x^2 + 2*(a^6 + 3*a^4 + 3*a^2 + 1)*b*x)*sqrt(b^2
*x^2 + 2*a*b*x + a^2 + 1))/((a^8 + 4*a^6 + 6*a^4 + 4*a^2 + 1)*x^4)
```

giac [B] time = 0.57, size = 709, normalized size = 4.25

$$-\frac{1}{24}\,b\left(\frac{3\left(2\,a^3b^3-3\,ab^3\right)\log\left(\frac{\left|-2\,x|b|+2\,\sqrt{b^2x^2+2\,abx+a^2+1}-2\,\sqrt{a^2+1}\right|}{\left|-2\,x|b|+2\,\sqrt{b^2x^2+2\,abx+a^2+1}+2\,\sqrt{a^2+1}\right|}\right)}{\left(a^6+3\,a^4+3\,a^2+1\right)\sqrt{a^2+1}}-\frac{2\left(6\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)^5a^3b^3-16\cdots\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/x^5,x, algorithm="giac")
```

```
[Out] -1/24*b*(3*(2*a^3*b^3 - 3*a*b^3)*log(abs(-2*x*abs(b) + 2*sqrt(b^2*x^2 + 2*a
*b*x + a^2 + 1) - 2*sqrt(a^2 + 1))/abs(-2*x*abs(b) + 2*sqrt(b^2*x^2 + 2*a*b
*x + a^2 + 1) + 2*sqrt(a^2 + 1)))/((a^6 + 3*a^4 + 3*a^2 + 1)*sqrt(a^2 + 1))
 - 2*(6*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^5*a^3*b^3 - 16*(x*ab
s(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^3*a^5*b^3 + 42*(x*abs(b) - sqrt(b
^2*x^2 + 2*a*b*x + a^2 + 1))*a^7*b^3 + 12*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*
x + a^2 + 1))^2*a^6*b^2*abs(b) + 20*a^8*b^2*abs(b) - 9*(x*abs(b) - sqrt(b^2
*x^2 + 2*a*b*x + a^2 + 1))^5*a*b^3 + 8*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x +
 a^2 + 1))^3*a^3*b^3 + 93*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*a^
5*b^3 + 36*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2*a^4*b^2*abs(b)
+ 56*a^6*b^2*abs(b) + 24*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^3*a
*b^3 + 60*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*a^3*b^3 + 36*(x*ab
s(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2*a^2*b^2*abs(b) + 48*a^4*b^2*abs
```

```
(b) + 9*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*a*b^3 + 12*(x*abs(b)
 - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2*b^2*abs(b) + 8*a^2*b^2*abs(b) - 4*b
^2*abs(b))/((a^6 + 3*a^4 + 3*a^2 + 1)*((x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x +
 a^2 + 1))^2 - a^2 - 1)^3)) - 1/4*log(b*x + a + sqrt((b*x + a)^2 + 1))/x^4
```

maple [A] time = 0.01, size = 275, normalized size = 1.65

$$-\frac{\operatorname{arcsinh}(bx+a)}{4x^4}-\frac{b\sqrt{b^2x^2+2abx+a^2+1}}{12\left(a^2+1\right)x^3}+\frac{5b^2a\sqrt{b^2x^2+2abx+a^2+1}}{24\left(a^2+1\right)^2x^2}-\frac{5b^3a^2\sqrt{b^2x^2+2abx+a^2+1}}{8\left(a^2+1\right)^3x}+\frac{5b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)/x^5,x)
```

```
[Out] -1/4*arcsinh(b*x+a)/x^4-1/12*b/(a^2+1)/x^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+5/
24*b^2*a/(a^2+1)^2/x^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-5/8*b^3*a^2/(a^2+1)^3/
x*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+5/8*b^4*a^3/(a^2+1)^(7/2)*ln((2*a^2+2+2*a*b
*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/b/x)-3/8*b^4*a/(a^2+1)^(5
/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/b/x)
+1/6*b^3/(a^2+1)^2/x*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)
```

maxima [B] time = 0.36, size = 357, normalized size = 2.14

$$\frac{1}{24}\left(\frac{15\,a^3b^3\operatorname{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}\,|x|}+\frac{2\,a^2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}\,|x|}+\frac{2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}\,|x|}\right)}{\left(a^2+1\right)^{\frac{7}{2}}}-\frac{9\,ab^3\operatorname{arsinh}\left(\frac{2}{\sqrt{-4\,a^2b^2\cdots}}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/x^5,x, algorithm="maxima")
```

```
[Out] 1/24*(15*a^3*b^3*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)
) + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2
+ 4*(a^2 + 1)*b^2)*abs(x)))/(a^2 + 1)^(7/2) - 9*a*b^3*arcsinh(2*a*b*x/(sqrt
(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 +
1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)))/(a^2 + 1)^
(5/2) - 15*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^2*b^2/((a^2 + 1)^3*x) + 4*sq
rt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^2/((a^2 + 1)^2*x) + 5*sqrt(b^2*x^2 + 2*a*
b*x + a^2 + 1)*a*b/((a^2 + 1)^2*x^2) - 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/
((a^2 + 1)*x^3))*b - 1/4*arcsinh(b*x + a)/x^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{asinh}(a+b\,x)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a + b*x)/x^5,x)
```

```
[Out] int(asinh(a + b*x)/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}(a+bx)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)/x**5,x)
```

```
[Out] Integral(asinh(a + b*x)/x**5, x)
```

3.67 $\int x^3 \sinh^{-1}(a+bx)^2 \, dx$

Optimal. Leaf size=331

$$-\frac{a^4\sinh^{-1}(a+bx)^2}{4b^4}+\frac{2a^3\sqrt{(a+bx)^2+1}\,\sinh^{-1}(a+bx)}{b^4}-\frac{2a^3x}{b^3}+\frac{3a^2(a+bx)^2}{4b^4}+\frac{3a^2\sinh^{-1}(a+bx)^2}{4b^4}-\frac{3a^2(a+bx)}{}$$

```
[Out] 4/3*a*x/b^3-2*a^3*x/b^3-3/32*(b*x+a)^2/b^4+3/4*a^2*(b*x+a)^2/b^4-2/9*a*(b*x
+a)^3/b^4+1/32*(b*x+a)^4/b^4-3/32*arcsinh(b*x+a)^2/b^4+3/4*a^2*arcsinh(b*x+
a)^2/b^4-1/4*a^4*arcsinh(b*x+a)^2/b^4+1/4*x^4*arcsinh(b*x+a)^2-4/3*a*arcsin
h(b*x+a)*(1+(b*x+a)^2)^(1/2)/b^4+2*a^3*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)/b
^4+3/16*(b*x+a)*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)/b^4-3/2*a^2*(b*x+a)*arcs
inh(b*x+a)*(1+(b*x+a)^2)^(1/2)/b^4+2/3*a*(b*x+a)^2*arcsinh(b*x+a)*(1+(b*x+a
)^2)^(1/2)/b^4-1/8*(b*x+a)^3*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)/b^4
```

Rubi [A] time = 0.55, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5865, 5801, 5821, 5675, 5717, 8, 5758, 30}

$$-\frac{2a^3x}{b^3}+\frac{3a^2(a+bx)^2}{4b^4}-\frac{a^4\sinh^{-1}(a+bx)^2}{4b^4}+\frac{2a^3\sqrt{(a+bx)^2+1}\,\sinh^{-1}(a+bx)}{b^4}+\frac{3a^2\sinh^{-1}(a+bx)^2}{4b^4}-\frac{3a^2(a+bx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcSinh[a + b*x]^2,x]
```

```
[Out] (4*a*x)/(3*b^3) - (2*a^3*x)/b^3 - (3*(a + b*x)^2)/(32*b^4) + (3*a^2*(a + b*
x)^2)/(4*b^4) - (2*a*(a + b*x)^3)/(9*b^4) + (a + b*x)^4/(32*b^4) - (4*a*Sqr
t[1 + (a + b*x)^2]*ArcSinh[a + b*x])/(3*b^4) + (2*a^3*Sqrt[1 + (a + b*x)^2]
*ArcSinh[a + b*x])/b^4 + (3*(a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x
])/(16*b^4) - (3*a^2*(a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x])/(2*b
^4) + (2*a*(a + b*x)^2*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x])/(3*b^4) - ((
a + b*x)^3*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x])/(8*b^4) - (3*ArcSinh[a +
 b*x]^2)/(32*b^4) + (3*a^2*ArcSinh[a + b*x]^2)/(4*b^4) - (a^4*ArcSinh[a + b
*x]^2)/(4*b^4) + (x^4*ArcSinh[a + b*x]^2)/4
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5801

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n
 - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int x^3 \sinh^{-1}(a+bx)^2\,dx = \frac{\text{Subst}\left(\int \left(-\frac{a}{b}+\frac{x}{b}\right)^3 \sinh^{-1}(x)^2\,dx, x, a+bx\right)}{b}$$

$$= \frac{1}{4}x^4\sinh^{-1}(a+bx)^2 - \frac{1}{2}\text{Subst}\left(\int \frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^4 \sinh^{-1}(x)}{\sqrt{1+x^2}}\,dx, x, a+bx\right)$$

$$= \frac{1}{4}x^4\sinh^{-1}(a+bx)^2 - \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^4\sinh^{-1}(x)}{b^4\sqrt{1+x^2}} - \frac{4a^3x\sinh^{-1}(x)}{b^4\sqrt{1+x^2}} + \frac{6a^2x^2\sinh^{-1}}{b^4\sqrt{1+x^2}}\right.\right.$$

$$= \frac{1}{4}x^4\sinh^{-1}(a+bx)^2 - \frac{\text{Subst}\left(\int \frac{x^4\sinh^{-1}(x)}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{2b^4} + \frac{(2a)\,\text{Subst}\left(\int \frac{x^3\sinh^{-1}(x)}{\sqrt{1+x^2}}\right.}{b^4}$$

$$= \frac{2a^3\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{b^4} - \frac{3a^2(a+bx)\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{2b^4} +$$

$$= -\frac{2a^3x}{b^3} + \frac{3a^2(a+bx)^2}{4b^4} - \frac{2a(a+bx)^3}{9b^4} + \frac{(a+bx)^4}{32b^4} - \frac{4a\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+b}{3b^4}$$

$$= \frac{4ax}{3b^3} - \frac{2a^3x}{b^3} - \frac{3(a+bx)^2}{32b^4} + \frac{3a^2(a+bx)^2}{4b^4} - \frac{2a(a+bx)^3}{9b^4} + \frac{(a+bx)^4}{32b^4} - \frac{4a\sqrt{1+(a-}{}$$

Mathematica [A] time = 0.19, size = 145, normalized size = 0.44

$$\frac{-9\left(8a^4-24a^2-8b^4x^4+3\right)\sinh^{-1}(a+bx)^2+bx\left(-300a^3+78a^2bx+a\left(330-28b^2x^2\right)+9bx\left(b^2x^2-3\right)\right)+6\sqrt{a}}{288b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcSinh[a + b*x]^2,x]
```

```
[Out] (b*x*(-300*a^3 + 78*a^2*b*x + a*(330 - 28*b^2*x^2) + 9*b*x*(-3 + b^2*x^2))
+ 6*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(50*a^3 + 9*b*x - 26*a^2*b*x - 6*b^3*
x^3 + a*(-55 + 14*b^2*x^2))*ArcSinh[a + b*x] - 9*(3 - 24*a^2 + 8*a^4 - 8*b^
4*x^4)*ArcSinh[a + b*x]^2)/(288*b^4)
```

fricas [A] time = 0.54, size = 182, normalized size = 0.55

$$9\,b^4x^4-28\,ab^3x^3+3\left(26\,a^2-9\right)b^2x^2-30\left(10\,a^3-11\,a\right)bx+9\left(8\,b^4x^4-8\,a^4+24\,a^2-3\right)\log\left(bx+a+\sqrt{b^2x^2+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/288*(9*b^4*x^4 - 28*a*b^3*x^3 + 3*(26*a^2 - 9)*b^2*x^2 - 30*(10*a^3 - 11*
a)*b*x + 9*(8*b^4*x^4 - 8*a^4 + 24*a^2 - 3)*log(b*x + a + sqrt(b^2*x^2 + 2*
a*b*x + a^2 + 1))^2 - 6*(6*b^3*x^3 - 14*a*b^2*x^2 - 50*a^3 + (26*a^2 - 9)*b
*x + 55*a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*log(b*x + a + sqrt(b^2*x^2 + 2
*a*b*x + a^2 + 1)))/b^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{arsinh}(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*arcsinh(b*x + a)^2, x)
```

maple [A] time = 0.11, size = 377, normalized size = 1.14

$$-a^3\left(\operatorname{arcsinh}(bx+a)^2(bx+a)-2\operatorname{arcsinh}(bx+a)\sqrt{1+(bx+a)^2}+2bx+2a\right)+\frac{3a^2\left(2\operatorname{arcsinh}(bx+a)^2(bx+a)^2-2\operatorname{arcsir}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arcsinh(b*x+a)^2,x)
```

```
[Out] 1/b^4*(-a^3*(arcsinh(b*x+a)^2*(b*x+a)-2*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)+
2*b*x+2*a)+3/4*a^2*(2*arcsinh(b*x+a)^2*(b*x+a)^2-2*arcsinh(b*x+a)*(1+(b*x+a
)^2)^(1/2)*(b*x+a)+arcsinh(b*x+a)^2+(b*x+a)^2+1)-1/9*a*(9*arcsinh(b*x+a)^2*
(b*x+a)^3-6*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)*(b*x+a)^2+27*arcsinh(b*x+a)^
2*(b*x+a)+2*(b*x+a)^3-42*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)+42*b*x+42*a)+1/
4*arcsinh(b*x+a)^2*(1+(b*x+a)^2)^2-1/8*arcsinh(b*x+a)*(b*x+a)*(1+(b*x+a)^2)
^(3/2)+5/16*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)*(b*x+a)+5/32*arcsinh(b*x+a)^
2+1/32*(1+(b*x+a)^2)^2-5/32*(b*x+a)^2-5/32+3*a*(arcsinh(b*x+a)^2*(b*x+a)-2*
arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)+2*b*x+2*a)-1/2*arcsinh(b*x+a)^2*(1+(b*x+
a)^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4}x^4\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^2-\int\frac{\left(b^3x^6+2\,ab^2x^5+\left(a^2b+b\right)x^4+\left(b^2x^5+abx^4\right)\sqrt{b^2x^2+2\,abx}\right.}{2\left(b^3x^3+3\,ab^2x^2+a^3+\left(3\,a^2b+b\right)x+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*x^4*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2 - integrate(1/2*
(b^3*x^6 + 2*a*b^2*x^5 + (a^2*b + b)*x^4 + (b^2*x^5 + a*b*x^4)*sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/(b^
3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b + b)*x + (b^2*x^2 + 2*a*b*x + a^2 + 1)
^(3/2) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,\mathrm{asinh}\left(a+b\,x\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*asinh(a + b*x)^2,x)
```

```
[Out] int(x^3*asinh(a + b*x)^2, x)
```

sympy [A] time = 3.11, size = 366, normalized size = 1.11

$$\begin{cases}-\frac{a^4\operatorname{asinh}^2(a+bx)}{4b^4}-\frac{25a^3x}{24b^3}+\frac{25a^3\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}(a+bx)}{24b^4}+\frac{13a^2x^2}{48b^2}-\frac{13a^2x\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}(a+bx)}{24b^3}+\frac{3a^2\operatorname{asinh}^2(a+\right.}{4b^4} \\ \frac{x^4\operatorname{asinh}^2(a)}{4}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*asinh(b*x+a)**2,x)
```

```
[Out] Piecewise((-a**4*asinh(a + b*x)**2/(4*b**4) - 25*a**3*x/(24*b**3) + 25*a**3
*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/(24*b**4) + 13*a**2*x*
*2/(48*b**2) - 13*a**2*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x
)/(24*b**3) + 3*a**2*asinh(a + b*x)**2/(4*b**4) - 7*a*x**3/(72*b) + 7*a*x**
2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/(24*b**2) + 55*a*x/(4
8*b**3) - 55*a*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/(48*b**4
) + x**4*asinh(a + b*x)**2/4 + x**4/32 - x**3*sqrt(a**2 + 2*a*b*x + b**2*x*
*2 + 1)*asinh(a + b*x)/(8*b) - 3*x**2/(32*b**2) + 3*x*sqrt(a**2 + 2*a*b*x +
 b**2*x**2 + 1)*asinh(a + b*x)/(16*b**3) - 3*asinh(a + b*x)**2/(32*b**4), N
e(b, 0)), (x**4*asinh(a)**2/4, True))
```

3.68 $\int x^2 \sinh^{-1}(a+bx)^2 \, dx$

Optimal. Leaf size=211

$$\frac{a^3 \sinh^{-1}(a+bx)^2}{3b^3}-\frac{2a^2\sqrt{(a+bx)^2+1}\,\sinh^{-1}(a+bx)}{b^3}+\frac{2a^2x}{b^2}-\frac{a(a+bx)^2}{2b^3}+\frac{2(a+bx)^3}{27b^3}-\frac{a\sinh^{-1}(a+bx)^2}{2b^3}+\frac{a(a+$$

```
[Out] -4/9*x/b^2+2*a^2*x/b^2-1/2*a*(b*x+a)^2/b^3+2/27*(b*x+a)^3/b^3-1/2*a*arcsinh
(b*x+a)^2/b^3+1/3*a^3*arcsinh(b*x+a)^2/b^3+1/3*x^3*arcsinh(b*x+a)^2+4/9*arc
sinh(b*x+a)*(1+(b*x+a)^2)^(1/2)/b^3-2*a^2*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2
)/b^3+a*(b*x+a)*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)/b^3-2/9*(b*x+a)^2*arcsin
h(b*x+a)*(1+(b*x+a)^2)^(1/2)/b^3
```

Rubi [A] time = 0.37, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5865, 5801, 5821, 5675, 5717, 8, 5758, 30}

$$\frac{2a^2x}{b^2}+\frac{a^3 \sinh^{-1}(a+bx)^2}{3b^3}-\frac{2a^2\sqrt{(a+bx)^2+1}\,\sinh^{-1}(a+bx)}{b^3}-\frac{a(a+bx)^2}{2b^3}+\frac{2(a+bx)^3}{27b^3}-\frac{a\sinh^{-1}(a+bx)^2}{2b^3}+\frac{a(a+$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcSinh[a + b*x]^2,x]
```

```
[Out] (-4*x)/(9*b^2) + (2*a^2*x)/b^2 - (a*(a + b*x)^2)/(2*b^3) + (2*(a + b*x)^3)/
(27*b^3) + (4*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x])/(9*b^3) - (2*a^2*Sqrt
[1 + (a + b*x)^2]*ArcSinh[a + b*x])/b^3 + (a*(a + b*x)*Sqrt[1 + (a + b*x)^2
]*ArcSinh[a + b*x])/b^3 - (2*(a + b*x)^2*Sqrt[1 + (a + b*x)^2]*ArcSinh[a +
b*x])/(9*b^3) - (a*ArcSinh[a + b*x]^2)/(2*b^3) + (a^3*ArcSinh[a + b*x]^2)/(
3*b^3) + (x^3*ArcSinh[a + b*x]^2)/3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
```

```
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5801

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n
 - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int x^2 \sinh^{-1}(a+bx)^2\,dx = \frac{\text{Subst}\left(\int \left(-\frac{a}{b}+\frac{x}{b}\right)^2 \sinh^{-1}(x)^2\,dx, x, a+bx\right)}{b}$$

$$= \frac{1}{3}x^3\sinh^{-1}(a+bx)^2 - \frac{2}{3}\text{Subst}\left(\int \frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^3 \sinh^{-1}(x)}{\sqrt{1+x^2}}\,dx, x, a+bx\right)$$

$$= \frac{1}{3}x^3\sinh^{-1}(a+bx)^2 - \frac{2}{3}\text{Subst}\left(\int \left(-\frac{a^3\sinh^{-1}(x)}{b^3\sqrt{1+x^2}} + \frac{3a^2x\sinh^{-1}(x)}{b^3\sqrt{1+x^2}} - \frac{3ax^2\sinh^{-1}}{b^3\sqrt{1+x}}\right.\right.$$

$$= \frac{1}{3}x^3\sinh^{-1}(a+bx)^2 - \frac{2\,\text{Subst}\left(\int \frac{x^3\sinh^{-1}(x)}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{3b^3} + \frac{(2a)\,\text{Subst}\left(\int \frac{x^2\sinh^{-1}}{\sqrt{1+x^2}}\right.}{b^3}$$

$$= -\frac{2a^2\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{b^3} + \frac{a(a+bx)\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{b^3} - $$

$$= \frac{2a^2x}{b^2} - \frac{a(a+bx)^2}{2b^3} + \frac{2(a+bx)^3}{27b^3} + \frac{4\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{9b^3} - \frac{2a^2\sqrt{1+(a}{}$$

$$= -\frac{4x}{9b^2} + \frac{2a^2x}{b^2} - \frac{a(a+bx)^2}{2b^3} + \frac{2(a+bx)^3}{27b^3} + \frac{4\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{9b^3} - \frac{2a^2}{}$$

Mathematica [A] time = 0.14, size = 107, normalized size = 0.51

$$\frac{9\left(2a^3-3a+2b^3x^3\right)\sinh^{-1}(a+bx)^2 + bx\left(66a^2-15abx+4b^2x^2-24\right) - 6\sqrt{a^2+2abx+b^2x^2+1}\,\left(11a^2-5a\right.}{54b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcSinh[a + b*x]^2,x]
```

```
[Out] (b*x*(-24 + 66*a^2 - 15*a*b*x + 4*b^2*x^2) - 6*Sqrt[1 + a^2 + 2*a*b*x + b^2
*x^2]*(-4 + 11*a^2 - 5*a*b*x + 2*b^2*x^2)*ArcSinh[a + b*x] + 9*(-3*a + 2*a^
3 + 2*b^3*x^3)*ArcSinh[a + b*x]^2)/(54*b^3)
```

fricas [A] time = 0.53, size = 146, normalized size = 0.69

$$\frac{4\,b^3x^3 - 15\,ab^2x^2 + 6\left(11\,a^2 - 4\right)bx + 9\left(2\,b^3x^3 + 2\,a^3 - 3\,a\right)\log\left(bx + a + \sqrt{b^2x^2 + 2\,abx + a^2 + 1}\right)^2 - 6\left(2\,b^2x^2 - \right.}{54\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/54*(4*b^3*x^3 - 15*a*b^2*x^2 + 6*(11*a^2 - 4)*b*x + 9*(2*b^3*x^3 + 2*a^3
- 3*a)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2 - 6*(2*b^2*x^2 -
5*a*b*x + 11*a^2 - 4)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*log(b*x + a + sqrt(
b^2*x^2 + 2*a*b*x + a^2 + 1)))/b^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{arsinh}\left(bx + a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*arcsinh(b*x + a)^2, x)
```

maple [A] time = 0.09, size = 212, normalized size = 1.00

$$\frac{-\frac{a\left(2\operatorname{arcsinh}(bx+a)^2(bx+a)^2 - 2\operatorname{arcsinh}(bx+a)\sqrt{1+(bx+a)^2}\,(bx+a) + \operatorname{arcsinh}(bx+a)^2 + (bx+a)^2 + 1\right)}{2} - \frac{\operatorname{arcsinh}(bx+a)^2(bx+a)}{3} + \frac{\operatorname{arcsinh}(bx+a)^2(bx+\ldots}{3}}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsinh(b*x+a)^2,x)
```

```
[Out] 1/b^3*(-1/2*a*(2*arcsinh(b*x+a)^2*(b*x+a)^2-2*arcsinh(b*x+a)*(1+(b*x+a)^2)^
(1/2)*(b*x+a)+arcsinh(b*x+a)^2+(b*x+a)^2+1)-1/3*arcsinh(b*x+a)^2*(b*x+a)+1/
3*arcsinh(b*x+a)^2*(b*x+a)*(1+(b*x+a)^2)+2/3*arcsinh(b*x+a)*(1+(b*x+a)^2)^(
1/2)-14/27*b*x-14/27*a-2/9*arcsinh(b*x+a)*(1+(b*x+a)^2)^(3/2)+2/27*(1+(b*x+
a)^2)*(b*x+a)+a^2*(arcsinh(b*x+a)^2*(b*x+a)-2*arcsinh(b*x+a)*(1+(b*x+a)^2)^
(1/2)+2*b*x+2*a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\,x^3\log\left(bx + a + \sqrt{b^2x^2 + 2\,abx + a^2 + 1}\right)^2 - \int \frac{2\left(b^3x^5 + 2\,ab^2x^4 + \left(a^2b + b\right)x^3 + \left(b^2x^4 + abx^3\right)\sqrt{b^2x^2 + 2\,abx + \ldots}\right.}{3\left(b^3x^3 + 3\,ab^2x^2 + a^3 + \left(3\,a^2b + b\right)x + \left(b\ldots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*x^3*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2 - integrate(2/3*
(b^3*x^5 + 2*a*b^2*x^4 + (a^2*b + b)*x^3 + (b^2*x^4 + a*b*x^3)*sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/(b^
```

```
3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b + b)*x + (b^2*x^2 + 2*a*b*x + a^2 + 1)
^(3/2) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \operatorname{asinh}(a+b\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*asinh(a + b*x)^2,x)
```

```
[Out] int(x^2*asinh(a + b*x)^2, x)
```

sympy [A] time = 1.38, size = 243, normalized size = 1.15

$$\begin{cases} \frac{a^3 \operatorname{asinh}^2(a+bx)}{3b^3} + \frac{11a^2x}{9b^2} - \frac{11a^2\sqrt{a^2+2abx+b^2x^2+1}\,\operatorname{asinh}(a+bx)}{9b^3} - \frac{5ax^2}{18b} + \frac{5ax\sqrt{a^2+2abx+b^2x^2+1}\,\operatorname{asinh}(a+bx)}{9b^2} - \frac{a\operatorname{asinh}^2(a+bx)}{2b^3} + \\ \frac{x^3\operatorname{asinh}^2(a)}{3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asinh(b*x+a)**2,x)
```

```
[Out] Piecewise((a**3*asinh(a + b*x)**2/(3*b**3) + 11*a**2*x/(9*b**2) - 11*a**2*s
qrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/(9*b**3) - 5*a*x**2/(18*
b) + 5*a*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/(9*b**2) - a
*asinh(a + b*x)**2/(2*b**3) + x**3*asinh(a + b*x)**2/3 + 2*x**3/27 - 2*x**2
*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/(9*b) - 4*x/(9*b**2) +
 4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/(9*b**3), Ne(b, 0)),
 (x**3*asinh(a)**2/3, True))
```

3.69 $\int x \sinh^{-1}(a+bx)^2\,dx$

Optimal. Leaf size=126

$$-\frac{a^2\sinh^{-1}(a+bx)^2}{2b^2}+\frac{(a+bx)^2}{4b^2}-\frac{\sqrt{(a+bx)^2+1}\,(a+bx)\sinh^{-1}(a+bx)}{2b^2}+\frac{\sinh^{-1}(a+bx)^2}{4b^2}+\frac{2a\sqrt{(a+bx)^2+1}\,\sin}{b^2}$$

```
[Out] -2*a*x/b+1/4*(b*x+a)^2/b^2+1/4*arcsinh(b*x+a)^2/b^2-1/2*a^2*arcsinh(b*x+a)^
2/b^2+1/2*x^2*arcsinh(b*x+a)^2+2*a*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)/b^2-1
/2*(b*x+a)*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)/b^2
```

Rubi [A] time = 0.24, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {5865, 5801, 5821, 5675, 5717, 8, 5758, 30}

$$-\frac{a^2\sinh^{-1}(a+bx)^2}{2b^2}+\frac{(a+bx)^2}{4b^2}-\frac{\sqrt{(a+bx)^2+1}\,(a+bx)\sinh^{-1}(a+bx)}{2b^2}+\frac{\sinh^{-1}(a+bx)^2}{4b^2}+\frac{2a\sqrt{(a+bx)^2+1}\,\sin}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcSinh[a + b*x]^2,x]
```

```
[Out] (-2*a*x)/b + (a + b*x)^2/(4*b^2) + (2*a*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b
*x])/b^2 - ((a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x])/(2*b^2) + Arc
Sinh[a + b*x]^2/(4*b^2) - (a^2*ArcSinh[a + b*x]^2)/(2*b^2) + (x^2*ArcSinh[a
 + b*x]^2)/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5801

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n
 - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}\int x\sinh^{-1}(a+bx)^2\,dx &= \frac{\text{Subst}\left(\int\left(-\frac{a}{b}+\frac{x}{b}\right)\sinh^{-1}(x)^2\,dx,x,a+bx\right)}{b}\\
&= \frac{1}{2}x^2\sinh^{-1}(a+bx)^2-\text{Subst}\left(\int\frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^2\sinh^{-1}(x)}{\sqrt{1+x^2}}\,dx,x,a+bx\right)\\
&= \frac{1}{2}x^2\sinh^{-1}(a+bx)^2-\text{Subst}\left(\int\left(\frac{a^2\sinh^{-1}(x)}{b^2\sqrt{1+x^2}}-\frac{2ax\sinh^{-1}(x)}{b^2\sqrt{1+x^2}}+\frac{x^2\sinh^{-1}(x)}{b^2\sqrt{1+x^2}}\right)dx\right.\\
&= \frac{1}{2}x^2\sinh^{-1}(a+bx)^2-\frac{\text{Subst}\left(\int\frac{x^2\sinh^{-1}(x)}{\sqrt{1+x^2}}\,dx,x,a+bx\right)}{b^2}+\frac{(2a)\,\text{Subst}\left(\int\frac{x\sinh^{-1}(x)}{\sqrt{1+x^2}}\right.}{b^2}\\
&= \frac{2a\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)}{b^2}-\frac{(a+bx)\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)}{2b^2}-\frac{a^2\sin}{}\\
&= -\frac{2ax}{b}+\frac{(a+bx)^2}{4b^2}+\frac{2a\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)}{b^2}-\frac{(a+bx)\sqrt{1+(a+bx)^2}\sin}{2b^2}\end{aligned}$$

Mathematica [A] time = 0.08, size = 79, normalized size = 0.63

$$\frac{\left(-2a^2+2b^2x^2+1\right)\sinh^{-1}(a+bx)^2+2(3a-bx)\sqrt{a^2+2abx+b^2x^2+1}\sinh^{-1}(a+bx)+bx(bx-6a)}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcSinh[a + b*x]^2,x]
```

```
[Out] (b*x*(-6*a + b*x) + 2*(3*a - b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh
[a + b*x] + (1 - 2*a^2 + 2*b^2*x^2)*ArcSinh[a + b*x]^2)/(4*b^2)
```

fricas [A] time = 0.57, size = 114, normalized size = 0.90

$$\frac{b^2x^2-6\,abx+\left(2\,b^2x^2-2\,a^2+1\right)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^2-2\,\sqrt{b^2x^2+2\,abx+a^2+1}\,(bx-3\,a}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*x^2 - 6*a*b*x + (2*b^2*x^2 - 2*a^2 + 1)*log(b*x + a + sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1))^2 - 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b*x - 3*a)*
log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)))/b^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{arsinh}(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x*arcsinh(b*x + a)^2, x)
```

maple [A] time = 0.08, size = 113, normalized size = 0.90

$$\frac{\frac{\operatorname{arcsinh}(bx+a)^2\left(1+(bx+a)^2\right)}{2}-\frac{\operatorname{arcsinh}(bx+a)\sqrt{1+(bx+a)^2}\,(bx+a)}{2}-\frac{\operatorname{arcsinh}(bx+a)^2}{4}+\frac{(bx+a)^2}{4}+\frac{1}{4}-a\left(\operatorname{arcsinh}(bx+a)^2(bx+a)-\right.}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsinh(b*x+a)^2,x)
```

```
[Out] 1/b^2*(1/2*arcsinh(b*x+a)^2*(1+(b*x+a)^2)-1/2*arcsinh(b*x+a)*(1+(b*x+a)^2)^
(1/2)*(b*x+a)-1/4*arcsinh(b*x+a)^2+1/4*(b*x+a)^2+1/4-a*(arcsinh(b*x+a)^2*(b
*x+a)-2*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)+2*b*x+2*a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}x^2\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^2-\int\frac{\left(b^3x^4+2\,ab^2x^3+\left(a^2b+b\right)x^2+\left(b^2x^3+abx^2\right)\sqrt{b^2x^2+2\,abx+\right.}{b^3x^3+3\,ab^2x^2+a^3+\left(3\,a^2b+b\right)x+\left(b^2x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x^2*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2 - integrate((b^3
*x^4 + 2*a*b^2*x^3 + (a^2*b + b)*x^2 + (b^2*x^3 + a*b*x^2)*sqrt(b^2*x^2 + 2
*a*b*x + a^2 + 1))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/(b^3*x^
3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b + b)*x + (b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/
2) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\operatorname{asinh}(a+b\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*asinh(a + b*x)^2,x)
```

```
[Out] int(x*asinh(a + b*x)^2, x)
```

sympy [A] time = 0.68, size = 138, normalized size = 1.10

$$\begin{cases} -\frac{a^2\operatorname{asinh}^2(a+bx)}{2b^2}-\frac{3ax}{2b}+\frac{3a\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}(a+bx)}{2b^2}+\frac{x^2\operatorname{asinh}^2(a+bx)}{2}+\frac{x^2}{4}-\frac{x\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}(a+bx)}{2b}+\frac{\operatorname{asinh}^2(a+\right.}{4b^2} \\ \frac{x^2\operatorname{asinh}^2(a)}{2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asinh(b*x+a)**2,x)
```

```
[Out] Piecewise((-a**2*asinh(a + b*x)**2/(2*b**2) - 3*a*x/(2*b) + 3*a*sqrt(a**2 +
 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/(2*b**2) + x**2*asinh(a + b*x)**2/
2 + x**2/4 - x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/(2*b) +
asinh(a + b*x)**2/(4*b**2), Ne(b, 0)), (x**2*asinh(a)**2/2, True))
```

3.70 $\int \sinh^{-1}(a+bx)^2\,dx$

Optimal. Leaf size=45

$$\frac{(a+bx)\sinh^{-1}(a+bx)^2}{b}-\frac{2\sqrt{(a+bx)^2+1}\,\sinh^{-1}(a+bx)}{b}+2x$$

```
[Out] 2*x+(b*x+a)*arcsinh(b*x+a)^2/b-2*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)/b
```

Rubi [A] time = 0.05, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5863, 5653, 5717, 8}

$$\frac{(a+bx)\sinh^{-1}(a+bx)^2}{b}-\frac{2\sqrt{(a+bx)^2+1}\,\sinh^{-1}(a+bx)}{b}+2x$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a + b*x]^2,x]
```

```
[Out] 2*x - (2*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x])/b + ((a + b*x)*ArcSinh[a +
 b*x]^2)/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5863

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\begin{aligned}\int \sinh^{-1}(a+bx)^2\,dx &= \frac{\text{Subst}\left(\int \sinh^{-1}(x)^2\,dx,x,a+bx\right)}{b}\\ &= \frac{(a+bx)\sinh^{-1}(a+bx)^2}{b}-\frac{2\,\text{Subst}\left(\int \frac{x\sinh^{-1}(x)}{\sqrt{1+x^2}}\,dx,x,a+bx\right)}{b}\\ &= -\frac{2\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{b}+\frac{(a+bx)\sinh^{-1}(a+bx)^2}{b}+\frac{2\,\text{Subst}(\int 1\,dx,x,a+b}{b}\\ &= 2x-\frac{2\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{b}+\frac{(a+bx)\sinh^{-1}(a+bx)^2}{b}\end{aligned}$$

Mathematica [A] time = 0.02, size = 47, normalized size = 1.04

$$\frac{2(a+bx)+(a+bx)\sinh^{-1}(a+bx)^2-2\sqrt{(a+bx)^2+1}\,\sinh^{-1}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a + b*x]^2,x]
```

```
[Out] (2*(a + b*x) - 2*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x] + (a + b*x)*ArcSinh
[a + b*x]^2)/b
```

fricas [B] time = 0.55, size = 88, normalized size = 1.96

$$\frac{(bx+a)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^2+2\,bx-2\,\sqrt{b^2x^2+2\,abx+a^2+1}\,\log\left(bx+a+\sqrt{b^2x^2+2\,a\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] ((b*x + a)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2 + 2*b*x - 2*s
qrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2
 + 1)))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{arsinh}(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(b*x + a)^2, x)
```

maple [A] time = 0.05, size = 46, normalized size = 1.02

$$\frac{\operatorname{arcsinh}(bx+a)^2(bx+a)-2\operatorname{arcsinh}(bx+a)\sqrt{1+(bx+a)^2}+2bx+2a}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)^2,x)
```

```
[Out] 1/b*(arcsinh(b*x+a)^2*(b*x+a)-2*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)+2*b*x+2*
a)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$x\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^2-\int\frac{2\left(b^3x^3+2\,ab^2x^2+\left(a^2b+b\right)x+\sqrt{b^2x^2+2\,abx+a^2+1}\left(b^2x^2\right.\right.}{b^3x^3+3\,ab^2x^2+a^3+\left(3\,a^2b+b\right)x+\left(b^2x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] x*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2 - integrate(2*(b^3*x^3
 + 2*a*b^2*x^2 + (a^2*b + b)*x + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b^2*x^2
 + a*b*x))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/(b^3*x^3 + 3*a*
```

```
b^2*x^2 + a^3 + (3*a^2*b + b)*x + (b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + a),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \operatorname{asinh}(a+b\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a + b*x)^2,x)
```

```
[Out] int(asinh(a + b*x)^2, x)
```

sympy [A] time = 0.26, size = 63, normalized size = 1.40

$$\begin{cases} \frac{a \operatorname{asinh}^{2}{\left(a+bx \right)}}{b} + x \operatorname{asinh}^{2}{\left(a + bx \right)} + 2x - \frac{2 \sqrt{a^{2}+2abx+b^{2}x^{2}+1} \operatorname{asinh}{\left(a+bx \right)}}{b} & \text{for}\: b \neq 0 \\ x \operatorname{asinh}^{2}{\left(a \right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)**2,x)
```

```
[Out] Piecewise((a*asinh(a + b*x)**2/b + x*asinh(a + b*x)**2 + 2*x - 2*sqrt(a**2
+ 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/b, Ne(b, 0)), (x*asinh(a)**2, Tru
e))
```

3.71 $\int \frac{\sinh^{-1}(a+bx)^2}{x}\,dx$

Optimal. Leaf size=205

$$2\sinh^{-1}(a+bx)\text{Li}_2\left(\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)+2\sinh^{-1}(a+bx)\text{Li}_2\left(\frac{e^{\sinh^{-1}(a+bx)}}{a+\sqrt{a^2+1}}\right)-2\text{Li}_3\left(\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)-2\text{Li}_3\left(\frac{e^{\sinh^{-1}(a+bx)}}{a+\sqrt{a^2+1}}\right)$$

```
[Out] -1/3*arcsinh(b*x+a)^3+arcsinh(b*x+a)^2*ln(1-(b*x+a+(1+(b*x+a)^2)^(1/2))/(a-
(a^2+1)^(1/2)))+arcsinh(b*x+a)^2*ln(1-(b*x+a+(1+(b*x+a)^2)^(1/2))/(a+(a^2+1
)^(1/2)))+2*arcsinh(b*x+a)*polylog(2,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a-(a^2+1)
^(1/2)))+2*arcsinh(b*x+a)*polylog(2,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^
(1/2)))-2*polylog(3,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a-(a^2+1)^(1/2)))-2*polylo
g(3,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))
```

Rubi [A] time = 0.35, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5865, 5799, 5561, 2190, 2531, 2282, 6589}

$$2\sinh^{-1}(a+bx)\text{PolyLog}\left(2,\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)+2\sinh^{-1}(a+bx)\text{PolyLog}\left(2,\frac{e^{\sinh^{-1}(a+bx)}}{\sqrt{a^2+1}+a}\right)-2\text{PolyLog}\left(3,\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a + b*x]^2/x,x]
```

```
[Out] -ArcSinh[a + b*x]^3/3 + ArcSinh[a + b*x]^2*Log[1 - E^ArcSinh[a + b*x]/(a -
Sqrt[1 + a^2])] + ArcSinh[a + b*x]^2*Log[1 - E^ArcSinh[a + b*x]/(a + Sqrt[1
 + a^2])] + 2*ArcSinh[a + b*x]*PolyLog[2, E^ArcSinh[a + b*x]/(a - Sqrt[1 +
a^2])] + 2*ArcSinh[a + b*x]*PolyLog[2, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2
])] - 2*PolyLog[3, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])] - 2*PolyLog[3, E
^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
```

```
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Cosh[x])/(c*d + e*Sinh[x]), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\sinh^{-1}(a+bx)^2}{x}\,dx = \frac{\text{Subst}\left(\int \frac{\sinh^{-1}(x)^2}{-\frac{a}{b}+\frac{x}{b}}\,dx, x, a+bx\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \frac{x^2\cosh(x)}{-\frac{a}{b}+\frac{\sinh(x)}{b}}\,dx, x, \sinh^{-1}(a+bx)\right)}{b}$$

$$= -\frac{1}{3}\sinh^{-1}(a+bx)^3 + \frac{\text{Subst}\left(\int \frac{e^x x^2}{-\frac{a}{b}-\frac{\sqrt{1+a^2}}{b}+\frac{e^x}{b}}\,dx, x, \sinh^{-1}(a+bx)\right)}{b} + \frac{\text{Subst}\left(\int \frac{e^x x^2}{-\frac{a}{b}+\frac{\sqrt{1+a}}{b}}\right.}{}$$

$$= -\frac{1}{3}\sinh^{-1}(a+bx)^3 + \sinh^{-1}(a+bx)^2\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{1+a^2}}\right) + \sinh^{-1}(a+bx)^2\log\left(1-\right.$$

$$= -\frac{1}{3}\sinh^{-1}(a+bx)^3 + \sinh^{-1}(a+bx)^2\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{1+a^2}}\right) + \sinh^{-1}(a+bx)^2\log\left(1-\right.$$

$$= -\frac{1}{3}\sinh^{-1}(a+bx)^3 + \sinh^{-1}(a+bx)^2\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{1+a^2}}\right) + \sinh^{-1}(a+bx)^2\log\left(1-\right.$$

$$= -\frac{1}{3}\sinh^{-1}(a+bx)^3 + \sinh^{-1}(a+bx)^2\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{1+a^2}}\right) + \sinh^{-1}(a+bx)^2\log\left(1-\right.$$

Mathematica [A] time = 0.03, size = 251, normalized size = 1.22

$$2\sinh^{-1}(a+bx)\text{Li}_2\left(-\frac{e^{\sinh^{-1}(a+bx)}}{\left(-\frac{a}{b}-\frac{\sqrt{a^2+1}}{b}\right)b}\right)+2\sinh^{-1}(a+bx)\text{Li}_2\left(-\frac{e^{\sinh^{-1}(a+bx)}}{\left(\frac{\sqrt{a^2+1}}{b}-\frac{a}{b}\right)b}\right)-2\text{Li}_3\left(\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)-2\text{Li}_3\left(\frac{e^{\sinh^{-1}}}{a+\sqrt{a}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a + b*x]^2/x,x]
```

```
[Out] -1/3*ArcSinh[a + b*x]^3 + ArcSinh[a + b*x]^2*Log[1 + E^ArcSinh[a + b*x]/((-
(a/b) - Sqrt[1 + a^2]/b)*b)] + ArcSinh[a + b*x]^2*Log[1 + E^ArcSinh[a + b*x
]/((-(a/b) + Sqrt[1 + a^2]/b)*b)] + 2*ArcSinh[a + b*x]*PolyLog[2, -(E^ArcSi
nh[a + b*x]/((-(a/b) - Sqrt[1 + a^2]/b)*b))] + 2*ArcSinh[a + b*x]*PolyLog[2
, -(E^ArcSinh[a + b*x]/((-(a/b) + Sqrt[1 + a^2]/b)*b))] - 2*PolyLog[3, E^Ar
cSinh[a + b*x]/(a - Sqrt[1 + a^2])] - 2*PolyLog[3, E^ArcSinh[a + b*x]/(a +
Sqrt[1 + a^2])]
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\operatorname{arsinh}(bx+a)^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2/x,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(b*x + a)^2/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}(bx+a)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2/x,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(b*x + a)^2/x, x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsinh}(bx+a)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)^2/x,x)
```

```
[Out] int(arcsinh(b*x+a)^2/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}(bx+a)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2/x,x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(b*x + a)^2/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{asinh}(a+b\,x)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a + b*x)^2/x,x)
```

```
[Out] int(asinh(a + b*x)^2/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}^2(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)**2/x,x)
```

```
[Out] Integral(asinh(a + b*x)**2/x, x)
```

3.72 $\int \frac{\sinh^{-1}(a+bx)^2}{x^2}\,dx$

Optimal. Leaf size=178

$$-\frac{2b\text{Li}_2\left(\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)}{\sqrt{a^2+1}}+\frac{2b\text{Li}_2\left(\frac{e^{\sinh^{-1}(a+bx)}}{a+\sqrt{a^2+1}}\right)}{\sqrt{a^2+1}}-\frac{2b\sinh^{-1}(a+bx)\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)}{\sqrt{a^2+1}}+\frac{2b\sinh^{-1}(a+bx)\log\left(1-\frac{e}{}\right.}{\sqrt{a^2+1}}$$

```
[Out] -arcsinh(b*x+a)^2/x-2*b*arcsinh(b*x+a)*ln(1-(b*x+a+(1+(b*x+a)^2)^(1/2))/(a-
(a^2+1)^(1/2)))/(a^2+1)^(1/2)+2*b*arcsinh(b*x+a)*ln(1-(b*x+a+(1+(b*x+a)^2)^
(1/2))/(a+(a^2+1)^(1/2)))/(a^2+1)^(1/2)-2*b*polylog(2,(b*x+a+(1+(b*x+a)^2)^
(1/2))/(a-(a^2+1)^(1/2)))/(a^2+1)^(1/2)+2*b*polylog(2,(b*x+a+(1+(b*x+a)^2)^
(1/2))/(a+(a^2+1)^(1/2)))/(a^2+1)^(1/2)
```

Rubi [A] time = 0.38, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5865, 5801, 5831, 3322, 2264, 2190, 2279, 2391}

$$-\frac{2b\text{PolyLog}\left(2,\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)}{\sqrt{a^2+1}}+\frac{2b\text{PolyLog}\left(2,\frac{e^{\sinh^{-1}(a+bx)}}{\sqrt{a^2+1}+a}\right)}{\sqrt{a^2+1}}-\frac{2b\sinh^{-1}(a+bx)\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)}{\sqrt{a^2+1}}+\frac{2b\sinh^{-1}(a}{}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a + b*x]^2/x^2,x]
```

```
[Out] -(ArcSinh[a + b*x]^2/x) - (2*b*ArcSinh[a + b*x]*Log[1 - E^ArcSinh[a + b*x]/
(a - Sqrt[1 + a^2])])/Sqrt[1 + a^2] + (2*b*ArcSinh[a + b*x]*Log[1 - E^ArcSi
nh[a + b*x]/(a + Sqrt[1 + a^2])])/Sqrt[1 + a^2] - (2*b*PolyLog[2, E^ArcSinh
[a + b*x]/(a - Sqrt[1 + a^2])])/Sqrt[1 + a^2] + (2*b*PolyLog[2, E^ArcSinh[a
 + b*x]/(a + Sqrt[1 + a^2])])/Sqrt[1 + a^2]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5801

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n
 - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5831

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\sinh^{-1}(a+bx)^2}{x^2}\,dx = \frac{\text{Subst}\left(\int \frac{\sinh^{-1}(x)^2}{\left(-\frac{a}{b}+\frac{x}{b}\right)^2}\,dx, x, a+bx\right)}{b}$$

$$= -\frac{\sinh^{-1}(a+bx)^2}{x} + 2\,\text{Subst}\left(\int \frac{\sinh^{-1}(x)}{\left(-\frac{a}{b}+\frac{x}{b}\right)\sqrt{1+x^2}}\,dx, x, a+bx\right)$$

$$= -\frac{\sinh^{-1}(a+bx)^2}{x} + 2\,\text{Subst}\left(\int \frac{x}{-\frac{a}{b}+\frac{\sinh(x)}{b}}\,dx, x, \sinh^{-1}(a+bx)\right)$$

$$= -\frac{\sinh^{-1}(a+bx)^2}{x} + 4\,\text{Subst}\left(\int \frac{e^x x}{-\frac{1}{b}-\frac{2ae^x}{b}+\frac{e^{2x}}{b}}\,dx, x, \sinh^{-1}(a+bx)\right)$$

$$= -\frac{\sinh^{-1}(a+bx)^2}{x} + \frac{4\,\text{Subst}\left(\int \frac{e^x x}{-\frac{2a}{b}-\frac{2\sqrt{1+a^2}}{b}+\frac{2e^x}{b}}\,dx, x, \sinh^{-1}(a+bx)\right)}{\sqrt{1+a^2}} - \frac{4\,\text{Subst}\left(\int -\right.}{}$$

$$= -\frac{\sinh^{-1}(a+bx)^2}{x} - \frac{2b\sinh^{-1}(a+bx)\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{\sqrt{1+a^2}} + \frac{2b\sinh^{-1}(a+bx)\log\left(\right.}{\sqrt{1+a^2}}$$

$$= -\frac{\sinh^{-1}(a+bx)^2}{x} - \frac{2b\sinh^{-1}(a+bx)\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{\sqrt{1+a^2}} + \frac{2b\sinh^{-1}(a+bx)\log\left(\right.}{\sqrt{1+a^2}}$$

$$= -\frac{\sinh^{-1}(a+bx)^2}{x} - \frac{2b\sinh^{-1}(a+bx)\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{\sqrt{1+a^2}} + \frac{2b\sinh^{-1}(a+bx)\log\left(\right.}{\sqrt{1+a^2}}$$

Mathematica [A] time = 0.12, size = 178, normalized size = 1.00

$$\frac{-2bx\text{Li}_2\left(\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)+2bx\text{Li}_2\left(\frac{e^{\sinh^{-1}(a+bx)}}{a+\sqrt{a^2+1}}\right)-\sinh^{-1}(a+bx)\left(\sqrt{a^2+1}\,\sinh^{-1}(a+bx)+2bx\left(\log\left(\frac{\sqrt{a^2+1}+e^{\sinh}}{\sqrt{a^2+1}}\right.\right.\right.}{\sqrt{a^2+1}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a + b*x]^2/x^2,x]
```

```
[Out] (-(ArcSinh[a + b*x]*(Sqrt[1 + a^2]*ArcSinh[a + b*x] + 2*b*x*(-Log[(a + Sqrt
[1 + a^2] - E^ArcSinh[a + b*x])/(a + Sqrt[1 + a^2])] + Log[(-a + Sqrt[1 + a
^2] + E^ArcSinh[a + b*x])/(-a + Sqrt[1 + a^2])]))) - 2*b*x*PolyLog[2, E^Arc
Sinh[a + b*x]/(a - Sqrt[1 + a^2])] + 2*b*x*PolyLog[2, E^ArcSinh[a + b*x]/(a
 + Sqrt[1 + a^2])])/(Sqrt[1 + a^2]*x)
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\operatorname{arsinh}(bx+a)^2}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(b*x + a)^2/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}(bx+a)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(b*x + a)^2/x^2, x)
```

maple [A] time = 0.39, size = 217, normalized size = 1.22

$$-\frac{\operatorname{arcsinh}(bx+a)^2}{x}+\frac{2b\operatorname{arcsinh}(bx+a)\ln\left(\frac{\sqrt{a^2+1}-bx-\sqrt{1+(bx+a)^2}}{a+\sqrt{a^2+1}}\right)}{\sqrt{a^2+1}}-\frac{2b\operatorname{arcsinh}(bx+a)\ln\left(\frac{\sqrt{a^2+1}+bx+\sqrt{1+(bx+a)^2}}{-a+\sqrt{a^2+1}}\right)}{\sqrt{a^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)^2/x^2,x)
```

```
[Out] -arcsinh(b*x+a)^2/x+2*b*arcsinh(b*x+a)/(a^2+1)^(1/2)*ln(((a^2+1)^(1/2)-b*x-
(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))-2*b*arcsinh(b*x+a)/(a^2+1)^(1/2)*ln
(((a^2+1)^(1/2)+b*x+(1+(b*x+a)^2)^(1/2))/(-a+(a^2+1)^(1/2)))+2*b/(a^2+1)^(1
/2)*dilog(((a^2+1)^(1/2)-b*x-(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))-2*b/(a
^2+1)^(1/2)*dilog(((a^2+1)^(1/2)+b*x+(1+(b*x+a)^2)^(1/2))/(-a+(a^2+1)^(1/2)
))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^2}{x}+\int \frac{2\left(b^3x^2+2\,ab^2x+a^2b+\sqrt{b^2x^2+2\,abx+a^2+1}\left(b^2x+ab\right)+b\right)\mathrm{lo}}{b^3x^4+3\,ab^2x^3+\left(3\,a^2b+b\right)x^2+\left(a^3+a\right)x+\left(b^2x^3+2\,abx^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] -log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2/x + integrate(2*(b^3*x^
2 + 2*a*b^2*x + a^2*b + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b^2*x + a*b) + b
)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/(b^3*x^4 + 3*a*b^2*x^3 +
 (3*a^2*b + b)*x^2 + (a^3 + a)*x + (b^2*x^3 + 2*a*b*x^2 + (a^2 + 1)*x)*sqrt
(b^2*x^2 + 2*a*b*x + a^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{asinh}\left(a+b\,x\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a + b*x)^2/x^2,x)
```

```
[Out] int(asinh(a + b*x)^2/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}^{2}{\left(a+bx\right)}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)**2/x**2,x)
```

```
[Out] Integral(asinh(a + b*x)**2/x**2, x)
```

3.73 $\int \frac{\sinh^{-1}(a+bx)^2}{x^3}\,dx$

Optimal. Leaf size=235

$$\frac{ab^2\text{Li}_2\left(\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)}{\left(a^2+1\right)^{3/2}}-\frac{ab^2\text{Li}_2\left(\frac{e^{\sinh^{-1}(a+bx)}}{a+\sqrt{a^2+1}}\right)}{\left(a^2+1\right)^{3/2}}+\frac{b^2\log(x)}{a^2+1}+\frac{ab^2\sinh^{-1}(a+bx)\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)}{\left(a^2+1\right)^{3/2}}-\frac{ab^2\sinh^{-1}(a+bx)}{\left(a^2\right.}$$

```
[Out] -1/2*arcsinh(b*x+a)^2/x^2+b^2*ln(x)/(a^2+1)+a*b^2*arcsinh(b*x+a)*ln(1-(b*x+
a+(1+(b*x+a)^2)^(1/2))/(a-(a^2+1)^(1/2)))/(a^2+1)^(3/2)-a*b^2*arcsinh(b*x+a
)*ln(1-(b*x+a+(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))/(a^2+1)^(3/2)+a*b^2*p
olylog(2,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a-(a^2+1)^(1/2)))/(a^2+1)^(3/2)-a*b^2
*polylog(2,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))/(a^2+1)^(3/2)-b*a
rcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)/(a^2+1)/x
```

Rubi [A] time = 0.49, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {5865, 5801, 5831, 3324, 3322, 2264, 2190, 2279, 2391, 2668, 31}

$$\frac{ab^2\text{PolyLog}\left(2,\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)}{\left(a^2+1\right)^{3/2}}-\frac{ab^2\text{PolyLog}\left(2,\frac{e^{\sinh^{-1}(a+bx)}}{\sqrt{a^2+1}+a}\right)}{\left(a^2+1\right)^{3/2}}+\frac{b^2\log(x)}{a^2+1}+\frac{ab^2\sinh^{-1}(a+bx)\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)}{\left(a^2+1\right)^{3/2}}-\ldots$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a + b*x]^2/x^3,x]
```

```
[Out] -((b*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x])/((1 + a^2)*x)) - ArcSinh[a + b
*x]^2/(2*x^2) + (a*b^2*ArcSinh[a + b*x]*Log[1 - E^ArcSinh[a + b*x]/(a - Sqr
t[1 + a^2])])/(1 + a^2)^(3/2) - (a*b^2*ArcSinh[a + b*x]*Log[1 - E^ArcSinh[a
 + b*x]/(a + Sqrt[1 + a^2])])/(1 + a^2)^(3/2) + (b^2*Log[x])/(1 + a^2) + (a
*b^2*PolyLog[2, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])])/(1 + a^2)^(3/2) -
(a*b^2*PolyLog[2, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])])/(1 + a^2)^(3/2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3324

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[(b*(c + d*x)^m*Cos[e + f*x])/(f*(a^2 - b^2)*(a + b*Sin[e +
f*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[(b*d*m)/(f*(a^2 - b^2)), Int[((c + d*x)^(m - 1)*Cos[e + f*x])/(a
 + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 5801

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n
 - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5831

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\sinh^{-1}(a+bx)^2}{x^3}\,dx = \frac{\text{Subst}\left(\int \frac{\sinh^{-1}(x)^2}{\left(-\frac{a}{b}+\frac{x}{b}\right)^3}\,dx, x, a+bx\right)}{b}$$

$$= -\frac{\sinh^{-1}(a+bx)^2}{2x^2} + \text{Subst}\left(\int \frac{\sinh^{-1}(x)}{\left(-\frac{a}{b}+\frac{x}{b}\right)^2\sqrt{1+x^2}}\,dx, x, a+bx\right)$$

$$= -\frac{\sinh^{-1}(a+bx)^2}{2x^2} + \text{Subst}\left(\int \frac{x}{\left(-\frac{a}{b}+\frac{\sinh(x)}{b}\right)^2}\,dx, x, \sinh^{-1}(a+bx)\right)$$

$$= -\frac{b\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{(1+a^2)\,x} - \frac{\sinh^{-1}(a+bx)^2}{2x^2} + \frac{b\,\text{Subst}\left(\int \frac{\cosh(x)}{-\frac{a}{b}+\frac{\sinh(x)}{b}}\,dx, x, \sinh\right.}{1+a^2}$$

$$= -\frac{b\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{(1+a^2)\,x} - \frac{\sinh^{-1}(a+bx)^2}{2x^2} - \frac{(2ab)\,\text{Subst}\left(\int \frac{e^x x}{-\frac{1}{b}-\frac{2ae^x}{b}+\frac{e^{2x}}{b}}\,dx, x,\right.}{1+a^2}$$

$$= -\frac{b\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{(1+a^2)\,x} - \frac{\sinh^{-1}(a+bx)^2}{2x^2} + \frac{b^2\log(x)}{1+a^2} - \frac{(2ab)\,\text{Subst}\left(\int \frac{}{-\frac{2a}{b}-}\right.}{}$$

$$= -\frac{b\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{(1+a^2)\,x} - \frac{\sinh^{-1}(a+bx)^2}{2x^2} + \frac{ab^2\sinh^{-1}(a+bx)\log\left(1-\frac{e^{\sinh}}{a-\sqrt{}}\right.}{(1+a^2)^{3/2}}$$

$$= -\frac{b\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{(1+a^2)\,x} - \frac{\sinh^{-1}(a+bx)^2}{2x^2} + \frac{ab^2\sinh^{-1}(a+bx)\log\left(1-\frac{e^{\sinh}}{a-\sqrt{}}\right.}{(1+a^2)^{3/2}}$$

$$= -\frac{b\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{(1+a^2)\,x} - \frac{\sinh^{-1}(a+bx)^2}{2x^2} + \frac{ab^2\sinh^{-1}(a+bx)\log\left(1-\frac{e^{\sinh}}{a-\sqrt{}}\right.}{(1+a^2)^{3/2}}$$

Mathematica [A] time = 0.13, size = 279, normalized size = 1.19

$$-\frac{-2ab^2x^2\text{Li}_2\left(\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right) + 2ab^2x^2\text{Li}_2\left(\frac{e^{\sinh^{-1}(a+bx)}}{a+\sqrt{a^2+1}}\right) - 2\sqrt{a^2+1}\,b^2x^2\log(x) + 2ab^2x^2\sinh^{-1}(a+bx)\log\left(\frac{\sqrt{a^2+1}-e}{\sqrt{a}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a + b*x]^2/x^3,x]
```

```
[Out] -1/2*(2*Sqrt[1 + a^2]*b*x*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x] + Sqrt[1 +
 a^2]*ArcSinh[a + b*x]^2 + a^2*Sqrt[1 + a^2]*ArcSinh[a + b*x]^2 + 2*a*b^2*x
^2*ArcSinh[a + b*x]*Log[(a + Sqrt[1 + a^2] - E^ArcSinh[a + b*x])/(a + Sqrt[
1 + a^2])] - 2*a*b^2*x^2*ArcSinh[a + b*x]*Log[(-a + Sqrt[1 + a^2] + E^ArcSi
```

```
nh[a + b*x])/(-a + Sqrt[1 + a^2])] - 2*Sqrt[1 + a^2]*b^2*x^2*Log[x] - 2*a*b
^2*x^2*PolyLog[2, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])] + 2*a*b^2*x^2*Pol
yLog[2, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])])/((1 + a^2)^(3/2)*x^2)
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\operatorname{arsinh}(bx+a)^2}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2/x^3,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(b*x + a)^2/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}(bx+a)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2/x^3,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(b*x + a)^2/x^3, x)
```

maple [A] time = 0.66, size = 384, normalized size = 1.63

$$\frac{b^2 \operatorname{arcsinh}(bx+a)}{a^2+1} - \frac{\operatorname{arcsinh}(bx+a)^2 a^2}{2x^2\left(a^2+1\right)} - \frac{b \operatorname{arcsinh}(bx+a)\sqrt{1+(bx+a)^2}}{\left(a^2+1\right)x} - \frac{\operatorname{arcsinh}(bx+a)^2}{2x^2\left(a^2+1\right)} - \frac{b^2 a \operatorname{arcsinh}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)^2/x^3,x)
```

```
[Out] b^2*arcsinh(b*x+a)/(a^2+1)-1/2*arcsinh(b*x+a)^2/x^2/(a^2+1)*a^2-b*arcsinh(b
*x+a)*(1+(b*x+a)^2)^(1/2)/(a^2+1)/x-1/2*arcsinh(b*x+a)^2/x^2/(a^2+1)-b^2/(a
^2+1)^(3/2)*a*arcsinh(b*x+a)*ln(((a^2+1)^(1/2)-b*x-(1+(b*x+a)^2)^(1/2))/(a+
(a^2+1)^(1/2)))+b^2/(a^2+1)^(3/2)*a*arcsinh(b*x+a)*ln(((a^2+1)^(1/2)+b*x+(1
+(b*x+a)^2)^(1/2))/(-a+(a^2+1)^(1/2)))-b^2/(a^2+1)^(3/2)*dilog(((a^2+1)^(1/
2)-b*x-(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))*a+b^2/(a^2+1)^(3/2)*dilog(((
a^2+1)^(1/2)+b*x+(1+(b*x+a)^2)^(1/2))/(-a+(a^2+1)^(1/2)))*a-2*b^2/(a^2+1)*l
n(b*x+a+(1+(b*x+a)^2)^(1/2))+b^2/(a^2+1)*ln((b*x+a+(1+(b*x+a)^2)^(1/2))^2-2
*a*(b*x+a+(1+(b*x+a)^2)^(1/2))-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^2}{2\,x^2} + \int \frac{\left(b^3x^2+2\,ab^2x+a^2b+\sqrt{b^2x^2+2\,abx+a^2+1}\left(b^2x+ab\right)+b\right)}{b^3x^5+3\,ab^2x^4+\left(3\,a^2b+b\right)x^3+\left(a^3+a\right)x^2+\left(b^2x^4+2\,abx^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2/x^2 + integrate((b^
3*x^2 + 2*a*b^2*x + a^2*b + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b^2*x + a*b)
 + b)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/(b^3*x^5 + 3*a*b^2*x
^4 + (3*a^2*b + b)*x^3 + (a^3 + a)*x^2 + (b^2*x^4 + 2*a*b*x^3 + (a^2 + 1)*x
^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{asinh}(a+b\,x)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a + b*x)^2/x^3,x)
```

```
[Out] int(asinh(a + b*x)^2/x^3, x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}^2(a+bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)**2/x**3,x)
```

```
[Out] Integral(asinh(a + b*x)**2/x**3, x)
```

3.74 $\int \frac{\sinh^{-1}(a+bx)^2}{x^4} dx$

Optimal. Leaf size=478

$$\frac{b^3\text{Li}_2\left(\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)}{3\left(a^2+1\right)^{3/2}}-\frac{a^2b^3\text{Li}_2\left(\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)}{\left(a^2+1\right)^{5/2}}-\frac{b^3\text{Li}_2\left(\frac{e^{\sinh^{-1}(a+bx)}}{a+\sqrt{a^2+1}}\right)}{3\left(a^2+1\right)^{3/2}}+\frac{a^2b^3\text{Li}_2\left(\frac{e^{\sinh^{-1}(a+bx)}}{a+\sqrt{a^2+1}}\right)}{\left(a^2+1\right)^{5/2}}-\frac{ab^3\log(x)}{\left(a^2+1\right)^2}+\frac{b^3\sinh^{-1}(a}{}$$

```
[Out] -1/3*b^2/(a^2+1)/x-1/3*arcsinh(b*x+a)^2/x^3-a*b^3*ln(x)/(a^2+1)^2-a^2*b^3*a
rcsinh(b*x+a)*ln(1-(b*x+a+(1+(b*x+a)^2)^(1/2))/(a-(a^2+1)^(1/2)))/(a^2+1)^(
5/2)+1/3*b^3*arcsinh(b*x+a)*ln(1-(b*x+a+(1+(b*x+a)^2)^(1/2))/(a-(a^2+1)^(1/
2)))/(a^2+1)^(3/2)+a^2*b^3*arcsinh(b*x+a)*ln(1-(b*x+a+(1+(b*x+a)^2)^(1/2))/
(a+(a^2+1)^(1/2)))/(a^2+1)^(5/2)-1/3*b^3*arcsinh(b*x+a)*ln(1-(b*x+a+(1+(b*x
+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))/(a^2+1)^(3/2)-a^2*b^3*polylog(2,(b*x+a+(1+
(b*x+a)^2)^(1/2))/(a-(a^2+1)^(1/2)))/(a^2+1)^(5/2)+1/3*b^3*polylog(2,(b*x+a
+(1+(b*x+a)^2)^(1/2))/(a-(a^2+1)^(1/2)))/(a^2+1)^(3/2)+a^2*b^3*polylog(2,(b
*x+a+(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))/(a^2+1)^(5/2)-1/3*b^3*polylog(
2,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))/(a^2+1)^(3/2)-1/3*b*arcsin
h(b*x+a)*(1+(b*x+a)^2)^(1/2)/(a^2+1)/x^2+a*b^2*arcsinh(b*x+a)*(1+(b*x+a)^2)
^(1/2)/(a^2+1)^2/x
```

Rubi [A] time = 1.57, antiderivative size = 478, normalized size of antiderivative = 1.00, number of steps used = 40, number of rules used = 16, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.333, Rules used = {5865, 5801, 5831, 3325, 3324, 3322, 2264, 2190, 2279, 2391, 2668, 31, 6741, 12, 6742, 32}

$$\frac{b^3\text{PolyLog}\left(2,\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)}{3\left(a^2+1\right)^{3/2}}-\frac{a^2b^3\text{PolyLog}\left(2,\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)}{\left(a^2+1\right)^{5/2}}-\frac{b^3\text{PolyLog}\left(2,\frac{e^{\sinh^{-1}(a+bx)}}{\sqrt{a^2+1}+a}\right)}{3\left(a^2+1\right)^{3/2}}+\frac{a^2b^3\text{PolyLog}\left(2,\frac{e^{\sinh^{-1}}}{\sqrt{a^2+}}\right)}{\left(a^2+1\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a + b*x]^2/x^4,x]
```

```
[Out] -b^2/(3*(1 + a^2)*x) - (b*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x])/(3*(1 + a
^2)*x^2) + (a*b^2*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x])/((1 + a^2)^2*x) -
 ArcSinh[a + b*x]^2/(3*x^3) - (a^2*b^3*ArcSinh[a + b*x]*Log[1 - E^ArcSinh[a
 + b*x]/(a - Sqrt[1 + a^2])])/(1 + a^2)^(5/2) + (b^3*ArcSinh[a + b*x]*Log[1
 - E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])])/(3*(1 + a^2)^(3/2)) + (a^2*b^3*
ArcSinh[a + b*x]*Log[1 - E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])])/(1 + a^2)
^(5/2) - (b^3*ArcSinh[a + b*x]*Log[1 - E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2
])])/(3*(1 + a^2)^(3/2)) - (a*b^3*Log[x])/(1 + a^2)^2 - (a^2*b^3*PolyLog[2,
 E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])])/(1 + a^2)^(5/2) + (b^3*PolyLog[2,
 E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])])/(3*(1 + a^2)^(3/2)) + (a^2*b^3*Po
lyLog[2, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])])/(1 + a^2)^(5/2) - (b^3*Po
lyLog[2, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])])/(3*(1 + a^2)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3324

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[(b*(c + d*x)^m*Cos[e + f*x])/(f*(a^2 - b^2)*(a + b*Sin[e +
f*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[(b*d*m)/(f*(a^2 - b^2)), Int[((c + d*x)^(m - 1)*Cos[e + f*x])/(a
 + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 3325

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> -Simp[(b*(c + d*x)^m*Cos[e + f*x]*(a + b*Sin[e + f*x])^(n + 1
))/(f*(n + 1)*(a^2 - b^2)), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m*(a +
b*Sin[e + f*x])^(n + 1), x], x] - Dist[(b*(n + 2))/((n + 1)*(a^2 - b^2)), I
```

```
nt[(c + d*x)^m*Sin[e + f*x]*(a + b*Sin[e + f*x])^(n + 1), x], x] + Dist[(b*
d*m)/(f*(n + 1)*(a^2 - b^2)), Int[(c + d*x)^(m - 1)*Cos[e + f*x]*(a + b*Sin
[e + f*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && ILtQ[n, -2] && IGtQ[m, 0]
```

Rule 5801

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n
 - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5831

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\sinh^{-1}(a+bx)^2}{x^4}\,dx = \frac{\text{Subst}\left(\int \frac{\sinh^{-1}(x)^2}{\left(-\frac{a}{b}+\frac{x}{b}\right)^4}\,dx, x, a+bx\right)}{b}$$

$$= -\frac{\sinh^{-1}(a+bx)^2}{3x^3} + \frac{2}{3}\text{Subst}\left(\int \frac{\sinh^{-1}(x)}{\left(-\frac{a}{b}+\frac{x}{b}\right)^3\sqrt{1+x^2}}\,dx, x, a+bx\right)$$

$$= -\frac{\sinh^{-1}(a+bx)^2}{3x^3} + \frac{2}{3}\text{Subst}\left(\int \frac{x}{\left(-\frac{a}{b}+\frac{\sinh(x)}{b}\right)^3}\,dx, x, \sinh^{-1}(a+bx)\right)$$

$$= -\frac{b\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{3\left(1+a^2\right)x^2} - \frac{\sinh^{-1}(a+bx)^2}{3x^3} + \frac{b\,\text{Subst}\left(\int \frac{\cosh(x)}{\left(-\frac{a}{b}+\frac{\sinh(x)}{b}\right)^2}\,dx, x, \sin\right.}{3\left(1+a^2\right)}$$

$$= -\frac{b\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{3\left(1+a^2\right)x^2} + \frac{2ab^2\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{3\left(1+a^2\right)^2x} - \frac{\sinh^{-1}(a+b}{3x^3}$$

$$= -\frac{b^2}{3\left(1+a^2\right)x} - \frac{b\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{3\left(1+a^2\right)x^2} + \frac{2ab^2\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{3\left(1+a^2\right)^2x}$$

$$= -\frac{b^2}{3\left(1+a^2\right)x} - \frac{b\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{3\left(1+a^2\right)x^2} + \frac{2ab^2\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{3\left(1+a^2\right)^2x}$$

$$= -\frac{b^2}{3\left(1+a^2\right)x} - \frac{b\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{3\left(1+a^2\right)x^2} + \frac{2ab^2\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{3\left(1+a^2\right)^2x}$$

$$= -\frac{b^2}{3\left(1+a^2\right)x} - \frac{b\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{3\left(1+a^2\right)x^2} + \frac{ab^2\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{\left(1+a^2\right)^2x} -$$

$$= -\frac{b^2}{3\left(1+a^2\right)x} - \frac{b\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{3\left(1+a^2\right)x^2} + \frac{ab^2\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{\left(1+a^2\right)^2x} -$$

$$= -\frac{b^2}{3\left(1+a^2\right)x} - \frac{b\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{3\left(1+a^2\right)x^2} + \frac{ab^2\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{\left(1+a^2\right)^2x} -$$

$$= -\frac{b^2}{3\left(1+a^2\right)x} - \frac{b\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{3\left(1+a^2\right)x^2} + \frac{ab^2\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{\left(1+a^2\right)^2x} -$$

$$= -\frac{b^2}{3\left(1+a^2\right)x} - \frac{b\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{3\left(1+a^2\right)x^2} + \frac{ab^2\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{\left(1+a^2\right)^2x} -$$

$$= -\frac{b^2}{\cdots} - \frac{b\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{\cdots} + \frac{ab^2\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{\cdots} -$$

Mathematica [C] time = 10.52, size = 1830, normalized size = 3.83

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSinh[a + b*x]^2/x^4,x]
```

```
[Out] (-((b*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x])/((1 + a^2)*x^2)) - ArcSinh[a
+ b*x]^2/x^3 - (b^2*(1 + a^2 - 3*a*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x]))
/((1 + a^2)^2*x) + (I*b^3*Pi*ArcTanh[(-1 - a*Tanh[ArcSinh[a + b*x]/2])/Sqrt
[1 + a^2]])/(1 + a^2)^(5/2) - ((2*I)*a^2*b^3*Pi*ArcTanh[(-1 - a*Tanh[ArcSin
h[a + b*x]/2])/Sqrt[1 + a^2]])/(1 + a^2)^(5/2) - (3*a*b^3*Log[-((b*x)/a)])/
(1 + a^2)^2 + (b^3*(-2*ArcCos[I*a]*ArcTanh[((-I + a)*Cot[(Pi + (2*I)*ArcSin
h[a + b*x])/4])/Sqrt[-1 - a^2]] - (Pi - (2*I)*ArcSinh[a + b*x])*ArcTanh[((I
 + a)*Tan[(Pi + (2*I)*ArcSinh[a + b*x])/4])/Sqrt[-1 - a^2]] + (ArcCos[I*a]
+ (2*I)*ArcTanh[((-I + a)*Cot[(Pi + (2*I)*ArcSinh[a + b*x])/4])/Sqrt[-1 - a
^2]] + (2*I)*ArcTanh[((I + a)*Tan[(Pi + (2*I)*ArcSinh[a + b*x])/4])/Sqrt[-1
 - a^2]])*Log[Sqrt[-1 - a^2]/(Sqrt[2]*E^(ArcSinh[a + b*x]/2)*Sqrt[b*x])] +
(ArcCos[I*a] - (2*I)*(ArcTanh[((-I + a)*Cot[(Pi + (2*I)*ArcSinh[a + b*x])/4
])/Sqrt[-1 - a^2]] + ArcTanh[((I + a)*Tan[(Pi + (2*I)*ArcSinh[a + b*x])/4])
/Sqrt[-1 - a^2]]))*Log[(I*Sqrt[-1 - a^2]*E^(ArcSinh[a + b*x]/2))/(Sqrt[2]*S
qrt[b*x])] - (ArcCos[I*a] + (2*I)*ArcTanh[((-I + a)*Cot[(Pi + (2*I)*ArcSinh
[a + b*x])/4])/Sqrt[-1 - a^2]])*Log[((I + a)*(a + I*(-1 + Sqrt[-1 - a^2]))*
(I + Cot[(Pi + (2*I)*ArcSinh[a + b*x])/4]))/(I + a - Sqrt[-1 - a^2]*Cot[(Pi
 + (2*I)*ArcSinh[a + b*x])/4])] - (ArcCos[I*a] - (2*I)*ArcTanh[((-I + a)*Co
t[(Pi + (2*I)*ArcSinh[a + b*x])/4])/Sqrt[-1 - a^2]])*Log[((I + a)*(a - I*(1
 + Sqrt[-1 - a^2]))*(-I + Cot[(Pi + (2*I)*ArcSinh[a + b*x])/4]))/(-I - a +
Sqrt[-1 - a^2]*Cot[(Pi + (2*I)*ArcSinh[a + b*x])/4])] + I*(PolyLog[2, -((((
-I)*a + Sqrt[-1 - a^2])*(I + a + Sqrt[-1 - a^2]*Cot[(Pi + (2*I)*ArcSinh[a +
 b*x])/4]))/(-I - a + Sqrt[-1 - a^2]*Cot[(Pi + (2*I)*ArcSinh[a + b*x])/4]))
] - PolyLog[2, ((I*a + Sqrt[-1 - a^2])*(I + a + Sqrt[-1 - a^2]*Cot[(Pi + (2
*I)*ArcSinh[a + b*x])/4]))/(-I - a + Sqrt[-1 - a^2]*Cot[(Pi + (2*I)*ArcSinh
[a + b*x])/4])])))/(-1 - a^2)^(5/2) - (2*a^2*b^3*(-2*ArcCos[I*a]*ArcTanh[((
-I + a)*Cot[(Pi + (2*I)*ArcSinh[a + b*x])/4])/Sqrt[-1 - a^2]] - (Pi - (2*I)
*ArcSinh[a + b*x])*ArcTanh[((I + a)*Tan[(Pi + (2*I)*ArcSinh[a + b*x])/4])/S
qrt[-1 - a^2]] + (ArcCos[I*a] + (2*I)*ArcTanh[((-I + a)*Cot[(Pi + (2*I)*Arc
Sinh[a + b*x])/4])/Sqrt[-1 - a^2]] + (2*I)*ArcTanh[((I + a)*Tan[(Pi + (2*I)
*ArcSinh[a + b*x])/4])/Sqrt[-1 - a^2]])*Log[Sqrt[-1 - a^2]/(Sqrt[2]*E^(ArcS
inh[a + b*x]/2)*Sqrt[b*x])] + (ArcCos[I*a] - (2*I)*(ArcTanh[((-I + a)*Cot[(
Pi + (2*I)*ArcSinh[a + b*x])/4])/Sqrt[-1 - a^2]] + ArcTanh[((I + a)*Tan[(Pi
 + (2*I)*ArcSinh[a + b*x])/4])/Sqrt[-1 - a^2]]))*Log[(I*Sqrt[-1 - a^2]*E^(A
rcSinh[a + b*x]/2))/(Sqrt[2]*Sqrt[b*x])] - (ArcCos[I*a] + (2*I)*ArcTanh[((-
I + a)*Cot[(Pi + (2*I)*ArcSinh[a + b*x])/4])/Sqrt[-1 - a^2]])*Log[((I + a)*
(a + I*(-1 + Sqrt[-1 - a^2]))*(I + Cot[(Pi + (2*I)*ArcSinh[a + b*x])/4]))/(
I + a - Sqrt[-1 - a^2]*Cot[(Pi + (2*I)*ArcSinh[a + b*x])/4])] - (ArcCos[I*a
] - (2*I)*ArcTanh[((-I + a)*Cot[(Pi + (2*I)*ArcSinh[a + b*x])/4])/Sqrt[-1 -
 a^2]])*Log[((I + a)*(a - I*(1 + Sqrt[-1 - a^2]))*(-I + Cot[(Pi + (2*I)*Arc
Sinh[a + b*x])/4]))/(-I - a + Sqrt[-1 - a^2]*Cot[(Pi + (2*I)*ArcSinh[a + b*
x])/4])] + I*(PolyLog[2, -((((-I)*a + Sqrt[-1 - a^2])*(I + a + Sqrt[-1 - a^
2]*Cot[(Pi + (2*I)*ArcSinh[a + b*x])/4]))/(-I - a + Sqrt[-1 - a^2]*Cot[(Pi
+ (2*I)*ArcSinh[a + b*x])/4]))] - PolyLog[2, ((I*a + Sqrt[-1 - a^2])*(I + a
 + Sqrt[-1 - a^2]*Cot[(Pi + (2*I)*ArcSinh[a + b*x])/4]))/(-I - a + Sqrt[-1
- a^2]*Cot[(Pi + (2*I)*ArcSinh[a + b*x])/4])])))/(-1 - a^2)^(5/2))/3
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\operatorname{arsinh}\left(bx+a\right)^2}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2/x^4,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(b*x + a)^2/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}(bx+a)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2/x^4,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(b*x + a)^2/x^4, x)
```

maple [A] time = 0.89, size = 730, normalized size = 1.53

$$-\frac{b^3 \operatorname{arcsinh}(bx+a)\,a}{\left(a^2+1\right)^2}+\frac{a\,b^2 \operatorname{arcsinh}(bx+a)\sqrt{1+(bx+a)^2}}{\left(a^2+1\right)^2 x}-\frac{\operatorname{arcsinh}(bx+a)^2 a^4}{3\left(a^2+1\right)^2 x^3}-\frac{b \operatorname{arcsinh}(bx+a)\sqrt{1+(bx}{3\left(a^2+1\right)^2 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)^2/x^4,x)
```

```
[Out] -b^3/(a^2+1)^2*arcsinh(b*x+a)*a+a*b^2*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)/(a
^2+1)^2/x-1/3/(a^2+1)^2/x^3*arcsinh(b*x+a)^2*a^4-1/3*b/(a^2+1)^2/x^2*arcsin
h(b*x+a)*(1+(b*x+a)^2)^(1/2)*a^2-1/3*b^2/(a^2+1)^2/x*a^2-2/3/(a^2+1)^2/x^3*
arcsinh(b*x+a)^2*a^2-1/3*b/(a^2+1)^2/x^2*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)
-1/3*b^2/(a^2+1)^2/x-1/3/(a^2+1)^2/x^3*arcsinh(b*x+a)^2+2*b^3/(a^2+1)^2*a*l
n(b*x+a+(1+(b*x+a)^2)^(1/2))-b^3/(a^2+1)^2*a*ln((b*x+a+(1+(b*x+a)^2)^(1/2))
^2-2*a*(b*x+a+(1+(b*x+a)^2)^(1/2))-1)-1/3*b^3/(a^2+1)^(5/2)*arcsinh(b*x+a)*
ln(((a^2+1)^(1/2)-b*x-(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))+1/3*b^3/(a^2+
1)^(5/2)*arcsinh(b*x+a)*ln(((a^2+1)^(1/2)+b*x+(1+(b*x+a)^2)^(1/2))/(-a+(a^2
+1)^(1/2)))-1/3*b^3/(a^2+1)^(5/2)*dilog(((a^2+1)^(1/2)-b*x-(1+(b*x+a)^2)^(1
/2))/(a+(a^2+1)^(1/2)))+1/3*b^3/(a^2+1)^(5/2)*dilog(((a^2+1)^(1/2)+b*x+(1+(
b*x+a)^2)^(1/2))/(-a+(a^2+1)^(1/2)))+2/3*b^3/(a^2+1)^(5/2)*a^2*arcsinh(b*x+
a)*ln(((a^2+1)^(1/2)-b*x-(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))-2/3*b^3/(a
^2+1)^(5/2)*a^2*arcsinh(b*x+a)*ln(((a^2+1)^(1/2)+b*x+(1+(b*x+a)^2)^(1/2))/(
-a+(a^2+1)^(1/2)))+2/3*b^3/(a^2+1)^(5/2)*a^2*dilog(((a^2+1)^(1/2)-b*x-(1+(b
*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))-2/3*b^3/(a^2+1)^(5/2)*a^2*dilog(((a^2+1)
^(1/2)+b*x+(1+(b*x+a)^2)^(1/2))/(-a+(a^2+1)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^2}{3\,x^3}+\int \frac{2\left(b^3x^2+2\,ab^2x+a^2b+\sqrt{b^2x^2+2\,abx+a^2+1}\left(b^2x+ab\right)+b\right)\mathrm{lo}}{3\left(b^3x^6+3\,ab^2x^5+\left(3\,a^2b+b\right)x^4+\left(a^3+a\right)x^3+\left(b^2x^5+2\,abx^4+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2/x^3 + integrate(2/3
*(b^3*x^2 + 2*a*b^2*x + a^2*b + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b^2*x +
a*b) + b)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/(b^3*x^6 + 3*a*b
^2*x^5 + (3*a^2*b + b)*x^4 + (a^3 + a)*x^3 + (b^2*x^5 + 2*a*b*x^4 + (a^2 +
1)*x^3)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{asinh}(a+b\,x)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a + b*x)^2/x^4,x)
```

```
[Out] int(asinh(a + b*x)^2/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}^2(a+bx)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)**2/x**4,x)
```

```
[Out] Integral(asinh(a + b*x)**2/x**4, x)
```

3.75 $\int x^2 \sinh^{-1}(a+bx)^3\,dx$

Optimal. Leaf size=355

$$\frac{a^3\sinh^{-1}(a+bx)^3}{3b^3}-\frac{6a^2\sqrt{(a+bx)^2+1}}{b^3}+\frac{6a^2(a+bx)\sinh^{-1}(a+bx)}{b^3}-\frac{3a^2\sqrt{(a+bx)^2+1}\,\sinh^{-1}(a+bx)^2}{b^3}+\frac{3a\sqrt{(}}{}$$

```
[Out] -2/27*(1+(b*x+a)^2)^(3/2)/b^3-3/4*a*arcsinh(b*x+a)/b^3-4/3*(b*x+a)*arcsinh(
b*x+a)/b^3+6*a^2*(b*x+a)*arcsinh(b*x+a)/b^3-3/2*a*(b*x+a)^2*arcsinh(b*x+a)/
b^3+2/9*(b*x+a)^3*arcsinh(b*x+a)/b^3-1/2*a*arcsinh(b*x+a)^3/b^3+1/3*a^3*arc
sinh(b*x+a)^3/b^3+1/3*x^3*arcsinh(b*x+a)^3+14/9*(1+(b*x+a)^2)^(1/2)/b^3-6*a
^2*(1+(b*x+a)^2)^(1/2)/b^3+3/4*a*(b*x+a)*(1+(b*x+a)^2)^(1/2)/b^3+2/3*arcsin
h(b*x+a)^2*(1+(b*x+a)^2)^(1/2)/b^3-3*a^2*arcsinh(b*x+a)^2*(1+(b*x+a)^2)^(1/
2)/b^3+3/2*a*(b*x+a)*arcsinh(b*x+a)^2*(1+(b*x+a)^2)^(1/2)/b^3-1/3*(b*x+a)^2
*arcsinh(b*x+a)^2*(1+(b*x+a)^2)^(1/2)/b^3
```

Rubi [A] time = 0.45, antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 11, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.917$, Rules used = {5865, 5801, 5831, 3317, 3296, 2638, 3311, 30, 2635, 8, 2633}

$$-\frac{6a^2\sqrt{(a+bx)^2+1}}{b^3}+\frac{6a^2(a+bx)\sinh^{-1}(a+bx)}{b^3}+\frac{a^3\sinh^{-1}(a+bx)^3}{3b^3}-\frac{3a^2\sqrt{(a+bx)^2+1}\,\sinh^{-1}(a+bx)^2}{b^3}+\frac{3a\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcSinh[a + b*x]^3,x]
```

```
[Out] (14*Sqrt[1 + (a + b*x)^2])/(9*b^3) - (6*a^2*Sqrt[1 + (a + b*x)^2])/b^3 + (3
*a*(a + b*x)*Sqrt[1 + (a + b*x)^2])/(4*b^3) - (2*(1 + (a + b*x)^2)^(3/2))/(
27*b^3) - (3*a*ArcSinh[a + b*x])/(4*b^3) - (4*(a + b*x)*ArcSinh[a + b*x])/(
3*b^3) + (6*a^2*(a + b*x)*ArcSinh[a + b*x])/b^3 - (3*a*(a + b*x)^2*ArcSinh[
a + b*x])/(2*b^3) + (2*(a + b*x)^3*ArcSinh[a + b*x])/(9*b^3) + (2*Sqrt[1 +
(a + b*x)^2]*ArcSinh[a + b*x]^2)/(3*b^3) - (3*a^2*Sqrt[1 + (a + b*x)^2]*Arc
Sinh[a + b*x]^2)/b^3 + (3*a*(a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x
]^2)/(2*b^3) - ((a + b*x)^2*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x]^2)/(3*b^
3) - (a*ArcSinh[a + b*x]^3)/(2*b^3) + (a^3*ArcSinh[a + b*x]^3)/(3*b^3) + (x
^3*ArcSinh[a + b*x]^3)/3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
```

]

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 5801

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n
 - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5831

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int x^2 \sinh^{-1}(a+bx)^3\,dx = \frac{\text{Subst}\left(\int \left(-\frac{a}{b}+\frac{x}{b}\right)^2 \sinh^{-1}(x)^3\,dx, x, a+bx\right)}{b}$$

$$= \frac{1}{3}x^3\sinh^{-1}(a+bx)^3 - \text{Subst}\left(\int \frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^3 \sinh^{-1}(x)^2}{\sqrt{1+x^2}}\,dx, x, a+bx\right)$$

$$= \frac{1}{3}x^3\sinh^{-1}(a+bx)^3 - \text{Subst}\left(\int x^2\left(-\frac{a}{b}+\frac{\sinh(x)}{b}\right)^3\,dx, x, \sinh^{-1}(a+bx)\right)$$

$$= \frac{1}{3}x^3\sinh^{-1}(a+bx)^3 - \text{Subst}\left(\int\left(-\frac{a^3x^2}{b^3}+\frac{3a^2x^2\sinh(x)}{b^3}-\frac{3ax^2\sinh^2(x)}{b^3}+\frac{x^2\sinh^3}{b^3}\right.\right.$$

$$= \frac{a^3\sinh^{-1}(a+bx)^3}{3b^3} + \frac{1}{3}x^3\sinh^{-1}(a+bx)^3 - \frac{\text{Subst}\left(\int x^2\sinh^3(x)\,dx, x, \sinh^{-1}(a+bx\right.}{b^3}$$

$$= -\frac{3a(a+bx)^2\sinh^{-1}(a+bx)}{2b^3} + \frac{2(a+bx)^3\sinh^{-1}(a+bx)}{9b^3} - \frac{3a^2\sqrt{1+(a+bx)^2}\,\sinh^{-}}{b^3}$$

$$= \frac{3a(a+bx)\sqrt{1+(a+bx)^2}}{4b^3} + \frac{6a^2(a+bx)\sinh^{-1}(a+bx)}{b^3} - \frac{3a(a+bx)^2\sinh^{-1}(a+bx)}{2b^3}$$

$$= \frac{2\sqrt{1+(a+bx)^2}}{9b^3} - \frac{6a^2\sqrt{1+(a+bx)^2}}{b^3} + \frac{3a(a+bx)\sqrt{1+(a+bx)^2}}{4b^3} - \frac{2\left(1+(a+bx)\right.}{27b^3}$$

$$= \frac{14\sqrt{1+(a+bx)^2}}{9b^3} - \frac{6a^2\sqrt{1+(a+bx)^2}}{b^3} + \frac{3a(a+bx)\sqrt{1+(a+bx)^2}}{4b^3} - \frac{2\left(1+(a+bx\right.}{27b^3}$$

Mathematica [A] time = 0.21, size = 175, normalized size = 0.49

$$18\left(2a^3-3a+2b^3x^3\right)\sinh^{-1}(a+bx)^3+\left(-575a^2+65abx-8b^2x^2+160\right)\sqrt{a^2+2abx+b^2x^2+1}\;-18\sqrt{a^2+2ab}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcSinh[a + b*x]^3,x]
```

```
[Out] ((160 - 575*a^2 + 65*a*b*x - 8*b^2*x^2)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] +
 3*(170*a^3 + 132*a^2*b*x + 8*b*x*(-6 + b^2*x^2) - 15*a*(5 + 2*b^2*x^2))*Ar
cSinh[a + b*x] - 18*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(-4 + 11*a^2 - 5*a*b*
x + 2*b^2*x^2)*ArcSinh[a + b*x]^2 + 18*(-3*a + 2*a^3 + 2*b^3*x^3)*ArcSinh[a
 + b*x]^3)/(108*b^3)
```

fricas [A] time = 0.49, size = 225, normalized size = 0.63

$$18\left(2\,b^3x^3+2\,a^3-3\,a\right)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^3-18\left(2\,b^2x^2-5\,abx+11\,a^2-4\right)\sqrt{b^2x^2+2\,abx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/108*(18*(2*b^3*x^3 + 2*a^3 - 3*a)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x +
a^2 + 1))^3 - 18*(2*b^2*x^2 - 5*a*b*x + 11*a^2 - 4)*sqrt(b^2*x^2 + 2*a*b*x
+ a^2 + 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2 + 3*(8*b^3*x^
3 - 30*a*b^2*x^2 + 170*a^3 + 12*(11*a^2 - 4)*b*x - 75*a)*log(b*x + a + sqrt
(b^2*x^2 + 2*a*b*x + a^2 + 1)) - (8*b^2*x^2 - 65*a*b*x + 575*a^2 - 160)*sqr
t(b^2*x^2 + 2*a*b*x + a^2 + 1))/b^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{arsinh}(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2*arcsinh(b*x + a)^3, x)
```

maple [A] time = 0.10, size = 311, normalized size = 0.88

$$-\frac{\frac{a\left(4\operatorname{arcsinh}(bx+a)^3(bx+a)^2-6\operatorname{arcsinh}(bx+a)^2\sqrt{1+(bx+a)^2}\,(bx+a)+2\operatorname{arcsinh}(bx+a)^3+6\operatorname{arcsinh}(bx+a)(bx+a)^2-3(bx+a)\sqrt{1+(bx+a)^2}+3\operatorname{arcsin}\right.}{4}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsinh(b*x+a)^3,x)
```

```
[Out] 1/b^3*(-1/4*a*(4*arcsinh(b*x+a)^3*(b*x+a)^2-6*arcsinh(b*x+a)^2*(1+(b*x+a)^2
)^(1/2)*(b*x+a)+2*arcsinh(b*x+a)^3+6*arcsinh(b*x+a)*(b*x+a)^2-3*(b*x+a)*(1+
(b*x+a)^2)^(1/2)+3*arcsinh(b*x+a))-1/3*arcsinh(b*x+a)^3*(b*x+a)+1/3*arcsinh
(b*x+a)^3*(b*x+a)*(1+(b*x+a)^2)+arcsinh(b*x+a)^2*(1+(b*x+a)^2)^(1/2)-14/9*(
b*x+a)*arcsinh(b*x+a)+14/9*(1+(b*x+a)^2)^(1/2)-1/3*arcsinh(b*x+a)^2*(1+(b*x
+a)^2)^(3/2)+2/9*(b*x+a)*(1+(b*x+a)^2)*arcsinh(b*x+a)-2/27*(1+(b*x+a)^2)^(3
/2)+a^2*(arcsinh(b*x+a)^3*(b*x+a)-3*arcsinh(b*x+a)^2*(1+(b*x+a)^2)^(1/2)+6*
(b*x+a)*arcsinh(b*x+a)-6*(1+(b*x+a)^2)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}x^3\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^3-\int\frac{\left(b^3x^5+2\,ab^2x^4+\left(a^2b+b\right)x^3+\left(b^2x^4+abx^3\right)\sqrt{b^2x^2+2\,abx}\right.}{b^3x^3+3\,ab^2x^2+a^3+\left(3\,a^2b+b\right)x+\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/3*x^3*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^3 - integrate((b^3
*x^5 + 2*a*b^2*x^4 + (a^2*b + b)*x^3 + (b^2*x^4 + a*b*x^3)*sqrt(b^2*x^2 + 2
*a*b*x + a^2 + 1))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2/(b^3*
x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b + b)*x + (b^2*x^2 + 2*a*b*x + a^2 + 1)^(
3/2) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\operatorname{asinh}(a+b\,x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*asinh(a + b*x)^3,x)
```

```
[Out] int(x^2*asinh(a + b*x)^3, x)
```

sympy [A] time = 3.04, size = 432, normalized size = 1.22

$$\begin{cases}\frac{a^3\operatorname{asinh}^3(a+bx)}{3b^3}+\frac{85a^3\operatorname{asinh}(a+bx)}{18b^3}+\frac{11a^2x\operatorname{asinh}(a+bx)}{3b^2}-\frac{11a^2\sqrt{a^2+2abx+b^2x^2+1}\,\operatorname{asinh}^2(a+bx)}{6b^3}-\frac{575a^2\sqrt{a^2+2abx+b^2x^2+1}}{108b^3}-\frac{5ax^2}{} \\ \frac{x^3\operatorname{asinh}^3(a)}{3}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asinh(b*x+a)**3,x)
```

```
[Out] Piecewise((a**3*asinh(a + b*x)**3/(3*b**3) + 85*a**3*asinh(a + b*x)/(18*b**
3) + 11*a**2*x*asinh(a + b*x)/(3*b**2) - 11*a**2*sqrt(a**2 + 2*a*b*x + b**2
*x**2 + 1)*asinh(a + b*x)**2/(6*b**3) - 575*a**2*sqrt(a**2 + 2*a*b*x + b**2
*x**2 + 1)/(108*b**3) - 5*a*x**2*asinh(a + b*x)/(6*b) + 5*a*x*sqrt(a**2 + 2
*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)**2/(6*b**2) + 65*a*x*sqrt(a**2 + 2*a
*b*x + b**2*x**2 + 1)/(108*b**2) - a*asinh(a + b*x)**3/(2*b**3) - 25*a*asin
h(a + b*x)/(12*b**3) + x**3*asinh(a + b*x)**3/3 + 2*x**3*asinh(a + b*x)/9 -
 x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)**2/(3*b) - 2*x**2
*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(27*b) - 4*x*asinh(a + b*x)/(3*b**2)
+ 2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)**2/(3*b**3) + 40*sq
rt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(27*b**3), Ne(b, 0)), (x**3*asinh(a)**3/
3, True))
```

3.76 $\int x \sinh^{-1}(a+bx)^3 \, dx$

Optimal. Leaf size=203

$$-\frac{a^2 \sinh^{-1}(a+bx)^3}{2b^2}-\frac{3(a+bx)\sqrt{(a+bx)^2+1}}{8b^2}+\frac{6a\sqrt{(a+bx)^2+1}}{b^2}+\frac{\sinh^{-1}(a+bx)^3}{4b^2}-\frac{3(a+bx)\sqrt{(a+bx)^2+1}}{4b^2}$$

```
[Out] 3/8*arcsinh(b*x+a)/b^2-6*a*(b*x+a)*arcsinh(b*x+a)/b^2+3/4*(b*x+a)^2*arcsinh
(b*x+a)/b^2+1/4*arcsinh(b*x+a)^3/b^2-1/2*a^2*arcsinh(b*x+a)^3/b^2+1/2*x^2*a
rcsinh(b*x+a)^3+6*a*(1+(b*x+a)^2)^(1/2)/b^2-3/8*(b*x+a)*(1+(b*x+a)^2)^(1/2)
/b^2+3*a*arcsinh(b*x+a)^2*(1+(b*x+a)^2)^(1/2)/b^2-3/4*(b*x+a)*arcsinh(b*x+a
)^2*(1+(b*x+a)^2)^(1/2)/b^2
```

Rubi [A] time = 0.30, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5865, 5801, 5831, 3317, 3296, 2638, 3311, 30, 2635, 8}

$$-\frac{a^2 \sinh^{-1}(a+bx)^3}{2b^2}-\frac{3(a+bx)\sqrt{(a+bx)^2+1}}{8b^2}+\frac{6a\sqrt{(a+bx)^2+1}}{b^2}+\frac{\sinh^{-1}(a+bx)^3}{4b^2}-\frac{3(a+bx)\sqrt{(a+bx)^2+1}}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcSinh[a + b*x]^3,x]
```

```
[Out] (6*a*Sqrt[1 + (a + b*x)^2])/b^2 - (3*(a + b*x)*Sqrt[1 + (a + b*x)^2])/(8*b^
2) + (3*ArcSinh[a + b*x])/(8*b^2) - (6*a*(a + b*x)*ArcSinh[a + b*x])/b^2 +
(3*(a + b*x)^2*ArcSinh[a + b*x])/(4*b^2) + (3*a*Sqrt[1 + (a + b*x)^2]*ArcSi
nh[a + b*x]^2)/b^2 - (3*(a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x]^2)
/(4*b^2) + ArcSinh[a + b*x]^3/(4*b^2) - (a^2*ArcSinh[a + b*x]^3)/(2*b^2) +
(x^2*ArcSinh[a + b*x]^3)/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 5801

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n
 - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5831

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int x\sinh^{-1}(a+bx)^3\,dx = \frac{\text{Subst}\left(\int\left(-\frac{a}{b}+\frac{x}{b}\right)\sinh^{-1}(x)^3\,dx,x,a+bx\right)}{b}$$
$$= \frac{1}{2}x^2\sinh^{-1}(a+bx)^3 - \frac{3}{2}\text{Subst}\left(\int\frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^2\sinh^{-1}(x)^2}{\sqrt{1+x^2}}dx,x,a+bx\right)$$
$$= \frac{1}{2}x^2\sinh^{-1}(a+bx)^3 - \frac{3}{2}\text{Subst}\left(\int x^2\left(-\frac{a}{b}+\frac{\sinh(x)}{b}\right)^2dx,x,\sinh^{-1}(a+bx)\right)$$
$$= \frac{1}{2}x^2\sinh^{-1}(a+bx)^3 - \frac{3}{2}\text{Subst}\left(\int\left(\frac{a^2x^2}{b^2}-\frac{2ax^2\sinh(x)}{b^2}+\frac{x^2\sinh^2(x)}{b^2}\right)dx,x,\text{sinh}$$
$$= -\frac{a^2\sinh^{-1}(a+bx)^3}{2b^2}+\frac{1}{2}x^2\sinh^{-1}(a+bx)^3-\frac{3\,\text{Subst}\left(\int x^2\sinh^2(x)\,dx,x,\sinh^{-1}(\right.}{2b^2}$$
$$= \frac{3(a+bx)^2\sinh^{-1}(a+bx)}{4b^2}+\frac{3a\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)^2}{b^2}-\frac{3(a+bx)\sqrt{1+(a}{}$$
$$= -\frac{3(a+bx)\sqrt{1+(a+bx)^2}}{8b^2}-\frac{6a(a+bx)\sinh^{-1}(a+bx)}{b^2}+\frac{3(a+bx)^2\sinh^{-1}(a+bx}{4b^2}$$
$$= \frac{6a\sqrt{1+(a+bx)^2}}{b^2}-\frac{3(a+bx)\sqrt{1+(a+bx)^2}}{8b^2}+\frac{3\sinh^{-1}(a+bx)}{8b^2}-\frac{6a(a+bx)\,\text{sinh}}{b^2}$$

Mathematica [A] time = 0.14, size = 129, normalized size = 0.64

$$\frac{3(15a-bx)\sqrt{a^2+2abx+b^2x^2+1}+\left(-4a^2+4b^2x^2+2\right)\sinh^{-1}(a+bx)^3+6(3a-bx)\sqrt{a^2+2abx+b^2x^2+1}}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcSinh[a + b*x]^3,x]
```

```
[Out] (3*(15*a - b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] + (3 - 42*a^2 - 36*a*b*x
+ 6*b^2*x^2)*ArcSinh[a + b*x] + 6*(3*a - b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*
x^2]*ArcSinh[a + b*x]^2 + (2 - 4*a^2 + 4*b^2*x^2)*ArcSinh[a + b*x]^3)/(8*b^
2)
```

fricas [A] time = 0.50, size = 180, normalized size = 0.89

$$\frac{2\left(2\,b^2x^2-2\,a^2+1\right)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^3-6\,\sqrt{b^2x^2+2\,abx+a^2+1}\,(bx-3\,a)\log\left(bx+a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(2*(2*b^2*x^2 - 2*a^2 + 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 +
 1))^3 - 6*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b*x - 3*a)*log(b*x + a + sqrt
(b^2*x^2 + 2*a*b*x + a^2 + 1))^2 + 3*(2*b^2*x^2 - 12*a*b*x - 14*a^2 + 1)*lo
g(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - 3*sqrt(b^2*x^2 + 2*a*b*x +
 a^2 + 1)*(b*x - 15*a))/b^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\,\text{arsinh}\left(bx+a\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x*arcsinh(b*x + a)^3, x)
```

maple [A] time = 0.08, size = 169, normalized size = 0.83

$$\frac{\frac{\operatorname{arcsinh}(bx+a)^3\left(1+(bx+a)^2\right)}{2}-\frac{3\operatorname{arcsinh}(bx+a)^2\sqrt{1+(bx+a)^2}\,(bx+a)}{4}-\frac{\operatorname{arcsinh}(bx+a)^3}{4}+\frac{3\operatorname{arcsinh}(bx+a)\left(1+(bx+a)^2\right)}{4}-\frac{3(bx+a)\sqrt{1+(bx+a)^2}}{8}\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsinh(b*x+a)^3,x)
```

```
[Out] 1/b^2*(1/2*arcsinh(b*x+a)^3*(1+(b*x+a)^2)-3/4*arcsinh(b*x+a)^2*(1+(b*x+a)^2
)^(1/2)*(b*x+a)-1/4*arcsinh(b*x+a)^3+3/4*arcsinh(b*x+a)*(1+(b*x+a)^2)-3/8*(
b*x+a)*(1+(b*x+a)^2)^(1/2)-3/8*arcsinh(b*x+a)-a*(arcsinh(b*x+a)^3*(b*x+a)-3
*arcsinh(b*x+a)^2*(1+(b*x+a)^2)^(1/2)+6*(b*x+a)*arcsinh(b*x+a)-6*(1+(b*x+a)
^2)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}x^2\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^3-\int\frac{3\left(b^3x^4+2\,ab^2x^3+\left(a^2b+b\right)x^2+\left(b^2x^3+abx^2\right)\sqrt{b^2x^2+2\,abx\cdots}\right)}{2\left(b^3x^3+3\,ab^2x^2+a^3+\left(3\,a^2b+b\right)x+\left(b\cdots\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*x^2*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^3 - integrate(3/2*
(b^3*x^4 + 2*a*b^2*x^3 + (a^2*b + b)*x^2 + (b^2*x^3 + a*b*x^2)*sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2/(
b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b + b)*x + (b^2*x^2 + 2*a*b*x + a^2 +
1)^(3/2) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\mathrm{asinh}\left(a+b\,x\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*asinh(a + b*x)^3,x)
```

```
[Out] int(x*asinh(a + b*x)^3, x)
```

sympy [A] time = 1.29, size = 248, normalized size = 1.22

$$\begin{cases}-\frac{a^2\operatorname{asinh}^3(a+bx)}{2b^2}-\frac{21a^2\operatorname{asinh}(a+bx)}{4b^2}-\frac{9ax\operatorname{asinh}(a+bx)}{2b}+\frac{9a\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}^2(a+bx)}{4b^2}+\frac{45a\sqrt{a^2+2abx+b^2x^2+1}}{8b^2}+\frac{x^2\operatorname{asinh}^3(a\cdots}{2} \\ \frac{x^2\operatorname{asinh}^3(a)}{2}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asinh(b*x+a)**3,x)
```

```
[Out] Piecewise((-a**2*asinh(a + b*x)**3/(2*b**2) - 21*a**2*asinh(a + b*x)/(4*b**
2) - 9*a*x*asinh(a + b*x)/(2*b) + 9*a*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*
asinh(a + b*x)**2/(4*b**2) + 45*a*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(8*b
**2) + x**2*asinh(a + b*x)**3/2 + 3*x**2*asinh(a + b*x)/4 - 3*x*sqrt(a**2 +
 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)**2/(4*b) - 3*x*sqrt(a**2 + 2*a*b*x
 + b**2*x**2 + 1)/(8*b) + asinh(a + b*x)**3/(4*b**2) + 3*asinh(a + b*x)/(8*
b**2), Ne(b, 0)), (x**2*asinh(a)**3/2, True))
```

3.77 $\int \sinh^{-1}(a+bx)^3\,dx$

Optimal. Leaf size=78

$$-\frac{6\sqrt{(a+bx)^2+1}}{b}+\frac{(a+bx)\sinh^{-1}(a+bx)^3}{b}-\frac{3\sqrt{(a+bx)^2+1}\,\sinh^{-1}(a+bx)^2}{b}+\frac{6(a+bx)\sinh^{-1}(a+bx)}{b}$$

```
[Out] 6*(b*x+a)*arcsinh(b*x+a)/b+(b*x+a)*arcsinh(b*x+a)^3/b-6*(1+(b*x+a)^2)^(1/2)
/b-3*arcsinh(b*x+a)^2*(1+(b*x+a)^2)^(1/2)/b
```

Rubi [A] time = 0.07, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5863, 5653, 5717, 261}

$$-\frac{6\sqrt{(a+bx)^2+1}}{b}+\frac{(a+bx)\sinh^{-1}(a+bx)^3}{b}-\frac{3\sqrt{(a+bx)^2+1}\,\sinh^{-1}(a+bx)^2}{b}+\frac{6(a+bx)\sinh^{-1}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a + b*x]^3,x]
```

```
[Out] (-6*Sqrt[1 + (a + b*x)^2])/b + (6*(a + b*x)*ArcSinh[a + b*x])/b - (3*Sqrt[1
 + (a + b*x)^2]*ArcSinh[a + b*x]^2)/b + ((a + b*x)*ArcSinh[a + b*x]^3)/b
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5863

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \sinh^{-1}(a+bx)^3\,dx = \frac{\text{Subst}\left(\int \sinh^{-1}(x)^3\,dx, x, a+bx\right)}{b}$$
$$= \frac{(a+bx)\sinh^{-1}(a+bx)^3}{b} - \frac{3\,\text{Subst}\left(\int \frac{x\sinh^{-1}(x)^2}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{b}$$
$$= -\frac{3\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)^2}{b} + \frac{(a+bx)\sinh^{-1}(a+bx)^3}{b} + \frac{6\,\text{Subst}\left(\int \sinh^{-1}(x)\,d\right.}{b}$$
$$= \frac{6(a+bx)\sinh^{-1}(a+bx)}{b} - \frac{3\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)^2}{b} + \frac{(a+bx)\sinh^{-1}(a+bx)^3}{b}$$
$$= -\frac{6\sqrt{1+(a+bx)^2}}{b} + \frac{6(a+bx)\sinh^{-1}(a+bx)}{b} - \frac{3\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)^2}{b} + \frac{(a}{}$$

Mathematica [A] time = 0.03, size = 70, normalized size = 0.90

$$\frac{-6\sqrt{(a+bx)^2+1} + (a+bx)\sinh^{-1}(a+bx)^3 - 3\sqrt{(a+bx)^2+1}\,\sinh^{-1}(a+bx)^2 + 6(a+bx)\sinh^{-1}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a + b*x]^3,x]
[Out] (-6*Sqrt[1 + (a + b*x)^2] + 6*(a + b*x)*ArcSinh[a + b*x] - 3*Sqrt[1 + (a +
b*x)^2]*ArcSinh[a + b*x]^2 + (a + b*x)*ArcSinh[a + b*x]^3)/b
```

fricas [A] time = 0.50, size = 139, normalized size = 1.78

$$\frac{(bx+a)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^3 - 3\sqrt{b^2x^2+2\,abx+a^2+1}\,\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^3,x, algorithm="fricas")
[Out] ((b*x + a)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^3 - 3*sqrt(b^2*
x^2 + 2*a*b*x + a^2 + 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2
 + 6*(b*x + a)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - 6*sqrt(b^
2*x^2 + 2*a*b*x + a^2 + 1))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{arsinh}(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^3,x, algorithm="giac")
[Out] integrate(arcsinh(b*x + a)^3, x)
```

maple [A] time = 0.05, size = 67, normalized size = 0.86

$$\frac{\operatorname{arcsinh}(bx+a)^3(bx+a) - 3\operatorname{arcsinh}(bx+a)^2\sqrt{1+(bx+a)^2} + 6(bx+a)\operatorname{arcsinh}(bx+a) - 6\sqrt{1+(bx+a)^2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)^3,x)
```

```
[Out] 1/b*(arcsinh(b*x+a)^3*(b*x+a)-3*arcsinh(b*x+a)^2*(1+(b*x+a)^2)^(1/2)+6*(b*x
+a)*arcsinh(b*x+a)-6*(1+(b*x+a)^2)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$x \log\left(bx + a + \sqrt{b^2x^2 + 2\,abx + a^2 + 1}\right)^3 - \int \frac{3\left(b^3x^3 + 2\,ab^2x^2 + \left(a^2b + b\right)x + \sqrt{b^2x^2 + 2\,abx + a^2 + 1}\left(b^2x^2 \cdots\right.\right.}{b^3x^3 + 3\,ab^2x^2 + a^3 + \left(3\,a^2b + b\right)x + \left(b^2x \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] x*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^3 - integrate(3*(b^3*x^3
 + 2*a*b^2*x^2 + (a^2*b + b)*x + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b^2*x^2
 + a*b*x))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2/(b^3*x^3 + 3*
a*b^2*x^2 + a^3 + (3*a^2*b + b)*x + (b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + a
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{asinh}(a + b\,x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a + b*x)^3,x)
```

```
[Out] int(asinh(a + b*x)^3, x)
```

sympy [A] time = 0.60, size = 109, normalized size = 1.40

$$\begin{cases} \frac{a \operatorname{asinh}^3(a+bx)}{b} + \frac{6a \operatorname{asinh}(a+bx)}{b} + x \operatorname{asinh}^3(a + bx) + 6x \operatorname{asinh}(a + bx) - \frac{3\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}^2(a+bx)}{b} - \frac{6\sqrt{a^2+2abx \cdots}}{b} \\ x \operatorname{asinh}^3(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)**3,x)
```

```
[Out] Piecewise((a*asinh(a + b*x)**3/b + 6*a*asinh(a + b*x)/b + x*asinh(a + b*x)*
*3 + 6*x*asinh(a + b*x) - 3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a +
b*x)**2/b - 6*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/b, Ne(b, 0)), (x*asinh(a
)**3, True))
```

3.78 $\int \frac{\sinh^{-1}(a+bx)^3}{x} \, dx$

Optimal. Leaf size=275

$$3\sinh^{-1}(a+bx)^2\text{Li}_2\left(\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)+3\sinh^{-1}(a+bx)^2\text{Li}_2\left(\frac{e^{\sinh^{-1}(a+bx)}}{a+\sqrt{a^2+1}}\right)-6\sinh^{-1}(a+bx)\text{Li}_3\left(\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)-6\sin$$

```
[Out] -1/4*arcsinh(b*x+a)^4+arcsinh(b*x+a)^3*ln(1-(b*x+a+(1+(b*x+a)^2)^(1/2))/(a-
(a^2+1)^(1/2)))+arcsinh(b*x+a)^3*ln(1-(b*x+a+(1+(b*x+a)^2)^(1/2))/(a+(a^2+1
)^(1/2)))+3*arcsinh(b*x+a)^2*polylog(2,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a-(a^2+
1)^(1/2)))+3*arcsinh(b*x+a)^2*polylog(2,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a+(a^2
+1)^(1/2)))-6*arcsinh(b*x+a)*polylog(3,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a-(a^2+
1)^(1/2)))-6*arcsinh(b*x+a)*polylog(3,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a+(a^2+1
)^(1/2)))+6*polylog(4,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a-(a^2+1)^(1/2)))+6*poly
log(4,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))
```

Rubi [A] time = 0.40, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5865, 5799, 5561, 2190, 2531, 6609, 2282, 6589}

$$3\sinh^{-1}(a+bx)^2\text{PolyLog}\left(2,\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)+3\sinh^{-1}(a+bx)^2\text{PolyLog}\left(2,\frac{e^{\sinh^{-1}(a+bx)}}{\sqrt{a^2+1}+a}\right)-6\sinh^{-1}(a+bx)\text{PolyLog}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a + b*x]^3/x,x]
```

```
[Out] -ArcSinh[a + b*x]^4/4 + ArcSinh[a + b*x]^3*Log[1 - E^ArcSinh[a + b*x]/(a -
Sqrt[1 + a^2])] + ArcSinh[a + b*x]^3*Log[1 - E^ArcSinh[a + b*x]/(a + Sqrt[1
 + a^2])] + 3*ArcSinh[a + b*x]^2*PolyLog[2, E^ArcSinh[a + b*x]/(a - Sqrt[1
+ a^2])] + 3*ArcSinh[a + b*x]^2*PolyLog[2, E^ArcSinh[a + b*x]/(a + Sqrt[1 +
 a^2])] - 6*ArcSinh[a + b*x]*PolyLog[3, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^
2])] - 6*ArcSinh[a + b*x]*PolyLog[3, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])
] + 6*PolyLog[4, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])] + 6*PolyLog[4, E^A
rcSinh[a + b*x]/(a + Sqrt[1 + a^2])]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Cosh[x])/(c*d + e*Sinh[x]), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{\sinh^{-1}(a+bx)^3}{x}\,dx = \frac{\text{Subst}\left(\int \frac{\sinh^{-1}(x)^3}{-\frac{a}{b}+\frac{x}{b}}\,dx, x, a+bx\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{x^3\cosh(x)}{-\frac{a}{b}+\frac{\sinh(x)}{b}}\,dx, x, \sinh^{-1}(a+bx)\right)}{b}$$
$$= -\frac{1}{4}\sinh^{-1}(a+bx)^4 + \frac{\text{Subst}\left(\int \frac{e^x x^3}{-\frac{a}{b}-\frac{\sqrt{1+a^2}}{b}+\frac{e^x}{b}}\,dx, x, \sinh^{-1}(a+bx)\right)}{b} + \frac{\text{Subst}\left(\int \frac{e^x x^3}{-\frac{a}{b}+\frac{\sqrt{1+a^2}}{b}\cdots}\right.}{b}$$
$$= -\frac{1}{4}\sinh^{-1}(a+bx)^4 + \sinh^{-1}(a+bx)^3\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{1+a^2}}\right) + \sinh^{-1}(a+bx)^3\log\left(1-\right.$$
$$= -\frac{1}{4}\sinh^{-1}(a+bx)^4 + \sinh^{-1}(a+bx)^3\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{1+a^2}}\right) + \sinh^{-1}(a+bx)^3\log\left(1-\right.$$
$$= -\frac{1}{4}\sinh^{-1}(a+bx)^4 + \sinh^{-1}(a+bx)^3\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{1+a^2}}\right) + \sinh^{-1}(a+bx)^3\log\left(1-\right.$$
$$= -\frac{1}{4}\sinh^{-1}(a+bx)^4 + \sinh^{-1}(a+bx)^3\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{1+a^2}}\right) + \sinh^{-1}(a+bx)^3\log\left(1-\right.$$
$$= -\frac{1}{4}\sinh^{-1}(a+bx)^4 + \sinh^{-1}(a+bx)^3\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{1+a^2}}\right) + \sinh^{-1}(a+bx)^3\log\left(1-\right.$$

Mathematica [A] time = 0.03, size = 346, normalized size = 1.26

$$3\sinh^{-1}(a+bx)^2\text{Li}_2\left(-\frac{e^{\sinh^{-1}(a+bx)}}{\left(-\frac{a}{b}-\frac{\sqrt{a^2+1}}{b}\right)b}\right)+3\sinh^{-1}(a+bx)^2\text{Li}_2\left(-\frac{e^{\sinh^{-1}(a+bx)}}{\left(\frac{\sqrt{a^2+1}}{b}-\frac{a}{b}\right)b}\right)-6\sinh^{-1}(a+bx)\text{Li}_3\left(-\frac{e^{\sinh^{-1}(a\cdots}}{\left(-\frac{a}{b}-\frac{\sqrt{a^2}\cdots}{b}\right.}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a + b*x]^3/x,x]
```

```
[Out] -1/4*ArcSinh[a + b*x]^4 + ArcSinh[a + b*x]^3*Log[1 + E^ArcSinh[a + b*x]/((-
(a/b) - Sqrt[1 + a^2]/b)*b)] + ArcSinh[a + b*x]^3*Log[1 + E^ArcSinh[a + b*x
]/((-(a/b) + Sqrt[1 + a^2]/b)*b)] + 3*ArcSinh[a + b*x]^2*PolyLog[2, -(E^Arc
Sinh[a + b*x]/((-(a/b) - Sqrt[1 + a^2]/b)*b))] + 3*ArcSinh[a + b*x]^2*PolyL
og[2, -(E^ArcSinh[a + b*x]/((-(a/b) + Sqrt[1 + a^2]/b)*b))] - 6*ArcSinh[a +
 b*x]*PolyLog[3, -(E^ArcSinh[a + b*x]/((-(a/b) - Sqrt[1 + a^2]/b)*b))] - 6*
ArcSinh[a + b*x]*PolyLog[3, -(E^ArcSinh[a + b*x]/((-(a/b) + Sqrt[1 + a^2]/b
)*b))] + 6*PolyLog[4, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])] + 6*PolyLog[4
, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])]
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\operatorname{arsinh}(bx+a)^3}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^3/x,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(b*x + a)^3/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}(bx+a)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^3/x,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(b*x + a)^3/x, x)
```

maple [F] time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsinh}(bx+a)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)^3/x,x)
```

```
[Out] int(arcsinh(b*x+a)^3/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}(bx+a)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^3/x,x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(b*x + a)^3/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{asinh}(a+b\,x)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a + b*x)^3/x,x)
```

```
[Out] int(asinh(a + b*x)^3/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}^3(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)**3/x,x)
```

```
[Out] Integral(asinh(a + b*x)**3/x, x)
```

3.79 $\int \frac{\sinh^{-1}(a+bx)^3}{x^2}\,dx$

Optimal. Leaf size=268

$$-\frac{6b\sinh^{-1}(a+bx)\text{Li}_2\left(\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)}{\sqrt{a^2+1}}+\frac{6b\sinh^{-1}(a+bx)\text{Li}_2\left(\frac{e^{\sinh^{-1}(a+bx)}}{a+\sqrt{a^2+1}}\right)}{\sqrt{a^2+1}}+\frac{6b\text{Li}_3\left(\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)}{\sqrt{a^2+1}}-\frac{6b\text{Li}_3\left(\frac{e^{\sinh^{-1}(a+bx)}}{a+\sqrt{a^2+1}}\right)}{\sqrt{a^2+1}}-$$

```
[Out] -arcsinh(b*x+a)^3/x-3*b*arcsinh(b*x+a)^2*ln(1-(b*x+a+(1+(b*x+a)^2)^(1/2))/(
a-(a^2+1)^(1/2)))/(a^2+1)^(1/2)+3*b*arcsinh(b*x+a)^2*ln(1-(b*x+a+(1+(b*x+a)
^2)^(1/2))/(a+(a^2+1)^(1/2)))/(a^2+1)^(1/2)-6*b*arcsinh(b*x+a)*polylog(2,(b
*x+a+(1+(b*x+a)^2)^(1/2))/(a-(a^2+1)^(1/2)))/(a^2+1)^(1/2)+6*b*arcsinh(b*x+
a)*polylog(2,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))/(a^2+1)^(1/2)+6
*b*polylog(3,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a-(a^2+1)^(1/2)))/(a^2+1)^(1/2)-6
*b*polylog(3,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))/(a^2+1)^(1/2)
```

Rubi [A] time = 0.58, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {5865, 5801, 5831, 3322, 2264, 2190, 2531, 2282, 6589}

$$-\frac{6b\sinh^{-1}(a+bx)\text{PolyLog}\left(2,\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)}{\sqrt{a^2+1}}+\frac{6b\sinh^{-1}(a+bx)\text{PolyLog}\left(2,\frac{e^{\sinh^{-1}(a+bx)}}{\sqrt{a^2+1}+a}\right)}{\sqrt{a^2+1}}+\frac{6b\text{PolyLog}\left(3,\frac{e^{\sinh^{-1}(a+b}}{a-\sqrt{a^2+1}}\right.}{\sqrt{a^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a + b*x]^3/x^2,x]
```

```
[Out] -(ArcSinh[a + b*x]^3/x) - (3*b*ArcSinh[a + b*x]^2*Log[1 - E^ArcSinh[a + b*x
]/(a - Sqrt[1 + a^2])])/Sqrt[1 + a^2] + (3*b*ArcSinh[a + b*x]^2*Log[1 - E^A
rcSinh[a + b*x]/(a + Sqrt[1 + a^2])])/Sqrt[1 + a^2] - (6*b*ArcSinh[a + b*x]
*PolyLog[2, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])])/Sqrt[1 + a^2] + (6*b*A
rcSinh[a + b*x]*PolyLog[2, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])])/Sqrt[1
+ a^2] + (6*b*PolyLog[3, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])])/Sqrt[1 +
a^2] - (6*b*PolyLog[3, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])])/Sqrt[1 + a^
2]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5801

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n
 - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5831

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\sinh^{-1}(a+bx)^3}{x^2}\,dx = \frac{\text{Subst}\left(\int \frac{\sinh^{-1}(x)^3}{\left(-\frac{a}{b}+\frac{x}{b}\right)^2}\,dx, x, a+bx\right)}{b}$$

$$= -\frac{\sinh^{-1}(a+bx)^3}{x} + 3\,\text{Subst}\left(\int \frac{\sinh^{-1}(x)^2}{\left(-\frac{a}{b}+\frac{x}{b}\right)\sqrt{1+x^2}}\,dx, x, a+bx\right)$$

$$= -\frac{\sinh^{-1}(a+bx)^3}{x} + 3\,\text{Subst}\left(\int \frac{x^2}{-\frac{a}{b}+\frac{\sinh(x)}{b}}\,dx, x, \sinh^{-1}(a+bx)\right)$$

$$= -\frac{\sinh^{-1}(a+bx)^3}{x} + 6\,\text{Subst}\left(\int \frac{e^x x^2}{-\frac{1}{b}-\frac{2ae^x}{b}+\frac{e^{2x}}{b}}\,dx, x, \sinh^{-1}(a+bx)\right)$$

$$= -\frac{\sinh^{-1}(a+bx)^3}{x} + \frac{6\,\text{Subst}\left(\int \frac{e^x x^2}{-\frac{2a}{b}-\frac{2\sqrt{1+a^2}}{b}+\frac{2e^x}{b}}\,dx, x, \sinh^{-1}(a+bx)\right)}{\sqrt{1+a^2}} - \frac{6\,\text{Subst}\left(\int \frac{}{-\frac{2a}{b}+}\right.}{}$$

$$= -\frac{\sinh^{-1}(a+bx)^3}{x} - \frac{3b\sinh^{-1}(a+bx)^2\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{\sqrt{1+a^2}} + \frac{3b\sinh^{-1}(a+bx)^2\log\left(1\right.}{\sqrt{1+a^2}}$$

$$= -\frac{\sinh^{-1}(a+bx)^3}{x} - \frac{3b\sinh^{-1}(a+bx)^2\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{\sqrt{1+a^2}} + \frac{3b\sinh^{-1}(a+bx)^2\log\left(1\right.}{\sqrt{1+a^2}}$$

$$= -\frac{\sinh^{-1}(a+bx)^3}{x} - \frac{3b\sinh^{-1}(a+bx)^2\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{\sqrt{1+a^2}} + \frac{3b\sinh^{-1}(a+bx)^2\log\left(1\right.}{\sqrt{1+a^2}}$$

$$= -\frac{\sinh^{-1}(a+bx)^3}{x} - \frac{3b\sinh^{-1}(a+bx)^2\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{\sqrt{1+a^2}} + \frac{3b\sinh^{-1}(a+bx)^2\log\left(1\right.}{\sqrt{1+a^2}}$$

Mathematica [A] time = 0.12, size = 259, normalized size = 0.97

$$-\frac{6bx\sinh^{-1}(a+bx)\text{Li}_2\left(\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right) - 6bx\sinh^{-1}(a+bx)\text{Li}_2\left(\frac{e^{\sinh^{-1}(a+bx)}}{a+\sqrt{a^2+1}}\right) - 6bx\text{Li}_3\left(\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right) + 6bx\text{Li}_3\left(\frac{e^{\sinh^{-1}(}}{a+\sqrt{a}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a + b*x]^3/x^2,x]
```

```
[Out] -((Sqrt[1 + a^2]*ArcSinh[a + b*x]^3 - 3*b*x*ArcSinh[a + b*x]^2*Log[(a + Sqr
t[1 + a^2] - E^ArcSinh[a + b*x])/(a + Sqrt[1 + a^2])] + 3*b*x*ArcSinh[a + b
*x]^2*Log[(-a + Sqrt[1 + a^2] + E^ArcSinh[a + b*x])/(-a + Sqrt[1 + a^2])] +
 6*b*x*ArcSinh[a + b*x]*PolyLog[2, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])]
- 6*b*x*ArcSinh[a + b*x]*PolyLog[2, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])]
 - 6*b*x*PolyLog[3, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])] + 6*b*x*PolyLog
[3, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])])/(Sqrt[1 + a^2]*x))
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{arsinh}\,(bx+a)^3}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^3/x^2,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(b*x + a)^3/x^2, x)
```

giac **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}\left(bx+a\right)^{3}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^3/x^2,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(b*x + a)^3/x^2, x)
```

maple **[F]** time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsinh}\left(bx+a\right)^{3}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)^3/x^2,x)
```

```
[Out] int(arcsinh(b*x+a)^3/x^2,x)
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\log\left(bx+a+\sqrt{b^{2}x^{2}+2\,abx+a^{2}+1}\right)^{3}}{x}+\int \frac{3\left(b^{3}x^{2}+2\,ab^{2}x+a^{2}b+\sqrt{b^{2}x^{2}+2\,abx+a^{2}+1}\left(b^{2}x+ab\right)+b}{b^{3}x^{4}+3\,ab^{2}x^{3}+\left(3\,a^{2}b+b\right)x^{2}+\left(a^{3}+a\right)x+\left(b^{2}x^{3}+2\,abx^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^3/x^2,x, algorithm="maxima")
```

```
[Out] -log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^3/x + integrate(3*(b^3*x^
2 + 2*a*b^2*x + a^2*b + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b^2*x + a*b) + b
)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2/(b^3*x^4 + 3*a*b^2*x^3
 + (3*a^2*b + b)*x^2 + (a^3 + a)*x + (b^2*x^3 + 2*a*b*x^2 + (a^2 + 1)*x)*sq
rt(b^2*x^2 + 2*a*b*x + a^2 + 1)), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{asinh}\left(a+b\,x\right)^{3}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a + b*x)^3/x^2,x)
```

```
[Out] int(asinh(a + b*x)^3/x^2, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}^{3}\left(a+bx\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)**3/x**2,x)
```

```
[Out] Integral(asinh(a + b*x)**3/x**2, x)
```

3.80 $\int \frac{\sinh^{-1}(a+bx)^3}{x^3}\,dx$

Optimal. Leaf size=514

$$\frac{3ab^2\sinh^{-1}(a+bx)\mathrm{Li}_2\left(\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)}{\left(a^2+1\right)^{3/2}}-\frac{3ab^2\sinh^{-1}(a+bx)\mathrm{Li}_2\left(\frac{e^{\sinh^{-1}(a+bx)}}{a+\sqrt{a^2+1}}\right)}{\left(a^2+1\right)^{3/2}}+\frac{3b^2\mathrm{Li}_2\left(\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)}{a^2+1}+\frac{3b^2\mathrm{Li}_2\left(\frac{e^{\sinh^{-1}(a+}}{a+\sqrt{a^2+}}\right.}{a^2+1}$$

```
[Out] -3/2*b^2*arcsinh(b*x+a)^2/(a^2+1)-1/2*arcsinh(b*x+a)^3/x^2+3*b^2*arcsinh(b*
x+a)*ln(1-(b*x+a+(1+(b*x+a)^2)^(1/2))/(a-(a^2+1)^(1/2)))/(a^2+1)+3/2*a*b^2*
arcsinh(b*x+a)^2*ln(1-(b*x+a+(1+(b*x+a)^2)^(1/2))/(a-(a^2+1)^(1/2)))/(a^2+1
)^(3/2)+3*b^2*arcsinh(b*x+a)*ln(1-(b*x+a+(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1
/2)))/(a^2+1)-3/2*a*b^2*arcsinh(b*x+a)^2*ln(1-(b*x+a+(1+(b*x+a)^2)^(1/2))/(
a+(a^2+1)^(1/2)))/(a^2+1)^(3/2)+3*b^2*polylog(2,(b*x+a+(1+(b*x+a)^2)^(1/2))
/(a-(a^2+1)^(1/2)))/(a^2+1)+3*a*b^2*arcsinh(b*x+a)*polylog(2,(b*x+a+(1+(b*x
+a)^2)^(1/2))/(a-(a^2+1)^(1/2)))/(a^2+1)^(3/2)+3*b^2*polylog(2,(b*x+a+(1+(b
*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))/(a^2+1)-3*a*b^2*arcsinh(b*x+a)*polylog(2
,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))/(a^2+1)^(3/2)-3*a*b^2*polyl
og(3,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a-(a^2+1)^(1/2)))/(a^2+1)^(3/2)+3*a*b^2*p
olylog(3,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))/(a^2+1)^(3/2)-3/2*b
*arcsinh(b*x+a)^2*(1+(b*x+a)^2)^(1/2)/(a^2+1)/x
```

Rubi [A] time = 0.88, antiderivative size = 514, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 13, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.083, Rules used = {5865, 5801, 5831, 3324, 3322, 2264, 2190, 2531, 2282, 6589, 5561, 2279, 2391}

$$\frac{3ab^2\sinh^{-1}(a+bx)\mathrm{PolyLog}\left(2,\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)}{\left(a^2+1\right)^{3/2}}-\frac{3ab^2\sinh^{-1}(a+bx)\mathrm{PolyLog}\left(2,\frac{e^{\sinh^{-1}(a+bx)}}{\sqrt{a^2+1}+a}\right)}{\left(a^2+1\right)^{3/2}}+\frac{3b^2\mathrm{PolyLog}\left(2,\frac{e^{\sinh^{-}}}{a-\sqrt{}}\right.}{a^2+1}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a + b*x]^3/x^3,x]
```

```
[Out] (-3*b^2*ArcSinh[a + b*x]^2)/(2*(1 + a^2)) - (3*b*Sqrt[1 + (a + b*x)^2]*ArcS
inh[a + b*x]^2)/(2*(1 + a^2)*x) - ArcSinh[a + b*x]^3/(2*x^2) + (3*b^2*ArcSi
nh[a + b*x]*Log[1 - E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])])/(1 + a^2) + (3
*a*b^2*ArcSinh[a + b*x]^2*Log[1 - E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])])/
(2*(1 + a^2)^(3/2)) + (3*b^2*ArcSinh[a + b*x]*Log[1 - E^ArcSinh[a + b*x]/(a
 + Sqrt[1 + a^2])])/(1 + a^2) - (3*a*b^2*ArcSinh[a + b*x]^2*Log[1 - E^ArcSi
nh[a + b*x]/(a + Sqrt[1 + a^2])])/(2*(1 + a^2)^(3/2)) + (3*b^2*PolyLog[2, E
^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])])/(1 + a^2) + (3*a*b^2*ArcSinh[a + b*
x]*PolyLog[2, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])])/(1 + a^2)^(3/2) + (3
*b^2*PolyLog[2, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])])/(1 + a^2) - (3*a*b
^2*ArcSinh[a + b*x]*PolyLog[2, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])])/(1
+ a^2)^(3/2) - (3*a*b^2*PolyLog[3, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])])
/(1 + a^2)^(3/2) + (3*a*b^2*PolyLog[3, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2
])])/(1 + a^2)^(3/2)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3324

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[(b*(c + d*x)^m*Cos[e + f*x])/(f*(a^2 - b^2)*(a + b*Sin[e +
f*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[(b*d*m)/(f*(a^2 - b^2)), Int[((c + d*x)^(m - 1)*Cos[e + f*x])/(a
 + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5801

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n
 - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5831

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\sinh^{-1}(a+bx)^3}{x^3}\,dx = \frac{\text{Subst}\left(\int \frac{\sinh^{-1}(x)^3}{\left(-\frac{a}{b}+\frac{x}{b}\right)^3}\,dx, x, a+bx\right)}{b}$$

$$= -\frac{\sinh^{-1}(a+bx)^3}{2x^2} + \frac{3}{2}\,\text{Subst}\left(\int \frac{\sinh^{-1}(x)^2}{\left(-\frac{a}{b}+\frac{x}{b}\right)^2\sqrt{1+x^2}}\,dx, x, a+bx\right)$$

$$= -\frac{\sinh^{-1}(a+bx)^3}{2x^2} + \frac{3}{2}\,\text{Subst}\left(\int \frac{x^2}{\left(-\frac{a}{b}+\frac{\sinh(x)}{b}\right)^2}\,dx, x, \sinh^{-1}(a+bx)\right)$$

$$= -\frac{3b\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)^2}{2\left(1+a^2\right)x} - \frac{\sinh^{-1}(a+bx)^3}{2x^2} + \frac{(3b)\,\text{Subst}\left(\int \frac{x\cosh(x)}{-\frac{a}{b}+\frac{\sinh(x)}{b}}\,dx,\right.}{1+a^2}$$

$$= -\frac{3b^2\sinh^{-1}(a+bx)^2}{2\left(1+a^2\right)} - \frac{3b\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)^2}{2\left(1+a^2\right)x} - \frac{\sinh^{-1}(a+bx)^3}{2x^2} + \frac{(3b)}{}$$

$$= -\frac{3b^2\sinh^{-1}(a+bx)^2}{2\left(1+a^2\right)} - \frac{3b\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)^2}{2\left(1+a^2\right)x} - \frac{\sinh^{-1}(a+bx)^3}{2x^2} + \frac{3b^2}{}$$

$$= -\frac{3b^2\sinh^{-1}(a+bx)^2}{2\left(1+a^2\right)} - \frac{3b\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)^2}{2\left(1+a^2\right)x} - \frac{\sinh^{-1}(a+bx)^3}{2x^2} + \frac{3b^2}{}$$

$$= -\frac{3b^2\sinh^{-1}(a+bx)^2}{2\left(1+a^2\right)} - \frac{3b\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)^2}{2\left(1+a^2\right)x} - \frac{\sinh^{-1}(a+bx)^3}{2x^2} + \frac{3b^2}{}$$

$$= -\frac{3b^2\sinh^{-1}(a+bx)^2}{2\left(1+a^2\right)} - \frac{3b\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)^2}{2\left(1+a^2\right)x} - \frac{\sinh^{-1}(a+bx)^3}{2x^2} + \frac{3b^2}{}$$

$$= -\frac{3b^2\sinh^{-1}(a+bx)^2}{2\left(1+a^2\right)} - \frac{3b\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)^2}{2\left(1+a^2\right)x} - \frac{\sinh^{-1}(a+bx)^3}{2x^2} + \frac{3b^2}{}$$

Mathematica [A] time = 0.24, size = 524, normalized size = 1.02

$$\frac{6b^2x^2\left(\sqrt{a^2+1}+a\sinh^{-1}(a+bx)\right)\text{Li}_2\left(\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right) + 6b^2x^2\left(\sqrt{a^2+1}-a\sinh^{-1}(a+bx)\right)\text{Li}_2\left(\frac{e^{\sinh^{-1}(a+bx)}}{a+\sqrt{a^2+1}}\right) -}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a + b*x]^3/x^3,x]
```

```
[Out] (-3*Sqrt[1 + a^2]*b^2*x^2*ArcSinh[a + b*x]^2 - 3*Sqrt[1 + a^2]*b*x*Sqrt[1 +
 a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x]^2 - Sqrt[1 + a^2]*ArcSinh[a + b*
x]^3 - a^2*Sqrt[1 + a^2]*ArcSinh[a + b*x]^3 + 6*Sqrt[1 + a^2]*b^2*x^2*ArcSi
nh[a + b*x]*Log[(a + Sqrt[1 + a^2] - E^ArcSinh[a + b*x])/(a + Sqrt[1 + a^2]
)] - 3*a*b^2*x^2*ArcSinh[a + b*x]^2*Log[(a + Sqrt[1 + a^2] - E^ArcSinh[a +
b*x])/(a + Sqrt[1 + a^2])] + 6*Sqrt[1 + a^2]*b^2*x^2*ArcSinh[a + b*x]*Log[(
-a + Sqrt[1 + a^2] + E^ArcSinh[a + b*x])/(-a + Sqrt[1 + a^2])] + 3*a*b^2*x^
2*ArcSinh[a + b*x]^2*Log[(-a + Sqrt[1 + a^2] + E^ArcSinh[a + b*x])/(-a + Sq
rt[1 + a^2])] + 6*b^2*x^2*(Sqrt[1 + a^2] + a*ArcSinh[a + b*x])*PolyLog[2, E
^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])] + 6*b^2*x^2*(Sqrt[1 + a^2] - a*ArcSi
nh[a + b*x])*PolyLog[2, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])] - 6*a*b^2*x
^2*PolyLog[3, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])] + 6*a*b^2*x^2*PolyLog
[3, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])])/(2*(1 + a^2)^(3/2)*x^2)
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\operatorname{arsinh}(bx+a)^3}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^3/x^3,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(b*x + a)^3/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}(bx+a)^3}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^3/x^3,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(b*x + a)^3/x^3, x)
```

maple [F] time = 0.76, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsinh}(bx+a)^3}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)^3/x^3,x)
```

```
[Out] int(arcsinh(b*x+a)^3/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^3}{2\,x^2}+\int \frac{3\left(b^3x^2+2\,ab^2x+a^2b+\sqrt{b^2x^2+2\,abx+a^2+1}\left(b^2x+ab\right)+b\right)\text{lo}}{2\left(b^3x^5+3\,ab^2x^4+\left(3\,a^2b+b\right)x^3+\left(a^3+a\right)x^2+\left(b^2x^4+2\,abx^3+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^3/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^3/x^2 + integrate(3/2
*(b^3*x^2 + 2*a*b^2*x + a^2*b + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b^2*x +
a*b) + b)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2/(b^3*x^5 + 3*a
```

```
*b^2*x^4 + (3*a^2*b + b)*x^3 + (a^3 + a)*x^2 + (b^2*x^4 + 2*a*b*x^3 + (a^2
+ 1)*x^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{asinh}(a+b\,x)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a + b*x)^3/x^3,x)
[Out] int(asinh(a + b*x)^3/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}^3(a+bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)**3/x**3,x)
[Out] Integral(asinh(a + b*x)**3/x**3, x)
```

3.81 $\int \frac{x^2}{\sinh^{-1}(a+bx)} \, dx$

Optimal. Leaf size=60

$$\frac{a^2 \text{Chi}\left(\sinh^{-1}(a+bx)\right)}{b^3}-\frac{\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{4b^3}+\frac{\text{Chi}\left(3\sinh^{-1}(a+bx)\right)}{4b^3}-\frac{a\text{Shi}\left(2\sinh^{-1}(a+bx)\right)}{b^3}$$

```
[Out] -1/4*Chi(arcsinh(b*x+a))/b^3+a^2*Chi(arcsinh(b*x+a))/b^3+1/4*Chi(3*arcsinh(
b*x+a))/b^3-a*Shi(2*arcsinh(b*x+a))/b^3
```

Rubi [A] time = 0.53, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5865, 5805, 6741, 12, 6742, 3301, 5448, 3298}

$$\frac{a^2 \text{Chi}\left(\sinh^{-1}(a+bx)\right)}{b^3}-\frac{\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{4b^3}+\frac{\text{Chi}\left(3\sinh^{-1}(a+bx)\right)}{4b^3}-\frac{a\text{Shi}\left(2\sinh^{-1}(a+bx)\right)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/ArcSinh[a + b*x],x]
```

```
[Out] -CoshIntegral[ArcSinh[a + b*x]]/(4*b^3) + (a^2*CoshIntegral[ArcSinh[a + b*x
]])/b^3 + CoshIntegral[3*ArcSinh[a + b*x]]/(4*b^3) - (a*SinhIntegral[2*ArcS
inh[a + b*x]])/b^3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5805

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]*(c*d + e*Sinh[x
])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m, 0
]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
```

```
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{x^2}{\sinh^{-1}(a+bx)}\,dx = \frac{\text{Subst}\left(\int \frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^2}{\sinh^{-1}(x)}\,dx, x, a+bx\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{\cosh(x)\left(-\frac{a}{b}+\frac{\sinh(x)}{b}\right)^2}{x}\,dx, x, \sinh^{-1}(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{\cosh(x)(a-\sinh(x))^2}{b^2x}\,dx, x, \sinh^{-1}(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{\cosh(x)(a-\sinh(x))^2}{x}\,dx, x, \sinh^{-1}(a+bx)\right)}{b^3}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^2\cosh(x)}{x} - \frac{2a\cosh(x)\sinh(x)}{x} + \frac{\cosh(x)\sinh^2(x)}{x}\right)dx, x, \sinh^{-1}(a+bx)\right)}{b^3}$$
$$= \frac{\text{Subst}\left(\int \frac{\cosh(x)\sinh^2(x)}{x}\,dx, x, \sinh^{-1}(a+bx)\right)}{b^3} - \frac{(2a)\,\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{x}\,dx, x, \text{sinh}\right.}{b^3}$$
$$= \frac{a^2\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{b^3} + \frac{\text{Subst}\left(\int \left(-\frac{\cosh(x)}{4x}+\frac{\cosh(3x)}{4x}\right)dx, x, \sinh^{-1}(a+bx)\right)}{b^3} - \frac{(2a}{}$$
$$= \frac{a^2\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{b^3} - \frac{\text{Subst}\left(\int \frac{\cosh(x)}{x}\,dx, x, \sinh^{-1}(a+bx)\right)}{4b^3} + \frac{\text{Subst}\left(\int \frac{\cosh(3x)}{x}\right.}{}$$
$$= -\frac{\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{4b^3} + \frac{a^2\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{b^3} + \frac{\text{Chi}\left(3\sinh^{-1}(a+bx)\right)}{4b^3} - \frac{a\text{Shi}\left(2\right.}{}$$

Mathematica [A] time = 0.15, size = 44, normalized size = 0.73

$$\frac{\left(4a^2-1\right)\text{Chi}\left(\sinh^{-1}(a+bx)\right)+\text{Chi}\left(3\sinh^{-1}(a+bx)\right)-4a\text{Shi}\left(2\sinh^{-1}(a+bx)\right)}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/ArcSinh[a + b*x],x]
```

```
[Out] ((-1 + 4*a^2)*CoshIntegral[ArcSinh[a + b*x]] + CoshIntegral[3*ArcSinh[a + b
*x]] - 4*a*SinhIntegral[2*ArcSinh[a + b*x]])/(4*b^3)
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^2}{\text{arsinh}\,(bx+a)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(x^2/arcsinh(b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\operatorname{arsinh}(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^2/arcsinh(b*x + a), x)
```

maple [A] time = 0.12, size = 49, normalized size = 0.82

$$\frac{-a\,\mathrm{Shi}\left(2\operatorname{arcsinh}(bx+a)\right)-\frac{\mathrm{X}(\operatorname{arcsinh}(bx+a))}{4}+\frac{\mathrm{X}(3\operatorname{arcsinh}(bx+a))}{4}+a^2\mathrm{X}\left(\operatorname{arcsinh}(bx+a)\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arcsinh(b*x+a),x)
```

```
[Out] 1/b^3*(-a*Shi(2*arcsinh(b*x+a))-1/4*Chi(arcsinh(b*x+a))+1/4*Chi(3*arcsinh(b
*x+a))+a^2*Chi(arcsinh(b*x+a)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\operatorname{arsinh}(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x^2/arcsinh(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{\operatorname{asinh}(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/asinh(a + b*x),x)
```

```
[Out] int(x^2/asinh(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\operatorname{asinh}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/asinh(b*x+a),x)
```

```
[Out] Integral(x**2/asinh(a + b*x), x)
```

3.82 $\int \frac{x}{\sinh^{-1}(a+bx)} dx$

Optimal. Leaf size=30

$$\frac{\text{Shi}\left(2\sinh^{-1}(a+bx)\right)}{2b^2}-\frac{a\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{b^2}$$

```
[Out] -a*Chi(arcsinh(b*x+a))/b^2+1/2*Shi(2*arcsinh(b*x+a))/b^2
```

Rubi [A] time = 0.21, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {5865, 5805, 6741, 12, 6742, 3301, 5448, 3298}

$$\frac{\text{Shi}\left(2\sinh^{-1}(a+bx)\right)}{2b^2}-\frac{a\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/ArcSinh[a + b*x],x]
```

```
[Out] -((a*CoshIntegral[ArcSinh[a + b*x]])/b^2) + SinhIntegral[2*ArcSinh[a + b*x]
]/(2*b^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5805

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]*(c*d + e*Sinh[x
])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m, 0
]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\sinh^{-1}(a+bx)}\,dx &= \frac{\text{Subst}\left(\int \frac{-\frac{a}{b}+\frac{x}{b}}{\sinh^{-1}(x)}\,dx, x, a+bx\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{\cosh(x)\left(-\frac{a}{b}+\frac{\sinh(x)}{b}\right)}{x}\,dx, x, \sinh^{-1}(a+bx)\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{\cosh(x)(-a+\sinh(x))}{bx}\,dx, x, \sinh^{-1}(a+bx)\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{\cosh(x)(-a+\sinh(x))}{x}\,dx, x, \sinh^{-1}(a+bx)\right)}{b^2}\\
&= \frac{\text{Subst}\left(\int \left(-\frac{a\cosh(x)}{x}+\frac{\cosh(x)\sinh(x)}{x}\right)dx, x, \sinh^{-1}(a+bx)\right)}{b^2}\\
&= \frac{\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{x}\,dx, x, \sinh^{-1}(a+bx)\right)}{b^2}-\frac{a\,\text{Subst}\left(\int \frac{\cosh(x)}{x}\,dx, x, \sinh^{-1}(a+bx)\right)}{b^2}\\
&= -\frac{a\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{b^2}+\frac{\text{Subst}\left(\int \frac{\sinh(2x)}{2x}\,dx, x, \sinh^{-1}(a+bx)\right)}{b^2}\\
&= -\frac{a\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{b^2}+\frac{\text{Subst}\left(\int \frac{\sinh(2x)}{x}\,dx, x, \sinh^{-1}(a+bx)\right)}{2b^2}\\
&= -\frac{a\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{b^2}+\frac{\text{Shi}\left(2\sinh^{-1}(a+bx)\right)}{2b^2}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 30, normalized size = 1.00

$$\frac{\text{Shi}\left(2\sinh^{-1}(a+bx)\right)}{2b^2}-\frac{a\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/ArcSinh[a + b*x],x]
```

```
[Out] -((a*CoshIntegral[ArcSinh[a + b*x]])/b^2) + SinhIntegral[2*ArcSinh[a + b*x]
]/(2*b^2)
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x}{\text{arsinh}\,(bx+a)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(x/arcsinh(b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\operatorname{arsinh}(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x/arcsinh(b*x + a), x)
```

maple [A] time = 0.06, size = 27, normalized size = 0.90

$$\frac{\frac{\operatorname{Shi}(2\operatorname{arcsinh}(bx+a))}{2} - a\mathrm{X}(\operatorname{arcsinh}(bx+a))}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arcsinh(b*x+a),x)
```

```
[Out] 1/b^2*(1/2*Shi(2*arcsinh(b*x+a))-a*Chi(arcsinh(b*x+a)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\operatorname{arsinh}(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x/arcsinh(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x}{\operatorname{asinh}(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/asinh(a + b*x),x)
```

```
[Out] int(x/asinh(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\operatorname{asinh}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/asinh(b*x+a),x)
```

```
[Out] Integral(x/asinh(a + b*x), x)
```

3.83 $\int \frac{1}{\sinh^{-1}(a+bx)} dx$

Optimal. Leaf size=11

$$\frac{\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{b}$$

```
[Out] Chi(arcsinh(b*x+a))/b
```

Rubi [A] time = 0.02, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5863, 5657, 3301}

$$\frac{\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a + b*x]^(-1),x]
```

```
[Out] CoshIntegral[ArcSinh[a + b*x]]/b
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 5863

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sinh^{-1}(a+bx)} dx &= \frac{\text{Subst}\left(\int \frac{1}{\sinh^{-1}(x)} dx, x, a+bx\right)}{b} \\
&= \frac{\text{Subst}\left(\int \frac{\cosh(x)}{x} dx, x, \sinh^{-1}(a+bx)\right)}{b} \\
&= \frac{\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{b}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 11, normalized size = 1.00

$$\frac{\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a + b*x]^(-1),x]
[Out] CoshIntegral[ArcSinh[a + b*x]]/b
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\operatorname{arsinh}(bx+a)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(b*x+a),x, algorithm="fricas")
[Out] integral(1/arcsinh(b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{arsinh}(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(b*x+a),x, algorithm="giac")
[Out] integrate(1/arcsinh(b*x + a), x)
```

maple [A] time = 0.05, size = 12, normalized size = 1.09

$$\frac{\mathrm{X}(\operatorname{arcsinh}(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arcsinh(b*x+a),x)
[Out] Chi(arcsinh(b*x+a))/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{arsinh}(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(b*x+a),x, algorithm="maxima")
[Out] integrate(1/arcsinh(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.09

$$\int \frac{1}{\operatorname{asinh}(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/asinh(a + b*x),x)
[Out] int(1/asinh(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{asinh}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/asinh(b*x+a),x)
[Out] Integral(1/asinh(a + b*x), x)
```

3.84 $\int \frac{1}{x \sinh^{-1}(a+bx)} dx$

Optimal. Leaf size=15

$$\text{Int}\left(\frac{1}{x \sinh^{-1}(a+bx)}, x\right)$$

[Out] Unintegrable(1/x/arcsinh(b*x+a),x)

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x \sinh^{-1}(a+bx)} dx$$

Verification is Not applicable to the result.

[In] Int[1/(x*ArcSinh[a + b*x]),x]

[Out] Defer[Subst][Defer[Int][1/((-(a/b) + x/b)*ArcSinh[x]), x], x, a + b*x]/b

Rubi steps

$$\int \frac{1}{x \sinh^{-1}(a+bx)} dx = \frac{\text{Subst}\left(\int \frac{1}{\left(-\frac{a}{b}+\frac{x}{b}\right)\sinh^{-1}(x)} dx, x, a+bx\right)}{b}$$

Mathematica [A] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{1}{x \sinh^{-1}(a+bx)} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x*ArcSinh[a + b*x]),x]

[Out] Integrate[1/(x*ArcSinh[a + b*x]), x]

fricas [A] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{x \operatorname{arsinh}(bx+a)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/arcsinh(b*x+a),x, algorithm="fricas")

[Out] integral(1/(x*arcsinh(b*x + a)), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{arsinh}(bx+a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/arcsinh(b*x+a),x, algorithm="giac")

[Out] integrate(1/(x*arcsinh(b*x + a)), x)

maple [A] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{arcsinh}(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arcsinh(b*x+a),x)
```

```
[Out] int(1/x/arcsinh(b*x+a),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{arsinh}(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*arcsinh(b*x + a)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{1}{x \operatorname{asinh}(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*asinh(a + b*x)),x)
```

```
[Out] int(1/(x*asinh(a + b*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{asinh}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/asinh(b*x+a),x)
```

```
[Out] Integral(1/(x*asinh(a + b*x)), x)
```

3.85 $\int \frac{x^2}{\sinh^{-1}(a+bx)^2} dx$

Optimal. Leaf size=154

$$\frac{a^2\text{Shi}\left(\sinh^{-1}(a+bx)\right)}{b^3}-\frac{a^2\sqrt{(a+bx)^2+1}}{b^3\sinh^{-1}(a+bx)}-\frac{2a\text{Chi}\left(2\sinh^{-1}(a+bx)\right)}{b^3}-\frac{\text{Shi}\left(\sinh^{-1}(a+bx)\right)}{4b^3}+\frac{3\text{Shi}\left(3\sinh^{-1}(a+\right.}{4b^3}$$

```
[Out] -2*a*Chi(2*arcsinh(b*x+a))/b^3-1/4*Shi(arcsinh(b*x+a))/b^3+a^2*Shi(arcsinh(
b*x+a))/b^3+3/4*Shi(3*arcsinh(b*x+a))/b^3-a^2*(1+(b*x+a)^2)^(1/2)/b^3/arcsi
nh(b*x+a)+2*a*(b*x+a)*(1+(b*x+a)^2)^(1/2)/b^3/arcsinh(b*x+a)-(b*x+a)^2*(1+(
b*x+a)^2)^(1/2)/b^3/arcsinh(b*x+a)
```

Rubi [A] time = 0.22, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5865, 5803, 5655, 5779, 3298, 5665, 3301}

$$\frac{a^2\text{Shi}\left(\sinh^{-1}(a+bx)\right)}{b^3}-\frac{a^2\sqrt{(a+bx)^2+1}}{b^3\sinh^{-1}(a+bx)}-\frac{2a\text{Chi}\left(2\sinh^{-1}(a+bx)\right)}{b^3}-\frac{\text{Shi}\left(\sinh^{-1}(a+bx)\right)}{4b^3}+\frac{3\text{Shi}\left(3\sinh^{-1}(a+\right.}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/ArcSinh[a + b*x]^2,x]
```

```
[Out] -((a^2*Sqrt[1 + (a + b*x)^2])/(b^3*ArcSinh[a + b*x])) + (2*a*(a + b*x)*Sqrt
[1 + (a + b*x)^2])/(b^3*ArcSinh[a + b*x]) - ((a + b*x)^2*Sqrt[1 + (a + b*x)
^2])/(b^3*ArcSinh[a + b*x]) - (2*a*CoshIntegral[2*ArcSinh[a + b*x]])/b^3 -
SinhIntegral[ArcSinh[a + b*x]]/(4*b^3) + (a^2*SinhIntegral[ArcSinh[a + b*x]
])/b^3 + (3*SinhIntegral[3*ArcSinh[a + b*x]])/(4*b^3)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 5803

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*ArcSinh[c*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[m, 0] && LtQ[n, -1]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{x^2}{\sinh^{-1}(a+bx)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^2}{\sinh^{-1}(x)^2}\,dx, x, a+bx\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a^2}{b^2\sinh^{-1}(x)^2} - \frac{2ax}{b^2\sinh^{-1}(x)^2} + \frac{x^2}{b^2\sinh^{-1}(x)^2}\right) dx, x, a+bx\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \frac{x^2}{\sinh^{-1}(x)^2}\,dx, x, a+bx\right)}{b^3} - \frac{(2a)\,\text{Subst}\left(\int \frac{x}{\sinh^{-1}(x)^2}\,dx, x, a+bx\right)}{b^3} + \frac{a^2\,\text{Subst}\left(\right.}{}$$

$$= -\frac{a^2\sqrt{1+(a+bx)^2}}{b^3\sinh^{-1}(a+bx)} + \frac{2a(a+bx)\sqrt{1+(a+bx)^2}}{b^3\sinh^{-1}(a+bx)} - \frac{(a+bx)^2\sqrt{1+(a+bx)^2}}{b^3\sinh^{-1}(a+bx)} + \frac{\text{Subst}}{}$$

$$= -\frac{a^2\sqrt{1+(a+bx)^2}}{b^3\sinh^{-1}(a+bx)} + \frac{2a(a+bx)\sqrt{1+(a+bx)^2}}{b^3\sinh^{-1}(a+bx)} - \frac{(a+bx)^2\sqrt{1+(a+bx)^2}}{b^3\sinh^{-1}(a+bx)} - \frac{2a\text{Ch}}{}$$

$$= -\frac{a^2\sqrt{1+(a+bx)^2}}{b^3\sinh^{-1}(a+bx)} + \frac{2a(a+bx)\sqrt{1+(a+bx)^2}}{b^3\sinh^{-1}(a+bx)} - \frac{(a+bx)^2\sqrt{1+(a+bx)^2}}{b^3\sinh^{-1}(a+bx)} - \frac{2a\text{Ch}}{}$$

Mathematica [A] time = 0.46, size = 83, normalized size = 0.54

$$\frac{-\frac{4b^2x^2\sqrt{a^2+2abx+b^2x^2+1}}{\sinh^{-1}(a+bx)} + \left(4a^2-1\right)\text{Shi}\left(\sinh^{-1}(a+bx)\right) - 8a\text{Chi}\left(2\sinh^{-1}(a+bx)\right) + 3\text{Shi}\left(3\sinh^{-1}(a+bx)\right)}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/ArcSinh[a + b*x]^2,x]
```

```
[Out] ((-4*b^2*x^2*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2])/ArcSinh[a + b*x] - 8*a*Cosh
Integral[2*ArcSinh[a + b*x]] + (-1 + 4*a^2)*SinhIntegral[ArcSinh[a + b*x]]
+ 3*SinhIntegral[3*ArcSinh[a + b*x]])/(4*b^3)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^2}{\operatorname{arsinh}\left(bx+a\right)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(x^2/arcsinh(b*x + a)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\operatorname{arsinh}(bx+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2/arcsinh(b*x + a)^2, x)
```

maple [A] time = 0.13, size = 146, normalized size = 0.95

$$\frac{-\frac{a(2\mathrm{X}(2\operatorname{arcsinh}(bx+a))\operatorname{arcsinh}(bx+a)-\sinh(2\operatorname{arcsinh}(bx+a)))}{\operatorname{arcsinh}(bx+a)}+\frac{\sqrt{1+(bx+a)^2}}{4\operatorname{arcsinh}(bx+a)}-\frac{\operatorname{Shi}(\operatorname{arcsinh}(bx+a))}{4}-\frac{\cosh(3\operatorname{arcsinh}(bx+a))}{4\operatorname{arcsinh}(bx+a)}+\frac{3\operatorname{Shi}(3\operatorname{arc}}{}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arcsinh(b*x+a)^2,x)
```

```
[Out] 1/b^3*(-a*(2*Chi(2*arcsinh(b*x+a))*arcsinh(b*x+a)-sinh(2*arcsinh(b*x+a)))/a
rcsinh(b*x+a)+1/4/arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)-1/4*Shi(arcsinh(b*x+a)
)-1/4/arcsinh(b*x+a)*cosh(3*arcsinh(b*x+a))+3/4*Shi(3*arcsinh(b*x+a))+a^2*(
Shi(arcsinh(b*x+a))*arcsinh(b*x+a)-(1+(b*x+a)^2)^(1/2))/arcsinh(b*x+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b^3x^5+3\,ab^2x^4+\left(3\,a^2b+b\right)x^3+\left(a^3+a\right)x^2+\left(b^2x^4+2\,abx^3+\left(a^2+1\right)x^2\right)\sqrt{b^2x^2+2\,abx+a^2+1}}{\left(b^3x^2+2\,ab^2x+a^2b+\sqrt{b^2x^2+2\,abx+a^2+1}\left(b^2x+ab\right)+b\right)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)}+\int\frac{3\,b^5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(b^3*x^5 + 3*a*b^2*x^4 + (3*a^2*b + b)*x^3 + (a^3 + a)*x^2 + (b^2*x^4 + 2*
a*b*x^3 + (a^2 + 1)*x^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/((b^3*x^2 + 2*a
*b^2*x + a^2*b + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b^2*x + a*b) + b)*log(b
*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))) + integrate((3*b^5*x^6 + 14*a*
b^4*x^5 + 2*(13*a^2*b^3 + 3*b^3)*x^4 + 8*(3*a^3*b^2 + 2*a*b^2)*x^3 + (11*a^
4*b + 14*a^2*b + 3*b)*x^2 + (3*b^3*x^4 + 8*a*b^2*x^3 + (7*a^2*b + b)*x^2 +
2*(a^3 + a)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + 2*(a^5 + 2*a^3 + a)*x + (6*b
^4*x^5 + 22*a*b^3*x^4 + (30*a^2*b^2 + 7*b^2)*x^3 + (18*a^3*b + 13*a*b)*x^2
+ 2*(2*a^4 + 3*a^2 + 1)*x)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/((b^5*x^4 + 4
*a*b^4*x^3 + a^4*b + 2*a^2*b + 2*(3*a^2*b^3 + b^3)*x^2 + (b^3*x^2 + 2*a*b^2
*x + a^2*b)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + 4*(a^3*b^2 + a*b^2)*x + 2*(b^4*
x^3 + 3*a*b^3*x^2 + a^3*b + a*b + (3*a^2*b^2 + b^2)*x)*sqrt(b^2*x^2 + 2*a*b
*x + a^2 + 1) + b)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\operatorname{asinh}(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/asinh(a + b*x)^2,x)
```

```
[Out] int(x^2/asinh(a + b*x)^2, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\operatorname{asinh}^2(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/asinh(b*x+a)**2,x)
```

```
[Out] Integral(x**2/asinh(a + b*x)**2, x)
```

3.86 $\int \frac{x}{\sinh^{-1}(a+bx)^2} \, dx$

Optimal. Leaf size=84

$$\frac{\text{Chi}\left(2\sinh^{-1}(a+bx)\right)}{b^2}-\frac{a\text{Shi}\left(\sinh^{-1}(a+bx)\right)}{b^2}+\frac{a\sqrt{(a+bx)^2+1}}{b^2\sinh^{-1}(a+bx)}-\frac{(a+bx)\sqrt{(a+bx)^2+1}}{b^2\sinh^{-1}(a+bx)}$$

```
[Out] Chi(2*arcsinh(b*x+a))/b^2-a*Shi(arcsinh(b*x+a))/b^2+a*(1+(b*x+a)^2)^(1/2)/b
^2/arcsinh(b*x+a)-(b*x+a)*(1+(b*x+a)^2)^(1/2)/b^2/arcsinh(b*x+a)
```

Rubi [A] time = 0.13, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {5865, 5803, 5655, 5779, 3298, 5665, 3301}

$$\frac{\text{Chi}\left(2\sinh^{-1}(a+bx)\right)}{b^2}-\frac{a\text{Shi}\left(\sinh^{-1}(a+bx)\right)}{b^2}+\frac{a\sqrt{(a+bx)^2+1}}{b^2\sinh^{-1}(a+bx)}-\frac{(a+bx)\sqrt{(a+bx)^2+1}}{b^2\sinh^{-1}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[x/ArcSinh[a + b*x]^2,x]
```

```
[Out] (a*Sqrt[1 + (a + b*x)^2])/(b^2*ArcSinh[a + b*x]) - ((a + b*x)*Sqrt[1 + (a +
 b*x)^2])/(b^2*ArcSinh[a + b*x]) + CoshIntegral[2*ArcSinh[a + b*x]]/b^2 - (
a*SinhIntegral[ArcSinh[a + b*x]])/b^2
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 5803

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*ArcSinh[c*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[m, 0] && LtQ[n, -1]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\sinh^{-1}(a+bx)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{-\frac{a}{b}+\frac{x}{b}}{\sinh^{-1}(x)^2}\,dx, x, a+bx\right)}{b}\\
&= \frac{\text{Subst}\left(\int \left(-\frac{a}{b\sinh^{-1}(x)^2}+\frac{x}{b\sinh^{-1}(x)^2}\right)dx, x, a+bx\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{x}{\sinh^{-1}(x)^2}\,dx, x, a+bx\right)}{b^2}-\frac{a\,\text{Subst}\left(\int \frac{1}{\sinh^{-1}(x)^2}\,dx, x, a+bx\right)}{b^2}\\
&= \frac{a\sqrt{1+(a+bx)^2}}{b^2\sinh^{-1}(a+bx)}-\frac{(a+bx)\sqrt{1+(a+bx)^2}}{b^2\sinh^{-1}(a+bx)}+\frac{\text{Subst}\left(\int \frac{\cosh(2x)}{x}\,dx, x, \sinh^{-1}(a+bx\right.}{b^2}\\
&= \frac{a\sqrt{1+(a+bx)^2}}{b^2\sinh^{-1}(a+bx)}-\frac{(a+bx)\sqrt{1+(a+bx)^2}}{b^2\sinh^{-1}(a+bx)}+\frac{\text{Chi}\left(2\sinh^{-1}(a+bx)\right)}{b^2}-\frac{a\,\text{Subst}\left(\int\right.}{}\\
&= \frac{a\sqrt{1+(a+bx)^2}}{b^2\sinh^{-1}(a+bx)}-\frac{(a+bx)\sqrt{1+(a+bx)^2}}{b^2\sinh^{-1}(a+bx)}+\frac{\text{Chi}\left(2\sinh^{-1}(a+bx)\right)}{b^2}-\frac{a\text{Shi}\left(\sinh\right.}{b}
\end{aligned}$$

Mathematica [A] time = 0.17, size = 62, normalized size = 0.74

$$-\frac{-\sinh^{-1}(a+bx)\text{Chi}\left(2\sinh^{-1}(a+bx)\right)+a\sinh^{-1}(a+bx)\text{Shi}\left(\sinh^{-1}(a+bx)\right)+bx\sqrt{(a+bx)^2+1}}{b^2\sinh^{-1}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/ArcSinh[a + b*x]^2,x]
```

```
[Out] -((b*x*Sqrt[1 + (a + b*x)^2] - ArcSinh[a + b*x]*CoshIntegral[2*ArcSinh[a +
b*x]] + a*ArcSinh[a + b*x]*SinhIntegral[ArcSinh[a + b*x]])/(b^2*ArcSinh[a +
 b*x]))
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x}{\text{arsinh}\,(bx+a)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(x/arcsinh(b*x + a)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\operatorname{arsinh}(bx+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x/arcsinh(b*x + a)^2, x)
```

maple [A] time = 0.08, size = 73, normalized size = 0.87

$$\frac{-\frac{\sinh(2\operatorname{arcsinh}(bx+a))}{2\operatorname{arcsinh}(bx+a)} + \operatorname{X}(2\operatorname{arcsinh}(bx+a)) - \frac{a\left(\operatorname{Shi}(\operatorname{arcsinh}(bx+a))\operatorname{arcsinh}(bx+a) - \sqrt{1+(bx+a)^2}\right)}{\operatorname{arcsinh}(bx+a)}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arcsinh(b*x+a)^2,x)
```

```
[Out] 1/b^2*(-1/2*sinh(2*arcsinh(b*x+a))/arcsinh(b*x+a)+Chi(2*arcsinh(b*x+a))-a*(
Shi(arcsinh(b*x+a))*arcsinh(b*x+a)-(1+(b*x+a)^2)^(1/2))/arcsinh(b*x+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b^3x^4 + 3\,ab^2x^3 + \left(3\,a^2b+b\right)x^2 + \left(a^3+a\right)x + \left(b^2x^3 + 2\,abx^2 + \left(a^2+1\right)x\right)\sqrt{b^2x^2+2\,abx+a^2+1}}{\left(b^3x^2 + 2\,ab^2x + a^2b + \sqrt{b^2x^2+2\,abx+a^2+1}\left(b^2x+ab\right)+b\right)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)} + \int \frac{2\,b^5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(b^3*x^4 + 3*a*b^2*x^3 + (3*a^2*b + b)*x^2 + (a^3 + a)*x + (b^2*x^3 + 2*a*
b*x^2 + (a^2 + 1)*x)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/((b^3*x^2 + 2*a*b^2
*x + a^2*b + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b^2*x + a*b) + b)*log(b*x +
 a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))) + integrate((2*b^5*x^5 + 9*a*b^4*x
^4 + a^5 + 4*(4*a^2*b^3 + b^3)*x^3 + 2*a^3 + 2*(7*a^3*b^2 + 5*a*b^2)*x^2 +
(2*b^3*x^3 + 5*a*b^2*x^2 + 4*a^2*b*x + a^3 + a)*(b^2*x^2 + 2*a*b*x + a^2 +
1) + 2*(3*a^4*b + 4*a^2*b + b)*x + (4*b^4*x^4 + 14*a*b^3*x^3 + 2*a^4 + 2*(9
*a^2*b^2 + 2*b^2)*x^2 + 3*a^2 + (10*a^3*b + 7*a*b)*x + 1)*sqrt(b^2*x^2 + 2*
a*b*x + a^2 + 1) + a)/((b^5*x^4 + 4*a*b^4*x^3 + a^4*b + 2*a^2*b + 2*(3*a^2*
b^3 + b^3)*x^2 + (b^3*x^2 + 2*a*b^2*x + a^2*b)*(b^2*x^2 + 2*a*b*x + a^2 + 1
) + 4*(a^3*b^2 + a*b^2)*x + 2*(b^4*x^3 + 3*a*b^3*x^2 + a^3*b + a*b + (3*a^2
*b^2 + b^2)*x)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + b)*log(b*x + a + sqrt(b^
2*x^2 + 2*a*b*x + a^2 + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\operatorname{asinh}(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/asinh(a + b*x)^2,x)
```

```
[Out] int(x/asinh(a + b*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\operatorname{asinh}^2(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/asinh(b*x+a)**2,x)
```

```
[Out] Integral(x/asinh(a + b*x)**2, x)
```

3.87 $\int \frac{1}{\sinh^{-1}(a+bx)^2} dx$

Optimal. Leaf size=38

$$\frac{\text{Shi}\left(\sinh^{-1}(a+bx)\right)}{b}-\frac{\sqrt{(a+bx)^2+1}}{b\sinh^{-1}(a+bx)}$$

```
[Out] Shi(arcsinh(b*x+a))/b-(1+(b*x+a)^2)^(1/2)/b/arcsinh(b*x+a)
```

Rubi [A] time = 0.07, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5863, 5655, 5779, 3298}

$$\frac{\text{Shi}\left(\sinh^{-1}(a+bx)\right)}{b}-\frac{\sqrt{(a+bx)^2+1}}{b\sinh^{-1}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a + b*x]^(-2),x]
```

```
[Out] -(Sqrt[1 + (a + b*x)^2]/(b*ArcSinh[a + b*x])) + SinhIntegral[ArcSinh[a + b*
x]]/b
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 5863

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \frac{1}{\sinh^{-1}(a+bx)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sinh^{-1}(x)^2}\,dx, x, a+bx\right)}{b}$$
$$= -\frac{\sqrt{1+(a+bx)^2}}{b\sinh^{-1}(a+bx)} + \frac{\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\sinh^{-1}(x)}\,dx, x, a+bx\right)}{b}$$
$$= -\frac{\sqrt{1+(a+bx)^2}}{b\sinh^{-1}(a+bx)} + \frac{\text{Subst}\left(\int \frac{\sinh(x)}{x}\,dx, x, \sinh^{-1}(a+bx)\right)}{b}$$
$$= -\frac{\sqrt{1+(a+bx)^2}}{b\sinh^{-1}(a+bx)} + \frac{\text{Shi}\left(\sinh^{-1}(a+bx)\right)}{b}$$

Mathematica [A] time = 0.02, size = 35, normalized size = 0.92

$$\frac{\text{Shi}\left(\sinh^{-1}(a+bx)\right) - \frac{\sqrt{(a+bx)^2+1}}{\sinh^{-1}(a+bx)}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a + b*x]^(-2),x]
[Out] (-(Sqrt[1 + (a + b*x)^2]/ArcSinh[a + b*x]) + SinhIntegral[ArcSinh[a + b*x]]
)/b
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\text{arsinh}\,(bx+a)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(b*x+a)^2,x, algorithm="fricas")
[Out] integral(arcsinh(b*x + a)^(-2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\text{arsinh}\,(bx+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(b*x+a)^2,x, algorithm="giac")
[Out] integrate(arcsinh(b*x + a)^(-2), x)
```

maple [A] time = 0.05, size = 34, normalized size = 0.89

$$\frac{-\frac{\sqrt{1+(bx+a)^2}}{\text{arcsinh}(bx+a)} + \text{Shi}\,(\text{arcsinh}\,(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arcsinh(b*x+a)^2,x)
[Out] 1/b*(-1/arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)+Shi(arcsinh(b*x+a)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b^3x^3 + 3\,ab^2x^2 + a^3 + \left(3\,a^2b + b\right)x + \left(b^2x^2 + 2\,abx + a^2 + 1\right)^{\frac{3}{2}} + a}{\left(b^3x^2 + 2\,ab^2x + a^2b + \sqrt{b^2x^2 + 2\,abx + a^2 + 1}\left(b^2x + ab\right) + b\right)\log\left(bx + a + \sqrt{b^2x^2 + 2\,abx + a^2 + 1}\right)} + \int \frac{}{\left(b^4x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b + b)*x + (b^2*x^2 + 2*a*b*x + a^2
+ 1)^(3/2) + a)/((b^3*x^2 + 2*a*b^2*x + a^2*b + sqrt(b^2*x^2 + 2*a*b*x + a^
2 + 1)*(b^2*x + a*b) + b)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)))
 + integrate((b^4*x^4 + 4*a*b^3*x^3 + a^4 + 2*(3*a^2*b^2 + b^2)*x^2 + (b^2*
x^2 + 2*a*b*x + a^2 + 1)*(b^2*x^2 + 2*a*b*x + a^2 - 1) + 2*a^2 + 4*(a^3*b +
 a*b)*x + (2*b^3*x^3 + 6*a*b^2*x^2 + 2*a^3 + (6*a^2*b + b)*x + a)*sqrt(b^2*
x^2 + 2*a*b*x + a^2 + 1) + 1)/((b^4*x^4 + 4*a*b^3*x^3 + a^4 + 2*(3*a^2*b^2
+ b^2)*x^2 + (b^2*x^2 + 2*a*b*x + a^2 + 1)*(b^2*x^2 + 2*a*b*x + a^2) + 2*a^
2 + 4*(a^3*b + a*b)*x + 2*(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b + b)*x +
a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*
b*x + a^2 + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\mathrm{asinh}\left(a + b\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/asinh(a + b*x)^2,x)
```

```
[Out] int(1/asinh(a + b*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{asinh}^{2}\left(a + bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/asinh(b*x+a)**2,x)
```

```
[Out] Integral(asinh(a + b*x)**(-2), x)
```

3.88 $\int \frac{1}{x \sinh^{-1}(a+bx)^2} dx$

Optimal. Leaf size=15

$$\text{Int}\left(\frac{1}{x \sinh^{-1}(a+bx)^2}, x\right)$$

```
[Out] Unintegrable(1/x/arcsinh(b*x+a)^2,x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x \sinh^{-1}(a+bx)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*ArcSinh[a + b*x]^2),x]
[Out] Defer[Subst][Defer[Int][1/((-(a/b) + x/b)*ArcSinh[x]^2), x], x, a + b*x]/b
```

Rubi steps

$$\int \frac{1}{x \sinh^{-1}(a+bx)^2} dx = \frac{\text{Subst}\left(\int \frac{1}{\left(-\frac{a}{b}+\frac{x}{b}\right) \sinh^{-1}(x)^2} dx, x, a+bx\right)}{b}$$

Mathematica [A] time = 2.14, size = 0, normalized size = 0.00

$$\int \frac{1}{x \sinh^{-1}(a+bx)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*ArcSinh[a + b*x]^2),x]
[Out] Integrate[1/(x*ArcSinh[a + b*x]^2), x]
```

fricas [A] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{x \operatorname{arsinh}(bx+a)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(b*x+a)^2,x, algorithm="fricas")
[Out] integral(1/(x*arcsinh(b*x + a)^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{arsinh}(bx+a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(b*x+a)^2,x, algorithm="giac")
[Out] integrate(1/(x*arcsinh(b*x + a)^2), x)
```

maple [A] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{arcsinh}(bx+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arcsinh(b*x+a)^2,x)
```

```
[Out] int(1/x/arcsinh(b*x+a)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b^3x^3 + 3\,ab^2x^2 + a^3 + \left(3\,a^2b + b\right)x + \left(b^2x^2 + 2\,abx + a^2 + 1\right)^{\frac{3}{2}} + a}{\left(b^3x^3 + 2\,ab^2x^2 + \left(a^2b + b\right)x + \sqrt{b^2x^2 + 2\,abx + a^2 + 1}\left(b^2x^2 + abx\right)\right)\log\left(bx + a + \sqrt{b^2x^2 + 2\,abx + a^2 + 1}\right)} - \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b + b)*x + (b^2*x^2 + 2*a*b*x + a^2
+ 1)^(3/2) + a)/((b^3*x^3 + 2*a*b^2*x^2 + (a^2*b + b)*x + sqrt(b^2*x^2 + 2*
a*b*x + a^2 + 1)*(b^2*x^2 + a*b*x))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x +
a^2 + 1))) - integrate((a*b^4*x^4 + 4*a^2*b^3*x^3 + a^5 + 2*a^3 + 2*(3*a^3*
b^2 + a*b^2)*x^2 + (a*b^2*x^2 + a^3 + 2*(a^2*b + b)*x + a)*(b^2*x^2 + 2*a*b
*x + a^2 + 1) + 4*(a^4*b + a^2*b)*x + (2*a*b^3*x^3 + 2*a^4 + 2*(3*a^2*b^2 +
 b^2)*x^2 + 3*a^2 + (6*a^3*b + 5*a*b)*x + 1)*sqrt(b^2*x^2 + 2*a*b*x + a^2 +
 1) + a)/((b^5*x^6 + 4*a*b^4*x^5 + 2*(3*a^2*b^3 + b^3)*x^4 + 4*(a^3*b^2 + a
*b^2)*x^3 + (a^4*b + 2*a^2*b + b)*x^2 + (b^3*x^4 + 2*a*b^2*x^3 + a^2*b*x^2)
*(b^2*x^2 + 2*a*b*x + a^2 + 1) + 2*(b^4*x^5 + 3*a*b^3*x^4 + (3*a^2*b^2 + b^
2)*x^3 + (a^3*b + a*b)*x^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*log(b*x + a
+ sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{1}{x\,\mathrm{asinh}\left(a+b\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*asinh(a + b*x)^2),x)
```

```
[Out] int(1/(x*asinh(a + b*x)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{asinh}^{2}{\left(a + bx \right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/asinh(b*x+a)**2,x)
```

```
[Out] Integral(1/(x*asinh(a + b*x)**2), x)
```

3.89 $\int \frac{x^2}{\sinh^{-1}(a+bx)^3} dx$

Optimal. Leaf size=257

$$\frac{a^2\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{2b^3}-\frac{a^2(a+bx)}{2b^3\sinh^{-1}(a+bx)}-\frac{a^2\sqrt{(a+bx)^2+1}}{2b^3\sinh^{-1}(a+bx)^2}-\frac{\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{8b^3}+\frac{9\text{Chi}\left(3\sinh^{-1}(a+\right.}{8b^3}$$

```
[Out] a/b^3/arcsinh(b*x+a)+(-b*x-a)/b^3/arcsinh(b*x+a)-1/2*a^2*(b*x+a)/b^3/arcsin
h(b*x+a)+2*a*(b*x+a)^2/b^3/arcsinh(b*x+a)-3/2*(b*x+a)^3/b^3/arcsinh(b*x+a)-
1/8*Chi(arcsinh(b*x+a))/b^3+1/2*a^2*Chi(arcsinh(b*x+a))/b^3+9/8*Chi(3*arcsi
nh(b*x+a))/b^3-2*a*Shi(2*arcsinh(b*x+a))/b^3-1/2*a^2*(1+(b*x+a)^2)^(1/2)/b^
3/arcsinh(b*x+a)^2+a*(b*x+a)*(1+(b*x+a)^2)^(1/2)/b^3/arcsinh(b*x+a)^2-1/2*(
b*x+a)^2*(1+(b*x+a)^2)^(1/2)/b^3/arcsinh(b*x+a)^2
```

Rubi [A] time = 0.50, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 12, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5865, 5803, 5655, 5774, 5657, 3301, 5667, 5669, 5448, 12, 3298, 5675}

$$\frac{a^2\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{2b^3}-\frac{a^2(a+bx)}{2b^3\sinh^{-1}(a+bx)}-\frac{a^2\sqrt{(a+bx)^2+1}}{2b^3\sinh^{-1}(a+bx)^2}-\frac{\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{8b^3}+\frac{9\text{Chi}\left(3\sinh^{-1}(a+\right.}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/ArcSinh[a + b*x]^3,x]
```

```
[Out] -(a^2*Sqrt[1 + (a + b*x)^2])/(2*b^3*ArcSinh[a + b*x]^2) + (a*(a + b*x)*Sqrt
[1 + (a + b*x)^2])/(b^3*ArcSinh[a + b*x]^2) - ((a + b*x)^2*Sqrt[1 + (a + b*
x)^2])/(2*b^3*ArcSinh[a + b*x]^2) + a/(b^3*ArcSinh[a + b*x]) - (a + b*x)/(b
^3*ArcSinh[a + b*x]) - (a^2*(a + b*x))/(2*b^3*ArcSinh[a + b*x]) + (2*a*(a +
 b*x)^2)/(b^3*ArcSinh[a + b*x]) - (3*(a + b*x)^3)/(2*b^3*ArcSinh[a + b*x])
- CoshIntegral[ArcSinh[a + b*x]]/(8*b^3) + (a^2*CoshIntegral[ArcSinh[a + b*
x]])/(2*b^3) + (9*CoshIntegral[3*ArcSinh[a + b*x]])/(8*b^3) - (2*a*SinhInte
gral[2*ArcSinh[a + b*x]])/b^3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5803

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*ArcSinh[c*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[m, 0] && LtQ[n, -1]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[(((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{x^2}{\sinh^{-1}(a+bx)^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^2}{\sinh^{-1}(x)^3}\,dx, x, a+bx\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a^2}{b^2\sinh^{-1}(x)^3} - \frac{2ax}{b^2\sinh^{-1}(x)^3} + \frac{x^2}{b^2\sinh^{-1}(x)^3}\right)dx, x, a+bx\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \frac{x^2}{\sinh^{-1}(x)^3}\,dx, x, a+bx\right)}{b^3} - \frac{(2a)\,\text{Subst}\left(\int \frac{x}{\sinh^{-1}(x)^3}\,dx, x, a+bx\right)}{b^3} + \frac{a^2\,\text{Subst}\left(\right.}{}$$

$$= -\frac{a^2\sqrt{1+(a+bx)^2}}{2b^3\sinh^{-1}(a+bx)^2} + \frac{a(a+bx)\sqrt{1+(a+bx)^2}}{b^3\sinh^{-1}(a+bx)^2} - \frac{(a+bx)^2\sqrt{1+(a+bx)^2}}{2b^3\sinh^{-1}(a+bx)^2} + \frac{\text{Subs}}{}$$

$$= -\frac{a^2\sqrt{1+(a+bx)^2}}{2b^3\sinh^{-1}(a+bx)^2} + \frac{a(a+bx)\sqrt{1+(a+bx)^2}}{b^3\sinh^{-1}(a+bx)^2} - \frac{(a+bx)^2\sqrt{1+(a+bx)^2}}{2b^3\sinh^{-1}(a+bx)^2} + \frac{}{b^3\sin}$$

$$= -\frac{a^2\sqrt{1+(a+bx)^2}}{2b^3\sinh^{-1}(a+bx)^2} + \frac{a(a+bx)\sqrt{1+(a+bx)^2}}{b^3\sinh^{-1}(a+bx)^2} - \frac{(a+bx)^2\sqrt{1+(a+bx)^2}}{2b^3\sinh^{-1}(a+bx)^2} + \frac{}{b^3\sin}$$

$$= -\frac{a^2\sqrt{1+(a+bx)^2}}{2b^3\sinh^{-1}(a+bx)^2} + \frac{a(a+bx)\sqrt{1+(a+bx)^2}}{b^3\sinh^{-1}(a+bx)^2} - \frac{(a+bx)^2\sqrt{1+(a+bx)^2}}{2b^3\sinh^{-1}(a+bx)^2} + \frac{}{b^3\sin}$$

$$= -\frac{a^2\sqrt{1+(a+bx)^2}}{2b^3\sinh^{-1}(a+bx)^2} + \frac{a(a+bx)\sqrt{1+(a+bx)^2}}{b^3\sinh^{-1}(a+bx)^2} - \frac{(a+bx)^2\sqrt{1+(a+bx)^2}}{2b^3\sinh^{-1}(a+bx)^2} + \frac{}{b^3\sin}$$

$$= -\frac{a^2\sqrt{1+(a+bx)^2}}{2b^3\sinh^{-1}(a+bx)^2} + \frac{a(a+bx)\sqrt{1+(a+bx)^2}}{b^3\sinh^{-1}(a+bx)^2} - \frac{(a+bx)^2\sqrt{1+(a+bx)^2}}{2b^3\sinh^{-1}(a+bx)^2} + \frac{}{b^3\sin}$$

Mathematica [A] time = 0.40, size = 110, normalized size = 0.43

$$\frac{-\frac{4bx\left(bx\sqrt{a^2+2abx+b^2x^2+1}+\left(2a^2+5abx+3b^2x^2+2\right)\sinh^{-1}(a+bx)\right)}{\sinh^{-1}(a+bx)^2} + \left(4a^2-1\right)\text{Chi}\left(\sinh^{-1}(a+bx)\right) + 9\text{Chi}\left(3\sinh^{-1}(a+bx\right.}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/ArcSinh[a + b*x]^3,x]
```

```
[Out] ((-4*b*x*(b*x*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] + (2 + 2*a^2 + 5*a*b*x + 3*
b^2*x^2)*ArcSinh[a + b*x]))/ArcSinh[a + b*x]^2 + (-1 + 4*a^2)*CoshIntegral[
ArcSinh[a + b*x]] + 9*CoshIntegral[3*ArcSinh[a + b*x]] - 16*a*SinhIntegral[
2*ArcSinh[a + b*x]])/(8*b^3)
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^2}{\text{arsinh}\,(bx+a)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] integral(x^2/arcsinh(b*x + a)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\text{arsinh}\,(bx+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2/arcsinh(b*x + a)^3, x)
```

maple [A] time = 0.16, size = 215, normalized size = 0.84

$$\frac{-\frac{a\left(4\,\mathrm{Shi}(2\operatorname{arcsinh}(bx+a))\operatorname{arcsinh}(bx+a)^2-2\cosh(2\operatorname{arcsinh}(bx+a))\operatorname{arcsinh}(bx+a)-\sinh(2\operatorname{arcsinh}(bx+a))\right)}{2\operatorname{arcsinh}(bx+a)^2}+\frac{\sqrt{1+(bx+a)^2}}{8\operatorname{arcsinh}(bx+a)^2}+\frac{bx+a}{8\operatorname{arcsinh}(bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arcsinh(b*x+a)^3,x)
```

```
[Out] 1/b^3*(-1/2*a*(4*Shi(2*arcsinh(b*x+a))*arcsinh(b*x+a)^2-2*cosh(2*arcsinh(b*
x+a))*arcsinh(b*x+a)-sinh(2*arcsinh(b*x+a)))/arcsinh(b*x+a)^2+1/8/arcsinh(b
*x+a)^2*(1+(b*x+a)^2)^(1/2)+1/8/arcsinh(b*x+a)*(b*x+a)-1/8*Chi(arcsinh(b*x+
a))-1/8/arcsinh(b*x+a)^2*cosh(3*arcsinh(b*x+a))-3/8/arcsinh(b*x+a)*sinh(3*a
rcsinh(b*x+a))+9/8*Chi(3*arcsinh(b*x+a))+1/2*a^2*(Chi(arcsinh(b*x+a))*arcsi
nh(b*x+a)^2-(b*x+a)*arcsinh(b*x+a)-(1+(b*x+a)^2)^(1/2))/arcsinh(b*x+a)^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(b^8*x^9 + 7*a*b^7*x^8 + 3*(7*a^2*b^6 + b^6)*x^7 + 5*(7*a^3*b^5 + 3*a*
b^5)*x^6 + (35*a^4*b^4 + 30*a^2*b^4 + 3*b^4)*x^5 + 3*(7*a^5*b^3 + 10*a^3*b^
3 + 3*a*b^3)*x^4 + (7*a^6*b^2 + 15*a^4*b^2 + 9*a^2*b^2 + b^2)*x^3 + (a^7*b
+ 3*a^5*b + 3*a^3*b + a*b)*x^2 + (b^5*x^6 + 4*a*b^4*x^5 + (6*a^2*b^3 + b^3)
*x^4 + 2*(2*a^3*b^2 + a*b^2)*x^3 + (a^4*b + a^2*b)*x^2)*(b^2*x^2 + 2*a*b*x
+ a^2 + 1)^(3/2) + (3*b^6*x^7 + 15*a*b^5*x^6 + 5*(6*a^2*b^4 + b^4)*x^5 + 15
*(2*a^3*b^3 + a*b^3)*x^4 + (15*a^4*b^2 + 15*a^2*b^2 + 2*b^2)*x^3 + (3*a^5*b
 + 5*a^3*b + 2*a*b)*x^2)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + (3*b^8*x^9 + 23*a*
b^7*x^8 + (77*a^2*b^6 + 9*b^6)*x^7 + 3*(49*a^3*b^5 + 17*a*b^5)*x^6 + (175*a
^4*b^4 + 120*a^2*b^4 + 9*b^4)*x^5 + (133*a^5*b^3 + 150*a^3*b^3 + 33*a*b^3)*
x^4 + 3*(21*a^6*b^2 + 35*a^4*b^2 + 15*a^2*b^2 + b^2)*x^3 + (17*a^7*b + 39*a
^5*b + 27*a^3*b + 5*a*b)*x^2 + (3*b^5*x^6 + 14*a*b^4*x^5 + 2*(13*a^2*b^3 +
2*b^3)*x^4 + 12*(2*a^3*b^2 + a*b^2)*x^3 + (11*a^4*b + 12*a^2*b + b)*x^2 + 2
*(a^5 + 2*a^3 + a)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + (9*b^6*x^7 + 51
*a*b^5*x^6 + (120*a^2*b^4 + 17*b^4)*x^5 + 5*(30*a^3*b^3 + 13*a*b^3)*x^4 + (
105*a^4*b^2 + 93*a^2*b^2 + 10*b^2)*x^3 + (39*a^5*b + 59*a^3*b + 20*a*b)*x^2
 + 2*(3*a^6 + 7*a^4 + 5*a^2 + 1)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + 2*(a^8
+ 3*a^6 + 3*a^4 + a^2)*x + (9*b^7*x^8 + 60*a*b^6*x^7 + (171*a^2*b^5 + 22*b^
5)*x^6 + 2*(135*a^3*b^4 + 52*a*b^4)*x^5 + (255*a^4*b^3 + 196*a^2*b^3 + 18*b
^3)*x^4 + 2*(72*a^5*b^2 + 92*a^3*b^2 + 25*a*b^2)*x^3 + (45*a^6*b + 86*a^4*b
 + 46*a^2*b + 5*b)*x^2 + 2*(3*a^7 + 8*a^5 + 7*a^3 + 2*a)*x)*sqrt(b^2*x^2 +
2*a*b*x + a^2 + 1))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + (3*b
^7*x^8 + 18*a*b^6*x^7 + (45*a^2*b^5 + 7*b^5)*x^6 + 4*(15*a^3*b^4 + 7*a*b^4)
*x^5 + (45*a^4*b^3 + 42*a^2*b^3 + 5*b^3)*x^4 + 2*(9*a^5*b^2 + 14*a^3*b^2 +
5*a*b^2)*x^3 + (3*a^6*b + 7*a^4*b + 5*a^2*b + b)*x^2)*sqrt(b^2*x^2 + 2*a*b*
x + a^2 + 1))/((b^8*x^6 + 6*a*b^7*x^5 + a^6*b^2 + 3*a^4*b^2 + 3*(5*a^2*b^6
+ b^6)*x^4 + 3*a^2*b^2 + 4*(5*a^3*b^5 + 3*a*b^5)*x^3 + 3*(5*a^4*b^4 + 6*a^2
*b^4 + b^4)*x^2 + (b^5*x^3 + 3*a*b^4*x^2 + 3*a^2*b^3*x + a^3*b^2)*(b^2*x^2
+ 2*a*b*x + a^2 + 1)^(3/2) + 3*(b^6*x^4 + 4*a*b^5*x^3 + a^4*b^2 + a^2*b^2 +
 (6*a^2*b^4 + b^4)*x^2 + 2*(2*a^3*b^3 + a*b^3)*x)*(b^2*x^2 + 2*a*b*x + a^2
```

```
+ 1) + b^2 + 6*(a^5*b^3 + 2*a^3*b^3 + a*b^3)*x + 3*(b^7*x^5 + 5*a*b^6*x^4 +
 a^5*b^2 + 2*a^3*b^2 + 2*(5*a^2*b^5 + b^5)*x^3 + a*b^2 + 2*(5*a^3*b^4 + 3*a
*b^4)*x^2 + (5*a^4*b^3 + 6*a^2*b^3 + b^3)*x)*sqrt(b^2*x^2 + 2*a*b*x + a^2 +
 1))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2) + integrate(1/2*(9
*b^10*x^10 + 82*a*b^9*x^9 + 2*a^10 + 2*(167*a^2*b^8 + 18*b^8)*x^8 + 8*a^8 +
 32*(25*a^3*b^7 + 8*a*b^7)*x^7 + 2*(623*a^4*b^6 + 394*a^2*b^6 + 27*b^6)*x^6
 + 12*a^6 + 4*(329*a^5*b^5 + 342*a^3*b^5 + 69*a*b^5)*x^5 + 4*(238*a^6*b^4 +
 365*a^4*b^4 + 144*a^2*b^4 + 9*b^4)*x^4 + 8*a^4 + 16*(29*a^7*b^3 + 61*a^5*b
^3 + 39*a^3*b^3 + 7*a*b^3)*x^3 + (9*b^6*x^6 + 46*a*b^5*x^5 + 2*a^6 + 4*(24*
a^2*b^4 + b^4)*x^4 + 4*a^4 + 8*(13*a^3*b^3 + 2*a*b^3)*x^3 + (61*a^4*b^2 + 2
4*a^2*b^2 - b^2)*x^2 + 2*a^2 + 2*(9*a^5*b + 8*a^3*b - a*b)*x)*(b^2*x^2 + 2*
a*b*x + a^2 + 1)^2 + (145*a^8*b^2 + 396*a^6*b^2 + 366*a^4*b^2 + 124*a^2*b^2
 + 9*b^2)*x^2 + (36*b^7*x^7 + 220*a*b^6*x^6 + 8*a^7 + 8*(71*a^2*b^5 + 6*b^5
)*x^5 + 20*a^5 + 16*(50*a^3*b^4 + 13*a*b^4)*x^4 + (660*a^4*b^3 + 356*a^2*b^
3 + 13*b^3)*x^3 + 16*a^3 + (316*a^5*b^2 + 300*a^3*b^2 + 39*a*b^2)*x^2 + 2*(
40*a^6*b + 62*a^4*b + 21*a^2*b - b)*x + 4*a)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^
(3/2) + (54*b^8*x^8 + 384*a*b^7*x^7 + 12*a^8 + 6*(197*a^2*b^6 + 20*b^6)*x^6
 + 36*a^6 + 12*(171*a^3*b^5 + 52*a*b^5)*x^5 + (2190*a^4*b^4 + 1332*a^2*b^4
+ 83*b^4)*x^4 + 38*a^4 + 4*(366*a^5*b^3 + 372*a^3*b^3 + 71*a*b^3)*x^3 + (59
4*a^6*b^2 + 912*a^4*b^2 + 357*a^2*b^2 + 19*b^2)*x^2 + 16*a^2 + 2*(66*a^7*b
+ 144*a^5*b + 97*a^3*b + 19*a*b)*x + 2)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + 2*a
^2 + 2*(13*a^9*b + 44*a^7*b + 54*a^5*b + 28*a^3*b + 5*a*b)*x + (36*b^9*x^9
+ 292*a*b^8*x^8 + 8*a^9 + 4*(261*a^2*b^7 + 28*b^7)*x^7 + 28*a^7 + 4*(539*a^
3*b^6 + 172*a*b^6)*x^6 + (2828*a^4*b^5 + 1788*a^2*b^5 + 123*b^5)*x^5 + 36*a
^5 + (2436*a^5*b^4 + 2540*a^3*b^4 + 519*a*b^4)*x^4 + (1372*a^6*b^3 + 2120*a
^4*b^3 + 855*a^2*b^3 + 57*b^3)*x^3 + 20*a^3 + (484*a^7*b^2 + 1032*a^5*b^2 +
 681*a^3*b^2 + 133*a*b^2)*x^2 + 2*(48*a^8*b + 134*a^6*b + 129*a^4*b + 48*a^
2*b + 5*b)*x + 4*a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/((b^10*x^8 + 8*a*b^9
*x^7 + a^8*b^2 + 4*a^6*b^2 + 4*(7*a^2*b^8 + b^8)*x^6 + 6*a^4*b^2 + 8*(7*a^3
*b^7 + 3*a*b^7)*x^5 + 2*(35*a^4*b^6 + 30*a^2*b^6 + 3*b^6)*x^4 + 4*a^2*b^2 +
 8*(7*a^5*b^5 + 10*a^3*b^5 + 3*a*b^5)*x^3 + (b^6*x^4 + 4*a*b^5*x^3 + 6*a^2*
b^4*x^2 + 4*a^3*b^3*x + a^4*b^2)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^2 + 4*(7*a^6
*b^4 + 15*a^4*b^4 + 9*a^2*b^4 + b^4)*x^2 + 4*(b^7*x^5 + 5*a*b^6*x^4 + a^5*b
^2 + a^3*b^2 + (10*a^2*b^5 + b^5)*x^3 + (10*a^3*b^4 + 3*a*b^4)*x^2 + (5*a^4
*b^3 + 3*a^2*b^3)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + 6*(b^8*x^6 + 6*a
*b^7*x^5 + a^6*b^2 + 2*a^4*b^2 + (15*a^2*b^6 + 2*b^6)*x^4 + a^2*b^2 + 4*(5*
a^3*b^5 + 2*a*b^5)*x^3 + (15*a^4*b^4 + 12*a^2*b^4 + b^4)*x^2 + 2*(3*a^5*b^3
 + 4*a^3*b^3 + a*b^3)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + b^2 + 8*(a^7*b^3 +
 3*a^5*b^3 + 3*a^3*b^3 + a*b^3)*x + 4*(b^9*x^7 + 7*a*b^8*x^6 + a^7*b^2 + 3*
a^5*b^2 + 3*(7*a^2*b^7 + b^7)*x^5 + 3*a^3*b^2 + 5*(7*a^3*b^6 + 3*a*b^6)*x^4
 + (35*a^4*b^5 + 30*a^2*b^5 + 3*b^5)*x^3 + a*b^2 + 3*(7*a^5*b^4 + 10*a^3*b^
4 + 3*a*b^4)*x^2 + (7*a^6*b^3 + 15*a^4*b^3 + 9*a^2*b^3 + b^3)*x)*sqrt(b^2*x
^2 + 2*a*b*x + a^2 + 1))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\mathrm{asinh}\left(a+b\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/asinh(a + b*x)^3,x)
```

```
[Out] int(x^2/asinh(a + b*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\operatorname{asinh}^3(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/asinh(b*x+a)**3,x)
```

```
[Out] Integral(x**2/asinh(a + b*x)**3, x)
```

3.90 $\int \frac{x}{\sinh^{-1}(a+bx)^3} dx$

Optimal. Leaf size=147

$$-\frac{a\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{2b^2}+\frac{\text{Shi}\left(2\sinh^{-1}(a+bx)\right)}{b^2}-\frac{(a+bx)^2}{b^2\sinh^{-1}(a+bx)}+\frac{a(a+bx)}{2b^2\sinh^{-1}(a+bx)}-\frac{\sqrt{(a+bx)^2+1}\,(a+$$

```
[Out] -1/2/b^2/arcsinh(b*x+a)+1/2*a*(b*x+a)/b^2/arcsinh(b*x+a)-(b*x+a)^2/b^2/arcs
inh(b*x+a)-1/2*a*Chi(arcsinh(b*x+a))/b^2+Shi(2*arcsinh(b*x+a))/b^2+1/2*a*(1
+(b*x+a)^2)^(1/2)/b^2/arcsinh(b*x+a)^2-1/2*(b*x+a)*(1+(b*x+a)^2)^(1/2)/b^2/
arcsinh(b*x+a)^2
```

Rubi [A] time = 0.25, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.200, Rules used = {5865, 5803, 5655, 5774, 5657, 3301, 5667, 5669, 5448, 12, 3298, 5675}

$$-\frac{a\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{2b^2}+\frac{\text{Shi}\left(2\sinh^{-1}(a+bx)\right)}{b^2}-\frac{(a+bx)^2}{b^2\sinh^{-1}(a+bx)}+\frac{a(a+bx)}{2b^2\sinh^{-1}(a+bx)}-\frac{\sqrt{(a+bx)^2+1}\,(a+$$

Antiderivative was successfully verified.

```
[In] Int[x/ArcSinh[a + b*x]^3,x]
```

```
[Out] (a*Sqrt[1 + (a + b*x)^2])/(2*b^2*ArcSinh[a + b*x]^2) - ((a + b*x)*Sqrt[1 +
(a + b*x)^2])/(2*b^2*ArcSinh[a + b*x]^2) - 1/(2*b^2*ArcSinh[a + b*x]) + (a*
(a + b*x))/(2*b^2*ArcSinh[a + b*x]) - (a + b*x)^2/(b^2*ArcSinh[a + b*x]) -
(a*CoshIntegral[ArcSinh[a + b*x]])/(2*b^2) + SinhIntegral[2*ArcSinh[a + b*x
]]/b^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
```

```
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5803

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*ArcSinh[c*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[m, 0] && LtQ[n, -1]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{x}{\sinh^{-1}(a+bx)^3}\,dx = \frac{\text{Subst}\left(\int \frac{-\frac{a}{b}+\frac{x}{b}}{\sinh^{-1}(x)^3}\,dx, x, a+bx\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{a}{b\sinh^{-1}(x)^3}+\frac{x}{b\sinh^{-1}(x)^3}\right)dx, x, a+bx\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \frac{x}{\sinh^{-1}(x)^3}\,dx, x, a+bx\right)}{b^2} - \frac{a\,\text{Subst}\left(\int \frac{1}{\sinh^{-1}(x)^3}\,dx, x, a+bx\right)}{b^2}$$

$$= \frac{a\sqrt{1+(a+bx)^2}}{2b^2\sinh^{-1}(a+bx)^2} - \frac{(a+bx)\sqrt{1+(a+bx)^2}}{2b^2\sinh^{-1}(a+bx)^2} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}\sinh^{-1}(x)^2}\,dx, x, a+b\right.}{2b^2}$$

$$= \frac{a\sqrt{1+(a+bx)^2}}{2b^2\sinh^{-1}(a+bx)^2} - \frac{(a+bx)\sqrt{1+(a+bx)^2}}{2b^2\sinh^{-1}(a+bx)^2} - \frac{1}{2b^2\sinh^{-1}(a+bx)} + \frac{a(a+bx}{2b^2\sinh^{-1}(a}$$

$$= \frac{a\sqrt{1+(a+bx)^2}}{2b^2\sinh^{-1}(a+bx)^2} - \frac{(a+bx)\sqrt{1+(a+bx)^2}}{2b^2\sinh^{-1}(a+bx)^2} - \frac{1}{2b^2\sinh^{-1}(a+bx)} + \frac{a(a+bx}{2b^2\sinh^{-1}(a}$$

$$= \frac{a\sqrt{1+(a+bx)^2}}{2b^2\sinh^{-1}(a+bx)^2} - \frac{(a+bx)\sqrt{1+(a+bx)^2}}{2b^2\sinh^{-1}(a+bx)^2} - \frac{1}{2b^2\sinh^{-1}(a+bx)} + \frac{a(a+bx}{2b^2\sinh^{-1}(a}$$

$$= \frac{a\sqrt{1+(a+bx)^2}}{2b^2\sinh^{-1}(a+bx)^2} - \frac{(a+bx)\sqrt{1+(a+bx)^2}}{2b^2\sinh^{-1}(a+bx)^2} - \frac{1}{2b^2\sinh^{-1}(a+bx)} + \frac{a(a+bx}{2b^2\sinh^{-1}(a}$$

$$= \frac{a\sqrt{1+(a+bx)^2}}{2b^2\sinh^{-1}(a+bx)^2} - \frac{(a+bx)\sqrt{1+(a+bx)^2}}{2b^2\sinh^{-1}(a+bx)^2} - \frac{1}{2b^2\sinh^{-1}(a+bx)} + \frac{a(a+bx}{2b^2\sinh^{-1}(a}$$

Mathematica [A] time = 0.11, size = 117, normalized size = 0.80

$$-\frac{bx\sqrt{a^2+2abx+b^2x^2+1}+a^2\sinh^{-1}(a+bx)+2b^2x^2\sinh^{-1}(a+bx)+a\sinh^{-1}(a+bx)^2\text{Chi}\left(\sinh^{-1}(a+b\right.}{2b^2\sinh^{-1}(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/ArcSinh[a + b*x]^3,x]
```

```
[Out] -1/2*(b*x*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] + ArcSinh[a + b*x] + a^2*ArcSin
h[a + b*x] + 3*a*b*x*ArcSinh[a + b*x] + 2*b^2*x^2*ArcSinh[a + b*x] + a*ArcS
inh[a + b*x]^2*CoshIntegral[ArcSinh[a + b*x]] - 2*ArcSinh[a + b*x]^2*SinhIn
tegral[2*ArcSinh[a + b*x]])/(b^2*ArcSinh[a + b*x]^2)
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x}{\text{arsinh}\,(bx+a)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] integral(x/arcsinh(b*x + a)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\text{arsinh}\,(bx+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x/arcsinh(b*x + a)^3, x)
```

maple [A] time = 0.08, size = 107, normalized size = 0.73

$$\frac{-\frac{\sinh(2\operatorname{arcsinh}(bx+a))}{4\operatorname{arcsinh}(bx+a)^2} - \frac{\cosh(2\operatorname{arcsinh}(bx+a))}{2\operatorname{arcsinh}(bx+a)} + \operatorname{Shi}(2\operatorname{arcsinh}(bx+a)) - \frac{a\left(X(\operatorname{arcsinh}(bx+a))\operatorname{arcsinh}(bx+a)^2-(bx+a)\operatorname{arcsinh}(bx+a)\right.}{2\operatorname{arcsinh}(bx+a)^2}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arcsinh(b*x+a)^3,x)
```

```
[Out] 1/b^2*(-1/4*sinh(2*arcsinh(b*x+a))/arcsinh(b*x+a)^2-1/2/arcsinh(b*x+a)*cosh
(2*arcsinh(b*x+a))+Shi(2*arcsinh(b*x+a))-1/2*a*(Chi(arcsinh(b*x+a))*arcsinh
(b*x+a)^2-(b*x+a)*arcsinh(b*x+a)-(1+(b*x+a)^2)^(1/2))/arcsinh(b*x+a)^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(b^8*x^8 + 7*a*b^7*x^7 + 3*(7*a^2*b^6 + b^6)*x^6 + 5*(7*a^3*b^5 + 3*a*
b^5)*x^5 + (35*a^4*b^4 + 30*a^2*b^4 + 3*b^4)*x^4 + 3*(7*a^5*b^3 + 10*a^3*b^
3 + 3*a*b^3)*x^3 + (7*a^6*b^2 + 15*a^4*b^2 + 9*a^2*b^2 + b^2)*x^2 + (b^5*x^
5 + 4*a*b^4*x^4 + (6*a^2*b^3 + b^3)*x^3 + 2*(2*a^3*b^2 + a*b^2)*x^2 + (a^4*
b + a^2*b)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + (3*b^6*x^6 + 15*a*b^5*x
^5 + 5*(6*a^2*b^4 + b^4)*x^4 + 15*(2*a^3*b^3 + a*b^3)*x^3 + (15*a^4*b^2 + 1
5*a^2*b^2 + 2*b^2)*x^2 + (3*a^5*b + 5*a^3*b + 2*a*b)*x)*(b^2*x^2 + 2*a*b*x
+ a^2 + 1) + (a^7*b + 3*a^5*b + 3*a^3*b + a*b)*x + (2*b^8*x^8 + 15*a*b^7*x^
7 + a^8 + (49*a^2*b^6 + 6*b^6)*x^6 + 3*a^6 + (91*a^3*b^5 + 33*a*b^5)*x^5 +
3*(35*a^4*b^4 + 25*a^2*b^4 + 2*b^4)*x^4 + 3*a^4 + (77*a^5*b^3 + 90*a^3*b^3
+ 21*a*b^3)*x^3 + (35*a^6*b^2 + 60*a^4*b^2 + 27*a^2*b^2 + 2*b^2)*x^2 + (2*b
^5*x^5 + 9*a*b^4*x^4 + a^5 + 2*(8*a^2*b^3 + b^3)*x^3 + 2*a^3 + 2*(7*a^3*b^2
 + 3*a*b^2)*x^2 + 6*(a^4*b + a^2*b)*x + a)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3
/2) + (6*b^6*x^6 + 33*a*b^5*x^5 + 3*a^6 + 5*(15*a^2*b^4 + 2*b^4)*x^4 + 7*a^
4 + (90*a^3*b^3 + 37*a*b^3)*x^3 + (60*a^4*b^2 + 51*a^2*b^2 + 5*b^2)*x^2 + 5
*a^2 + (21*a^5*b + 31*a^3*b + 10*a*b)*x + 1)*(b^2*x^2 + 2*a*b*x + a^2 + 1)
+ a^2 + 3*(3*a^7*b + 7*a^5*b + 5*a^3*b + a*b)*x + (6*b^7*x^7 + 39*a*b^6*x^6
 + 3*a^7 + 2*(54*a^2*b^5 + 7*b^5)*x^5 + 8*a^5 + (165*a^3*b^4 + 64*a*b^4)*x^
4 + (150*a^4*b^3 + 116*a^2*b^3 + 11*b^3)*x^3 + 7*a^3 + (81*a^5*b^2 + 104*a^
3*b^2 + 29*a*b^2)*x^2 + (24*a^6*b + 46*a^4*b + 25*a^2*b + 3*b)*x + 2*a)*sqr
t(b^2*x^2 + 2*a*b*x + a^2 + 1))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2
+ 1)) + (3*b^7*x^7 + 18*a*b^6*x^6 + (45*a^2*b^5 + 7*b^5)*x^5 + 4*(15*a^3*b^
4 + 7*a*b^4)*x^4 + (45*a^4*b^3 + 42*a^2*b^3 + 5*b^3)*x^3 + 2*(9*a^5*b^2 + 1
4*a^3*b^2 + 5*a*b^2)*x^2 + (3*a^6*b + 7*a^4*b + 5*a^2*b + b)*x)*sqrt(b^2*x^
2 + 2*a*b*x + a^2 + 1))/((b^8*x^6 + 6*a*b^7*x^5 + a^6*b^2 + 3*a^4*b^2 + 3*(
5*a^2*b^6 + b^6)*x^4 + 3*a^2*b^2 + 4*(5*a^3*b^5 + 3*a*b^5)*x^3 + 3*(5*a^4*b
^4 + 6*a^2*b^4 + b^4)*x^2 + (b^5*x^3 + 3*a*b^4*x^2 + 3*a^2*b^3*x + a^3*b^2)
*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + 3*(b^6*x^4 + 4*a*b^5*x^3 + a^4*b^2 +
 a^2*b^2 + (6*a^2*b^4 + b^4)*x^2 + 2*(2*a^3*b^3 + a*b^3)*x)*(b^2*x^2 + 2*a*
b*x + a^2 + 1) + b^2 + 6*(a^5*b^3 + 2*a^3*b^3 + a*b^3)*x + 3*(b^7*x^5 + 5*a
*b^6*x^4 + a^5*b^2 + 2*a^3*b^2 + 2*(5*a^2*b^5 + b^5)*x^3 + a*b^2 + 2*(5*a^3
*b^4 + 3*a*b^4)*x^2 + (5*a^4*b^3 + 6*a^2*b^3 + b^3)*x)*sqrt(b^2*x^2 + 2*a*b
*x + a^2 + 1))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2) + integr
```

```
ate(1/2*(4*b^9*x^9 + 35*a*b^8*x^8 + 3*a^9 + 8*(17*a^2*b^7 + 2*b^7)*x^7 + 12
*a^7 + 4*(77*a^3*b^6 + 27*a*b^6)*x^6 + 8*(56*a^4*b^5 + 39*a^2*b^5 + 3*b^5)*
x^5 + 18*a^5 + 2*(217*a^5*b^4 + 250*a^3*b^4 + 57*a*b^4)*x^4 + 8*(35*a^6*b^3
 + 60*a^4*b^3 + 27*a^2*b^3 + 2*b^3)*x^3 + (4*b^5*x^5 + 19*a*b^4*x^4 + 36*a^
2*b^3*x^3 + 34*a^3*b^2*x^2 + 16*a^4*b*x + 3*a^5 - 3*a)*(b^2*x^2 + 2*a*b*x +
 a^2 + 1)^2 + 12*a^3 + 4*(29*a^7*b^2 + 69*a^5*b^2 + 51*a^3*b^2 + 11*a*b^2)*
x^2 + (16*b^6*x^6 + 92*a*b^5*x^5 + 12*a^6 + 4*(55*a^2*b^4 + 4*b^4)*x^4 + 12
*a^4 + 20*(14*a^3*b^3 + 3*a*b^3)*x^3 + 4*(50*a^4*b^2 + 21*a^2*b^2)*x^2 - 3*
a^2 + (76*a^5*b + 52*a^3*b - 3*a*b)*x - 3)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3
/2) + 3*(8*b^7*x^7 + 54*a*b^6*x^6 + 6*a^7 + 4*(39*a^2*b^5 + 4*b^5)*x^5 + 12
*a^5 + 2*(125*a^3*b^4 + 38*a*b^4)*x^4 + 8*(30*a^4*b^3 + 18*a^2*b^3 + b^3)*x
^3 + 7*a^3 + (138*a^5*b^2 + 136*a^3*b^2 + 23*a*b^2)*x^2 + 2*(22*a^6*b + 32*
a^4*b + 11*a^2*b)*x + a)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + 4*(7*a^8*b + 22*a^
6*b + 24*a^4*b + 10*a^2*b + b)*x + (16*b^8*x^8 + 124*a*b^7*x^7 + 12*a^8 + 1
2*(35*a^2*b^6 + 4*b^6)*x^6 + 36*a^6 + 4*(203*a^3*b^5 + 69*a*b^5)*x^5 + 4*(2
45*a^4*b^4 + 165*a^2*b^4 + 12*b^4)*x^4 + 39*a^4 + 3*(252*a^5*b^3 + 280*a^3*
b^3 + 61*a*b^3)*x^3 + (364*a^6*b^2 + 600*a^4*b^2 + 261*a^2*b^2 + 19*b^2)*x^
2 + 18*a^2 + (100*a^7*b + 228*a^5*b + 165*a^3*b + 37*a*b)*x + 3)*sqrt(b^2*x
^2 + 2*a*b*x + a^2 + 1) + 3*a)/((b^9*x^8 + 8*a*b^8*x^7 + a^8*b + 4*a^6*b +
4*(7*a^2*b^7 + b^7)*x^6 + 8*(7*a^3*b^6 + 3*a*b^6)*x^5 + 6*a^4*b + 2*(35*a^4
*b^5 + 30*a^2*b^5 + 3*b^5)*x^4 + 8*(7*a^5*b^4 + 10*a^3*b^4 + 3*a*b^4)*x^3 +
 (b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b)*(b^2*x^2 + 2
*a*b*x + a^2 + 1)^2 + 4*a^2*b + 4*(7*a^6*b^3 + 15*a^4*b^3 + 9*a^2*b^3 + b^3
)*x^2 + 4*(b^6*x^5 + 5*a*b^5*x^4 + a^5*b + a^3*b + (10*a^2*b^4 + b^4)*x^3 +
 (10*a^3*b^3 + 3*a*b^3)*x^2 + (5*a^4*b^2 + 3*a^2*b^2)*x)*(b^2*x^2 + 2*a*b*x
 + a^2 + 1)^(3/2) + 6*(b^7*x^6 + 6*a*b^6*x^5 + a^6*b + 2*a^4*b + (15*a^2*b^
5 + 2*b^5)*x^4 + 4*(5*a^3*b^4 + 2*a*b^4)*x^3 + a^2*b + (15*a^4*b^3 + 12*a^2
*b^3 + b^3)*x^2 + 2*(3*a^5*b^2 + 4*a^3*b^2 + a*b^2)*x)*(b^2*x^2 + 2*a*b*x +
 a^2 + 1) + 8*(a^7*b^2 + 3*a^5*b^2 + 3*a^3*b^2 + a*b^2)*x + 4*(b^8*x^7 + 7*
a*b^7*x^6 + a^7*b + 3*a^5*b + 3*(7*a^2*b^6 + b^6)*x^5 + 5*(7*a^3*b^5 + 3*a*
b^5)*x^4 + 3*a^3*b + (35*a^4*b^4 + 30*a^2*b^4 + 3*b^4)*x^3 + 3*(7*a^5*b^3 +
 10*a^3*b^3 + 3*a*b^3)*x^2 + a*b + (7*a^6*b^2 + 15*a^4*b^2 + 9*a^2*b^2 + b^
2)*x)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + b)*log(b*x + a + sqrt(b^2*x^2 + 2
*a*b*x + a^2 + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\operatorname{asinh}(a+b\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/asinh(a + b*x)^3,x)
[Out] int(x/asinh(a + b*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\operatorname{asinh}^3(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/asinh(b*x+a)**3,x)
[Out] Integral(x/asinh(a + b*x)**3, x)
```

3.91 $\int \frac{1}{\sinh^{-1}(a+bx)^3} dx$

Optimal. Leaf size=63

$$\frac{\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{2b}-\frac{a+bx}{2b\sinh^{-1}(a+bx)}-\frac{\sqrt{(a+bx)^2+1}}{2b\sinh^{-1}(a+bx)^2}$$

```
[Out] 1/2*(-b*x-a)/b/arcsinh(b*x+a)+1/2*Chi(arcsinh(b*x+a))/b-1/2*(1+(b*x+a)^2)^(
1/2)/b/arcsinh(b*x+a)^2
```

Rubi [A] time = 0.08, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5863, 5655, 5774, 5657, 3301}

$$\frac{\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{2b}-\frac{a+bx}{2b\sinh^{-1}(a+bx)}-\frac{\sqrt{(a+bx)^2+1}}{2b\sinh^{-1}(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a + b*x]^(-3),x]
```

```
[Out] -Sqrt[1 + (a + b*x)^2]/(2*b*ArcSinh[a + b*x]^2) - (a + b*x)/(2*b*ArcSinh[a
+ b*x]) + CoshIntegral[ArcSinh[a + b*x]]/(2*b)
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5863

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \frac{1}{\sinh^{-1}(a+bx)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sinh^{-1}(x)^3}\,dx, x, a+bx\right)}{b}$$
$$= -\frac{\sqrt{1+(a+bx)^2}}{2b\sinh^{-1}(a+bx)^2} + \frac{\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\sinh^{-1}(x)^2}\,dx, x, a+bx\right)}{2b}$$
$$= -\frac{\sqrt{1+(a+bx)^2}}{2b\sinh^{-1}(a+bx)^2} - \frac{a+bx}{2b\sinh^{-1}(a+bx)} + \frac{\text{Subst}\left(\int \frac{1}{\sinh^{-1}(x)}\,dx, x, a+bx\right)}{2b}$$
$$= -\frac{\sqrt{1+(a+bx)^2}}{2b\sinh^{-1}(a+bx)^2} - \frac{a+bx}{2b\sinh^{-1}(a+bx)} + \frac{\text{Subst}\left(\int \frac{\cosh(x)}{x}\,dx, x, \sinh^{-1}(a+bx)\right)}{2b}$$
$$= -\frac{\sqrt{1+(a+bx)^2}}{2b\sinh^{-1}(a+bx)^2} - \frac{a+bx}{2b\sinh^{-1}(a+bx)} + \frac{\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{2b}$$

Mathematica [A] time = 0.04, size = 53, normalized size = 0.84

$$\frac{\text{Chi}\left(\sinh^{-1}(a+bx)\right) - \frac{a+bx}{\sinh^{-1}(a+bx)} - \frac{\sqrt{(a+bx)^2+1}}{\sinh^{-1}(a+bx)^2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a + b*x]^(-3),x]
[Out] (-(Sqrt[1 + (a + b*x)^2]/ArcSinh[a + b*x]^2) - (a + b*x)/ArcSinh[a + b*x] +
 CoshIntegral[ArcSinh[a + b*x]])/(2*b)
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\text{arsinh}\,(bx+a)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(b*x+a)^3,x, algorithm="fricas")
[Out] integral(arcsinh(b*x + a)^(-3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\text{arsinh}\,(bx+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(b*x+a)^3,x, algorithm="giac")
[Out] integrate(arcsinh(b*x + a)^(-3), x)
```

maple [A] time = 0.03, size = 51, normalized size = 0.81

$$\frac{-\frac{\sqrt{1+(bx+a)^2}}{2\,\text{arcsinh}(bx+a)^2} - \frac{bx+a}{2\,\text{arcsinh}(bx+a)} + \frac{\text{X}(\text{arcsinh}(bx+a))}{2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arcsinh(b*x+a)^3,x)
```

```
[Out] 1/b*(-1/2/arcsinh(b*x+a)^2*(1+(b*x+a)^2)^(1/2)-1/2/arcsinh(b*x+a)*(b*x+a)+1
/2*Chi(arcsinh(b*x+a)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(b^7*x^7 + 7*a*b^6*x^6 + a^7 + 3*(7*a^2*b^5 + b^5)*x^5 + 3*a^5 + 5*(7*
a^3*b^4 + 3*a*b^4)*x^4 + (35*a^4*b^3 + 30*a^2*b^3 + 3*b^3)*x^3 + 3*a^3 + 3*
(7*a^5*b^2 + 10*a^3*b^2 + 3*a*b^2)*x^2 + (b^4*x^4 + 4*a*b^3*x^3 + a^4 + (6*
a^2*b^2 + b^2)*x^2 + a^2 + 2*(2*a^3*b + a*b)*x)*(b^2*x^2 + 2*a*b*x + a^2 +
1)^(3/2) + (3*b^5*x^5 + 15*a*b^4*x^4 + 3*a^5 + 5*(6*a^2*b^3 + b^3)*x^3 + 5*
a^3 + 15*(2*a^3*b^2 + a*b^2)*x^2 + (15*a^4*b + 15*a^2*b + 2*b)*x + 2*a)*(b^
2*x^2 + 2*a*b*x + a^2 + 1) + (7*a^6*b + 15*a^4*b + 9*a^2*b + b)*x + (b^7*x^
7 + 7*a*b^6*x^6 + a^7 + 3*(7*a^2*b^5 + b^5)*x^5 + 3*a^5 + 5*(7*a^3*b^4 + 3*
a*b^4)*x^4 + (35*a^4*b^3 + 30*a^2*b^3 + 3*b^3)*x^3 + 3*a^3 + 3*(7*a^5*b^2 +
 10*a^3*b^2 + 3*a*b^2)*x^2 + (b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3
*b*x + a^4 - 1)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + 3*(b^5*x^5 + 5*a*b^4*
x^4 + a^5 + (10*a^2*b^3 + b^3)*x^3 + a^3 + (10*a^3*b^2 + 3*a*b^2)*x^2 + (5*
a^4*b + 3*a^2*b)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + (7*a^6*b + 15*a^4*b + 9
*a^2*b + b)*x + (3*b^6*x^6 + 18*a*b^5*x^5 + 3*a^6 + 3*(15*a^2*b^4 + 2*b^4)*
x^4 + 6*a^4 + 12*(5*a^3*b^3 + 2*a*b^3)*x^3 + (45*a^4*b^2 + 36*a^2*b^2 + 4*b
^2)*x^2 + 4*a^2 + 2*(9*a^5*b + 12*a^3*b + 4*a*b)*x + 1)*sqrt(b^2*x^2 + 2*a*
b*x + a^2 + 1) + a)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + (3*b
^6*x^6 + 18*a*b^5*x^5 + 3*a^6 + (45*a^2*b^4 + 7*b^4)*x^4 + 7*a^4 + 4*(15*a^
3*b^3 + 7*a*b^3)*x^3 + (45*a^4*b^2 + 42*a^2*b^2 + 5*b^2)*x^2 + 5*a^2 + 2*(9
*a^5*b + 14*a^3*b + 5*a*b)*x + 1)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + a)/((
b^7*x^6 + 6*a*b^6*x^5 + a^6*b + 3*a^4*b + 3*(5*a^2*b^5 + b^5)*x^4 + 4*(5*a^
3*b^4 + 3*a*b^4)*x^3 + 3*a^2*b + 3*(5*a^4*b^3 + 6*a^2*b^3 + b^3)*x^2 + (b^4
*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*x + a^3*b)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/
2) + 3*(b^5*x^4 + 4*a*b^4*x^3 + a^4*b + a^2*b + (6*a^2*b^3 + b^3)*x^2 + 2*(
2*a^3*b^2 + a*b^2)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + 6*(a^5*b^2 + 2*a^3*b^
2 + a*b^2)*x + 3*(b^6*x^5 + 5*a*b^5*x^4 + a^5*b + 2*a^3*b + 2*(5*a^2*b^4 +
b^4)*x^3 + 2*(5*a^3*b^3 + 3*a*b^3)*x^2 + a*b + (5*a^4*b^2 + 6*a^2*b^2 + b^2
)*x)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + b)*log(b*x + a + sqrt(b^2*x^2 + 2*
a*b*x + a^2 + 1))^2) + integrate(1/2*(b^8*x^8 + 8*a*b^7*x^7 + a^8 + 4*(7*a^
2*b^6 + b^6)*x^6 + 4*a^6 + 8*(7*a^3*b^5 + 3*a*b^5)*x^5 + 2*(35*a^4*b^4 + 30
*a^2*b^4 + 3*b^4)*x^4 + 6*a^4 + 8*(7*a^5*b^3 + 10*a^3*b^3 + 3*a*b^3)*x^3 +
(b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x + a^4 + 3)*(b^2*x^2 + 2*
a*b*x + a^2 + 1)^2 + 4*(7*a^6*b^2 + 15*a^4*b^2 + 9*a^2*b^2 + b^2)*x^2 + (4*
b^5*x^5 + 20*a*b^4*x^4 + 4*a^5 + 4*(10*a^2*b^3 + b^3)*x^3 + 4*a^3 + 4*(10*a
^3*b^2 + 3*a*b^2)*x^2 + (20*a^4*b + 12*a^2*b + 3*b)*x + 3*a)*(b^2*x^2 + 2*a
*b*x + a^2 + 1)^(3/2) + 3*(2*b^6*x^6 + 12*a*b^5*x^5 + 2*a^6 + 2*(15*a^2*b^4
 + 2*b^4)*x^4 + 4*a^4 + 8*(5*a^3*b^3 + 2*a*b^3)*x^3 + (30*a^4*b^2 + 24*a^2*
b^2 + b^2)*x^2 + a^2 + 2*(6*a^5*b + 8*a^3*b + a*b)*x - 1)*(b^2*x^2 + 2*a*b*
x + a^2 + 1) + 4*a^2 + 8*(a^7*b + 3*a^5*b + 3*a^3*b + a*b)*x + (4*b^7*x^7 +
 28*a*b^6*x^6 + 4*a^7 + 12*(7*a^2*b^5 + b^5)*x^5 + 12*a^5 + 20*(7*a^3*b^4 +
 3*a*b^4)*x^4 + (140*a^4*b^3 + 120*a^2*b^3 + 9*b^3)*x^3 + 9*a^3 + 3*(28*a^5
*b^2 + 40*a^3*b^2 + 9*a*b^2)*x^2 + (28*a^6*b + 60*a^4*b + 27*a^2*b + b)*x +
 a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 1)/((b^8*x^8 + 8*a*b^7*x^7 + a^8 +
4*(7*a^2*b^6 + b^6)*x^6 + 4*a^6 + 8*(7*a^3*b^5 + 3*a*b^5)*x^5 + 2*(35*a^4*b
^4 + 30*a^2*b^4 + 3*b^4)*x^4 + 6*a^4 + 8*(7*a^5*b^3 + 10*a^3*b^3 + 3*a*b^3)
*x^3 + (b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x + a^4)*(b^2*x^2 +
 2*a*b*x + a^2 + 1)^2 + 4*(7*a^6*b^2 + 15*a^4*b^2 + 9*a^2*b^2 + b^2)*x^2 +
4*(b^5*x^5 + 5*a*b^4*x^4 + a^5 + (10*a^2*b^3 + b^3)*x^3 + a^3 + (10*a^3*b^2
```

```
 + 3*a*b^2)*x^2 + (5*a^4*b + 3*a^2*b)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2
) + 6*(b^6*x^6 + 6*a*b^5*x^5 + a^6 + (15*a^2*b^4 + 2*b^4)*x^4 + 2*a^4 + 4*(
5*a^3*b^3 + 2*a*b^3)*x^3 + (15*a^4*b^2 + 12*a^2*b^2 + b^2)*x^2 + a^2 + 2*(3
*a^5*b + 4*a^3*b + a*b)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + 4*a^2 + 8*(a^7*b
 + 3*a^5*b + 3*a^3*b + a*b)*x + 4*(b^7*x^7 + 7*a*b^6*x^6 + a^7 + 3*(7*a^2*b
^5 + b^5)*x^5 + 3*a^5 + 5*(7*a^3*b^4 + 3*a*b^4)*x^4 + (35*a^4*b^3 + 30*a^2*
b^3 + 3*b^3)*x^3 + 3*a^3 + 3*(7*a^5*b^2 + 10*a^3*b^2 + 3*a*b^2)*x^2 + (7*a^
6*b + 15*a^4*b + 9*a^2*b + b)*x + a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 1)
*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\operatorname{asinh}\left(a+b\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/asinh(a + b*x)^3,x)
```

```
[Out] int(1/asinh(a + b*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{asinh}^{3}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/asinh(b*x+a)**3,x)
```

```
[Out] Integral(asinh(a + b*x)**(-3), x)
```

3.92 $\int \frac{1}{x\sinh^{-1}(a+bx)^3}\,dx$

Optimal. Leaf size=15

$$\mathrm{Int}\left(\frac{1}{x\sinh^{-1}(a+bx)^3},x\right)$$

```
[Out] Unintegrable(1/x/arcsinh(b*x+a)^3,x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\sinh^{-1}(a+bx)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*ArcSinh[a + b*x]^3),x]
[Out] Defer[Subst][Defer[Int][1/((-(a/b) + x/b)*ArcSinh[x]^3), x], x, a + b*x]/b
```

Rubi steps

$$\int \frac{1}{x\sinh^{-1}(a+bx)^3}\,dx = \frac{\mathrm{Subst}\left(\int \frac{1}{\left(-\frac{a}{b}+\frac{x}{b}\right)\sinh^{-1}(x)^3}\,dx,x,a+bx\right)}{b}$$

Mathematica [A] time = 2.14, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sinh^{-1}(a+bx)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*ArcSinh[a + b*x]^3),x]
[Out] Integrate[1/(x*ArcSinh[a + b*x]^3), x]
```

fricas [A] time = 0.52, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{1}{x\,\mathrm{arsinh}\left(bx+a\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(b*x+a)^3,x, algorithm="fricas")
[Out] integral(1/(x*arcsinh(b*x + a)^3), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\,\mathrm{arsinh}\left(bx+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(b*x+a)^3,x, algorithm="giac")
[Out] integrate(1/(x*arcsinh(b*x + a)^3), x)
```

maple [A] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{arcsinh}(bx+a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arcsinh(b*x+a)^3,x)
```

```
[Out] int(1/x/arcsinh(b*x+a)^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(b^8*x^8 + 7*a*b^7*x^7 + 3*(7*a^2*b^6 + b^6)*x^6 + 5*(7*a^3*b^5 + 3*a*
b^5)*x^5 + (35*a^4*b^4 + 30*a^2*b^4 + 3*b^4)*x^4 + 3*(7*a^5*b^3 + 10*a^3*b^
3 + 3*a*b^3)*x^3 + (7*a^6*b^2 + 15*a^4*b^2 + 9*a^2*b^2 + b^2)*x^2 + (b^5*x^
5 + 4*a*b^4*x^4 + (6*a^2*b^3 + b^3)*x^3 + 2*(2*a^3*b^2 + a*b^2)*x^2 + (a^4*
b + a^2*b)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + (3*b^6*x^6 + 15*a*b^5*x
^5 + 5*(6*a^2*b^4 + b^4)*x^4 + 15*(2*a^3*b^3 + a*b^3)*x^3 + (15*a^4*b^2 + 1
5*a^2*b^2 + 2*b^2)*x^2 + (3*a^5*b + 5*a^3*b + 2*a*b)*x)*(b^2*x^2 + 2*a*b*x
+ a^2 + 1) + (a^7*b + 3*a^5*b + 3*a^3*b + a*b)*x - (a*b^7*x^7 + 7*a^2*b^6*x
^6 + a^8 + 3*a^6 + 3*(7*a^3*b^5 + a*b^5)*x^5 + 5*(7*a^4*b^4 + 3*a^2*b^4)*x^
4 + 3*a^4 + (35*a^5*b^3 + 30*a^3*b^3 + 3*a*b^3)*x^3 + 3*(7*a^6*b^2 + 10*a^4
*b^2 + 3*a^2*b^2)*x^2 + (a*b^4*x^4 + a^5 + 2*(2*a^2*b^3 + b^3)*x^3 + 2*a^3
+ 6*(a^3*b^2 + a*b^2)*x^2 + 2*(2*a^4*b + 3*a^2*b + b)*x + a)*(b^2*x^2 + 2*a
*b*x + a^2 + 1)^(3/2) + (3*a*b^5*x^5 + 3*a^6 + (15*a^2*b^4 + 4*b^4)*x^4 + 7
*a^4 + (30*a^3*b^3 + 19*a*b^3)*x^3 + (30*a^4*b^2 + 33*a^2*b^2 + 5*b^2)*x^2
+ 5*a^2 + 5*(3*a^5*b + 5*a^3*b + 2*a*b)*x + 1)*(b^2*x^2 + 2*a*b*x + a^2 + 1
) + a^2 + (7*a^7*b + 15*a^5*b + 9*a^3*b + a*b)*x + (3*a*b^6*x^6 + 3*a^7 + 2
*(9*a^2*b^5 + b^5)*x^5 + 8*a^5 + (45*a^3*b^4 + 16*a*b^4)*x^4 + (60*a^4*b^3
+ 44*a^2*b^3 + 3*b^3)*x^3 + 7*a^3 + (45*a^5*b^2 + 56*a^3*b^2 + 13*a*b^2)*x^
2 + (18*a^6*b + 34*a^4*b + 17*a^2*b + b)*x + 2*a)*sqrt(b^2*x^2 + 2*a*b*x +
a^2 + 1))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + (3*b^7*x^7 + 1
8*a*b^6*x^6 + (45*a^2*b^5 + 7*b^5)*x^5 + 4*(15*a^3*b^4 + 7*a*b^4)*x^4 + (45
*a^4*b^3 + 42*a^2*b^3 + 5*b^3)*x^3 + 2*(9*a^5*b^2 + 14*a^3*b^2 + 5*a*b^2)*x
^2 + (3*a^6*b + 7*a^4*b + 5*a^2*b + b)*x)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)
)/((b^8*x^8 + 6*a*b^7*x^7 + 3*(5*a^2*b^6 + b^6)*x^6 + 4*(5*a^3*b^5 + 3*a*b^
5)*x^5 + 3*(5*a^4*b^4 + 6*a^2*b^4 + b^4)*x^4 + 6*(a^5*b^3 + 2*a^3*b^3 + a*b
^3)*x^3 + (a^6*b^2 + 3*a^4*b^2 + 3*a^2*b^2 + b^2)*x^2 + (b^5*x^5 + 3*a*b^4*
x^4 + 3*a^2*b^3*x^3 + a^3*b^2*x^2)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + 3*
(b^6*x^6 + 4*a*b^5*x^5 + (6*a^2*b^4 + b^4)*x^4 + 2*(2*a^3*b^3 + a*b^3)*x^3
+ (a^4*b^2 + a^2*b^2)*x^2)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + 3*(b^7*x^7 + 5*a
*b^6*x^6 + 2*(5*a^2*b^5 + b^5)*x^5 + 2*(5*a^3*b^4 + 3*a*b^4)*x^4 + (5*a^4*b
^3 + 6*a^2*b^3 + b^3)*x^3 + (a^5*b^2 + 2*a^3*b^2 + a*b^2)*x^2)*sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2)
+ integrate(1/2*(a*b^9*x^9 + 10*a^2*b^8*x^8 + 2*a^10 + 8*a^8 + 4*(11*a^3*b^
7 + a*b^7)*x^7 + 16*(7*a^4*b^6 + 2*a^2*b^6)*x^6 + 12*a^6 + 2*(91*a^5*b^5 +
54*a^3*b^5 + 3*a*b^5)*x^5 + 4*(49*a^6*b^4 + 50*a^4*b^4 + 9*a^2*b^4)*x^4 + 8
*a^4 + 4*(35*a^7*b^3 + 55*a^5*b^3 + 21*a^3*b^3 + a*b^3)*x^3 + (a*b^5*x^5 +
2*a^6 + 2*(3*a^2*b^4 + 2*b^4)*x^4 + 4*a^4 + 2*(7*a^3*b^3 + 8*a*b^3)*x^3 + 8
*(2*a^4*b^2 + 3*a^2*b^2 + b^2)*x^2 + 2*a^2 + (9*a^5*b + 16*a^3*b + 7*a*b)*x
)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^2 + 16*(4*a^8*b^2 + 9*a^6*b^2 + 6*a^4*b^2 +
 a^2*b^2)*x^2 + (4*a*b^6*x^6 + 8*a^7 + 4*(7*a^2*b^5 + 3*b^5)*x^5 + 20*a^5 +
 16*(5*a^3*b^4 + 4*a*b^4)*x^4 + 2*(60*a^4*b^3 + 70*a^2*b^3 + 11*b^3)*x^3 +
16*a^3 + (100*a^5*b^2 + 156*a^3*b^2 + 57*a*b^2)*x^2 + (44*a^6*b + 88*a^4*b
```

```
+ 51*a^2*b + 7*b)*x + 4*a)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + (6*a*b^7*x
^7 + 12*a^8 + 12*(4*a^2*b^6 + b^6)*x^6 + 36*a^6 + 6*(27*a^3*b^5 + 14*a*b^5)
*x^5 + 4*(75*a^4*b^4 + 63*a^2*b^4 + 5*b^4)*x^4 + 38*a^4 + (330*a^5*b^3 + 40
8*a^3*b^3 + 95*a*b^3)*x^3 + 2*(108*a^6*b^2 + 186*a^4*b^2 + 84*a^2*b^2 + 5*b
^2)*x^2 + 16*a^2 + (78*a^7*b + 180*a^5*b + 131*a^3*b + 29*a*b)*x + 2)*(b^2*
x^2 + 2*a*b*x + a^2 + 1) + 2*a^2 + (17*a^9*b + 52*a^7*b + 54*a^5*b + 20*a^3
*b + a*b)*x + (4*a*b^8*x^8 + 8*a^9 + 4*(9*a^2*b^7 + b^7)*x^7 + 28*a^7 + 20*
(7*a^3*b^6 + 2*a*b^6)*x^6 + 2*(154*a^4*b^5 + 84*a^2*b^5 + 3*b^5)*x^5 + 36*a
^5 + (420*a^5*b^4 + 380*a^3*b^4 + 51*a*b^4)*x^4 + (364*a^6*b^3 + 500*a^4*b^
3 + 153*a^2*b^3 + 3*b^3)*x^3 + 20*a^3 + (196*a^7*b^2 + 384*a^5*b^2 + 213*a^
3*b^2 + 25*a*b^2)*x^2 + (60*a^8*b + 160*a^6*b + 141*a^4*b + 42*a^2*b + b)*x
 + 4*a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/((b^10*x^11 + 8*a*b^9*x^10 + 4*(
7*a^2*b^8 + b^8)*x^9 + 8*(7*a^3*b^7 + 3*a*b^7)*x^8 + 2*(35*a^4*b^6 + 30*a^2
*b^6 + 3*b^6)*x^7 + 8*(7*a^5*b^5 + 10*a^3*b^5 + 3*a*b^5)*x^6 + 4*(7*a^6*b^4
 + 15*a^4*b^4 + 9*a^2*b^4 + b^4)*x^5 + 8*(a^7*b^3 + 3*a^5*b^3 + 3*a^3*b^3 +
 a*b^3)*x^4 + (a^8*b^2 + 4*a^6*b^2 + 6*a^4*b^2 + 4*a^2*b^2 + b^2)*x^3 + (b^
6*x^7 + 4*a*b^5*x^6 + 6*a^2*b^4*x^5 + 4*a^3*b^3*x^4 + a^4*b^2*x^3)*(b^2*x^2
 + 2*a*b*x + a^2 + 1)^2 + 4*(b^7*x^8 + 5*a*b^6*x^7 + (10*a^2*b^5 + b^5)*x^6
 + (10*a^3*b^4 + 3*a*b^4)*x^5 + (5*a^4*b^3 + 3*a^2*b^3)*x^4 + (a^5*b^2 + a^
3*b^2)*x^3)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + 6*(b^8*x^9 + 6*a*b^7*x^8
+ (15*a^2*b^6 + 2*b^6)*x^7 + 4*(5*a^3*b^5 + 2*a*b^5)*x^6 + (15*a^4*b^4 + 12
*a^2*b^4 + b^4)*x^5 + 2*(3*a^5*b^3 + 4*a^3*b^3 + a*b^3)*x^4 + (a^6*b^2 + 2*
a^4*b^2 + a^2*b^2)*x^3)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + 4*(b^9*x^10 + 7*a*b
^8*x^9 + 3*(7*a^2*b^7 + b^7)*x^8 + 5*(7*a^3*b^6 + 3*a*b^6)*x^7 + (35*a^4*b^
5 + 30*a^2*b^5 + 3*b^5)*x^6 + 3*(7*a^5*b^4 + 10*a^3*b^4 + 3*a*b^4)*x^5 + (7
*a^6*b^3 + 15*a^4*b^3 + 9*a^2*b^3 + b^3)*x^4 + (a^7*b^2 + 3*a^5*b^2 + 3*a^3
*b^2 + a*b^2)*x^3)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*log(b*x + a + sqrt(b^
2*x^2 + 2*a*b*x + a^2 + 1))), x)
```

mupad [**A**] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{1}{x\,\mathrm{asinh}\,(a+b\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*asinh(a + b*x)^3),x)
[Out] int(1/(x*asinh(a + b*x)^3), x)
```

sympy [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\,\mathrm{asinh}^3\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/asinh(b*x+a)**3,x)
[Out] Integral(1/(x*asinh(a + b*x)**3), x)
```

3.93 $\int x^m \left(a+b\sinh^{-1}(c+dx)\right)^n\,dx$

Optimal. Leaf size=19

$$\mathrm{Int}\left(x^m\left(a+b\sinh^{-1}(c+dx)\right)^n,x\right)$$

```
[Out] Unintegrable(x^m*(a+b*arcsinh(d*x+c))^n,x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^m\left(a+b\sinh^{-1}(c+dx)\right)^n\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*(a + b*ArcSinh[c + d*x])^n,x]
```

```
[Out] Defer[Subst][Defer[Int][(-(c/d) + x/d)^m*(a + b*ArcSinh[x])^n, x], x, c + d
*x]/d
```

Rubi steps

$$\int x^m\left(a+b\sinh^{-1}(c+dx)\right)^n\,dx=\frac{\mathrm{Subst}\left(\int\left(-\frac{c}{d}+\frac{x}{d}\right)^m\left(a+b\sinh^{-1}(x)\right)^n\,dx,x,c+dx\right)}{d}$$

Mathematica [A] time = 0.44, size = 0, normalized size = 0.00

$$\int x^m\left(a+b\sinh^{-1}(c+dx)\right)^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*(a + b*ArcSinh[c + d*x])^n,x]
```

```
[Out] Integrate[x^m*(a + b*ArcSinh[c + d*x])^n, x]
```

fricas [A] time = 0.57, size = 0, normalized size = 0.00

$$\mathrm{integral}\left((b\,\mathrm{arsinh}\,(dx+c)+a)^n x^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(d*x + c) + a)^n*x^m, x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(d*x+c))^n,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.19, size = 0, normalized size = 0.00

$$\int x^m\left(a+b\,\mathrm{arcsinh}\,(dx+c)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a+b*arcsinh(d*x+c))^n,x)
```

```
[Out] int(x^m*(a+b*arcsinh(d*x+c))^n,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arsinh}(dx + c) + a)^n x^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^n*x^m, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int x^m \left(a + b \operatorname{asinh}(c + d\,x)\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*asinh(c + d*x))^n,x)
```

```
[Out] int(x^m*(a + b*asinh(c + d*x))^n, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \left(a + b \operatorname{asinh}(c + dx)\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a+b*asinh(d*x+c))**n,x)
```

```
[Out] Integral(x**m*(a + b*asinh(c + d*x))**n, x)
```

3.94 $\int x^2 \left(a + b\sinh^{-1}(c + dx)\right)^n \, dx$

Optimal. Leaf size=545

$$\frac{c^2 e^{-\frac{a}{b}} \left(a + b\sinh^{-1}(c + dx)\right)^n \left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{-n} \Gamma\left(n+1, -\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{2d^3} - \frac{c^2 e^{a/b} \left(a + b\sinh^{-1}(c + dx)\right)^n \left(\frac{a-}{}\right.}{}$$

```
[Out] 1/8*3^(-1-n)*(a+b*arcsinh(d*x+c))^n*GAMMA(1+n,-3*(a+b*arcsinh(d*x+c))/b)/d^
3/exp(3*a/b)/(((-a-b*arcsinh(d*x+c))/b)^n)-2^(-2-n)*c*(a+b*arcsinh(d*x+c))^
n*GAMMA(1+n,-2*(a+b*arcsinh(d*x+c))/b)/d^3/exp(2*a/b)/(((-a-b*arcsinh(d*x+c
))/b)^n)-1/8*(a+b*arcsinh(d*x+c))^n*GAMMA(1+n,(-a-b*arcsinh(d*x+c))/b)/d^3/
exp(a/b)/(((-a-b*arcsinh(d*x+c))/b)^n)+1/2*c^2*(a+b*arcsinh(d*x+c))^n*GAMMA
(1+n,(-a-b*arcsinh(d*x+c))/b)/d^3/exp(a/b)/(((-a-b*arcsinh(d*x+c))/b)^n)+1/
8*exp(a/b)*(a+b*arcsinh(d*x+c))^n*GAMMA(1+n,(a+b*arcsinh(d*x+c))/b)/d^3/(((
a+b*arcsinh(d*x+c))/b)^n)-1/2*c^2*exp(a/b)*(a+b*arcsinh(d*x+c))^n*GAMMA(1+n
,(a+b*arcsinh(d*x+c))/b)/d^3/(((a+b*arcsinh(d*x+c))/b)^n)-2^(-2-n)*c*exp(2*
a/b)*(a+b*arcsinh(d*x+c))^n*GAMMA(1+n,2*(a+b*arcsinh(d*x+c))/b)/d^3/(((a+b*
arcsinh(d*x+c))/b)^n)-1/8*3^(-1-n)*exp(3*a/b)*(a+b*arcsinh(d*x+c))^n*GAMMA(
1+n,3*(a+b*arcsinh(d*x+c))/b)/d^3/(((a+b*arcsinh(d*x+c))/b)^n)
```

Rubi [A] time = 1.15, antiderivative size = 545, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {5865, 5805, 6741, 12, 6742, 3307, 2181, 5448, 3308}

$$\frac{c^2 e^{-\frac{a}{b}} \left(a + b\sinh^{-1}(c + dx)\right)^n \left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{-n} \text{Gamma}\left(n+1, -\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{2d^3} - \frac{c^2 e^{a/b} \left(a + b\sinh^{-1}(c + d\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcSinh[c + d*x])^n,x]
```

```
[Out] (3^(-1 - n)*(a + b*ArcSinh[c + d*x])^n*Gamma[1 + n, (-3*(a + b*ArcSinh[c +
d*x]))/b])/(8*d^3*E^((3*a)/b)*(-((a + b*ArcSinh[c + d*x])/b))^n) - (2^(-2 -
 n)*c*(a + b*ArcSinh[c + d*x])^n*Gamma[1 + n, (-2*(a + b*ArcSinh[c + d*x]))
/b])/(d^3*E^((2*a)/b)*(-((a + b*ArcSinh[c + d*x])/b))^n) - ((a + b*ArcSinh[
c + d*x])^n*Gamma[1 + n, -((a + b*ArcSinh[c + d*x])/b)])/(8*d^3*E^(a/b)*(-(
(a + b*ArcSinh[c + d*x])/b))^n) + (c^2*(a + b*ArcSinh[c + d*x])^n*Gamma[1 +
 n, -((a + b*ArcSinh[c + d*x])/b)])/(2*d^3*E^(a/b)*(-((a + b*ArcSinh[c + d*
x])/b))^n) + (E^(a/b)*(a + b*ArcSinh[c + d*x])^n*Gamma[1 + n, (a + b*ArcSin
h[c + d*x])/b])/(8*d^3*((a + b*ArcSinh[c + d*x])/b)^n) - (c^2*E^(a/b)*(a +
b*ArcSinh[c + d*x])^n*Gamma[1 + n, (a + b*ArcSinh[c + d*x])/b])/(2*d^3*((a
+ b*ArcSinh[c + d*x])/b)^n) - (2^(-2 - n)*c*E^((2*a)/b)*(a + b*ArcSinh[c +
d*x])^n*Gamma[1 + n, (2*(a + b*ArcSinh[c + d*x]))/b])/(d^3*((a + b*ArcSinh[
c + d*x])/b)^n) - (3^(-1 - n)*E^((3*a)/b)*(a + b*ArcSinh[c + d*x])^n*Gamma[
1 + n, (3*(a + b*ArcSinh[c + d*x]))/b])/(8*d^3*((a + b*ArcSinh[c + d*x])/b)
^n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
```

```
ntegerQ[m]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5805

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]*(c*d + e*Sinh[x
])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m, 0
]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x^2 \left(a+b\sinh^{-1}(c+dx)\right)^n dx = \frac{\text{Subst}\left(\int \left(-\frac{c}{d}+\frac{x}{d}\right)^2 \left(a+b\sinh^{-1}(x)\right)^n dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int (a+bx)^n \cosh(x) \left(-\frac{c}{d}+\frac{\sinh(x)}{d}\right)^2 dx, x, \sinh^{-1}(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^n \cosh(x)(c-\sinh(x))^2}{d^2} dx, x, \sinh^{-1}(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int (a+bx)^n \cosh(x)(c-\sinh(x))^2 dx, x, \sinh^{-1}(c+dx)\right)}{d^3}$$

$$= \frac{\text{Subst}\left(\int \left(c^2(a+bx)^n \cosh(x) - 2c(a+bx)^n \cosh(x)\sinh(x) + (a+bx)^n \text{c}\right.\right.}{d^3}$$

$$= \frac{\text{Subst}\left(\int (a+bx)^n \cosh(x)\sinh^2(x) dx, x, \sinh^{-1}(c+dx)\right)}{d^3} - \frac{(2c)\,\text{Subst}\left(\int\right.}{}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{4}(a+bx)^n \cosh(x) + \frac{1}{4}(a+bx)^n \cosh(3x)\right) dx, x, \sinh^{-1}(c+d\right.}{d^3}$$

$$= \frac{c^2 e^{-\frac{a}{b}} \left(a+b\sinh^{-1}(c+dx)\right)^n \left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{-n} \Gamma\left(1+n, -\frac{a+b\sinh^{-1}(c+dx)}{b}\right.}{2d^3}$$

$$= \frac{c^2 e^{-\frac{a}{b}} \left(a+b\sinh^{-1}(c+dx)\right)^n \left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{-n} \Gamma\left(1+n, -\frac{a+b\sinh^{-1}(c+dx)}{b}\right.}{2d^3}$$

$$= \frac{3^{-1-n} e^{-\frac{3a}{b}} \left(a+b\sinh^{-1}(c+dx)\right)^n \left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{-n} \Gamma\left(1+n, -\frac{3(a+b\sinh^{-}}{b}\right.}{8d^3}$$

Mathematica [A] time = 1.03, size = 345, normalized size = 0.63

$$\frac{2^{-n-3} 3^{-n-1} e^{-\frac{3a}{b}} \left(a+b\sinh^{-1}(c+dx)\right)^n \left(-\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{b^2}\right)^{-n} \left(\left(4c^2-1\right) 2^n 3^{n+1} e^{\frac{2a}{b}} \left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)^n \Gamma\left(n\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcSinh[c + d*x])^n,x]
```

```
[Out] (2^(-3 - n)*3^(-1 - n)*(a + b*ArcSinh[c + d*x])^n*(-(2^n*3^(1 + n)*(-1 + 4*
c^2)*E^((4*a)/b)*(-((a + b*ArcSinh[c + d*x])/b))^n*Gamma[1 + n, a/b + ArcSi
nh[c + d*x]]) + 2^n*(a/b + ArcSinh[c + d*x])^n*Gamma[1 + n, (-3*(a + b*ArcS
inh[c + d*x]))/b] - 2*3^(1 + n)*c*E^(a/b)*(a/b + ArcSinh[c + d*x])^n*Gamma[
1 + n, (-2*(a + b*ArcSinh[c + d*x]))/b] + 2^n*3^(1 + n)*(-1 + 4*c^2)*E^((2*
a)/b)*(a/b + ArcSinh[c + d*x])^n*Gamma[1 + n, -((a + b*ArcSinh[c + d*x])/b)
] - E^((5*a)/b)*(-((a + b*ArcSinh[c + d*x])/b))^n*(2*3^(1 + n)*c*Gamma[1 +
n, (2*(a + b*ArcSinh[c + d*x]))/b] + 2^n*E^(a/b)*Gamma[1 + n, (3*(a + b*Arc
Sinh[c + d*x]))/b])))/(d^3*E^((3*a)/b)*(-((a + b*ArcSinh[c + d*x])^2/b^2))^
n)
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left((b \operatorname{arsinh}(dx+c)+a)^n x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(d*x + c) + a)^n*x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arsinh}(dx + c) + a)^n x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*arcsinh(d*x + c) + a)^n*x^2, x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int x^2 (a + b \operatorname{arcsinh}(dx + c))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsinh(d*x+c))^n,x)
[Out] int(x^2*(a+b*arcsinh(d*x+c))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arsinh}(dx + c) + a)^n x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*arcsinh(d*x + c) + a)^n*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 (a + b \operatorname{asinh}(c + d\,x))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*asinh(c + d*x))^n,x)
[Out] int(x^2*(a + b*asinh(c + d*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 (a + b \operatorname{asinh}(c + dx))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asinh(d*x+c))**n,x)
[Out] Integral(x**2*(a + b*asinh(c + d*x))**n, x)
```

3.95 $\int x\left(a+b\sinh^{-1}(c+dx)\right)^n\,dx$

Optimal. Leaf size=267

$$\frac{2^{-n-3}e^{-\frac{2a}{b}}\left(a+b\sinh^{-1}(c+dx)\right)^n\left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{d^2}-\frac{ce^{-\frac{a}{b}}\left(a+b\sinh^{-1}(c+dx\right.}{}$$

```
[Out] 2^(-3-n)*(a+b*arcsinh(d*x+c))^n*GAMMA(1+n,-2*(a+b*arcsinh(d*x+c))/b)/d^2/ex
p(2*a/b)/(((-a-b*arcsinh(d*x+c))/b)^n)-1/2*c*(a+b*arcsinh(d*x+c))^n*GAMMA(1
+n,(-a-b*arcsinh(d*x+c))/b)/d^2/exp(a/b)/(((-a-b*arcsinh(d*x+c))/b)^n)+1/2*
c*exp(a/b)*(a+b*arcsinh(d*x+c))^n*GAMMA(1+n,(a+b*arcsinh(d*x+c))/b)/d^2/(((
a+b*arcsinh(d*x+c))/b)^n)+2^(-3-n)*exp(2*a/b)*(a+b*arcsinh(d*x+c))^n*GAMMA(
1+n,2*(a+b*arcsinh(d*x+c))/b)/d^2/(((a+b*arcsinh(d*x+c))/b)^n)
```

Rubi [A] time = 0.48, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {5865, 5805, 6741, 12, 6742, 3307, 2181, 5448, 3308}

$$\frac{2^{-n-3}e^{-\frac{2a}{b}}\left(a+b\sinh^{-1}(c+dx)\right)^n\left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{d^2}-\frac{ce^{-\frac{a}{b}}\left(a+b\sinh^{-1}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcSinh[c + d*x])^n,x]
```

```
[Out] (2^(-3 - n)*(a + b*ArcSinh[c + d*x])^n*Gamma[1 + n, (-2*(a + b*ArcSinh[c +
d*x]))/b])/(d^2*E^((2*a)/b)*(-((a + b*ArcSinh[c + d*x])/b))^n) - (c*(a + b*
ArcSinh[c + d*x])^n*Gamma[1 + n, -((a + b*ArcSinh[c + d*x])/b)])/(2*d^2*E^(
a/b)*(-((a + b*ArcSinh[c + d*x])/b))^n) + (c*E^(a/b)*(a + b*ArcSinh[c + d*x
])^n*Gamma[1 + n, (a + b*ArcSinh[c + d*x])/b])/(2*d^2*((a + b*ArcSinh[c + d
*x])/b)^n) + (2^(-3 - n)*E^((2*a)/b)*(a + b*ArcSinh[c + d*x])^n*Gamma[1 + n
, (2*(a + b*ArcSinh[c + d*x]))/b])/(d^2*((a + b*ArcSinh[c + d*x])/b)^n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5805

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]*(c*d + e*Sinh[x
])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m, 0
]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x\left(a+b\sinh^{-1}(c+dx)\right)^n\,dx=\frac{\text{Subst}\left(\int\left(-\frac{c}{d}+\frac{x}{d}\right)\left(a+b\sinh^{-1}(x)\right)^n\,dx,x,c+dx\right)}{d}$$
$$=\frac{\text{Subst}\left(\int(a+bx)^n\cosh(x)\left(-\frac{c}{d}+\frac{\sinh(x)}{d}\right)\,dx,x,\sinh^{-1}(c+dx)\right)}{d}$$
$$=\frac{\text{Subst}\left(\int\frac{(a+bx)^n\cosh(x)(-c+\sinh(x))}{d}\,dx,x,\sinh^{-1}(c+dx)\right)}{d}$$
$$=\frac{\text{Subst}\left(\int(a+bx)^n\cosh(x)(-c+\sinh(x))\,dx,x,\sinh^{-1}(c+dx)\right)}{d^2}$$
$$=\frac{\text{Subst}\left(\int\left(-c(a+bx)^n\cosh(x)+(a+bx)^n\cosh(x)\sinh(x)\right)\,dx,x,\sinh^{-1}(c$$
$$=\frac{\text{Subst}\left(\int(a+bx)^n\cosh(x)\sinh(x)\,dx,x,\sinh^{-1}(c+dx)\right)}{d^2}-\frac{c\,\text{Subst}\left(\int(a+$$
$$=\frac{\text{Subst}\left(\int\frac{1}{2}(a+bx)^n\sinh(2x)\,dx,x,\sinh^{-1}(c+dx)\right)}{d^2}-\frac{c\,\text{Subst}\left(\int e^{-x}(a+b$$
$$=-\frac{ce^{-\frac{a}{b}}\left(a+b\sinh^{-1}(c+dx)\right)^n\left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{a+b\sinh^{-1}(c+dx)}{b}\right.}{2d^2}$$
$$=-\frac{ce^{-\frac{a}{b}}\left(a+b\sinh^{-1}(c+dx)\right)^n\left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{a+b\sinh^{-1}(c+dx)}{b}\right.}{2d^2}$$
$$=\frac{2^{-3-n}e^{-\frac{2a}{b}}\left(a+b\sinh^{-1}(c+dx)\right)^n\left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{2\left(a+b\sinh^{-1}\right.}{b}\right.}{d^2}$$

Mathematica [A] time = 0.19, size = 228, normalized size = 0.85

$$\frac{2^{-n-3}e^{-\frac{2a}{b}}\left(a+b\sinh^{-1}(c+dx)\right)^n\left(-\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{b^2}\right)^{-n}\left(\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)^n\Gamma\left(n+1,-\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcSinh[c + d*x])^n,x]
```

```
[Out] (2^(-3 - n)*(a + b*ArcSinh[c + d*x])^n*(2^(2 + n)*c*E^((3*a)/b)*(-((a + b*A
rcSinh[c + d*x])/b))^n*Gamma[1 + n, a/b + ArcSinh[c + d*x]] + (a/b + ArcSin
h[c + d*x])^n*Gamma[1 + n, (-2*(a + b*ArcSinh[c + d*x]))/b] - 2^(2 + n)*c*E
^(a/b)*(a/b + ArcSinh[c + d*x])^n*Gamma[1 + n, -((a + b*ArcSinh[c + d*x])/b
)] + E^((4*a)/b)*(-((a + b*ArcSinh[c + d*x])/b))^n*Gamma[1 + n, (2*(a + b*A
rcSinh[c + d*x]))/b]))/(d^2*E^((2*a)/b)*(-((a + b*ArcSinh[c + d*x])^2/b^2))
^n)
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\,\text{arsinh}\left(dx+c\right)+a\right)^n x,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(d*x + c) + a)^n*x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arsinh}(dx + c) + a)^n x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*arcsinh(d*x + c) + a)^n*x, x)
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int x (a + b \operatorname{arcsinh}(dx + c))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsinh(d*x+c))^n,x)
[Out] int(x*(a+b*arcsinh(d*x+c))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arsinh}(dx + c) + a)^n x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*arcsinh(d*x + c) + a)^n*x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x (a + b \operatorname{asinh}(c + d\,x))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*asinh(c + d*x))^n,x)
[Out] int(x*(a + b*asinh(c + d*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x (a + b \operatorname{asinh}(c + dx))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(d*x+c))**n,x)
[Out] Integral(x*(a + b*asinh(c + d*x))**n, x)
```

3.96 $\int \left(a+b\sinh^{-1}(c+dx)\right)^n\,dx$

Optimal. Leaf size=128

$$\frac{e^{-\frac{a}{b}}\left(a+b\sinh^{-1}(c+dx)\right)^n\left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{2d}-\frac{e^{a/b}\left(a+b\sinh^{-1}(c+dx)\right)^n\left(\frac{a+b\sin}{}\right.}{2}$$

```
[Out] 1/2*(a+b*arcsinh(d*x+c))^n*GAMMA(1+n,(-a-b*arcsinh(d*x+c))/b)/d/exp(a/b)/((
(-a-b*arcsinh(d*x+c))/b)^n)-1/2*exp(a/b)*(a+b*arcsinh(d*x+c))^n*GAMMA(1+n,(
a+b*arcsinh(d*x+c))/b)/d/(((a+b*arcsinh(d*x+c))/b)^n)
```

Rubi [A] time = 0.12, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5863, 5657, 3307, 2181}

$$\frac{e^{-\frac{a}{b}}\left(a+b\sinh^{-1}(c+dx)\right)^n\left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{2d}-\frac{e^{a/b}\left(a+b\sinh^{-1}(c+dx)\right)^n}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^n,x]
```

```
[Out] ((a + b*ArcSinh[c + d*x])^n*Gamma[1 + n, -((a + b*ArcSinh[c + d*x])/b)])/(2
*d*E^(a/b)*(-((a + b*ArcSinh[c + d*x])/b))^n) - (E^(a/b)*(a + b*ArcSinh[c +
 d*x])^n*Gamma[1 + n, (a + b*ArcSinh[c + d*x])/b])/(2*d*((a + b*ArcSinh[c +
 d*x])/b)^n)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 5863

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \left(a+b\sinh^{-1}(c+dx)\right)^n\,dx = \frac{\text{Subst}\left(\int \left(a+b\sinh^{-1}(x)\right)^n\,dx,x,c+dx\right)}{d}$$
$$= \frac{\text{Subst}\left(\int x^n\cosh\left(\frac{a}{b}-\frac{x}{b}\right)\,dx,x,a+b\sinh^{-1}(c+dx)\right)}{bd}$$
$$= \frac{\text{Subst}\left(\int e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}x^n\,dx,x,a+b\sinh^{-1}(c+dx)\right)}{2bd} + \frac{\text{Subst}\left(\int e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}x^n\,dx,x,a-\right.}{2bd}$$
$$= \frac{e^{-\frac{a}{b}}\left(a+b\sinh^{-1}(c+dx)\right)^n\left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{2d} - \frac{e^{a/b}}{}$$

Mathematica [A] time = 0.12, size = 109, normalized size = 0.85

$$\frac{e^{-\frac{a}{b}}\left(a+b\sinh^{-1}(c+dx)\right)^n\left(\left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)-e^{\frac{2a}{b}}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)^{-n}\Gamma\left(n+1,\right.\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^n,x]
```

```
[Out] ((a + b*ArcSinh[c + d*x])^n*(-((E^((2*a)/b)*Gamma[1 + n, a/b + ArcSinh[c +
d*x]])/(a/b + ArcSinh[c + d*x])^n) + Gamma[1 + n, -((a + b*ArcSinh[c + d*x]
)/b)]/(-((a + b*ArcSinh[c + d*x])/b))^n))/(2*d*E^(a/b))
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left((b\,\text{arsinh}\,(dx+c)+a)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(d*x + c) + a)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\,\text{arsinh}\,(dx+c)+a)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^n, x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int (a+b\,\text{arcsinh}\,(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^n,x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\,\text{arsinh}\,(dx+c)+a)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b \operatorname{asinh}(c + d x))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^n,x)
```

```
[Out] int((a + b*asinh(c + d*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{asinh}(c + dx))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**n,x)
```

```
[Out] Integral((a + b*asinh(c + d*x))**n, x)
```

3.97 $\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^n}{x}\,dx$

Optimal. Leaf size=19

$$\mathrm{Int}\left(\frac{\left(a+b\sinh^{-1}(c+dx)\right)^n}{x},x\right)$$

[Out] Unintegrable((a+b*arcsinh(d*x+c))^n/x,x)

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^n}{x}\,dx$$

Verification is Not applicable to the result.

[In] Int[(a + b*ArcSinh[c + d*x])^n/x,x]

[Out] Defer[Subst][Defer[Int][(a + b*ArcSinh[x])^n/(-(c/d) + x/d), x], x, c + d*x]/d

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^n}{x}\,dx=\frac{\mathrm{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^n}{-\frac{c}{d}+\frac{x}{d}}\,dx,x,c+dx\right)}{d}$$

Mathematica [A] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^n}{x}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b*ArcSinh[c + d*x])^n/x,x]

[Out] Integrate[(a + b*ArcSinh[c + d*x])^n/x, x]

fricas [A] time = 0.53, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{(b\,\mathrm{arsinh}\,(dx+c)+a)^n}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsinh(d*x+c))^n/x,x, algorithm="fricas")

[Out] integral((b*arcsinh(d*x + c) + a)^n/x, x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\mathrm{arsinh}\,(dx+c)+a)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsinh(d*x+c))^n/x,x, algorithm="giac")

```
[Out] integrate((b*arcsinh(d*x + c) + a)^n/x, x)
```

maple [A] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{arcsinh}(dx + c))^n}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^n/x,x)
[Out] int((a+b*arcsinh(d*x+c))^n/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arsinh}(dx + c) + a)^n}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^n/x,x, algorithm="maxima")
[Out] integrate((b*arcsinh(d*x + c) + a)^n/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(a + b \operatorname{asinh}(c + d\,x))^n}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^n/x,x)
[Out] int((a + b*asinh(c + d*x))^n/x, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{asinh}(c + dx))^n}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**n/x,x)
[Out] Integral((a + b*asinh(c + d*x))**n/x, x)
```

3.98 $\int x^2\sqrt{a+b\sinh^{-1}(c+dx)}\,dx$

Optimal. Leaf size=496

$$\frac{\sqrt{\pi}\sqrt{b}\,c^2e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4d^3}-\frac{\sqrt{\pi}\sqrt{b}\,c^2e^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4d^3}+\frac{c^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d^3}-\frac{\sqrt{\pi}\ \cdots}{\cdots}$$

```
[Out] 1/144*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*3^
(1/2)*Pi^(1/2)/d^3-1/144*erfi(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b
^(1/2)*3^(1/2)*Pi^(1/2)/d^3/exp(3*a/b)+1/16*c*exp(2*a/b)*erf(2^(1/2)*(a+b*a
rcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*2^(1/2)*Pi^(1/2)/d^3+1/16*c*erfi(2^(1
/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*2^(1/2)*Pi^(1/2)/d^3/exp(2*
a/b)-1/16*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)
/d^3+1/4*c^2*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1
/2)/d^3+1/16*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/d^3/
exp(a/b)-1/4*c^2*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/
d^3/exp(a/b)+c^2*(d*x+c)*(a+b*arcsinh(d*x+c))^(1/2)/d^3+1/3*(d*x+c)^3*(a+b*
arcsinh(d*x+c))^(1/2)/d^3-1/2*c*cosh(2*arcsinh(d*x+c))*(a+b*arcsinh(d*x+c))
^(1/2)/d^3
```

Rubi [A] time = 1.85, antiderivative size = 496, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 12, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5865, 5805, 6741, 6742, 5325, 5298, 2205, 2204, 5324, 5299, 5372, 5300}

$$\frac{\sqrt{\pi}\sqrt{b}\,c^2e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4d^3}-\frac{\sqrt{\pi}\sqrt{b}\,c^2e^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4d^3}+\frac{c^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d^3}-\frac{\sqrt{\pi}\ \cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (c^2*(c + d*x)*Sqrt[a + b*ArcSinh[c + d*x]])/d^3 + ((c + d*x)^3*Sqrt[a + b*
ArcSinh[c + d*x]])/(3*d^3) - (c*Sqrt[a + b*ArcSinh[c + d*x]]*Cosh[2*ArcSinh
[c + d*x]])/(2*d^3) - (Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c +
d*x]]/Sqrt[b]])/(16*d^3) + (Sqrt[b]*c^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*Arc
Sinh[c + d*x]]/Sqrt[b]])/(4*d^3) + (Sqrt[b]*c*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(S
qrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(8*d^3) + (Sqrt[b]*E^((3*a)/
b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(48*d^3)
 + (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(16*d^3*E^
(a/b)) - (Sqrt[b]*c^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/
(4*d^3*E^(a/b)) + (Sqrt[b]*c*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c
+ d*x]])/Sqrt[b]])/(8*d^3*E^((2*a)/b)) - (Sqrt[b]*Sqrt[Pi/3]*Erfi[(Sqrt[3]*
Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(48*d^3*E^((3*a)/b))
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5298

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5299

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5300

```
Int[((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_), x_Symbol] :> Int[Ex
pandTrigReduce[(a + b*Sinh[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[n, 1] && IGtQ[p, 1]
```

Rule 5324

```
Int[((e_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cosh[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1
))/(d*n), Int[(e*x)^(m - n)*Cosh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[0, n, m + 1]
```

Rule 5325

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Sinh[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1
))/(d*n), Int[(e*x)^(m - n)*Sinh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[0, n, m + 1]
```

Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 5805

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]*(c*d + e*Sinh[x
])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m, 0
]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x^2\sqrt{a+b\sinh^{-1}(c+dx)}\,dx = \frac{\text{Subst}\left(\int\left(-\frac{c}{d}+\frac{x}{d}\right)^2\sqrt{a+b\sinh^{-1}(x)}\,dx,x,c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int\sqrt{a+bx}\cosh(x)\left(-\frac{c}{d}+\frac{\sinh(x)}{d}\right)^2dx,x,\sinh^{-1}(c+dx)\right)}{d}$$

$$= \frac{2\,\text{Subst}\left(\int x^2\cosh\left(\frac{a-x^2}{b}\right)\left(c+\sinh\left(\frac{a-x^2}{b}\right)\right)^2dx,x,\sqrt{a+b\sinh^{-1}(c+dx)}\right)}{bd^3}$$

$$= \frac{2\,\text{Subst}\left(\int x^2\cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)\left(c+\sinh\left(\frac{a-x^2}{b}\right)\right)^2dx,x,\sqrt{a+b\sinh^{-1}(c+dx)}\right)}{bd^3}$$

$$= \frac{2\,\text{Subst}\left(\int\left(c^2x^2\cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)+cx^2\sinh\left(\frac{2a}{b}-\frac{2x^2}{b}\right)+x^2\cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)\sinh^2\right.}{bd^3}$$

$$= \frac{2\,\text{Subst}\left(\int x^2\cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{x^2}{b}\right)dx,x,\sqrt{a+b\sinh^{-1}(c+dx)}\right)}{bd^3} + \frac{(2}{}$$

$$= \frac{c^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d^3} + \frac{(c+dx)^3\sqrt{a+b\sinh^{-1}(c+dx)}}{3d^3} - \frac{c\sqrt{a+b}}{}$$

$$= \frac{c^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d^3} + \frac{(c+dx)^3\sqrt{a+b\sinh^{-1}(c+dx)}}{3d^3} - \frac{c\sqrt{a+b}}{}$$

$$= \frac{c^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d^3} + \frac{(c+dx)^3\sqrt{a+b\sinh^{-1}(c+dx)}}{3d^3} - \frac{c\sqrt{a+b}}{}$$

$$= \frac{c^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d^3} + \frac{(c+dx)^3\sqrt{a+b\sinh^{-1}(c+dx)}}{3d^3} - \frac{c\sqrt{a+b}}{}$$

$$= \frac{c^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d^3} + \frac{(c+dx)^3\sqrt{a+b\sinh^{-1}(c+dx)}}{3d^3} - \frac{c\sqrt{a+b}}{}$$

Mathematica [A] time = 1.77, size = 656, normalized size = 1.32

$$\frac{9\sqrt{\pi}\sqrt{b}\left(4c^2-1\right)\left(\sinh\left(\frac{a}{b}\right)+\cosh\left(\frac{a}{b}\right)\right)\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)+36\sqrt{\pi}\sqrt{b}\,c^2\sinh\left(\frac{a}{b}\right)\text{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)-3}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (-36*(c + d*x)*Sqrt[a + b*ArcSinh[c + d*x]] + 144*c^2*(c + d*x)*Sqrt[a + b*
ArcSinh[c + d*x]] - 72*c*Sqrt[a + b*ArcSinh[c + d*x]]*Cosh[2*ArcSinh[c + d*
x]] + Sqrt[b]*Sqrt[3*Pi]*Cosh[(3*a)/b]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c +
```

```
d*x]])/Sqrt[b]] + 9*Sqrt[b]*Sqrt[Pi]*Cosh[a/b]*Erfi[Sqrt[a + b*ArcSinh[c +
d*x]]/Sqrt[b]] - 36*Sqrt[b]*c^2*Sqrt[Pi]*Cosh[a/b]*Erfi[Sqrt[a + b*ArcSinh[
c + d*x]]/Sqrt[b]] + 9*Sqrt[b]*c*Sqrt[2*Pi]*Cosh[(2*a)/b]*Erfi[(Sqrt[2]*Sqr
t[a + b*ArcSinh[c + d*x]])/Sqrt[b]] - Sqrt[b]*Sqrt[3*Pi]*Cosh[(3*a)/b]*Erfi
[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]] - 9*Sqrt[b]*Sqrt[Pi]*Erfi[
Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]*Sinh[a/b] + 36*Sqrt[b]*c^2*Sqrt[Pi]*E
rfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]*Sinh[a/b] + 9*Sqrt[b]*(-1 + 4*c^2
)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b]
) - 9*Sqrt[b]*c*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt
[b]]*Sinh[(2*a)/b] + 9*Sqrt[b]*c*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh
[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b] + Sinh[(2*a)/b]) + Sqrt[b]*Sqrt[3*Pi]*E
rf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*Sinh[(3*a)/b] + Sqrt[b]*
Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*Sinh[(3*a)/
b] + 12*Sqrt[a + b*ArcSinh[c + d*x]]*Sinh[3*ArcSinh[c + d*x]])/(144*d^3)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \operatorname{arsinh}\left(dx+c\right)+a}\, x^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*arcsinh(d*x + c) + a)*x^2, x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt{a+b \operatorname{arcsinh}\left(dx+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsinh(d*x+c))^(1/2),x)
```

```
[Out] int(x^2*(a+b*arcsinh(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \operatorname{arsinh}\left(dx+c\right)+a}\, x^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*arcsinh(d*x + c) + a)*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\sqrt{a+b\,\mathrm{asinh}\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*asinh(c + d*x))^(1/2),x)
```

```
[Out] int(x^2*(a + b*asinh(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt{a + b \operatorname{asinh}(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asinh(d*x+c))**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(a + b*asinh(c + d*x)), x)
```

3.99 $\int x\sqrt{a+b\sinh^{-1}(c+dx)}\,dx$

Optimal. Leaf size=259

$$-\frac{\sqrt{\pi}\sqrt{b}\,ce^{a/b}\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4d^2}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}\,e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d^2}+\frac{\sqrt{\pi}\sqrt{b}\,ce^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4d^2}-$$

```
[Out] -1/32*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*2^
(1/2)*Pi^(1/2)/d^2-1/32*erfi(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^
(1/2)*2^(1/2)*Pi^(1/2)/d^2/exp(2*a/b)-1/4*c*exp(a/b)*erf((a+b*arcsinh(d*x+c
))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/d^2+1/4*c*erfi((a+b*arcsinh(d*x+c))^(1/2
)/b^(1/2))*b^(1/2)*Pi^(1/2)/d^2/exp(a/b)-c*(d*x+c)*(a+b*arcsinh(d*x+c))^(1/
2)/d^2+1/4*cosh(2*arcsinh(d*x+c))*(a+b*arcsinh(d*x+c))^(1/2)/d^2
```

Rubi [A] time = 0.70, antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5865, 5805, 6741, 6742, 5325, 5298, 2205, 2204, 5324, 5299}

$$-\frac{\sqrt{\pi}\sqrt{b}\,ce^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4d^2}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}\,e^{\frac{2a}{b}}\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d^2}+\frac{\sqrt{\pi}\sqrt{b}\,ce^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4d^2}-$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] -((c*(c + d*x)*Sqrt[a + b*ArcSinh[c + d*x]])/d^2) + (Sqrt[a + b*ArcSinh[c +
 d*x]]*Cosh[2*ArcSinh[c + d*x]])/(4*d^2) - (Sqrt[b]*c*E^(a/b)*Sqrt[Pi]*Erf[
Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(4*d^2) - (Sqrt[b]*E^((2*a)/b)*Sqrt[
Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(16*d^2) + (Sqrt
[b]*c*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(4*d^2*E^(a/b))
- (Sqrt[b]*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])
/(16*d^2*E^((2*a)/b))
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5298

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5299

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
```

```
[n, 1]
```

Rule 5324

```
Int[((e_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cosh[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1
))/(d*n), Int[(e*x)^(m - n)*Cosh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[0, n, m + 1]
```

Rule 5325

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Sinh[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1
))/(d*n), Int[(e*x)^(m - n)*Sinh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[0, n, m + 1]
```

Rule 5805

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]*(c*d + e*Sinh[x
])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m, 0
]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x\sqrt{a+b\sinh^{-1}(c+dx)}\,dx = \frac{\text{Subst}\left(\int\left(-\frac{c}{d}+\frac{x}{d}\right)\sqrt{a+b\sinh^{-1}(x)}\,dx,x,c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int\sqrt{a+bx}\cosh(x)\left(-\frac{c}{d}+\frac{\sinh(x)}{d}\right)dx,x,\sinh^{-1}(c+dx)\right)}{d}$$

$$= -\frac{2\,\text{Subst}\left(\int x^2\cosh\left(\frac{a-x^2}{b}\right)\left(c+\sinh\left(\frac{a-x^2}{b}\right)\right)dx,x,\sqrt{a+b\sinh^{-1}(c+dx)}\right)}{bd^2}$$

$$= -\frac{2\,\text{Subst}\left(\int x^2\cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)\left(c+\sinh\left(\frac{a-x^2}{b}\right)\right)dx,x,\sqrt{a+b\sinh^{-1}(c+dx)}\right.}{bd^2}$$

$$= -\frac{2\,\text{Subst}\left(\int\left(cx^2\cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)+\frac{1}{2}x^2\sinh\left(\frac{2a}{b}-\frac{2x^2}{b}\right)\right)dx,x,\sqrt{a+b\sinh^{-1}(\right.}{bd^2}$$

$$= -\frac{\text{Subst}\left(\int x^2\sinh\left(\frac{2a}{b}-\frac{2x^2}{b}\right)dx,x,\sqrt{a+b\sinh^{-1}(c+dx)}\right)}{bd^2} - \frac{(2c)\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{c(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d^2} + \frac{\sqrt{a+b\sinh^{-1}(c+dx)}\cosh\left(2\sinh^{-1}(c\right.}{4d^2}$$

$$= -\frac{c(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d^2} + \frac{\sqrt{a+b\sinh^{-1}(c+dx)}\cosh\left(2\sinh^{-1}(c\right.}{4d^2}$$

$$= -\frac{c(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d^2} + \frac{\sqrt{a+b\sinh^{-1}(c+dx)}\cosh\left(2\sinh^{-1}(c\right.}{4d^2}$$

Mathematica [A] time = 1.82, size = 251, normalized size = 0.97

$$\frac{-\sqrt{2\pi}\sqrt{b}\left(\sinh\left(\frac{2a}{b}\right)+\cosh\left(\frac{2a}{b}\right)\right)\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)+\sqrt{2\pi}\sqrt{b}\left(\sinh\left(\frac{2a}{b}\right)-\cosh\left(\frac{2a}{b}\right)\right)\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\,\text{s}}}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (8*Sqrt[a + b*ArcSinh[c + d*x]]*Cosh[2*ArcSinh[c + d*x]] - (16*c*Sqrt[a + b
*ArcSinh[c + d*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcSinh[c + d*x]])/Sqrt
[a/b + ArcSinh[c + d*x]]) + Gamma[3/2, -((a + b*ArcSinh[c + d*x])/b)]/Sqrt[
-((a + b*ArcSinh[c + d*x])/b)]))/E^(a/b) + Sqrt[b]*Sqrt[2*Pi]*Erfi[(Sqrt[2]
*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*(-Cosh[(2*a)/b] + Sinh[(2*a)/b]) -
Sqrt[b]*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*(Cos
h[(2*a)/b] + Sinh[(2*a)/b]))/(32*d^2)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \operatorname{arsinh}(dx + c) + a}\, x\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*arcsinh(d*x + c) + a)*x, x)
```

maple [F] time = 0.52, size = 0, normalized size = 0.00

$$\int x\sqrt{a + b \operatorname{arcsinh}(dx + c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsinh(d*x+c))^(1/2),x)
```

```
[Out] int(x*(a+b*arcsinh(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \operatorname{arsinh}(dx + c) + a}\, x\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*arcsinh(d*x + c) + a)*x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\sqrt{a + b \operatorname{asinh}(c + d\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*asinh(c + d*x))^(1/2),x)
```

```
[Out] int(x*(a + b*asinh(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{a + b \operatorname{asinh}(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(d*x+c))**(1/2),x)
```

```
[Out] Integral(x*sqrt(a + b*asinh(c + d*x)), x)
```

3.100 $\int \sqrt{a+b\sinh^{-1}(c+dx)}\,dx$

Optimal. Leaf size=115

$$\frac{\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4d}-\frac{\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4d}+\frac{(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d}$$

```
[Out] 1/4*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/d-1/4
*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/d/exp(a/b)+(d*x+
c)*(a+b*arcsinh(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.25, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5863, 5653, 5779, 3308, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}\sqrt{b}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4d}-\frac{\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4d}+\frac{(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] ((c + d*x)*Sqrt[a + b*ArcSinh[c + d*x]])/d + (Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[
Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(4*d) - (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[
a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(4*d*E^(a/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 5863

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \sqrt{a+b\sinh^{-1}(c+dx)}\,dx = \frac{\text{Subst}\left(\int \sqrt{a+b\sinh^{-1}(x)}\,dx, x, c+dx\right)}{d}$$
$$= \frac{(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d} - \frac{b\,\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\sqrt{a+b\sinh^{-1}(x)}}\,dx, x, c+dx\right)}{2d}$$
$$= \frac{(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d} - \frac{b\,\text{Subst}\left(\int \frac{\sinh(x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx)\right)}{2d}$$
$$= \frac{(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d} + \frac{b\,\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx)\right)}{4d} - b\,\text{Su}$$
$$= \frac{(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d} + \frac{\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(c+dx)}\right)}{2d} -$$
$$= \frac{(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d} + \frac{\sqrt{b}\,e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4d} - \frac{\sqrt{b}\,e^{-\frac{a}{b}}\sqrt{\pi}\,\text{e}}{}$$

Mathematica [A] time = 0.08, size = 111, normalized size = 0.97

$$\frac{e^{-\frac{a}{b}}\sqrt{a+b\sinh^{-1}(c+dx)}\left(\frac{\Gamma\left(\frac{3}{2},-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}} - \frac{e^{\frac{2a}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)}{\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}}\right)}{2d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (Sqrt[a + b*ArcSinh[c + d*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcSinh[c +
d*x]])/Sqrt[a/b + ArcSinh[c + d*x]]) + Gamma[3/2, -((a + b*ArcSinh[c + d*x]
)/b)]/Sqrt[-((a + b*ArcSinh[c + d*x])/b)]))/(2*d*E^(a/b))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \operatorname{arsinh}(dx + c) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*arcsinh(d*x + c) + a), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \operatorname{arcsinh}(dx + c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^(1/2),x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \operatorname{arsinh}(dx + c) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*arcsinh(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a + b \operatorname{asinh}(c + d\, x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^(1/2),x)
```

```
[Out] int((a + b*asinh(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \operatorname{asinh}(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*asinh(c + d*x)), x)
```

3.101 $\int x \left(a+b\sinh^{-1}(c+dx)\right)^{3/2} \, dx$

Optimal. Leaf size=326

$$-\frac{3\sqrt{\pi}\, b^{3/2} c e^{a/b} \operatorname{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8d^2}-\frac{3\sqrt{\frac{\pi}{2}}\, b^{3/2} e^{\frac{2a}{b}} \operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{64d^2}-\frac{3\sqrt{\pi}\, b^{3/2} c e^{-\frac{a}{b}} \operatorname{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8d^2}-$$

```
[Out] -c*(d*x+c)*(a+b*arcsinh(d*x+c))^(3/2)/d^2+1/4*(a+b*arcsinh(d*x+c))^(3/2)*co
sh(2*arcsinh(d*x+c))/d^2-3/128*b^(3/2)*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(
d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d^2+3/128*b^(3/2)*erfi(2^(1/2)*(a+b
*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d^2/exp(2*a/b)-3/8*b^(3/2)
*c*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d^2-3/8*b^(3/2
)*c*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d^2/exp(a/b)-3/16*b*s
inh(2*arcsinh(d*x+c))*(a+b*arcsinh(d*x+c))^(1/2)/d^2+3/2*b*c*(1+(d*x+c)^2)^
(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/d^2
```

Rubi [A] time = 0.96, antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5865, 5805, 6741, 6742, 5325, 5324, 5299, 2205, 2204, 5298}

$$-\frac{3\sqrt{\pi}\, b^{3/2} c e^{a/b} \operatorname{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8d^2}-\frac{3\sqrt{\frac{\pi}{2}}\, b^{3/2} e^{\frac{2a}{b}} \operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{64d^2}-\frac{3\sqrt{\pi}\, b^{3/2} c e^{-\frac{a}{b}} \operatorname{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (3*b*c*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/(2*d^2) - (c*(c
+ d*x)*(a + b*ArcSinh[c + d*x])^(3/2))/d^2 + ((a + b*ArcSinh[c + d*x])^(3/2
)*Cosh[2*ArcSinh[c + d*x]])/(4*d^2) - (3*b^(3/2)*c*E^(a/b)*Sqrt[Pi]*Erf[Sqr
t[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(8*d^2) - (3*b^(3/2)*E^((2*a)/b)*Sqrt[P
i/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(64*d^2) - (3*b^(
3/2)*c*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(8*d^2*E^(a/b))
 + (3*b^(3/2)*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b
]])/(64*d^2*E^((2*a)/b)) - (3*b*Sqrt[a + b*ArcSinh[c + d*x]]*Sinh[2*ArcSinh
[c + d*x]])/(16*d^2)
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5298

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5299

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5324

```
Int[((e_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cosh[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1
))/(d*n), Int[(e*x)^(m - n)*Cosh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[0, n, m + 1]
```

Rule 5325

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Sinh[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1
))/(d*n), Int[(e*x)^(m - n)*Sinh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[0, n, m + 1]
```

Rule 5805

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]*(c*d + e*Sinh[x
])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m, 0
]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}dx=\frac{\text{Subst}\left(\int\left(-\frac{c}{d}+\frac{x}{d}\right)\left(a+b\sinh^{-1}(x)\right)^{3/2}dx,x,c+dx\right)}{d}$$

$$=\frac{\text{Subst}\left(\int(a+bx)^{3/2}\cosh(x)\left(-\frac{c}{d}+\frac{\sinh(x)}{d}\right)dx,x,\sinh^{-1}(c+dx)\right)}{d}$$

$$=-\frac{2\,\text{Subst}\left(\int x^4\cosh\left(\frac{a-x^2}{b}\right)\left(c+\sinh\left(\frac{a-x^2}{b}\right)\right)dx,x,\sqrt{a+b\sinh^{-1}(c+dx)}\right)}{bd^2}$$

$$=-\frac{2\,\text{Subst}\left(\int x^4\cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)\left(c+\sinh\left(\frac{a-x^2}{b}\right)\right)dx,x,\sqrt{a+b\sinh^{-1}(c+dx)}\right.}{bd^2}$$

$$=-\frac{2\,\text{Subst}\left(\int\left(cx^4\cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)+\frac{1}{2}x^4\sinh\left(\frac{2a}{b}-\frac{2x^2}{b}\right)\right)dx,x,\sqrt{a+b\sinh^{-1}(\right.}{bd^2}$$

$$=-\frac{\text{Subst}\left(\int x^4\sinh\left(\frac{2a}{b}-\frac{2x^2}{b}\right)dx,x,\sqrt{a+b\sinh^{-1}(c+dx)}\right)}{bd^2}-\frac{(2c)\,\text{Subst}\left(\int\right.}{}$$

$$=-\frac{c(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{d^2}+\frac{\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}\cosh\left(2\sinh\right.}{4d^2}$$

$$=\frac{3bc\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{2d^2}-\frac{c(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{3}}{d^2}$$

$$=\frac{3bc\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{2d^2}-\frac{c(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{3}}{d^2}$$

$$=\frac{3bc\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{2d^2}-\frac{c(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{3}}{d^2}$$

Mathematica [A] time = 5.19, size = 582, normalized size = 1.79

$$\frac{-16\sqrt{b}\,c\left(\sqrt{\pi}\,(3b-2a)\left(\sinh\left(\frac{a}{b}\right)+\cosh\left(\frac{a}{b}\right)\right)\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)+\sqrt{\pi}\,(2a+3b)\left(\cosh\left(\frac{a}{b}\right)-\sinh\left(\frac{a}{b}\right)\right)\text{erfi}\left(\frac{\sqrt{}}{}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] ((-64*a*c*Sqrt[a + b*ArcSinh[c + d*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + Arc
Sinh[c + d*x]])/Sqrt[a/b + ArcSinh[c + d*x]]) + Gamma[3/2, -((a + b*ArcSinh
[c + d*x])/b)]/Sqrt[-((a + b*ArcSinh[c + d*x])/b)]))/E^(a/b) - 16*Sqrt[b]*c
*(4*Sqrt[b]*Sqrt[a + b*ArcSinh[c + d*x]]*(-3*Sqrt[1 + (c + d*x)^2] + 2*(c +
 d*x)*ArcSinh[c + d*x]) + (2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c +
d*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + (-2*a + 3*b)*Sqrt[Pi]*Erf[Sqrt[a +
 b*ArcSinh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])) + 4*a*(8*Sqrt[a + b*
ArcSinh[c + d*x]]*Cosh[2*ArcSinh[c + d*x]] + Sqrt[b]*Sqrt[2*Pi]*Erfi[(Sqrt[
2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*(-Cosh[(2*a)/b] + Sinh[(2*a)/b])
- Sqrt[b]*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*(C
osh[(2*a)/b] + Sinh[(2*a)/b])) + Sqrt[b]*((4*a + 3*b)*Sqrt[2*Pi]*Erfi[(Sqrt
[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b] - Sinh[(2*a)/b])
```

```
+ (4*a - 3*b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]
]*(Cosh[(2*a)/b] + Sinh[(2*a)/b]) + 8*Sqrt[b]*Sqrt[a + b*ArcSinh[c + d*x]]*
(4*ArcSinh[c + d*x]*Cosh[2*ArcSinh[c + d*x]] - 3*Sinh[2*ArcSinh[c + d*x]]))
)/(128*d^2)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\,\text{arsinh}\,(dx+c)+a)^{\frac{3}{2}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(3/2)*x, x)
```

maple [F] time = 0.50, size = 0, normalized size = 0.00

$$\int x\,(a+b\,\text{arcsinh}\,(dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsinh(d*x+c))^(3/2),x)
```

```
[Out] int(x*(a+b*arcsinh(d*x+c))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\,\text{arsinh}\,(dx+c)+a)^{\frac{3}{2}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(3/2)*x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\,(a+b\,\text{asinh}\,(c+d\,x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*asinh(c + d*x))^(3/2),x)
```

```
[Out] int(x*(a + b*asinh(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\,(a+b\,\text{asinh}\,(c+dx))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(d*x+c))**(3/2),x)
```

```
[Out] Integral(x*(a + b*asinh(c + d*x))**(3/2), x)
```

3.102 $\int \left(a+b\sinh^{-1}(c+dx)\right)^{3/2}\,dx$

Optimal. Leaf size=150

$$\frac{3\sqrt{\pi}\,b^{3/2}e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8d}+\frac{3\sqrt{\pi}\,b^{3/2}e^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8d}-\frac{3b\sqrt{(c+dx)^2+1}\,\sqrt{a+b\sinh^{-1}(c+dx)}}{2d}$$

```
[Out] (d*x+c)*(a+b*arcsinh(d*x+c))^(3/2)/d+3/8*b^(3/2)*exp(a/b)*erf((a+b*arcsinh(
d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d+3/8*b^(3/2)*erfi((a+b*arcsinh(d*x+c))^(1/
2)/b^(1/2))*Pi^(1/2)/d/exp(a/b)-3/2*b*(1+(d*x+c)^2)^(1/2)*(a+b*arcsinh(d*x+
c))^(1/2)/d
```

Rubi [A] time = 0.25, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5863, 5653, 5717, 5657, 3307, 2180, 2205, 2204}

$$\frac{3\sqrt{\pi}\,b^{3/2}e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8d}+\frac{3\sqrt{\pi}\,b^{3/2}e^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8d}-\frac{3b\sqrt{(c+dx)^2+1}\,\sqrt{a+b\sinh^{-1}(c+dx)}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (-3*b*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/(2*d) + ((c + d*x
)*(a + b*ArcSinh[c + d*x])^(3/2))/d + (3*b^(3/2)*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[
a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(8*d) + (3*b^(3/2)*Sqrt[Pi]*Erfi[Sqrt[a +
 b*ArcSinh[c + d*x]]/Sqrt[b]])/(8*d*E^(a/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
```

```
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5863

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \left(a+b\sinh^{-1}(c+dx)\right)^{3/2}\,dx=\frac{\text{Subst}\left(\int \left(a+b\sinh^{-1}(x)\right)^{3/2}\,dx,x,c+dx\right)}{d}$$

$$=\frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{d}-\frac{(3b)\,\text{Subst}\left(\int \frac{x\sqrt{a+b\sinh^{-1}(x)}}{\sqrt{1+x^2}}\,dx,x,c+d\right.}{2d}$$

$$=-\frac{3b\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{2d}+\frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)}{d}$$

$$=-\frac{3b\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{2d}+\frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)}{d}$$

$$=-\frac{3b\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{2d}+\frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)}{d}$$

$$=-\frac{3b\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{2d}+\frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)}{d}$$

$$=-\frac{3b\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{2d}+\frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)}{d}$$

Mathematica [A] time = 0.15, size = 272, normalized size = 1.81

$$\frac{\sqrt{b}\left(\sqrt{\pi}\,(3b-2a)\left(\sinh\left(\frac{a}{b}\right)+\cosh\left(\frac{a}{b}\right)\right)\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)+\sqrt{\pi}\,(2a+3b)\left(\cosh\left(\frac{a}{b}\right)-\sinh\left(\frac{a}{b}\right)\right)\text{erfi}\left(\frac{\sqrt{a+}}{}\right.\right.}{8d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (a*Sqrt[a + b*ArcSinh[c + d*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcSinh[c
+ d*x]])/Sqrt[a/b + ArcSinh[c + d*x]]) + Gamma[3/2, -((a + b*ArcSinh[c + d*
x])/b)]/Sqrt[-((a + b*ArcSinh[c + d*x])/b)]))/(2*d*E^(a/b)) + (Sqrt[b]*(4*S
qrt[b]*Sqrt[a + b*ArcSinh[c + d*x]]*(-3*Sqrt[1 + (c + d*x)^2] + 2*(c + d*x)
*ArcSinh[c + d*x]) + (2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]
/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + (-2*a + 3*b)*Sqrt[Pi]*Erf[Sqrt[a + b*Ar
cSinh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])))/(8*d)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arsinh}(dx + c) + a)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(3/2), x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{arcsinh}(dx + c))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^(3/2),x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arsinh}(dx + c) + a)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b \operatorname{asinh}(c + d\,x))^{3/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^(3/2),x)
```

```
[Out] int((a + b*asinh(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{asinh}(c + dx))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))**(3/2), x)
```

3.103 $\int x \left(a + b \sinh^{-1}(c + dx)\right)^{5/2} dx$

Optimal. Leaf size=389

$$-\frac{15\sqrt{\pi}\, b^{5/2} c e^{a/b} \operatorname{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d^2} - \frac{15\sqrt{\frac{\pi}{2}}\, b^{5/2} e^{\frac{2a}{b}} \operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{256d^2} + \frac{15\sqrt{\pi}\, b^{5/2} c e^{-\frac{a}{b}} \operatorname{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d^2}$$

```
[Out] -c*(d*x+c)*(a+b*arcsinh(d*x+c))^(5/2)/d^2+1/4*(a+b*arcsinh(d*x+c))^(5/2)*co
sh(2*arcsinh(d*x+c))/d^2-5/16*b*(a+b*arcsinh(d*x+c))^(3/2)*sinh(2*arcsinh(d
*x+c))/d^2-15/512*b^(5/2)*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)
/b^(1/2))*2^(1/2)*Pi^(1/2)/d^2-15/512*b^(5/2)*erfi(2^(1/2)*(a+b*arcsinh(d*x
+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d^2/exp(2*a/b)-15/16*b^(5/2)*c*exp(a/b
)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d^2+15/16*b^(5/2)*c*erfi
((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d^2/exp(a/b)+5/2*b*c*(a+b*arc
sinh(d*x+c))^(3/2)*(1+(d*x+c)^2)^(1/2)/d^2-15/4*b^2*c*(d*x+c)*(a+b*arcsinh(
d*x+c))^(1/2)/d^2+15/64*b^2*cosh(2*arcsinh(d*x+c))*(a+b*arcsinh(d*x+c))^(1/
2)/d^2
```

Rubi [A] time = 1.13, antiderivative size = 389, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5865, 5805, 6741, 6742, 5325, 5324, 5298, 2205, 2204, 5299}

$$-\frac{15\sqrt{\pi}\, b^{5/2} c e^{a/b} \operatorname{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d^2} - \frac{15\sqrt{\frac{\pi}{2}}\, b^{5/2} e^{\frac{2a}{b}} \operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{256d^2} + \frac{15\sqrt{\pi}\, b^{5/2} c e^{-\frac{a}{b}} \operatorname{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] (-15*b^2*c*(c + d*x)*Sqrt[a + b*ArcSinh[c + d*x]])/(4*d^2) + (5*b*c*Sqrt[1
+ (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^(3/2))/(2*d^2) - (c*(c + d*x)*(a +
b*ArcSinh[c + d*x])^(5/2))/d^2 + (15*b^2*Sqrt[a + b*ArcSinh[c + d*x]]*Cosh[
2*ArcSinh[c + d*x]])/(64*d^2) + ((a + b*ArcSinh[c + d*x])^(5/2)*Cosh[2*ArcS
inh[c + d*x]])/(4*d^2) - (15*b^(5/2)*c*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcS
inh[c + d*x]]/Sqrt[b]])/(16*d^2) - (15*b^(5/2)*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(
Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(256*d^2) + (15*b^(5/2)*c*S
qrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(16*d^2*E^(a/b)) - (15*
b^(5/2)*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(2
56*d^2*E^((2*a)/b)) - (5*b*(a + b*ArcSinh[c + d*x])^(3/2)*Sinh[2*ArcSinh[c
+ d*x]])/(16*d^2)
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5298

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
```

```
[n, 1]
```

Rule 5299

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5324

```
Int[((e_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cosh[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1
))/(d*n), Int[(e*x)^(m - n)*Cosh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[0, n, m + 1]
```

Rule 5325

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Sinh[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1
))/(d*n), Int[(e*x)^(m - n)*Sinh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[0, n, m + 1]
```

Rule 5805

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]*(c*d + e*Sinh[x
])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m, 0
]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}dx=\frac{\text{Subst}\left(\int\left(-\frac{c}{d}+\frac{x}{d}\right)\left(a+b\sinh^{-1}(x)\right)^{5/2}dx,x,c+dx\right)}{d}$$

$$=\frac{\text{Subst}\left(\int(a+bx)^{5/2}\cosh(x)\left(-\frac{c}{d}+\frac{\sinh(x)}{d}\right)dx,x,\sinh^{-1}(c+dx)\right)}{d}$$

$$=-\frac{2\,\text{Subst}\left(\int x^6\cosh\left(\frac{a-x^2}{b}\right)\left(c+\sinh\left(\frac{a-x^2}{b}\right)\right)dx,x,\sqrt{a+b\sinh^{-1}(c+dx)}\right)}{bd^2}$$

$$=-\frac{2\,\text{Subst}\left(\int x^6\cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)\left(c+\sinh\left(\frac{a-x^2}{b}\right)\right)dx,x,\sqrt{a+b\sinh^{-1}(c+dx)}\right.}{bd^2}$$

$$=-\frac{2\,\text{Subst}\left(\int\left(cx^6\cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)+\frac{1}{2}x^6\sinh\left(\frac{2a}{b}-\frac{2x^2}{b}\right)\right)dx,x,\sqrt{a+b\sinh^{-1}(\right.}{bd^2}$$

$$=-\frac{\text{Subst}\left(\int x^6\sinh\left(\frac{2a}{b}-\frac{2x^2}{b}\right)dx,x,\sqrt{a+b\sinh^{-1}(c+dx)}\right)}{bd^2}-\frac{(2c)\,\text{Subst}\left(\int\right.}{}$$

$$=-\frac{c(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}{d^2}+\frac{\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}\cosh\left(2\sinh\right.}{4d^2}$$

$$=\frac{5bc\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{2d^2}-\frac{c(c+dx)\left(a+b\sinh^{-1}(c+dx\right.}{d^2}$$

$$=-\frac{15b^2c(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{4d^2}+\frac{5bc\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+\right.}{2d^2}$$

$$=-\frac{15b^2c(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{4d^2}+\frac{5bc\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+\right.}{2d^2}$$

$$=-\frac{15b^2c(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{4d^2}+\frac{5bc\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+\right.}{2d^2}$$

Mathematica [B] time = 10.19, size = 939, normalized size = 2.41

$$\frac{480c\sqrt{\pi}\cosh\left(\frac{a}{b}\right)\text{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)b^{5/2}-15\sqrt{2\pi}\cosh\left(\frac{2a}{b}\right)\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)b^{5/2}-480c\sqrt{\pi}\,\text{erfi}\left(\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] (-1920*b^2*c^2*Sqrt[a + b*ArcSinh[c + d*x]] - 1920*b^2*c*d*x*Sqrt[a + b*Arc
Sinh[c + d*x]] + 1280*a*b*c*Sqrt[1 + c^2 + 2*c*d*x + d^2*x^2]*Sqrt[a + b*Ar
cSinh[c + d*x]] - 1024*a*b*c^2*ArcSinh[c + d*x]*Sqrt[a + b*ArcSinh[c + d*x]
] - 1024*a*b*c*d*x*ArcSinh[c + d*x]*Sqrt[a + b*ArcSinh[c + d*x]] + 1280*b^2
*c*Sqrt[1 + c^2 + 2*c*d*x + d^2*x^2]*ArcSinh[c + d*x]*Sqrt[a + b*ArcSinh[c
+ d*x]] - 512*b^2*c^2*ArcSinh[c + d*x]^2*Sqrt[a + b*ArcSinh[c + d*x]] - 512
*b^2*c*d*x*ArcSinh[c + d*x]^2*Sqrt[a + b*ArcSinh[c + d*x]] + 128*a^2*Sqrt[a
 + b*ArcSinh[c + d*x]]*Cosh[2*ArcSinh[c + d*x]] + 120*b^2*Sqrt[a + b*ArcSin
h[c + d*x]]*Cosh[2*ArcSinh[c + d*x]] + 256*a*b*ArcSinh[c + d*x]*Sqrt[a + b*
```

```
ArcSinh[c + d*x]]*Cosh[2*ArcSinh[c + d*x]] + 128*b^2*ArcSinh[c + d*x]^2*Sqr
t[a + b*ArcSinh[c + d*x]]*Cosh[2*ArcSinh[c + d*x]] - 128*a^2*Sqrt[b]*c*Sqrt
[Pi]*Cosh[a/b]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]] + 480*b^(5/2)*c*S
qrt[Pi]*Cosh[a/b]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]] - 15*b^(5/2)*S
qrt[2*Pi]*Cosh[(2*a)/b]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]
] + (256*a^2*b*c*E^(a/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[3/2, a/b + ArcS
inh[c + d*x]])/Sqrt[a + b*ArcSinh[c + d*x]] + (256*a^2*b*c*Sqrt[-((a + b*Ar
cSinh[c + d*x])/b)]*Gamma[3/2, -((a + b*ArcSinh[c + d*x])/b)])/(E^(a/b)*Sqr
t[a + b*ArcSinh[c + d*x]]) + 128*a^2*Sqrt[b]*c*Sqrt[Pi]*Erfi[Sqrt[a + b*Arc
Sinh[c + d*x]]/Sqrt[b]]*Sinh[a/b] - 480*b^(5/2)*c*Sqrt[Pi]*Erfi[Sqrt[a + b*
ArcSinh[c + d*x]]/Sqrt[b]]*Sinh[a/b] + 32*Sqrt[b]*(4*a^2 - 15*b^2)*c*Sqrt[P
i]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b]) + 15*b
^(5/2)*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*Sinh
[(2*a)/b] - 15*b^(5/2)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]]
)/Sqrt[b]]*(Cosh[(2*a)/b] + Sinh[(2*a)/b]) - 160*a*b*Sqrt[a + b*ArcSinh[c +
 d*x]]*Sinh[2*ArcSinh[c + d*x]] - 160*b^2*ArcSinh[c + d*x]*Sqrt[a + b*ArcSi
nh[c + d*x]]*Sinh[2*ArcSinh[c + d*x]])/(512*d^2)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arsinh}(dx + c) + a)^{\frac{5}{2}} x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(5/2)*x, x)
```

maple [F] time = 0.51, size = 0, normalized size = 0.00

$$\int x (a + b \operatorname{arcsinh}(dx + c))^{\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsinh(d*x+c))^(5/2),x)
```

```
[Out] int(x*(a+b*arcsinh(d*x+c))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arsinh}(dx + c) + a)^{\frac{5}{2}} x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(5/2)*x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x \, (a + b \operatorname{asinh}(c + d\, x))^{5/2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*asinh(c + d*x))^(5/2),x)
```

```
[Out] int(x*(a + b*asinh(c + d*x))^(5/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+b\operatorname{asinh}\left(c+dx\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(d*x+c))**(5/2),x)
```

```
[Out] Integral(x*(a + b*asinh(c + d*x))**(5/2), x)
```

3.104 $\int \left(a+b\sinh^{-1}(c+dx)\right)^{5/2}\,dx$

Optimal. Leaf size=179

$$\frac{15\sqrt{\pi}\,b^{5/2}e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}-\frac{15\sqrt{\pi}\,b^{5/2}e^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}+\frac{15b^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{4d}$$

```
[Out] (d*x+c)*(a+b*arcsinh(d*x+c))^(5/2)/d+15/16*b^(5/2)*exp(a/b)*erf((a+b*arcsin
h(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d-15/16*b^(5/2)*erfi((a+b*arcsinh(d*x+c))
^(1/2)/b^(1/2))*Pi^(1/2)/d/exp(a/b)-5/2*b*(a+b*arcsinh(d*x+c))^(3/2)*(1+(d*
x+c)^2)^(1/2)/d+15/4*b^2*(d*x+c)*(a+b*arcsinh(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.39, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5863, 5653, 5717, 5779, 3308, 2180, 2204, 2205}

$$\frac{15\sqrt{\pi}\,b^{5/2}e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}-\frac{15\sqrt{\pi}\,b^{5/2}e^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}+\frac{15b^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] (15*b^2*(c + d*x)*Sqrt[a + b*ArcSinh[c + d*x]])/(4*d) - (5*b*Sqrt[1 + (c +
d*x)^2]*(a + b*ArcSinh[c + d*x])^(3/2))/(2*d) + ((c + d*x)*(a + b*ArcSinh[c
 + d*x])^(5/2))/d + (15*b^(5/2)*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c +
 d*x]]/Sqrt[b]])/(16*d) - (15*b^(5/2)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c +
d*x]]/Sqrt[b]])/(16*d*E^(a/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
```

```
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 5863

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \left(a+b\sinh^{-1}(c+dx)\right)^{5/2}\,dx=\frac{\text{Subst}\left(\int \left(a+b\sinh^{-1}(x)\right)^{5/2}\,dx,x,c+dx\right)}{d}$$

$$=\frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}{d}-\frac{(5b)\,\text{Subst}\left(\int \frac{x\left(a+b\sinh^{-1}(x)\right)^{3/2}}{\sqrt{1+x^2}}\,dx,x,c+dx\right.}{2d}$$

$$=-\frac{5b\sqrt{1+(c+dx)^2}\,\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{2d}+\frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{5}}{d}$$

$$=\frac{15b^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{4d}-\frac{5b\sqrt{1+(c+dx)^2}\,\left(a+b\sinh^{-1}(c+dx)\right)}{2d}$$

$$=\frac{15b^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{4d}-\frac{5b\sqrt{1+(c+dx)^2}\,\left(a+b\sinh^{-1}(c+dx)\right)}{2d}$$

$$=\frac{15b^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{4d}-\frac{5b\sqrt{1+(c+dx)^2}\,\left(a+b\sinh^{-1}(c+dx)\right)}{2d}$$

$$=\frac{15b^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{4d}-\frac{5b\sqrt{1+(c+dx)^2}\,\left(a+b\sinh^{-1}(c+dx)\right)}{2d}$$

$$=\frac{15b^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{4d}-\frac{5b\sqrt{1+(c+dx)^2}\,\left(a+b\sinh^{-1}(c+dx)\right)}{2d}$$

Mathematica [B] time = 2.45, size = 458, normalized size = 2.56

$$\frac{\sqrt{b}\left(\sqrt{\pi}\left(4a^2-12ab+15b^2\right)\left(\sinh\left(\frac{a}{b}\right)+\cosh\left(\frac{a}{b}\right)\right)\operatorname{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)+\sqrt{\pi}\left(4a^2+12ab+15b^2\right)\left(\sinh\left(\frac{a}{b}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] ((8*a^2*Sqrt[a + b*ArcSinh[c + d*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcSi
nh[c + d*x]])/Sqrt[a/b + ArcSinh[c + d*x]]) + Gamma[3/2, -((a + b*ArcSinh[c
 + d*x])/b)]/Sqrt[-((a + b*ArcSinh[c + d*x])/b)]))/E^(a/b) + 4*a*Sqrt[b]*(4
*Sqrt[b]*Sqrt[a + b*ArcSinh[c + d*x]]*(-3*Sqrt[1 + (c + d*x)^2] + 2*(c + d*
x)*ArcSinh[c + d*x]) + (2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x
]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + (-2*a + 3*b)*Sqrt[Pi]*Erf[Sqrt[a + b*
ArcSinh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])) + Sqrt[b]*(4*Sqrt[b]*Sq
rt[a + b*ArcSinh[c + d*x]]*(2*Sqrt[1 + (c + d*x)^2]*(a - 5*b*ArcSinh[c + d*
x]) + b*(c + d*x)*(15 + 4*ArcSinh[c + d*x]^2)) + (4*a^2 + 12*a*b + 15*b^2)*
Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]*(-Cosh[a/b] + Sinh[a/b]
) + (4*a^2 - 12*a*b + 15*b^2)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqr
t[b]]*(Cosh[a/b] + Sinh[a/b])))/(16*d)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arsinh}(dx+c)+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(5/2), x)
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int (a+b \operatorname{arcsinh}(dx+c))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^(5/2),x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arsinh}(dx+c)+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b\,\mathrm{asinh}\,(c + d\,x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^(5/2),x)
```

```
[Out] int((a + b*asinh(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\,\mathrm{asinh}\,(c + dx))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**(5/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))**(5/2), x)
```

3.105 $\int \frac{x^2}{\sqrt{a+b\sinh^{-1}(c+dx)}}\,dx$

Optimal. Leaf size=411

$$\frac{\sqrt{\pi}\,c^2e^{a/b}\mathrm{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d^3}+\frac{\sqrt{\pi}\,c^2e^{-\frac{a}{b}}\mathrm{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d^3}+\frac{\sqrt{\frac{\pi}{2}}\,ce^{\frac{2a}{b}}\mathrm{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d^3}-\frac{\sqrt{\pi}\,e^{a/b}\mathrm{e}}{}$$

```
[Out] 1/24*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^
(1/2)/d^3/b^(1/2)+1/24*erfi(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(
1/2)*Pi^(1/2)/d^3/exp(3*a/b)/b^(1/2)+1/4*c*exp(2*a/b)*erf(2^(1/2)*(a+b*arcs
inh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d^3/b^(1/2)-1/4*c*erfi(2^(1/2)*
(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d^3/exp(2*a/b)/b^(1/2)
-1/8*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d^3/b^(1/2)+
1/2*c^2*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d^3/b^(1/
2)-1/8*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d^3/exp(a/b)/b^(1/
2)+1/2*c^2*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d^3/exp(a/b)/b
^(1/2)
```

Rubi [A] time = 0.86, antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5865, 5805, 6741, 6742, 5299, 2205, 2204, 5298, 5618}

$$\frac{\sqrt{\pi}\,c^2e^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d^3}+\frac{\sqrt{\pi}\,c^2e^{-\frac{a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d^3}+\frac{\sqrt{\frac{\pi}{2}}\,ce^{\frac{2a}{b}}\mathrm{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d^3}-\frac{\sqrt{\pi}\,e^{a/b}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] -(E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(8*Sqrt[b]*d^
3) + (c^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(2*Sq
rt[b]*d^3) + (c*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c +
d*x]])/Sqrt[b]])/(2*Sqrt[b]*d^3) + (E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqr
t[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(8*Sqrt[b]*d^3) - (Sqrt[Pi]*Erfi[Sqrt[
a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(8*Sqrt[b]*d^3*E^(a/b)) + (c^2*Sqrt[Pi]*E
rfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(2*Sqrt[b]*d^3*E^(a/b)) - (c*Sqr
t[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(2*Sqrt[b]*d^
3*E^((2*a)/b)) + (Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sq
rt[b]])/(8*Sqrt[b]*d^3*E^((3*a)/b))
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5298

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
```

```
[n, 1]
```

Rule 5299

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5618

```
Int[Cosh[w_]^(q_.)*Sinh[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rule 5805

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]*(c*d + e*Sinh[x
])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m, 0
]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a+b\sinh^{-1}(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(-\frac{c}{d}+\frac{x}{d}\right)^2}{\sqrt{a+b\sinh^{-1}(x)}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\cosh(x)\left(-\frac{c}{d}+\frac{\sinh(x)}{d}\right)^2}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx)\right)}{d}$$

$$= \frac{2\,\text{Subst}\left(\int \cosh\left(\frac{a-x^2}{b}\right)\left(c+\sinh\left(\frac{a-x^2}{b}\right)\right)^2\,dx, x, \sqrt{a+b\sinh^{-1}(c+dx)}\right)}{bd^3}$$

$$= \frac{2\,\text{Subst}\left(\int \cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)\left(c+\sinh\left(\frac{a-x^2}{b}\right)\right)^2\,dx, x, \sqrt{a+b\sinh^{-1}(c+dx)}\right)}{bd^3}$$

$$= \frac{2\,\text{Subst}\left(\int \left(c^2\cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)+c\sinh\left(\frac{2a}{b}-\frac{2x^2}{b}\right)+\cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{x}{b}\right.\right.}{bd^3}$$

$$= \frac{2\,\text{Subst}\left(\int \cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{x^2}{b}\right)dx, x, \sqrt{a+b\sinh^{-1}(c+dx)}\right)}{bd^3} + \frac{(2c)}{}$$

$$= \frac{2\,\text{Subst}\left(\int \left(\frac{1}{4}\cosh\left(\frac{3a}{b}-\frac{3x^2}{b}\right)-\frac{1}{4}\cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)\right)dx, x, \sqrt{a+b\sinh^{-1}(c+dx}\right.}{bd^3}$$

$$= \frac{c^2e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d^3} + \frac{ce^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d^3} + \frac{c^2e^{-\frac{a}{b}}\sqrt{}}{}$$

$$= \frac{c^2e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d^3} + \frac{ce^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d^3} + \frac{c^2e^{-\frac{a}{b}}\sqrt{}}{}$$

$$= -\frac{e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,d^3} + \frac{c^2e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d^3} + \frac{ce^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{er}}{}$$

Mathematica [A] time = 1.10, size = 471, normalized size = 1.15

$$\frac{\sqrt{\pi}\left(3\left(4c^2-1\right)\left(\sinh\left(\frac{a}{b}\right)+\cosh\left(\frac{a}{b}\right)\right)\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)-12c^2\sinh\left(\frac{a}{b}\right)\text{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)+12c^2\cos\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (Sqrt[Pi]*(Sqrt[3]*Cosh[(3*a)/b]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])
/Sqrt[b]] - 3*Cosh[a/b]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]] + 12*c^2
*Cosh[a/b]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]] - 6*Sqrt[2]*c*Cosh[(2
*a)/b]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]] + Sqrt[3]*Cosh[
(3*a)/b]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]] + 3*Erfi[Sqrt
[a + b*ArcSinh[c + d*x]]/Sqrt[b]]*Sinh[a/b] - 12*c^2*Erfi[Sqrt[a + b*ArcSin
h[c + d*x]]/Sqrt[b]]*Sinh[a/b] + 3*(-1 + 4*c^2)*Erf[Sqrt[a + b*ArcSinh[c +
d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b]) + 6*Sqrt[2]*c*Erfi[(Sqrt[2]*Sqrt[a +
```

```
 b*ArcSinh[c + d*x]])/Sqrt[b]]*Sinh[(2*a)/b] + 6*Sqrt[2]*c*Erf[(Sqrt[2]*Sqr
t[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b] + Sinh[(2*a)/b]) + Sqrt[
3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*Sinh[(3*a)/b] - Sqrt
[3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*Sinh[(3*a)/b]))/(2
4*Sqrt[b]*d^3)
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{b \operatorname{arsinh}(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt(b*arcsinh(d*x + c) + a), x)
```

maple [**F(-2)**] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a+b \operatorname{arcsinh}(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*arcsinh(d*x+c))^(1/2),x)
```

```
[Out] int(x^2/(a+b*arcsinh(d*x+c))^(1/2),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{b \operatorname{arsinh}(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(b*arcsinh(d*x + c) + a), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\sqrt{a+b \operatorname{asinh}(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*asinh(c + d*x))^(1/2),x)
```

```
[Out] int(x^2/(a + b*asinh(c + d*x))^(1/2), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a+b \operatorname{asinh}(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*asinh(d*x+c))**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(a + b*asinh(c + d*x)), x)
```

3.106 $\int \frac{x}{\sqrt{a+b\sinh^{-1}(c+dx)}}\,dx$

Optimal. Leaf size=204

$$-\frac{\sqrt{\pi}\,ce^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d^2}-\frac{\sqrt{\frac{\pi}{2}}\,e^{\frac{2a}{b}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4\sqrt{b}\,d^2}-\frac{\sqrt{\pi}\,ce^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d^2}+\frac{\sqrt{\frac{\pi}{2}}\,e^{-\frac{2a}{b}}\operatorname{erfi}\left(\right.}{}$$

```
[Out] -1/8*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^
(1/2)/d^2/b^(1/2)+1/8*erfi(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(1
/2)*Pi^(1/2)/d^2/exp(2*a/b)/b^(1/2)-1/2*c*exp(a/b)*erf((a+b*arcsinh(d*x+c))
^(1/2)/b^(1/2))*Pi^(1/2)/d^2/b^(1/2)-1/2*c*erfi((a+b*arcsinh(d*x+c))^(1/2)/
b^(1/2))*Pi^(1/2)/d^2/exp(a/b)/b^(1/2)
```

Rubi [A] time = 0.39, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5865, 5805, 6741, 6742, 5299, 2205, 2204, 5298}

$$-\frac{\sqrt{\pi}\,ce^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d^2}-\frac{\sqrt{\frac{\pi}{2}}\,e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4\sqrt{b}\,d^2}-\frac{\sqrt{\pi}\,ce^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d^2}+\frac{\sqrt{\frac{\pi}{2}}\,e^{-\frac{2a}{b}}\operatorname{Erfi}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] -(c*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(2*Sqrt[b]*
d^2) - (E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/S
qrt[b]])/(4*Sqrt[b]*d^2) - (c*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sq
rt[b]])/(2*Sqrt[b]*d^2*E^(a/b)) + (Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcS
inh[c + d*x]])/Sqrt[b]])/(4*Sqrt[b]*d^2*E^((2*a)/b))
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5298

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5299

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5805

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]*(c*d + e*Sinh[x
])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m, 0
]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{\sqrt{a+b\sinh^{-1}(c+dx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{-\frac{c}{d}+\frac{x}{d}}{\sqrt{a+b\sinh^{-1}(x)}}\,dx, x, c+dx\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{\cosh(x)\left(-\frac{c}{d}+\frac{\sinh(x)}{d}\right)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx)\right)}{d} \\
&= -\frac{2\,\text{Subst}\left(\int \cosh\left(\frac{a-x^2}{b}\right)\left(c+\sinh\left(\frac{a-x^2}{b}\right)\right)dx, x, \sqrt{a+b\sinh^{-1}(c+dx)}\right)}{bd^2} \\
&= -\frac{2\,\text{Subst}\left(\int \cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)\left(c+\sinh\left(\frac{a-x^2}{b}\right)\right)dx, x, \sqrt{a+b\sinh^{-1}(c+dx)}\right)}{bd^2} \\
&= -\frac{2\,\text{Subst}\left(\int \left(c\cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)+\frac{1}{2}\sinh\left(\frac{2a}{b}-\frac{2x^2}{b}\right)\right)dx, x, \sqrt{a+b\sinh^{-1}(c+d}\right.}{bd^2} \\
&= -\frac{\text{Subst}\left(\int \sinh\left(\frac{2a}{b}-\frac{2x^2}{b}\right)dx, x, \sqrt{a+b\sinh^{-1}(c+dx)}\right)}{bd^2} - \frac{(2c)\,\text{Subst}\left(\int \cos\right.}{} \\
&= -\frac{\text{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(c+dx)}\right)}{2bd^2} + \frac{\text{Subst}\left(\int e^{-\frac{2a}{b}+\frac{2x^2}{b}}\,dx, x, \sqrt{}\right.}{2bd^2} \\
&= -\frac{ce^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d^2} - \frac{e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4\sqrt{b}\,d^2} - \frac{ce^{-\frac{a}{b}}\sqrt{\pi}}{}
\end{aligned}
$$

Mathematica [A] time = 1.02, size = 217, normalized size = 1.06

$$\frac{\frac{e^{-\frac{a}{b}}\left(4ce^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)-4c\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)\right)}{\sqrt{a+b\sinh^{-1}(c+dx)}}-\frac{\sqrt{2\pi}\left(\left(\sinh\left(\frac{2a}{b}\right)+\cosh\left(\frac{2a}{b}\right)\right)\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh}}{\sqrt{b}}\right.\right.}{}}{8d^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] ((4*c*E^((2*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2, a/b + ArcSinh[c +
 d*x]] - 4*c*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/2, -((a + b*ArcSin
h[c + d*x])/b)])/(E^(a/b)*Sqrt[a + b*ArcSinh[c + d*x]]) - (Sqrt[2*Pi]*(Erfi
[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*(-Cosh[(2*a)/b] + Sinh[(2*
a)/b]) + Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b]
 + Sinh[(2*a)/b])))/Sqrt[b])/(8*d^2)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{b\,\text{arsinh}\,(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt(b*arcsinh(d*x + c) + a), x)
```

maple [F] time = 0.49, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a+b\,\text{arcsinh}\,(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*arcsinh(d*x+c))^(1/2),x)
```

```
[Out] int(x/(a+b*arcsinh(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{b\,\text{arsinh}\,(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(b*arcsinh(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{a+b\,\text{asinh}\,(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*asinh(c + d*x))^(1/2),x)
```

```
[Out] int(x/(a + b*asinh(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a + b \operatorname{asinh}(c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*asinh(d*x+c))**(1/2),x)
```

```
[Out] Integral(x/sqrt(a + b*asinh(c + d*x)), x)
```

3.107 $\int \frac{1}{\sqrt{a+b\sinh^{-1}(c+dx)}}\,dx$

Optimal. Leaf size=92

$$\frac{\sqrt{\pi}\,e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d}+\frac{\sqrt{\pi}\,e^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d}$$

```
[Out] 1/2*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/b^(1/2)+1/2
*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/exp(a/b)/b^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5863, 5657, 3307, 2180, 2205, 2204}

$$\frac{\sqrt{\pi}\,e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d}+\frac{\sqrt{\pi}\,e^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(2*Sqrt[b]*d)
+ (Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(2*Sqrt[b]*d*E^(a/b
))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 5863

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\sinh^{-1}(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+b\sinh^{-1}(x)}}\,dx, x, c+dx\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\sinh^{-1}(c+dx)\right)}{bd}$$
$$= \frac{\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\sinh^{-1}(c+dx)\right)}{2bd} + \frac{\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\right.}{2bd}$$
$$= \frac{\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(c+dx)}\right)}{bd} + \frac{\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\right.}{bd}$$
$$= \frac{e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d} + \frac{e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d}$$

Mathematica [A] time = 0.11, size = 111, normalized size = 1.21

$$\frac{e^{-\frac{a}{b}}\left(\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)-e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)}{2d\sqrt{a+b\sinh^{-1}(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (-(E^((2*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2, a/b + ArcSinh[c + d*
x]]) + Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c +
d*x])/b)])/(2*d*E^(a/b)*Sqrt[a + b*ArcSinh[c + d*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\,\text{arsinh}\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*arcsinh(d*x + c) + a), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b \operatorname{arcsinh}(dx + c)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(d*x+c))^(1/2),x)
```

```
[Out] int(1/(a+b*arcsinh(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b \operatorname{arsinh}(dx + c) + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*arcsinh(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a + b \operatorname{asinh}(c + d\,x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*asinh(c + d*x))^(1/2),x)
```

```
[Out] int(1/(a + b*asinh(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b \operatorname{asinh}(c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*asinh(c + d*x)), x)
```

3.108 $\int \frac{x}{\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}\,dx$

Optimal. Leaf size=269

$$\frac{\sqrt{\pi}\,ce^{a/b}\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d^2}+\frac{\sqrt{\frac{\pi}{2}}\,e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d^2}-\frac{\sqrt{\pi}\,ce^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d^2}+\frac{\sqrt{\frac{\pi}{2}}\,e^{-\frac{2a}{b}}\text{erfi}}{}$$

```
[Out] 1/2*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(
1/2)/b^(3/2)/d^2+1/2*erfi(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(1/
2)*Pi^(1/2)/b^(3/2)/d^2/exp(2*a/b)+c*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2
)/b^(1/2))*Pi^(1/2)/b^(3/2)/d^2-c*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*
Pi^(1/2)/b^(3/2)/d^2/exp(a/b)+2*c*(1+(d*x+c)^2)^(1/2)/b/d^2/(a+b*arcsinh(d*
x+c))^(1/2)-2*(d*x+c)*(1+(d*x+c)^2)^(1/2)/b/d^2/(a+b*arcsinh(d*x+c))^(1/2)
```

Rubi [A] time = 0.54, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5865, 5803, 5655, 5779, 3308, 2180, 2204, 2205, 5665, 3307}

$$\frac{\sqrt{\pi}\,ce^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d^2}+\frac{\sqrt{\frac{\pi}{2}}\,e^{\frac{2a}{b}}\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d^2}-\frac{\sqrt{\pi}\,ce^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d^2}+\frac{\sqrt{\frac{\pi}{2}}\,e^{-\frac{2a}{b}}\text{Er}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (2*c*Sqrt[1 + (c + d*x)^2])/(b*d^2*Sqrt[a + b*ArcSinh[c + d*x]]) - (2*(c +
d*x)*Sqrt[1 + (c + d*x)^2])/(b*d^2*Sqrt[a + b*ArcSinh[c + d*x]]) + (c*E^(a/
b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(b^(3/2)*d^2) + (E^(
(2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(b
^(3/2)*d^2) - (c*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(b^(3
/2)*d^2*E^(a/b)) + (Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/
Sqrt[b]])/(b^(3/2)*d^2*E^((2*a)/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
```

```
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 5803

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*ArcSinh[c*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[m, 0] && LtQ[n, -1]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{x}{\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{-\frac{c}{d}+\frac{x}{d}}{\left(a+b\sinh^{-1}(x)\right)^{3/2}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{c}{d\left(a+b\sinh^{-1}(x)\right)^{3/2}}+\frac{x}{d\left(a+b\sinh^{-1}(x)\right)^{3/2}}\right)dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{x}{\left(a+b\sinh^{-1}(x)\right)^{3/2}}\,dx, x, c+dx\right)}{d^2} - \frac{c\,\text{Subst}\left(\int \frac{1}{\left(a+b\sinh^{-1}(x)\right)^{3/2}}\,dx, x, c+\right.}{d^2}$$

$$= \frac{2c\sqrt{1+(c+dx)^2}}{bd^2\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{bd^2\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{2\,\text{Subst}\left(\int \frac{\cosh(2}{\sqrt{a+b}}\right.}{}$$

$$= \frac{2c\sqrt{1+(c+dx)^2}}{bd^2\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{bd^2\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{a+bx}}\right.}{}$$

$$= \frac{2c\sqrt{1+(c+dx)^2}}{bd^2\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{bd^2\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{2\,\text{Subst}\left(\int e^{\frac{2a}{b}-2}\right.}{}$$

$$= \frac{2c\sqrt{1+(c+dx)^2}}{bd^2\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{bd^2\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{}}{}\right.}{b^{3/2}}$$

$$= \frac{2c\sqrt{1+(c+dx)^2}}{bd^2\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{bd^2\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{ce^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+}}{}\right.}{b^{3/2}d}$$

Mathematica [A] time = 2.69, size = 301, normalized size = 1.12

$$\frac{\sqrt{2\pi}\left(\sinh\left(\frac{2a}{b}\right)+\cosh\left(\frac{2a}{b}\right)\right)\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)+\sqrt{2\pi}\left(\cosh\left(\frac{2a}{b}\right)-\sinh\left(\frac{2a}{b}\right)\right)\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx}}{\sqrt{b}}\right.}{2b^{3/2}d^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] -((c*(-(E^(a/b)*(1 + E^(2*ArcSinh[c + d*x]))) + E^((2*a)/b + ArcSinh[c + d*
x])*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2, a/b + ArcSinh[c + d*x]] + E^Arc
Sinh[c + d*x]*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/2, -((a + b*ArcSi
nh[c + d*x])/b)]))/(b*d^2*E^((a + b*ArcSinh[c + d*x])/b)*Sqrt[a + b*ArcSinh
[c + d*x]])) + (Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt
[b]]*(Cosh[(2*a)/b] - Sinh[(2*a)/b]) + Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*A
rcSinh[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b] + Sinh[(2*a)/b]) - (2*Sqrt[b]*Sin
h[2*ArcSinh[c + d*x]])/Sqrt[a + b*ArcSinh[c + d*x]])/(2*b^(3/2)*d^2)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(b\,\mathrm{arsinh}\,(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x/(b*arcsinh(d*x + c) + a)^(3/2), x)
```

maple [F]   time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{x}{(a+b\,\mathrm{arcsinh}\,(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*arcsinh(d*x+c))^(3/2),x)
```

```
[Out] int(x/(a+b*arcsinh(d*x+c))^(3/2),x)
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(b\,\mathrm{arsinh}\,(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(b*arcsinh(d*x + c) + a)^(3/2), x)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*asinh(c + d*x))^(3/2),x)
```

```
[Out] int(x/(a + b*asinh(c + d*x))^(3/2), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(a+b\,\mathrm{asinh}\,(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*asinh(d*x+c))**(3/2),x)
```

```
[Out] Integral(x/(a + b*asinh(c + d*x))**(3/2), x)
```

3.109 $\int \frac{1}{\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}\,dx$

Optimal. Leaf size=122

$$-\frac{\sqrt{\pi}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}+\frac{\sqrt{\pi}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}-\frac{2\sqrt{(c+dx)^2+1}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}}$$

```
[Out] -exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/d+erfi((
a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/d/exp(a/b)-2*(1+(d*x+c)
^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5863, 5655, 5779, 3308, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}+\frac{\sqrt{\pi}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}-\frac{2\sqrt{(c+dx)^2+1}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^(-3/2),x]
```

```
[Out] (-2*Sqrt[1 + (c + d*x)^2])/(b*d*Sqrt[a + b*ArcSinh[c + d*x]]) - (E^(a/b)*Sq
rt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(b^(3/2)*d) + (Sqrt[Pi]*E
rfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(b^(3/2)*d*E^(a/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
```

```
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 5863

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+b\sinh^{-1}(x)\right)^{3/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{2\,\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\sqrt{a+b\sinh^{-1}(x)}}\,dx, x, c+dx\right)}{bd}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{2\,\text{Subst}\left(\int \frac{\sinh(x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx)\right)}{bd}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx)\right)}{bd} + \frac{\text{Subst}\left(\right.}{}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{2\,\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(c+dx)}\right)}{b^2d} -$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d} + \frac{e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+}}{}\right.}{b^{3/2}d}$$

Mathematica [A] time = 0.10, size = 155, normalized size = 1.27

$$\frac{e^{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\left(-e^{a/b}\left(e^{2\sinh^{-1}(c+dx)}+1\right)+e^{\frac{2a}{b}+\sinh^{-1}(c+dx)}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)+e^{\sinh^{-1}(}\right.}{bd\sqrt{a+b\sinh^{-1}(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^(-3/2),x]
```

```
[Out] (-(E^(a/b)*(1 + E^(2*ArcSinh[c + d*x]))) + E^((2*a)/b + ArcSinh[c + d*x])*S
qrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2, a/b + ArcSinh[c + d*x]] + E^ArcSinh[
```

```
c + d*x]*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c
+ d*x])/b)])/(b*d*E^((a + b*ArcSinh[c + d*x])/b)*Sqrt[a + b*ArcSinh[c + d*x
]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \operatorname{arsinh}(dx + c) + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(-3/2), x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b \operatorname{arcsinh}(dx + c))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(d*x+c))^(3/2),x)
```

```
[Out] int(1/(a+b*arcsinh(d*x+c))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \operatorname{arsinh}(dx + c) + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a + b \operatorname{asinh}(c + d\,x))^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*asinh(c + d*x))^(3/2),x)
```

```
[Out] int(1/(a + b*asinh(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b \operatorname{asinh}(c + dx))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(d*x+c))**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))**(-3/2), x)
```

3.110 $\int \frac{x}{\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}\,dx$

Optimal. Leaf size=365

$$-\frac{2\sqrt{\pi}\,ce^{a/b}\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d^2}-\frac{2\sqrt{2\pi}\,e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d^2}-\frac{2\sqrt{\pi}\,ce^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d^2}+\frac{2\sqrt{2\pi}}{}$$

```
[Out] -2/3*c*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(5/2)/d^
2-2/3*c*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(5/2)/d^2/exp(a
/b)-2/3*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*
Pi^(1/2)/b^(5/2)/d^2+2/3*erfi(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2
^(1/2)*Pi^(1/2)/b^(5/2)/d^2/exp(2*a/b)+2/3*c*(1+(d*x+c)^2)^(1/2)/b/d^2/(a+b
*arcsinh(d*x+c))^(3/2)-2/3*(d*x+c)*(1+(d*x+c)^2)^(1/2)/b/d^2/(a+b*arcsinh(d
*x+c))^(3/2)-4/3/b^2/d^2/(a+b*arcsinh(d*x+c))^(1/2)+4/3*c*(d*x+c)/b^2/d^2/(
a+b*arcsinh(d*x+c))^(1/2)-8/3*(d*x+c)^2/b^2/d^2/(a+b*arcsinh(d*x+c))^(1/2)
```

Rubi [A] time = 0.88, antiderivative size = 365, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 15, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.938, Rules used = {5865, 5803, 5655, 5774, 5657, 3307, 2180, 2205, 2204, 5667, 5669, 5448, 12, 3308, 5675}

$$-\frac{2\sqrt{\pi}\,ce^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d^2}-\frac{2\sqrt{2\pi}\,e^{\frac{2a}{b}}\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d^2}-\frac{2\sqrt{\pi}\,ce^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d^2}+\frac{2\sqrt{2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] (2*c*Sqrt[1 + (c + d*x)^2])/(3*b*d^2*(a + b*ArcSinh[c + d*x])^(3/2)) - (2*(
c + d*x)*Sqrt[1 + (c + d*x)^2])/(3*b*d^2*(a + b*ArcSinh[c + d*x])^(3/2)) -
4/(3*b^2*d^2*Sqrt[a + b*ArcSinh[c + d*x]]) + (4*c*(c + d*x))/(3*b^2*d^2*Sqr
t[a + b*ArcSinh[c + d*x]]) - (8*(c + d*x)^2)/(3*b^2*d^2*Sqrt[a + b*ArcSinh[
c + d*x]]) - (2*c*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]
])/(3*b^(5/2)*d^2) - (2*E^((2*a)/b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcS
inh[c + d*x]])/Sqrt[b]])/(3*b^(5/2)*d^2) - (2*c*Sqrt[Pi]*Erfi[Sqrt[a + b*Ar
cSinh[c + d*x]]/Sqrt[b]])/(3*b^(5/2)*d^2*E^(a/b)) + (2*Sqrt[2*Pi]*Erfi[(Sqr
t[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(3*b^(5/2)*d^2*E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5803

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*ArcSinh[c*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[m, 0] && LtQ[n, -1]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{x}{\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{-\frac{c}{d}+\frac{x}{d}}{\left(a+b\sinh^{-1}(x)\right)^{5/2}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{c}{d\left(a+b\sinh^{-1}(x)\right)^{5/2}}+\frac{x}{d\left(a+b\sinh^{-1}(x)\right)^{5/2}}\right)dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{x}{\left(a+b\sinh^{-1}(x)\right)^{5/2}}\,dx, x, c+dx\right)}{d^2} - \frac{c\,\text{Subst}\left(\int \frac{1}{\left(a+b\sinh^{-1}(x)\right)^{5/2}}\,dx, x, c+dx\right.}{d^2}$$

$$= \frac{2c\sqrt{1+(c+dx)^2}}{3bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{3bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} + \frac{2\,\text{Subst}\left(\int \right.}{}$$

$$= \frac{2c\sqrt{1+(c+dx)^2}}{3bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{3bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{}{3b^2d^2\sqrt{a+b}}$$

$$= \frac{2c\sqrt{1+(c+dx)^2}}{3bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{3bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{}{3b^2d^2\sqrt{a+b}}$$

$$= \frac{2c\sqrt{1+(c+dx)^2}}{3bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{3bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{}{3b^2d^2\sqrt{a+b}}$$

$$= \frac{2c\sqrt{1+(c+dx)^2}}{3bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{3bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{}{3b^2d^2\sqrt{a+b}}$$

$$= \frac{2c\sqrt{1+(c+dx)^2}}{3bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{3bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{}{3b^2d^2\sqrt{a+b}}$$

$$= \frac{2c\sqrt{1+(c+dx)^2}}{3bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{3bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{}{3b^2d^2\sqrt{a+b}}$$

$$= \frac{2c\sqrt{1+(c+dx)^2}}{3bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{3bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{}{3b^2d^2\sqrt{a+b}}$$

Mathematica [A] time = 2.73, size = 375, normalized size = 1.03

$$\frac{-2\sqrt{2\pi}\left(\sinh\left(\frac{2a}{b}\right)+\cosh\left(\frac{2a}{b}\right)\right)\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)+2\sqrt{2\pi}\left(\cosh\left(\frac{2a}{b}\right)-\sinh\left(\frac{2a}{b}\right)\right)\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+}}{\sqrt{b}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] ((Sqrt[b]*c*(E^(a/b)*(-2*a + b + 2*a*E^(2*ArcSinh[c + d*x]) + b*E^(2*ArcSin
h[c + d*x]) + 2*b*(-1 + E^(2*ArcSinh[c + d*x]))*ArcSinh[c + d*x]) + 2*E^((2
*a)/b + ArcSinh[c + d*x])*Sqrt[a/b + ArcSinh[c + d*x]]*(a + b*ArcSinh[c + d
*x])*Gamma[1/2, a/b + ArcSinh[c + d*x]] + 2*b*E^ArcSinh[c + d*x]*(-((a + b*
ArcSinh[c + d*x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcSinh[c + d*x])/b)]))/(E^
((a + b*ArcSinh[c + d*x])/b)*(a + b*ArcSinh[c + d*x])^(3/2)) + 2*Sqrt[2*Pi]
*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b] - Sinh
[(2*a)/b]) - 2*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b
]]*(Cosh[(2*a)/b] + Sinh[(2*a)/b]) - (Sqrt[b]*(4*(a + b*ArcSinh[c + d*x])*C
osh[2*ArcSinh[c + d*x]] + b*Sinh[2*ArcSinh[c + d*x]]))/(a + b*ArcSinh[c + d
*x])^(3/2))/(3*b^(5/2)*d^2)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(b \operatorname{arsinh}(dx + c) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x/(b*arcsinh(d*x + c) + a)^(5/2), x)
```

maple [F] time = 0.48, size = 0, normalized size = 0.00

$$\int \frac{x}{(a + b \operatorname{arcsinh}(dx + c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*arcsinh(d*x+c))^(5/2),x)
```

```
[Out] int(x/(a+b*arcsinh(d*x+c))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(b \operatorname{arsinh}(dx + c) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(b*arcsinh(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{(a + b \operatorname{asinh}(c + d\,x))^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*asinh(c + d*x))^(5/2),x)
[Out] int(x/(a + b*asinh(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(a + b\operatorname{asinh}(c + dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*asinh(d*x+c))**(5/2),x)
[Out] Integral(x/(a + b*asinh(c + d*x))**(5/2), x)
```

3.111 $\int \frac{1}{\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}\,dx$

Optimal. Leaf size=158

$$\frac{2\sqrt{\pi}\,e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}+\frac{2\sqrt{\pi}\,e^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}-\frac{4(c+dx)}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}}-\frac{2\sqrt{(c+d}}{3bd\left(a+b\sinh\right.}$$

```
[Out] 2/3*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(5/2)/d+2/3
*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(5/2)/d/exp(a/b)-2/3*(
1+(d*x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))^(3/2)-4/3*(d*x+c)/b^2/d/(a+b*ar
csinh(d*x+c))^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5863, 5655, 5774, 5657, 3307, 2180, 2205, 2204}

$$\frac{2\sqrt{\pi}\,e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}+\frac{2\sqrt{\pi}\,e^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}-\frac{4(c+dx)}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}}-\frac{2\sqrt{(c+c}}{3bd\left(a+b\sinh\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^(-5/2),x]
```

```
[Out] (-2*Sqrt[1 + (c + d*x)^2])/(3*b*d*(a + b*ArcSinh[c + d*x])^(3/2)) - (4*(c +
 d*x))/(3*b^2*d*Sqrt[a + b*ArcSinh[c + d*x]]) + (2*E^(a/b)*Sqrt[Pi]*Erf[Sqr
t[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(3*b^(5/2)*d) + (2*Sqrt[Pi]*Erfi[Sqrt[a
 + b*ArcSinh[c + d*x]]/Sqrt[b]])/(3*b^(5/2)*d*E^(a/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5863

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+b\sinh^{-1}(x)\right)^{5/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} + \frac{2\,\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\left(a+b\sinh^{-1}(x)\right)^{3/2}}\,dx, x, c+\ldots\right)}{3bd}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4(c+dx)}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{4\,\text{Subst}\left(\int \ldots\right.}{\ldots}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4(c+dx)}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{4\,\text{Subst}\left(\int \ldots\right.}{\ldots}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4(c+dx)}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{2\,\text{Subst}\left(\int \ldots\right.}{\ldots}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4(c+dx)}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{4\,\text{Subst}\left(\int \ldots\right.}{\ldots}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4(c+dx)}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{2e^{a/b}\sqrt{\pi}\,\text{erf}\ldots}{\ldots}$$

Mathematica [A] time = 0.56, size = 207, normalized size = 1.31

$$\frac{e^{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\left(-e^{a/b}\left(2a\left(e^{2\sinh^{-1}(c+dx)}-1\right)-2b\sinh^{-1}(c+dx)+be^{2\sinh^{-1}(c+dx)}\left(2\sinh^{-1}(c+dx)+1\right)+b\right)-\ldots\right.}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^(-5/2),x]
```

```
[Out] (-(E^(a/b)*(b + 2*a*(-1 + E^(2*ArcSinh[c + d*x])) - 2*b*ArcSinh[c + d*x] +
b*E^(2*ArcSinh[c + d*x])*(1 + 2*ArcSinh[c + d*x]))) - 2*E^((2*a)/b + ArcSin
h[c + d*x])*Sqrt[a/b + ArcSinh[c + d*x]]*(a + b*ArcSinh[c + d*x])*Gamma[1/2
, a/b + ArcSinh[c + d*x]] - 2*b*E^ArcSinh[c + d*x]*(-((a + b*ArcSinh[c + d*
x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcSinh[c + d*x])/b)])/(3*b^2*d*E^((a + b
*ArcSinh[c + d*x])/b)*(a + b*ArcSinh[c + d*x])^(3/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \operatorname{arsinh}(dx + c) + a)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(d*x + c) + a)^(-5/2), x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b \operatorname{arcsinh}(dx + c))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(d*x+c))^(5/2),x)
[Out] int(1/(a+b*arcsinh(d*x+c))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \operatorname{arsinh}(dx + c) + a)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*arcsinh(d*x + c) + a)^(-5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a + b \operatorname{asinh}(c + d\,x))^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*asinh(c + d*x))^(5/2),x)
[Out] int(1/(a + b*asinh(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b \operatorname{asinh}(c + dx))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(d*x+c))**(5/2),x)
[Out] Integral((a + b*asinh(c + d*x))**(-5/2), x)
```

3.112 $\int \frac{x}{\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}\,dx$

Optimal. Leaf size=445

$$\frac{4\sqrt{\pi}\,c e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d^2}+\frac{8\sqrt{2\pi}\,e^{\frac{2a}{b}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d^2}-\frac{4\sqrt{\pi}\,c e^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d^2}+\frac{8\sqrt{2\pi}\,e}{}$$

```
[Out] -4/15/b^2/d^2/(a+b*arcsinh(d*x+c))^(3/2)+4/15*c*(d*x+c)/b^2/d^2/(a+b*arcsin
h(d*x+c))^(3/2)-8/15*(d*x+c)^2/b^2/d^2/(a+b*arcsinh(d*x+c))^(3/2)+4/15*c*ex
p(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(7/2)/d^2-4/15*c*
erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(7/2)/d^2/exp(a/b)+8/15
*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2
)/b^(7/2)/d^2+8/15*erfi(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)
*Pi^(1/2)/b^(7/2)/d^2/exp(2*a/b)+2/5*c*(1+(d*x+c)^2)^(1/2)/b/d^2/(a+b*arcsi
nh(d*x+c))^(5/2)-2/5*(d*x+c)*(1+(d*x+c)^2)^(1/2)/b/d^2/(a+b*arcsinh(d*x+c))
^(5/2)+8/15*c*(1+(d*x+c)^2)^(1/2)/b^3/d^2/(a+b*arcsinh(d*x+c))^(1/2)-32/15*
(d*x+c)*(1+(d*x+c)^2)^(1/2)/b^3/d^2/(a+b*arcsinh(d*x+c))^(1/2)
```

Rubi [A] time = 1.05, antiderivative size = 445, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 13, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.812, Rules used = {5865, 5803, 5655, 5774, 5779, 3308, 2180, 2204, 2205, 5667, 5665, 3307, 5675}

$$\frac{4\sqrt{\pi}\,c e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d^2}+\frac{8\sqrt{2\pi}\,e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d^2}-\frac{4\sqrt{\pi}\,c e^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d^2}+\frac{8\sqrt{2\pi}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*ArcSinh[c + d*x])^(7/2),x]
```

```
[Out] (2*c*Sqrt[1 + (c + d*x)^2])/(5*b*d^2*(a + b*ArcSinh[c + d*x])^(5/2)) - (2*(
c + d*x)*Sqrt[1 + (c + d*x)^2])/(5*b*d^2*(a + b*ArcSinh[c + d*x])^(5/2)) -
4/(15*b^2*d^2*(a + b*ArcSinh[c + d*x])^(3/2)) + (4*c*(c + d*x))/(15*b^2*d^2
*(a + b*ArcSinh[c + d*x])^(3/2)) - (8*(c + d*x)^2)/(15*b^2*d^2*(a + b*ArcSi
nh[c + d*x])^(3/2)) + (8*c*Sqrt[1 + (c + d*x)^2])/(15*b^3*d^2*Sqrt[a + b*Ar
cSinh[c + d*x]]) - (32*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(15*b^3*d^2*Sqrt[a
+ b*ArcSinh[c + d*x]]) + (4*c*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d
*x]]/Sqrt[b]])/(15*b^(7/2)*d^2) + (8*E^((2*a)/b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sq
rt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(15*b^(7/2)*d^2) - (4*c*Sqrt[Pi]*Erfi
[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(15*b^(7/2)*d^2*E^(a/b)) + (8*Sqrt[
2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(15*b^(7/2)*d^2
*E^((2*a)/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 5803

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*ArcSinh[c*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[m, 0] && LtQ[n, -1]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{x}{\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}\,dx = \frac{\text{Subst}\left(\int \frac{-\frac{c}{d}+\frac{x}{d}}{\left(a+b\sinh^{-1}(x)\right)^{7/2}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{c}{d\left(a+b\sinh^{-1}(x)\right)^{7/2}}+\frac{x}{d\left(a+b\sinh^{-1}(x)\right)^{7/2}}\right)dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{x}{\left(a+b\sinh^{-1}(x)\right)^{7/2}}\,dx, x, c+dx\right)}{d^2} - \frac{c\,\text{Subst}\left(\int \frac{1}{\left(a+b\sinh^{-1}(x)\right)^{7/2}}\,dx, x, c+dx \right.}{d^2}$$

$$= \frac{2c\sqrt{1+(c+dx)^2}}{5bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{5bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} + \frac{2\,\text{Subst}\left(\int - \right.}{}$$

$$= \frac{2c\sqrt{1+(c+dx)^2}}{5bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{5bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{}{15b^2d^2\left(a+b\right.}$$

$$= \frac{2c\sqrt{1+(c+dx)^2}}{5bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{5bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{}{15b^2d^2\left(a+b\right.}$$

$$= \frac{2c\sqrt{1+(c+dx)^2}}{5bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{5bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{}{15b^2d^2\left(a+b\right.}$$

$$= \frac{2c\sqrt{1+(c+dx)^2}}{5bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{5bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{}{15b^2d^2\left(a+b\right.}$$

$$= \frac{2c\sqrt{1+(c+dx)^2}}{5bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{5bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{}{15b^2d^2\left(a+b\right.}$$

$$= \frac{2c\sqrt{1+(c+dx)^2}}{5bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{5bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{}{15b^2d^2\left(a+b\right.}$$

Mathematica [A] time = 2.59, size = 429, normalized size = 0.96

$$\frac{\frac{\sqrt{b}\left(-\sinh\left(2\sinh^{-1}(c+dx)\right)\left(16\left(a+b\sinh^{-1}(c+dx)\right)^2+3b^2\right)-4b\cosh\left(2\sinh^{-1}(c+dx)\right)\left(a+b\sinh^{-1}(c+dx)\right)\right)}{\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}+8\sqrt{2\pi}\left(\sinh\left(\frac{2a}{b}\right)+\cosh\left(\frac{2a}{b}\right)\right.}{15b^{7/2}d^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(a + b*ArcSinh[c + d*x])^(7/2),x]
```

```
[Out] -1/30*(c*(-6*b^2*E^ArcSinh[c + d*x] - (2*(4*a^2 + 2*a*b*(-1 + 4*ArcSinh[c +
 d*x]) + b^2*(3 - 2*ArcSinh[c + d*x] + 4*ArcSinh[c + d*x]^2)))/E^ArcSinh[c
```

```
+ d*x] + 8*E^(a/b)*Sqrt[a/b + ArcSinh[c + d*x]]*(a + b*ArcSinh[c + d*x])^2*
Gamma[1/2, a/b + ArcSinh[c + d*x]] - (4*(a + b*ArcSinh[c + d*x])*(E^(a/b +
ArcSinh[c + d*x])*(2*a + b + 2*b*ArcSinh[c + d*x]) + 2*b*(-((a + b*ArcSinh[
c + d*x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcSinh[c + d*x])/b)]))/E^(a/b)))/(
b^3*d^2*(a + b*ArcSinh[c + d*x])^(5/2)) + (8*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[
a + b*ArcSinh[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b] - Sinh[(2*a)/b]) + 8*Sqrt[
2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b] +
Sinh[(2*a)/b]) + (Sqrt[b]*(-4*b*(a + b*ArcSinh[c + d*x])*Cosh[2*ArcSinh[c +
 d*x]] - (3*b^2 + 16*(a + b*ArcSinh[c + d*x])^2)*Sinh[2*ArcSinh[c + d*x]]))
/(a + b*ArcSinh[c + d*x])^(5/2))/(15*b^(7/2)*d^2)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(b \operatorname{arsinh}(dx + c) + a)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(x/(b*arcsinh(d*x + c) + a)^(7/2), x)
```

maple [F] time = 0.48, size = 0, normalized size = 0.00

$$\int \frac{x}{(a + b \operatorname{arcsinh}(dx + c))^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*arcsinh(d*x+c))^(7/2),x)
```

```
[Out] int(x/(a+b*arcsinh(d*x+c))^(7/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(b \operatorname{arsinh}(dx + c) + a)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(b*arcsinh(d*x + c) + a)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{(a + b \operatorname{asinh}(c + d\,x))^{7/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*asinh(c + d*x))^(7/2),x)
```

```
[Out] int(x/(a + b*asinh(c + d*x))^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(a + b\,\mathrm{asinh}\,(c + dx))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*asinh(d*x+c))**(7/2),x)
[Out] Integral(x/(a + b*asinh(c + d*x))**(7/2), x)
```

3.113 $\int \frac{1}{\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}\,dx$

Optimal. Leaf size=195

$$-\frac{4\sqrt{\pi}\,e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}+\frac{4\sqrt{\pi}\,e^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}-\frac{8\sqrt{(c+dx)^2+1}}{15b^3d\sqrt{a+b\sinh^{-1}(c+dx)}}-\frac{4(}{15b^2d\left(a+b\right.}$$

```
[Out] -4/15*(d*x+c)/b^2/d/(a+b*arcsinh(d*x+c))^(3/2)-4/15*exp(a/b)*erf((a+b*arcsi
nh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(7/2)/d+4/15*erfi((a+b*arcsinh(d*x+c))
^(1/2)/b^(1/2))*Pi^(1/2)/b^(7/2)/d/exp(a/b)-2/5*(1+(d*x+c)^2)^(1/2)/b/d/(a+
b*arcsinh(d*x+c))^(5/2)-8/15*(1+(d*x+c)^2)^(1/2)/b^3/d/(a+b*arcsinh(d*x+c))
^(1/2)
```

Rubi [A] time = 0.41, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5863, 5655, 5774, 5779, 3308, 2180, 2204, 2205}

$$-\frac{4\sqrt{\pi}\,e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}+\frac{4\sqrt{\pi}\,e^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}-\frac{4(c+dx)}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}-\frac{8\sqrt{(}}{15b^3d\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^(-7/2),x]
```

```
[Out] (-2*Sqrt[1 + (c + d*x)^2])/(5*b*d*(a + b*ArcSinh[c + d*x])^(5/2)) - (4*(c +
 d*x))/(15*b^2*d*(a + b*ArcSinh[c + d*x])^(3/2)) - (8*Sqrt[1 + (c + d*x)^2]
)/(15*b^3*d*Sqrt[a + b*ArcSinh[c + d*x]]) - (4*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a
+ b*ArcSinh[c + d*x]]/Sqrt[b]])/(15*b^(7/2)*d) + (4*Sqrt[Pi]*Erfi[Sqrt[a +
b*ArcSinh[c + d*x]]/Sqrt[b]])/(15*b^(7/2)*d*E^(a/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 5863

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+b\sinh^{-1}(x)\right)^{7/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} + \frac{2\,\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\left(a+b\sinh^{-1}(x)\right)^{5/2}}\,dx, x, c+\right.}{5bd}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4(c+dx)}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} + \frac{4\,\text{Subst}}{}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4(c+dx)}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{8}{15b^3d\sqrt{}}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4(c+dx)}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{8}{15b^3d\sqrt{}}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4(c+dx)}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{8}{15b^3d\sqrt{}}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4(c+dx)}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{8}{15b^3d\sqrt{}}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4(c+dx)}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{8}{15b^3d\sqrt{}}$$

Mathematica [A] time = 0.54, size = 238, normalized size = 1.22

$$\frac{-2e^{-\sinh^{-1}(c+dx)}\left(4a^2+2ab\left(4\sinh^{-1}(c+dx)-1\right)+b^2\left(4\sinh^{-1}(c+dx)^2-2\sinh^{-1}(c+dx)+3\right)\right)+8e^{a/b}\sqrt{\frac{a}{b}}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^(-7/2),x]
```

```
[Out] (-6*b^2*E^ArcSinh[c + d*x] - (2*(4*a^2 + 2*a*b*(-1 + 4*ArcSinh[c + d*x]) +
b^2*(3 - 2*ArcSinh[c + d*x] + 4*ArcSinh[c + d*x]^2)))/E^ArcSinh[c + d*x] +
8*E^(a/b)*Sqrt[a/b + ArcSinh[c + d*x]]*(a + b*ArcSinh[c + d*x])^2*Gamma[1/2
, a/b + ArcSinh[c + d*x]] - (4*(a + b*ArcSinh[c + d*x])*(E^(a/b + ArcSinh[c
 + d*x])*(2*a + b + 2*b*ArcSinh[c + d*x]) + 2*b*(-((a + b*ArcSinh[c + d*x])
/b))^(3/2)*Gamma[1/2, -((a + b*ArcSinh[c + d*x])/b)]))/E^(a/b))/(30*b^3*d*(
a + b*ArcSinh[c + d*x])^(5/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\operatorname{arsinh}(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(-7/2), x)
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\operatorname{arcsinh}(dx+c))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(d*x+c))^(7/2),x)
```

```
[Out] int(1/(a+b*arcsinh(d*x+c))^(7/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\operatorname{arsinh}(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(-7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+b\operatorname{asinh}(c+d\,x))^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*asinh(c + d*x))^(7/2),x)
```

```
[Out] int(1/(a + b*asinh(c + d*x))^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\operatorname{asinh}(c+dx))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(d*x+c))**(7/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))**(-7/2), x)
```

3.114 $\int (ce+dex)^m \left(a+b\sinh^{-1}(c+dx)\right)\,dx$

Optimal. Leaf size=91

$$\frac{(e(c+dx))^{m+1}\left(a+b\sinh^{-1}(c+dx)\right)}{de(m+1)}-\frac{b(e(c+dx))^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};-(c+dx)^2\right)}{de^2(m+1)(m+2)}$$

```
[Out] (e*(d*x+c))^(1+m)*(a+b*arcsinh(d*x+c))/d/e/(1+m)-b*(e*(d*x+c))^(2+m)*hyperg
eom([1/2, 1+1/2*m],[2+1/2*m],-(d*x+c)^2)/d/e^2/(1+m)/(2+m)
```

Rubi [A] time = 0.07, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5865, 5661, 364}

$$\frac{(e(c+dx))^{m+1}\left(a+b\sinh^{-1}(c+dx)\right)}{de(m+1)}-\frac{b(e(c+dx))^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};-(c+dx)^2\right)}{de^2(m+1)(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^m*(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] ((e*(c + d*x))^(1 + m)*(a + b*ArcSinh[c + d*x]))/(d*e*(1 + m)) - (b*(e*(c +
 d*x))^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, -(c + d*x)^2])/
(d*e^2*(1 + m)*(2 + m))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^m\left(a+b\sinh^{-1}(c+dx)\right)dx=\frac{\text{Subst}\left(\int (ex)^m\left(a+b\sinh^{-1}(x)\right)dx,x,c+dx\right)}{d}$$

$$=\frac{(e(c+dx))^{1+m}\left(a+b\sinh^{-1}(c+dx)\right)}{de(1+m)}-\frac{b\,\text{Subst}\left(\int\frac{(ex)^{1+m}}{\sqrt{1+x^2}}\,dx,x,c+\right.}{de(1+m)}$$

$$=\frac{(e(c+dx))^{1+m}\left(a+b\sinh^{-1}(c+dx)\right)}{de(1+m)}-\frac{b(e(c+dx))^{2+m}\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2}\right.}{de^2(1+m)(2}$$

Mathematica [A] time = 0.05, size = 79, normalized size = 0.87

$$-\frac{(c+dx)(e(c+dx))^m\left(b(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};-(c+dx)^2\right)-(m+2)\left(a+b\sinh^{-1}(c+dx)\right)\right)}{d(m+1)(m+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^m*(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] -(((c + d*x)*(e*(c + d*x))^m*(-((2 + m)*(a + b*ArcSinh[c + d*x])) + b*(c +
d*x)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, -(c + d*x)^2]))/(d*(1 + m
)*(2 + m)))
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\mathrm{integral}\left((b\,\mathrm{arsinh}\,(dx+c)+a)(dex+ce)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(d*x + c) + a)*(d*e*x + c*e)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(b\,\mathrm{arsinh}\,(dx+c)+a)(dex+ce)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)*(d*e*x + c*e)^m, x)
```

maple [F] time = 2.15, size = 0, normalized size = 0.00

$$\int(dex+ce)^m\,(a+b\,\mathrm{arcsinh}\,(dx+c))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c)),x)
```

```
[Out] int((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b\left(\frac{(de^mx+ce^m)(dx+c)^m\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)}{d(m+1)}-\int\frac{\left(d^2e^mx^2+2\,cde^mx+c^2e^m\right)(dx+c)^m}{d^2(m+1)x^2+2\,cd(m+1)x+c^2(m+1)+m}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] b*((d*e^m*x + c*e^m)*(d*x + c)^m*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2
 + 1))/(d*(m + 1)) - integrate((d^2*e^m*x^2 + 2*c*d*e^m*x + c^2*e^m)*(d*x +
 c)^m/(d^2*(m + 1)*x^2 + 2*c*d*(m + 1)*x + c^2*(m + 1) + m + 1), x) - integ
rate((d*e^m*x + c*e^m)*(d*x + c)^m/(d^3*(m + 1)*x^3 + 3*c*d^2*(m + 1)*x^2 +
 c^3*(m + 1) + c*(m + 1) + (3*c^2*d*(m + 1) + d*(m + 1))*x + (d^2*(m + 1)*x
^2 + 2*c*d*(m + 1)*x + c^2*(m + 1) + m + 1)*sqrt(d^2*x^2 + 2*c*d*x + c^2 +
1)), x)) + (d*e*x + c*e)^(m + 1)*a/(d*e*(m + 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e+d\,e\,x)^m\,\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^m*(a + b*asinh(c + d*x)),x)
```

```
[Out] int((c*e + d*e*x)^m*(a + b*asinh(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(e\left(c+dx\right)\right)^{m}\left(a+b\,\mathrm{asinh}\left(c+dx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**m*(a+b*asinh(d*x+c)),x)
```

```
[Out] Integral((e*(c + d*x))**m*(a + b*asinh(c + d*x)), x)
```

3.115 $\int (ce+dex)^4 \left(a+b\sinh^{-1}(c+dx)\right)\,dx$

Optimal. Leaf size=100

$$\frac{e^4(c+dx)^5\left(a+b\sinh^{-1}(c+dx)\right)}{5d}-\frac{be^4\left((c+dx)^2+1\right)^{5/2}}{25d}+\frac{2be^4\left((c+dx)^2+1\right)^{3/2}}{15d}-\frac{be^4\sqrt{(c+dx)^2+1}}{5d}$$

```
[Out] 2/15*b*e^4*(1+(d*x+c)^2)^(3/2)/d-1/25*b*e^4*(1+(d*x+c)^2)^(5/2)/d+1/5*e^4*(
d*x+c)^5*(a+b*arcsinh(d*x+c))/d-1/5*b*e^4*(1+(d*x+c)^2)^(1/2)/d
```

Rubi [A] time = 0.08, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5865, 12, 5661, 266, 43}

$$\frac{e^4(c+dx)^5\left(a+b\sinh^{-1}(c+dx)\right)}{5d}-\frac{be^4\left((c+dx)^2+1\right)^{5/2}}{25d}+\frac{2be^4\left((c+dx)^2+1\right)^{3/2}}{15d}-\frac{be^4\sqrt{(c+dx)^2+1}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4*(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] -(b*e^4*Sqrt[1 + (c + d*x)^2])/(5*d) + (2*b*e^4*(1 + (c + d*x)^2)^(3/2))/(1
5*d) - (b*e^4*(1 + (c + d*x)^2)^(5/2))/(25*d) + (e^4*(c + d*x)^5*(a + b*Arc
Sinh[c + d*x]))/(5*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^4 \left(a+b\sinh^{-1}(c+dx)\right) dx = \frac{\text{Subst}\left(\int e^4x^4\left(a+b\sinh^{-1}(x)\right) dx, x, c+dx\right)}{d}$$
$$= \frac{e^4\,\text{Subst}\left(\int x^4\left(a+b\sinh^{-1}(x)\right) dx, x, c+dx\right)}{d}$$
$$= \frac{e^4(c+dx)^5\left(a+b\sinh^{-1}(c+dx)\right)}{5d} - \frac{\left(be^4\right)\text{Subst}\left(\int \frac{x^5}{\sqrt{1+x^2}} dx, x, c+\right.}{5d}$$
$$= \frac{e^4(c+dx)^5\left(a+b\sinh^{-1}(c+dx)\right)}{5d} - \frac{\left(be^4\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1+x}} dx, x, (c+\right.}{10d}$$
$$= \frac{e^4(c+dx)^5\left(a+b\sinh^{-1}(c+dx)\right)}{5d} - \frac{\left(be^4\right)\text{Subst}\left(\int\left(\frac{1}{\sqrt{1+x}} - 2\sqrt{1+\right.\right.}{1}$$
$$= -\frac{be^4\sqrt{1+(c+dx)^2}}{5d} + \frac{2be^4\left(1+(c+dx)^2\right)^{3/2}}{15d} - \frac{be^4\left(1+(c+dx)^2\right)^{5}}{25d}$$

Mathematica [A] time = 0.10, size = 71, normalized size = 0.71

$$\frac{e^4\left(\frac{1}{5}(c+dx)^5\left(a+b\sinh^{-1}(c+dx)\right) - \frac{1}{75}b\sqrt{(c+dx)^2+1}\left(-10(c+dx)^2+3\left((c+dx)^2+1\right)^2+5\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^4*(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (e^4*(-1/75*(b*Sqrt[1 + (c + d*x)^2]*(5 - 10*(c + d*x)^2 + 3*(1 + (c + d*x)
^2)^2)) + ((c + d*x)^5*(a + b*ArcSinh[c + d*x]))/5))/d
```

fricas [B] time = 0.56, size = 279, normalized size = 2.79

$$15\,ad^5e^4x^5 + 75\,acd^4e^4x^4 + 150\,ac^2d^3e^4x^3 + 150\,ac^3d^2e^4x^2 + 75\,ac^4de^4x + 15\left(bd^5e^4x^5 + 5\,bcd^4e^4x^4 + 10\,bc^2d^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/75*(15*a*d^5*e^4*x^5 + 75*a*c*d^4*e^4*x^4 + 150*a*c^2*d^3*e^4*x^3 + 150*a
*c^3*d^2*e^4*x^2 + 75*a*c^4*d*e^4*x + 15*(b*d^5*e^4*x^5 + 5*b*c*d^4*e^4*x^4
 + 10*b*c^2*d^3*e^4*x^3 + 10*b*c^3*d^2*e^4*x^2 + 5*b*c^4*d*e^4*x + b*c^5*e^
4)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - (3*b*d^4*e^4*x^4 + 12
*b*c*d^3*e^4*x^3 + 2*(9*b*c^2 - 2*b)*d^2*e^4*x^2 + 4*(3*b*c^3 - 2*b*c)*d*e^
4*x + (3*b*c^4 - 4*b*c^2 + 8*b)*e^4)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/d
```

giac [B] time = 1.93, size = 817, normalized size = 8.17

$$\frac{1}{600}\left(120\,ad^4x^5 + 600\,acd^3x^4 + 1200\,ac^2d^2x^3 + 1200\,ac^3dx^2 - 600\left(d\left(\frac{c\log\left(-cd - \left(x|d| - \sqrt{d^2x^2 + 2\,cdx + c^2}\right.\right.}{d|d|}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/600*(120*a*d^4*x^5 + 600*a*c*d^3*x^4 + 1200*a*c^2*d^2*x^3 + 1200*a*c^3*d*
x^2 - 600*(d*(c*log(-c*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*a
```

```
bs(d))/(d*abs(d)) + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^2) - x*log(d*x + c
+ sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)))*b*c^4 + 600*(2*x^2*log(d*x + c + sqrt
(d^2*x^2 + 2*c*d*x + c^2 + 1)) - (sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(x/d^2
- 3*c/d^3) - (2*c^2 - 1)*log(-c*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^
2 + 1))*abs(d))/(d^2*abs(d)))*d)*b*c^3*d + 200*(6*x^3*log(d*x + c + sqrt(d^
2*x^2 + 2*c*d*x + c^2 + 1)) - (sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(x*(2*x/d^
2 - 5*c/d^3) + (11*c^2*d - 4*d)/d^5) + 3*(2*c^3 - 3*c)*log(-c*d - (x*abs(d)
 - sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*abs(d))/(d^3*abs(d)))*d)*b*c^2*d^2 +
25*(24*x^4*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - (sqrt(d^2*x^2
 + 2*c*d*x + c^2 + 1)*((2*x*(3*x/d^2 - 7*c/d^3) + (26*c^2*d^3 - 9*d^3)/d^7)
*x - 5*(10*c^3*d^2 - 11*c*d^2)/d^7) - 3*(8*c^4 - 24*c^2 + 3)*log(-c*d - (x*
abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*abs(d))/(d^4*abs(d)))*d)*b*c*d^
3 + (120*x^5*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - (sqrt(d^2*x
^2 + 2*c*d*x + c^2 + 1)*((2*(3*x*(4*x/d^2 - 9*c/d^3) + (47*c^2*d^5 - 16*d^5
)/d^9)*x - 7*(22*c^3*d^4 - 23*c*d^4)/d^9)*x + (274*c^4*d^3 - 607*c^2*d^3 +
64*d^3)/d^9) + 15*(8*c^5 - 40*c^3 + 15*c)*log(-c*d - (x*abs(d) - sqrt(d^2*x
^2 + 2*c*d*x + c^2 + 1))*abs(d))/(d^5*abs(d)))*d)*b*d^4 + 600*a*c^4*x)*e^4
```

maple [A] time = 0.01, size = 93, normalized size = 0.93

$$\frac{\frac{(dx+c)^5 e^4 a}{5}+e^4 b\left(\frac{(dx+c)^5 \operatorname{arcsinh}(dx+c)}{5}-\frac{(dx+c)^4\sqrt{1+(dx+c)^2}}{25}+\frac{4(dx+c)^2\sqrt{1+(dx+c)^2}}{75}-\frac{8\sqrt{1+(dx+c)^2}}{75}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c)),x)
```

```
[Out] 1/d*(1/5*(d*x+c)^5*e^4*a+e^4*b*(1/5*(d*x+c)^5*arcsinh(d*x+c)-1/25*(d*x+c)^4
*(1+(d*x+c)^2)^(1/2)+4/75*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)-8/75*(1+(d*x+c)^2)^
(1/2)))
```

maxima [B] time = 0.48, size = 1231, normalized size = 12.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/5*a*d^4*e^4*x^5 + a*c*d^3*e^4*x^4 + 2*a*c^2*d^2*e^4*x^3 + 2*a*c^3*d*e^4*x
^2 + (2*x^2*arcsinh(d*x + c) - d*(3*c^2*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2
*d^2 + 4*(c^2 + 1)*d^2))/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*x/d^2 - (c
^2 + 1)*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 - 3
*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c/d^3))*b*c^3*d*e^4 + 1/3*(6*x^3*arcsinh
(d*x + c) - d*(2*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*x^2/d^2 - 15*c^3*arcsinh
(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^4 - 5*sqrt(d^2*x^2 +
 2*c*d*x + c^2 + 1)*c*x/d^3 + 9*(c^2 + 1)*c*arcsinh(2*(d^2*x + c*d)/sqrt(-4
*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^4 + 15*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c^2
/d^4 - 4*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)/d^4))*b*c^2*d^2*e^4 +
1/24*(24*x^4*arcsinh(d*x + c) - (6*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*x^3/d^
2 - 14*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c*x^2/d^3 + 105*c^4*arcsinh(2*(d^2
*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^5 + 35*sqrt(d^2*x^2 + 2*c*d
*x + c^2 + 1)*c^2*x/d^4 - 90*(c^2 + 1)*c^2*arcsinh(2*(d^2*x + c*d)/sqrt(-4*
c^2*d^2 + 4*(c^2 + 1)*d^2))/d^5 - 105*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c^3
/d^5 - 9*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)*x/d^4 + 9*(c^2 + 1)^2*
arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^5 + 55*sqrt(d
^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)*c/d^5)*d)*b*c*d^3*e^4 + 1/600*(120*x^
5*arcsinh(d*x + c) - (24*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*x^4/d^2 - 54*sqr
t(d^2*x^2 + 2*c*d*x + c^2 + 1)*c*x^3/d^3 + 126*sqrt(d^2*x^2 + 2*c*d*x + c^2
 + 1)*c^2*x^2/d^4 - 945*c^5*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^
```

```
2 + 1)*d^2))/d^6 - 315*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c^3*x/d^5 - 32*sqr
t(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)*x^2/d^4 + 1050*(c^2 + 1)*c^3*arcsi
nh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^6 + 945*sqrt(d^2*x
^2 + 2*c*d*x + c^2 + 1)*c^4/d^6 + 161*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^
2 + 1)*c*x/d^5 - 225*(c^2 + 1)^2*c*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2
+ 4*(c^2 + 1)*d^2))/d^6 - 735*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)*c
^2/d^6 + 64*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)^2/d^6)*d)*b*d^4*e^4
 + a*c^4*e^4*x + ((d*x + c)*arcsinh(d*x + c) - sqrt((d*x + c)^2 + 1))*b*c^4
*e^4/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e+d\,e\,x)^4\,\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^4*(a + b*asinh(c + d*x)),x)
```

```
[Out] int((c*e + d*e*x)^4*(a + b*asinh(c + d*x)), x)
```

sympy [A] time = 3.50, size = 527, normalized size = 5.27

$$\begin{cases} ac^4e^4x + 2ac^3de^4x^2 + 2ac^2d^2e^4x^3 + acd^3e^4x^4 + \frac{ad^4e^4x^5}{5} + \frac{bc^5e^4\operatorname{asinh}(c+dx)}{5d} + bc^4e^4x\operatorname{asinh}(c+dx) - \frac{bc^4e^4\sqrt{c^2+2cd\ldots}}{25d} \\ c^4e^4x\left(a+b\operatorname{asinh}(c)\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4*(a+b*asinh(d*x+c)),x)
```

```
[Out] Piecewise((a*c**4*e**4*x + 2*a*c**3*d*e**4*x**2 + 2*a*c**2*d**2*e**4*x**3 +
 a*c*d**3*e**4*x**4 + a*d**4*e**4*x**5/5 + b*c**5*e**4*asinh(c + d*x)/(5*d)
 + b*c**4*e**4*x*asinh(c + d*x) - b*c**4*e**4*sqrt(c**2 + 2*c*d*x + d**2*x*
*2 + 1)/(25*d) + 2*b*c**3*d*e**4*x**2*asinh(c + d*x) - 4*b*c**3*e**4*x*sqrt
(c**2 + 2*c*d*x + d**2*x**2 + 1)/25 + 2*b*c**2*d**2*e**4*x**3*asinh(c + d*x
) - 6*b*c**2*d*e**4*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/25 + 4*b*c**2
*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(75*d) + b*c*d**3*e**4*x**4*asin
h(c + d*x) - 4*b*c*d**2*e**4*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/25 +
 8*b*c*e**4*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/75 + b*d**4*e**4*x**5*as
inh(c + d*x)/5 - b*d**3*e**4*x**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/25 +
 4*b*d*e**4*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/75 - 8*b*e**4*sqrt(c*
*2 + 2*c*d*x + d**2*x**2 + 1)/(75*d), Ne(d, 0)), (c**4*e**4*x*(a + b*asinh(
c)), True))
```

3.116 $\int (ce+dex)^3 \left(a+b\sinh^{-1}(c+dx)\right) dx$

Optimal. Leaf size=105

$$\frac{e^3(c+dx)^4\left(a+b\sinh^{-1}(c+dx)\right)}{4d}-\frac{be^3\sqrt{(c+dx)^2+1}\,(c+dx)^3}{16d}+\frac{3be^3\sqrt{(c+dx)^2+1}\,(c+dx)}{32d}-\frac{3be^3\sinh^{-1}(c+dx)}{32d}$$

```
[Out] -3/32*b*e^3*arcsinh(d*x+c)/d+1/4*e^3*(d*x+c)^4*(a+b*arcsinh(d*x+c))/d+3/32*
b*e^3*(d*x+c)*(1+(d*x+c)^2)^(1/2)/d-1/16*b*e^3*(d*x+c)^3*(1+(d*x+c)^2)^(1/2
)/d
```

Rubi [A] time = 0.07, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = $\{5865, 12, 5661, 321, 215\}$

$$\frac{e^3(c+dx)^4\left(a+b\sinh^{-1}(c+dx)\right)}{4d}-\frac{be^3\sqrt{(c+dx)^2+1}\,(c+dx)^3}{16d}+\frac{3be^3\sqrt{(c+dx)^2+1}\,(c+dx)}{32d}-\frac{3be^3\sinh^{-1}(c+dx)}{32d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3*(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (3*b*e^3*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(32*d) - (b*e^3*(c + d*x)^3*Sqrt[
1 + (c + d*x)^2])/(16*d) - (3*b*e^3*ArcSinh[c + d*x])/(32*d) + (e^3*(c + d*
x)^4*(a + b*ArcSinh[c + d*x]))/(4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^3 \left(a+b\sinh^{-1}(c+dx)\right) dx = \frac{\text{Subst}\left(\int e^3x^3 \left(a+b\sinh^{-1}(x)\right) dx, x, c+dx\right)}{d}$$
$$= \frac{e^3 \,\text{Subst}\left(\int x^3 \left(a+b\sinh^{-1}(x)\right) dx, x, c+dx\right)}{d}$$
$$= \frac{e^3(c+dx)^4 \left(a+b\sinh^{-1}(c+dx)\right)}{4d} - \frac{\left(be^3\right) \text{Subst}\left(\int \frac{x^4}{\sqrt{1+x^2}} dx, x, c+\right.}{4d}$$
$$= -\frac{be^3(c+dx)^3\sqrt{1+(c+dx)^2}}{16d} + \frac{e^3(c+dx)^4 \left(a+b\sinh^{-1}(c+dx)\right)}{4d}$$
$$= \frac{3be^3(c+dx)\sqrt{1+(c+dx)^2}}{32d} - \frac{be^3(c+dx)^3\sqrt{1+(c+dx)^2}}{16d} + \frac{e^3(c+}{}$$
$$= \frac{3be^3(c+dx)\sqrt{1+(c+dx)^2}}{32d} - \frac{be^3(c+dx)^3\sqrt{1+(c+dx)^2}}{16d} - \frac{3be^3 \text{s}}{}$$

Mathematica [A] time = 0.08, size = 83, normalized size = 0.79

$$\frac{e^3 \left(8(c+dx)^4 \left(a+b\sinh^{-1}(c+dx)\right) - 2b\sqrt{(c+dx)^2+1}\,(c+dx)^3 + 3b\sqrt{(c+dx)^2+1}\,(c+dx) - 3b\sinh^{-1}(c+\right.}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^3*(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (e^3*(3*b*(c + d*x)*Sqrt[1 + (c + d*x)^2] - 2*b*(c + d*x)^3*Sqrt[1 + (c + d
*x)^2] - 3*b*ArcSinh[c + d*x] + 8*(c + d*x)^4*(a + b*ArcSinh[c + d*x])))/(3
2*d)
```

fricas [B] time = 0.49, size = 228, normalized size = 2.17

$$\frac{8\,ad^4e^3x^4 + 32\,acd^3e^3x^3 + 48\,ac^2d^2e^3x^2 + 32\,ac^3de^3x + \left(8\,bd^4e^3x^4 + 32\,bcd^3e^3x^3 + 48\,bc^2d^2e^3x^2 + 32\,bc^3de^3x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/32*(8*a*d^4*e^3*x^4 + 32*a*c*d^3*e^3*x^3 + 48*a*c^2*d^2*e^3*x^2 + 32*a*c^
3*d*e^3*x + (8*b*d^4*e^3*x^4 + 32*b*c*d^3*e^3*x^3 + 48*b*c^2*d^2*e^3*x^2 +
32*b*c^3*d*e^3*x + (8*b*c^4 - 3*b)*e^3)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*
x + c^2 + 1)) - (2*b*d^3*e^3*x^3 + 6*b*c*d^2*e^3*x^2 + 3*(2*b*c^2 - b)*d*e^
3*x + (2*b*c^3 - 3*b*c)*e^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/d
```

giac [B] time = 1.63, size = 594, normalized size = 5.66

$$\frac{1}{96}\left(24\,ad^3x^4 + 96\,acd^2x^3 + 144\,ac^2dx^2 - 96\left(d\left(\frac{c\log\left(-cd - \left(x|d| - \sqrt{d^2x^2+2\,cdx+c^2+1}\right)|d|\right)}{d|d|} + \frac{\sqrt{d^2x^2+2}}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/96*(24*a*d^3*x^4 + 96*a*c*d^2*x^3 + 144*a*c^2*d*x^2 - 96*(d*(c*log(-c*d -
 (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*abs(d))/(d*abs(d)) + sqrt(d
```

```
^2*x^2 + 2*c*d*x + c^2 + 1)/d^2) - x*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x +
 c^2 + 1)))*b*c^3 + 72*(2*x^2*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 +
1)) - (sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(x/d^2 - 3*c/d^3) - (2*c^2 - 1)*lo
g(-c*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*abs(d))/(d^2*abs(d)
))*d)*b*c^2*d + 16*(6*x^3*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))
- (sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(x*(2*x/d^2 - 5*c/d^3) + (11*c^2*d - 4
*d)/d^5) + 3*(2*c^3 - 3*c)*log(-c*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x +
c^2 + 1))*abs(d))/(d^3*abs(d)))*d)*b*c*d^2 + (24*x^4*log(d*x + c + sqrt(d^2
*x^2 + 2*c*d*x + c^2 + 1)) - (sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*((2*x*(3*x/
d^2 - 7*c/d^3) + (26*c^2*d^3 - 9*d^3)/d^7)*x - 5*(10*c^3*d^2 - 11*c*d^2)/d^
7) - 3*(8*c^4 - 24*c^2 + 3)*log(-c*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x +
 c^2 + 1))*abs(d))/(d^4*abs(d)))*d)*b*d^3 + 96*a*c^3*x)*e^3
```

maple [A] time = 0.01, size = 86, normalized size = 0.82

$$\frac{\frac{(dx+c)^4 e^3 a}{4} + e^3 b\left(\frac{(dx+c)^4 \operatorname{arcsinh}(dx+c)}{4} - \frac{(dx+c)^3\sqrt{1+(dx+c)^2}}{16} + \frac{3(dx+c)\sqrt{1+(dx+c)^2}}{32} - \frac{3\operatorname{arcsinh}(dx+c)}{32}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c)),x)
```

```
[Out] 1/d*(1/4*(d*x+c)^4*e^3*a+e^3*b*(1/4*(d*x+c)^4*arcsinh(d*x+c)-1/16*(d*x+c)^3
*(1+(d*x+c)^2)^(1/2)+3/32*(d*x+c)*(1+(d*x+c)^2)^(1/2)-3/32*arcsinh(d*x+c)))
```

maxima [B] time = 0.39, size = 790, normalized size = 7.52

$$\frac{1}{4}\,ad^3e^3x^4 + acd^2e^3x^3 + \frac{3}{2}\,ac^2de^3x^2 + \frac{3}{4}\left(2\,x^2\operatorname{arsinh}\left(dx+c\right) - d\left(\frac{3\,c^2\operatorname{arsinh}\left(\frac{2\left(d^2x+cd\right)}{\sqrt{-4\,c^2d^2+4\left(c^2+1\right)d^2}}\right)}{d^3} + \frac{\sqrt{d^2x^2+2\,cdx+}}{d^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*a*d^3*e^3*x^4 + a*c*d^2*e^3*x^3 + 3/2*a*c^2*d*e^3*x^2 + 3/4*(2*x^2*arcs
inh(d*x + c) - d*(3*c^2*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 +
1)*d^2))/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*x/d^2 - (c^2 + 1)*arcsinh(
2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 - 3*sqrt(d^2*x^2 +
2*c*d*x + c^2 + 1)*c/d^3))*b*c^2*d*e^3 + 1/6*(6*x^3*arcsinh(d*x + c) - d*(2
*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*x^2/d^2 - 15*c^3*arcsinh(2*(d^2*x + c*d)
/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^4 - 5*sqrt(d^2*x^2 + 2*c*d*x + c^2 +
 1)*c*x/d^3 + 9*(c^2 + 1)*c*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^
2 + 1)*d^2))/d^4 + 15*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c^2/d^4 - 4*sqrt(d^
2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)/d^4))*b*c*d^2*e^3 + 1/96*(24*x^4*arcsi
nh(d*x + c) - (6*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*x^3/d^2 - 14*sqrt(d^2*x^
2 + 2*c*d*x + c^2 + 1)*c*x^2/d^3 + 105*c^4*arcsinh(2*(d^2*x + c*d)/sqrt(-4*
c^2*d^2 + 4*(c^2 + 1)*d^2))/d^5 + 35*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c^2*
x/d^4 - 90*(c^2 + 1)*c^2*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 +
 1)*d^2))/d^5 - 105*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c^3/d^5 - 9*sqrt(d^2*
x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)*x/d^4 + 9*(c^2 + 1)^2*arcsinh(2*(d^2*x +
 c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^5 + 55*sqrt(d^2*x^2 + 2*c*d*x +
 c^2 + 1)*(c^2 + 1)*c/d^5)*d)*b*d^3*e^3 + a*c^3*e^3*x + ((d*x + c)*arcsinh(
d*x + c) - sqrt((d*x + c)^2 + 1))*b*c^3*e^3/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(c\,e+d\,e\,x\right)}^3\,\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3*(a + b*asinh(c + d*x)),x)
```

```
[Out] int((c*e + d*e*x)^3*(a + b*asinh(c + d*x)), x)
```

sympy [A] time = 1.69, size = 394, normalized size = 3.75

$$\begin{cases} ac^3e^3x + \frac{3ac^2de^3x^2}{2} + acd^2e^3x^3 + \frac{ad^3e^3x^4}{4} + \frac{bc^4e^3\operatorname{asinh}(c+dx)}{4d} + bc^3e^3x\operatorname{asinh}(c+dx) - \frac{bc^3e^3\sqrt{c^2+2cdx+d^2x^2+1}}{16d} + \frac{3bc^2d \\ c^3e^3x(a+b\operatorname{asinh}(c)) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3*(a+b*asinh(d*x+c)),x)
```

```
[Out] Piecewise((a*c**3*e**3*x + 3*a*c**2*d*e**3*x**2/2 + a*c*d**2*e**3*x**3 + a*
d**3*e**3*x**4/4 + b*c**4*e**3*asinh(c + d*x)/(4*d) + b*c**3*e**3*x*asinh(c
 + d*x) - b*c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(16*d) + 3*b*c**
2*d*e**3*x**2*asinh(c + d*x)/2 - 3*b*c**2*e**3*x*sqrt(c**2 + 2*c*d*x + d**2
*x**2 + 1)/16 + b*c*d**2*e**3*x**3*asinh(c + d*x) - 3*b*c*d*e**3*x**2*sqrt(
c**2 + 2*c*d*x + d**2*x**2 + 1)/16 + 3*b*c*e**3*sqrt(c**2 + 2*c*d*x + d**2*
x**2 + 1)/(32*d) + b*d**3*e**3*x**4*asinh(c + d*x)/4 - b*d**2*e**3*x**3*sqr
t(c**2 + 2*c*d*x + d**2*x**2 + 1)/16 + 3*b*e**3*x*sqrt(c**2 + 2*c*d*x + d**
2*x**2 + 1)/32 - 3*b*e**3*asinh(c + d*x)/(32*d), Ne(d, 0)), (c**3*e**3*x*(a
 + b*asinh(c)), True))
```

3.117 $\int (ce+dex)^2 \left(a+b\sinh^{-1}(c+dx)\right) dx$

Optimal. Leaf size=76

$$\frac{e^2(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)}{3d}-\frac{be^2\left((c+dx)^2+1\right)^{3/2}}{9d}+\frac{be^2\sqrt{(c+dx)^2+1}}{3d}$$

```
[Out] -1/9*b*e^2*(1+(d*x+c)^2)^(3/2)/d+1/3*e^2*(d*x+c)^3*(a+b*arcsinh(d*x+c))/d+1
/3*b*e^2*(1+(d*x+c)^2)^(1/2)/d
```

Rubi [A] time = 0.07, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5865, 12, 5661, 266, 43}

$$\frac{e^2(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)}{3d}-\frac{be^2\left((c+dx)^2+1\right)^{3/2}}{9d}+\frac{be^2\sqrt{(c+dx)^2+1}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2*(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (b*e^2*Sqrt[1 + (c + d*x)^2])/(3*d) - (b*e^2*(1 + (c + d*x)^2)^(3/2))/(9*d)
 + (e^2*(c + d*x)^3*(a + b*ArcSinh[c + d*x]))/(3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^2 \left(a+b\sinh^{-1}(c+dx)\right) dx = \frac{\text{Subst}\left(\int e^2x^2 \left(a+b\sinh^{-1}(x)\right) dx, x, c+dx\right)}{d}$$
$$= \frac{e^2\,\text{Subst}\left(\int x^2 \left(a+b\sinh^{-1}(x)\right) dx, x, c+dx\right)}{d}$$
$$= \frac{e^2(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)}{3d} - \frac{\left(be^2\right)\text{Subst}\left(\int \frac{x^3}{\sqrt{1+x^2}} dx, x, c+\right.}{3d}$$
$$= \frac{e^2(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)}{3d} - \frac{\left(be^2\right)\text{Subst}\left(\int \frac{x}{\sqrt{1+x}} dx, x, (c+\right.}{6d}$$
$$= \frac{e^2(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)}{3d} - \frac{\left(be^2\right)\text{Subst}\left(\int \left(-\frac{1}{\sqrt{1+x}} + \sqrt{1+\right.\right.}{6d}$$
$$= \frac{be^2\sqrt{1+(c+dx)^2}}{3d} - \frac{be^2\left(1+(c+dx)^2\right)^{3/2}}{9d} + \frac{e^2(c+dx)^3\left(a+b\sinh\right.}{3d}$$

Mathematica [A] time = 0.05, size = 64, normalized size = 0.84

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right) - \frac{1}{9}b\left(c^2+2cdx+d^2x^2-2\right)\sqrt{(c+dx)^2+1}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^2*(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (e^2*(-1/9*(b*(-2 + c^2 + 2*c*d*x + d^2*x^2)*Sqrt[1 + (c + d*x)^2]) + ((c +
 d*x)^3*(a + b*ArcSinh[c + d*x]))/3))/d
```

fricas [B] time = 0.62, size = 168, normalized size = 2.21

$$\frac{3\,ad^3e^2x^3 + 9\,acd^2e^2x^2 + 9\,ac^2de^2x + 3\left(bd^3e^2x^3 + 3\,bcd^2e^2x^2 + 3\,bc^2de^2x + bc^3e^2\right)\log\left(dx+c+\sqrt{d^2x^2+2\,cd\right.}{9\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/9*(3*a*d^3*e^2*x^3 + 9*a*c*d^2*e^2*x^2 + 9*a*c^2*d*e^2*x + 3*(b*d^3*e^2*x
^3 + 3*b*c*d^2*e^2*x^2 + 3*b*c^2*d*e^2*x + b*c^3*e^2)*log(d*x + c + sqrt(d^
2*x^2 + 2*c*d*x + c^2 + 1)) - (b*d^2*e^2*x^2 + 2*b*c*d*e^2*x + (b*c^2 - 2*b
)*e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/d
```

giac [B] time = 1.31, size = 403, normalized size = 5.30

$$\frac{1}{18}\left(6\,ad^2x^3 + 18\,acdx^2 - 18\left(d\left(\frac{c\log\left(-cd-\left(x|d|-\sqrt{d^2x^2+2\,cdx+c^2+1}\right)|d|\right)}{d|d|} + \frac{\sqrt{d^2x^2+2\,cdx+c^2+1}}{d^2}\right) - \right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/18*(6*a*d^2*x^3 + 18*a*c*d*x^2 - 18*(d*(c*log(-c*d - (x*abs(d) - sqrt(d^2
*x^2 + 2*c*d*x + c^2 + 1))*abs(d))/(d*abs(d)) + sqrt(d^2*x^2 + 2*c*d*x + c^
2 + 1)/d^2) - x*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)))*b*c^2 + 9
*(2*x^2*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - (sqrt(d^2*x^2 +
```

```
2*c*d*x + c^2 + 1)*(x/d^2 - 3*c/d^3) - (2*c^2 - 1)*log(-c*d - (x*abs(d) - s
qrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*abs(d))/(d^2*abs(d)))*d)*b*c*d + (6*x^3*l
og(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - (sqrt(d^2*x^2 + 2*c*d*x +
 c^2 + 1)*(x*(2*x/d^2 - 5*c/d^3) + (11*c^2*d - 4*d)/d^5) + 3*(2*c^3 - 3*c)*
log(-c*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*abs(d))/(d^3*abs(
d)))*d)*b*d^2 + 18*a*c^2*x)*e^2
```

maple [A] time = 0.01, size = 73, normalized size = 0.96

$$\frac{\frac{(dx+c)^3 e^2 a}{3} + e^2 b\left(\frac{(dx+c)^3 \operatorname{arcsinh}(dx+c)}{3} - \frac{(dx+c)^2\sqrt{1+(dx+c)^2}}{9} + \frac{2\sqrt{1+(dx+c)^2}}{9}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c)),x)
```

```
[Out] 1/d*(1/3*(d*x+c)^3*e^2*a+e^2*b*(1/3*(d*x+c)^3*arcsinh(d*x+c)-1/9*(d*x+c)^2*
(1+(d*x+c)^2)^(1/2)+2/9*(1+(d*x+c)^2)^(1/2)))
```

maxima [B] time = 0.36, size = 445, normalized size = 5.86

$$\frac{1}{3}\,ad^2e^2x^3+acde^2x^2+\frac{1}{2}\left(2\,x^2\operatorname{arsinh}\left(dx+c\right)-d\left(\frac{3\,c^2\operatorname{arsinh}\left(\frac{2\left(d^2x+cd\right)}{\sqrt{-4\,c^2d^2+4\left(c^2+1\right)d^2}}\right)}{d^3}+\frac{\sqrt{d^2x^2+2\,cdx+c^2+1}\,x}{d^2}-\frac{(c}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3*a*d^2*e^2*x^3 + a*c*d*e^2*x^2 + 1/2*(2*x^2*arcsinh(d*x + c) - d*(3*c^2*
arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 + sqrt(d^2*
x^2 + 2*c*d*x + c^2 + 1)*x/d^2 - (c^2 + 1)*arcsinh(2*(d^2*x + c*d)/sqrt(-4*
c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c/d^3
))*b*c*d*e^2 + 1/18*(6*x^3*arcsinh(d*x + c) - d*(2*sqrt(d^2*x^2 + 2*c*d*x +
 c^2 + 1)*x^2/d^2 - 15*c^3*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2
 + 1)*d^2))/d^4 - 5*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c*x/d^3 + 9*(c^2 + 1)
*c*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^4 + 15*sqr
t(d^2*x^2 + 2*c*d*x + c^2 + 1)*c^2/d^4 - 4*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1
)*(c^2 + 1)/d^4))*b*d^2*e^2 + a*c^2*e^2*x + ((d*x + c)*arcsinh(d*x + c) - s
qrt((d*x + c)^2 + 1))*b*c^2*e^2/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e + d\,e\,x)^2\,\left(a + b\,\operatorname{asinh}\left(c + d\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2*(a + b*asinh(c + d*x)),x)
```

```
[Out] int((c*e + d*e*x)^2*(a + b*asinh(c + d*x)), x)
```

sympy [A] time = 0.77, size = 258, normalized size = 3.39

$$\begin{cases} ac^2e^2x + acde^2x^2 + \frac{ad^2e^2x^3}{3} + \frac{bc^3e^2\operatorname{asinh}\left(c+dx\right)}{3d} + bc^2e^2x\operatorname{asinh}\left(c+dx\right) - \frac{bc^2e^2\sqrt{c^2+2cdx+d^2x^2+1}}{9d} + bcde^2x^2\operatorname{asinh}\left(c+d\right. \\ c^2e^2x\left(a+b\operatorname{asinh}\left(c\right)\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2*(a+b*asinh(d*x+c)),x)
```

```
[Out] Piecewise((a*c**2*e**2*x + a*c*d*e**2*x**2 + a*d**2*e**2*x**3/3 + b*c**3*e*
*2*asinh(c + d*x)/(3*d) + b*c**2*e**2*x*asinh(c + d*x) - b*c**2*e**2*sqrt(c
**2 + 2*c*d*x + d**2*x**2 + 1)/(9*d) + b*c*d*e**2*x**2*asinh(c + d*x) - 2*b
*c*e**2*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/9 + b*d**2*e**2*x**3*asinh(c
 + d*x)/3 - b*d*e**2*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/9 + 2*b*e**2
*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(9*d), Ne(d, 0)), (c**2*e**2*x*(a + b
*asinh(c)), True))
```

3.118 $\int (ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)\,dx$

Optimal. Leaf size=68

$$\frac{e(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)}{2d}-\frac{be\sqrt{(c+dx)^2+1}\,(c+dx)}{4d}+\frac{be\sinh^{-1}(c+dx)}{4d}$$

```
[Out] 1/4*b*e*arcsinh(d*x+c)/d+1/2*e*(d*x+c)^2*(a+b*arcsinh(d*x+c))/d-1/4*b*e*(d*
x+c)*(1+(d*x+c)^2)^(1/2)/d
```

Rubi [A] time = 0.04, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {5865, 12, 5661, 321, 215}

$$\frac{e(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)}{2d}-\frac{be\sqrt{(c+dx)^2+1}\,(c+dx)}{4d}+\frac{be\sinh^{-1}(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)*(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] -(b*e*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(4*d) + (b*e*ArcSinh[c + d*x])/(4*d)
 + (e*(c + d*x)^2*(a + b*ArcSinh[c + d*x]))/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)\,dx = \frac{\text{Subst}\left(\int ex\left(a+b\sinh^{-1}(x)\right)\,dx,x,c+dx\right)}{d}$$
$$= \frac{e\,\text{Subst}\left(\int x\left(a+b\sinh^{-1}(x)\right)\,dx,x,c+dx\right)}{d}$$
$$= \frac{e(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)}{2d} - \frac{(be)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{1+x^2}}\,dx,x,c+dx\right.}{2d}$$
$$= -\frac{be(c+dx)\sqrt{1+(c+dx)^2}}{4d} + \frac{e(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)}{2d} + \frac{(b\ldots}{}$$
$$= -\frac{be(c+dx)\sqrt{1+(c+dx)^2}}{4d} + \frac{be\sinh^{-1}(c+dx)}{4d} + \frac{e(c+dx)^2\left(a+b\,\text{s}\ldots\right.}{2d}$$

Mathematica [A] time = 0.06, size = 57, normalized size = 0.84

$$\frac{e\left(2(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)-b\sqrt{(c+dx)^2+1}\,(c+dx)+b\sinh^{-1}(c+dx)\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)*(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (e*(-(b*(c + d*x)*Sqrt[1 + (c + d*x)^2]) + b*ArcSinh[c + d*x] + 2*(c + d*x)
^2*(a + b*ArcSinh[c + d*x])))/(4*d)
```

fricas [A] time = 0.55, size = 108, normalized size = 1.59

$$\frac{2\,ad^2ex^2+4\,acdex+\left(2\,bd^2ex^2+4\,bcdex+\left(2\,bc^2+b\right)e\right)\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)-(bdex+bce\ldots}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*d^2*e*x^2 + 4*a*c*d*e*x + (2*b*d^2*e*x^2 + 4*b*c*d*e*x + (2*b*c^2
+ b)*e)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - (b*d*e*x + b*c*e
)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/d
```

giac [B] time = 0.96, size = 243, normalized size = 3.57

$$\frac{1}{4}\left(2\,adx^2-4\left(d\left(\frac{c\log\left(-cd-\left(x|d|-\sqrt{d^2x^2+2\,cdx+c^2+1}\right)|d|\right)}{d|d|}+\frac{\sqrt{d^2x^2+2\,cdx+c^2+1}}{d^2}\right)-x\log\left(dx+c\ldots\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/4*(2*a*d*x^2 - 4*(d*(c*log(-c*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^
2 + 1))*abs(d))/(d*abs(d)) + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^2) - x*log
(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)))*b*c + (2*x^2*log(d*x + c + s
qrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - (sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(x/d
^2 - 3*c/d^3) - (2*c^2 - 1)*log(-c*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x +
 c^2 + 1))*abs(d))/(d^2*abs(d)))*d)*b*d + 4*a*c*x)*e
```

maple [A] time = 0.01, size = 62, normalized size = 0.91

$$\frac{\frac{(dx+c)^2 ea}{2}+eb\left(\frac{(dx+c)^2 \operatorname{arcsinh}(dx+c)}{2}-\frac{(dx+c)\sqrt{1+(dx+c)^2}}{4}+\frac{\operatorname{arcsinh}(dx+c)}{4}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)*(a+b*arcsinh(d*x+c)),x)
```

```
[Out] 1/d*(1/2*(d*x+c)^2*e*a+e*b*(1/2*(d*x+c)^2*arcsinh(d*x+c)-1/4*(d*x+c)*(1+(d*
x+c)^2)^(1/2)+1/4*arcsinh(d*x+c)))
```

maxima [B] time = 0.44, size = 201, normalized size = 2.96

$$\frac{1}{2}\,adex^2+\frac{1}{4}\left(2\,x^2\operatorname{arsinh}\left(dx+c\right)-d\left(\frac{3\,c^2\operatorname{arsinh}\left(\frac{2\left(d^2x+cd\right)}{\sqrt{-4\,c^2d^2+4\left(c^2+1\right)d^2}}\right)}{d^3}+\frac{\sqrt{d^2x^2+2\,cdx+c^2+1}\,x}{d^2}-\frac{\left(c^2+1\right)\operatorname{arsir}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*a*d*e*x^2 + 1/4*(2*x^2*arcsinh(d*x + c) - d*(3*c^2*arcsinh(2*(d^2*x + c
*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c^2
+ 1)*x/d^2 - (c^2 + 1)*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1
)*d^2))/d^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c/d^3))*b*d*e + a*c*e*x +
 ((d*x + c)*arcsinh(d*x + c) - sqrt((d*x + c)^2 + 1))*b*c*e/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e+d\,e\,x)\,\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)*(a + b*asinh(c + d*x)),x)
```

```
[Out] int((c*e + d*e*x)*(a + b*asinh(c + d*x)), x)
```

sympy [A] time = 0.33, size = 148, normalized size = 2.18

$$\begin{cases} acex+\frac{adex^2}{2}+\frac{bc^2e\operatorname{asinh}(c+dx)}{2d}+bcex\operatorname{asinh}(c+dx)-\frac{bce\sqrt{c^2+2cdx+d^2x^2+1}}{4d}+\frac{bdex^2\operatorname{asinh}(c+dx)}{2}-\frac{bex\sqrt{c^2+2cdx+d^2x^2+1}}{4}+ \\ cex\left(a+b\operatorname{asinh}(c)\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*asinh(d*x+c)),x)
```

```
[Out] Piecewise((a*c*e*x + a*d*e*x**2/2 + b*c**2*e*asinh(c + d*x)/(2*d) + b*c*e*x
*asinh(c + d*x) - b*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(4*d) + b*d*e*
x**2*asinh(c + d*x)/2 - b*e*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/4 + b*e*
asinh(c + d*x)/(4*d), Ne(d, 0)), (c*e*x*(a + b*asinh(c)), True))
```

3.119 $\int \left(a+b \sinh^{-1}(c+dx)\right) dx$

Optimal. Leaf size=39

$$ax-\frac{b\sqrt{(c+dx)^2+1}}{d}+\frac{b(c+dx)\sinh^{-1}(c+dx)}{d}$$

```
[Out] a*x+b*(d*x+c)*arcsinh(d*x+c)/d-b*(1+(d*x+c)^2)^(1/2)/d
```

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5863, 5653, 261}

$$ax-\frac{b\sqrt{(c+dx)^2+1}}{d}+\frac{b(c+dx)\sinh^{-1}(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcSinh[c + d*x],x]
```

```
[Out] a*x - (b*Sqrt[1 + (c + d*x)^2])/d + (b*(c + d*x)*ArcSinh[c + d*x])/d
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5863

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\begin{aligned}
\int \left(a+b\sinh^{-1}(c+dx)\right) dx &= ax+b\int \sinh^{-1}(c+dx)\,dx\\
&= ax+\frac{b\,\text{Subst}\left(\int \sinh^{-1}(x)\,dx,x,c+dx\right)}{d}\\
&= ax+\frac{b(c+dx)\sinh^{-1}(c+dx)}{d}-\frac{b\,\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}}\,dx,x,c+dx\right)}{d}\\
&= ax-\frac{b\sqrt{1+(c+dx)^2}}{d}+\frac{b(c+dx)\sinh^{-1}(c+dx)}{d}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 50, normalized size = 1.28

$$ax-\frac{b\left(\sqrt{c^2+2cdx+d^2x^2+1}-c\sinh^{-1}(c+dx)\right)}{d}+bx\sinh^{-1}(c+dx)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcSinh[c + d*x],x]
```

```
[Out] a*x + b*x*ArcSinh[c + d*x] - (b*(Sqrt[1 + c^2 + 2*c*d*x + d^2*x^2] - c*ArcS
inh[c + d*x]))/d
```

fricas [A] time = 0.51, size = 65, normalized size = 1.67

$$\frac{adx + (bdx + bc)\log\left(dx + c + \sqrt{d^2x^2 + 2\,cdx + c^2 + 1}\right) - \sqrt{d^2x^2 + 2\,cdx + c^2 + 1}\,b}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arcsinh(d*x+c),x, algorithm="fricas")
```

```
[Out] (a*d*x + (b*d*x + b*c)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - s
qrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*b)/d
```

giac [B] time = 0.36, size = 99, normalized size = 2.54

$$-\left(d\left(\frac{c\log\left(-cd - \left(x|d| - \sqrt{d^2x^2 + 2\,cdx + c^2 + 1}\right)|d|\right)}{d|d|} + \frac{\sqrt{d^2x^2 + 2\,cdx + c^2 + 1}}{d^2}\right) - x\log\left(dx + c + \sqrt{(dx + c)^2 +}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arcsinh(d*x+c),x, algorithm="giac")
```

```
[Out] -(d*(c*log(-c*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*abs(d))/(d
*abs(d)) + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^2) - x*log(d*x + c + sqrt((d
*x + c)^2 + 1)))*b + a*x
```

maple [A] time = 0.00, size = 36, normalized size = 0.92

$$ax + \frac{b\left((dx + c)\operatorname{arcsinh}(dx + c) - \sqrt{1 + (dx + c)^2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arcsinh(d*x+c),x)
```

```
[Out] a*x+b/d*((d*x+c)*arcsinh(d*x+c)-(1+(d*x+c)^2)^(1/2))
```

maxima [A] time = 0.34, size = 35, normalized size = 0.90

$$ax + \frac{\left((dx + c)\operatorname{arsinh}(dx + c) - \sqrt{(dx + c)^2 + 1}\right)b}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arcsinh(d*x+c),x, algorithm="maxima")
```

```
[Out] a*x + ((d*x + c)*arcsinh(d*x + c) - sqrt((d*x + c)^2 + 1))*b/d
```

mupad [B] time = 0.48, size = 85, normalized size = 2.18

$$a\,x + b\,x\,\operatorname{asinh}(c + d\,x) - \frac{b\,\sqrt{c^2 + 2\,c\,d\,x + d^2\,x^2 + 1}}{d} + \frac{b\,c\,d^2\,\ln\left(\sqrt{c^2 + 2\,c\,d\,x + d^2\,x^2 + 1} + \frac{x\,d^2 + c\,d}{\sqrt{d^2}}\right)}{\left(d^2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*asinh(c + d*x),x)
```

```
[Out] a*x + b*x*asinh(c + d*x) - (b*(c^2 + d^2*x^2 + 2*c*d*x + 1)^(1/2))/d + (b*c
*d^2*log((c^2 + d^2*x^2 + 2*c*d*x + 1)^(1/2) + (c*d + d^2*x)/(d^2)^(1/2)))/
(d^2)^(3/2)
```

sympy [A] time = 0.16, size = 51, normalized size = 1.31

$$ax + b\left(\begin{cases} \frac{c \operatorname{asinh}(c+dx)}{d} + x \operatorname{asinh}(c + dx) - \frac{\sqrt{c^2+2cdx+d^2x^2+1}}{d} & \text{for } d \neq 0 \\ x \operatorname{asinh}(c) & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*asinh(d*x+c),x)
```

```
[Out] a*x + b*Piecewise((c*asinh(c + d*x)/d + x*asinh(c + d*x) - sqrt(c**2 + 2*c*
d*x + d**2*x**2 + 1)/d, Ne(d, 0)), (x*asinh(c), True))
```

3.120 $\int \frac{a+b\sinh^{-1}(c+dx)}{ce+dex}\,dx$

Optimal. Leaf size=81

$$\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{2bde}+\frac{\log\left(1-e^{-2\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)}{de}-\frac{b\text{Li}_2\left(e^{-2\sinh^{-1}(c+dx)}\right)}{2de}$$

```
[Out] 1/2*(a+b*arcsinh(d*x+c))^2/b/d/e+(a+b*arcsinh(d*x+c))*ln(1-1/(d*x+c+(1+(d*x
+c)^2)^(1/2))^2)/d/e-1/2*b*polylog(2,1/(d*x+c+(1+(d*x+c)^2)^(1/2))^2)/d/e
```

Rubi [A] time = 0.11, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5865, 12, 5659, 3716, 2190, 2279, 2391}

$$\frac{b\text{PolyLog}\left(2,e^{2\sinh^{-1}(c+dx)}\right)}{2de}-\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{2bde}+\frac{\log\left(1-e^{2\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)}{de}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x),x]
```

```
[Out] -(a + b*ArcSinh[c + d*x])^2/(2*b*d*e) + ((a + b*ArcSinh[c + d*x])*Log[1 - E
^(2*ArcSinh[c + d*x])])/(d*e) + (b*PolyLog[2, E^(2*ArcSinh[c + d*x])])/(2*d
*e)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\sinh^{-1}(c+dx)}{ce+dex}\,dx &= \frac{\text{Subst}\left(\int \frac{a+b\sinh^{-1}(x)}{ex}\,dx, x, c+dx\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{a+b\sinh^{-1}(x)}{x}\,dx, x, c+dx\right)}{de}\\ &= \frac{\text{Subst}\left(\int (a+bx)\coth(x)\,dx, x, \sinh^{-1}(c+dx)\right)}{de}\\ &= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{2bde} - \frac{2\,\text{Subst}\left(\int \frac{e^{2x}(a+bx)}{1-e^{2x}}\,dx, x, \sinh^{-1}(c+dx)\right)}{de}\\ &= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{2bde} + \frac{\left(a+b\sinh^{-1}(c+dx)\right)\log\left(1-e^{2\sinh^{-1}(c+dx)}\right)}{de} - \frac{b\,\text{Su}\ldots}{\ldots}\\ &= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{2bde} + \frac{\left(a+b\sinh^{-1}(c+dx)\right)\log\left(1-e^{2\sinh^{-1}(c+dx)}\right)}{de} - \frac{b\,\text{Su}\ldots}{\ldots}\\ &= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{2bde} + \frac{\left(a+b\sinh^{-1}(c+dx)\right)\log\left(1-e^{2\sinh^{-1}(c+dx)}\right)}{de} + \frac{b\text{Li}\ldots}{\ldots}\end{aligned}$$

Mathematica [A] time = 0.03, size = 70, normalized size = 0.86

$$\frac{b^2\text{Li}_2\left(e^{2\sinh^{-1}(c+dx)}\right) - \left(a+b\sinh^{-1}(c+dx)\right)\left(a+b\sinh^{-1}(c+dx) - 2b\log\left(1-e^{2\sinh^{-1}(c+dx)}\right)\right)}{2bde}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x),x]
```

```
[Out] (-((a + b*ArcSinh[c + d*x])*(a + b*ArcSinh[c + d*x] - 2*b*Log[1 - E^(2*ArcS
inh[c + d*x])])) + b^2*PolyLog[2, E^(2*ArcSinh[c + d*x])])/(2*b*d*e)
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\,\text{arsinh}\,(dx+c)+a}{dex+ce}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e),x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(d*x + c) + a)/(d*e*x + c*e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{arsinh}\,(dx+c)+a}{dex+ce}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)/(d*e*x + c*e), x)
```

maple [A] time = 0.10, size = 159, normalized size = 1.96

$$\frac{a\ln(dx+c)}{de}-\frac{b\,\mathrm{arcsinh}(dx+c)^2}{2de}+\frac{b\,\mathrm{arcsinh}(dx+c)\ln\left(1+dx+c+\sqrt{1+(dx+c)^2}\right)}{de}+\frac{b\,\mathrm{polylog}\left(2,-dx-c-\right.}{de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))/(d*e*x+c*e),x)
```

```
[Out] 1/d*a/e*ln(d*x+c)-1/2/d*b/e*arcsinh(d*x+c)^2+1/d*b/e*arcsinh(d*x+c)*ln(1+d*
x+c+(1+(d*x+c)^2)^(1/2))+1/d*b/e*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))+1/d*
b/e*arcsinh(d*x+c)*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))+1/d*b/e*polylog(2,d*x+c+
(1+(d*x+c)^2)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b\int\frac{\log\left(dx+c+\sqrt{(dx+c)^2+1}\right)}{dex+ce}\,dx+\frac{a\log(dex+ce)}{de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e),x, algorithm="maxima")
```

```
[Out] b*integrate(log(d*x + c + sqrt((d*x + c)^2 + 1))/(d*e*x + c*e), x) + a*log(
d*e*x + c*e)/(d*e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{a+b\,\mathrm{asinh}(c+d\,x)}{c\,e+d\,e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))/(c*e + d*e*x),x)
```

```
[Out] int((a + b*asinh(c + d*x))/(c*e + d*e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{a}{c+dx}\,dx+\int\frac{b\,\mathrm{asinh}(c+dx)}{c+dx}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))/(d*e*x+c*e),x)
```

```
[Out] (Integral(a/(c + d*x), x) + Integral(b*asinh(c + d*x)/(c + d*x), x))/e
```

3.121 $\int \frac{a+b\sinh^{-1}(c+dx)}{(ce+dex)^2}\,dx$

Optimal. Leaf size=49

$$-\frac{a+b\sinh^{-1}(c+dx)}{de^2(c+dx)}-\frac{b\tanh^{-1}\left(\sqrt{(c+dx)^2+1}\right)}{de^2}$$

```
[Out] (-a-b*arcsinh(d*x+c))/d/e^2/(d*x+c)-b*arctanh((1+(d*x+c)^2)^(1/2))/d/e^2
```

Rubi [A] time = 0.05, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5865, 12, 5661, 266, 63, 207}

$$-\frac{a+b\sinh^{-1}(c+dx)}{de^2(c+dx)}-\frac{b\tanh^{-1}\left(\sqrt{(c+dx)^2+1}\right)}{de^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^2,x]
```

```
[Out] -((a + b*ArcSinh[c + d*x])/(d*e^2*(c + d*x))) - (b*ArcTanh[Sqrt[1 + (c + d*
x)^2]])/(d*e^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
```

```
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\sinh^{-1}(c+dx)}{(ce+dex)^2}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{a+b\sinh^{-1}(x)}{e^2x^2}\,dx, x, c+dx\right)}{d}\\
&= \frac{\operatorname{Subst}\left(\int \frac{a+b\sinh^{-1}(x)}{x^2}\,dx, x, c+dx\right)}{de^2}\\
&= -\frac{a+b\sinh^{-1}(c+dx)}{de^2(c+dx)} + \frac{b\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1+x^2}}\,dx, x, c+dx\right)}{de^2}\\
&= -\frac{a+b\sinh^{-1}(c+dx)}{de^2(c+dx)} + \frac{b\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, (c+dx)^2\right)}{2de^2}\\
&= -\frac{a+b\sinh^{-1}(c+dx)}{de^2(c+dx)} + \frac{b\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+(c+dx)^2}\right)}{de^2}\\
&= -\frac{a+b\sinh^{-1}(c+dx)}{de^2(c+dx)} - \frac{b\tanh^{-1}\left(\sqrt{1+(c+dx)^2}\right)}{de^2}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 43, normalized size = 0.88

$$
-\frac{\frac{a+b\sinh^{-1}(c+dx)}{c+dx} + b\tanh^{-1}\left(\sqrt{(c+dx)^2+1}\right)}{de^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^2,x]
[Out] -(((a + b*ArcSinh[c + d*x])/(c + d*x) + b*ArcTanh[Sqrt[1 + (c + d*x)^2]])/(
d*e^2))
```

fricas [B] time = 0.58, size = 175, normalized size = 3.57

$$
\frac{bdx\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right) - ac - \left(bcdx+bc^2\right)\log\left(-dx-c+\sqrt{d^2x^2+2\,cdx+c^2+1}+1\right) + (bd}{cd^2e^2x+c}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^2,x, algorithm="fricas")
[Out] (b*d*x*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - a*c - (b*c*d*x +
b*c^2)*log(-d*x - c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1) + 1) + (b*d*x + b*c
)*log(-d*x - c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + (b*c*d*x + b*c^2)*log
(-d*x - c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1) - 1))/(c*d^2*e^2*x + c^2*d*e^
2)
```

giac [B] time = 0.49, size = 131, normalized size = 2.67

$$
-b\left(\frac{e^{(-1)}\log\left(dx+c+\sqrt{(dx+c)^2+1}\right)}{(dxe+ce)d} + \frac{de^{(-2)}\log\left(\sqrt{\frac{e^2}{(dxe+ce)^2}+1}+\frac{\sqrt{d^2}\,e}{(dxe+ce)d}\right)}{|d|^2\operatorname{sgn}\left(\frac{1}{dxe+ce}\right)\operatorname{sgn}(d)}\right) - \frac{ae^{(-1)}}{(dxe+ce)d}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^2,x, algorithm="giac")
```

```
[Out] -b*(e^(-1)*log(d*x + c + sqrt((d*x + c)^2 + 1))/((d*x*e + c*e)*d) + d*e^(-2
)*log(sqrt(e^2/(d*x*e + c*e)^2 + 1) + sqrt(d^2)*e/((d*x*e + c*e)*d))/(abs(d
)^2*sgn(1/(d*x*e + c*e))*sgn(d))) - a*e^(-1)/((d*x*e + c*e)*d)
```

maple [A] time = 0.00, size = 54, normalized size = 1.10

$$\frac{-\frac{a}{e^2(dx+c)}+\frac{b\left(-\frac{\operatorname{arcsinh}(dx+c)}{dx+c}-\operatorname{arctanh}\left(\frac{1}{\sqrt{1+(dx+c)^2}}\right)\right)}{e^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^2,x)
```

```
[Out] 1/d*(-a/e^2/(d*x+c)+b/e^2*(-1/(d*x+c)*arcsinh(d*x+c)-arctanh(1/(1+(d*x+c)^2
)^(1/2))))
```

maxima [A] time = 0.37, size = 80, normalized size = 1.63

$$-b\left(\frac{\operatorname{arsinh}\left(dx+c\right)}{d^2e^2x+cde^2}+\frac{\operatorname{arsinh}\left(\frac{de^2}{|d^2e^2x+cde^2|}\right)}{de^2}\right)-\frac{a}{d^2e^2x+cde^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^2,x, algorithm="maxima")
```

```
[Out] -b*(arcsinh(d*x + c)/(d^2*e^2*x + c*d*e^2) + arcsinh(d*e^2/abs(d^2*e^2*x +
c*d*e^2))/(d*e^2)) - a/(d^2*e^2*x + c*d*e^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a+b\operatorname{asinh}\left(c+d\,x\right)}{{\left(c\,e+d\,e\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))/(c*e + d*e*x)^2,x)
```

```
[Out] int((a + b*asinh(c + d*x))/(c*e + d*e*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^2+2cdx+d^2x^2}\,dx+\int \frac{b\operatorname{asinh}\left(c+dx\right)}{c^2+2cdx+d^2x^2}\,dx}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))/(d*e*x+c*e)**2,x)
```

```
[Out] (Integral(a/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(b*asinh(c + d*x)/(c
**2 + 2*c*d*x + d**2*x**2), x))/e**2
```

3.122 $\int \frac{a+b\sinh^{-1}(c+dx)}{(ce+dex)^3}\,dx$

Optimal. Leaf size=59

$$-\frac{a+b\sinh^{-1}(c+dx)}{2de^3(c+dx)^2}-\frac{b\sqrt{(c+dx)^2+1}}{2de^3(c+dx)}$$

```
[Out] 1/2*(-a-b*arcsinh(d*x+c))/d/e^3/(d*x+c)^2-1/2*b*(1+(d*x+c)^2)^(1/2)/d/e^3/(
d*x+c)
```

Rubi [A] time = 0.05, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {5865, 12, 5661, 264}

$$-\frac{a+b\sinh^{-1}(c+dx)}{2de^3(c+dx)^2}-\frac{b\sqrt{(c+dx)^2+1}}{2de^3(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^3,x]
```

```
[Out] -(b*Sqrt[1 + (c + d*x)^2])/(2*d*e^3*(c + d*x)) - (a + b*ArcSinh[c + d*x])/(
2*d*e^3*(c + d*x)^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(c+dx)}{(ce+dex)^3}\,dx = \frac{\text{Subst}\left(\int \frac{a+b\sinh^{-1}(x)}{e^3x^3}\,dx,x,c+dx\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{a+b\sinh^{-1}(x)}{x^3}\,dx,x,c+dx\right)}{de^3}$$
$$= -\frac{a+b\sinh^{-1}(c+dx)}{2de^3(c+dx)^2}+\frac{b\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{1+x^2}}\,dx,x,c+dx\right)}{2de^3}$$
$$= -\frac{b\sqrt{1+(c+dx)^2}}{2de^3(c+dx)}-\frac{a+b\sinh^{-1}(c+dx)}{2de^3(c+dx)^2}$$

Mathematica [A] time = 0.06, size = 57, normalized size = 0.97

$$\frac{\frac{-a-b\sinh^{-1}(c+dx)}{2(c+dx)^2}-\frac{b\sqrt{(c+dx)^2+1}}{2(c+dx)}}{de^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^3,x]
```

```
[Out] (-1/2*(b*Sqrt[1 + (c + d*x)^2])/(c + d*x) + (-a - b*ArcSinh[c + d*x])/(2*(c
 + d*x)^2))/(d*e^3)
```

fricas [B] time = 0.55, size = 118, normalized size = 2.00

$$\frac{ad^2x^2+2\,acdx-bc^2\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)-\left(bc^2dx+bc^3\right)\sqrt{d^2x^2+2\,cdx+c^2+1}}{2\left(c^2d^3e^3x^2+2\,c^3d^2e^3x+c^4de^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(a*d^2*x^2 + 2*a*c*d*x - b*c^2*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c
^2 + 1)) - (b*c^2*d*x + b*c^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(c^2*d^3*
e^3*x^2 + 2*c^3*d^2*e^3*x + c^4*d*e^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{arsinh}\left(dx+c\right)+a}{\left(dex+ce\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)/(d*e*x + c*e)^3, x)
```

maple [A] time = 0.01, size = 60, normalized size = 1.02

$$\frac{-\frac{a}{2e^3(dx+c)^2}+\frac{b\left(-\frac{\text{arcsinh}(dx+c)}{2(dx+c)^2}-\frac{\sqrt{1+(dx+c)^2}}{2(dx+c)}\right)}{e^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^3,x)
```

```
[Out] 1/d*(-1/2*a/e^3/(d*x+c)^2+b/e^3*(-1/2/(d*x+c)^2*arcsinh(d*x+c)-1/2/(d*x+c)*
(1+(d*x+c)^2)^(1/2)))
```

maxima [B] time = 0.41, size = 117, normalized size = 1.98

$$-\frac{1}{2}\,b\left(\frac{\sqrt{d^2x^2+2\,cdx+c^2+1}\,d}{d^3e^3x+cd^2e^3}+\frac{\operatorname{arsinh}\left(dx+c\right)}{d^3e^3x^2+2\,cd^2e^3x+c^2de^3}\right)-\frac{a}{2\left(d^3e^3x^2+2\,cd^2e^3x+c^2de^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^3,x, algorithm="maxima")
```

```
[Out] -1/2*b*(sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*d/(d^3*e^3*x + c*d^2*e^3) + arcsi
nh(d*x + c)/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3)) - 1/2*a/(d^3*e^3*x^2
 + 2*c*d^2*e^3*x + c^2*d*e^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a+b\,\mathrm{asinh}\left(c+d\,x\right)}{{\left(c\,e+d\,e\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))/(c*e + d*e*x)^3,x)
```

```
[Out] int((a + b*asinh(c + d*x))/(c*e + d*e*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx+\int \frac{b\operatorname{asinh}\left(c+dx\right)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))/(d*e*x+c*e)**3,x)
```

```
[Out] (Integral(a/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integral(
b*asinh(c + d*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x))/e**3
```

3.123 $\int \frac{a+b\sinh^{-1}(c+dx)}{(ce+dex)^4} dx$

Optimal. Leaf size=84

$$-\frac{a+b\sinh^{-1}(c+dx)}{3de^4(c+dx)^3}-\frac{b\sqrt{(c+dx)^2+1}}{6de^4(c+dx)^2}+\frac{b\tanh^{-1}\left(\sqrt{(c+dx)^2+1}\right)}{6de^4}$$

```
[Out] 1/3*(-a-b*arcsinh(d*x+c))/d/e^4/(d*x+c)^3+1/6*b*arctanh((1+(d*x+c)^2)^(1/2)
)/d/e^4-1/6*b*(1+(d*x+c)^2)^(1/2)/d/e^4/(d*x+c)^2
```

Rubi [A] time = 0.07, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5865, 12, 5661, 266, 51, 63, 207}

$$-\frac{a+b\sinh^{-1}(c+dx)}{3de^4(c+dx)^3}-\frac{b\sqrt{(c+dx)^2+1}}{6de^4(c+dx)^2}+\frac{b\tanh^{-1}\left(\sqrt{(c+dx)^2+1}\right)}{6de^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^4,x]
```

```
[Out] -(b*Sqrt[1 + (c + d*x)^2])/(6*d*e^4*(c + d*x)^2) - (a + b*ArcSinh[c + d*x])
/(3*d*e^4*(c + d*x)^3) + (b*ArcTanh[Sqrt[1 + (c + d*x)^2]])/(6*d*e^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\sinh^{-1}(c+dx)}{(ce+dex)^4}\,dx &= \frac{\text{Subst}\left(\int \frac{a+b\sinh^{-1}(x)}{e^4x^4}\,dx,x,c+dx\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{a+b\sinh^{-1}(x)}{x^4}\,dx,x,c+dx\right)}{de^4} \\
&= -\frac{a+b\sinh^{-1}(c+dx)}{3de^4(c+dx)^3}+\frac{b\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{1+x^2}}\,dx,x,c+dx\right)}{3de^4} \\
&= -\frac{a+b\sinh^{-1}(c+dx)}{3de^4(c+dx)^3}+\frac{b\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{1+x}}\,dx,x,(c+dx)^2\right)}{6de^4} \\
&= -\frac{b\sqrt{1+(c+dx)^2}}{6de^4(c+dx)^2}-\frac{a+b\sinh^{-1}(c+dx)}{3de^4(c+dx)^3}-\frac{b\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx,x,(c+dx)^2\right)}{12de^4} \\
&= -\frac{b\sqrt{1+(c+dx)^2}}{6de^4(c+dx)^2}-\frac{a+b\sinh^{-1}(c+dx)}{3de^4(c+dx)^3}-\frac{b\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\sqrt{1+(c+dx)^2}\right)}{6de^4} \\
&= -\frac{b\sqrt{1+(c+dx)^2}}{6de^4(c+dx)^2}-\frac{a+b\sinh^{-1}(c+dx)}{3de^4(c+dx)^3}+\frac{b\tanh^{-1}\left(\sqrt{1+(c+dx)^2}\right)}{6de^4}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 82, normalized size = 0.98

$$-\frac{2a+b\sqrt{c^2+2cdx+d^2x^2+1}\,(c+dx)+2b\sinh^{-1}(c+dx)-b(c+dx)^3\tanh^{-1}\left(\sqrt{(c+dx)^2+1}\right)}{6de^4(c+dx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^4,x]
```

```
[Out] -1/6*(2*a + b*(c + d*x)*Sqrt[1 + c^2 + 2*c*d*x + d^2*x^2] + 2*b*ArcSinh[c +
 d*x] - b*(c + d*x)^3*ArcTanh[Sqrt[1 + (c + d*x)^2]])/(d*e^4*(c + d*x)^3)
```

fricas [B] time = 0.56, size = 343, normalized size = 4.08

$$-\frac{2\,ac^3-2\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx\right)\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)-\left(bc^3d^3x^3+3\,bc^4d^2x^2+3\,bc^5dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^4,x, algorithm="fricas")
```

```
[Out] -1/6*(2*a*c^3 - 2*(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x)*log(d*x + c + s
qrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - (b*c^3*d^3*x^3 + 3*b*c^4*d^2*x^2 + 3*b*
c^5*d*x + b*c^6)*log(-d*x - c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1) + 1) - 2*
(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*log(-d*x - c + sqrt(d^2*x
^2 + 2*c*d*x + c^2 + 1)) + (b*c^3*d^3*x^3 + 3*b*c^4*d^2*x^2 + 3*b*c^5*d*x +
 b*c^6)*log(-d*x - c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1) - 1) + (b*c^3*d*x
+ b*c^4)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(c^3*d^4*e^4*x^3 + 3*c^4*d^3*e^
4*x^2 + 3*c^5*d^2*e^4*x + c^6*d*e^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arsinh}(dx + c) + a}{(dex + ce)^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^4,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)/(d*e*x + c*e)^4, x)
```

maple [A] time = 0.01, size = 74, normalized size = 0.88

$$\frac{-\frac{a}{3e^4(dx+c)^3} + \frac{b\left(-\frac{\operatorname{arcsinh}(dx+c)}{3(dx+c)^3} - \frac{\sqrt{1+(dx+c)^2}}{6(dx+c)^2} + \frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{1+(dx+c)^2}}\right)}{6}\right)}{e^4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^4,x)
```

```
[Out] 1/d*(-1/3*a/e^4/(d*x+c)^3+b/e^4*(-1/3/(d*x+c)^3*arcsinh(d*x+c)-1/6/(d*x+c)^
2*(1+(d*x+c)^2)^(1/2)+1/6*arctanh(1/(1+(d*x+c)^2)^(1/2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{6}\,b\left(\frac{2\left(d^2x^2 + 2\,cdx + c^2 + \log\left(dx + c + \sqrt{d^2x^2 + 2\,cdx + c^2 + 1}\right)\right)}{d^4e^4x^3 + 3\,cd^3e^4x^2 + 3\,c^2d^2e^4x + c^3de^4} - \frac{i\left(\log\left(\frac{i\left(d^2x+cd\right)}{d} + 1\right) - \log\left(-\frac{i\left(d^2x+cd\right)}{d}\right.\right.}{de^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^4,x, algorithm="maxima")
```

```
[Out] -1/6*b*(2*(d^2*x^2 + 2*c*d*x + c^2 + log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x +
 c^2 + 1)))/(d^4*e^4*x^3 + 3*c*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^4) -
 I*(log(I*(d^2*x + c*d)/d + 1) - log(-I*(d^2*x + c*d)/d + 1))/(d*e^4) - 6*i
ntegrate(1/3/(d^6*e^4*x^6 + 6*c*d^5*e^4*x^5 + c^6*e^4 + c^4*e^4 + (15*c^2*d
^4*e^4 + d^4*e^4)*x^4 + 4*(5*c^3*d^3*e^4 + c*d^3*e^4)*x^3 + 3*(5*c^4*d^2*e^
4 + 2*c^2*d^2*e^4)*x^2 + 2*(3*c^5*d*e^4 + 2*c^3*d*e^4)*x + (d^5*e^4*x^5 + 5
*c*d^4*e^4*x^4 + c^5*e^4 + c^3*e^4 + (10*c^2*d^3*e^4 + d^3*e^4)*x^3 + (10*c
^3*d^2*e^4 + 3*c*d^2*e^4)*x^2 + (5*c^4*d*e^4 + 3*c^2*d*e^4)*x)*sqrt(d^2*x^2
 + 2*c*d*x + c^2 + 1)), x)) - 1/3*a/(d^4*e^4*x^3 + 3*c*d^3*e^4*x^2 + 3*c^2*
d^2*e^4*x + c^3*d*e^4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b\,\mathrm{asinh}\left(c + d\,x\right)}{{\left(c\,e + d\,e\,x\right)}^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))/(c*e + d*e*x)^4,x)
```

```
[Out] int((a + b*asinh(c + d*x))/(c*e + d*e*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{b\,\text{asinh}\,(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))/(d*e*x+c*e)**4,x)
```

```
[Out] (Integral(a/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x*
*4), x) + Integral(b*asinh(c + d*x)/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 +
 4*c*d**3*x**3 + d**4*x**4), x))/e**4
```

3.124 $\int \frac{a+b\sinh^{-1}(c+dx)}{(ce+dex)^5}\,dx$

Optimal. Leaf size=90

$$-\frac{a+b\sinh^{-1}(c+dx)}{4de^5(c+dx)^4}+\frac{b\sqrt{(c+dx)^2+1}}{6de^5(c+dx)}-\frac{b\sqrt{(c+dx)^2+1}}{12de^5(c+dx)^3}$$

```
[Out] 1/4*(-a-b*arcsinh(d*x+c))/d/e^5/(d*x+c)^4-1/12*b*(1+(d*x+c)^2)^(1/2)/d/e^5/
(d*x+c)^3+1/6*b*(1+(d*x+c)^2)^(1/2)/d/e^5/(d*x+c)
```

Rubi [A] time = 0.07, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5865, 12, 5661, 271, 264}

$$-\frac{a+b\sinh^{-1}(c+dx)}{4de^5(c+dx)^4}+\frac{b\sqrt{(c+dx)^2+1}}{6de^5(c+dx)}-\frac{b\sqrt{(c+dx)^2+1}}{12de^5(c+dx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^5,x]
```

```
[Out] -(b*Sqrt[1 + (c + d*x)^2])/(12*d*e^5*(c + d*x)^3) + (b*Sqrt[1 + (c + d*x)^2
])/(6*d*e^5*(c + d*x)) - (a + b*ArcSinh[c + d*x])/(4*d*e^5*(c + d*x)^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(c+dx)}{(ce+dex)^5}\,dx = \frac{\text{Subst}\left(\int \frac{a+b\sinh^{-1}(x)}{e^5x^5}\,dx,x,c+dx\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{a+b\sinh^{-1}(x)}{x^5}\,dx,x,c+dx\right)}{de^5}$$
$$= -\frac{a+b\sinh^{-1}(c+dx)}{4de^5(c+dx)^4}+\frac{b\,\text{Subst}\left(\int \frac{1}{x^4\sqrt{1+x^2}}\,dx,x,c+dx\right)}{4de^5}$$
$$= -\frac{b\sqrt{1+(c+dx)^2}}{12de^5(c+dx)^3}-\frac{a+b\sinh^{-1}(c+dx)}{4de^5(c+dx)^4}-\frac{b\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{1+x^2}}\,dx,x,c+dx\right)}{6de^5}$$
$$= -\frac{b\sqrt{1+(c+dx)^2}}{12de^5(c+dx)^3}+\frac{b\sqrt{1+(c+dx)^2}}{6de^5(c+dx)}-\frac{a+b\sinh^{-1}(c+dx)}{4de^5(c+dx)^4}$$

Mathematica [A] time = 0.08, size = 61, normalized size = 0.68

$$-\frac{3\left(a+b\sinh^{-1}(c+dx)\right)+b(c+dx)\sqrt{(c+dx)^2+1}\,\left(1-2(c+dx)^2\right)}{12de^5(c+dx)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^5,x]
```

```
[Out] -1/12*(b*(c + d*x)*(1 - 2*(c + d*x)^2)*Sqrt[1 + (c + d*x)^2] + 3*(a + b*Arc
Sinh[c + d*x]))/(d*e^5*(c + d*x)^4)
```

fricas [B] time = 0.57, size = 210, normalized size = 2.33

$$\frac{3\,ad^4x^4+12\,acd^3x^3+18\,ac^2d^2x^2+12\,ac^3dx-3\,bc^4\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)+\left(2\,bc^4d^3x^3+6\,bc^5d\right.}{12\left(c^4d^5e^5x^4+4\,c^5d^4e^5x^3+6\,c^6d^3e^5x^2+4\,c^7d^2e^5x+c^8d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^5,x, algorithm="fricas")
```

```
[Out] 1/12*(3*a*d^4*x^4 + 12*a*c*d^3*x^3 + 18*a*c^2*d^2*x^2 + 12*a*c^3*d*x - 3*b*
c^4*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + (2*b*c^4*d^3*x^3 + 6
*b*c^5*d^2*x^2 + 2*b*c^7 - b*c^5 + (6*b*c^6 - b*c^4)*d*x)*sqrt(d^2*x^2 + 2*
c*d*x + c^2 + 1))/(c^4*d^5*e^5*x^4 + 4*c^5*d^4*e^5*x^3 + 6*c^6*d^3*e^5*x^2
+ 4*c^7*d^2*e^5*x + c^8*d*e^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{arsinh}\left(dx+c\right)+a}{\left(dex+ce\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^5,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)/(d*e*x + c*e)^5, x)
```

maple [A] time = 0.01, size = 80, normalized size = 0.89

$$\frac{-\frac{a}{4e^5(dx+c)^4}+\frac{b\left(-\frac{\text{arcsinh}(dx+c)}{4(dx+c)^4}-\frac{\sqrt{1+(dx+c)^2}}{12(dx+c)^3}+\frac{\sqrt{1+(dx+c)^2}}{6dx+6c}\right)}{e^5}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^5,x)
```

```
[Out] 1/d*(-1/4*a/e^5/(d*x+c)^4+b/e^5*(-1/4/(d*x+c)^4*arcsinh(d*x+c)-1/12/(d*x+c)
^3*(1+(d*x+c)^2)^(1/2)+1/6/(d*x+c)*(1+(d*x+c)^2)^(1/2)))
```

maxima [B] time = 0.36, size = 258, normalized size = 2.87

$$\frac{1}{12}b\left(\frac{\left(2d^4x^4+8cd^3x^3+2c^4+\left(12c^2d^2+d^2\right)x^2+c^2+2\left(4c^3d+cd\right)x-1\right)d}{\left(d^5e^5x^3+3cd^4e^5x^2+3c^2d^3e^5x+c^3d^2e^5\right)\sqrt{d^2x^2+2cdx+c^2+1}}-\frac{3\operatorname{arsinh}(dx}{d^5e^5x^4+4cd^4e^5x^3+6c^2d^3e^5x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^5,x, algorithm="maxima")
```

```
[Out] 1/12*b*((2*d^4*x^4 + 8*c*d^3*x^3 + 2*c^4 + (12*c^2*d^2 + d^2)*x^2 + c^2 + 2
*(4*c^3*d + c*d)*x - 1)*d/((d^5*e^5*x^3 + 3*c*d^4*e^5*x^2 + 3*c^2*d^3*e^5*x
 + c^3*d^2*e^5)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - 3*arcsinh(d*x + c)/(d^
5*e^5*x^4 + 4*c*d^4*e^5*x^3 + 6*c^2*d^3*e^5*x^2 + 4*c^3*d^2*e^5*x + c^4*d*e
^5)) - 1/4*a/(d^5*e^5*x^4 + 4*c*d^4*e^5*x^3 + 6*c^2*d^3*e^5*x^2 + 4*c^3*d^2
*e^5*x + c^4*d*e^5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\operatorname{asinh}(c+d\,x)}{(c\,e+d\,e\,x)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))/(c*e + d*e*x)^5,x)
```

```
[Out] int((a + b*asinh(c + d*x))/(c*e + d*e*x)^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^5+5c^4dx+10c^3d^2x^2+10c^2d^3x^3+5cd^4x^4+d^5x^5}\,dx+\int \frac{b\operatorname{asinh}(c+dx)}{c^5+5c^4dx+10c^3d^2x^2+10c^2d^3x^3+5cd^4x^4+d^5x^5}\,dx}{e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))/(d*e*x+c*e)**5,x)
```

```
[Out] (Integral(a/(c**5 + 5*c**4*d*x + 10*c**3*d**2*x**2 + 10*c**2*d**3*x**3 + 5*
c*d**4*x**4 + d**5*x**5), x) + Integral(b*asinh(c + d*x)/(c**5 + 5*c**4*d*x
 + 10*c**3*d**2*x**2 + 10*c**2*d**3*x**3 + 5*c*d**4*x**4 + d**5*x**5), x))/
e**5
```

3.125 $\int \frac{a+b\sinh^{-1}(c+dx)}{(ce+dex)^6}\,dx$

Optimal. Leaf size=115

$$-\frac{a+b\sinh^{-1}(c+dx)}{5de^6(c+dx)^5}+\frac{3b\sqrt{(c+dx)^2+1}}{40de^6(c+dx)^2}-\frac{b\sqrt{(c+dx)^2+1}}{20de^6(c+dx)^4}-\frac{3b\tanh^{-1}\left(\sqrt{(c+dx)^2+1}\right)}{40de^6}$$

```
[Out] 1/5*(-a-b*arcsinh(d*x+c))/d/e^6/(d*x+c)^5-3/40*b*arctanh((1+(d*x+c)^2)^(1/2
))/d/e^6-1/20*b*(1+(d*x+c)^2)^(1/2)/d/e^6/(d*x+c)^4+3/40*b*(1+(d*x+c)^2)^(1
/2)/d/e^6/(d*x+c)^2
```

Rubi [A] time = 0.09, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5865, 12, 5661, 266, 51, 63, 207}

$$-\frac{a+b\sinh^{-1}(c+dx)}{5de^6(c+dx)^5}+\frac{3b\sqrt{(c+dx)^2+1}}{40de^6(c+dx)^2}-\frac{b\sqrt{(c+dx)^2+1}}{20de^6(c+dx)^4}-\frac{3b\tanh^{-1}\left(\sqrt{(c+dx)^2+1}\right)}{40de^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^6,x]
```

```
[Out] -(b*Sqrt[1 + (c + d*x)^2])/(20*d*e^6*(c + d*x)^4) + (3*b*Sqrt[1 + (c + d*x)
^2])/(40*d*e^6*(c + d*x)^2) - (a + b*ArcSinh[c + d*x])/(5*d*e^6*(c + d*x)^5
) - (3*b*ArcTanh[Sqrt[1 + (c + d*x)^2]])/(40*d*e^6)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\sinh^{-1}(c+dx)}{(ce+dex)^6}\,dx &= \frac{\text{Subst}\left(\int \frac{a+b\sinh^{-1}(x)}{e^6x^6}\,dx,x,c+dx\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{a+b\sinh^{-1}(x)}{x^6}\,dx,x,c+dx\right)}{de^6}\\
&= -\frac{a+b\sinh^{-1}(c+dx)}{5de^6(c+dx)^5}+\frac{b\,\text{Subst}\left(\int \frac{1}{x^5\sqrt{1+x^2}}\,dx,x,c+dx\right)}{5de^6}\\
&= -\frac{a+b\sinh^{-1}(c+dx)}{5de^6(c+dx)^5}+\frac{b\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{1+x}}\,dx,x,(c+dx)^2\right)}{10de^6}\\
&= -\frac{b\sqrt{1+(c+dx)^2}}{20de^6(c+dx)^4}-\frac{a+b\sinh^{-1}(c+dx)}{5de^6(c+dx)^5}-\frac{(3b)\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{1+x}}\,dx,x,(c+dx)\right.}{40de^6}\\
&= -\frac{b\sqrt{1+(c+dx)^2}}{20de^6(c+dx)^4}+\frac{3b\sqrt{1+(c+dx)^2}}{40de^6(c+dx)^2}-\frac{a+b\sinh^{-1}(c+dx)}{5de^6(c+dx)^5}+\frac{(3b)\,\text{Subst}\left(\int\right.}{}\\
&= -\frac{b\sqrt{1+(c+dx)^2}}{20de^6(c+dx)^4}+\frac{3b\sqrt{1+(c+dx)^2}}{40de^6(c+dx)^2}-\frac{a+b\sinh^{-1}(c+dx)}{5de^6(c+dx)^5}+\frac{(3b)\,\text{Subst}\left(\int\right.}{}\\
&= -\frac{b\sqrt{1+(c+dx)^2}}{20de^6(c+dx)^4}+\frac{3b\sqrt{1+(c+dx)^2}}{40de^6(c+dx)^2}-\frac{a+b\sinh^{-1}(c+dx)}{5de^6(c+dx)^5}-\frac{3b\tanh^{-1}\left(\sqrt{1\right.}}{40d}
\end{aligned}$$

Mathematica [C] time = 0.08, size = 61, normalized size = 0.53

$$-\frac{\frac{a+b\sinh^{-1}(c+dx)}{(c+dx)^5}+b\sqrt{(c+dx)^2+1}\;{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};(c+dx)^2+1\right)}{5de^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^6,x]
```

```
[Out] -1/5*((a + b*ArcSinh[c + d*x])/(c + d*x)^5 + b*Sqrt[1 + (c + d*x)^2]*Hyperg
eometric2F1[1/2, 3, 3/2, 1 + (c + d*x)^2])/(d*e^6)
```

fricas [B] time = 0.72, size = 509, normalized size = 4.43

$$-\frac{8\,ac^5-8\left(bd^5x^5+5\,bcd^4x^4+10\,bc^2d^3x^3+10\,bc^3d^2x^2+5\,bc^4dx\right)\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)+3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^6,x, algorithm="fricas")
```

```
[Out] -1/40*(8*a*c^5 - 8*(b*d^5*x^5 + 5*b*c*d^4*x^4 + 10*b*c^2*d^3*x^3 + 10*b*c^3
*d^2*x^2 + 5*b*c^4*d*x)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) +
3*(b*c^5*d^5*x^5 + 5*b*c^6*d^4*x^4 + 10*b*c^7*d^3*x^3 + 10*b*c^8*d^2*x^2 +
5*b*c^9*d*x + b*c^10)*log(-d*x - c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1) + 1)
 - 8*(b*d^5*x^5 + 5*b*c*d^4*x^4 + 10*b*c^2*d^3*x^3 + 10*b*c^3*d^2*x^2 + 5*b
*c^4*d*x + b*c^5)*log(-d*x - c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - 3*(b*
c^5*d^5*x^5 + 5*b*c^6*d^4*x^4 + 10*b*c^7*d^3*x^3 + 10*b*c^8*d^2*x^2 + 5*b*c
^9*d*x + b*c^10)*log(-d*x - c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1) - 1) - (3
*b*c^5*d^3*x^3 + 9*b*c^6*d^2*x^2 + 3*b*c^8 - 2*b*c^6 + (9*b*c^7 - 2*b*c^5)*
d*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(c^5*d^6*e^6*x^5 + 5*c^6*d^5*e^6*x^
4 + 10*c^7*d^4*e^6*x^3 + 10*c^8*d^3*e^6*x^2 + 5*c^9*d^2*e^6*x + c^10*d*e^6)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arsinh}(dx+c)+a}{(dex+ce)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^6,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)/(d*e*x + c*e)^6, x)
```

maple [A] time = 0.01, size = 94, normalized size = 0.82

$$\frac{-\frac{a}{5e^6(dx+c)^5}+\frac{b\left(-\frac{\operatorname{arcsinh}(dx+c)}{5(dx+c)^5}-\frac{\sqrt{1+(dx+c)^2}}{20(dx+c)^4}+\frac{3\sqrt{1+(dx+c)^2}}{40(dx+c)^2}-\frac{3\operatorname{arctanh}\left(\frac{1}{\sqrt{1+(dx+c)^2}}\right)}{40}\right)}{e^6}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^6,x)
```

```
[Out] 1/d*(-1/5*a/e^6/(d*x+c)^5+b/e^6*(-1/5/(d*x+c)^5*arcsinh(d*x+c)-1/20/(d*x+c)
^4*(1+(d*x+c)^2)^(1/2)+3/40/(d*x+c)^2*(1+(d*x+c)^2)^(1/2)-3/40*arctanh(1/(1
+(d*x+c)^2)^(1/2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{30}b\left(\frac{2\left(3d^4x^4+12cd^3x^3+3c^4+\left(18c^2d^2-d^2\right)x^2-c^2+2\left(6c^3d-cd\right)x-3\log\left(dx+c+\sqrt{d^2x^2+2cdx+c^2+\cdots}\right.\right.}{d^6e^6x^5+5cd^5e^6x^4+10c^2d^4e^6x^3+10c^3d^3e^6x^2+5c^4d^2e^6x+c^5de^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^6,x, algorithm="maxima")
```

```
[Out] 1/30*b*(2*(3*d^4*x^4 + 12*c*d^3*x^3 + 3*c^4 + (18*c^2*d^2 - d^2)*x^2 - c^2
+ 2*(6*c^3*d - c*d)*x - 3*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)))
/(d^6*e^6*x^5 + 5*c*d^5*e^6*x^4 + 10*c^2*d^4*e^6*x^3 + 10*c^3*d^3*e^6*x^2 +
 5*c^4*d^2*e^6*x + c^5*d*e^6) - 3*I*(log(I*(d^2*x + c*d)/d + 1) - log(-I*(d
^2*x + c*d)/d + 1))/(d*e^6) + 30*integrate(1/5/(d^8*e^6*x^8 + 8*c*d^7*e^6*x
^7 + c^8*e^6 + c^6*e^6 + (28*c^2*d^6*e^6 + d^6*e^6)*x^6 + 2*(28*c^3*d^5*e^6
 + 3*c*d^5*e^6)*x^5 + 5*(14*c^4*d^4*e^6 + 3*c^2*d^4*e^6)*x^4 + 4*(14*c^5*d^
```

```
3*e^6 + 5*c^3*d^3*e^6)*x^3 + (28*c^6*d^2*e^6 + 15*c^4*d^2*e^6)*x^2 + 2*(4*c
^7*d*e^6 + 3*c^5*d*e^6)*x + (d^7*e^6*x^7 + 7*c*d^6*e^6*x^6 + c^7*e^6 + c^5*
e^6 + (21*c^2*d^5*e^6 + d^5*e^6)*x^5 + 5*(7*c^3*d^4*e^6 + c*d^4*e^6)*x^4 +
5*(7*c^4*d^3*e^6 + 2*c^2*d^3*e^6)*x^3 + (21*c^5*d^2*e^6 + 10*c^3*d^2*e^6)*x
^2 + (7*c^6*d*e^6 + 5*c^4*d*e^6)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)), x))
 - 1/5*a/(d^6*e^6*x^5 + 5*c*d^5*e^6*x^4 + 10*c^2*d^4*e^6*x^3 + 10*c^3*d^3*e
^6*x^2 + 5*c^4*d^2*e^6*x + c^5*d*e^6)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b\,\text{asinh}\,(c + d\,x)}{(c\,e + d\,e\,x)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))/(c*e + d*e*x)^6,x)
```

```
[Out] int((a + b*asinh(c + d*x))/(c*e + d*e*x)^6, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^6+6c^5dx+15c^4d^2x^2+20c^3d^3x^3+15c^2d^4x^4+6cd^5x^5+d^6x^6}\,dx + \int \frac{b\,\text{asinh}\,(c+dx)}{c^6+6c^5dx+15c^4d^2x^2+20c^3d^3x^3+15c^2d^4x^4+6cd^5x^5+d^6x^6}\,dx}{e^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))/(d*e*x+c*e)**6,x)
```

```
[Out] (Integral(a/(c**6 + 6*c**5*d*x + 15*c**4*d**2*x**2 + 20*c**3*d**3*x**3 + 15
*c**2*d**4*x**4 + 6*c*d**5*x**5 + d**6*x**6), x) + Integral(b*asinh(c + d*x
)/(c**6 + 6*c**5*d*x + 15*c**4*d**2*x**2 + 20*c**3*d**3*x**3 + 15*c**2*d**4
*x**4 + 6*c*d**5*x**5 + d**6*x**6), x))/e**6
```

3.126 $\int (ce + dex)^m \left(a + b \sinh^{-1}(c + dx)\right)^2 dx$

Optimal. Leaf size=187

$$\frac{2b^2(e(c+dx))^{m+3}\,{}_3F_2\left(1,\frac{m}{2}+\frac{3}{2},\frac{m}{2}+\frac{3}{2};\frac{m}{2}+2,\frac{m}{2}+\frac{5}{2};-(c+dx)^2\right)}{de^3(m+1)(m+2)(m+3)}-\frac{2b(e(c+dx))^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};-(c+dx)^2\right)}{de^2(m+1)(m+2)}$$

```
[Out] (e*(d*x+c))^(1+m)*(a+b*arcsinh(d*x+c))^2/d/e/(1+m)-2*b*(e*(d*x+c))^(2+m)*(a
+b*arcsinh(d*x+c))*hypergeom([1/2, 1+1/2*m],[2+1/2*m],-(d*x+c)^2)/d/e^2/(1+
m)/(2+m)+2*b^2*(e*(d*x+c))^(3+m)*HypergeometricPFQ([1, 3/2+1/2*m, 3/2+1/2*m
],[2+1/2*m, 5/2+1/2*m],-(d*x+c)^2)/d/e^3/(3+m)/(m^2+3*m+2)
```

Rubi [A] time = 0.21, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {5865, 5661, 5762}

$$\frac{2b^2(e(c+dx))^{m+3}\,{}_3F_2\left(1,\frac{m}{2}+\frac{3}{2},\frac{m}{2}+\frac{3}{2};\frac{m}{2}+2,\frac{m}{2}+\frac{5}{2};-(c+dx)^2\right)}{de^3(m+1)(m+2)(m+3)}-\frac{2b(e(c+dx))^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};-(c+dx)^2\right)}{de^2(m+1)(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^m*(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] ((e*(c + d*x))^(1 + m)*(a + b*ArcSinh[c + d*x])^2)/(d*e*(1 + m)) - (2*b*(e*
(c + d*x))^(2 + m)*(a + b*ArcSinh[c + d*x])*Hypergeometric2F1[1/2, (2 + m)/
2, (4 + m)/2, -(c + d*x)^2])/(d*e^2*(1 + m)*(2 + m)) + (2*b^2*(e*(c + d*x))
^(3 + m)*HypergeometricPFQ[{1, 3/2 + m/2, 3/2 + m/2}, {2 + m/2, 5/2 + m/2},
 -(c + d*x)^2])/(d*e^3*(1 + m)*(2 + m)*(3 + m))
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5762

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*ArcSinh[c*x])*Hypergeom
etric2F1[1/2, (1 + m)/2, (3 + m)/2, -(c^2*x^2)])/(Sqrt[d]*f*(m + 1)), x] -
Simp[(b*c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2
, 2 + m/2}, -(c^2*x^2)])/(Sqrt[d]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b,
c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[d, 0] &&  !IntegerQ[m]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^m \left(a+b\sinh^{-1}(c+dx)\right)^2\,dx = \frac{\mathrm{Subst}\left(\int (ex)^m \left(a+b\sinh^{-1}(x)\right)^2 dx, x, c+dx\right)}{d}$$

$$= \frac{(e(c+dx))^{1+m}\left(a+b\sinh^{-1}(c+dx)\right)^2}{de(1+m)} - \frac{(2b)\,\mathrm{Subst}\left(\int \frac{(ex)^{1+m}(a+b\text{s}}{\sqrt{1+x}}\right.}{de(1+}$$

$$= \frac{(e(c+dx))^{1+m}\left(a+b\sinh^{-1}(c+dx)\right)^2}{de(1+m)} - \frac{2b(e(c+dx))^{2+m}\left(a+b\,\text{s}\right.}{}$$

Mathematica [A] time = 0.12, size = 155, normalized size = 0.83

$$\frac{(c+dx)(e(c+dx))^m \left(\frac{2b^2(c+dx)^2\,{}_3F_2\left(1,\frac{m}{2}+\frac{3}{2},\frac{m}{2}+\frac{3}{2};\frac{m}{2}+2,\frac{m}{2}+\frac{5}{2};-(c+dx)^2\right)}{(m+2)(m+3)} - \frac{2b(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};-(c+dx)^2\right)\left(a+b\sinh^{-1}(c+dx)\right)}{m+2} + \right.}{d(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^m*(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] ((c + d*x)*(e*(c + d*x))^m*((a + b*ArcSinh[c + d*x])^2 - (2*b*(c + d*x)*(a
+ b*ArcSinh[c + d*x])*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, -(c + d*
x)^2])/(2 + m) + (2*b^2*(c + d*x)^2*HypergeometricPFQ[{1, 3/2 + m/2, 3/2 +
m/2}, {2 + m/2, 5/2 + m/2}, -(c + d*x)^2])/((2 + m)*(3 + m))))/(d*(1 + m))
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b^2\operatorname{arsinh}(dx+c)^2 + 2\,ab\operatorname{arsinh}(dx+c) + a^2\right)(dex+ce)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(d*x + c)^2 + 2*a*b*arcsinh(d*x + c) + a^2)*(d*e*x + c
*e)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\operatorname{arsinh}(dx+c)+a)^2(dex+ce)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^2*(d*e*x + c*e)^m, x)
```

maple [F] time = 1.95, size = 0, normalized size = 0.00

$$\int (dex+ce)^m \left(a+b\operatorname{arcsinh}(dx+c)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^2,x)
```

```
[Out] int((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(b^2de^mx + b^2ce^m\right)(dx+c)^m \log\left(dx + c + \sqrt{d^2x^2 + 2\,cdx + c^2 + 1}\right)^2}{d(m+1)} + \frac{(dex+ce)^{m+1}a^2}{de(m+1)} + \int -\frac{2\left(\left(b^2c^2e^m - \left(c^2e^m(m\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] (b^2*d*e^m*x + b^2*c*e^m)*(d*x + c)^m*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x
+ c^2 + 1))^2/(d*(m + 1)) + (d*e*x + c*e)^(m + 1)*a^2/(d*e*(m + 1)) + integ
rate(-2*((b^2*c^2*e^m - (c^2*e^m*(m + 1) + e^m*(m + 1))*a*b - (a*b*d^2*e^m*
(m + 1) - b^2*d^2*e^m)*x^2 - 2*(a*b*c*d*e^m*(m + 1) - b^2*c*d*e^m)*x)*sqrt(
d^2*x^2 + 2*c*d*x + c^2 + 1)*(d*x + c)^m - ((a*b*d^3*e^m*(m + 1) - b^2*d^3*
e^m)*x^3 + (c^3*e^m*(m + 1) + c*e^m*(m + 1))*a*b - (c^3*e^m + c*e^m)*b^2 +
3*(a*b*c*d^2*e^m*(m + 1) - b^2*c*d^2*e^m)*x^2 + ((3*c^2*d*e^m*(m + 1) + d*e
^m*(m + 1))*a*b - (3*c^2*d*e^m + d*e^m)*b^2)*x)*(d*x + c)^m)*log(d*x + c +
sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(d^3*(m + 1)*x^3 + 3*c*d^2*(m + 1)*x^2 +
 c^3*(m + 1) + c*(m + 1) + (3*c^2*d*(m + 1) + d*(m + 1))*x + (d^2*(m + 1)*x
^2 + 2*c*d*(m + 1)*x + c^2*(m + 1) + m + 1)*sqrt(d^2*x^2 + 2*c*d*x + c^2 +
1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e + d\,e\,x)^m\,(a + b\,\mathrm{asinh}\,(c + d\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^m*(a + b*asinh(c + d*x))^2,x)
```

```
[Out] int((c*e + d*e*x)^m*(a + b*asinh(c + d*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\,(c + dx))^m\,(a + b\,\mathrm{asinh}\,(c + dx))^2\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**m*(a+b*asinh(d*x+c))**2,x)
```

```
[Out] Integral((e*(c + d*x))**m*(a + b*asinh(c + d*x))**2, x)
```

3.127 $\int (ce+dex)^4 \left(a+b\sinh^{-1}(c+dx)\right)^2 dx$

Optimal. Leaf size=197

$$\frac{e^4(c+dx)^5\left(a+b\sinh^{-1}(c+dx)\right)^2}{5d}-\frac{2be^4\sqrt{(c+dx)^2+1}\,(c+dx)^4\left(a+b\sinh^{-1}(c+dx)\right)}{25d}+\frac{8be^4\sqrt{(c+dx)^2+1}}{}$$

```
[Out] 16/75*b^2*e^4*x-8/225*b^2*e^4*(d*x+c)^3/d+2/125*b^2*e^4*(d*x+c)^5/d+1/5*e^4
*(d*x+c)^5*(a+b*arcsinh(d*x+c))^2/d-16/75*b*e^4*(a+b*arcsinh(d*x+c))*(1+(d*
x+c)^2)^(1/2)/d+8/75*b*e^4*(d*x+c)^2*(a+b*arcsinh(d*x+c))*(1+(d*x+c)^2)^(1/
2)/d-2/25*b*e^4*(d*x+c)^4*(a+b*arcsinh(d*x+c))*(1+(d*x+c)^2)^(1/2)/d
```

Rubi [A] time = 0.31, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {5865, 12, 5661, 5758, 5717, 8, 30}

$$\frac{e^4(c+dx)^5\left(a+b\sinh^{-1}(c+dx)\right)^2}{5d}-\frac{2be^4\sqrt{(c+dx)^2+1}\,(c+dx)^4\left(a+b\sinh^{-1}(c+dx)\right)}{25d}+\frac{8be^4\sqrt{(c+dx)^2+1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4*(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] (16*b^2*e^4*x)/75 - (8*b^2*e^4*(c + d*x)^3)/(225*d) + (2*b^2*e^4*(c + d*x)^
5)/(125*d) - (16*b*e^4*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x]))/(75*
d) + (8*b*e^4*(c + d*x)^2*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x]))/(
75*d) - (2*b*e^4*(c + d*x)^4*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])
)/(25*d) + (e^4*(c + d*x)^5*(a + b*ArcSinh[c + d*x])^2)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^4\left(a+b\sinh^{-1}(c+dx)\right)^2\,dx=\frac{\text{Subst}\left(\int e^4x^4\left(a+b\sinh^{-1}(x)\right)^2\,dx,x,c+dx\right)}{d}$$
$$=\frac{e^4\,\text{Subst}\left(\int x^4\left(a+b\sinh^{-1}(x)\right)^2\,dx,x,c+dx\right)}{d}$$
$$=\frac{e^4(c+dx)^5\left(a+b\sinh^{-1}(c+dx)\right)^2}{5d}-\frac{\left(2be^4\right)\text{Subst}\left(\int\frac{x^5\left(a+b\sinh^{-1}(x)\right)}{\sqrt{1+x^2}}\right.}{5d}$$
$$=-\frac{2be^4(c+dx)^4\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)}{25d}+\frac{e^4(c+dx)^5\left(\right.}{}$$
$$=\frac{2b^2e^4(c+dx)^5}{125d}+\frac{8be^4(c+dx)^2\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)}{75d}$$
$$=-\frac{8b^2e^4(c+dx)^3}{225d}+\frac{2b^2e^4(c+dx)^5}{125d}-\frac{16be^4\sqrt{1+(c+dx)^2}\left(a+b\sinh\right.}{75d}$$
$$=\frac{16}{75}b^2e^4x-\frac{8b^2e^4(c+dx)^3}{225d}+\frac{2b^2e^4(c+dx)^5}{125d}-\frac{16be^4\sqrt{1+(c+dx)^2}\left(\right.}{75d}$$

Mathematica [A] time = 0.26, size = 192, normalized size = 0.97

$$\frac{e^4\left(9\left(25a^2+2b^2\right)(c+dx)^5+30ab\sqrt{(c+dx)^2+1}\left(-3(c+dx)^4+4(c+dx)^2-8\right)+30b\sinh^{-1}(c+dx)\left(15a(c+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^4*(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] (e^4*(240*b^2*(c + d*x) - 40*b^2*(c + d*x)^3 + 9*(25*a^2 + 2*b^2)*(c + d*x)
^5 + 30*a*b*Sqrt[1 + (c + d*x)^2]*(-8 + 4*(c + d*x)^2 - 3*(c + d*x)^4) + 30
*b*(15*a*(c + d*x)^5 - 8*b*Sqrt[1 + (c + d*x)^2] + 4*b*(c + d*x)^2*Sqrt[1 +
 (c + d*x)^2] - 3*b*(c + d*x)^4*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x] + 2
25*b^2*(c + d*x)^5*ArcSinh[c + d*x]^2))/(1125*d)
```

fricas [B] time = 0.76, size = 618, normalized size = 3.14

$$\frac{9\left(25\,a^2+2\,b^2\right)d^5e^4x^5+45\left(25\,a^2+2\,b^2\right)cd^4e^4x^4+10\left(9\left(25\,a^2+2\,b^2\right)c^2-4\,b^2\right)d^3e^4x^3+30\left(3\left(25\,a^2+2\,b^2\right)c^3-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/1125*(9*(25*a^2 + 2*b^2)*d^5*e^4*x^5 + 45*(25*a^2 + 2*b^2)*c*d^4*e^4*x^4
+ 10*(9*(25*a^2 + 2*b^2)*c^2 - 4*b^2)*d^3*e^4*x^3 + 30*(3*(25*a^2 + 2*b^2)*
c^3 - 4*b^2*c)*d^2*e^4*x^2 + 15*(3*(25*a^2 + 2*b^2)*c^4 - 8*b^2*c^2 + 16*b^
2)*d*e^4*x + 225*(b^2*d^5*e^4*x^5 + 5*b^2*c*d^4*e^4*x^4 + 10*b^2*c^2*d^3*e^
4*x^3 + 10*b^2*c^3*d^2*e^4*x^2 + 5*b^2*c^4*d*e^4*x + b^2*c^5*e^4)*log(d*x +
 c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 + 30*(15*a*b*d^5*e^4*x^5 + 75*a*b
*c*d^4*e^4*x^4 + 150*a*b*c^2*d^3*e^4*x^3 + 150*a*b*c^3*d^2*e^4*x^2 + 75*a*b
*c^4*d*e^4*x + 15*a*b*c^5*e^4 - (3*b^2*d^4*e^4*x^4 + 12*b^2*c*d^3*e^4*x^3 +
 2*(9*b^2*c^2 - 2*b^2)*d^2*e^4*x^2 + 4*(3*b^2*c^3 - 2*b^2*c)*d*e^4*x + (3*b
^2*c^4 - 4*b^2*c^2 + 8*b^2)*e^4)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x
 + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - 30*(3*a*b*d^4*e^4*x^4 + 12*a*b*
c*d^3*e^4*x^3 + 2*(9*a*b*c^2 - 2*a*b)*d^2*e^4*x^2 + 4*(3*a*b*c^3 - 2*a*b*c)
*d*e^4*x + (3*a*b*c^4 - 4*a*b*c^2 + 8*a*b)*e^4)*sqrt(d^2*x^2 + 2*c*d*x + c^
2 + 1))/d
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex + ce)^4 (b \operatorname{arsinh}(dx + c) + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^4*(b*arcsinh(d*x + c) + a)^2, x)
```

maple [A] time = 0.05, size = 218, normalized size = 1.11

$$\frac{\frac{(dx+c)^5 e^4 a^2}{5} + e^4 b^2 \left(\frac{(dx+c)^5 \operatorname{arcsinh}(dx+c)^2}{5} - \frac{16 \operatorname{arcsinh}(dx+c)\sqrt{1+(dx+c)^2}}{75} - \frac{2(dx+c)^4 \operatorname{arcsinh}(dx+c)\sqrt{1+(dx+c)^2}}{25} + \frac{8 \operatorname{arcsinh}(dx+c)\sqrt{}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^2,x)
```

```
[Out] 1/d*(1/5*(d*x+c)^5*e^4*a^2+e^4*b^2*(1/5*(d*x+c)^5*arcsinh(d*x+c)^2-16/75*ar
csinh(d*x+c)*(1+(d*x+c)^2)^(1/2)-2/25*(d*x+c)^4*arcsinh(d*x+c)*(1+(d*x+c)^2
)^(1/2)+8/75*arcsinh(d*x+c)*(1+(d*x+c)^2)^(1/2)*(d*x+c)^2+16/75*d*x+16/75*c
+2/125*(d*x+c)^5-8/225*(d*x+c)^3)+2*e^4*a*b*(1/5*(d*x+c)^5*arcsinh(d*x+c)-1
/25*(d*x+c)^4*(1+(d*x+c)^2)^(1/2)+4/75*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)-8/75*(
1+(d*x+c)^2)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/5*a^2*d^4*e^4*x^5 + a^2*c*d^3*e^4*x^4 + 2*a^2*c^2*d^2*e^4*x^3 + 2*a^2*c^3
*d*e^4*x^2 + 2*(2*x^2*arcsinh(d*x + c) - d*(3*c^2*arcsinh(2*(d^2*x + c*d)/s
qrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*
x/d^2 - (c^2 + 1)*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2
))/d^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c/d^3))*a*b*c^3*d*e^4 + 2/3*(6
*x^3*arcsinh(d*x + c) - d*(2*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*x^2/d^2 - 15
*c^3*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^4 - 5*sq
rt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c*x/d^3 + 9*(c^2 + 1)*c*arcsinh(2*(d^2*x +
```

```
c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^4 + 15*sqrt(d^2*x^2 + 2*c*d*x +
c^2 + 1)*c^2/d^4 - 4*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)/d^4))*a*b*
c^2*d^2*e^4 + 1/12*(24*x^4*arcsinh(d*x + c) - (6*sqrt(d^2*x^2 + 2*c*d*x + c
^2 + 1)*x^3/d^2 - 14*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c*x^2/d^3 + 105*c^4*
arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^5 + 35*sqrt(d
^2*x^2 + 2*c*d*x + c^2 + 1)*c^2*x/d^4 - 90*(c^2 + 1)*c^2*arcsinh(2*(d^2*x +
 c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^5 - 105*sqrt(d^2*x^2 + 2*c*d*x
+ c^2 + 1)*c^3/d^5 - 9*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)*x/d^4 +
9*(c^2 + 1)^2*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d
^5 + 55*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)*c/d^5)*d)*a*b*c*d^3*e^4
 + 1/300*(120*x^5*arcsinh(d*x + c) - (24*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*
x^4/d^2 - 54*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c*x^3/d^3 + 126*sqrt(d^2*x^2
 + 2*c*d*x + c^2 + 1)*c^2*x^2/d^4 - 945*c^5*arcsinh(2*(d^2*x + c*d)/sqrt(-4
*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^6 - 315*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c^
3*x/d^5 - 32*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)*x^2/d^4 + 1050*(c^
2 + 1)*c^3*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^6
+ 945*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c^4/d^6 + 161*sqrt(d^2*x^2 + 2*c*d*
x + c^2 + 1)*(c^2 + 1)*c*x/d^5 - 225*(c^2 + 1)^2*c*arcsinh(2*(d^2*x + c*d)/
sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^6 - 735*sqrt(d^2*x^2 + 2*c*d*x + c^2
+ 1)*(c^2 + 1)*c^2/d^6 + 64*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)^2/d
^6)*d)*a*b*d^4*e^4 + a^2*c^4*e^4*x + 2*((d*x + c)*arcsinh(d*x + c) - sqrt((
d*x + c)^2 + 1))*a*b*c^4*e^4/d + 1/5*(b^2*d^4*e^4*x^5 + 5*b^2*c*d^3*e^4*x^4
 + 10*b^2*c^2*d^2*e^4*x^3 + 10*b^2*c^3*d*e^4*x^2 + 5*b^2*c^4*e^4*x)*log(d*x
 + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 - integrate(2/5*(b^2*d^7*e^4*x^
7 + 7*b^2*c*d^6*e^4*x^6 + (21*c^2*d^5*e^4 + d^5*e^4)*b^2*x^5 + 5*(7*c^3*d^4
*e^4 + c*d^4*e^4)*b^2*x^4 + 5*(7*c^4*d^3*e^4 + 2*c^2*d^3*e^4)*b^2*x^3 + 10*
(2*c^5*d^2*e^4 + c^3*d^2*e^4)*b^2*x^2 + 5*(c^6*d*e^4 + c^4*d*e^4)*b^2*x + (
b^2*d^6*e^4*x^6 + 6*b^2*c*d^5*e^4*x^5 + 15*b^2*c^2*d^4*e^4*x^4 + 20*b^2*c^3
*d^3*e^4*x^3 + 15*b^2*c^4*d^2*e^4*x^2 + 5*b^2*c^5*d*e^4*x)*sqrt(d^2*x^2 + 2
*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(d^3*x^
3 + 3*c*d^2*x^2 + c^3 + (3*c^2*d + d)*x + (d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/
2) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e+d\,e\,x)^4\,(a+b\,\mathrm{asinh}\,(c+d\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^4*(a + b*asinh(c + d*x))^2,x)
```

```
[Out] int((c*e + d*e*x)^4*(a + b*asinh(c + d*x))^2, x)
```

sympy [A] time = 7.46, size = 1268, normalized size = 6.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4*(a+b*asinh(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*c**4*e**4*x + 2*a**2*c**3*d*e**4*x**2 + 2*a**2*c**2*d**2*e*
*4*x**3 + a**2*c*d**3*e**4*x**4 + a**2*d**4*e**4*x**5/5 + 2*a*b*c**5*e**4*a
sinh(c + d*x)/(5*d) + 2*a*b*c**4*e**4*x*asinh(c + d*x) - 2*a*b*c**4*e**4*sq
rt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(25*d) + 4*a*b*c**3*d*e**4*x**2*asinh(c
+ d*x) - 8*a*b*c**3*e**4*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/25 + 4*a*b*
c**2*d**2*e**4*x**3*asinh(c + d*x) - 12*a*b*c**2*d*e**4*x**2*sqrt(c**2 + 2*
c*d*x + d**2*x**2 + 1)/25 + 8*a*b*c**2*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2
 + 1)/(75*d) + 2*a*b*c*d**3*e**4*x**4*asinh(c + d*x) - 8*a*b*c*d**2*e**4*x*
*3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/25 + 16*a*b*c*e**4*x*sqrt(c**2 + 2*
c*d*x + d**2*x**2 + 1)/75 + 2*a*b*d**4*e**4*x**5*asinh(c + d*x)/5 - 2*a*b*d
```

```
**3*e**4*x**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/25 + 8*a*b*d*e**4*x**2*s
qrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/75 - 16*a*b*e**4*sqrt(c**2 + 2*c*d*x +
d**2*x**2 + 1)/(75*d) + b**2*c**5*e**4*asinh(c + d*x)**2/(5*d) + b**2*c**4*
e**4*x*asinh(c + d*x)**2 + 2*b**2*c**4*e**4*x/25 - 2*b**2*c**4*e**4*sqrt(c*
*2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/(25*d) + 2*b**2*c**3*d*e**4*x*
*2*asinh(c + d*x)**2 + 4*b**2*c**3*d*e**4*x**2/25 - 8*b**2*c**3*e**4*x*sqrt
(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/25 + 2*b**2*c**2*d**2*e**4*
x**3*asinh(c + d*x)**2 + 4*b**2*c**2*d**2*e**4*x**3/25 - 12*b**2*c**2*d*e**
4*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/25 - 8*b**2*c**2
*e**4*x/75 + 8*b**2*c**2*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c
+ d*x)/(75*d) + b**2*c*d**3*e**4*x**4*asinh(c + d*x)**2 + 2*b**2*c*d**3*e**
4*x**4/25 - 8*b**2*c*d**2*e**4*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*as
inh(c + d*x)/25 - 8*b**2*c*d*e**4*x**2/75 + 16*b**2*c*e**4*x*sqrt(c**2 + 2*
c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/75 + b**2*d**4*e**4*x**5*asinh(c + d*
x)**2/5 + 2*b**2*d**4*e**4*x**5/125 - 2*b**2*d**3*e**4*x**4*sqrt(c**2 + 2*c
*d*x + d**2*x**2 + 1)*asinh(c + d*x)/25 - 8*b**2*d**2*e**4*x**3/225 + 8*b**
2*d*e**4*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/75 + 16*b
**2*e**4*x/75 - 16*b**2*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c +
 d*x)/(75*d), Ne(d, 0)), (c**4*e**4*x*(a + b*asinh(c))**2, True))
```

3.128 $\int (ce+dex)^3 \left(a+b\sinh^{-1}(c+dx)\right)^2\,dx$

Optimal. Leaf size=172

$$\frac{e^3(c+dx)^4\left(a+b\sinh^{-1}(c+dx)\right)^2}{4d}-\frac{be^3\sqrt{(c+dx)^2+1}\,(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)}{8d}+\frac{3be^3\sqrt{(c+dx)^2+1}\,(c+$$

```
[Out] -3/32*b^2*e^3*(d*x+c)^2/d+1/32*b^2*e^3*(d*x+c)^4/d-3/32*e^3*(a+b*arcsinh(d*
x+c))^2/d+1/4*e^3*(d*x+c)^4*(a+b*arcsinh(d*x+c))^2/d+3/16*b*e^3*(d*x+c)*(a+
b*arcsinh(d*x+c))*(1+(d*x+c)^2)^(1/2)/d-1/8*b*e^3*(d*x+c)^3*(a+b*arcsinh(d*
x+c))*(1+(d*x+c)^2)^(1/2)/d
```

Rubi [A] time = 0.26, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {5865, 12, 5661, 5758, 5675, 30}

$$\frac{e^3(c+dx)^4\left(a+b\sinh^{-1}(c+dx)\right)^2}{4d}-\frac{be^3\sqrt{(c+dx)^2+1}\,(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)}{8d}+\frac{3be^3\sqrt{(c+dx)^2+1}\,(c+$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3*(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] (-3*b^2*e^3*(c + d*x)^2)/(32*d) + (b^2*e^3*(c + d*x)^4)/(32*d) + (3*b*e^3*(
c + d*x)*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x]))/(16*d) - (b*e^3*(c
 + d*x)^3*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x]))/(8*d) - (3*e^3*(a
 + b*ArcSinh[c + d*x])^2)/(32*d) + (e^3*(c + d*x)^4*(a + b*ArcSinh[c + d*x]
)^2)/(4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
```

```
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^3 \left(a+b\sinh^{-1}(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int e^3x^3\left(a+b\sinh^{-1}(x)\right)^2\,dx,x,c+dx\right)}{d}$$

$$= \frac{e^3\,\text{Subst}\left(\int x^3\left(a+b\sinh^{-1}(x)\right)^2\,dx,x,c+dx\right)}{d}$$

$$= \frac{e^3(c+dx)^4\left(a+b\sinh^{-1}(c+dx)\right)^2}{4d} - \frac{\left(be^3\right)\text{Subst}\left(\int \frac{x^4\left(a+b\sinh^{-1}(x)\right.}{\sqrt{1+x^2}}\right.}{2d}$$

$$= -\frac{be^3(c+dx)^3\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)}{8d} + \frac{e^3(c+dx)^4}{}$$

$$= \frac{b^2e^3(c+dx)^4}{32d} + \frac{3be^3(c+dx)\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)}{16d}$$

$$= -\frac{3b^2e^3(c+dx)^2}{32d} + \frac{b^2e^3(c+dx)^4}{32d} + \frac{3be^3(c+dx)\sqrt{1+(c+dx)^2}\left(a\right.}{16d}$$

Mathematica [A] time = 0.20, size = 170, normalized size = 0.99

$$\frac{e^3\left(\left(8a^2+b^2\right)(c+dx)^4+2ab\left(3-2(c+dx)^2\right)\sqrt{(c+dx)^2+1}\,(c+dx)+2b(c+dx)\sinh^{-1}(c+dx)\left(8a(c+dx)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^3*(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] (e^3*(-3*b^2*(c + d*x)^2 + (8*a^2 + b^2)*(c + d*x)^4 + 2*a*b*(c + d*x)*(3 -
 2*(c + d*x)^2)*Sqrt[1 + (c + d*x)^2] - 6*a*b*ArcSinh[c + d*x] + 2*b*(c + d
*x)*(8*a*(c + d*x)^3 + 3*b*Sqrt[1 + (c + d*x)^2] - 2*b*(c + d*x)^2*Sqrt[1 +
 (c + d*x)^2])*ArcSinh[c + d*x] + b^2*(-3 + 8*(c + d*x)^4)*ArcSinh[c + d*x]
^2))/(32*d)
```

fricas [B] time = 0.56, size = 486, normalized size = 2.83

$$\frac{\left(8\,a^2+b^2\right)d^4e^3x^4+4\left(8\,a^2+b^2\right)cd^3e^3x^3+3\left(2\left(8\,a^2+b^2\right)c^2-b^2\right)d^2e^3x^2+2\left(2\left(8\,a^2+b^2\right)c^3-3\,b^2c\right)de^3x+\left(8\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/32*((8*a^2 + b^2)*d^4*e^3*x^4 + 4*(8*a^2 + b^2)*c*d^3*e^3*x^3 + 3*(2*(8*a
^2 + b^2)*c^2 - b^2)*d^2*e^3*x^2 + 2*(2*(8*a^2 + b^2)*c^3 - 3*b^2*c)*d*e^3*
x + (8*b^2*d^4*e^3*x^4 + 32*b^2*c*d^3*e^3*x^3 + 48*b^2*c^2*d^2*e^3*x^2 + 32
```

```
*b^2*c^3*d*e^3*x + (8*b^2*c^4 - 3*b^2)*e^3)*log(d*x + c + sqrt(d^2*x^2 + 2*
c*d*x + c^2 + 1))^2 + 2*(8*a*b*d^4*e^3*x^4 + 32*a*b*c*d^3*e^3*x^3 + 48*a*b*
c^2*d^2*e^3*x^2 + 32*a*b*c^3*d*e^3*x + (8*a*b*c^4 - 3*a*b)*e^3 - (2*b^2*d^3
*e^3*x^3 + 6*b^2*c*d^2*e^3*x^2 + 3*(2*b^2*c^2 - b^2)*d*e^3*x + (2*b^2*c^3 -
 3*b^2*c)*e^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^
2 + 2*c*d*x + c^2 + 1)) - 2*(2*a*b*d^3*e^3*x^3 + 6*a*b*c*d^2*e^3*x^2 + 3*(2
*a*b*c^2 - a*b)*d*e^3*x + (2*a*b*c^3 - 3*a*b*c)*e^3)*sqrt(d^2*x^2 + 2*c*d*x
 + c^2 + 1))/d
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex + ce)^3 (b \operatorname{arsinh}(dx + c) + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^3*(b*arcsinh(d*x + c) + a)^2, x)
```

maple [A] time = 0.04, size = 194, normalized size = 1.13

$$\frac{\frac{(dx+c)^4 e^3 a^2}{4} + e^3 b^2 \left(\frac{(dx+c)^4 \operatorname{arcsinh}(dx+c)^2}{4} - \frac{(dx+c)^3 \operatorname{arcsinh}(dx+c) \sqrt{1+(dx+c)^2}}{8} + \frac{3 \operatorname{arcsinh}(dx+c) \sqrt{1+(dx+c)^2}\,(dx+c)}{16} - \frac{3 \operatorname{arcsinh}(dx+c)^2}{32} \right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^2,x)
```

```
[Out] 1/d*(1/4*(d*x+c)^4*e^3*a^2+e^3*b^2*(1/4*(d*x+c)^4*arcsinh(d*x+c)^2-1/8*(d*x
+c)^3*arcsinh(d*x+c)*(1+(d*x+c)^2)^(1/2)+3/16*arcsinh(d*x+c)*(1+(d*x+c)^2)^
(1/2)*(d*x+c)-3/32*arcsinh(d*x+c)^2+1/32*(d*x+c)^4-3/32*(d*x+c)^2-3/32)+2*e
^3*a*b*(1/4*(d*x+c)^4*arcsinh(d*x+c)-1/16*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)+3/3
2*(d*x+c)*(1+(d*x+c)^2)^(1/2)-3/32*arcsinh(d*x+c)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/4*a^2*d^3*e^3*x^4 + a^2*c*d^2*e^3*x^3 + 3/2*a^2*c^2*d*e^3*x^2 + 3/2*(2*x^
2*arcsinh(d*x + c) - d*(3*c^2*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(
c^2 + 1)*d^2))/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*x/d^2 - (c^2 + 1)*ar
csinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 - 3*sqrt(d^2*
x^2 + 2*c*d*x + c^2 + 1)*c/d^3))*a*b*c^2*d*e^3 + 1/3*(6*x^3*arcsinh(d*x + c
) - d*(2*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*x^2/d^2 - 15*c^3*arcsinh(2*(d^2*
x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^4 - 5*sqrt(d^2*x^2 + 2*c*d*x
 + c^2 + 1)*c*x/d^3 + 9*(c^2 + 1)*c*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2
 + 4*(c^2 + 1)*d^2))/d^4 + 15*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c^2/d^4 - 4
*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)/d^4))*a*b*c*d^2*e^3 + 1/48*(24
*x^4*arcsinh(d*x + c) - (6*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*x^3/d^2 - 14*s
qrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c*x^2/d^3 + 105*c^4*arcsinh(2*(d^2*x + c*d
)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^5 + 35*sqrt(d^2*x^2 + 2*c*d*x + c^2
 + 1)*c^2*x/d^4 - 90*(c^2 + 1)*c^2*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2
+ 4*(c^2 + 1)*d^2))/d^5 - 105*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c^3/d^5 - 9
*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)*x/d^4 + 9*(c^2 + 1)^2*arcsinh(
2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^5 + 55*sqrt(d^2*x^2 +
 2*c*d*x + c^2 + 1)*(c^2 + 1)*c/d^5)*d)*a*b*d^3*e^3 + a^2*c^3*e^3*x + 2*((d
```

```
*x + c)*arcsinh(d*x + c) - sqrt((d*x + c)^2 + 1))*a*b*c^3*e^3/d + 1/4*(b^2*
d^3*e^3*x^4 + 4*b^2*c*d^2*e^3*x^3 + 6*b^2*c^2*d*e^3*x^2 + 4*b^2*c^3*e^3*x)*
log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 - integrate(1/2*(b^2*d^6
*e^3*x^6 + 6*b^2*c*d^5*e^3*x^5 + (15*c^2*d^4*e^3 + d^4*e^3)*b^2*x^4 + 4*(5*
c^3*d^3*e^3 + c*d^3*e^3)*b^2*x^3 + 2*(7*c^4*d^2*e^3 + 3*c^2*d^2*e^3)*b^2*x^
2 + 4*(c^5*d*e^3 + c^3*d*e^3)*b^2*x + (b^2*d^5*e^3*x^5 + 5*b^2*c*d^4*e^3*x^
4 + 10*b^2*c^2*d^3*e^3*x^3 + 10*b^2*c^3*d^2*e^3*x^2 + 4*b^2*c^4*d*e^3*x)*sq
rt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2
 + 1))/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (3*c^2*d + d)*x + (d^2*x^2 + 2*c*d*x
+ c^2 + 1)^(3/2) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e + d\,e\,x)^3\,(a + b\,\mathrm{asinh}\,(c + d\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3*(a + b*asinh(c + d*x))^2,x)
```

```
[Out] int((c*e + d*e*x)^3*(a + b*asinh(c + d*x))^2, x)
```

sympy [A] time = 4.59, size = 916, normalized size = 5.33

$$\begin{cases} a^2c^3e^3x + \frac{3a^2c^2de^3x^2}{2} + a^2cd^2e^3x^3 + \frac{a^2d^3e^3x^4}{4} + \frac{abc^4e^3\,\mathrm{asinh}\,(c+dx)}{2d} + 2abc^3e^3x\,\mathrm{asinh}\,(c+dx) - \frac{abc^3e^3\sqrt{c^2+2cdx+d^2x^2+1}}{8d} \\ c^3e^3x\,(a + b\,\mathrm{asinh}\,(c))^2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3*(a+b*asinh(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*c**3*e**3*x + 3*a**2*c**2*d*e**3*x**2/2 + a**2*c*d**2*e**3*
x**3 + a**2*d**3*e**3*x**4/4 + a*b*c**4*e**3*asinh(c + d*x)/(2*d) + 2*a*b*c
**3*e**3*x*asinh(c + d*x) - a*b*c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 +
 1)/(8*d) + 3*a*b*c**2*d*e**3*x**2*asinh(c + d*x) - 3*a*b*c**2*e**3*x*sqrt(
c**2 + 2*c*d*x + d**2*x**2 + 1)/8 + 2*a*b*c*d**2*e**3*x**3*asinh(c + d*x) -
 3*a*b*c*d*e**3*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/8 + 3*a*b*c*e**3*
sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(16*d) + a*b*d**3*e**3*x**4*asinh(c +
d*x)/2 - a*b*d**2*e**3*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/8 + 3*a*b*
e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/16 - 3*a*b*e**3*asinh(c + d*x)/
(16*d) + b**2*c**4*e**3*asinh(c + d*x)**2/(4*d) + b**2*c**3*e**3*x*asinh(c
+ d*x)**2 + b**2*c**3*e**3*x/8 - b**2*c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*
x**2 + 1)*asinh(c + d*x)/(8*d) + 3*b**2*c**2*d*e**3*x**2*asinh(c + d*x)**2/
2 + 3*b**2*c**2*d*e**3*x**2/16 - 3*b**2*c**2*e**3*x*sqrt(c**2 + 2*c*d*x + d
**2*x**2 + 1)*asinh(c + d*x)/8 + b**2*c*d**2*e**3*x**3*asinh(c + d*x)**2 +
b**2*c*d**2*e**3*x**3/8 - 3*b**2*c*d*e**3*x**2*sqrt(c**2 + 2*c*d*x + d**2*x
**2 + 1)*asinh(c + d*x)/8 - 3*b**2*c*e**3*x/16 + 3*b**2*c*e**3*sqrt(c**2 +
2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/(16*d) + b**2*d**3*e**3*x**4*asinh(
c + d*x)**2/4 + b**2*d**3*e**3*x**4/32 - b**2*d**2*e**3*x**3*sqrt(c**2 + 2*
c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/8 - 3*b**2*d*e**3*x**2/32 + 3*b**2*e*
*3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/16 - 3*b**2*e**3*a
sinh(c + d*x)**2/(32*d), Ne(d, 0)), (c**3*e**3*x*(a + b*asinh(c))**2, True)
)
```

3.129 $\int (ce+dex)^2 \left(a+b\sinh^{-1}(c+dx)\right)^2 \, dx$

Optimal. Leaf size=136

$$\frac{e^2(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)^2}{3d}-\frac{2be^2\sqrt{(c+dx)^2+1}\,(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)}{9d}+\frac{4be^2\sqrt{(c+dx)^2+1}\,\left(a\right.}{9d}$$

```
[Out] -4/9*b^2*e^2*x+2/27*b^2*e^2*(d*x+c)^3/d+1/3*e^2*(d*x+c)^3*(a+b*arcsinh(d*x+
c))^2/d+4/9*b*e^2*(a+b*arcsinh(d*x+c))*(1+(d*x+c)^2)^(1/2)/d-2/9*b*e^2*(d*x
+c)^2*(a+b*arcsinh(d*x+c))*(1+(d*x+c)^2)^(1/2)/d
```

Rubi [A] time = 0.20, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {5865, 12, 5661, 5758, 5717, 8, 30}

$$\frac{e^2(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)^2}{3d}-\frac{2be^2\sqrt{(c+dx)^2+1}\,(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)}{9d}+\frac{4be^2\sqrt{(c+dx)^2+1}\,\left(a\right.}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2*(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] (-4*b^2*e^2*x)/9 + (2*b^2*e^2*(c + d*x)^3)/(27*d) + (4*b*e^2*Sqrt[1 + (c +
d*x)^2]*(a + b*ArcSinh[c + d*x]))/(9*d) - (2*b*e^2*(c + d*x)^2*Sqrt[1 + (c
+ d*x)^2]*(a + b*ArcSinh[c + d*x]))/(9*d) + (e^2*(c + d*x)^3*(a + b*ArcSinh
[c + d*x])^2)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^2\left(a+b\sinh^{-1}(c+dx)\right)^2\,dx=\frac{\text{Subst}\left(\int e^2x^2\left(a+b\sinh^{-1}(x)\right)^2\,dx,x,c+dx\right)}{d}$$
$$=\frac{e^2\,\text{Subst}\left(\int x^2\left(a+b\sinh^{-1}(x)\right)^2\,dx,x,c+dx\right)}{d}$$
$$=\frac{e^2(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)^2}{3d}-\frac{\left(2be^2\right)\text{Subst}\left(\int \frac{x^3\left(a+b\sinh^{-1}(\right.}{\sqrt{1+x^2}}\right.}{3d}$$
$$=-\frac{2be^2(c+dx)^2\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)}{9d}+\frac{e^2(c+dx)}{}$$
$$=\frac{2b^2e^2(c+dx)^3}{27d}+\frac{4be^2\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)}{9d}-\frac{2be}{}$$
$$=-\frac{4}{9}b^2e^2x+\frac{2b^2e^2(c+dx)^3}{27d}+\frac{4be^2\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+\right.}{9d}$$

Mathematica [A] time = 0.18, size = 147, normalized size = 1.08

$$\frac{e^2\left(\left(9a^2+2b^2\right)(c+dx)^3+6ab\left(2-(c+dx)^2\right)\sqrt{(c+dx)^2+1}+6b\sinh^{-1}(c+dx)\left(3a(c+dx)^3-b\sqrt{(c+dx)^2}\right.\right.}{27d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^2*(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] (e^2*(-12*b^2*(c + d*x) + (9*a^2 + 2*b^2)*(c + d*x)^3 + 6*a*b*(2 - (c + d*x
)^2)*Sqrt[1 + (c + d*x)^2] + 6*b*(3*a*(c + d*x)^3 + 2*b*Sqrt[1 + (c + d*x)^
2] - b*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x] + 9*b^2*(c + d*x
)^3*ArcSinh[c + d*x]^2))/(27*d)
```

fricas [B] time = 0.57, size = 358, normalized size = 2.63

$$\frac{\left(9\,a^2+2\,b^2\right)d^3e^2x^3+3\left(9\,a^2+2\,b^2\right)cd^2e^2x^2+3\left(\left(9\,a^2+2\,b^2\right)c^2-4\,b^2\right)de^2x+9\left(b^2d^3e^2x^3+3\,b^2cd^2e^2x^2+3\,b^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/27*((9*a^2 + 2*b^2)*d^3*e^2*x^3 + 3*(9*a^2 + 2*b^2)*c*d^2*e^2*x^2 + 3*((9
*a^2 + 2*b^2)*c^2 - 4*b^2)*d*e^2*x + 9*(b^2*d^3*e^2*x^3 + 3*b^2*c*d^2*e^2*x
^2 + 3*b^2*c^2*d*e^2*x + b^2*c^3*e^2)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x
+ c^2 + 1))^2 + 6*(3*a*b*d^3*e^2*x^3 + 9*a*b*c*d^2*e^2*x^2 + 9*a*b*c^2*d*e^
2*x + 3*a*b*c^3*e^2 - (b^2*d^2*e^2*x^2 + 2*b^2*c*d*e^2*x + (b^2*c^2 - 2*b^2
)*e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*
d*x + c^2 + 1)) - 6*(a*b*d^2*e^2*x^2 + 2*a*b*c*d*e^2*x + (a*b*c^2 - 2*a*b)*
e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/d
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex + ce)^2 (b \operatorname{arsinh}(dx + c) + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2*(b*arcsinh(d*x + c) + a)^2, x)
```

maple [A] time = 0.04, size = 163, normalized size = 1.20

$$\frac{\frac{(dx+c)^3 e^2 a^2}{3} + e^2 b^2 \left(\frac{(dx+c)^3 \operatorname{arcsinh}(dx+c)^2}{3} + \frac{4 \operatorname{arcsinh}(dx+c) \sqrt{1+(dx+c)^2}}{9} - \frac{2 \operatorname{arcsinh}(dx+c) \sqrt{1+(dx+c)^2}\,(dx+c)^2}{9} - \frac{4dx}{9} - \frac{4c}{9} + \frac{2(dx+c)^3}{27} \right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^2,x)
```

```
[Out] 1/d*(1/3*(d*x+c)^3*e^2*a^2+e^2*b^2*(1/3*(d*x+c)^3*arcsinh(d*x+c)^2+4/9*arcs
inh(d*x+c)*(1+(d*x+c)^2)^(1/2)-2/9*arcsinh(d*x+c)*(1+(d*x+c)^2)^(1/2)*(d*x+
c)^2-4/9*d*x-4/9*c+2/27*(d*x+c)^3)+2*e^2*a*b*(1/3*(d*x+c)^3*arcsinh(d*x+c)-
1/9*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+2/9*(1+(d*x+c)^2)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3} a^2 d^2 e^2 x^3 + a^2 c d e^2 x^2 + \left(2 x^2 \operatorname{arsinh}(dx + c) - d \left(\frac{3 c^2 \operatorname{arsinh}\left(\frac{2 (d^2 x + cd)}{\sqrt{-4 c^2 d^2 + 4 (c^2+1) d^2}} \right)}{d^3} + \frac{\sqrt{d^2 x^2 + 2 cdx + c^2 + 1}\, x}{d^2} - \cdots \right. \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*d^2*e^2*x^3 + a^2*c*d*e^2*x^2 + (2*x^2*arcsinh(d*x + c) - d*(3*c^2*
arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 + sqrt(d^2*
x^2 + 2*c*d*x + c^2 + 1)*x/d^2 - (c^2 + 1)*arcsinh(2*(d^2*x + c*d)/sqrt(-4*
c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c/d^3
))*a*b*c*d*e^2 + 1/9*(6*x^3*arcsinh(d*x + c) - d*(2*sqrt(d^2*x^2 + 2*c*d*x
+ c^2 + 1)*x^2/d^2 - 15*c^3*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^
2 + 1)*d^2))/d^4 - 5*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c*x/d^3 + 9*(c^2 + 1
)*c*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^4 + 15*sq
rt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c^2/d^4 - 4*sqrt(d^2*x^2 + 2*c*d*x + c^2 +
1)*(c^2 + 1)/d^4))*a*b*d^2*e^2 + a^2*c^2*e^2*x + 2*((d*x + c)*arcsinh(d*x +
 c) - sqrt((d*x + c)^2 + 1))*a*b*c^2*e^2/d + 1/3*(b^2*d^2*e^2*x^3 + 3*b^2*c
*d*e^2*x^2 + 3*b^2*c^2*e^2*x)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 +
1))^2 - integrate(2/3*(b^2*d^5*e^2*x^5 + 5*b^2*c*d^4*e^2*x^4 + (10*c^2*d^3*
e^2 + d^3*e^2)*b^2*x^3 + 3*(3*c^3*d^2*e^2 + c*d^2*e^2)*b^2*x^2 + 3*(c^4*d*e
^2 + c^2*d*e^2)*b^2*x + (b^2*d^4*e^2*x^4 + 4*b^2*c*d^3*e^2*x^3 + 6*b^2*c^2*
```

```
d^2*e^2*x^2 + 3*b^2*c^3*d*e^2*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x
 + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (3
*c^2*d + d)*x + (d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e+d\,e\,x)^2\,(a+b\,\mathrm{asinh}\,(c+d\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2*(a + b*asinh(c + d*x))^2,x)
```

```
[Out] int((c*e + d*e*x)^2*(a + b*asinh(c + d*x))^2, x)
```

sympy [A] time = 1.86, size = 610, normalized size = 4.49

$$\begin{cases} a^2c^2e^2x + a^2cde^2x^2 + \frac{a^2d^2e^2x^3}{3} + \frac{2abc^3e^2\,\mathrm{asinh}\,(c+dx)}{3d} + 2abc^2e^2x\,\mathrm{asinh}\,(c+dx) - \frac{2abc^2e^2\sqrt{c^2+2cdx+d^2x^2+1}}{9d} + 2abcde^2 \\ c^2e^2x\,(a+b\,\mathrm{asinh}\,(c))^2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2*(a+b*asinh(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*c**2*e**2*x + a**2*c*d*e**2*x**2 + a**2*d**2*e**2*x**3/3 +
2*a*b*c**3*e**2*asinh(c + d*x)/(3*d) + 2*a*b*c**2*e**2*x*asinh(c + d*x) - 2
*a*b*c**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(9*d) + 2*a*b*c*d*e**2*
x**2*asinh(c + d*x) - 4*a*b*c*e**2*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/9
 + 2*a*b*d**2*e**2*x**3*asinh(c + d*x)/3 - 2*a*b*d*e**2*x**2*sqrt(c**2 + 2*
c*d*x + d**2*x**2 + 1)/9 + 4*a*b*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/
(9*d) + b**2*c**3*e**2*asinh(c + d*x)**2/(3*d) + b**2*c**2*e**2*x*asinh(c +
 d*x)**2 + 2*b**2*c**2*e**2*x/9 - 2*b**2*c**2*e**2*sqrt(c**2 + 2*c*d*x + d*
*2*x**2 + 1)*asinh(c + d*x)/(9*d) + b**2*c*d*e**2*x**2*asinh(c + d*x)**2 +
2*b**2*c*d*e**2*x**2/9 - 4*b**2*c*e**2*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 +
1)*asinh(c + d*x)/9 + b**2*d**2*e**2*x**3*asinh(c + d*x)**2/3 + 2*b**2*d**2
*e**2*x**3/27 - 2*b**2*d*e**2*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asi
nh(c + d*x)/9 - 4*b**2*e**2*x/9 + 4*b**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x*
*2 + 1)*asinh(c + d*x)/(9*d), Ne(d, 0)), (c**2*e**2*x*(a + b*asinh(c))**2,
True))
```

3.130 $\int (ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^2\,dx$

Optimal. Leaf size=103

$$\frac{e(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)^2}{2d}-\frac{be\sqrt{(c+dx)^2+1}\,(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)}{2d}+\frac{e\left(a+b\sinh^{-1}(c+dx)\right)^2}{4d}+$$

```
[Out] 1/4*b^2*e*(d*x+c)^2/d+1/4*e*(a+b*arcsinh(d*x+c))^2/d+1/2*e*(d*x+c)^2*(a+b*a
rcsinh(d*x+c))^2/d-1/2*b*e*(d*x+c)*(a+b*arcsinh(d*x+c))*(1+(d*x+c)^2)^(1/2)
/d
```

Rubi [A] time = 0.14, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5865, 12, 5661, 5758, 5675, 30}

$$\frac{e(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)^2}{2d}-\frac{be\sqrt{(c+dx)^2+1}\,(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)}{2d}+\frac{e\left(a+b\sinh^{-1}(c+dx)\right)^2}{4d}+$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] (b^2*e*(c + d*x)^2)/(4*d) - (b*e*(c + d*x)*Sqrt[1 + (c + d*x)^2]*(a + b*Arc
Sinh[c + d*x]))/(2*d) + (e*(a + b*ArcSinh[c + d*x])^2)/(4*d) + (e*(c + d*x)
^2*(a + b*ArcSinh[c + d*x])^2)/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^2\,dx=\frac{\text{Subst}\left(\int ex\left(a+b\sinh^{-1}(x)\right)^2\,dx,x,c+dx\right)}{d}$$
$$=\frac{e\,\text{Subst}\left(\int x\left(a+b\sinh^{-1}(x)\right)^2\,dx,x,c+dx\right)}{d}$$
$$=\frac{e(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)^2}{2d}-\frac{(be)\,\text{Subst}\left(\int\frac{x^2\left(a+b\sinh^{-1}(x)\right)}{\sqrt{1+x^2}}\,dx\cdots\right)}{d}$$
$$=-\frac{be(c+dx)\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)}{2d}+\frac{e(c+dx)^2\left(a+\cdots\right.}{\cdots}$$
$$=\frac{b^2e(c+dx)^2}{4d}-\frac{be(c+dx)\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)}{2d}+\cdots$$

Mathematica [A] time = 0.18, size = 120, normalized size = 1.17

$$\frac{e\left(\left(2a^2+b^2\right)(c+dx)^2-2ab\sqrt{(c+dx)^2+1}\,(c+dx)+2b(c+dx)\sinh^{-1}(c+dx)\left(2a(c+dx)-b\sqrt{(c+dx)^2+1}\cdots\right.\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] (e*((2*a^2 + b^2)*(c + d*x)^2 - 2*a*b*(c + d*x)*Sqrt[1 + (c + d*x)^2] + 2*a
*b*ArcSinh[c + d*x] + 2*b*(c + d*x)*(2*a*(c + d*x) - b*Sqrt[1 + (c + d*x)^2
])*ArcSinh[c + d*x] + b^2*(1 + 2*(c + d*x)^2)*ArcSinh[c + d*x]^2))/(4*d)
```

fricas [B] time = 0.55, size = 230, normalized size = 2.23

$$\frac{\left(2\,a^2+b^2\right)d^2ex^2+2\left(2\,a^2+b^2\right)cdex+\left(2\,b^2d^2ex^2+4\,b^2cdex+\left(2\,b^2c^2+b^2\right)e\right)\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/4*((2*a^2 + b^2)*d^2*e*x^2 + 2*(2*a^2 + b^2)*c*d*e*x + (2*b^2*d^2*e*x^2 +
 4*b^2*c*d*e*x + (2*b^2*c^2 + b^2)*e)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x
+ c^2 + 1))^2 + 2*(2*a*b*d^2*e*x^2 + 4*a*b*c*d*e*x + (2*a*b*c^2 + a*b)*e -
(b^2*d*e*x + b^2*c*e)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt
(d^2*x^2 + 2*c*d*x + c^2 + 1)) - 2*(a*b*d*e*x + a*b*c*e)*sqrt(d^2*x^2 + 2*c
*d*x + c^2 + 1))/d
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex+ce)(b\,\text{arsinh}\,(dx+c)+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^2, x)
```

maple [A] time = 0.03, size = 135, normalized size = 1.31

$$\frac{\frac{(dx+c)^2 e a^2}{2} + e b^2 \left(\frac{\operatorname{arcsinh}(dx+c)^2 (1+(dx+c)^2)}{2} - \frac{\operatorname{arcsinh}(dx+c)\sqrt{1+(dx+c)^2}\,(dx+c)}{2} - \frac{\operatorname{arcsinh}(dx+c)^2}{4} + \frac{(dx+c)^2}{4} + \frac{1}{4}\right) + 2eab \left(\frac{(dx+c)^2 a}{}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^2,x)
```

```
[Out] 1/d*(1/2*(d*x+c)^2*e*a^2+e*b^2*(1/2*arcsinh(d*x+c)^2*(1+(d*x+c)^2)-1/2*arcs
inh(d*x+c)*(1+(d*x+c)^2)^(1/2)*(d*x+c)-1/4*arcsinh(d*x+c)^2+1/4*(d*x+c)^2+1
/4)+2*e*a*b*(1/2*(d*x+c)^2*arcsinh(d*x+c)-1/4*(d*x+c)*(1+(d*x+c)^2)^(1/2)+1
/4*arcsinh(d*x+c)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}a^2dex^2 + \frac{1}{2}\left(2x^2 \operatorname{arsinh}(dx+c) - d\left(\frac{3c^2 \operatorname{arsinh}\left(\frac{2(d^2x+cd)}{\sqrt{-4c^2d^2+4(c^2+1)d^2}}\right)}{d^3} + \frac{\sqrt{d^2x^2+2cdx+c^2+1}\,x}{d^2} - \frac{(c^2+1)\operatorname{arsi}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*d*e*x^2 + 1/2*(2*x^2*arcsinh(d*x + c) - d*(3*c^2*arcsinh(2*(d^2*x +
 c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c^
2 + 1)*x/d^2 - (c^2 + 1)*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 +
 1)*d^2))/d^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c/d^3))*a*b*d*e + a^2*c
*e*x + 2*((d*x + c)*arcsinh(d*x + c) - sqrt((d*x + c)^2 + 1))*a*b*c*e/d + 1
/2*(b^2*d*e*x^2 + 2*b^2*c*e*x)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 +
 1))^2 - integrate((b^2*d^4*e*x^4 + 4*b^2*c*d^3*e*x^3 + (5*c^2*d^2*e + d^2*
e)*b^2*x^2 + 2*(c^3*d*e + c*d*e)*b^2*x + (b^2*d^3*e*x^3 + 3*b^2*c*d^2*e*x^2
 + 2*b^2*c^2*d*e*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d
^2*x^2 + 2*c*d*x + c^2 + 1))/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (3*c^2*d + d)*x
 + (d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e + d\,e\,x)\,\left(a + b\,\mathrm{asinh}\,(c + d\,x)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)*(a + b*asinh(c + d*x))^2,x)
```

```
[Out] int((c*e + d*e*x)*(a + b*asinh(c + d*x))^2, x)
```

sympy [A] time = 0.86, size = 335, normalized size = 3.25

$$\begin{cases} a^2cex + \frac{a^2dex^2}{2} + \frac{abc^2e\,\mathrm{asinh}\,(c+dx)}{d} + 2abcex\,\mathrm{asinh}\,(c+dx) - \frac{abce\sqrt{c^2+2cdx+d^2x^2+1}}{2d} + abdex^2\,\mathrm{asinh}\,(c+dx) - \frac{abex\sqrt{c^2+}}{} \\ cex\,(a + b\,\mathrm{asinh}\,(c))^2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*asinh(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*c*e*x + a**2*d*e*x**2/2 + a*b*c**2*e*asinh(c + d*x)/d + 2*a
*b*c*e*x*asinh(c + d*x) - a*b*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(2*d
) + a*b*d*e*x**2*asinh(c + d*x) - a*b*e*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 +
 1)/2 + a*b*e*asinh(c + d*x)/(2*d) + b**2*c**2*e*asinh(c + d*x)**2/(2*d) +
b**2*c*e*x*asinh(c + d*x)**2 + b**2*c*e*x/2 - b**2*c*e*sqrt(c**2 + 2*c*d*x
+ d**2*x**2 + 1)*asinh(c + d*x)/(2*d) + b**2*d*e*x**2*asinh(c + d*x)**2/2 +
 b**2*d*e*x**2/4 - b**2*e*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c +
d*x)/2 + b**2*e*asinh(c + d*x)**2/(4*d), Ne(d, 0)), (c*e*x*(a + b*asinh(c))
**2, True))
```

3.131 $\int \left(a+b\sinh^{-1}(c+dx)\right)^2 \, dx$

Optimal. Leaf size=57

$$-\frac{2b\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)}{d}+\frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^2}{d}+2b^2x$$

```
[Out] 2*b^2*x+(d*x+c)*(a+b*arcsinh(d*x+c))^2/d-2*b*(a+b*arcsinh(d*x+c))*(1+(d*x+c
)^2)^(1/2)/d
```

Rubi [A] time = 0.07, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5863, 5653, 5717, 8}

$$-\frac{2b\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)}{d}+\frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^2}{d}+2b^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] 2*b^2*x - (2*b*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x]))/d + ((c + d*
x)*(a + b*ArcSinh[c + d*x])^2)/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5863

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \left(a+b\sinh^{-1}(c+dx)\right)^2 dx = \frac{\text{Subst}\left(\int \left(a+b\sinh^{-1}(x)\right)^2 dx, x, c+dx\right)}{d}$$

$$= \frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^2}{d} - \frac{(2b)\,\text{Subst}\left(\int \frac{x\left(a+b\sinh^{-1}(x)\right)}{\sqrt{1+x^2}} dx, x, c+dx\right)}{d}$$

$$= -\frac{2b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)}{d} + \frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^2}{d}$$

$$= 2b^2x - \frac{2b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)}{d} + \frac{(c+dx)\left(a+b\sinh^{-1}(c+\right.}{d}$$

Mathematica [A] time = 0.10, size = 87, normalized size = 1.53

$$\frac{\left(a^2+2b^2\right)(c+dx) - 2ab\sqrt{(c+dx)^2+1} + 2b\sinh^{-1}(c+dx)\left(ac+adx+b\left(-\sqrt{(c+dx)^2+1}\right)\right) + b^2(c+dx)\,\text{si}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] ((a^2 + 2*b^2)*(c + d*x) - 2*a*b*Sqrt[1 + (c + d*x)^2] + 2*b*(a*c + a*d*x -
 b*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x] + b^2*(c + d*x)*ArcSinh[c + d*x]
^2)/d
```

fricas [B] time = 0.56, size = 141, normalized size = 2.47

$$\frac{\left(a^2+2\,b^2\right)dx + \left(b^2dx+b^2c\right)\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)^2 - 2\,\sqrt{d^2x^2+2\,cdx+c^2+1}\,ab + 2\left(abdx\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] ((a^2 + 2*b^2)*d*x + (b^2*d*x + b^2*c)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x
 + c^2 + 1))^2 - 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*a*b + 2*(a*b*d*x + a*b
*c - sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*b^2)*log(d*x + c + sqrt(d^2*x^2 + 2*
c*d*x + c^2 + 1)))/d
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\,\text{arsinh}\,(dx+c)+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^2, x)
```

maple [A] time = 0.05, size = 90, normalized size = 1.58

$$\frac{a^2(dx+c) + b^2\left((dx+c)\,\text{arcsinh}\,(dx+c)^2 - 2\,\text{arcsinh}\,(dx+c)\sqrt{1+(dx+c)^2} + 2dx + 2c\right) + 2ab\left((dx+c)\,\text{a}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(d*x+c)+b^2*((d*x+c)*arcsinh(d*x+c)^2-2*arcsinh(d*x+c)*(1+(d*x+c)^
2)^(1/2)+2*d*x+2*c)+2*a*b*((d*x+c)*arcsinh(d*x+c)-(1+(d*x+c)^2)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\left(x \log\left(dx + c + \sqrt{d^2x^2 + 2\,cdx + c^2 + 1}\right)^2 - \int \frac{2\left(d^3x^3 + 2\,cd^2x^2 + \left(c^2d + d\right)x + \sqrt{d^2x^2 + 2\,cdx + c^2 + 1}\left(d^2x^2 + \right.\right.}{d^3x^3 + 3\,cd^2x^2 + c^3 + \left(3\,c^2d + d\right)x + \left(d^2x^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] (x*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 - integrate(2*(d^3*x^
3 + 2*c*d^2*x^2 + (c^2*d + d)*x + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(d^2*x^
2 + c*d*x))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(d^3*x^3 + 3*c
*d^2*x^2 + c^3 + (3*c^2*d + d)*x + (d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + c)
, x))*b^2 + a^2*x + 2*((d*x + c)*arcsinh(d*x + c) - sqrt((d*x + c)^2 + 1))*
a*b/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int (a + b\,\mathrm{asinh}\,(c + d\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^2,x)
```

```
[Out] int((a + b*asinh(c + d*x))^2, x)
```

sympy [A] time = 0.32, size = 143, normalized size = 2.51

$$\begin{cases} a^2x + \frac{2abc\,\mathrm{asinh}\,(c+dx)}{d} + 2abx\,\mathrm{asinh}\,(c + dx) - \frac{2ab\sqrt{c^2+2cdx+d^2x^2+1}}{d} + \frac{b^2c\,\mathrm{asinh}^2\,(c+dx)}{d} + b^2x\,\mathrm{asinh}^2\,(c + dx) + 2b^2x - \\ x\,(a + b\,\mathrm{asinh}\,(c))^2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*x + 2*a*b*c*asinh(c + d*x)/d + 2*a*b*x*asinh(c + d*x) - 2*a
*b*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/d + b**2*c*asinh(c + d*x)**2/d + b*
*2*x*asinh(c + d*x)**2 + 2*b**2*x - 2*b**2*sqrt(c**2 + 2*c*d*x + d**2*x**2
+ 1)*asinh(c + d*x)/d, Ne(d, 0)), (x*(a + b*asinh(c))**2, True))
```

3.132 $\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{ce+dex}\,dx$

Optimal. Leaf size=116

$$-\frac{b\text{Li}_2\left(e^{-2\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)}{de}+\frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{3bde}+\frac{\log\left(1-e^{-2\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}\right.}{de}$$

```
[Out] 1/3*(a+b*arcsinh(d*x+c))^3/b/d/e+(a+b*arcsinh(d*x+c))^2*ln(1-1/(d*x+c+(1+(d
*x+c)^2)^(1/2))^2)/d/e-b*(a+b*arcsinh(d*x+c))*polylog(2,1/(d*x+c+(1+(d*x+c)
^2)^(1/2))^2)/d/e-1/2*b^2*polylog(3,1/(d*x+c+(1+(d*x+c)^2)^(1/2))^2)/d/e
```

Rubi [A] time = 0.20, antiderivative size = 115, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {5865, 12, 5659, 3716, 2190, 2531, 2282, 6589}

$$\frac{b\text{PolyLog}\left(2,e^{2\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)}{de}-\frac{b^2\text{PolyLog}\left(3,e^{2\sinh^{-1}(c+dx)}\right)}{2de}-\frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{3bde}+\frac{\text{lo}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcSinh[c + d*x])^2/(c*e + d*e*x),x]
```

```
[Out] -(a + b*ArcSinh[c + d*x])^3/(3*b*d*e) + ((a + b*ArcSinh[c + d*x])^2*Log[1 -
 E^(2*ArcSinh[c + d*x])])/(d*e) + (b*(a + b*ArcSinh[c + d*x])*PolyLog[2, E^
(2*ArcSinh[c + d*x])])/(d*e) - (b^2*PolyLog[3, E^(2*ArcSinh[c + d*x])])/(2*
d*e)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{ce+dex}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^2}{ex}\,dx,x,c+dx\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^2}{x}\,dx,x,c+dx\right)}{de}$$
$$= \frac{\text{Subst}\left(\int (a+bx)^2\coth(x)\,dx,x,\sinh^{-1}(c+dx)\right)}{de}$$
$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{3bde} - \frac{2\,\text{Subst}\left(\int \frac{e^{2x}(a+bx)^2}{1-e^{2x}}\,dx,x,\sinh^{-1}(c+dx)\right)}{de}$$
$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{3bde} + \frac{\left(a+b\sinh^{-1}(c+dx)\right)^2\log\left(1-e^{2\sinh^{-1}(c+dx)}\right)}{de} - \frac{(2b}{}$$
$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{3bde} + \frac{\left(a+b\sinh^{-1}(c+dx)\right)^2\log\left(1-e^{2\sinh^{-1}(c+dx)}\right)}{de} + \frac{b\left(\right.}{}$$
$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{3bde} + \frac{\left(a+b\sinh^{-1}(c+dx)\right)^2\log\left(1-e^{2\sinh^{-1}(c+dx)}\right)}{de} + \frac{b\left(\right.}{}$$
$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{3bde} + \frac{\left(a+b\sinh^{-1}(c+dx)\right)^2\log\left(1-e^{2\sinh^{-1}(c+dx)}\right)}{de} + \frac{b\left(\right.}{}$$

Mathematica [A] time = 0.05, size = 100, normalized size = 0.86

$$\frac{6b^2\text{Li}_2\left(e^{2\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right) - 2\left(a+b\sinh^{-1}(c+dx)\right)^2\left(a+b\sinh^{-1}(c+dx) - 3b\log\left(1-e^{2\sinh}\right.\right.}{6bde}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^2/(c*e + d*e*x),x]
```

```
[Out] (-2*(a + b*ArcSinh[c + d*x])^2*(a + b*ArcSinh[c + d*x] - 3*b*Log[1 - E^(2*A
rcSinh[c + d*x])]) + 6*b^2*(a + b*ArcSinh[c + d*x])*PolyLog[2, E^(2*ArcSinh
[c + d*x])] - 3*b^3*PolyLog[3, E^(2*ArcSinh[c + d*x])])/(6*b*d*e)
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\operatorname{arsinh}(dx+c)^2+2\,ab\operatorname{arsinh}(dx+c)+a^2}{dex+ce},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e),x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(d*x + c)^2 + 2*a*b*arcsinh(d*x + c) + a^2)/(d*e*x + c
*e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(b\operatorname{arsinh}(dx+c)+a)^2}{dex+ce}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^2/(d*e*x + c*e), x)
```

maple [B] time = 0.08, size = 404, normalized size = 3.48

$$\frac{a^2\ln(dx+c)}{de}-\frac{b^2\operatorname{arcsinh}(dx+c)^3}{3de}+\frac{b^2\operatorname{arcsinh}(dx+c)^2\ln\left(1+dx+c+\sqrt{1+(dx+c)^2}\right)}{de}+\frac{2b^2\operatorname{arcsinh}(dx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e),x)
```

```
[Out] 1/d*a^2/e*ln(d*x+c)-1/3/d*b^2/e*arcsinh(d*x+c)^3+1/d*b^2/e*arcsinh(d*x+c)^2
*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))+2/d*b^2/e*arcsinh(d*x+c)*polylog(2,-d*x-c-
(1+(d*x+c)^2)^(1/2))-2/d*b^2/e*polylog(3,-d*x-c-(1+(d*x+c)^2)^(1/2))+1/d*b^
2/e*arcsinh(d*x+c)^2*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))+2/d*b^2/e*arcsinh(d*x+
c)*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2))-2/d*b^2/e*polylog(3,d*x+c+(1+(d*x+c
)^2)^(1/2))-1/d*a*b/e*arcsinh(d*x+c)^2+2/d*a*b/e*arcsinh(d*x+c)*ln(1+d*x+c+
(1+(d*x+c)^2)^(1/2))+2/d*a*b/e*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))+2/d*a*
b/e*arcsinh(d*x+c)*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))+2/d*a*b/e*polylog(2,d*x+
c+(1+(d*x+c)^2)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^2\log(dex+ce)}{de}+\int\frac{b^2\log\left(dx+c+\sqrt{(dx+c)^2+1}\right)^2}{dex+ce}+\frac{2\,ab\log\left(dx+c+\sqrt{(dx+c)^2+1}\right)}{dex+ce}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e),x, algorithm="maxima")
```

```
[Out] a^2*log(d*e*x + c*e)/(d*e) + integrate(b^2*log(d*x + c + sqrt((d*x + c)^2 +
 1))^2/(d*e*x + c*e) + 2*a*b*log(d*x + c + sqrt((d*x + c)^2 + 1))/(d*e*x +
c*e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{asinh}\,(c+d\,x))^2}{c\,e+d\,e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^2/(c*e + d*e*x),x)
```

```
[Out] int((a + b*asinh(c + d*x))^2/(c*e + d*e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{c+dx}\,dx + \int \frac{b^2\,\mathrm{asinh}^2\,(c+dx)}{c+dx}\,dx + \int \frac{2ab\,\mathrm{asinh}\,(c+dx)}{c+dx}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**2/(d*e*x+c*e),x)
```

```
[Out] (Integral(a**2/(c + d*x), x) + Integral(b**2*asinh(c + d*x)**2/(c + d*x), x
) + Integral(2*a*b*asinh(c + d*x)/(c + d*x), x))/e
```

3.133 $\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{(ce+dex)^2}\,dx$

Optimal. Leaf size=100

$$-\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{de^2(c+dx)}-\frac{4b\tanh^{-1}\left(e^{\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)}{de^2}-\frac{2b^2\mathrm{Li}_2\left(-e^{\sinh^{-1}(c+dx)}\right)}{de^2}+\frac{2b^2\mathrm{Li}_2\left(e^{\si...}\right)}{de...}$$

```
[Out] -(a+b*arcsinh(d*x+c))^2/d/e^2/(d*x+c)-4*b*(a+b*arcsinh(d*x+c))*arctanh(d*x+
c+(1+(d*x+c)^2)^(1/2))/d/e^2-2*b^2*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))/d/
e^2+2*b^2*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2))/d/e^2
```

Rubi [A] time = 0.18, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {5865, 12, 5661, 5760, 4182, 2279, 2391}

$$-\frac{2b^2\text{PolyLog}\left(2,-e^{\sinh^{-1}(c+dx)}\right)}{de^2}+\frac{2b^2\text{PolyLog}\left(2,e^{\sinh^{-1}(c+dx)}\right)}{de^2}-\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{de^2(c+dx)}-\frac{4b\tanh^{-1}\left(e^{\sinh^{-1}(c+...}\right)}{d...}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^2/(c*e + d*e*x)^2,x]
```

```
[Out] -((a + b*ArcSinh[c + d*x])^2/(d*e^2*(c + d*x))) - (4*b*(a + b*ArcSinh[c + d
*x])*ArcTanh[E^ArcSinh[c + d*x]])/(d*e^2) - (2*b^2*PolyLog[2, -E^ArcSinh[c
+ d*x]])/(d*e^2) + (2*b^2*PolyLog[2, E^ArcSinh[c + d*x]])/(d*e^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{(ce+dex)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^2}{e^2x^2}\,dx, x, c+dx\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^2}{x^2}\,dx, x, c+dx\right)}{de^2}$$
$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{de^2(c+dx)} + \frac{(2b)\,\text{Subst}\left(\int \frac{a+b\sinh^{-1}(x)}{x\sqrt{1+x^2}}\,dx, x, c+dx\right)}{de^2}$$
$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{de^2(c+dx)} + \frac{(2b)\,\text{Subst}\left(\int (a+bx)\text{csch}(x)\,dx, x, \sinh^{-1}(c+dx)\right)}{de^2}$$
$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{de^2(c+dx)} - \frac{4b\left(a+b\sinh^{-1}(c+dx)\right)\tanh^{-1}\left(e^{\sinh^{-1}(c+dx)}\right)}{de^2} - \frac{(2\ldots}{\ldots}$$
$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{de^2(c+dx)} - \frac{4b\left(a+b\sinh^{-1}(c+dx)\right)\tanh^{-1}\left(e^{\sinh^{-1}(c+dx)}\right)}{de^2} - \frac{(2\ldots}{\ldots}$$
$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{de^2(c+dx)} - \frac{4b\left(a+b\sinh^{-1}(c+dx)\right)\tanh^{-1}\left(e^{\sinh^{-1}(c+dx)}\right)}{de^2} - \frac{2b\ldots}{\ldots}$$

Mathematica [A] time = 0.60, size = 154, normalized size = 1.54

$$\frac{-\frac{a^2}{c+dx} + ab\left(2\log\left(\frac{2\sinh^2\left(\frac{1}{2}\sinh^{-1}(c+dx)\right)}{c+dx}\right) - \frac{2\sinh^{-1}(c+dx)}{c+dx}\right) + b^2\left(2\text{Li}_2\left(-e^{-\sinh^{-1}(c+dx)}\right) - 2\text{Li}_2\left(e^{-\sinh^{-1}(c+dx)}\right) + \sinh\ldots\right.}{de^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^2/(c*e + d*e*x)^2,x]
```

```
[Out] (-(a^2/(c + d*x)) + a*b*((-2*ArcSinh[c + d*x])/(c + d*x) + 2*Log[(2*Sinh[Ar
cSinh[c + d*x]/2]^2)/(c + d*x)]) + b^2*(ArcSinh[c + d*x]*(-(ArcSinh[c + d*x
]/(c + d*x)) + 2*Log[1 - E^(-ArcSinh[c + d*x])] - 2*Log[1 + E^(-ArcSinh[c +
 d*x])]) + 2*PolyLog[2, -E^(-ArcSinh[c + d*x])] - 2*PolyLog[2, E^(-ArcSinh[
c + d*x])]))/(d*e^2)
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\operatorname{arsinh}(dx+c)^2 + 2\,ab\operatorname{arsinh}(dx+c) + a^2}{d^2e^2x^2 + 2\,cde^2x + c^2e^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(d*x + c)^2 + 2*a*b*arcsinh(d*x + c) + a^2)/(d^2*e^2*x
^2 + 2*c*d*e^2*x + c^2*e^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arsinh}(dx + c) + a)^2}{(dex + ce)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^2/(d*e*x + c*e)^2, x)
```

maple [A] time = 0.14, size = 229, normalized size = 2.29

$$-\frac{a^2}{d\,e^2\,(dx+c)}-\frac{b^2 \operatorname{arcsinh}(dx+c)^2}{d\,e^2\,(dx+c)}-\frac{2b^2 \operatorname{arcsinh}(dx+c)\ln\left(1+dx+c+\sqrt{1+(dx+c)^2}\right)}{d\,e^2}-\frac{2b^2 \operatorname{polylog}\left(2,\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^2,x)
```

```
[Out] -1/d*a^2/e^2/(d*x+c)-1/d*b^2/e^2*arcsinh(d*x+c)^2/(d*x+c)-2/d*b^2/e^2*arcsi
nh(d*x+c)*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))-2*b^2*polylog(2,-d*x-c-(1+(d*x+c)
^2)^(1/2))/d/e^2+2/d*b^2/e^2*arcsinh(d*x+c)*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))
+2*b^2*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2))/d/e^2-2/d*a*b/e^2/(d*x+c)*arcsi
nh(d*x+c)-2/d*a*b/e^2*arctanh(1/(1+(d*x+c)^2)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-b^2\left(\frac{\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)^2}{d^2e^2x+cde^2}-\int \frac{2\left(d^2x^2+2\,cdx+c^2+\sqrt{d^2x^2+}\right.}{d^4e^2x^4+4\,cd^3e^2x^3+c^4e^2+c^2e^2+\left(6\,c^2d^2e^2+d^2e^2\right)x^2+2\left(\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^2,x, algorithm="maxima")
```

```
[Out] -b^2*(log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2/(d^2*e^2*x + c*d*e
^2) - integrate(2*(d^2*x^2 + 2*c*d*x + c^2 + sqrt(d^2*x^2 + 2*c*d*x + c^2 +
 1)*(d*x + c) + 1)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(d^4*e^
2*x^4 + 4*c*d^3*e^2*x^3 + c^4*e^2 + c^2*e^2 + (6*c^2*d^2*e^2 + d^2*e^2)*x^2
 + 2*(2*c^3*d*e^2 + c*d*e^2)*x + (d^3*e^2*x^3 + 3*c*d^2*e^2*x^2 + c^3*e^2 +
 c*e^2 + (3*c^2*d*e^2 + d*e^2)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)), x)) -
 2*a*b*(arcsinh(d*x + c)/(d^2*e^2*x + c*d*e^2) + arcsinh(d*e^2/abs(d^2*e^2*
x + c*d*e^2))/(d*e^2)) - a^2/(d^2*e^2*x + c*d*e^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{asinh}\,(c+d\,x))^2}{(c\,e+d\,e\,x)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^2/(c*e + d*e*x)^2,x)
```

```
[Out] int((a + b*asinh(c + d*x))^2/(c*e + d*e*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{c^2+2cdx+d^2x^2}\,dx + \int \frac{b^2\operatorname{asinh}^2(c+dx)}{c^2+2cdx+d^2x^2}\,dx + \int \frac{2ab\operatorname{asinh}(c+dx)}{c^2+2cdx+d^2x^2}\,dx}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**2/(d*e*x+c*e)**2,x)
```

```
[Out] (Integral(a**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(b**2*asinh(c + d
*x)**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(2*a*b*asinh(c + d*x)/(c*
*2 + 2*c*d*x + d**2*x**2), x))/e**2
```

3.134 $\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{(ce+dex)^3} dx$

Optimal. Leaf size=85

$$-\frac{b\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)}{de^3(c+dx)}-\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de^3(c+dx)^2}+\frac{b^2\log(c+dx)}{de^3}$$

```
[Out] -1/2*(a+b*arcsinh(d*x+c))^2/d/e^3/(d*x+c)^2+b^2*ln(d*x+c)/d/e^3-b*(a+b*arcs
inh(d*x+c))*(1+(d*x+c)^2)^(1/2)/d/e^3/(d*x+c)
```

Rubi [A] time = 0.14, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {5865, 12, 5661, 5723, 29}

$$-\frac{b\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)}{de^3(c+dx)}-\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de^3(c+dx)^2}+\frac{b^2\log(c+dx)}{de^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^2/(c*e + d*e*x)^3,x]
```

```
[Out] -((b*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x]))/(d*e^3*(c + d*x))) - (
a + b*ArcSinh[c + d*x])^2/(2*d*e^3*(c + d*x)^2) + (b^2*Log[c + d*x])/(d*e^3
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5723

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)
^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 +
 c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
 d, e, f, m, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] &&
 NeQ[m, -1]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{(ce+dex)^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^2}{e^3x^3}\,dx,x,c+dx\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^2}{x^3}\,dx,x,c+dx\right)}{de^3}$$
$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de^3(c+dx)^2} + \frac{b\,\text{Subst}\left(\int \frac{a+b\sinh^{-1}(x)}{x^2\sqrt{1+x^2}}\,dx,x,c+dx\right)}{de^3}$$
$$= -\frac{b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)}{de^3(c+dx)} - \frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de^3(c+dx)^2} + \frac{b^2\,\text{Subst}\left(\right.}{}$$
$$= -\frac{b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)}{de^3(c+dx)} - \frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de^3(c+dx)^2} + \frac{b^2\log(c+}{de^3}$$

Mathematica [**A**] time = 0.25, size = 120, normalized size = 1.41

$$-\frac{a\left(a+2b(c+dx)\sqrt{c^2+2cdx+d^2x^2+1}\right)+2b\sinh^{-1}(c+dx)\left(a+b(c+dx)\sqrt{c^2+2cdx+d^2x^2+1}\right)-2b^2(c+}{2de^3(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^2/(c*e + d*e*x)^3,x]
```

```
[Out] -1/2*(a*(a + 2*b*(c + d*x)*Sqrt[1 + c^2 + 2*c*d*x + d^2*x^2]) + 2*b*(a + b*
(c + d*x)*Sqrt[1 + c^2 + 2*c*d*x + d^2*x^2])*ArcSinh[c + d*x] + b^2*ArcSinh
[c + d*x]^2 - 2*b^2*(c + d*x)^2*Log[c + d*x])/(d*e^3*(c + d*x)^2)
```

fricas [**B**] time = 0.66, size = 319, normalized size = 3.75

$$-\frac{2\,abc^2d^2x^2+4\,abc^3dx+2\,abc^4+b^2c^2\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)^2+a^2c^2-2\left(abd^2x^2+2\,abcdx-\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*b*c^2*d^2*x^2 + 4*a*b*c^3*d*x + 2*a*b*c^4 + b^2*c^2*log(d*x + c +
 sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 + a^2*c^2 - 2*(a*b*d^2*x^2 + 2*a*b*c*
d*x - (b^2*c^2*d*x + b^2*c^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x +
c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - 2*(b^2*c^2*d^2*x^2 + 2*b^2*c^3*d*x
 + b^2*c^4)*log(d*x + c) - 2*(a*b*d^2*x^2 + 2*a*b*c*d*x + a*b*c^2)*log(-d*x
 - c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + 2*(a*b*c^2*d*x + a*b*c^3)*sqrt(
d^2*x^2 + 2*c*d*x + c^2 + 1))/(c^2*d^3*e^3*x^2 + 2*c^3*d^2*e^3*x + c^4*d*e^
3)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,\text{arsinh}\left(dx+c\right)+a\right)^2}{\left(dex+ce\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^2/(d*e*x + c*e)^3, x)
```

maple [B] time = 0.20, size = 180, normalized size = 2.12

$$-\frac{a^2}{2d\,e^3\,(dx+c)^2}-\frac{b^2\,\mathrm{arcsinh}\,(dx+c)}{d\,e^3}-\frac{b^2\,\mathrm{arcsinh}\,(dx+c)\,\sqrt{1+(dx+c)^2}}{d\,e^3\,(dx+c)}-\frac{b^2\,\mathrm{arcsinh}\,(dx+c)^2}{2d\,e^3\,(dx+c)^2}+\frac{b^2\ln\left(\left(dx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^3,x)
```

```
[Out] -1/2/d*a^2/e^3/(d*x+c)^2-1/d*b^2/e^3*arcsinh(d*x+c)-1/d*b^2/e^3*arcsinh(d*x
+c)/(d*x+c)*(1+(d*x+c)^2)^(1/2)-1/2/d*b^2/e^3*arcsinh(d*x+c)^2/(d*x+c)^2+1/
d*b^2/e^3*ln((d*x+c+(1+(d*x+c)^2)^(1/2))^2-1)-1/d*a*b/e^3/(d*x+c)^2*arcsinh
(d*x+c)-1/d*a*b/e^3/(d*x+c)*(1+(d*x+c)^2)^(1/2)
```

maxima [B] time = 0.42, size = 230, normalized size = 2.71

$$-\left(\frac{\sqrt{d^2x^2+2\,cdx+c^2+1}\,d\,\mathrm{arsinh}\,(dx+c)}{d^3e^3x+cd^2e^3}-\frac{\log\,(dx+c)}{de^3}\right)b^2-ab\left(\frac{\sqrt{d^2x^2+2\,cdx+c^2+1}\,d}{d^3e^3x+cd^2e^3}+\frac{\mathrm{arsinh}\,(d}{d^3e^3x^2+2\,cd^2e}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^3,x, algorithm="maxima")
```

```
[Out] -(sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*d*arcsinh(d*x + c)/(d^3*e^3*x + c*d^2*e
^3) - log(d*x + c)/(d*e^3))*b^2 - a*b*(sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*d/
(d^3*e^3*x + c*d^2*e^3) + arcsinh(d*x + c)/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c
^2*d*e^3)) - 1/2*b^2*arcsinh(d*x + c)^2/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*
d*e^3) - 1/2*a^2/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)^2}{\left(c\,e+d\,e\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^2/(c*e + d*e*x)^3,x)
```

```
[Out] int((a + b*asinh(c + d*x))^2/(c*e + d*e*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx+\int \frac{b^2\,\mathrm{asinh}^2\,(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx+\int \frac{2ab\,\mathrm{asinh}\,(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**2/(d*e*x+c*e)**3,x)
```

```
[Out] (Integral(a**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integr
al(b**2*asinh(c + d*x)**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),
x) + Integral(2*a*b*asinh(c + d*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**
3*x**3), x))/e**3
```

3.135 $\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{(ce+dex)^4}\,dx$

Optimal. Leaf size=169

$$-\frac{b\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)}{3de^4(c+dx)^2}-\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{3de^4(c+dx)^3}+\frac{2b\tanh^{-1}\left(e^{\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx\right.}{3de^4}$$

```
[Out] -1/3*b^2/d/e^4/(d*x+c)-1/3*(a+b*arcsinh(d*x+c))^2/d/e^4/(d*x+c)^3+2/3*b*(a+
b*arcsinh(d*x+c))*arctanh(d*x+c+(1+(d*x+c)^2)^(1/2))/d/e^4+1/3*b^2*polylog(
2,-d*x-c-(1+(d*x+c)^2)^(1/2))/d/e^4-1/3*b^2*polylog(2,d*x+c+(1+(d*x+c)^2)^(
1/2))/d/e^4-1/3*b*(a+b*arcsinh(d*x+c))*(1+(d*x+c)^2)^(1/2)/d/e^4/(d*x+c)^2
```

Rubi [A] time = 0.25, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {5865, 12, 5661, 5747, 5760, 4182, 2279, 2391, 30}

$$\frac{b^2\text{PolyLog}\left(2,-e^{\sinh^{-1}(c+dx)}\right)}{3de^4}-\frac{b^2\text{PolyLog}\left(2,e^{\sinh^{-1}(c+dx)}\right)}{3de^4}-\frac{b\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)}{3de^4(c+dx)^2}-\frac{\left(a+b\sinh\right.}{3de^4(c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^2/(c*e + d*e*x)^4,x]
```

```
[Out] -b^2/(3*d*e^4*(c + d*x)) - (b*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x]
))/(3*d*e^4*(c + d*x)^2) - (a + b*ArcSinh[c + d*x])^2/(3*d*e^4*(c + d*x)^3)
 + (2*b*(a + b*ArcSinh[c + d*x])*ArcTanh[E^ArcSinh[c + d*x]])/(3*d*e^4) + (
b^2*PolyLog[2, -E^ArcSinh[c + d*x]])/(3*d*e^4) - (b^2*PolyLog[2, E^ArcSinh[
c + d*x]])/(3*d*e^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{(ce+dex)^4}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^2}{e^4x^4}\,dx,x,c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^2}{x^4}\,dx,x,c+dx\right)}{de^4}$$

$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{3de^4(c+dx)^3} + \frac{(2b)\,\text{Subst}\left(\int \frac{a+b\sinh^{-1}(x)}{x^3\sqrt{1+x^2}}\,dx,x,c+dx\right)}{3de^4}$$

$$= -\frac{b\sqrt{1+(c+dx)^2}\,\left(a+b\sinh^{-1}(c+dx)\right)}{3de^4(c+dx)^2} - \frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{3de^4(c+dx)^3} - \frac{b\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{b^2}{3de^4(c+dx)} - \frac{b\sqrt{1+(c+dx)^2}\,\left(a+b\sinh^{-1}(c+dx)\right)}{3de^4(c+dx)^2} - \frac{\left(a+b\sinh^{-1}(c+dx\right.}{3de^4(c+dx)^3}$$

$$= -\frac{b^2}{3de^4(c+dx)} - \frac{b\sqrt{1+(c+dx)^2}\,\left(a+b\sinh^{-1}(c+dx)\right)}{3de^4(c+dx)^2} - \frac{\left(a+b\sinh^{-1}(c+dx\right.}{3de^4(c+dx)^3}$$

$$= -\frac{b^2}{3de^4(c+dx)} - \frac{b\sqrt{1+(c+dx)^2}\,\left(a+b\sinh^{-1}(c+dx)\right)}{3de^4(c+dx)^2} - \frac{\left(a+b\sinh^{-1}(c+dx\right.}{3de^4(c+dx)^3}$$

$$= -\frac{b^2}{3de^4(c+dx)} - \frac{b\sqrt{1+(c+dx)^2}\,\left(a+b\sinh^{-1}(c+dx)\right)}{3de^4(c+dx)^2} - \frac{\left(a+b\sinh^{-1}(c+dx\right.}{3de^4(c+dx)^3}$$

Mathematica [A] time = 1.72, size = 212, normalized size = 1.25

$$-\frac{4a^2+ab\left(8\sinh^{-1}(c+dx)+2\sinh\left(2\sinh^{-1}(c+dx)\right)+\left(\sinh\left(3\sinh^{-1}(c+dx)\right)-3(c+dx)\right)\log\left(\tanh\left(\frac{1}{2}\sin\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^2/(c*e + d*e*x)^4,x]
```

```
[Out] -1/12*(4*a^2 + a*b*(8*ArcSinh[c + d*x] + 2*Sinh[2*ArcSinh[c + d*x]] + Log[T
anh[ArcSinh[c + d*x]/2]]*(-3*(c + d*x) + Sinh[3*ArcSinh[c + d*x]])) + b^2*(
4*(c + d*x)^2 + 4*ArcSinh[c + d*x]^2 + 4*(c + d*x)^3*PolyLog[2, -E^(-ArcSin
h[c + d*x])] - 4*(c + d*x)^3*PolyLog[2, E^(-ArcSinh[c + d*x])] + ArcSinh[c
+ d*x]*(2*Sinh[2*ArcSinh[c + d*x]] + (Log[1 - E^(-ArcSinh[c + d*x])] - Log[
1 + E^(-ArcSinh[c + d*x])])*(-3*(c + d*x) + Sinh[3*ArcSinh[c + d*x]]))))/(d
*e^4*(c + d*x)^3)
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\,\text{arsinh}\,(dx+c)^2+2\,ab\,\text{arsinh}\,(dx+c)+a^2}{d^4e^4x^4+4\,cd^3e^4x^3+6\,c^2d^2e^4x^2+4\,c^3de^4x+c^4e^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^4,x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(d*x + c)^2 + 2*a*b*arcsinh(d*x + c) + a^2)/(d^4*e^4*x
^4 + 4*c*d^3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^3*d*e^4*x + c^4*e^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{arsinh}\,(dx+c)+a)^2}{(dex+ce)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^4,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^2/(d*e*x + c*e)^4, x)
```

maple [A] time = 0.28, size = 310, normalized size = 1.83

$$-\frac{a^2}{3d\,e^4\,(dx+c)^3}-\frac{b^2\,\mathrm{arcsinh}\,(dx+c)\,\sqrt{1+(dx+c)^2}}{3d\,e^4\,(dx+c)^2}-\frac{b^2\,\mathrm{arcsinh}\,(dx+c)^2}{3d\,e^4\,(dx+c)^3}-\frac{b^2}{3d\,e^4\,(dx+c)}+\frac{b^2\,\mathrm{arcsinh}\,(dx+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^4,x)
```

```
[Out] -1/3/d*a^2/e^4/(d*x+c)^3-1/3/d*b^2/e^4/(d*x+c)^2*arcsinh(d*x+c)*(1+(d*x+c)^
2)^(1/2)-1/3/d*b^2/e^4/(d*x+c)^3*arcsinh(d*x+c)^2-1/3*b^2/d/e^4/(d*x+c)+1/3
/d*b^2/e^4*arcsinh(d*x+c)*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))+1/3*b^2*polylog(2
,-d*x-c-(1+(d*x+c)^2)^(1/2))/d/e^4-1/3/d*b^2/e^4*arcsinh(d*x+c)*ln(1-d*x-c-
(1+(d*x+c)^2)^(1/2))-1/3*b^2*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2))/d/e^4-2/3
/d*a*b/e^4/(d*x+c)^3*arcsinh(d*x+c)-1/3/d*a*b/e^4/(d*x+c)^2*(1+(d*x+c)^2)^(
1/2)+1/3/d*a*b/e^4*arctanh(1/(1+(d*x+c)^2)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b^2\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)^2}{3\left(d^4e^4x^3+3\,cd^3e^4x^2+3\,c^2d^2e^4x+c^3de^4\right)}-\frac{a^2}{3\left(d^4e^4x^3+3\,cd^3e^4x^2+3\,c^2d^2e^4x+c^3de^4\right)}+\int\frac{}{3\left(d^7e^4x^7+7\,cd^6\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^4,x, algorithm="maxima")
```

```
[Out] -1/3*b^2*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2/(d^4*e^4*x^3 +
3*c*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^4) - 1/3*a^2/(d^4*e^4*x^3 + 3*c
*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^4) + integrate(2/3*((3*a*b*d^3 + b
^2*d^3)*x^3 + 3*(c^3 + c)*a*b + (c^3 + c)*b^2 + 3*(3*a*b*c*d^2 + b^2*c*d^2)
*x^2 + (3*(3*c^2*d + d)*a*b + (3*c^2*d + d)*b^2)*x + (b^2*c^2 + 3*(c^2 + 1)
*a*b + (3*a*b*d^2 + b^2*d^2)*x^2 + 2*(3*a*b*c*d + b^2*c*d)*x)*sqrt(d^2*x^2
+ 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(d^7
*e^4*x^7 + 7*c*d^6*e^4*x^6 + c^7*e^4 + c^5*e^4 + (21*c^2*d^5*e^4 + d^5*e^4)
*x^5 + 5*(7*c^3*d^4*e^4 + c*d^4*e^4)*x^4 + 5*(7*c^4*d^3*e^4 + 2*c^2*d^3*e^4
)*x^3 + (21*c^5*d^2*e^4 + 10*c^3*d^2*e^4)*x^2 + (7*c^6*d*e^4 + 5*c^4*d*e^4)
*x + (d^6*e^4*x^6 + 6*c*d^5*e^4*x^5 + c^6*e^4 + c^4*e^4 + (15*c^2*d^4*e^4 +
 d^4*e^4)*x^4 + 4*(5*c^3*d^3*e^4 + c*d^3*e^4)*x^3 + 3*(5*c^4*d^2*e^4 + 2*c^
2*d^2*e^4)*x^2 + 2*(3*c^5*d*e^4 + 2*c^3*d*e^4)*x)*sqrt(d^2*x^2 + 2*c*d*x +
c^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^2}{{\left(c\,e+d\,e\,x\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^2/(c*e + d*e*x)^4,x)
```

```
[Out] int((a + b*asinh(c + d*x))^2/(c*e + d*e*x)^4, x)
```

<u>**sympy**</u> **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{b^2\operatorname{asinh}^2(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{2ab\operatorname{asinh}(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**2/(d*e*x+c*e)**4,x)
```

```
[Out] (Integral(a**2/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4
*x**4), x) + Integral(b**2*asinh(c + d*x)**2/(c**4 + 4*c**3*d*x + 6*c**2*d*
*2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x) + Integral(2*a*b*asinh(c + d*x)/(c
**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x))/e**4
```

3.136 $\int (ce+dex)^m \left(a+b\sinh^{-1}(c+dx)\right)^3 \, dx$

Optimal. Leaf size=87

$$\frac{(e(c+dx))^{m+1}\left(a+b\sinh^{-1}(c+dx)\right)^3}{de(m+1)}-\frac{3b\text{Int}\left(\frac{(e(c+dx))^{m+1}\left(a+b\sinh^{-1}(c+dx)\right)^2}{\sqrt{(c+dx)^2+1}},x\right)}{e(m+1)}$$

```
[Out] (e*(d*x+c))^(1+m)*(a+b*arcsinh(d*x+c))^3/d/e/(1+m)-3*b*Unintegrable((e*(d*x
+c))^(1+m)*(a+b*arcsinh(d*x+c))^2/(1+(d*x+c)^2)^(1/2),x)/e/(1+m)
```

Rubi [A] time = 0.17, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (ce+dex)^m \left(a+b\sinh^{-1}(c+dx)\right)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(c*e + d*e*x)^m*(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] ((e*(c + d*x))^(1 + m)*(a + b*ArcSinh[c + d*x])^3)/(d*e*(1 + m)) - (3*b*Def
er[Subst][Defer[Int][((e*x)^(1 + m)*(a + b*ArcSinh[x])^2)/Sqrt[1 + x^2], x]
, x, c + d*x])/(d*e*(1 + m))
```

Rubi steps

$$\int (ce+dex)^m \left(a+b\sinh^{-1}(c+dx)\right)^3 \, dx = \frac{\text{Subst}\left(\int (ex)^m \left(a+b\sinh^{-1}(x)\right)^3 \, dx, x, c+dx\right)}{d}$$

$$= \frac{(e(c+dx))^{1+m}\left(a+b\sinh^{-1}(c+dx)\right)^3}{de(1+m)} - \frac{(3b)\,\text{Subst}\left(\int \frac{(ex)^{1+m}\left(a+b\,\mathrm{s}\right.}{\sqrt{1+}}\right.}{de(1+}$$

Mathematica [A] time = 3.75, size = 0, normalized size = 0.00

$$\int (ce+dex)^m \left(a+b\sinh^{-1}(c+dx)\right)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c*e + d*e*x)^m*(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] Integrate[(c*e + d*e*x)^m*(a + b*ArcSinh[c + d*x])^3, x]
```

fricas [A] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^3\operatorname{arsinh}(dx+c)^3+3\,ab^2\operatorname{arsinh}(dx+c)^2+3\,a^2b\operatorname{arsinh}(dx+c)+a^3\right)(dex+ce)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*arcsinh(d*x + c)^3 + 3*a*b^2*arcsinh(d*x + c)^2 + 3*a^2*b*arc
sinh(d*x + c) + a^3)*(d*e*x + c*e)^m, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\operatorname{arsinh}(dx+c)+a)^3(dex+ce)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^3*(d*e*x + c*e)^m, x)
```

maple [A] time = 1.96, size = 0, normalized size = 0.00

$$\int (dex+ce)^m\,(a+b\operatorname{arcsinh}(dx+c))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^3,x)
```

```
[Out] int((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(b^3de^mx+b^3ce^m\right)(dx+c)^m\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)^3}{d(m+1)}+\frac{(dex+ce)^{m+1}a^3}{de(m+1)}+\int -\frac{3\left(\left(\left(b^3c^2e^m-\left(c^2e^m(\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] (b^3*d*e^m*x + b^3*c*e^m)*(d*x + c)^m*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x
+ c^2 + 1))^3/(d*(m + 1)) + (d*e*x + c*e)^(m + 1)*a^3/(d*e*(m + 1)) + integ
rate(-3*(((b^3*c^2*e^m - (c^2*e^m*(m + 1) + e^m*(m + 1))*a*b^2 - (a*b^2*d^2
*e^m*(m + 1) - b^3*d^2*e^m)*x^2 - 2*(a*b^2*c*d*e^m*(m + 1) - b^3*c*d*e^m)*x
)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(d*x + c)^m - ((c^3*e^m*(m + 1) + c*e^m
*(m + 1))*a*b^2 - (c^3*e^m + c*e^m)*b^3 + (a*b^2*d^3*e^m*(m + 1) - b^3*d^3*
e^m)*x^3 + 3*(a*b^2*c*d^2*e^m*(m + 1) - b^3*c*d^2*e^m)*x^2 + ((3*c^2*d*e^m*
(m + 1) + d*e^m*(m + 1))*a*b^2 - (3*c^2*d*e^m + d*e^m)*b^3)*x)*(d*x + c)^m)
*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 - ((a^2*b*d^2*e^m*(m +
1)*x^2 + 2*a^2*b*c*d*e^m*(m + 1)*x + (c^2*e^m*(m + 1) + e^m*(m + 1))*a^2*b)
*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(d*x + c)^m + (a^2*b*d^3*e^m*(m + 1)*x^3
 + 3*a^2*b*c*d^2*e^m*(m + 1)*x^2 + (3*c^2*d*e^m*(m + 1) + d*e^m*(m + 1))*a^
2*b*x + (c^3*e^m*(m + 1) + c*e^m*(m + 1))*a^2*b)*(d*x + c)^m)*log(d*x + c +
 sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)))/(d^3*(m + 1)*x^3 + 3*c*d^2*(m + 1)*x^2
 + c^3*(m + 1) + c*(m + 1) + (3*c^2*d*(m + 1) + d*(m + 1))*x + (d^2*(m + 1)
*x^2 + 2*c*d*(m + 1)*x + c^2*(m + 1) + m + 1)*sqrt(d^2*x^2 + 2*c*d*x + c^2
+ 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(c\,e+d\,e\,x\right)}^m\,{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^m*(a + b*asinh(c + d*x))^3,x)
```

```
[Out] int((c*e + d*e*x)^m*(a + b*asinh(c + d*x))^3, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (e(c+dx))^m\,(a+b\operatorname{asinh}(c+dx))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**m*(a+b*asinh(d*x+c))**3,x)
```

```
[Out] Integral((e*(c + d*x))**m*(a + b*asinh(c + d*x))**3, x)
```

3.137 $\int (ce+dex)^4 \left(a+b\sinh^{-1}(c+dx)\right)^3 \, dx$

Optimal. Leaf size=326

$$\frac{6b^2e^4(c+dx)^5\left(a+b\sinh^{-1}(c+dx)\right)}{125d}-\frac{8b^2e^4(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)}{75d}+\frac{16}{25}ab^2e^4x+\frac{e^4(c+dx)^5\left(a+b\sinh^{-1}\right.}{5d}$$

```
[Out] 16/25*a*b^2*e^4*x+76/1125*b^3*e^4*(1+(d*x+c)^2)^(3/2)/d-6/625*b^3*e^4*(1+(d
*x+c)^2)^(5/2)/d+16/25*b^3*e^4*(d*x+c)*arcsinh(d*x+c)/d-8/75*b^2*e^4*(d*x+c
)^3*(a+b*arcsinh(d*x+c))/d+6/125*b^2*e^4*(d*x+c)^5*(a+b*arcsinh(d*x+c))/d+1
/5*e^4*(d*x+c)^5*(a+b*arcsinh(d*x+c))^3/d-298/375*b^3*e^4*(1+(d*x+c)^2)^(1/
2)/d-8/25*b*e^4*(a+b*arcsinh(d*x+c))^2*(1+(d*x+c)^2)^(1/2)/d+4/25*b*e^4*(d*
x+c)^2*(a+b*arcsinh(d*x+c))^2*(1+(d*x+c)^2)^(1/2)/d-3/25*b*e^4*(d*x+c)^4*(a
+b*arcsinh(d*x+c))^2*(1+(d*x+c)^2)^(1/2)/d
```

Rubi [A] time = 0.47, antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {5865, 12, 5661, 5758, 5717, 5653, 261, 266, 43}

$$\frac{6b^2e^4(c+dx)^5\left(a+b\sinh^{-1}(c+dx)\right)}{125d}-\frac{8b^2e^4(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)}{75d}+\frac{16}{25}ab^2e^4x+\frac{e^4(c+dx)^5\left(a+b\sinh^{-1}\right.}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4*(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] (16*a*b^2*e^4*x)/25 - (298*b^3*e^4*Sqrt[1 + (c + d*x)^2])/(375*d) + (76*b^3
*e^4*(1 + (c + d*x)^2)^(3/2))/(1125*d) - (6*b^3*e^4*(1 + (c + d*x)^2)^(5/2)
)/(625*d) + (16*b^3*e^4*(c + d*x)*ArcSinh[c + d*x])/(25*d) - (8*b^2*e^4*(c
+ d*x)^3*(a + b*ArcSinh[c + d*x]))/(75*d) + (6*b^2*e^4*(c + d*x)^5*(a + b*A
rcSinh[c + d*x]))/(125*d) - (8*b*e^4*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c
 + d*x])^2)/(25*d) + (4*b*e^4*(c + d*x)^2*Sqrt[1 + (c + d*x)^2]*(a + b*ArcS
inh[c + d*x])^2)/(25*d) - (3*b*e^4*(c + d*x)^4*Sqrt[1 + (c + d*x)^2]*(a + b
*ArcSinh[c + d*x])^2)/(25*d) + (e^4*(c + d*x)^5*(a + b*ArcSinh[c + d*x])^3)
/(5*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^4 \left(a+b\sinh^{-1}(c+dx)\right)^3 dx = \frac{\text{Subst}\left(\int e^4x^4\left(a+b\sinh^{-1}(x)\right)^3 dx, x, c+dx\right)}{d}$$
$$= \frac{e^4\,\text{Subst}\left(\int x^4\left(a+b\sinh^{-1}(x)\right)^3 dx, x, c+dx\right)}{d}$$
$$= \frac{e^4(c+dx)^5\left(a+b\sinh^{-1}(c+dx)\right)^3}{5d} - \frac{\left(3be^4\right)\text{Subst}\left(\int \frac{x^5\left(a+b\sinh^{-1}(x)\right)^2}{\sqrt{1+x^2}}\right.}{5d}$$
$$= -\frac{3be^4(c+dx)^4\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{25d} + \frac{e^4(c+dx)^5\left(\right.}{}$$
$$= \frac{6b^2e^4(c+dx)^5\left(a+b\sinh^{-1}(c+dx)\right)}{125d} + \frac{4be^4(c+dx)^2\sqrt{1+(c+dx)^2}}{25d}$$
$$= -\frac{8b^2e^4(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)}{75d} + \frac{6b^2e^4(c+dx)^5\left(a+b\sinh^{-}\right.}{125d}$$
$$= \frac{16}{25}ab^2e^4x - \frac{8b^2e^4(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)}{75d} + \frac{6b^2e^4(c+dx)^5\left(a\right.}{1}$$
$$= \frac{16}{25}ab^2e^4x - \frac{6b^3e^4\sqrt{1+(c+dx)^2}}{125d} + \frac{4b^3e^4\left(1+(c+dx)^2\right)^{3/2}}{125d} - \frac{6b^3e^4\left(\right.}{}$$
$$= \frac{16}{25}ab^2e^4x - \frac{298b^3e^4\sqrt{1+(c+dx)^2}}{375d} + \frac{76b^3e^4\left(1+(c+dx)^2\right)^{3/2}}{1125d} - \frac{6b^3}{}$$

Mathematica [A] time = 0.48, size = 355, normalized size = 1.09

$$\frac{e^4\left(3a\left(25a^2+6b^2\right)(c+dx)^5 + \frac{1}{15}b\sqrt{(c+dx)^2+1}\left(-27\left(25a^2+2b^2\right)(c+dx)^4 + 4\left(225a^2+68b^2\right)(c+dx)^2 - 8\left(\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^4*(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] (e^4*(240*a*b^2*(c + d*x) - 40*a*b^2*(c + d*x)^3 + 3*a*(25*a^2 + 6*b^2)*(c
+ d*x)^5 + (b*Sqrt[1 + (c + d*x)^2]*(-8*(225*a^2 + 518*b^2) + 4*(225*a^2 +
68*b^2)*(c + d*x)^2 - 27*(25*a^2 + 2*b^2)*(c + d*x)^4))/15 - b*(-240*b^2*(c
 + d*x) + 40*b^2*(c + d*x)^3 - 225*a^2*(c + d*x)^5 - 18*b^2*(c + d*x)^5 + 2
40*a*b*Sqrt[1 + (c + d*x)^2] - 120*a*b*(c + d*x)^2*Sqrt[1 + (c + d*x)^2] +
90*a*b*(c + d*x)^4*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x] - 15*b^2*(-15*a*
(c + d*x)^5 + 8*b*Sqrt[1 + (c + d*x)^2] - 4*b*(c + d*x)^2*Sqrt[1 + (c + d*x
)^2] + 3*b*(c + d*x)^4*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x]^2 + 75*b^3*(
c + d*x)^5*ArcSinh[c + d*x]^3))/(375*d)
```

fricas [B] time = 0.61, size = 1077, normalized size = 3.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/5625*(45*(25*a^3 + 6*a*b^2)*d^5*e^4*x^5 + 225*(25*a^3 + 6*a*b^2)*c*d^4*e^
4*x^4 - 150*(4*a*b^2 - 3*(25*a^3 + 6*a*b^2)*c^2)*d^3*e^4*x^3 - 450*(4*a*b^2
```

```
*c - (25*a^3 + 6*a*b^2)*c^3)*d^2*e^4*x^2 - 225*(8*a*b^2*c^2 - (25*a^3 + 6*a
*b^2)*c^4 - 16*a*b^2)*d*e^4*x + 1125*(b^3*d^5*e^4*x^5 + 5*b^3*c*d^4*e^4*x^4
 + 10*b^3*c^2*d^3*e^4*x^3 + 10*b^3*c^3*d^2*e^4*x^2 + 5*b^3*c^4*d*e^4*x + b^
3*c^5*e^4)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3 + 225*(15*a*b
^2*d^5*e^4*x^5 + 75*a*b^2*c*d^4*e^4*x^4 + 150*a*b^2*c^2*d^3*e^4*x^3 + 150*a
*b^2*c^3*d^2*e^4*x^2 + 75*a*b^2*c^4*d*e^4*x + 15*a*b^2*c^5*e^4 - (3*b^3*d^4
*e^4*x^4 + 12*b^3*c*d^3*e^4*x^3 + 2*(9*b^3*c^2 - 2*b^3)*d^2*e^4*x^2 + 4*(3*
b^3*c^3 - 2*b^3*c)*d*e^4*x + (3*b^3*c^4 - 4*b^3*c^2 + 8*b^3)*e^4)*sqrt(d^2*
x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^
2 + 15*(9*(25*a^2*b + 2*b^3)*d^5*e^4*x^5 + 45*(25*a^2*b + 2*b^3)*c*d^4*e^4*
x^4 - 10*(4*b^3 - 9*(25*a^2*b + 2*b^3)*c^2)*d^3*e^4*x^3 - 30*(4*b^3*c - 3*(
25*a^2*b + 2*b^3)*c^3)*d^2*e^4*x^2 - 15*(8*b^3*c^2 - 3*(25*a^2*b + 2*b^3)*c
^4 - 16*b^3)*d*e^4*x - (40*b^3*c^3 - 9*(25*a^2*b + 2*b^3)*c^5 - 240*b^3*c)*
e^4 - 30*(3*a*b^2*d^4*e^4*x^4 + 12*a*b^2*c*d^3*e^4*x^3 + 2*(9*a*b^2*c^2 - 2
*a*b^2)*d^2*e^4*x^2 + 4*(3*a*b^2*c^3 - 2*a*b^2*c)*d*e^4*x + (3*a*b^2*c^4 -
4*a*b^2*c^2 + 8*a*b^2)*e^4)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c
+ sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - (27*(25*a^2*b + 2*b^3)*d^4*e^4*x^4 +
 108*(25*a^2*b + 2*b^3)*c*d^3*e^4*x^3 - 2*(450*a^2*b + 136*b^3 - 81*(25*a^2
*b + 2*b^3)*c^2)*d^2*e^4*x^2 + 4*(27*(25*a^2*b + 2*b^3)*c^3 - 2*(225*a^2*b
+ 68*b^3)*c)*d*e^4*x + (27*(25*a^2*b + 2*b^3)*c^4 + 1800*a^2*b + 4144*b^3 -
 4*(225*a^2*b + 68*b^3)*c^2)*e^4)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/d
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex + ce)^4 (b \operatorname{arsinh}(dx + c) + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^4*(b*arcsinh(d*x + c) + a)^3, x)
```

maple [A] time = 0.05, size = 420, normalized size = 1.29

$$\frac{\frac{(dx+c)^5 e^4 a^3}{5} + e^4 b^3 \left(\frac{(dx+c)^5 \operatorname{arcsinh}(dx+c)^3}{5} - \frac{8 \operatorname{arcsinh}(dx+c)^2 \sqrt{1+(dx+c)^2}}{25} - \frac{3(dx+c)^4 \operatorname{arcsinh}(dx+c)^2 \sqrt{1+(dx+c)^2}}{25} + \frac{4(dx+c)^2 \operatorname{arcsinh}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^3,x)
```

```
[Out] 1/d*(1/5*(d*x+c)^5*e^4*a^3+e^4*b^3*(1/5*(d*x+c)^5*arcsinh(d*x+c)^3-8/25*arc
sinh(d*x+c)^2*(1+(d*x+c)^2)^(1/2)-3/25*(d*x+c)^4*arcsinh(d*x+c)^2*(1+(d*x+c
)^2)^(1/2)+4/25*(d*x+c)^2*arcsinh(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+16/25*(d*x+c
)*arcsinh(d*x+c)-4144/5625*(1+(d*x+c)^2)^(1/2)+6/125*(d*x+c)^5*arcsinh(d*x+
c)-6/625*(d*x+c)^4*(1+(d*x+c)^2)^(1/2)+272/5625*(d*x+c)^2*(1+(d*x+c)^2)^(1/
2)-8/75*(d*x+c)^3*arcsinh(d*x+c))+3*e^4*a*b^2*(1/5*(d*x+c)^5*arcsinh(d*x+c)
^2-16/75*arcsinh(d*x+c)*(1+(d*x+c)^2)^(1/2)-2/25*(d*x+c)^4*arcsinh(d*x+c)*(
1+(d*x+c)^2)^(1/2)+8/75*arcsinh(d*x+c)*(1+(d*x+c)^2)^(1/2)*(d*x+c)^2+16/75*
d*x+16/75*c+2/125*(d*x+c)^5-8/225*(d*x+c)^3)+3*e^4*a^2*b*(1/5*(d*x+c)^5*arc
sinh(d*x+c)-1/25*(d*x+c)^4*(1+(d*x+c)^2)^(1/2)+4/75*(d*x+c)^2*(1+(d*x+c)^2)
^(1/2)-8/75*(1+(d*x+c)^2)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/5*a^3*d^4*e^4*x^5 + a^3*c*d^3*e^4*x^4 + 2*a^3*c^2*d^2*e^4*x^3 + 2*a^3*c^3
*d*e^4*x^2 + 3*(2*x^2*arcsinh(d*x + c) - d*(3*c^2*arcsinh(2*(d^2*x + c*d)/s
qrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*
x/d^2 - (c^2 + 1)*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2
))/d^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c/d^3))*a^2*b*c^3*d*e^4 + (6*x
^3*arcsinh(d*x + c) - d*(2*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*x^2/d^2 - 15*c
^3*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^4 - 5*sqrt
(d^2*x^2 + 2*c*d*x + c^2 + 1)*c*x/d^3 + 9*(c^2 + 1)*c*arcsinh(2*(d^2*x + c*
d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^4 + 15*sqrt(d^2*x^2 + 2*c*d*x + c^
2 + 1)*c^2/d^4 - 4*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)/d^4))*a^2*b*
c^2*d^2*e^4 + 1/8*(24*x^4*arcsinh(d*x + c) - (6*sqrt(d^2*x^2 + 2*c*d*x + c^
2 + 1)*x^3/d^2 - 14*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c*x^2/d^3 + 105*c^4*a
rcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^5 + 35*sqrt(d^
2*x^2 + 2*c*d*x + c^2 + 1)*c^2*x/d^4 - 90*(c^2 + 1)*c^2*arcsinh(2*(d^2*x +
c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^5 - 105*sqrt(d^2*x^2 + 2*c*d*x +
 c^2 + 1)*c^3/d^5 - 9*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)*x/d^4 + 9
*(c^2 + 1)^2*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^
5 + 55*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)*c/d^5)*d)*a^2*b*c*d^3*e^
4 + 1/200*(120*x^5*arcsinh(d*x + c) - (24*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)
*x^4/d^2 - 54*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c*x^3/d^3 + 126*sqrt(d^2*x^
2 + 2*c*d*x + c^2 + 1)*c^2*x^2/d^4 - 945*c^5*arcsinh(2*(d^2*x + c*d)/sqrt(-
4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^6 - 315*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c
^3*x/d^5 - 32*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)*x^2/d^4 + 1050*(c
^2 + 1)*c^3*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^6
 + 945*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c^4/d^6 + 161*sqrt(d^2*x^2 + 2*c*d
*x + c^2 + 1)*(c^2 + 1)*c*x/d^5 - 225*(c^2 + 1)^2*c*arcsinh(2*(d^2*x + c*d)
/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^6 - 735*sqrt(d^2*x^2 + 2*c*d*x + c^2
 + 1)*(c^2 + 1)*c^2/d^6 + 64*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)^2/
d^6)*d)*a^2*b*d^4*e^4 + a^3*c^4*e^4*x + 3*((d*x + c)*arcsinh(d*x + c) - sqr
t((d*x + c)^2 + 1))*a^2*b*c^4*e^4/d + 1/5*(b^3*d^4*e^4*x^5 + 5*b^3*c*d^3*e^
4*x^4 + 10*b^3*c^2*d^2*e^4*x^3 + 10*b^3*c^3*d*e^4*x^2 + 5*b^3*c^4*e^4*x)*lo
g(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3 + integrate(3/5*((5*a*b^2*
d^7*e^4 - b^3*d^7*e^4)*x^7 + 7*(5*a*b^2*c*d^6*e^4 - b^3*c*d^6*e^4)*x^6 + (5
*(21*c^2*d^5*e^4 + d^5*e^4)*a*b^2 - (21*c^2*d^5*e^4 + d^5*e^4)*b^3)*x^5 + 5
*(5*(7*c^3*d^4*e^4 + c*d^4*e^4)*a*b^2 - (7*c^3*d^4*e^4 + c*d^4*e^4)*b^3)*x^
4 + 5*(c^7*e^4 + c^5*e^4)*a*b^2 + 5*(5*(7*c^4*d^3*e^4 + 2*c^2*d^3*e^4)*a*b^
2 - (7*c^4*d^3*e^4 + 2*c^2*d^3*e^4)*b^3)*x^3 + 5*((21*c^5*d^2*e^4 + 10*c^3*
d^2*e^4)*a*b^2 - 2*(2*c^5*d^2*e^4 + c^3*d^2*e^4)*b^3)*x^2 + 5*((7*c^6*d*e^4
 + 5*c^4*d*e^4)*a*b^2 - (c^6*d*e^4 + c^4*d*e^4)*b^3)*x + ((5*a*b^2*d^6*e^4
- b^3*d^6*e^4)*x^6 + 6*(5*a*b^2*c*d^5*e^4 - b^3*c*d^5*e^4)*x^5 - 5*(3*b^3*c
^2*d^4*e^4 - (15*c^2*d^4*e^4 + d^4*e^4)*a*b^2)*x^4 + 5*(c^6*e^4 + c^4*e^4)*
a*b^2 - 20*(b^3*c^3*d^3*e^4 - (5*c^3*d^3*e^4 + c*d^3*e^4)*a*b^2)*x^3 - 15*(
b^3*c^4*d^2*e^4 - (5*c^4*d^2*e^4 + 2*c^2*d^2*e^4)*a*b^2)*x^2 - 5*(b^3*c^5*d
*e^4 - 2*(3*c^5*d*e^4 + 2*c^3*d*e^4)*a*b^2)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2
 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2/(d^3*x^3 + 3*c*d^
2*x^2 + c^3 + (3*c^2*d + d)*x + (d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + c), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e+d\,e\,x)^4\,(a+b\,\mathrm{asinh}\,(c+d\,x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^4*(a + b*asinh(c + d*x))^3,x)
```

```
[Out] int((c*e + d*e*x)^4*(a + b*asinh(c + d*x))^3, x)
```

sympy [A] time = 17.08, size = 2518, normalized size = 7.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4*(a+b*asinh(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*c**4*e**4*x + 2*a**3*c**3*d*e**4*x**2 + 2*a**3*c**2*d**2*e*
*4*x**3 + a**3*c*d**3*e**4*x**4 + a**3*d**4*e**4*x**5/5 + 3*a**2*b*c**5*e**
4*asinh(c + d*x)/(5*d) + 3*a**2*b*c**4*e**4*x*asinh(c + d*x) - 3*a**2*b*c**
4*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(25*d) + 6*a**2*b*c**3*d*e**4*x
**2*asinh(c + d*x) - 12*a**2*b*c**3*e**4*x*sqrt(c**2 + 2*c*d*x + d**2*x**2
+ 1)/25 + 6*a**2*b*c**2*d**2*e**4*x**3*asinh(c + d*x) - 18*a**2*b*c**2*d*e*
*4*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/25 + 4*a**2*b*c**2*e**4*sqrt(c
**2 + 2*c*d*x + d**2*x**2 + 1)/(25*d) + 3*a**2*b*c*d**3*e**4*x**4*asinh(c +
 d*x) - 12*a**2*b*c*d**2*e**4*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/25
+ 8*a**2*b*c*e**4*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/25 + 3*a**2*b*d**4
*e**4*x**5*asinh(c + d*x)/5 - 3*a**2*b*d**3*e**4*x**4*sqrt(c**2 + 2*c*d*x +
 d**2*x**2 + 1)/25 + 4*a**2*b*d*e**4*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 +
 1)/25 - 8*a**2*b*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(25*d) + 3*a*b*
*2*c**5*e**4*asinh(c + d*x)**2/(5*d) + 3*a*b**2*c**4*e**4*x*asinh(c + d*x)*
*2 + 6*a*b**2*c**4*e**4*x/25 - 6*a*b**2*c**4*e**4*sqrt(c**2 + 2*c*d*x + d**
2*x**2 + 1)*asinh(c + d*x)/(25*d) + 6*a*b**2*c**3*d*e**4*x**2*asinh(c + d*x
)**2 + 12*a*b**2*c**3*d*e**4*x**2/25 - 24*a*b**2*c**3*e**4*x*sqrt(c**2 + 2*
c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/25 + 6*a*b**2*c**2*d**2*e**4*x**3*asi
nh(c + d*x)**2 + 12*a*b**2*c**2*d**2*e**4*x**3/25 - 36*a*b**2*c**2*d*e**4*x
**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/25 - 8*a*b**2*c**2*
e**4*x/25 + 8*a*b**2*c**2*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c
 + d*x)/(25*d) + 3*a*b**2*c*d**3*e**4*x**4*asinh(c + d*x)**2 + 6*a*b**2*c*d
**3*e**4*x**4/25 - 24*a*b**2*c*d**2*e**4*x**3*sqrt(c**2 + 2*c*d*x + d**2*x*
*2 + 1)*asinh(c + d*x)/25 - 8*a*b**2*c*d*e**4*x**2/25 + 16*a*b**2*c*e**4*x*
sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/25 + 3*a*b**2*d**4*e**4
*x**5*asinh(c + d*x)**2/5 + 6*a*b**2*d**4*e**4*x**5/125 - 6*a*b**2*d**3*e**
4*x**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/25 - 8*a*b**2*d*
*2*e**4*x**3/75 + 8*a*b**2*d*e**4*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)
*asinh(c + d*x)/25 + 16*a*b**2*e**4*x/25 - 16*a*b**2*e**4*sqrt(c**2 + 2*c*d
*x + d**2*x**2 + 1)*asinh(c + d*x)/(25*d) + b**3*c**5*e**4*asinh(c + d*x)**
3/(5*d) + 6*b**3*c**5*e**4*asinh(c + d*x)/(125*d) + b**3*c**4*e**4*x*asinh(
c + d*x)**3 + 6*b**3*c**4*e**4*x*asinh(c + d*x)/25 - 3*b**3*c**4*e**4*sqrt(
c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/(25*d) - 6*b**3*c**4*e**4
*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(625*d) + 2*b**3*c**3*d*e**4*x**2*asi
nh(c + d*x)**3 + 12*b**3*c**3*d*e**4*x**2*asinh(c + d*x)/25 - 12*b**3*c**3*
e**4*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/25 - 24*b**3*
c**3*e**4*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/625 - 8*b**3*c**3*e**4*asi
nh(c + d*x)/(75*d) + 2*b**3*c**2*d**2*e**4*x**3*asinh(c + d*x)**3 + 12*b**3
*c**2*d**2*e**4*x**3*asinh(c + d*x)/25 - 18*b**3*c**2*d*e**4*x**2*sqrt(c**2
 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/25 - 36*b**3*c**2*d*e**4*x**2
*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/625 - 8*b**3*c**2*e**4*x*asinh(c + d*
x)/25 + 4*b**3*c**2*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x
)**2/(25*d) + 272*b**3*c**2*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(5625
*d) + b**3*c*d**3*e**4*x**4*asinh(c + d*x)**3 + 6*b**3*c*d**3*e**4*x**4*asi
nh(c + d*x)/25 - 12*b**3*c*d**2*e**4*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 +
 1)*asinh(c + d*x)**2/25 - 24*b**3*c*d**2*e**4*x**3*sqrt(c**2 + 2*c*d*x + d
**2*x**2 + 1)/625 - 8*b**3*c*d*e**4*x**2*asinh(c + d*x)/25 + 8*b**3*c*e**4*
x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/25 + 544*b**3*c*e*
*4*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/5625 + 16*b**3*c*e**4*asinh(c + d
*x)/(25*d) + b**3*d**4*e**4*x**5*asinh(c + d*x)**3/5 + 6*b**3*d**4*e**4*x**
5*asinh(c + d*x)/125 - 3*b**3*d**3*e**4*x**4*sqrt(c**2 + 2*c*d*x + d**2*x**
2 + 1)*asinh(c + d*x)**2/25 - 6*b**3*d**3*e**4*x**4*sqrt(c**2 + 2*c*d*x + d
**2*x**2 + 1)/625 - 8*b**3*d**2*e**4*x**3*asinh(c + d*x)/75 + 4*b**3*d*e**4
*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/25 + 272*b**3*
d*e**4*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/5625 + 16*b**3*e**4*x*asin
h(c + d*x)/25 - 8*b**3*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c +
```

```
d*x)**2/(25*d) - 4144*b**3*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(5625*
d), Ne(d, 0)), (c**4*e**4*x*(a + b*asinh(c))**3, True))
```

3.138 $\int (ce+dex)^3 \left(a+b\sinh^{-1}(c+dx)\right)^3 \, dx$

Optimal. Leaf size=279

$$\frac{3b^2e^3(c+dx)^4\left(a+b\sinh^{-1}(c+dx)\right)}{32d}-\frac{9b^2e^3(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)}{32d}+\frac{e^3(c+dx)^4\left(a+b\sinh^{-1}(c+dx\right.}{4d}$$

```
[Out] -45/256*b^3*e^3*arcsinh(d*x+c)/d-9/32*b^2*e^3*(d*x+c)^2*(a+b*arcsinh(d*x+c)
)/d+3/32*b^2*e^3*(d*x+c)^4*(a+b*arcsinh(d*x+c))/d-3/32*e^3*(a+b*arcsinh(d*x
+c))^3/d+1/4*e^3*(d*x+c)^4*(a+b*arcsinh(d*x+c))^3/d+45/256*b^3*e^3*(d*x+c)*
(1+(d*x+c)^2)^(1/2)/d-3/128*b^3*e^3*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)/d+9/32*b*
e^3*(d*x+c)*(a+b*arcsinh(d*x+c))^2*(1+(d*x+c)^2)^(1/2)/d-3/16*b*e^3*(d*x+c)
^3*(a+b*arcsinh(d*x+c))^2*(1+(d*x+c)^2)^(1/2)/d
```

Rubi [A] time = 0.39, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {5865, 12, 5661, 5758, 5675, 321, 215}

$$\frac{3b^2e^3(c+dx)^4\left(a+b\sinh^{-1}(c+dx)\right)}{32d}-\frac{9b^2e^3(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)}{32d}+\frac{e^3(c+dx)^4\left(a+b\sinh^{-1}(c+dx\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3*(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] (45*b^3*e^3*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(256*d) - (3*b^3*e^3*(c + d*x)
^3*Sqrt[1 + (c + d*x)^2])/(128*d) - (45*b^3*e^3*ArcSinh[c + d*x])/(256*d) -
 (9*b^2*e^3*(c + d*x)^2*(a + b*ArcSinh[c + d*x]))/(32*d) + (3*b^2*e^3*(c +
d*x)^4*(a + b*ArcSinh[c + d*x]))/(32*d) + (9*b*e^3*(c + d*x)*Sqrt[1 + (c +
d*x)^2]*(a + b*ArcSinh[c + d*x])^2)/(32*d) - (3*b*e^3*(c + d*x)^3*Sqrt[1 +
(c + d*x)^2]*(a + b*ArcSinh[c + d*x])^2)/(16*d) - (3*e^3*(a + b*ArcSinh[c +
 d*x])^3)/(32*d) + (e^3*(c + d*x)^4*(a + b*ArcSinh[c + d*x])^3)/(4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^3\left(a+b\sinh^{-1}(c+dx)\right)^3 dx = \frac{\text{Subst}\left(\int e^3x^3\left(a+b\sinh^{-1}(x)\right)^3 dx, x, c+dx\right)}{d}$$
$$= \frac{e^3\,\text{Subst}\left(\int x^3\left(a+b\sinh^{-1}(x)\right)^3 dx, x, c+dx\right)}{d}$$
$$= \frac{e^3(c+dx)^4\left(a+b\sinh^{-1}(c+dx)\right)^3}{4d} - \frac{\left(3be^3\right)\text{Subst}\left(\int \frac{x^4\left(a+b\sinh^{-1}(x)\right)^2}{\sqrt{1+x^2}}\right.}{4d}$$
$$= -\frac{3be^3(c+dx)^3\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{16d} + \frac{e^3(c+dx)^4\left(\right.}{}$$
$$= \frac{3b^2e^3(c+dx)^4\left(a+b\sinh^{-1}(c+dx)\right)}{32d} + \frac{9be^3(c+dx)\sqrt{1+(c+dx)^2}\left(\right.}{32d}$$
$$= -\frac{3b^3e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{128d} - \frac{9b^2e^3(c+dx)^2\left(a+b\sinh^{-1}(c+dx\right.}{32d}$$
$$= \frac{45b^3e^3(c+dx)\sqrt{1+(c+dx)^2}}{256d} - \frac{3b^3e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{128d} - \frac{9b^2}{}$$
$$= \frac{45b^3e^3(c+dx)\sqrt{1+(c+dx)^2}}{256d} - \frac{3b^3e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{128d} - \frac{45b}{}$$

Mathematica [A] time = 0.40, size = 303, normalized size = 1.09

$$\frac{e^3\left(8a\left(8a^2+3b^2\right)(c+dx)^4+3b\sqrt{(c+dx)^2+1}\,(c+dx)\left(3\left(8a^2+5b^2\right)-2\left(8a^2+b^2\right)(c+dx)^2\right)-24b(c+dx)\sin\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^3*(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] (e^3*(-72*a*b^2*(c + d*x)^2 + 8*a*(8*a^2 + 3*b^2)*(c + d*x)^4 + 3*b*(c + d*
x)*Sqrt[1 + (c + d*x)^2]*(3*(8*a^2 + 5*b^2) - 2*(8*a^2 + b^2)*(c + d*x)^2)
- 9*b*(8*a^2 + 5*b^2)*ArcSinh[c + d*x] - 24*b*(c + d*x)*(3*b^2*(c + d*x) -
8*a^2*(c + d*x)^3 - b^2*(c + d*x)^3 - 6*a*b*Sqrt[1 + (c + d*x)^2] + 4*a*b*(
c + d*x)^2*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x] + 24*b^2*(-3*a + 8*a*(c
+ d*x)^4 + 3*b*(c + d*x)*Sqrt[1 + (c + d*x)^2] - 2*b*(c + d*x)^3*Sqrt[1 + (
c + d*x)^2])*ArcSinh[c + d*x]^2 + 8*b^3*(-3 + 8*(c + d*x)^4)*ArcSinh[c + d*
x]^3))/(256*d)
```

fricas [B] time = 0.63, size = 832, normalized size = 2.98

$$\frac{8\left(8\,a^3+3\,ab^2\right)d^4e^3x^4+32\left(8\,a^3+3\,ab^2\right)cd^3e^3x^3-24\left(3\,ab^2-2\left(8\,a^3+3\,ab^2\right)c^2\right)d^2e^3x^2-16\left(9\,ab^2c-2\left(8\,a^3\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/256*(8*(8*a^3 + 3*a*b^2)*d^4*e^3*x^4 + 32*(8*a^3 + 3*a*b^2)*c*d^3*e^3*x^3
 - 24*(3*a*b^2 - 2*(8*a^3 + 3*a*b^2)*c^2)*d^2*e^3*x^2 - 16*(9*a*b^2*c - 2*(
8*a^3 + 3*a*b^2)*c^3)*d*e^3*x + 8*(8*b^3*d^4*e^3*x^4 + 32*b^3*c*d^3*e^3*x^3
 + 48*b^3*c^2*d^2*e^3*x^2 + 32*b^3*c^3*d*e^3*x + (8*b^3*c^4 - 3*b^3)*e^3)*l
og(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3 + 24*(8*a*b^2*d^4*e^3*x^4
 + 32*a*b^2*c*d^3*e^3*x^3 + 48*a*b^2*c^2*d^2*e^3*x^2 + 32*a*b^2*c^3*d*e^3*x
 + (8*a*b^2*c^4 - 3*a*b^2)*e^3 - (2*b^3*d^3*e^3*x^3 + 6*b^3*c*d^2*e^3*x^2 +
 3*(2*b^3*c^2 - b^3)*d*e^3*x + (2*b^3*c^3 - 3*b^3*c)*e^3)*sqrt(d^2*x^2 + 2*
c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 + 3*(8
*(8*a^2*b + b^3)*d^4*e^3*x^4 + 32*(8*a^2*b + b^3)*c*d^3*e^3*x^3 - 24*(b^3 -
 2*(8*a^2*b + b^3)*c^2)*d^2*e^3*x^2 - 16*(3*b^3*c - 2*(8*a^2*b + b^3)*c^3)*
d*e^3*x - (24*b^3*c^2 - 8*(8*a^2*b + b^3)*c^4 + 24*a^2*b + 15*b^3)*e^3 - 16
*(2*a*b^2*d^3*e^3*x^3 + 6*a*b^2*c*d^2*e^3*x^2 + 3*(2*a*b^2*c^2 - a*b^2)*d*e
^3*x + (2*a*b^2*c^3 - 3*a*b^2*c)*e^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*lo
g(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - 3*(2*(8*a^2*b + b^3)*d^3*e
^3*x^3 + 6*(8*a^2*b + b^3)*c*d^2*e^3*x^2 - 3*(8*a^2*b + 5*b^3 - 2*(8*a^2*b
+ b^3)*c^2)*d*e^3*x + (2*(8*a^2*b + b^3)*c^3 - 3*(8*a^2*b + 5*b^3)*c)*e^3)*
sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/d
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex+ce)^3(b\,\mathrm{arsinh}\,(dx+c)+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^3*(b*arcsinh(d*x + c) + a)^3, x)
```

maple [A] time = 0.06, size = 365, normalized size = 1.31

$$\frac{\frac{(dx+c)^4e^3a^3}{4}+e^3b^3\left(\frac{(dx+c)^4\,\mathrm{arcsinh}(dx+c)^3}{4}-\frac{3(dx+c)^3\,\mathrm{arcsinh}(dx+c)^2\sqrt{1+(dx+c)^2}}{16}+\frac{9\,\mathrm{arcsinh}(dx+c)^2\sqrt{1+(dx+c)^2}\,(dx+c)}{32}-\frac{3\,\mathrm{arcsinh}(}{32}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^3,x)
```

```
[Out] 1/d*(1/4*(d*x+c)^4*e^3*a^3+e^3*b^3*(1/4*(d*x+c)^4*arcsinh(d*x+c)^3-3/16*(d*
x+c)^3*arcsinh(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+9/32*arcsinh(d*x+c)^2*(1+(d*x+c
)^2)^(1/2)*(d*x+c)-3/32*arcsinh(d*x+c)^3+3/32*(d*x+c)^4*arcsinh(d*x+c)-3/12
```

```
8*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)+45/256*(d*x+c)*(1+(d*x+c)^2)^(1/2)+27/256*a
rcsinh(d*x+c)-9/32*arcsinh(d*x+c)*(1+(d*x+c)^2))+3*e^3*a*b^2*(1/4*(d*x+c)^4
*arcsinh(d*x+c)^2-1/8*(d*x+c)^3*arcsinh(d*x+c)*(1+(d*x+c)^2)^(1/2)+3/16*arc
sinh(d*x+c)*(1+(d*x+c)^2)^(1/2)*(d*x+c)-3/32*arcsinh(d*x+c)^2+1/32*(d*x+c)^
4-3/32*(d*x+c)^2-3/32)+3*e^3*a^2*b*(1/4*(d*x+c)^4*arcsinh(d*x+c)-1/16*(d*x+
c)^3*(1+(d*x+c)^2)^(1/2)+3/32*(d*x+c)*(1+(d*x+c)^2)^(1/2)-3/32*arcsinh(d*x+
c)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/4*a^3*d^3*e^3*x^4 + a^3*c*d^2*e^3*x^3 + 3/2*a^3*c^2*d*e^3*x^2 + 9/4*(2*x^
2*arcsinh(d*x + c) - d*(3*c^2*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(
c^2 + 1)*d^2))/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*x/d^2 - (c^2 + 1)*ar
csinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 - 3*sqrt(d^2*
x^2 + 2*c*d*x + c^2 + 1)*c/d^3))*a^2*b*c^2*d*e^3 + 1/2*(6*x^3*arcsinh(d*x +
 c) - d*(2*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*x^2/d^2 - 15*c^3*arcsinh(2*(d^
2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^4 - 5*sqrt(d^2*x^2 + 2*c*d
*x + c^2 + 1)*c*x/d^3 + 9*(c^2 + 1)*c*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d
^2 + 4*(c^2 + 1)*d^2))/d^4 + 15*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c^2/d^4 -
 4*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)/d^4))*a^2*b*c*d^2*e^3 + 1/32
*(24*x^4*arcsinh(d*x + c) - (6*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*x^3/d^2 -
14*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c*x^2/d^3 + 105*c^4*arcsinh(2*(d^2*x +
 c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^5 + 35*sqrt(d^2*x^2 + 2*c*d*x +
 c^2 + 1)*c^2*x/d^4 - 90*(c^2 + 1)*c^2*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*
d^2 + 4*(c^2 + 1)*d^2))/d^5 - 105*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c^3/d^5
 - 9*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)*x/d^4 + 9*(c^2 + 1)^2*arcs
inh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^5 + 55*sqrt(d^2*x
^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)*c/d^5)*d)*a^2*b*d^3*e^3 + a^3*c^3*e^3*x +
 3*((d*x + c)*arcsinh(d*x + c) - sqrt((d*x + c)^2 + 1))*a^2*b*c^3*e^3/d + 1
/4*(b^3*d^3*e^3*x^4 + 4*b^3*c*d^2*e^3*x^3 + 6*b^3*c^2*d*e^3*x^2 + 4*b^3*c^3
*e^3*x)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3 + integrate(3/4*
((4*a*b^2*d^6*e^3 - b^3*d^6*e^3)*x^6 + 6*(4*a*b^2*c*d^5*e^3 - b^3*c*d^5*e^3
)*x^5 + (4*(15*c^2*d^4*e^3 + d^4*e^3)*a*b^2 - (15*c^2*d^4*e^3 + d^4*e^3)*b^
3)*x^4 + 4*(c^6*e^3 + c^4*e^3)*a*b^2 + 4*(4*(5*c^3*d^3*e^3 + c*d^3*e^3)*a*b
^2 - (5*c^3*d^3*e^3 + c*d^3*e^3)*b^3)*x^3 + 2*(6*(5*c^4*d^2*e^3 + 2*c^2*d^2
*e^3)*a*b^2 - (7*c^4*d^2*e^3 + 3*c^2*d^2*e^3)*b^3)*x^2 + 4*(2*(3*c^5*d*e^3
+ 2*c^3*d*e^3)*a*b^2 - (c^5*d*e^3 + c^3*d*e^3)*b^3)*x + ((4*a*b^2*d^5*e^3 -
 b^3*d^5*e^3)*x^5 + 5*(4*a*b^2*c*d^4*e^3 - b^3*c*d^4*e^3)*x^4 + 4*(c^5*e^3
+ c^3*e^3)*a*b^2 - 2*(5*b^3*c^2*d^3*e^3 - 2*(10*c^2*d^3*e^3 + d^3*e^3)*a*b^
2)*x^3 - 2*(5*b^3*c^3*d^2*e^3 - 2*(10*c^3*d^2*e^3 + 3*c*d^2*e^3)*a*b^2)*x^2
 - 4*(b^3*c^4*d*e^3 - (5*c^4*d*e^3 + 3*c^2*d*e^3)*a*b^2)*x)*sqrt(d^2*x^2 +
2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2/(d^3
*x^3 + 3*c*d^2*x^2 + c^3 + (3*c^2*d + d)*x + (d^2*x^2 + 2*c*d*x + c^2 + 1)^
(3/2) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e+d\,e\,x)^3\,(a+b\,\mathrm{asinh}\,(c+d\,x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3*(a + b*asinh(c + d*x))^3,x)
```

```
[Out] int((c*e + d*e*x)^3*(a + b*asinh(c + d*x))^3, x)
```

sympy [A] time = 10.07, size = 1828, normalized size = 6.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3*(a+b*asinh(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*c**3*e**3*x + 3*a**3*c**2*d*e**3*x**2/2 + a**3*c*d**2*e**3*
x**3 + a**3*d**3*e**3*x**4/4 + 3*a**2*b*c**4*e**3*asinh(c + d*x)/(4*d) + 3*
a**2*b*c**3*e**3*x*asinh(c + d*x) - 3*a**2*b*c**3*e**3*sqrt(c**2 + 2*c*d*x
+ d**2*x**2 + 1)/(16*d) + 9*a**2*b*c**2*d*e**3*x**2*asinh(c + d*x)/2 - 9*a*
*2*b*c**2*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/16 + 3*a**2*b*c*d**2*
e**3*x**3*asinh(c + d*x) - 9*a**2*b*c*d*e**3*x**2*sqrt(c**2 + 2*c*d*x + d**
2*x**2 + 1)/16 + 9*a**2*b*c*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(32*d
) + 3*a**2*b*d**3*e**3*x**4*asinh(c + d*x)/4 - 3*a**2*b*d**2*e**3*x**3*sqrt
(c**2 + 2*c*d*x + d**2*x**2 + 1)/16 + 9*a**2*b*e**3*x*sqrt(c**2 + 2*c*d*x +
 d**2*x**2 + 1)/32 - 9*a**2*b*e**3*asinh(c + d*x)/(32*d) + 3*a*b**2*c**4*e*
*3*asinh(c + d*x)**2/(4*d) + 3*a*b**2*c**3*e**3*x*asinh(c + d*x)**2 + 3*a*b
**2*c**3*e**3*x/8 - 3*a*b**2*c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)
*asinh(c + d*x)/(8*d) + 9*a*b**2*c**2*d*e**3*x**2*asinh(c + d*x)**2/2 + 9*a
*b**2*c**2*d*e**3*x**2/16 - 9*a*b**2*c**2*e**3*x*sqrt(c**2 + 2*c*d*x + d**2
*x**2 + 1)*asinh(c + d*x)/8 + 3*a*b**2*c*d**2*e**3*x**3*asinh(c + d*x)**2 +
 3*a*b**2*c*d**2*e**3*x**3/8 - 9*a*b**2*c*d*e**3*x**2*sqrt(c**2 + 2*c*d*x +
 d**2*x**2 + 1)*asinh(c + d*x)/8 - 9*a*b**2*c*e**3*x/16 + 9*a*b**2*c*e**3*s
qrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/(16*d) + 3*a*b**2*d**3*e
**3*x**4*asinh(c + d*x)**2/4 + 3*a*b**2*d**3*e**3*x**4/32 - 3*a*b**2*d**2*e
**3*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/8 - 9*a*b**2*d
*e**3*x**2/32 + 9*a*b**2*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(
c + d*x)/16 - 9*a*b**2*e**3*asinh(c + d*x)**2/(32*d) + b**3*c**4*e**3*asinh
(c + d*x)**3/(4*d) + 3*b**3*c**4*e**3*asinh(c + d*x)/(32*d) + b**3*c**3*e**
3*x*asinh(c + d*x)**3 + 3*b**3*c**3*e**3*x*asinh(c + d*x)/8 - 3*b**3*c**3*e
**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/(16*d) - 3*b**3*
c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(128*d) + 3*b**3*c**2*d*e**3
*x**2*asinh(c + d*x)**3/2 + 9*b**3*c**2*d*e**3*x**2*asinh(c + d*x)/16 - 9*b
**3*c**2*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/16 -
 9*b**3*c**2*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/128 - 9*b**3*c**2*
e**3*asinh(c + d*x)/(32*d) + b**3*c*d**2*e**3*x**3*asinh(c + d*x)**3 + 3*b*
*3*c*d**2*e**3*x**3*asinh(c + d*x)/8 - 9*b**3*c*d*e**3*x**2*sqrt(c**2 + 2*c
*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/16 - 9*b**3*c*d*e**3*x**2*sqrt(c**2
 + 2*c*d*x + d**2*x**2 + 1)/128 - 9*b**3*c*e**3*x*asinh(c + d*x)/16 + 9*b**
3*c*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/(32*d) + 45
*b**3*c*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(256*d) + b**3*d**3*e**3*
x**4*asinh(c + d*x)**3/4 + 3*b**3*d**3*e**3*x**4*asinh(c + d*x)/32 - 3*b**3
*d**2*e**3*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/16 -
 3*b**3*d**2*e**3*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/128 - 9*b**3*d*
e**3*x**2*asinh(c + d*x)/32 + 9*b**3*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2
 + 1)*asinh(c + d*x)**2/32 + 45*b**3*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2
 + 1)/256 - 3*b**3*e**3*asinh(c + d*x)**3/(32*d) - 45*b**3*e**3*asinh(c + d
*x)/(256*d), Ne(d, 0)), (c**3*e**3*x*(a + b*asinh(c))**3, True))
```

3.139 $\int (ce+dex)^2 \left(a+b\sinh^{-1}(c+dx)\right)^3 \, dx$

Optimal. Leaf size=227

$$\frac{2b^2e^2(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)}{9d}-\frac{4}{3}ab^2e^2x+\frac{2be^2\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)^2}{3d}-\frac{be^2(c+dx)^2\sqrt{(c+$$

```
[Out] -4/3*a*b^2*e^2*x-2/27*b^3*e^2*(1+(d*x+c)^2)^(3/2)/d-4/3*b^3*e^2*(d*x+c)*arc
sinh(d*x+c)/d+2/9*b^2*e^2*(d*x+c)^3*(a+b*arcsinh(d*x+c))/d+1/3*e^2*(d*x+c)^
3*(a+b*arcsinh(d*x+c))^3/d+14/9*b^3*e^2*(1+(d*x+c)^2)^(1/2)/d+2/3*b*e^2*(a+
b*arcsinh(d*x+c))^2*(1+(d*x+c)^2)^(1/2)/d-1/3*b*e^2*(d*x+c)^2*(a+b*arcsinh(
d*x+c))^2*(1+(d*x+c)^2)^(1/2)/d
```

Rubi [A] time = 0.30, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {5865, 12, 5661, 5758, 5717, 5653, 261, 266, 43}

$$\frac{2b^2e^2(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)}{9d}-\frac{4}{3}ab^2e^2x+\frac{2be^2\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)^2}{3d}-\frac{be^2(c+dx)^2\sqrt{(c+$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2*(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] (-4*a*b^2*e^2*x)/3 + (14*b^3*e^2*Sqrt[1 + (c + d*x)^2])/(9*d) - (2*b^3*e^2*
(1 + (c + d*x)^2)^(3/2))/(27*d) - (4*b^3*e^2*(c + d*x)*ArcSinh[c + d*x])/(3
*d) + (2*b^2*e^2*(c + d*x)^3*(a + b*ArcSinh[c + d*x]))/(9*d) + (2*b*e^2*Sqr
t[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^2)/(3*d) - (b*e^2*(c + d*x)^2*S
qrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^2)/(3*d) + (e^2*(c + d*x)^3*(
a + b*ArcSinh[c + d*x])^3)/(3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
```

```
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^2\left(a+b\sinh^{-1}(c+dx)\right)^3 dx = \frac{\text{Subst}\left(\int e^2x^2\left(a+b\sinh^{-1}(x)\right)^3 dx, x, c+dx\right)}{d}$$

$$= \frac{e^2\,\text{Subst}\left(\int x^2\left(a+b\sinh^{-1}(x)\right)^3 dx, x, c+dx\right)}{d}$$

$$= \frac{e^2(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)^3}{3d} - \frac{\left(be^2\right)\text{Subst}\left(\int \frac{x^3\left(a+b\sinh^{-1}(x)\right)^2}{\sqrt{1+x^2}} \right.}{d}$$

$$= -\frac{be^2(c+dx)^2\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{3d} + \frac{e^2(c+dx)^3\left(a\right.}{}$$

$$= \frac{2b^2e^2(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)}{9d} + \frac{2be^2\sqrt{1+(c+dx)^2}\left(a+b\sin\right.}{3d}$$

$$= -\frac{4}{3}ab^2e^2x + \frac{2b^2e^2(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)}{9d} + \frac{2be^2\sqrt{1+(c+d}}{}$$

$$= -\frac{4}{3}ab^2e^2x - \frac{4b^3e^2(c+dx)\sinh^{-1}(c+dx)}{3d} + \frac{2b^2e^2(c+dx)^3\left(a+b\sin\right.}{9d}$$

$$= -\frac{4}{3}ab^2e^2x + \frac{14b^3e^2\sqrt{1+(c+dx)^2}}{9d} - \frac{2b^3e^2\left(1+(c+dx)^2\right)^{3/2}}{27d} - \frac{4b^3e^2}{}$$

Mathematica [A] time = 0.33, size = 258, normalized size = 1.14

$$\frac{e^2\left(a\left(3a^2+2b^2\right)(c+dx)^3+\frac{1}{3}b\sqrt{(c+dx)^2+1}\left(-\left(9a^2+2b^2\right)(c+dx)^2+18a^2+40b^2\right)-b\sinh^{-1}(c+dx)\left(-9a^2\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^2*(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] (e^2*(-12*a*b^2*(c + d*x) + a*(3*a^2 + 2*b^2)*(c + d*x)^3 + (b*Sqrt[1 + (c
+ d*x)^2]*(18*a^2 + 40*b^2 - (9*a^2 + 2*b^2)*(c + d*x)^2))/3 - b*(12*b^2*(c
 + d*x) - 9*a^2*(c + d*x)^3 - 2*b^2*(c + d*x)^3 - 12*a*b*Sqrt[1 + (c + d*x)
^2] + 6*a*b*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x] - 3*b^2*(-3
*a*(c + d*x)^3 - 2*b*Sqrt[1 + (c + d*x)^2] + b*(c + d*x)^2*Sqrt[1 + (c + d*
x)^2])*ArcSinh[c + d*x]^2 + 3*b^3*(c + d*x)^3*ArcSinh[c + d*x]^3))/(9*d)
```

fricas [B] time = 0.63, size = 613, normalized size = 2.70

$$\frac{3\left(3\,a^3+2\,ab^2\right)d^3e^2x^3+9\left(3\,a^3+2\,ab^2\right)cd^2e^2x^2-9\left(4\,ab^2-\left(3\,a^3+2\,ab^2\right)c^2\right)de^2x+9\left(b^3d^3e^2x^3+3\,b^3cd^2e^2x^2+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/27*(3*(3*a^3 + 2*a*b^2)*d^3*e^2*x^3 + 9*(3*a^3 + 2*a*b^2)*c*d^2*e^2*x^2 -
 9*(4*a*b^2 - (3*a^3 + 2*a*b^2)*c^2)*d*e^2*x + 9*(b^3*d^3*e^2*x^3 + 3*b^3*c
*d^2*e^2*x^2 + 3*b^3*c^2*d*e^2*x + b^3*c^3*e^2)*log(d*x + c + sqrt(d^2*x^2
+ 2*c*d*x + c^2 + 1))^3 + 9*(3*a*b^2*d^3*e^2*x^3 + 9*a*b^2*c*d^2*e^2*x^2 +
9*a*b^2*c^2*d*e^2*x + 3*a*b^2*c^3*e^2 - (b^3*d^2*e^2*x^2 + 2*b^3*c*d*e^2*x
+ (b^3*c^2 - 2*b^3)*e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + s
```

```
qrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 + 3*((9*a^2*b + 2*b^3)*d^3*e^2*x^3 + 3*
(9*a^2*b + 2*b^3)*c*d^2*e^2*x^2 - 3*(4*b^3 - (9*a^2*b + 2*b^3)*c^2)*d*e^2*x
 - (12*b^3*c - (9*a^2*b + 2*b^3)*c^3)*e^2 - 6*(a*b^2*d^2*e^2*x^2 + 2*a*b^2*
c*d*e^2*x + (a*b^2*c^2 - 2*a*b^2)*e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*l
og(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - ((9*a^2*b + 2*b^3)*d^2*e^
2*x^2 + 2*(9*a^2*b + 2*b^3)*c*d*e^2*x - (18*a^2*b + 40*b^3 - (9*a^2*b + 2*b
^3)*c^2)*e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/d
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex + ce)^2 (b \operatorname{arsinh}(dx + c) + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2*(b*arcsinh(d*x + c) + a)^3, x)
```

maple [A] time = 0.04, size = 302, normalized size = 1.33

$$\frac{\frac{(dx+c)^3 e^2 a^3}{3} + e^2 b^3 \left(\frac{(dx+c)^3 \operatorname{arcsinh}(dx+c)^3}{3} + \frac{2 \operatorname{arcsinh}(dx+c)^2 \sqrt{1+(dx+c)^2}}{3} - \frac{(dx+c)^2 \operatorname{arcsinh}(dx+c)^2 \sqrt{1+(dx+c)^2}}{3} - \frac{4(dx+c) \operatorname{arcsinh}(d}{3}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^3,x)
```

```
[Out] 1/d*(1/3*(d*x+c)^3*e^2*a^3+e^2*b^3*(1/3*(d*x+c)^3*arcsinh(d*x+c)^3+2/3*arcs
inh(d*x+c)^2*(1+(d*x+c)^2)^(1/2)-1/3*(d*x+c)^2*arcsinh(d*x+c)^2*(1+(d*x+c)^
2)^(1/2)-4/3*(d*x+c)*arcsinh(d*x+c)+40/27*(1+(d*x+c)^2)^(1/2)+2/9*(d*x+c)^3
*arcsinh(d*x+c)-2/27*(d*x+c)^2*(1+(d*x+c)^2)^(1/2))+3*e^2*a*b^2*(1/3*(d*x+c
)^3*arcsinh(d*x+c)^2+4/9*arcsinh(d*x+c)*(1+(d*x+c)^2)^(1/2)-2/9*arcsinh(d*x
+c)*(1+(d*x+c)^2)^(1/2)*(d*x+c)^2-4/9*d*x-4/9*c+2/27*(d*x+c)^3)+3*e^2*a^2*b
*(1/3*(d*x+c)^3*arcsinh(d*x+c)-1/9*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+2/9*(1+(d*
x+c)^2)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/3*a^3*d^2*e^2*x^3 + a^3*c*d*e^2*x^2 + 3/2*(2*x^2*arcsinh(d*x + c) - d*(3*
c^2*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 + sqrt(
d^2*x^2 + 2*c*d*x + c^2 + 1)*x/d^2 - (c^2 + 1)*arcsinh(2*(d^2*x + c*d)/sqrt
(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c
/d^3))*a^2*b*c*d*e^2 + 1/6*(6*x^3*arcsinh(d*x + c) - d*(2*sqrt(d^2*x^2 + 2*
c*d*x + c^2 + 1)*x^2/d^2 - 15*c^3*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 +
 4*(c^2 + 1)*d^2))/d^4 - 5*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c*x/d^3 + 9*(c
^2 + 1)*c*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^4 +
 15*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c^2/d^4 - 4*sqrt(d^2*x^2 + 2*c*d*x +
c^2 + 1)*(c^2 + 1)/d^4))*a^2*b*d^2*e^2 + a^3*c^2*e^2*x + 3*((d*x + c)*arcsi
nh(d*x + c) - sqrt((d*x + c)^2 + 1))*a^2*b*c^2*e^2/d + 1/3*(b^3*d^2*e^2*x^3
 + 3*b^3*c*d*e^2*x^2 + 3*b^3*c^2*e^2*x)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*
x + c^2 + 1))^3 + integrate(((3*a*b^2*d^5*e^2 - b^3*d^5*e^2)*x^5 + 5*(3*a*b
^2*c*d^4*e^2 - b^3*c*d^4*e^2)*x^4 + 3*(c^5*e^2 + c^3*e^2)*a*b^2 + (3*(10*c^
2*d^3*e^2 + d^3*e^2)*a*b^2 - (10*c^2*d^3*e^2 + d^3*e^2)*b^3)*x^3 + 3*((10*c
^3*d^2*e^2 + 3*c*d^2*e^2)*a*b^2 - (3*c^3*d^2*e^2 + c*d^2*e^2)*b^3)*x^2 + 3*
```

```
((5*c^4*d*e^2 + 3*c^2*d*e^2)*a*b^2 - (c^4*d*e^2 + c^2*d*e^2)*b^3)*x + ((3*a
*b^2*d^4*e^2 - b^3*d^4*e^2)*x^4 + 3*(c^4*e^2 + c^2*e^2)*a*b^2 + 4*(3*a*b^2*
c*d^3*e^2 - b^3*c*d^3*e^2)*x^3 - 3*(2*b^3*c^2*d^2*e^2 - (6*c^2*d^2*e^2 + d^
2*e^2)*a*b^2)*x^2 - 3*(b^3*c^3*d*e^2 - 2*(2*c^3*d*e^2 + c*d*e^2)*a*b^2)*x)*
sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c
^2 + 1))^2/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (3*c^2*d + d)*x + (d^2*x^2 + 2*c*
d*x + c^2 + 1)^(3/2) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e + d\,e\,x)^2\,(a + b\,\mathrm{asinh}\,(c + d\,x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2*(a + b*asinh(c + d*x))^3,x)
```

```
[Out] int((c*e + d*e*x)^2*(a + b*asinh(c + d*x))^3, x)
```

sympy [A] time = 4.99, size = 1173, normalized size = 5.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2*(a+b*asinh(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*c**2*e**2*x + a**3*c*d*e**2*x**2 + a**3*d**2*e**2*x**3/3 +
a**2*b*c**3*e**2*asinh(c + d*x)/d + 3*a**2*b*c**2*e**2*x*asinh(c + d*x) - a
**2*b*c**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(3*d) + 3*a**2*b*c*d*e
**2*x**2*asinh(c + d*x) - 2*a**2*b*c*e**2*x*sqrt(c**2 + 2*c*d*x + d**2*x**2
 + 1)/3 + a**2*b*d**2*e**2*x**3*asinh(c + d*x) - a**2*b*d*e**2*x**2*sqrt(c*
*2 + 2*c*d*x + d**2*x**2 + 1)/3 + 2*a**2*b*e**2*sqrt(c**2 + 2*c*d*x + d**2*
x**2 + 1)/(3*d) + a*b**2*c**3*e**2*asinh(c + d*x)**2/d + 3*a*b**2*c**2*e**2
*x*asinh(c + d*x)**2 + 2*a*b**2*c**2*e**2*x/3 - 2*a*b**2*c**2*e**2*sqrt(c**
2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/(3*d) + 3*a*b**2*c*d*e**2*x**2*
asinh(c + d*x)**2 + 2*a*b**2*c*d*e**2*x**2/3 - 4*a*b**2*c*e**2*x*sqrt(c**2
+ 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/3 + a*b**2*d**2*e**2*x**3*asinh(c
 + d*x)**2 + 2*a*b**2*d**2*e**2*x**3/9 - 2*a*b**2*d*e**2*x**2*sqrt(c**2 + 2
*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/3 - 4*a*b**2*e**2*x/3 + 4*a*b**2*e**
2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/(3*d) + b**3*c**3*e**
2*asinh(c + d*x)**3/(3*d) + 2*b**3*c**3*e**2*asinh(c + d*x)/(9*d) + b**3*c*
*2*e**2*x*asinh(c + d*x)**3 + 2*b**3*c**2*e**2*x*asinh(c + d*x)/3 - b**3*c*
*2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/(3*d) - 2*b*
*3*c**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(27*d) + b**3*c*d*e**2*x*
*2*asinh(c + d*x)**3 + 2*b**3*c*d*e**2*x**2*asinh(c + d*x)/3 - 2*b**3*c*e**
2*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/3 - 4*b**3*c*e**
2*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/27 - 4*b**3*c*e**2*asinh(c + d*x)/
(3*d) + b**3*d**2*e**2*x**3*asinh(c + d*x)**3/3 + 2*b**3*d**2*e**2*x**3*asi
nh(c + d*x)/9 - b**3*d*e**2*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh
(c + d*x)**2/3 - 2*b**3*d*e**2*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/27
 - 4*b**3*e**2*x*asinh(c + d*x)/3 + 2*b**3*e**2*sqrt(c**2 + 2*c*d*x + d**2*
x**2 + 1)*asinh(c + d*x)**2/(3*d) + 40*b**3*e**2*sqrt(c**2 + 2*c*d*x + d**2
*x**2 + 1)/(27*d), Ne(d, 0)), (c**2*e**2*x*(a + b*asinh(c))**3, True))
```

3.140 $\int (ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^3\,dx$

Optimal. Leaf size=161

$$\frac{3b^2e(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)}{4d}-\frac{3be(c+dx)\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)^2}{4d}+\frac{e(c+dx)^2\left(a+b\sin\right.}{2d}$$

```
[Out] 3/8*b^3*e*arcsinh(d*x+c)/d+3/4*b^2*e*(d*x+c)^2*(a+b*arcsinh(d*x+c))/d+1/4*e
*(a+b*arcsinh(d*x+c))^3/d+1/2*e*(d*x+c)^2*(a+b*arcsinh(d*x+c))^3/d-3/8*b^3*
e*(d*x+c)*(1+(d*x+c)^2)^(1/2)/d-3/4*b*e*(d*x+c)*(a+b*arcsinh(d*x+c))^2*(1+(
d*x+c)^2)^(1/2)/d
```

Rubi [A] time = 0.21, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5865, 12, 5661, 5758, 5675, 321, 215}

$$\frac{3b^2e(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)}{4d}-\frac{3be(c+dx)\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)^2}{4d}+\frac{e(c+dx)^2\left(a+b\sin\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] (-3*b^3*e*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(8*d) + (3*b^3*e*ArcSinh[c + d*x
])/(8*d) + (3*b^2*e*(c + d*x)^2*(a + b*ArcSinh[c + d*x]))/(4*d) - (3*b*e*(c
 + d*x)*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^2)/(4*d) + (e*(a + b
*ArcSinh[c + d*x])^3)/(4*d) + (e*(c + d*x)^2*(a + b*ArcSinh[c + d*x])^3)/(2
*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^3\,dx=\frac{\text{Subst}\left(\int ex\left(a+b\sinh^{-1}(x)\right)^3\,dx,x,c+dx\right)}{d}$$
$$=\frac{e\,\text{Subst}\left(\int x\left(a+b\sinh^{-1}(x)\right)^3\,dx,x,c+dx\right)}{d}$$
$$=\frac{e(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)^3}{2d}-\frac{(3be)\,\text{Subst}\left(\int\frac{x^2\left(a+b\sinh^{-1}(x)\right)^2}{\sqrt{1+x^2}}\,dx,\right.}{2d}$$
$$=-\frac{3be(c+dx)\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{4d}+\frac{e(c+dx)^2\left(a+\right.}{}$$
$$=\frac{3b^2e(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)}{4d}-\frac{3be(c+dx)\sqrt{1+(c+dx)^2}\left(a+\right.}{4d}$$
$$=-\frac{3b^3e(c+dx)\sqrt{1+(c+dx)^2}}{8d}+\frac{3b^2e(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)}{4d}-$$
$$=-\frac{3b^3e(c+dx)\sqrt{1+(c+dx)^2}}{8d}+\frac{3b^3e\sinh^{-1}(c+dx)}{8d}+\frac{3b^2e(c+dx)^2\left(\right.}{}$$

Mathematica [A] time = 0.21, size = 200, normalized size = 1.24

$$e\left(2a\left(2a^2+3b^2\right)(c+dx)^2-3b\left(2a^2+b^2\right)(c+dx)\sqrt{(c+dx)^2+1}+3b\left(2a^2+b^2\right)\sinh^{-1}(c+dx)-6b(c+dx)\sin$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] (e*(2*a*(2*a^2 + 3*b^2)*(c + d*x)^2 - 3*b*(2*a^2 + b^2)*(c + d*x)*Sqrt[1 +
(c + d*x)^2] + 3*b*(2*a^2 + b^2)*ArcSinh[c + d*x] - 6*b*(c + d*x)*(-2*a^2*(
c + d*x) - b^2*(c + d*x) + 2*a*b*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x] +
6*b^2*(a + 2*a*(c + d*x)^2 - b*(c + d*x)*Sqrt[1 + (c + d*x)^2])*ArcSinh[c +
 d*x]^2 + 2*b^3*(1 + 2*(c + d*x)^2)*ArcSinh[c + d*x]^3))/(8*d)
```

fricas [B] time = 0.54, size = 391, normalized size = 2.43

$$2\left(2a^3+3ab^2\right)d^2ex^2+4\left(2a^3+3ab^2\right)cdex+2\left(2b^3d^2ex^2+4b^3cdex+\left(2b^3c^2+b^3\right)e\right)\log\left(dx+c+\sqrt{d^2x^2+2c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/8*(2*(2*a^3 + 3*a*b^2)*d^2*e*x^2 + 4*(2*a^3 + 3*a*b^2)*c*d*e*x + 2*(2*b^3
*d^2*e*x^2 + 4*b^3*c*d*e*x + (2*b^3*c^2 + b^3)*e)*log(d*x + c + sqrt(d^2*x^
2 + 2*c*d*x + c^2 + 1))^3 + 6*(2*a*b^2*d^2*e*x^2 + 4*a*b^2*c*d*e*x + (2*a*b
^2*c^2 + a*b^2)*e - (b^3*d*e*x + b^3*c*e)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)
)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 + 3*(2*(2*a^2*b + b^3)
*d^2*e*x^2 + 4*(2*a^2*b + b^3)*c*d*e*x + (2*a^2*b + b^3 + 2*(2*a^2*b + b^3)
*c^2)*e - 4*(a*b^2*d*e*x + a*b^2*c*e)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*lo
g(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - 3*((2*a^2*b + b^3)*d*e*x +
 (2*a^2*b + b^3)*c*e)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/d
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex + ce)(b \operatorname{arsinh}(dx + c) + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^3, x)
```

maple [A] time = 0.06, size = 243, normalized size = 1.51

$$\frac{(dx+c)^2 e a^3}{2} + e b^3 \left(\frac{\operatorname{arcsinh}(dx+c)^3 \left(1+(dx+c)^2\right)}{2} - \frac{3 \operatorname{arcsinh}(dx+c)^2 \sqrt{1+(dx+c)^2}\,(dx+c)}{4} - \frac{\operatorname{arcsinh}(dx+c)^3}{4} + \frac{3 \operatorname{arcsinh}(dx+c)\left(1+(dx+c)^2\right)}{4} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^3,x)
```

```
[Out] 1/d*(1/2*(d*x+c)^2*e*a^3+e*b^3*(1/2*arcsinh(d*x+c)^3*(1+(d*x+c)^2)-3/4*arcs
inh(d*x+c)^2*(1+(d*x+c)^2)^(1/2)*(d*x+c)-1/4*arcsinh(d*x+c)^3+3/4*arcsinh(d
*x+c)*(1+(d*x+c)^2)-3/8*(d*x+c)*(1+(d*x+c)^2)^(1/2)-3/8*arcsinh(d*x+c))+3*e
*a*b^2*(1/2*arcsinh(d*x+c)^2*(1+(d*x+c)^2)-1/2*arcsinh(d*x+c)*(1+(d*x+c)^2)
^(1/2)*(d*x+c)-1/4*arcsinh(d*x+c)^2+1/4*(d*x+c)^2+1/4)+3*e*a^2*b*(1/2*(d*x+
c)^2*arcsinh(d*x+c)-1/4*(d*x+c)*(1+(d*x+c)^2)^(1/2)+1/4*arcsinh(d*x+c)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2} a^3 dex^2 + \frac{3}{4} \left(2 x^2 \operatorname{arsinh}(dx + c) - d \left(\frac{3 c^2 \operatorname{arsinh}\left(\frac{2\left(d^2 x + cd\right)}{\sqrt{-4 c^2 d^2 + 4 \left(c^2 + 1\right) d^2}} \right)}{d^3} + \frac{\sqrt{d^2 x^2 + 2 cdx + c^2 + 1}\, x}{d^2} - \frac{\left(c^2 + 1\right) a}{} \right. \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*a^3*d*e*x^2 + 3/4*(2*x^2*arcsinh(d*x + c) - d*(3*c^2*arcsinh(2*(d^2*x +
 c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c^
2 + 1)*x/d^2 - (c^2 + 1)*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 +
 1)*d^2))/d^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c/d^3))*a^2*b*d*e + a^3
*c*e*x + 3*((d*x + c)*arcsinh(d*x + c) - sqrt((d*x + c)^2 + 1))*a^2*b*c*e/d
 + 1/2*(b^3*d*e*x^2 + 2*b^3*c*e*x)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c
^2 + 1))^3 + integrate(3/2*((2*a*b^2*d^4*e - b^3*d^4*e)*x^4 + 2*(c^4*e + c^
2*e)*a*b^2 + 4*(2*a*b^2*c*d^3*e - b^3*c*d^3*e)*x^3 + (2*(6*c^2*d^2*e + d^2*
```

```
e)*a*b^2 - (5*c^2*d^2*e + d^2*e)*b^3)*x^2 + 2*(2*(2*c^3*d*e + c*d*e)*a*b^2
- (c^3*d*e + c*d*e)*b^3)*x + (2*(c^3*e + c*e)*a*b^2 + (2*a*b^2*d^3*e - b^3*
d^3*e)*x^3 + 3*(2*a*b^2*c*d^2*e - b^3*c*d^2*e)*x^2 - 2*(b^3*c^2*d*e - (3*c^
2*d*e + d*e)*a*b^2)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqr
t(d^2*x^2 + 2*c*d*x + c^2 + 1))^2/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (3*c^2*d +
 d)*x + (d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e+d\,e\,x)\,\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)*(a + b*asinh(c + d*x))^3,x)
```

```
[Out] int((c*e + d*e*x)*(a + b*asinh(c + d*x))^3, x)
```

sympy [A] time = 2.08, size = 685, normalized size = 4.25

$$\begin{cases} a^3cex + \frac{a^3dex^2}{2} + \frac{3a^2bc^2e\,\mathrm{asinh}\,(c+dx)}{2d} + 3a^2bcex\,\mathrm{asinh}\,(c+dx) - \frac{3a^2bce\sqrt{c^2+2cdx+d^2x^2+1}}{4d} + \frac{3a^2bdex^2\,\mathrm{asinh}\,(c+dx)}{2} - \frac{3a^2bex\sqrt{c^2 \\ cex\,(a+b\,\mathrm{asinh}\,(c))^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*asinh(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*c*e*x + a**3*d*e*x**2/2 + 3*a**2*b*c**2*e*asinh(c + d*x)/(2
*d) + 3*a**2*b*c*e*x*asinh(c + d*x) - 3*a**2*b*c*e*sqrt(c**2 + 2*c*d*x + d*
*2*x**2 + 1)/(4*d) + 3*a**2*b*d*e*x**2*asinh(c + d*x)/2 - 3*a**2*b*e*x*sqrt
(c**2 + 2*c*d*x + d**2*x**2 + 1)/4 + 3*a**2*b*e*asinh(c + d*x)/(4*d) + 3*a*
b**2*c**2*e*asinh(c + d*x)**2/(2*d) + 3*a*b**2*c*e*x*asinh(c + d*x)**2 + 3*
a*b**2*c*e*x/2 - 3*a*b**2*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c
+ d*x)/(2*d) + 3*a*b**2*d*e*x**2*asinh(c + d*x)**2/2 + 3*a*b**2*d*e*x**2/4
- 3*a*b**2*e*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/2 + 3*a*
b**2*e*asinh(c + d*x)**2/(4*d) + b**3*c**2*e*asinh(c + d*x)**3/(2*d) + 3*b*
*3*c**2*e*asinh(c + d*x)/(4*d) + b**3*c*e*x*asinh(c + d*x)**3 + 3*b**3*c*e*
x*asinh(c + d*x)/2 - 3*b**3*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(
c + d*x)**2/(4*d) - 3*b**3*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(8*d) +
 b**3*d*e*x**2*asinh(c + d*x)**3/2 + 3*b**3*d*e*x**2*asinh(c + d*x)/4 - 3*b
**3*e*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/4 - 3*b**3*e
*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/8 + b**3*e*asinh(c + d*x)**3/(4*d)
+ 3*b**3*e*asinh(c + d*x)/(8*d), Ne(d, 0)), (c*e*x*(a + b*asinh(c))**3, Tru
e))
```

3.141 $\int \left(a + b\sinh^{-1}(c + dx)\right)^3 \, dx$

Optimal. Leaf size=100

$$6ab^2x - \frac{3b\sqrt{(c+dx)^2+1}\,\left(a+b\sinh^{-1}(c+dx)\right)^2}{d} + \frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^3}{d} - \frac{6b^3\sqrt{(c+dx)^2+1}}{d} + \frac{6b^3(}{}$$

```
[Out] 6*a*b^2*x+6*b^3*(d*x+c)*arcsinh(d*x+c)/d+(d*x+c)*(a+b*arcsinh(d*x+c))^3/d-6
*b^3*(1+(d*x+c)^2)^(1/2)/d-3*b*(a+b*arcsinh(d*x+c))^2*(1+(d*x+c)^2)^(1/2)/d
```

Rubi [A] time = 0.11, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5863, 5653, 5717, 261}

$$6ab^2x - \frac{3b\sqrt{(c+dx)^2+1}\,\left(a+b\sinh^{-1}(c+dx)\right)^2}{d} + \frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^3}{d} - \frac{6b^3\sqrt{(c+dx)^2+1}}{d} + \frac{6b^3(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] 6*a*b^2*x - (6*b^3*Sqrt[1 + (c + d*x)^2])/d + (6*b^3*(c + d*x)*ArcSinh[c +
d*x])/d - (3*b*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^2)/d + ((c +
d*x)*(a + b*ArcSinh[c + d*x])^3)/d
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5863

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \left(a+b\sinh^{-1}(c+dx)\right)^3\,dx=\frac{\text{Subst}\left(\int \left(a+b\sinh^{-1}(x)\right)^3\,dx,x,c+dx\right)}{d}$$
$$=\frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^3}{d}-\frac{(3b)\,\text{Subst}\left(\int \frac{x\left(a+b\sinh^{-1}(x)\right)^2}{\sqrt{1+x^2}}\,dx,x,c+dx\right)}{d}$$
$$=-\frac{3b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{d}+\frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^3}{d}+$$
$$=6ab^2x-\frac{3b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{d}+\frac{(c+dx)\left(a+b\sinh^{-1}(c+\right.}{d}$$
$$=6ab^2x+\frac{6b^3(c+dx)\sinh^{-1}(c+dx)}{d}-\frac{3b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{d}$$
$$=6ab^2x-\frac{6b^3\sqrt{1+(c+dx)^2}}{d}+\frac{6b^3(c+dx)\sinh^{-1}(c+dx)}{d}-\frac{3b\sqrt{1+(c+dx)^2}\left(a\right.}{}$$

Mathematica [A] time = 0.17, size = 147, normalized size = 1.47

$$\frac{a\left(a^2+6b^2\right)(c+dx)-3b\left(a^2+2b^2\right)\sqrt{(c+dx)^2+1}-3b\sinh^{-1}(c+dx)\left(-\left(a^2(c+dx)\right)+2ab\sqrt{(c+dx)^2+1}-2\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] (a*(a^2 + 6*b^2)*(c + d*x) - 3*b*(a^2 + 2*b^2)*Sqrt[1 + (c + d*x)^2] - 3*b*
(-(a^2*(c + d*x)) - 2*b^2*(c + d*x) + 2*a*b*Sqrt[1 + (c + d*x)^2])*ArcSinh[
c + d*x] - 3*b^2*(-(a*(c + d*x)) + b*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x
]^2 + b^3*(c + d*x)*ArcSinh[c + d*x]^3)/d
```

fricas [B] time = 0.66, size = 239, normalized size = 2.39

$$\left(b^3dx+b^3c\right)\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)^3+\left(a^3+6\,ab^2\right)dx+3\left(ab^2dx+ab^2c-\sqrt{d^2x^2+2\,cdx+c^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] ((b^3*d*x + b^3*c)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3 + (a^
3 + 6*a*b^2)*d*x + 3*(a*b^2*d*x + a*b^2*c - sqrt(d^2*x^2 + 2*c*d*x + c^2 +
1)*b^3)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 - 3*(2*sqrt(d^2*
x^2 + 2*c*d*x + c^2 + 1)*a*b^2 - (a^2*b + 2*b^3)*d*x - (a^2*b + 2*b^3)*c)*l
og(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - 3*sqrt(d^2*x^2 + 2*c*d*x
+ c^2 + 1)*(a^2*b + 2*b^3))/d
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\,\text{arsinh}\,(dx+c)+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^3, x)
```

maple [A] time = 0.07, size = 160, normalized size = 1.60

$$a^3(dx+c)+b^3\left((dx+c)\operatorname{arcsinh}(dx+c)^3-3\operatorname{arcsinh}(dx+c)^2\sqrt{1+(dx+c)^2}+6(dx+c)\operatorname{arcsinh}(dx+c)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(d*x+c)+b^3*((d*x+c)*arcsinh(d*x+c)^3-3*arcsinh(d*x+c)^2*(1+(d*x+c
)^2)^(1/2)+6*(d*x+c)*arcsinh(d*x+c)-6*(1+(d*x+c)^2)^(1/2))+3*a*b^2*((d*x+c)
*arcsinh(d*x+c)^2-2*arcsinh(d*x+c)*(1+(d*x+c)^2)^(1/2)+2*d*x+2*c)+3*a^2*b*(
(d*x+c)*arcsinh(d*x+c)-(1+(d*x+c)^2)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b^3x\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)^3+a^3x+\frac{3\left((dx+c)\operatorname{arsinh}(dx+c)-\sqrt{(dx+c)^2+1}\right)a^2b}{d}+\int\frac{3\left((c^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] b^3*x*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3 + a^3*x + 3*((d*x
+ c)*arcsinh(d*x + c) - sqrt((d*x + c)^2 + 1))*a^2*b/d + integrate(3*((c^3
+ c)*a*b^2 + (a*b^2*d^3 - b^3*d^3)*x^3 + (3*a*b^2*c*d^2 - 2*b^3*c*d^2)*x^2
+ ((3*c^2*d + d)*a*b^2 - (c^2*d + d)*b^3)*x + ((c^2 + 1)*a*b^2 + (a*b^2*d^2
 - b^3*d^2)*x^2 + (2*a*b^2*c*d - b^3*c*d)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 +
 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2/(d^3*x^3 + 3*c*d^2*
x^2 + c^3 + (3*c^2*d + d)*x + (d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\operatorname{asinh}(c+d\,x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^3,x)
```

```
[Out] int((a + b*asinh(c + d*x))^3, x)
```

sympy [A] time = 0.80, size = 282, normalized size = 2.82

$$\begin{cases} a^3x+\frac{3a^2bc\operatorname{asinh}(c+dx)}{d}+3a^2bx\operatorname{asinh}(c+dx)-\frac{3a^2b\sqrt{c^2+2cdx+d^2x^2+1}}{d}+\frac{3ab^2c\operatorname{asinh}^2(c+dx)}{d}+3ab^2x\operatorname{asinh}^2(c+dx) \\ x\,(a+b\operatorname{asinh}(c))^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*x + 3*a**2*b*c*asinh(c + d*x)/d + 3*a**2*b*x*asinh(c + d*x)
 - 3*a**2*b*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/d + 3*a*b**2*c*asinh(c + d
*x)**2/d + 3*a*b**2*x*asinh(c + d*x)**2 + 6*a*b**2*x - 6*a*b**2*sqrt(c**2 +
 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/d + b**3*c*asinh(c + d*x)**3/d + 6
*b**3*c*asinh(c + d*x)/d + b**3*x*asinh(c + d*x)**3 + 6*b**3*x*asinh(c + d*
x) - 3*b**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/d - 6*b*
*3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/d, Ne(d, 0)), (x*(a + b*asinh(c))**
3, True))
```

3.142 $\int \frac{(a+b\sinh^{-1}(c+dx))^3}{ce+dex}\,dx$

Optimal. Leaf size=155

$$-\frac{3b^2\text{Li}_3\left(e^{-2\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)}{2de}-\frac{3b\text{Li}_2\left(e^{-2\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de}+\frac{\left(a+b\sinh^{-1}(c\right.}{4bde}$$

```
[Out] 1/4*(a+b*arcsinh(d*x+c))^4/b/d/e+(a+b*arcsinh(d*x+c))^3*ln(1-1/(d*x+c+(1+(d
*x+c)^2)^(1/2))^2)/d/e-3/2*b*(a+b*arcsinh(d*x+c))^2*polylog(2,1/(d*x+c+(1+(
d*x+c)^2)^(1/2))^2)/d/e-3/2*b^2*(a+b*arcsinh(d*x+c))*polylog(3,1/(d*x+c+(1+
(d*x+c)^2)^(1/2))^2)/d/e-3/4*b^3*polylog(4,1/(d*x+c+(1+(d*x+c)^2)^(1/2))^2)
/d/e
```

Rubi [A] time = 0.22, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {5865, 12, 5659, 3716, 2190, 2531, 6609, 2282, 6589}

$$-\frac{3b^2\text{PolyLog}\left(3,e^{2\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)}{2de}+\frac{3b\text{PolyLog}\left(2,e^{2\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de}+\frac{3b}{}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x),x]
```

```
[Out] -(a + b*ArcSinh[c + d*x])^4/(4*b*d*e) + ((a + b*ArcSinh[c + d*x])^3*Log[1 -
 E^(2*ArcSinh[c + d*x])])/(d*e) + (3*b*(a + b*ArcSinh[c + d*x])^2*PolyLog[2
, E^(2*ArcSinh[c + d*x])])/(2*d*e) - (3*b^2*(a + b*ArcSinh[c + d*x])*PolyLo
g[3, E^(2*ArcSinh[c + d*x])])/(2*d*e) + (3*b^3*PolyLog[4, E^(2*ArcSinh[c +
d*x])])/(4*d*e)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{ce+dex}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^3}{ex}\,dx,x,c+dx\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^3}{x}\,dx,x,c+dx\right)}{de}\\
&= \frac{\text{Subst}\left(\int (a+bx)^3\coth(x)\,dx,x,\sinh^{-1}(c+dx)\right)}{de}\\
&= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{4bde} - \frac{2\,\text{Subst}\left(\int \frac{e^{2x}(a+bx)^3}{1-e^{2x}}\,dx,x,\sinh^{-1}(c+dx)\right)}{de}\\
&= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{4bde} + \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3\log\left(1-e^{2\sinh^{-1}(c+dx)}\right)}{de} - \frac{(3b}{}\\
&= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{4bde} + \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3\log\left(1-e^{2\sinh^{-1}(c+dx)}\right)}{de} + \frac{3b}{}\\
&= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{4bde} + \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3\log\left(1-e^{2\sinh^{-1}(c+dx)}\right)}{de} + \frac{3b}{}\\
&= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{4bde} + \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3\log\left(1-e^{2\sinh^{-1}(c+dx)}\right)}{de} + \frac{3b}{}\\
&= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{4bde} + \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3\log\left(1-e^{2\sinh^{-1}(c+dx)}\right)}{de} + \frac{3b}{}
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 128, normalized size = 0.83

$$
\frac{-6b^2\text{Li}_3\left(e^{2\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)+6b\text{Li}_2\left(e^{2\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)^2-\frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{b}}{4de}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x),x]
```

```
[Out] (-((a + b*ArcSinh[c + d*x])^4/b) + 4*(a + b*ArcSinh[c + d*x])^3*Log[1 - E^(
2*ArcSinh[c + d*x])] + 6*b*(a + b*ArcSinh[c + d*x])^2*PolyLog[2, E^(2*ArcSi
nh[c + d*x])] - 6*b^2*(a + b*ArcSinh[c + d*x])*PolyLog[3, E^(2*ArcSinh[c +
d*x])] + 3*b^3*PolyLog[4, E^(2*ArcSinh[c + d*x])])/(4*d*e)
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$
\text{integral}\left(\frac{b^3\,\text{arsinh}\left(dx+c\right)^3+3\,ab^2\,\text{arsinh}\left(dx+c\right)^2+3\,a^2b\,\text{arsinh}\left(dx+c\right)+a^3}{dex+ce},x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e),x, algorithm="fricas")
```

```
[Out] integral((b^3*arcsinh(d*x + c)^3 + 3*a*b^2*arcsinh(d*x + c)^2 + 3*a^2*b*arc
sinh(d*x + c) + a^3)/(d*e*x + c*e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{\left(b\,\text{arsinh}\left(dx+c\right)+a\right)^3}{dex+ce}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^3/(d*e*x + c*e), x)
```

maple [B] time = 0.09, size = 736, normalized size = 4.75

$$\frac{a^3 \ln(dx+c)}{de} - \frac{b^3 \operatorname{arcsinh}(dx+c)^4}{4de} + \frac{b^3 \operatorname{arcsinh}(dx+c)^3 \ln\left(1+dx+c+\sqrt{1+(dx+c)^2}\right)}{de} + \frac{3b^3 \operatorname{arcsinh}(dx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e),x)
```

```
[Out] 1/d*a^3/e*ln(d*x+c)-1/4/d*b^3/e*arcsinh(d*x+c)^4+1/d*b^3/e*arcsinh(d*x+c)^3
*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))+3/d*b^3/e*arcsinh(d*x+c)^2*polylog(2,-d*x-
c-(1+(d*x+c)^2)^(1/2))-6/d*b^3/e*arcsinh(d*x+c)*polylog(3,-d*x-c-(1+(d*x+c)
^2)^(1/2))+6/d*b^3/e*polylog(4,-d*x-c-(1+(d*x+c)^2)^(1/2))+1/d*b^3/e*arcsin
h(d*x+c)^3*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))+3/d*b^3/e*arcsinh(d*x+c)^2*polyl
og(2,d*x+c+(1+(d*x+c)^2)^(1/2))-6/d*b^3/e*arcsinh(d*x+c)*polylog(3,d*x+c+(1
+(d*x+c)^2)^(1/2))+6/d*b^3/e*polylog(4,d*x+c+(1+(d*x+c)^2)^(1/2))-1/d*a*b^2
/e*arcsinh(d*x+c)^3+3/d*a*b^2/e*arcsinh(d*x+c)^2*ln(1+d*x+c+(1+(d*x+c)^2)^(
1/2))+6/d*a*b^2/e*arcsinh(d*x+c)*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))-6/d*
a*b^2/e*polylog(3,-d*x-c-(1+(d*x+c)^2)^(1/2))+3/d*a*b^2/e*arcsinh(d*x+c)^2*
ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))+6/d*a*b^2/e*arcsinh(d*x+c)*polylog(2,d*x+c+
(1+(d*x+c)^2)^(1/2))-6/d*a*b^2/e*polylog(3,d*x+c+(1+(d*x+c)^2)^(1/2))-3/2/d
*a^2*b/e*arcsinh(d*x+c)^2+3/d*a^2*b/e*arcsinh(d*x+c)*ln(1+d*x+c+(1+(d*x+c)^
2)^(1/2))+3/d*a^2*b/e*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))+3/d*a^2*b/e*arc
sinh(d*x+c)*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))+3/d*a^2*b/e*polylog(2,d*x+c+(1+
(d*x+c)^2)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^3 \log(dex+ce)}{de} + \int \frac{b^3 \log\left(dx+c+\sqrt{(dx+c)^2+1}\right)^3}{dex+ce} + \frac{3\,ab^2 \log\left(dx+c+\sqrt{(dx+c)^2+1}\right)^2}{dex+ce} + \frac{3\,a^2b \log\left(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e),x, algorithm="maxima")
```

```
[Out] a^3*log(d*e*x + c*e)/(d*e) + integrate(b^3*log(d*x + c + sqrt((d*x + c)^2 +
 1))^3/(d*e*x + c*e) + 3*a*b^2*log(d*x + c + sqrt((d*x + c)^2 + 1))^2/(d*e*
x + c*e) + 3*a^2*b*log(d*x + c + sqrt((d*x + c)^2 + 1))/(d*e*x + c*e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\operatorname{asinh}(c+d\,x))^3}{c\,e+d\,e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^3/(c*e + d*e*x),x)
```

```
[Out] int((a + b*asinh(c + d*x))^3/(c*e + d*e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^3}{c+dx}\,dx + \int \frac{b^3 \operatorname{asinh}^3(c+dx)}{c+dx}\,dx + \int \frac{3ab^2 \operatorname{asinh}^2(c+dx)}{c+dx}\,dx + \int \frac{3a^2b \operatorname{asinh}(c+dx)}{c+dx}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**3/(d*e*x+c*e),x)
```

```
[Out] (Integral(a**3/(c + d*x), x) + Integral(b**3*asinh(c + d*x)**3/(c + d*x), x
) + Integral(3*a*b**2*asinh(c + d*x)**2/(c + d*x), x) + Integral(3*a**2*b*a
sinh(c + d*x)/(c + d*x), x))/e
```

3.143 $\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{(ce+dex)^2}\,dx$

Optimal. Leaf size=166

$$-\frac{6b^2\text{Li}_2\left(-e^{\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)}{de^2}+\frac{6b^2\text{Li}_2\left(e^{\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)}{de^2}-\frac{\left(a+b\sinh^{-1}(c\right.}{de^2(c+dx}$$

```
[Out] -(a+b*arcsinh(d*x+c))^3/d/e^2/(d*x+c)-6*b*(a+b*arcsinh(d*x+c))^2*arctanh(d*
x+c+(1+(d*x+c)^2)^(1/2))/d/e^2-6*b^2*(a+b*arcsinh(d*x+c))*polylog(2,-d*x-c-
(1+(d*x+c)^2)^(1/2))/d/e^2+6*b^2*(a+b*arcsinh(d*x+c))*polylog(2,d*x+c+(1+(d
*x+c)^2)^(1/2))/d/e^2+6*b^3*polylog(3,-d*x-c-(1+(d*x+c)^2)^(1/2))/d/e^2-6*b
^3*polylog(3,d*x+c+(1+(d*x+c)^2)^(1/2))/d/e^2
```

Rubi [A] time = 0.26, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {5865, 12, 5661, 5760, 4182, 2531, 2282, 6589}

$$-\frac{6b^2\text{PolyLog}\left(2,-e^{\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)}{de^2}+\frac{6b^2\text{PolyLog}\left(2,e^{\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)}{de^2}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^2,x]
```

```
[Out] -((a + b*ArcSinh[c + d*x])^3/(d*e^2*(c + d*x))) - (6*b*(a + b*ArcSinh[c + d
*x])^2*ArcTanh[E^ArcSinh[c + d*x]])/(d*e^2) - (6*b^2*(a + b*ArcSinh[c + d*x
])*PolyLog[2, -E^ArcSinh[c + d*x]])/(d*e^2) + (6*b^2*(a + b*ArcSinh[c + d*x
])*PolyLog[2, E^ArcSinh[c + d*x]])/(d*e^2) + (6*b^3*PolyLog[3, -E^ArcSinh[c
 + d*x]])/(d*e^2) - (6*b^3*PolyLog[3, E^ArcSinh[c + d*x]])/(d*e^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
```

```
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{(ce+dex)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^3}{e^2x^2}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^3}{x^2}\,dx, x, c+dx\right)}{de^2}$$

$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{de^2(c+dx)} + \frac{(3b)\,\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^2}{x\sqrt{1+x^2}}\,dx, x, c+dx\right)}{de^2}$$

$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{de^2(c+dx)} + \frac{(3b)\,\text{Subst}\left(\int (a+bx)^2\text{csch}(x)\,dx, x, \sinh^{-1}(c+dx)\right)}{de^2}$$

$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{de^2(c+dx)} - \frac{6b\left(a+b\sinh^{-1}(c+dx)\right)^2\tanh^{-1}\left(e^{\sinh^{-1}(c+dx)}\right)}{de^2} - \ldots$$

$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{de^2(c+dx)} - \frac{6b\left(a+b\sinh^{-1}(c+dx)\right)^2\tanh^{-1}\left(e^{\sinh^{-1}(c+dx)}\right)}{de^2} - \ldots$$

$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{de^2(c+dx)} - \frac{6b\left(a+b\sinh^{-1}(c+dx)\right)^2\tanh^{-1}\left(e^{\sinh^{-1}(c+dx)}\right)}{de^2} - \ldots$$

$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{de^2(c+dx)} - \frac{6b\left(a+b\sinh^{-1}(c+dx)\right)^2\tanh^{-1}\left(e^{\sinh^{-1}(c+dx)}\right)}{de^2} - \ldots$$

Mathematica [A] time = 0.72, size = 315, normalized size = 1.90

$$-\frac{a^3}{c+dx} - 3a^2 b \log\left(\sqrt{c^2+2cdx+d^2x^2+1}+1\right) + 3a^2 b\log(c+dx) - \frac{3a^2 b \sinh^{-1}(c+dx)}{c+dx} + 3ab^2\left(2\text{Li}_2\left(-e^{-\sinh^{-1}(c+c}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^2,x]
```

```
[Out] (-(a^3/(c + d*x)) - (3*a^2*b*ArcSinh[c + d*x])/(c + d*x) + 3*a^2*b*Log[c +
d*x] - 3*a^2*b*Log[1 + Sqrt[1 + c^2 + 2*c*d*x + d^2*x^2]] + 3*a*b^2*(ArcSin
h[c + d*x]*(-(ArcSinh[c + d*x]/(c + d*x)) + 2*Log[1 - E^(-ArcSinh[c + d*x])
] - 2*Log[1 + E^(-ArcSinh[c + d*x])]) + 2*PolyLog[2, -E^(-ArcSinh[c + d*x])
] - 2*PolyLog[2, E^(-ArcSinh[c + d*x])]) + b^3*(-(ArcSinh[c + d*x]^3/(c + d
*x)) + 3*ArcSinh[c + d*x]^2*Log[1 - E^(-ArcSinh[c + d*x])] - 3*ArcSinh[c +
d*x]^2*Log[1 + E^(-ArcSinh[c + d*x])] + 6*ArcSinh[c + d*x]*PolyLog[2, -E^(-
ArcSinh[c + d*x])] - 6*ArcSinh[c + d*x]*PolyLog[2, E^(-ArcSinh[c + d*x])] +
 6*PolyLog[3, -E^(-ArcSinh[c + d*x])] - 6*PolyLog[3, E^(-ArcSinh[c + d*x])]
))/(d*e^2)
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3 \operatorname{arsinh}(dx+c)^3 + 3\,ab^2 \operatorname{arsinh}(dx+c)^2 + 3\,a^2 b \operatorname{arsinh}(dx+c) + a^3}{d^2e^2x^2 + 2\,cde^2x + c^2e^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^2,x, algorithm="fricas")
```

```
[Out] integral((b^3*arcsinh(d*x + c)^3 + 3*a*b^2*arcsinh(d*x + c)^2 + 3*a^2*b*arc
sinh(d*x + c) + a^3)/(d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arsinh}(dx+c)+a)^3}{(dex+ce)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^3/(d*e*x + c*e)^2, x)
```

maple [B] time = 0.15, size = 481, normalized size = 2.90

$$-\frac{a^3}{d\,e^2\,(dx+c)} - \frac{b^3 \operatorname{arcsinh}(dx+c)^3}{d\,e^2\,(dx+c)} - \frac{3b^3 \operatorname{arcsinh}(dx+c)^2 \ln\left(1+dx+c+\sqrt{1+(dx+c)^2}\right)}{d\,e^2} - \frac{6b^3 \operatorname{arcsinh}(dx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^2,x)
```

```
[Out] -1/d*a^3/e^2/(d*x+c)-1/d*b^3/e^2*arcsinh(d*x+c)^3/(d*x+c)-3/d*b^3/e^2*arcsi
nh(d*x+c)^2*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))-6/d*b^3/e^2*arcsinh(d*x+c)*poly
log(2,-d*x-c-(1+(d*x+c)^2)^(1/2))+6*b^3*polylog(3,-d*x-c-(1+(d*x+c)^2)^(1/2
))/d/e^2+3/d*b^3/e^2*arcsinh(d*x+c)^2*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))+6/d*b
^3/e^2*arcsinh(d*x+c)*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2))-6*b^3*polylog(3,
d*x+c+(1+(d*x+c)^2)^(1/2))/d/e^2-3/d*a*b^2/e^2/(d*x+c)*arcsinh(d*x+c)^2-6/d
*a*b^2/e^2*arcsinh(d*x+c)*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))-6/d*a*b^2/e^2*pol
ylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))+6/d*a*b^2/e^2*arcsinh(d*x+c)*ln(1-d*x-c-
```

```
(1+(d*x+c)^2)^(1/2))+6/d*a*b^2/e^2*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2))-3/d
*a^2*b/e^2/(d*x+c)*arcsinh(d*x+c)-3/d*a^2*b/e^2*arctanh(1/(1+(d*x+c)^2)^(1/
2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b^3\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)^3}{d^2e^2x+cde^2}-3\,a^2b\left(\frac{\operatorname{arsinh}(dx+c)}{d^2e^2x+cde^2}+\frac{\operatorname{arsinh}\left(\frac{de^2}{|d^2e^2x+cde^2|}\right)}{de^2}\right)-\frac{a^3}{d^2e^2x+cde^2}+\int -\frac{3(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^2,x, algorithm="maxima")
```

```
[Out] -b^3*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3/(d^2*e^2*x + c*d*e^
2) - 3*a^2*b*(arcsinh(d*x + c)/(d^2*e^2*x + c*d*e^2) + arcsinh(d*e^2/abs(d^
2*e^2*x + c*d*e^2))/(d*e^2)) - a^3/(d^2*e^2*x + c*d*e^2) + integrate(3*((c^
3 + c)*a*b^2 + (c^3 + c)*b^3 + (a*b^2*d^3 + b^3*d^3)*x^3 + 3*(a*b^2*c*d^2 +
 b^3*c*d^2)*x^2 + ((3*c^2*d + d)*a*b^2 + (3*c^2*d + d)*b^3)*x + (b^3*c^2 +
(c^2 + 1)*a*b^2 + (a*b^2*d^2 + b^3*d^2)*x^2 + 2*(a*b^2*c*d + b^3*c*d)*x)*sq
rt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2
 + 1))^2/(d^5*e^2*x^5 + 5*c*d^4*e^2*x^4 + c^5*e^2 + c^3*e^2 + (10*c^2*d^3*e
^2 + d^3*e^2)*x^3 + (10*c^3*d^2*e^2 + 3*c*d^2*e^2)*x^2 + (5*c^4*d*e^2 + 3*c
^2*d*e^2)*x + (d^4*e^2*x^4 + 4*c*d^3*e^2*x^3 + c^4*e^2 + c^2*e^2 + (6*c^2*d
^2*e^2 + d^2*e^2)*x^2 + 2*(2*c^3*d*e^2 + c*d*e^2)*x)*sqrt(d^2*x^2 + 2*c*d*x
 + c^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\operatorname{asinh}(c+d\,x))^3}{(c\,e+d\,e\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^3/(c*e + d*e*x)^2,x)
```

```
[Out] int((a + b*asinh(c + d*x))^3/(c*e + d*e*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^3}{c^2+2cdx+d^2x^2}\,dx+\int \frac{b^3\operatorname{asinh}^3(c+dx)}{c^2+2cdx+d^2x^2}\,dx+\int \frac{3ab^2\operatorname{asinh}^2(c+dx)}{c^2+2cdx+d^2x^2}\,dx+\int \frac{3a^2b\operatorname{asinh}(c+dx)}{c^2+2cdx+d^2x^2}\,dx}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**3/(d*e*x+c*e)**2,x)
```

```
[Out] (Integral(a**3/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(b**3*asinh(c + d
*x)**3/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(3*a*b**2*asinh(c + d*x)*
*2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(3*a**2*b*asinh(c + d*x)/(c**
2 + 2*c*d*x + d**2*x**2), x))/e**2
```

3.144 $\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{(ce+dex)^3}\,dx$

Optimal. Leaf size=157

$$\frac{3b^2\log\left(1-e^{-2\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)}{de^3}-\frac{3b\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de^3(c+dx)}+\frac{3b\left(a+b\sinh\right.}{2de}$$

```
[Out] 3/2*b*(a+b*arcsinh(d*x+c))^2/d/e^3-1/2*(a+b*arcsinh(d*x+c))^3/d/e^3/(d*x+c)
^2+3*b^2*(a+b*arcsinh(d*x+c))*ln(1-1/(d*x+c+(1+(d*x+c)^2)^(1/2))^2)/d/e^3-3
/2*b^3*polylog(2,1/(d*x+c+(1+(d*x+c)^2)^(1/2))^2)/d/e^3-3/2*b*(a+b*arcsinh(
d*x+c))^2*(1+(d*x+c)^2)^(1/2)/d/e^3/(d*x+c)
```

Rubi [A] time = 0.26, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {5865, 12, 5661, 5723, 5659, 3716, 2190, 2279, 2391}

$$\frac{3b^3\text{PolyLog}\left(2,e^{2\sinh^{-1}(c+dx)}\right)}{2de^3}+\frac{3b^2\log\left(1-e^{2\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)}{de^3}-\frac{3b\sqrt{(c+dx)^2+1}\left(a+b\sin\right.}{2de^3(c+dx)}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^3,x]
```

```
[Out] (-3*b*(a + b*ArcSinh[c + d*x])^2)/(2*d*e^3) - (3*b*Sqrt[1 + (c + d*x)^2]*(a
 + b*ArcSinh[c + d*x])^2)/(2*d*e^3*(c + d*x)) - (a + b*ArcSinh[c + d*x])^3/
(2*d*e^3*(c + d*x)^2) + (3*b^2*(a + b*ArcSinh[c + d*x])*Log[1 - E^(2*ArcSin
h[c + d*x])])/(d*e^3) + (3*b^3*PolyLog[2, E^(2*ArcSinh[c + d*x])])/(2*d*e^3
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
```

```
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5723

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)
^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 +
 c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
 d, e, f, m, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] &&
 NeQ[m, -1]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{(ce+dex)^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^3}{e^3x^3}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^3}{x^3}\,dx, x, c+dx\right)}{de^3}$$

$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{2de^3(c+dx)^2} + \frac{(3b)\,\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^2}{x^2\sqrt{1+x^2}}\,dx, x, c+dx\right)}{2de^3}$$

$$= -\frac{3b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de^3(c+dx)} - \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{2de^3(c+dx)^2} + \frac{(3b^2}{}$$

$$= -\frac{3b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de^3(c+dx)} - \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{2de^3(c+dx)^2} + \frac{(3b^2}{}$$

$$= -\frac{3b\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de^3} - \frac{3b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de^3(c+dx)} - \frac{(a}{}$$

$$= -\frac{3b\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de^3} - \frac{3b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de^3(c+dx)} - \frac{(a}{}$$

$$= -\frac{3b\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de^3} - \frac{3b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de^3(c+dx)} - \frac{(a}{}$$

$$= -\frac{3b\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de^3} - \frac{3b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de^3(c+dx)} - \frac{(a}{}$$

Mathematica [A] time = 0.81, size = 229, normalized size = 1.46

$$-\frac{a\left(a\left(a+3b(c+dx)\sqrt{c^2+2cdx+d^2x^2+1}\right)-6b^2(c+dx)^2\log(c+dx)\right)+3b^2\sinh^{-1}(c+dx)^2\left(a+b(c+dx\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^3,x]
```

```
[Out] -1/2*(3*b^2*(a + b*(c + d*x)*(-c - d*x + Sqrt[1 + c^2 + 2*c*d*x + d^2*x^2])
)*ArcSinh[c + d*x]^2 + b^3*ArcSinh[c + d*x]^3 + 3*b*ArcSinh[c + d*x]*(a*(a
+ 2*b*(c + d*x)*Sqrt[1 + c^2 + 2*c*d*x + d^2*x^2]) - 2*b^2*(c + d*x)^2*Log[
1 - E^(-2*ArcSinh[c + d*x])]) + a*(a*(a + 3*b*(c + d*x)*Sqrt[1 + c^2 + 2*c*
d*x + d^2*x^2]) - 6*b^2*(c + d*x)^2*Log[c + d*x]) + 3*b^3*(c + d*x)^2*PolyL
og[2, E^(-2*ArcSinh[c + d*x])])/(d*e^3*(c + d*x)^2)
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3 \operatorname{arsinh}(dx+c)^3 + 3\,ab^2 \operatorname{arsinh}(dx+c)^2 + 3\,a^2b \operatorname{arsinh}(dx+c) + a^3}{d^3e^3x^3 + 3\,cd^2e^3x^2 + 3\,c^2de^3x + c^3e^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*arcsinh(d*x + c)^3 + 3*a*b^2*arcsinh(d*x + c)^2 + 3*a^2*b*arc
sinh(d*x + c) + a^3)/(d^3*e^3*x^3 + 3*c*d^2*e^3*x^2 + 3*c^2*d*e^3*x + c^3*e
^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arsinh}(dx+c)+a)^3}{(dex+ce)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^3/(d*e*x + c*e)^3, x)
```

maple [B] time = 0.22, size = 409, normalized size = 2.61

$$-\frac{a^3}{2d\,e^3\,(dx+c)^2}-\frac{3b^3 \operatorname{arcsinh}(dx+c)^2\sqrt{1+(dx+c)^2}}{2d\,e^3\,(dx+c)}-\frac{3b^3 \operatorname{arcsinh}(dx+c)^2}{2d\,e^3}-\frac{b^3 \operatorname{arcsinh}(dx+c)^3}{2d\,e^3\,(dx+c)^2}+\frac{3b^3 \operatorname{arcsin}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^3,x)
```

```
[Out] -1/2/d*a^3/e^3/(d*x+c)^2-3/2/d*b^3/e^3*arcsinh(d*x+c)^2/(d*x+c)*(1+(d*x+c)^
2)^(1/2)-3/2/d*b^3/e^3*arcsinh(d*x+c)^2-1/2/d*b^3/e^3*arcsinh(d*x+c)^3/(d*x
+c)^2+3/d*b^3/e^3*arcsinh(d*x+c)*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))+3/d*b^3/e^
3*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))+3/d*b^3/e^3*arcsinh(d*x+c)*ln(1-d*x
-c-(1+(d*x+c)^2)^(1/2))+3/d*b^3/e^3*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2))-3/
d*a*b^2/e^3*arcsinh(d*x+c)-3/d*a*b^2/e^3*arcsinh(d*x+c)/(d*x+c)*(1+(d*x+c)^
2)^(1/2)-3/2/d*a*b^2/e^3*arcsinh(d*x+c)^2/(d*x+c)^2+3/d*a*b^2/e^3*ln((d*x+c
+(1+(d*x+c)^2)^(1/2))^2-1)-3/2/d*a^2*b/e^3/(d*x+c)^2*arcsinh(d*x+c)-3/2/d*a
^2*b/e^3/(d*x+c)*(1+(d*x+c)^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-3\left(\frac{\sqrt{d^2x^2+2\,cdx+c^2+1}\,d \operatorname{arsinh}(dx+c)}{d^3e^3x+cd^2e^3}-\frac{\log(dx+c)}{de^3}\right)ab^2-\frac{1}{2}\left(\frac{\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)^3}{d^3e^3x^2+2\,cd^2e^3x+c^2de^3}-2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^3,x, algorithm="maxima")
```

```
[Out] -3*(sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*d*arcsinh(d*x + c)/(d^3*e^3*x + c*d^2
*e^3) - log(d*x + c)/(d*e^3))*a*b^2 - 1/2*(log(d*x + c + sqrt(d^2*x^2 + 2*c
*d*x + c^2 + 1))^3/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3) - 2*integrate(
3/2*(d^2*x^2 + 2*c*d*x + c^2 + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(d*x + c)
+ 1)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2/(d^5*e^3*x^5 + 5*c*
d^4*e^3*x^4 + c^5*e^3 + c^3*e^3 + (10*c^2*d^3*e^3 + d^3*e^3)*x^3 + (10*c^3*
d^2*e^3 + 3*c*d^2*e^3)*x^2 + (5*c^4*d*e^3 + 3*c^2*d*e^3)*x + (d^4*e^3*x^4 +
 4*c*d^3*e^3*x^3 + c^4*e^3 + c^2*e^3 + (6*c^2*d^2*e^3 + d^2*e^3)*x^2 + 2*(2
*c^3*d*e^3 + c*d*e^3)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)), x))*b^3 - 3/2*
a^2*b*(sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*d/(d^3*e^3*x + c*d^2*e^3) + arcsin
h(d*x + c)/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3)) - 3/2*a*b^2*arcsinh(d
*x + c)^2/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3) - 1/2*a^3/(d^3*e^3*x^2
+ 2*c*d^2*e^3*x + c^2*d*e^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b \operatorname{asinh}(c+d\,x))^3}{(c\,e+d\,e\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^3/(c*e + d*e*x)^3,x)
```

```
[Out] int((a + b*asinh(c + d*x))^3/(c*e + d*e*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^3}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{b^3\operatorname{asinh}^3(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{3ab^2\operatorname{asinh}^2(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{3a^2b\operatorname{asinh}(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**3/(d*e*x+c*e)**3,x)
```

```
[Out] (Integral(a**3/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integr
al(b**3*asinh(c + d*x)**3/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),
x) + Integral(3*a*b**2*asinh(c + d*x)**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2
 + d**3*x**3), x) + Integral(3*a**2*b*asinh(c + d*x)/(c**3 + 3*c**2*d*x + 3
*c*d**2*x**2 + d**3*x**3), x))/e**3
```

3.145 $\int \frac{(a+b\sinh^{-1}(c+dx))^3}{(ce+dex)^4}\,dx$

Optimal. Leaf size=261

$$\frac{b^2\text{Li}_2\left(-e^{\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)}{de^4}-\frac{b^2\text{Li}_2\left(e^{\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)}{de^4}-\frac{b^2\left(a+b\sinh^{-1}(c+dx\right.}{de^4(c+dx)}$$

```
[Out] -b^2*(a+b*arcsinh(d*x+c))/d/e^4/(d*x+c)-1/3*(a+b*arcsinh(d*x+c))^3/d/e^4/(d
*x+c)^3+b*(a+b*arcsinh(d*x+c))^2*arctanh(d*x+c+(1+(d*x+c)^2)^(1/2))/d/e^4-b
^3*arctanh((1+(d*x+c)^2)^(1/2))/d/e^4+b^2*(a+b*arcsinh(d*x+c))*polylog(2,-d
*x-c-(1+(d*x+c)^2)^(1/2))/d/e^4-b^2*(a+b*arcsinh(d*x+c))*polylog(2,d*x+c+(1
+(d*x+c)^2)^(1/2))/d/e^4-b^3*polylog(3,-d*x-c-(1+(d*x+c)^2)^(1/2))/d/e^4+b^
3*polylog(3,d*x+c+(1+(d*x+c)^2)^(1/2))/d/e^4-1/2*b*(a+b*arcsinh(d*x+c))^2*(
1+(d*x+c)^2)^(1/2)/d/e^4/(d*x+c)^2
```

Rubi [A] time = 0.41, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {5865, 12, 5661, 5747, 5760, 4182, 2531, 2282, 6589, 266, 63, 207}

$$\frac{b^2\text{PolyLog}\left(2,-e^{\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)}{de^4}-\frac{b^2\text{PolyLog}\left(2,e^{\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)}{de^4}-b^3\text{Poly}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^4,x]
```

```
[Out] -((b^2*(a + b*ArcSinh[c + d*x]))/(d*e^4*(c + d*x))) - (b*Sqrt[1 + (c + d*x)
^2]*(a + b*ArcSinh[c + d*x])^2)/(2*d*e^4*(c + d*x)^2) - (a + b*ArcSinh[c +
d*x])^3/(3*d*e^4*(c + d*x)^3) + (b*(a + b*ArcSinh[c + d*x])^2*ArcTanh[E^Arc
Sinh[c + d*x]])/(d*e^4) - (b^3*ArcTanh[Sqrt[1 + (c + d*x)^2]])/(d*e^4) + (b
^2*(a + b*ArcSinh[c + d*x])*PolyLog[2, -E^ArcSinh[c + d*x]])/(d*e^4) - (b^2
*(a + b*ArcSinh[c + d*x])*PolyLog[2, E^ArcSinh[c + d*x]])/(d*e^4) - (b^3*Po
lyLog[3, -E^ArcSinh[c + d*x]])/(d*e^4) + (b^3*PolyLog[3, E^ArcSinh[c + d*x]
])/(d*e^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{(ce+dex)^4}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^3}{e^4x^4}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^3}{x^4}\,dx, x, c+dx\right)}{de^4}$$

$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{3de^4(c+dx)^3} + \frac{b\,\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^2}{x^3\sqrt{1+x^2}}\,dx, x, c+dx\right)}{de^4}$$

$$= -\frac{b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de^4(c+dx)^2} - \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{3de^4(c+dx)^3} - \frac{b\,\text{Subst}\big(}{}$$

$$= -\frac{b^2\left(a+b\sinh^{-1}(c+dx)\right)}{de^4(c+dx)} - \frac{b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de^4(c+dx)^2} - \frac{\big(a+b\,\text{s}}{3d}$$

$$= -\frac{b^2\left(a+b\sinh^{-1}(c+dx)\right)}{de^4(c+dx)} - \frac{b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de^4(c+dx)^2} - \frac{\big(a+b\,\text{s}}{3d}$$

$$= -\frac{b^2\left(a+b\sinh^{-1}(c+dx)\right)}{de^4(c+dx)} - \frac{b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de^4(c+dx)^2} - \frac{\big(a+b\,\text{s}}{3d}$$

$$= -\frac{b^2\left(a+b\sinh^{-1}(c+dx)\right)}{de^4(c+dx)} - \frac{b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de^4(c+dx)^2} - \frac{\big(a+b\,\text{s}}{3d}$$

$$= -\frac{b^2\left(a+b\sinh^{-1}(c+dx)\right)}{de^4(c+dx)} - \frac{b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de^4(c+dx)^2} - \frac{\big(a+b\,\text{s}}{3d}$$

Mathematica [B] time = 7.14, size = 694, normalized size = 2.66

$$-\frac{a^3}{3de^4(c+dx)^3} - \frac{a^2b\sqrt{c^2+2cdx+d^2x^2+1}}{2de^4(c+dx)^2} + \frac{a^2b\log\left(\sqrt{c^2+2cdx+d^2x^2+1}+1\right)}{2de^4} - \frac{a^2b\log(c+dx)}{2de^4} - \frac{a^2b\sinh^{-1}(}{de^4(c+}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^4,x]
```

```
[Out] -1/3*a^3/(d*e^4*(c + d*x)^3) - (a^2*b*Sqrt[1 + c^2 + 2*c*d*x + d^2*x^2])/(2
*d*e^4*(c + d*x)^2) - (a^2*b*ArcSinh[c + d*x])/(d*e^4*(c + d*x)^3) - (a^2*b
*Log[c + d*x])/(2*d*e^4) + (a^2*b*Log[1 + Sqrt[1 + c^2 + 2*c*d*x + d^2*x^2]
])/(2*d*e^4) + (a*b^2*(-8*PolyLog[2, -E^(-ArcSinh[c + d*x])] - (2*(-2 + 4*A
rcSinh[c + d*x]^2 + 2*Cosh[2*ArcSinh[c + d*x]] - 3*(c + d*x)*ArcSinh[c + d*
x]*Log[1 - E^(-ArcSinh[c + d*x])] + 3*(c + d*x)*ArcSinh[c + d*x]*Log[1 + E^
(-ArcSinh[c + d*x])] - 4*(c + d*x)^3*PolyLog[2, E^(-ArcSinh[c + d*x])] + 2*
```

```
ArcSinh[c + d*x]*Sinh[2*ArcSinh[c + d*x]] + ArcSinh[c + d*x]*Log[1 - E^(-Ar
cSinh[c + d*x])]*Sinh[3*ArcSinh[c + d*x]] - ArcSinh[c + d*x]*Log[1 + E^(-Ar
cSinh[c + d*x])]*Sinh[3*ArcSinh[c + d*x]]))/(c + d*x)^3))/(8*d*e^4) + (b^3*
(-24*ArcSinh[c + d*x]*Coth[ArcSinh[c + d*x]/2] + 4*ArcSinh[c + d*x]^3*Coth[
ArcSinh[c + d*x]/2] - 6*ArcSinh[c + d*x]^2*Csch[ArcSinh[c + d*x]/2]^2 - (c
+ d*x)*ArcSinh[c + d*x]^3*Csch[ArcSinh[c + d*x]/2]^4 - 24*ArcSinh[c + d*x]^
2*Log[1 - E^(-ArcSinh[c + d*x])] + 24*ArcSinh[c + d*x]^2*Log[1 + E^(-ArcSin
h[c + d*x])] + 48*Log[Tanh[ArcSinh[c + d*x]/2]] - 48*ArcSinh[c + d*x]*PolyL
og[2, -E^(-ArcSinh[c + d*x])] + 48*ArcSinh[c + d*x]*PolyLog[2, E^(-ArcSinh[
c + d*x])] - 48*PolyLog[3, -E^(-ArcSinh[c + d*x])] + 48*PolyLog[3, E^(-ArcS
inh[c + d*x])] - 6*ArcSinh[c + d*x]^2*Sech[ArcSinh[c + d*x]/2]^2 - (16*ArcS
inh[c + d*x]^3*Sinh[ArcSinh[c + d*x]/2]^4)/(c + d*x)^3 + 24*ArcSinh[c + d*x
]*Tanh[ArcSinh[c + d*x]/2] - 4*ArcSinh[c + d*x]^3*Tanh[ArcSinh[c + d*x]/2])
)/(48*d*e^4)
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3 \operatorname{arsinh}(dx+c)^3 + 3\,ab^2 \operatorname{arsinh}(dx+c)^2 + 3\,a^2 b \operatorname{arsinh}(dx+c) + a^3}{d^4e^4x^4 + 4\,cd^3e^4x^3 + 6\,c^2d^2e^4x^2 + 4\,c^3de^4x + c^4e^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^4,x, algorithm="fricas")
```

```
[Out] integral((b^3*arcsinh(d*x + c)^3 + 3*a*b^2*arcsinh(d*x + c)^2 + 3*a^2*b*arc
sinh(d*x + c) + a^3)/(d^4*e^4*x^4 + 4*c*d^3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4
*c^3*d*e^4*x + c^4*e^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arsinh}(dx+c)+a)^3}{(dex+ce)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^4,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^3/(d*e*x + c*e)^4, x)
```

maple [B] time = 0.31, size = 651, normalized size = 2.49

$$-\frac{a^3}{3d\,e^4\,(dx+c)^3} - \frac{b^3 \operatorname{arcsinh}(dx+c)^2 \sqrt{1+(dx+c)^2}}{2d\,e^4\,(dx+c)^2} - \frac{b^3 \operatorname{arcsinh}(dx+c)^3}{3d\,e^4\,(dx+c)^3} - \frac{b^3 \operatorname{arcsinh}(dx+c)}{d\,e^4\,(dx+c)} + \frac{b^3 \operatorname{arcsinh}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^4,x)
```

```
[Out] -1/3/d*a^3/e^4/(d*x+c)^3-1/2/d*b^3/e^4/(d*x+c)^2*arcsinh(d*x+c)^2*(1+(d*x+c
)^2)^(1/2)-1/3/d*b^3/e^4/(d*x+c)^3*arcsinh(d*x+c)^3-1/d*b^3/e^4/(d*x+c)*arc
sinh(d*x+c)+1/2/d*b^3/e^4*arcsinh(d*x+c)^2*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))+
1/d*b^3/e^4*arcsinh(d*x+c)*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))-b^3*polylo
g(3,-d*x-c-(1+(d*x+c)^2)^(1/2))/d/e^4-1/2/d*b^3/e^4*arcsinh(d*x+c)^2*ln(1-d
*x-c-(1+(d*x+c)^2)^(1/2))-1/d*b^3/e^4*arcsinh(d*x+c)*polylog(2,d*x+c+(1+(d*
x+c)^2)^(1/2))+b^3*polylog(3,d*x+c+(1+(d*x+c)^2)^(1/2))/d/e^4-2/d*b^3/e^4*a
rctanh(d*x+c+(1+(d*x+c)^2)^(1/2))-1/d*a*b^2/e^4/(d*x+c)^2*arcsinh(d*x+c)*(1
+(d*x+c)^2)^(1/2)-1/d*a*b^2/e^4/(d*x+c)^3*arcsinh(d*x+c)^2-1/d*a*b^2/e^4/(d
*x+c)+1/d*a*b^2/e^4*arcsinh(d*x+c)*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))+1/d*a*b^
2/e^4*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))-1/d*a*b^2/e^4*arcsinh(d*x+c)*ln
(1-d*x-c-(1+(d*x+c)^2)^(1/2))-1/d*a*b^2/e^4*polylog(2,d*x+c+(1+(d*x+c)^2)^(
```

```
1/2))-1/d*a^2*b/e^4/(d*x+c)^3*arcsinh(d*x+c)-1/2/d*a^2*b/e^4/(d*x+c)^2*(1+(
d*x+c)^2)^(1/2)+1/2/d*a^2*b/e^4*arctanh(1/(1+(d*x+c)^2)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b^3\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)^3}{3\left(d^4e^4x^3+3\,cd^3e^4x^2+3\,c^2d^2e^4x+c^3de^4\right)}-\frac{a^3}{3\left(d^4e^4x^3+3\,cd^3e^4x^2+3\,c^2d^2e^4x+c^3de^4\right)}+\int\frac{\left(3\left(c^3+c\right)ab^2+\left(c^3\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^4,x, algorithm="maxima")
```

```
[Out] -1/3*b^3*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3/(d^4*e^4*x^3 +
3*c*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^4) - 1/3*a^3/(d^4*e^4*x^3 + 3*c
*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^4) + integrate(((3*(c^3 + c)*a*b^2
 + (c^3 + c)*b^3 + (3*a*b^2*d^3 + b^3*d^3)*x^3 + 3*(3*a*b^2*c*d^2 + b^3*c*d
^2)*x^2 + (3*(3*c^2*d + d)*a*b^2 + (3*c^2*d + d)*b^3)*x + (b^3*c^2 + 3*(c^2
 + 1)*a*b^2 + (3*a*b^2*d^2 + b^3*d^2)*x^2 + 2*(3*a*b^2*c*d + b^3*c*d)*x)*sq
rt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2
 + 1))^2 + 3*(a^2*b*d^3*x^3 + 3*a^2*b*c*d^2*x^2 + (3*c^2*d + d)*a^2*b*x + (
c^3 + c)*a^2*b + (a^2*b*d^2*x^2 + 2*a^2*b*c*d*x + (c^2 + 1)*a^2*b)*sqrt(d^2
*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))
)/(d^7*e^4*x^7 + 7*c*d^6*e^4*x^6 + c^7*e^4 + c^5*e^4 + (21*c^2*d^5*e^4 + d^
5*e^4)*x^5 + 5*(7*c^3*d^4*e^4 + c*d^4*e^4)*x^4 + 5*(7*c^4*d^3*e^4 + 2*c^2*d
^3*e^4)*x^3 + (21*c^5*d^2*e^4 + 10*c^3*d^2*e^4)*x^2 + (7*c^6*d*e^4 + 5*c^4*
d*e^4)*x + (d^6*e^4*x^6 + 6*c*d^5*e^4*x^5 + c^6*e^4 + c^4*e^4 + (15*c^2*d^4
*e^4 + d^4*e^4)*x^4 + 4*(5*c^3*d^3*e^4 + c*d^3*e^4)*x^3 + 3*(5*c^4*d^2*e^4
+ 2*c^2*d^2*e^4)*x^2 + 2*(3*c^5*d*e^4 + 2*c^3*d*e^4)*x)*sqrt(d^2*x^2 + 2*c*
d*x + c^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^3}{{\left(c\,e+d\,e\,x\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^3/(c*e + d*e*x)^4,x)
```

```
[Out] int((a + b*asinh(c + d*x))^3/(c*e + d*e*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{a^3}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx+\int\frac{b^3\operatorname{asinh}^3(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx+\int\frac{3ab^2\operatorname{asinh}^2(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx+\int\frac{3a^2b}{c^4+4c^3dx+6}}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**3/(d*e*x+c*e)**4,x)
```

```
[Out] (Integral(a**3/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4
*x**4), x) + Integral(b**3*asinh(c + d*x)**3/(c**4 + 4*c**3*d*x + 6*c**2*d*
*2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x) + Integral(3*a*b**2*asinh(c + d*x)
**2/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x)
+ Integral(3*a**2*b*asinh(c + d*x)/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 +
4*c*d**3*x**3 + d**4*x**4), x))/e**4
```

3.146 $\int (ce+dex)^m \left(a+b\sinh^{-1}(c+dx)\right)^4 \, dx$

Optimal. Leaf size=87

$$\frac{(e(c+dx))^{m+1}\left(a+b\sinh^{-1}(c+dx)\right)^4}{de(m+1)}-\frac{4b\text{Int}\left(\frac{(e(c+dx))^{m+1}\left(a+b\sinh^{-1}(c+dx)\right)^3}{\sqrt{(c+dx)^2+1}},x\right)}{e(m+1)}$$

```
[Out] (e*(d*x+c))^(1+m)*(a+b*arcsinh(d*x+c))^4/d/e/(1+m)-4*b*Unintegrable((e*(d*x
+c))^(1+m)*(a+b*arcsinh(d*x+c))^3/(1+(d*x+c)^2)^(1/2),x)/e/(1+m)
```

Rubi [A] time = 0.18, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (ce+dex)^m \left(a+b\sinh^{-1}(c+dx)\right)^4 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(c*e + d*e*x)^m*(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] ((e*(c + d*x))^(1 + m)*(a + b*ArcSinh[c + d*x])^4)/(d*e*(1 + m)) - (4*b*Def
er[Subst][Defer[Int][((e*x)^(1 + m)*(a + b*ArcSinh[x])^3)/Sqrt[1 + x^2], x]
, x, c + d*x])/(d*e*(1 + m))
```

Rubi steps

$$\int (ce+dex)^m \left(a+b\sinh^{-1}(c+dx)\right)^4 \, dx = \frac{\text{Subst}\left(\int (ex)^m \left(a+b\sinh^{-1}(x)\right)^4 \, dx, x, c+dx\right)}{d}$$

$$= \frac{(e(c+dx))^{1+m}\left(a+b\sinh^{-1}(c+dx)\right)^4}{de(1+m)} - \frac{(4b)\,\text{Subst}\left(\int \frac{(ex)^{1+m}(a+b\,\text{s}}{\sqrt{1+}}\right.}{de(1+}$$

Mathematica [A] time = 1.92, size = 0, normalized size = 0.00

$$\int (ce+dex)^m \left(a+b\sinh^{-1}(c+dx)\right)^4 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c*e + d*e*x)^m*(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] Integrate[(c*e + d*e*x)^m*(a + b*ArcSinh[c + d*x])^4, x]
```

fricas [A] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^4 \operatorname{arsinh}(dx+c)^4 + 4\,ab^3 \operatorname{arsinh}(dx+c)^3 + 6\,a^2b^2 \operatorname{arsinh}(dx+c)^2 + 4\,a^3b \operatorname{arsinh}(dx+c) + a^4\right)(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral((b^4*arcsinh(d*x + c)^4 + 4*a*b^3*arcsinh(d*x + c)^3 + 6*a^2*b^2*a
rcsinh(d*x + c)^2 + 4*a^3*b*arcsinh(d*x + c) + a^4)*(d*e*x + c*e)^m, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\operatorname{arsinh}(dx+c)+a)^4(dex+ce)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^4,x, algorithm="giac")
[Out] integrate((b*arcsinh(d*x + c) + a)^4*(d*e*x + c*e)^m, x)
```

maple [A] time = 2.04, size = 0, normalized size = 0.00

$$\int (dex+ce)^m\,(a+b\operatorname{arcsinh}(dx+c))^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^4,x)
[Out] int((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^4,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^4,x, algorithm="maxima")
[Out] (b^4*d*e^m*x + b^4*c*e^m)*(d*x + c)^m*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x
+ c^2 + 1))^4/(d*(m + 1)) + (d*e*x + c*e)^(m + 1)*a^4/(d*e*(m + 1)) + integ
rate(-2*(2*((b^4*c^2*e^m - (c^2*e^m*(m + 1) + e^m*(m + 1))*a*b^3 - (a*b^3*d
^2*e^m*(m + 1) - b^4*d^2*e^m)*x^2 - 2*(a*b^3*c*d*e^m*(m + 1) - b^4*c*d*e^m)
*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(d*x + c)^m - ((c^3*e^m*(m + 1) + c*e
^m*(m + 1))*a*b^3 - (c^3*e^m + c*e^m)*b^4 + (a*b^3*d^3*e^m*(m + 1) - b^4*d^
3*e^m)*x^3 + 3*(a*b^3*c*d^2*e^m*(m + 1) - b^4*c*d^2*e^m)*x^2 + ((3*c^2*d*e^
m*(m + 1) + d*e^m*(m + 1))*a*b^3 - (3*c^2*d*e^m + d*e^m)*b^4)*x)*(d*x + c)^
m)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3 - 3*((a^2*b^2*d^2*e^m
*(m + 1)*x^2 + 2*a^2*b^2*c*d*e^m*(m + 1)*x + (c^2*e^m*(m + 1) + e^m*(m + 1)
)*a^2*b^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(d*x + c)^m + (a^2*b^2*d^3*e^m
*(m + 1)*x^3 + 3*a^2*b^2*c*d^2*e^m*(m + 1)*x^2 + (3*c^2*d*e^m*(m + 1) + d*e
^m*(m + 1))*a^2*b^2*x + (c^3*e^m*(m + 1) + c*e^m*(m + 1))*a^2*b^2)*(d*x + c
)^m)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 - 2*((a^3*b*d^2*e^m
*(m + 1)*x^2 + 2*a^3*b*c*d*e^m*(m + 1)*x + (c^2*e^m*(m + 1) + e^m*(m + 1))*
a^3*b)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(d*x + c)^m + (a^3*b*d^3*e^m*(m +
1)*x^3 + 3*a^3*b*c*d^2*e^m*(m + 1)*x^2 + (3*c^2*d*e^m*(m + 1) + d*e^m*(m +
1))*a^3*b*x + (c^3*e^m*(m + 1) + c*e^m*(m + 1))*a^3*b)*(d*x + c)^m)*log(d*x
 + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)))/(d^3*(m + 1)*x^3 + 3*c*d^2*(m +
1)*x^2 + c^3*(m + 1) + c*(m + 1) + (3*c^2*d*(m + 1) + d*(m + 1))*x + (d^2*(
m + 1)*x^2 + 2*c*d*(m + 1)*x + c^2*(m + 1) + m + 1)*sqrt(d^2*x^2 + 2*c*d*x
+ c^2 + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e+d\,e\,x)^m\,(a+b\operatorname{asinh}(c+d\,x))^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^m*(a + b*asinh(c + d*x))^4,x)
[Out] int((c*e + d*e*x)^m*(a + b*asinh(c + d*x))^4, x)
```

sympy [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int (e(c+dx))^m (a+b\,\mathrm{asinh}(c+dx))^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**m*(a+b*asinh(d*x+c))**4,x)
```

```
[Out] Integral((e*(c + d*x))**m*(a + b*asinh(c + d*x))**4, x)
```

3.147 $\int (ce+dex)^3 \left(a+b\sinh^{-1}(c+dx)\right)^4 dx$

Optimal. Leaf size=349

$$-\frac{3b^3e^3(c+dx)^3\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)}{32d}+\frac{45b^3e^3(c+dx)\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)}{64d}+\frac{3b}{}$$

```
[Out] -45/128*b^4*e^3*(d*x+c)^2/d+3/128*b^4*e^3*(d*x+c)^4/d-45/128*b^2*e^3*(a+b*a
rcsinh(d*x+c))^2/d-9/16*b^2*e^3*(d*x+c)^2*(a+b*arcsinh(d*x+c))^2/d+3/16*b^2
*e^3*(d*x+c)^4*(a+b*arcsinh(d*x+c))^2/d-3/32*e^3*(a+b*arcsinh(d*x+c))^4/d+1
/4*e^3*(d*x+c)^4*(a+b*arcsinh(d*x+c))^4/d+45/64*b^3*e^3*(d*x+c)*(a+b*arcsin
h(d*x+c))*(1+(d*x+c)^2)^(1/2)/d-3/32*b^3*e^3*(d*x+c)^3*(a+b*arcsinh(d*x+c))
*(1+(d*x+c)^2)^(1/2)/d+3/8*b*e^3*(d*x+c)*(a+b*arcsinh(d*x+c))^3*(1+(d*x+c)^
2)^(1/2)/d-1/4*b*e^3*(d*x+c)^3*(a+b*arcsinh(d*x+c))^3*(1+(d*x+c)^2)^(1/2)/d
```

Rubi [A] time = 0.67, antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {5865, 12, 5661, 5758, 5675, 30}

$$-\frac{3b^3e^3(c+dx)^3\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)}{32d}+\frac{45b^3e^3(c+dx)\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)}{64d}+\frac{3b}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3*(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] (-45*b^4*e^3*(c + d*x)^2)/(128*d) + (3*b^4*e^3*(c + d*x)^4)/(128*d) + (45*b
^3*e^3*(c + d*x)*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x]))/(64*d) - (
3*b^3*e^3*(c + d*x)^3*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x]))/(32*d
) - (45*b^2*e^3*(a + b*ArcSinh[c + d*x])^2)/(128*d) - (9*b^2*e^3*(c + d*x)^
2*(a + b*ArcSinh[c + d*x])^2)/(16*d) + (3*b^2*e^3*(c + d*x)^4*(a + b*ArcSin
h[c + d*x])^2)/(16*d) + (3*b*e^3*(c + d*x)*Sqrt[1 + (c + d*x)^2]*(a + b*Arc
Sinh[c + d*x])^3)/(8*d) - (b*e^3*(c + d*x)^3*Sqrt[1 + (c + d*x)^2]*(a + b*A
rcSinh[c + d*x])^3)/(4*d) - (3*e^3*(a + b*ArcSinh[c + d*x])^4)/(32*d) + (e^
3*(c + d*x)^4*(a + b*ArcSinh[c + d*x])^4)/(4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^3 \left(a+b\sinh^{-1}(c+dx)\right)^4 dx = \frac{\text{Subst}\left(\int e^3x^3\left(a+b\sinh^{-1}(x)\right)^4 dx, x, c+dx\right)}{d}$$

$$= \frac{e^3\,\text{Subst}\left(\int x^3\left(a+b\sinh^{-1}(x)\right)^4 dx, x, c+dx\right)}{d}$$

$$= \frac{e^3(c+dx)^4\left(a+b\sinh^{-1}(c+dx)\right)^4}{4d} - \frac{\left(be^3\right)\text{Subst}\left(\int \frac{x^4\left(a+b\sinh^{-1}(x)\cdots\right.}{\sqrt{1+x^2}}\cdots\right.}{d}\cdots$$

$$= -\frac{be^3(c+dx)^3\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{4d} + \frac{e^3(c+dx)^4\cdots}{\cdots}$$

$$= \frac{3b^2e^3(c+dx)^4\left(a+b\sinh^{-1}(c+dx)\right)^2}{16d} + \frac{3be^3(c+dx)\sqrt{1+(c+dx\cdots}}{8\cdots}$$

$$= -\frac{3b^3e^3(c+dx)^3\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)}{32d} - \frac{9b^2e^3(c+\cdots}{\cdots}$$

$$= \frac{3b^4e^3(c+dx)^4}{128d} + \frac{45b^3e^3(c+dx)\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+\cdots\right.}{64d}\cdots$$

$$= -\frac{45b^4e^3(c+dx)^2}{128d} + \frac{3b^4e^3(c+dx)^4}{128d} + \frac{45b^3e^3(c+dx)\sqrt{1+(c+dx\cdots}}{64d}\cdots$$

Mathematica [A] time = 0.64, size = 475, normalized size = 1.36

$$\frac{e^3\left(-9b^2\left(8a^2+5b^2\right)(c+dx)^2+2ab\sqrt{(c+dx)^2+1}\,(c+dx)\left(-2\left(8a^2+3b^2\right)(c+dx)^2+24a^2+45b^2\right)+3b^2\sin\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^3*(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] (e^3*(-9*b^2*(8*a^2 + 5*b^2)*(c + d*x)^2 + (32*a^4 + 24*a^2*b^2 + 3*b^4)*(c
 + d*x)^4 + 2*a*b*(c + d*x)*Sqrt[1 + (c + d*x)^2]*(24*a^2 + 45*b^2 - 2*(8*a
^2 + 3*b^2)*(c + d*x)^2) - 6*a*b*(8*a^2 + 15*b^2)*ArcSinh[c + d*x] + 2*b*(c
 + d*x)*(-72*a*b^2*(c + d*x) + 64*a^3*(c + d*x)^3 + 24*a*b^2*(c + d*x)^3 +
```

```
72*a^2*b*Sqrt[1 + (c + d*x)^2] + 45*b^3*Sqrt[1 + (c + d*x)^2] - 48*a^2*b*(c
 + d*x)^2*Sqrt[1 + (c + d*x)^2] - 6*b^3*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])*
ArcSinh[c + d*x] + 3*b^2*(-24*a^2 - 15*b^2 - 24*b^2*(c + d*x)^2 + 64*a^2*(c
 + d*x)^4 + 8*b^2*(c + d*x)^4 + 48*a*b*(c + d*x)*Sqrt[1 + (c + d*x)^2] - 32
*a*b*(c + d*x)^3*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x]^2 + 16*b^3*(-3*a +
 8*a*(c + d*x)^4 + 3*b*(c + d*x)*Sqrt[1 + (c + d*x)^2] - 2*b*(c + d*x)^3*Sq
rt[1 + (c + d*x)^2])*ArcSinh[c + d*x]^3 + 4*b^4*(-3 + 8*(c + d*x)^4)*ArcSin
h[c + d*x]^4))/(128*d)
```

fricas [B] time = 0.59, size = 1241, normalized size = 3.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/128*((32*a^4 + 24*a^2*b^2 + 3*b^4)*d^4*e^3*x^4 + 4*(32*a^4 + 24*a^2*b^2 +
 3*b^4)*c*d^3*e^3*x^3 - 3*(24*a^2*b^2 + 15*b^4 - 2*(32*a^4 + 24*a^2*b^2 + 3
*b^4)*c^2)*d^2*e^3*x^2 + 2*(2*(32*a^4 + 24*a^2*b^2 + 3*b^4)*c^3 - 9*(8*a^2*
b^2 + 5*b^4)*c)*d*e^3*x + 4*(8*b^4*d^4*e^3*x^4 + 32*b^4*c*d^3*e^3*x^3 + 48*
b^4*c^2*d^2*e^3*x^2 + 32*b^4*c^3*d*e^3*x + (8*b^4*c^4 - 3*b^4)*e^3)*log(d*x
 + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^4 + 16*(8*a*b^3*d^4*e^3*x^4 + 32*
a*b^3*c*d^3*e^3*x^3 + 48*a*b^3*c^2*d^2*e^3*x^2 + 32*a*b^3*c^3*d*e^3*x + (8*
a*b^3*c^4 - 3*a*b^3)*e^3 - (2*b^4*d^3*e^3*x^3 + 6*b^4*c*d^2*e^3*x^2 + 3*(2*
b^4*c^2 - b^4)*d*e^3*x + (2*b^4*c^3 - 3*b^4*c)*e^3)*sqrt(d^2*x^2 + 2*c*d*x
+ c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3 + 3*(8*(8*a^
2*b^2 + b^4)*d^4*e^3*x^4 + 32*(8*a^2*b^2 + b^4)*c*d^3*e^3*x^3 - 24*(b^4 - 2
*(8*a^2*b^2 + b^4)*c^2)*d^2*e^3*x^2 - 16*(3*b^4*c - 2*(8*a^2*b^2 + b^4)*c^3
)*d*e^3*x - (24*b^4*c^2 - 8*(8*a^2*b^2 + b^4)*c^4 + 24*a^2*b^2 + 15*b^4)*e^
3 - 16*(2*a*b^3*d^3*e^3*x^3 + 6*a*b^3*c*d^2*e^3*x^2 + 3*(2*a*b^3*c^2 - a*b^
3)*d*e^3*x + (2*a*b^3*c^3 - 3*a*b^3*c)*e^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 +
1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 + 2*(8*(8*a^3*b + 3*
a*b^3)*d^4*e^3*x^4 + 32*(8*a^3*b + 3*a*b^3)*c*d^3*e^3*x^3 - 24*(3*a*b^3 - 2
*(8*a^3*b + 3*a*b^3)*c^2)*d^2*e^3*x^2 - 16*(9*a*b^3*c - 2*(8*a^3*b + 3*a*b^
3)*c^3)*d*e^3*x - (72*a*b^3*c^2 - 8*(8*a^3*b + 3*a*b^3)*c^4 + 24*a^3*b + 45
*a*b^3)*e^3 - 3*(2*(8*a^2*b^2 + b^4)*d^3*e^3*x^3 + 6*(8*a^2*b^2 + b^4)*c*d^
2*e^3*x^2 - 3*(8*a^2*b^2 + 5*b^4 - 2*(8*a^2*b^2 + b^4)*c^2)*d*e^3*x + (2*(8
*a^2*b^2 + b^4)*c^3 - 3*(8*a^2*b^2 + 5*b^4)*c)*e^3)*sqrt(d^2*x^2 + 2*c*d*x
+ c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - 2*(2*(8*a^3*
b + 3*a*b^3)*d^3*e^3*x^3 + 6*(8*a^3*b + 3*a*b^3)*c*d^2*e^3*x^2 - 3*(8*a^3*b
 + 15*a*b^3 - 2*(8*a^3*b + 3*a*b^3)*c^2)*d*e^3*x + (2*(8*a^3*b + 3*a*b^3)*c
^3 - 3*(8*a^3*b + 15*a*b^3)*c)*e^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/d
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex + ce)^3 (b \operatorname{arsinh}(dx + c) + a)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^3*(b*arcsinh(d*x + c) + a)^4, x)
```

maple [A] time = 0.05, size = 573, normalized size = 1.64

$$\frac{\frac{(dx+c)^4 e^3 a^4}{4} + e^3 b^4 \left(\frac{(dx+c)^4 \operatorname{arcsinh}(dx+c)^4}{4} - \frac{(dx+c)^3 \operatorname{arcsinh}(dx+c)^3 \sqrt{1+(dx+c)^2}}{4} + \frac{3 \operatorname{arcsinh}(dx+c)^3 (dx+c) \sqrt{1+(dx+c)^2}}{8} - \frac{3 \operatorname{arcsinh}(dx+c}{32}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^4,x)
```

```
[Out] 1/d*(1/4*(d*x+c)^4*e^3*a^4+e^3*b^4*(1/4*(d*x+c)^4*arcsinh(d*x+c)^4-1/4*(d*x
+c)^3*arcsinh(d*x+c)^3*(1+(d*x+c)^2)^(1/2)+3/8*arcsinh(d*x+c)^3*(d*x+c)*(1+
(d*x+c)^2)^(1/2)-3/32*arcsinh(d*x+c)^4+3/16*(d*x+c)^4*arcsinh(d*x+c)^2-3/32
*(d*x+c)^3*arcsinh(d*x+c)*(1+(d*x+c)^2)^(1/2)+45/64*arcsinh(d*x+c)*(1+(d*x+
c)^2)^(1/2)*(d*x+c)+27/128*arcsinh(d*x+c)^2+3/128*(d*x+c)^4-45/128*(d*x+c)^
2-45/128-9/16*arcsinh(d*x+c)^2*(1+(d*x+c)^2))+4*e^3*a*b^3*(1/4*(d*x+c)^4*ar
csinh(d*x+c)^3-3/16*(d*x+c)^3*arcsinh(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+9/32*arc
sinh(d*x+c)^2*(1+(d*x+c)^2)^(1/2)*(d*x+c)-3/32*arcsinh(d*x+c)^3+3/32*(d*x+c
)^4*arcsinh(d*x+c)-3/128*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)+45/256*(d*x+c)*(1+(d
*x+c)^2)^(1/2)+27/256*arcsinh(d*x+c)-9/32*arcsinh(d*x+c)*(1+(d*x+c)^2))+6*e
^3*a^2*b^2*(1/4*(d*x+c)^4*arcsinh(d*x+c)^2-1/8*(d*x+c)^3*arcsinh(d*x+c)*(1+
(d*x+c)^2)^(1/2)+3/16*arcsinh(d*x+c)*(1+(d*x+c)^2)^(1/2)*(d*x+c)-3/32*arcsi
nh(d*x+c)^2+1/32*(d*x+c)^4-3/32*(d*x+c)^2-3/32)+4*e^3*a^3*b*(1/4*(d*x+c)^4*
arcsinh(d*x+c)-1/16*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)+3/32*(d*x+c)*(1+(d*x+c)^2
)^(1/2)-3/32*arcsinh(d*x+c)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/4*a^4*d^3*e^3*x^4 + a^4*c*d^2*e^3*x^3 + 3/2*a^4*c^2*d*e^3*x^2 + 3*(2*x^2*
arcsinh(d*x + c) - d*(3*c^2*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^
2 + 1)*d^2))/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*x/d^2 - (c^2 + 1)*arcs
inh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 - 3*sqrt(d^2*x^
2 + 2*c*d*x + c^2 + 1)*c/d^3))*a^3*b*c^2*d*e^3 + 2/3*(6*x^3*arcsinh(d*x + c
) - d*(2*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*x^2/d^2 - 15*c^3*arcsinh(2*(d^2*
x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^4 - 5*sqrt(d^2*x^2 + 2*c*d*x
 + c^2 + 1)*c*x/d^3 + 9*(c^2 + 1)*c*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2
 + 4*(c^2 + 1)*d^2))/d^4 + 15*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c^2/d^4 - 4
*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)/d^4))*a^3*b*c*d^2*e^3 + 1/24*(
24*x^4*arcsinh(d*x + c) - (6*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*x^3/d^2 - 14
*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c*x^2/d^3 + 105*c^4*arcsinh(2*(d^2*x + c
*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^5 + 35*sqrt(d^2*x^2 + 2*c*d*x + c
^2 + 1)*c^2*x/d^4 - 90*(c^2 + 1)*c^2*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^
2 + 4*(c^2 + 1)*d^2))/d^5 - 105*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c^3/d^5 -
 9*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)*x/d^4 + 9*(c^2 + 1)^2*arcsin
h(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^5 + 55*sqrt(d^2*x^2
 + 2*c*d*x + c^2 + 1)*(c^2 + 1)*c/d^5)*d)*a^3*b*d^3*e^3 + a^4*c^3*e^3*x + 4
*((d*x + c)*arcsinh(d*x + c) - sqrt((d*x + c)^2 + 1))*a^3*b*c^3*e^3/d + 1/4
*(b^4*d^3*e^3*x^4 + 4*b^4*c*d^2*e^3*x^3 + 6*b^4*c^2*d*e^3*x^2 + 4*b^4*c^3*e
^3*x)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^4 + integrate((((4*a
*b^3*d^6*e^3 - b^4*d^6*e^3)*x^6 + 6*(4*a*b^3*c*d^5*e^3 - b^4*c*d^5*e^3)*x^5
 + 4*(c^6*e^3 + c^4*e^3)*a*b^3 + (4*(15*c^2*d^4*e^3 + d^4*e^3)*a*b^3 - (15*
c^2*d^4*e^3 + d^4*e^3)*b^4)*x^4 + 4*(4*(5*c^3*d^3*e^3 + c*d^3*e^3)*a*b^3 -
(5*c^3*d^3*e^3 + c*d^3*e^3)*b^4)*x^3 + 2*(6*(5*c^4*d^2*e^3 + 2*c^2*d^2*e^3)
*a*b^3 - (7*c^4*d^2*e^3 + 3*c^2*d^2*e^3)*b^4)*x^2 + 4*(2*(3*c^5*d*e^3 + 2*c
^3*d*e^3)*a*b^3 - (c^5*d*e^3 + c^3*d*e^3)*b^4)*x + ((4*a*b^3*d^5*e^3 - b^4*
d^5*e^3)*x^5 + 4*(c^5*e^3 + c^3*e^3)*a*b^3 + 5*(4*a*b^3*c*d^4*e^3 - b^4*c*d
^4*e^3)*x^4 - 2*(5*b^4*c^2*d^3*e^3 - 2*(10*c^2*d^3*e^3 + d^3*e^3)*a*b^3)*x^
3 - 2*(5*b^4*c^3*d^2*e^3 - 2*(10*c^3*d^2*e^3 + 3*c*d^2*e^3)*a*b^3)*x^2 - 4*
(b^4*c^4*d*e^3 - (5*c^4*d*e^3 + 3*c^2*d*e^3)*a*b^3)*x)*sqrt(d^2*x^2 + 2*c*d
*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3 + 6*(a^2*
b^2*d^6*e^3*x^6 + 6*a^2*b^2*c*d^5*e^3*x^5 + (15*c^2*d^4*e^3 + d^4*e^3)*a^2*
b^2*x^4 + 4*(5*c^3*d^3*e^3 + c*d^3*e^3)*a^2*b^2*x^3 + 3*(5*c^4*d^2*e^3 + 2*
c^2*d^2*e^3)*a^2*b^2*x^2 + 2*(3*c^5*d*e^3 + 2*c^3*d*e^3)*a^2*b^2*x + (c^6*e
```

```
^3 + c^4*e^3)*a^2*b^2 + (a^2*b^2*d^5*e^3*x^5 + 5*a^2*b^2*c*d^4*e^3*x^4 + (1
0*c^2*d^3*e^3 + d^3*e^3)*a^2*b^2*x^3 + (10*c^3*d^2*e^3 + 3*c*d^2*e^3)*a^2*b
^2*x^2 + (5*c^4*d*e^3 + 3*c^2*d*e^3)*a^2*b^2*x + (c^5*e^3 + c^3*e^3)*a^2*b^
2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x
+ c^2 + 1))^2)/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (3*c^2*d + d)*x + (d^2*x^2 +
2*c*d*x + c^2 + 1)^(3/2) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (ce + dex)^3 (a + b\,\mathrm{asinh}(c + dx))^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3*(a + b*asinh(c + d*x))^4,x)
```

```
[Out] int((c*e + d*e*x)^3*(a + b*asinh(c + d*x))^4, x)
```

sympy [A] time = 20.60, size = 2876, normalized size = 8.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3*(a+b*asinh(d*x+c))**4,x)
```

```
[Out] Piecewise((a**4*c**3*e**3*x + 3*a**4*c**2*d*e**3*x**2/2 + a**4*c*d**2*e**3*
x**3 + a**4*d**3*e**3*x**4/4 + a**3*b*c**4*e**3*asinh(c + d*x)/d + 4*a**3*b
*c**3*e**3*x*asinh(c + d*x) - a**3*b*c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*x
**2 + 1)/(4*d) + 6*a**3*b*c**2*d*e**3*x**2*asinh(c + d*x) - 3*a**3*b*c**2*e
**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/4 + 4*a**3*b*c*d**2*e**3*x**3*as
inh(c + d*x) - 3*a**3*b*c*d*e**3*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/
4 + 3*a**3*b*c*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(8*d) + a**3*b*d**
3*e**3*x**4*asinh(c + d*x) - a**3*b*d**2*e**3*x**3*sqrt(c**2 + 2*c*d*x + d*
*2*x**2 + 1)/4 + 3*a**3*b*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/8 - 3
*a**3*b*e**3*asinh(c + d*x)/(8*d) + 3*a**2*b**2*c**4*e**3*asinh(c + d*x)**2
/(2*d) + 6*a**2*b**2*c**3*e**3*x*asinh(c + d*x)**2 + 3*a**2*b**2*c**3*e**3*
x/4 - 3*a**2*b**2*c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c +
d*x)/(4*d) + 9*a**2*b**2*c**2*d*e**3*x**2*asinh(c + d*x)**2 + 9*a**2*b**2*c
**2*d*e**3*x**2/8 - 9*a**2*b**2*c**2*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2
 + 1)*asinh(c + d*x)/4 + 6*a**2*b**2*c*d**2*e**3*x**3*asinh(c + d*x)**2 + 3
*a**2*b**2*c*d**2*e**3*x**3/4 - 9*a**2*b**2*c*d*e**3*x**2*sqrt(c**2 + 2*c*d
*x + d**2*x**2 + 1)*asinh(c + d*x)/4 - 9*a**2*b**2*c*e**3*x/8 + 9*a**2*b**2
*c*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/(8*d) + 3*a**2*
b**2*d**3*e**3*x**4*asinh(c + d*x)**2/2 + 3*a**2*b**2*d**3*e**3*x**4/16 - 3
*a**2*b**2*d**2*e**3*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*
x)/4 - 9*a**2*b**2*d*e**3*x**2/16 + 9*a**2*b**2*e**3*x*sqrt(c**2 + 2*c*d*x
+ d**2*x**2 + 1)*asinh(c + d*x)/8 - 9*a**2*b**2*e**3*asinh(c + d*x)**2/(16*
d) + a*b**3*c**4*e**3*asinh(c + d*x)**3/d + 3*a*b**3*c**4*e**3*asinh(c + d*
x)/(8*d) + 4*a*b**3*c**3*e**3*x*asinh(c + d*x)**3 + 3*a*b**3*c**3*e**3*x*as
inh(c + d*x)/2 - 3*a*b**3*c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*as
inh(c + d*x)**2/(4*d) - 3*a*b**3*c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2
+ 1)/(32*d) + 6*a*b**3*c**2*d*e**3*x**2*asinh(c + d*x)**3 + 9*a*b**3*c**2*d
*e**3*x**2*asinh(c + d*x)/4 - 9*a*b**3*c**2*e**3*x*sqrt(c**2 + 2*c*d*x + d*
*2*x**2 + 1)*asinh(c + d*x)**2/4 - 9*a*b**3*c**2*e**3*x*sqrt(c**2 + 2*c*d*x
 + d**2*x**2 + 1)/32 - 9*a*b**3*c**2*e**3*asinh(c + d*x)/(8*d) + 4*a*b**3*c
*d**2*e**3*x**3*asinh(c + d*x)**3 + 3*a*b**3*c*d**2*e**3*x**3*asinh(c + d*x
)/2 - 9*a*b**3*c*d*e**3*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c +
 d*x)**2/4 - 9*a*b**3*c*d*e**3*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/32
 - 9*a*b**3*c*e**3*x*asinh(c + d*x)/4 + 9*a*b**3*c*e**3*sqrt(c**2 + 2*c*d*x
 + d**2*x**2 + 1)*asinh(c + d*x)**2/(8*d) + 45*a*b**3*c*e**3*sqrt(c**2 + 2*
c*d*x + d**2*x**2 + 1)/(64*d) + a*b**3*d**3*e**3*x**4*asinh(c + d*x)**3 + 3
```

```
*a*b**3*d**3*e**3*x**4*asinh(c + d*x)/8 - 3*a*b**3*d**2*e**3*x**3*sqrt(c**2
 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/4 - 3*a*b**3*d**2*e**3*x**3*s
qrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/32 - 9*a*b**3*d*e**3*x**2*asinh(c + d*x
)/8 + 9*a*b**3*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**
2/8 + 45*a*b**3*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/64 - 3*a*b**3*e
**3*asinh(c + d*x)**3/(8*d) - 45*a*b**3*e**3*asinh(c + d*x)/(64*d) + b**4*c
**4*e**3*asinh(c + d*x)**4/(4*d) + 3*b**4*c**4*e**3*asinh(c + d*x)**2/(16*d
) + b**4*c**3*e**3*x*asinh(c + d*x)**4 + 3*b**4*c**3*e**3*x*asinh(c + d*x)*
*2/4 + 3*b**4*c**3*e**3*x/32 - b**4*c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*x*
*2 + 1)*asinh(c + d*x)**3/(4*d) - 3*b**4*c**3*e**3*sqrt(c**2 + 2*c*d*x + d*
*2*x**2 + 1)*asinh(c + d*x)/(32*d) + 3*b**4*c**2*d*e**3*x**2*asinh(c + d*x)
**4/2 + 9*b**4*c**2*d*e**3*x**2*asinh(c + d*x)**2/8 + 9*b**4*c**2*d*e**3*x*
*2/64 - 3*b**4*c**2*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d
*x)**3/4 - 9*b**4*c**2*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c
+ d*x)/32 - 9*b**4*c**2*e**3*asinh(c + d*x)**2/(16*d) + b**4*c*d**2*e**3*x*
*3*asinh(c + d*x)**4 + 3*b**4*c*d**2*e**3*x**3*asinh(c + d*x)**2/4 + 3*b**4
*c*d**2*e**3*x**3/32 - 3*b**4*c*d*e**3*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2
 + 1)*asinh(c + d*x)**3/4 - 9*b**4*c*d*e**3*x**2*sqrt(c**2 + 2*c*d*x + d**2
*x**2 + 1)*asinh(c + d*x)/32 - 9*b**4*c*e**3*x*asinh(c + d*x)**2/8 - 45*b**
4*c*e**3*x/64 + 3*b**4*c*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c
+ d*x)**3/(8*d) + 45*b**4*c*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh
(c + d*x)/(64*d) + b**4*d**3*e**3*x**4*asinh(c + d*x)**4/4 + 3*b**4*d**3*e*
*3*x**4*asinh(c + d*x)**2/16 + 3*b**4*d**3*e**3*x**4/128 - b**4*d**2*e**3*x
**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**3/4 - 3*b**4*d**2*
e**3*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/32 - 9*b**4*d
*e**3*x**2*asinh(c + d*x)**2/16 - 45*b**4*d*e**3*x**2/128 + 3*b**4*e**3*x*s
qrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**3/8 + 45*b**4*e**3*x*sq
rt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/64 - 3*b**4*e**3*asinh(c
+ d*x)**4/(32*d) - 45*b**4*e**3*asinh(c + d*x)**2/(128*d), Ne(d, 0)), (c**3
*e**3*x*(a + b*asinh(c))**4, True))
```

3.148 $\int (ce+dex)^2 \left(a+b\sinh^{-1}(c+dx)\right)^4 \, dx$

Optimal. Leaf size=281

$$\frac{160b^3e^2\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)}{27d}-\frac{8b^3e^2(c+dx)^2\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)}{27d}+\frac{4b^2e^2(c+d}{}$$

```
[Out] -160/27*b^4*e^2*x+8/81*b^4*e^2*(d*x+c)^3/d-8/3*b^2*e^2*(d*x+c)*(a+b*arcsinh
(d*x+c))^2/d+4/9*b^2*e^2*(d*x+c)^3*(a+b*arcsinh(d*x+c))^2/d+1/3*e^2*(d*x+c)
^3*(a+b*arcsinh(d*x+c))^4/d+160/27*b^3*e^2*(a+b*arcsinh(d*x+c))*(1+(d*x+c)^
2)^(1/2)/d-8/27*b^3*e^2*(d*x+c)^2*(a+b*arcsinh(d*x+c))*(1+(d*x+c)^2)^(1/2)/
d+8/9*b*e^2*(a+b*arcsinh(d*x+c))^3*(1+(d*x+c)^2)^(1/2)/d-4/9*b*e^2*(d*x+c)^
2*(a+b*arcsinh(d*x+c))^3*(1+(d*x+c)^2)^(1/2)/d
```

Rubi [A] time = 0.48, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {5865, 12, 5661, 5758, 5717, 5653, 8, 30}

$$\frac{160b^3e^2\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)}{27d}-\frac{8b^3e^2(c+dx)^2\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)}{27d}+\frac{4b^2e^2(c+d}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2*(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] (-160*b^4*e^2*x)/27 + (8*b^4*e^2*(c + d*x)^3)/(81*d) + (160*b^3*e^2*Sqrt[1
+ (c + d*x)^2]*(a + b*ArcSinh[c + d*x]))/(27*d) - (8*b^3*e^2*(c + d*x)^2*Sq
rt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x]))/(27*d) - (8*b^2*e^2*(c + d*x)
*(a + b*ArcSinh[c + d*x])^2)/(3*d) + (4*b^2*e^2*(c + d*x)^3*(a + b*ArcSinh[
c + d*x])^2)/(9*d) + (8*b*e^2*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x]
)^3)/(9*d) - (4*b*e^2*(c + d*x)^2*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c +
d*x])^3)/(9*d) + (e^2*(c + d*x)^3*(a + b*ArcSinh[c + d*x])^4)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^2 \left(a+b\sinh^{-1}(c+dx)\right)^4 \, dx = \frac{\text{Subst}\left(\int e^2x^2 \left(a+b\sinh^{-1}(x)\right)^4 \, dx, x, c+dx\right)}{d}$$

$$= \frac{e^2\,\text{Subst}\left(\int x^2 \left(a+b\sinh^{-1}(x)\right)^4 \, dx, x, c+dx\right)}{d}$$

$$= \frac{e^2(c+dx)^3 \left(a+b\sinh^{-1}(c+dx)\right)^4}{3d} - \frac{\left(4be^2\right)\text{Subst}\left(\int \frac{x^3\left(a+b\sinh^{-1}(\right.}{\sqrt{1+x^2}}\right.}{3d}$$

$$= -\frac{4be^2(c+dx)^2\sqrt{1+(c+dx)^2} \left(a+b\sinh^{-1}(c+dx)\right)^3}{9d} + \frac{e^2(c+dx}{}$$

$$= \frac{4b^2e^2(c+dx)^3 \left(a+b\sinh^{-1}(c+dx)\right)^2}{9d} + \frac{8be^2\sqrt{1+(c+dx)^2} \left(a+\right.}{9d}$$

$$= -\frac{8b^3e^2(c+dx)^2\sqrt{1+(c+dx)^2} \left(a+b\sinh^{-1}(c+dx)\right)}{27d} - \frac{8b^2e^2(c+}{}$$

$$= \frac{8b^4e^2(c+dx)^3}{81d} + \frac{160b^3e^2\sqrt{1+(c+dx)^2} \left(a+b\sinh^{-1}(c+dx)\right)}{27d} -$$

$$= -\frac{160}{27}b^4e^2x + \frac{8b^4e^2(c+dx)^3}{81d} + \frac{160b^3e^2\sqrt{1+(c+dx)^2} \left(a+b\sinh\right.}{27d}$$

Mathematica [A] time = 0.45, size = 412, normalized size = 1.47

$$\frac{e^2 \left(-24b^2\left(9a^2+20b^2\right)(c+dx)+12ab\sqrt{(c+dx)^2+1}\left(-\left(3a^2+2b^2\right)(c+dx)^2+6a^2+40b^2\right)+18b^2\sinh^{-1}(c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^2*(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] (e^2*(-24*b^2*(9*a^2 + 20*b^2)*(c + d*x) + (27*a^4 + 36*a^2*b^2 + 8*b^4)*(c
 + d*x)^3 + 12*a*b*Sqrt[1 + (c + d*x)^2]*(6*a^2 + 40*b^2 - (3*a^2 + 2*b^2)*
(c + d*x)^2) + 12*b*(-36*a*b^2*(c + d*x) + 9*a^3*(c + d*x)^3 + 6*a*b^2*(c +
 d*x)^3 + 18*a^2*b*Sqrt[1 + (c + d*x)^2] + 40*b^3*Sqrt[1 + (c + d*x)^2] - 9
*a^2*b*(c + d*x)^2*Sqrt[1 + (c + d*x)^2] - 2*b^3*(c + d*x)^2*Sqrt[1 + (c +
d*x)^2])*ArcSinh[c + d*x] + 18*b^2*(-12*b^2*(c + d*x) + 9*a^2*(c + d*x)^3 +
 2*b^2*(c + d*x)^3 + 12*a*b*Sqrt[1 + (c + d*x)^2] - 6*a*b*(c + d*x)^2*Sqrt[
1 + (c + d*x)^2])*ArcSinh[c + d*x]^2 - 36*b^3*(-3*a*(c + d*x)^3 - 2*b*Sqrt[
1 + (c + d*x)^2] + b*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x]^3
+ 27*b^4*(c + d*x)^3*ArcSinh[c + d*x]^4))/(81*d)
```

fricas [B] time = 0.75, size = 900, normalized size = 3.20

$$\left(27\,a^4+36\,a^2b^2+8\,b^4\right)d^3e^2x^3+3\left(27\,a^4+36\,a^2b^2+8\,b^4\right)cd^2e^2x^2-3\left(72\,a^2b^2+160\,b^4-\left(27\,a^4+36\,a^2b^2+8\,b^4\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/81*((27*a^4 + 36*a^2*b^2 + 8*b^4)*d^3*e^2*x^3 + 3*(27*a^4 + 36*a^2*b^2 +
8*b^4)*c*d^2*e^2*x^2 - 3*(72*a^2*b^2 + 160*b^4 - (27*a^4 + 36*a^2*b^2 + 8*b
^4)*c^2)*d*e^2*x + 27*(b^4*d^3*e^2*x^3 + 3*b^4*c*d^2*e^2*x^2 + 3*b^4*c^2*d*
e^2*x + b^4*c^3*e^2)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^4 + 3
6*(3*a*b^3*d^3*e^2*x^3 + 9*a*b^3*c*d^2*e^2*x^2 + 9*a*b^3*c^2*d*e^2*x + 3*a*
b^3*c^3*e^2 - (b^4*d^2*e^2*x^2 + 2*b^4*c*d*e^2*x + (b^4*c^2 - 2*b^4)*e^2)*s
qrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^
2 + 1))^3 + 18*((9*a^2*b^2 + 2*b^4)*d^3*e^2*x^3 + 3*(9*a^2*b^2 + 2*b^4)*c*d
^2*e^2*x^2 - 3*(4*b^4 - (9*a^2*b^2 + 2*b^4)*c^2)*d*e^2*x - (12*b^4*c - (9*a
^2*b^2 + 2*b^4)*c^3)*e^2 - 6*(a*b^3*d^2*e^2*x^2 + 2*a*b^3*c*d*e^2*x + (a*b^
3*c^2 - 2*a*b^3)*e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt
(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 + 12*(3*(3*a^3*b + 2*a*b^3)*d^3*e^2*x^3 +
9*(3*a^3*b + 2*a*b^3)*c*d^2*e^2*x^2 - 9*(4*a*b^3 - (3*a^3*b + 2*a*b^3)*c^2)
*d*e^2*x - 3*(12*a*b^3*c - (3*a^3*b + 2*a*b^3)*c^3)*e^2 - ((9*a^2*b^2 + 2*b
^4)*d^2*e^2*x^2 + 2*(9*a^2*b^2 + 2*b^4)*c*d*e^2*x - (18*a^2*b^2 + 40*b^4 -
(9*a^2*b^2 + 2*b^4)*c^2)*e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x +
c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - 12*((3*a^3*b + 2*a*b^3)*d^2*e^2*x^
2 + 2*(3*a^3*b + 2*a*b^3)*c*d*e^2*x - (6*a^3*b + 40*a*b^3 - (3*a^3*b + 2*a*
b^3)*c^2)*e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/d
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex+ce)^2(b\operatorname{arsinh}(dx+c)+a)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2*(b*arcsinh(d*x + c) + a)^4, x)
```

maple [A] time = 0.05, size = 473, normalized size = 1.68

$$\frac{(dx+c)^3e^2a^4}{3}+e^2b^4\left(\frac{(dx+c)^3\operatorname{arcsinh}(dx+c)^4}{3}+\frac{8\operatorname{arcsinh}(dx+c)^3\sqrt{1+(dx+c)^2}}{9}-\frac{4(dx+c)^2\operatorname{arcsinh}(dx+c)^3\sqrt{1+(dx+c)^2}}{9}-\frac{8(dx+c)\operatorname{arcsinh}(dx+}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^4,x)
```

```
[Out] 1/d*(1/3*(d*x+c)^3*e^2*a^4+e^2*b^4*(1/3*(d*x+c)^3*arcsinh(d*x+c)^4+8/9*arcs
inh(d*x+c)^3*(1+(d*x+c)^2)^(1/2)-4/9*(d*x+c)^2*arcsinh(d*x+c)^3*(1+(d*x+c)^
2)^(1/2)-8/3*(d*x+c)*arcsinh(d*x+c)^2+160/27*arcsinh(d*x+c)*(1+(d*x+c)^2)^(
1/2)-160/27*d*x-160/27*c+4/9*(d*x+c)^3*arcsinh(d*x+c)^2-8/27*arcsinh(d*x+c)
*(1+(d*x+c)^2)^(1/2)*(d*x+c)^2+8/81*(d*x+c)^3)+4*e^2*a*b^3*(1/3*(d*x+c)^3*a
rcsinh(d*x+c)^3+2/3*arcsinh(d*x+c)^2*(1+(d*x+c)^2)^(1/2)-1/3*(d*x+c)^2*arcs
inh(d*x+c)^2*(1+(d*x+c)^2)^(1/2)-4/3*(d*x+c)*arcsinh(d*x+c)+40/27*(1+(d*x+c
)^2)^(1/2)+2/9*(d*x+c)^3*arcsinh(d*x+c)-2/27*(d*x+c)^2*(1+(d*x+c)^2)^(1/2))
+6*e^2*a^2*b^2*(1/3*(d*x+c)^3*arcsinh(d*x+c)^2+4/9*arcsinh(d*x+c)*(1+(d*x+c
)^2)^(1/2)-2/9*arcsinh(d*x+c)*(1+(d*x+c)^2)^(1/2)*(d*x+c)^2-4/9*d*x-4/9*c+2
/27*(d*x+c)^3)+4*e^2*a^3*b*(1/3*(d*x+c)^3*arcsinh(d*x+c)-1/9*(d*x+c)^2*(1+(
d*x+c)^2)^(1/2)+2/9*(1+(d*x+c)^2)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/3*a^4*d^2*e^2*x^3 + a^4*c*d*e^2*x^2 + 2*(2*x^2*arcsinh(d*x + c) - d*(3*c^
2*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 + sqrt(d^
2*x^2 + 2*c*d*x + c^2 + 1)*x/d^2 - (c^2 + 1)*arcsinh(2*(d^2*x + c*d)/sqrt(-
4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c/d
^3))*a^3*b*c*d*e^2 + 2/9*(6*x^3*arcsinh(d*x + c) - d*(2*sqrt(d^2*x^2 + 2*c*
d*x + c^2 + 1)*x^2/d^2 - 15*c^3*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4
*(c^2 + 1)*d^2))/d^4 - 5*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c*x/d^3 + 9*(c^2
 + 1)*c*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^4 + 1
5*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c^2/d^4 - 4*sqrt(d^2*x^2 + 2*c*d*x + c^
2 + 1)*(c^2 + 1)/d^4))*a^3*b*d^2*e^2 + a^4*c^2*e^2*x + 4*((d*x + c)*arcsinh
(d*x + c) - sqrt((d*x + c)^2 + 1))*a^3*b*c^2*e^2/d + 1/3*(b^4*d^2*e^2*x^3 +
 3*b^4*c*d*e^2*x^2 + 3*b^4*c^2*e^2*x)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x
+ c^2 + 1))^4 + integrate(2/3*(2*((3*a*b^3*d^5*e^2 - b^4*d^5*e^2)*x^5 + 3*(
c^5*e^2 + c^3*e^2)*a*b^3 + 5*(3*a*b^3*c*d^4*e^2 - b^4*c*d^4*e^2)*x^4 + (3*(
10*c^2*d^3*e^2 + d^3*e^2)*a*b^3 - (10*c^2*d^3*e^2 + d^3*e^2)*b^4)*x^3 + 3*(
(10*c^3*d^2*e^2 + 3*c*d^2*e^2)*a*b^3 - (3*c^3*d^2*e^2 + c*d^2*e^2)*b^4)*x^2
 + 3*((5*c^4*d*e^2 + 3*c^2*d*e^2)*a*b^3 - (c^4*d*e^2 + c^2*d*e^2)*b^4)*x +
(3*(c^4*e^2 + c^2*e^2)*a*b^3 + (3*a*b^3*d^4*e^2 - b^4*d^4*e^2)*x^4 + 4*(3*a
*b^3*c*d^3*e^2 - b^4*c*d^3*e^2)*x^3 - 3*(2*b^4*c^2*d^2*e^2 - (6*c^2*d^2*e^2
 + d^2*e^2)*a*b^3)*x^2 - 3*(b^4*c^3*d*e^2 - 2*(2*c^3*d*e^2 + c*d*e^2)*a*b^3
)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*
x + c^2 + 1))^3 + 9*(a^2*b^2*d^5*e^2*x^5 + 5*a^2*b^2*c*d^4*e^2*x^4 + (10*c^
2*d^3*e^2 + d^3*e^2)*a^2*b^2*x^3 + (10*c^3*d^2*e^2 + 3*c*d^2*e^2)*a^2*b^2*x
^2 + (5*c^4*d*e^2 + 3*c^2*d*e^2)*a^2*b^2*x + (c^5*e^2 + c^3*e^2)*a^2*b^2 +
(a^2*b^2*d^4*e^2*x^4 + 4*a^2*b^2*c*d^3*e^2*x^3 + (6*c^2*d^2*e^2 + d^2*e^2)*
a^2*b^2*x^2 + 2*(2*c^3*d*e^2 + c*d*e^2)*a^2*b^2*x + (c^4*e^2 + c^2*e^2)*a^2
*b^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d
*x + c^2 + 1))^2)/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (3*c^2*d + d)*x + (d^2*x^2
 + 2*c*d*x + c^2 + 1)^(3/2) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e+d\,e\,x)^2\,(a+b\,\mathrm{asinh}\,(c+d\,x))^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2*(a + b*asinh(c + d*x))^4,x)
```

```
[Out] int((c*e + d*e*x)^2*(a + b*asinh(c + d*x))^4, x)
```

sympy [A] time = 9.65, size = 1889, normalized size = 6.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2*(a+b*asinh(d*x+c))**4,x)
```

```
[Out] Piecewise((a**4*c**2*e**2*x + a**4*c*d*e**2*x**2 + a**4*d**2*e**2*x**3/3 +
4*a**3*b*c**3*e**2*asinh(c + d*x)/(3*d) + 4*a**3*b*c**2*e**2*x*asinh(c + d*
x) - 4*a**3*b*c**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(9*d) + 4*a**3
*b*c*d*e**2*x**2*asinh(c + d*x) - 8*a**3*b*c*e**2*x*sqrt(c**2 + 2*c*d*x + d
**2*x**2 + 1)/9 + 4*a**3*b*d**2*e**2*x**3*asinh(c + d*x)/3 - 4*a**3*b*d*e**
2*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/9 + 8*a**3*b*e**2*sqrt(c**2 + 2
*c*d*x + d**2*x**2 + 1)/(9*d) + 2*a**2*b**2*c**3*e**2*asinh(c + d*x)**2/d +
 6*a**2*b**2*c**2*e**2*x*asinh(c + d*x)**2 + 4*a**2*b**2*c**2*e**2*x/3 - 4*
a**2*b**2*c**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/(3*
d) + 6*a**2*b**2*c*d*e**2*x**2*asinh(c + d*x)**2 + 4*a**2*b**2*c*d*e**2*x**
2/3 - 8*a**2*b**2*c*e**2*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d
*x)/3 + 2*a**2*b**2*d**2*e**2*x**3*asinh(c + d*x)**2 + 4*a**2*b**2*d**2*e**
2*x**3/9 - 4*a**2*b**2*d*e**2*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asi
nh(c + d*x)/3 - 8*a**2*b**2*e**2*x/3 + 8*a**2*b**2*e**2*sqrt(c**2 + 2*c*d*x
 + d**2*x**2 + 1)*asinh(c + d*x)/(3*d) + 4*a*b**3*c**3*e**2*asinh(c + d*x)*
*3/(3*d) + 8*a*b**3*c**3*e**2*asinh(c + d*x)/(9*d) + 4*a*b**3*c**2*e**2*x*a
sinh(c + d*x)**3 + 8*a*b**3*c**2*e**2*x*asinh(c + d*x)/3 - 4*a*b**3*c**2*e*
*2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/(3*d) - 8*a*b**3*
c**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(27*d) + 4*a*b**3*c*d*e**2*x
**2*asinh(c + d*x)**3 + 8*a*b**3*c*d*e**2*x**2*asinh(c + d*x)/3 - 8*a*b**3*
c*e**2*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/3 - 16*a*b*
*3*c*e**2*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/27 - 16*a*b**3*c*e**2*asin
h(c + d*x)/(3*d) + 4*a*b**3*d**2*e**2*x**3*asinh(c + d*x)**3/3 + 8*a*b**3*d
**2*e**2*x**3*asinh(c + d*x)/9 - 4*a*b**3*d*e**2*x**2*sqrt(c**2 + 2*c*d*x +
 d**2*x**2 + 1)*asinh(c + d*x)**2/3 - 8*a*b**3*d*e**2*x**2*sqrt(c**2 + 2*c*
d*x + d**2*x**2 + 1)/27 - 16*a*b**3*e**2*x*asinh(c + d*x)/3 + 8*a*b**3*e**2
*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/(3*d) + 160*a*b**3*
e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(27*d) + b**4*c**3*e**2*asinh(c +
 d*x)**4/(3*d) + 4*b**4*c**3*e**2*asinh(c + d*x)**2/(9*d) + b**4*c**2*e**2*
x*asinh(c + d*x)**4 + 4*b**4*c**2*e**2*x*asinh(c + d*x)**2/3 + 8*b**4*c**2*
e**2*x/27 - 4*b**4*c**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c +
 d*x)**3/(9*d) - 8*b**4*c**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asin
h(c + d*x)/(27*d) + b**4*c*d*e**2*x**2*asinh(c + d*x)**4 + 4*b**4*c*d*e**2*
x**2*asinh(c + d*x)**2/3 + 8*b**4*c*d*e**2*x**2/27 - 8*b**4*c*e**2*x*sqrt(c
**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**3/9 - 16*b**4*c*e**2*x*sqrt(
c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/27 - 8*b**4*c*e**2*asinh(c +
 d*x)**2/(3*d) + b**4*d**2*e**2*x**3*asinh(c + d*x)**4/3 + 4*b**4*d**2*e**2
*x**3*asinh(c + d*x)**2/9 + 8*b**4*d**2*e**2*x**3/81 - 4*b**4*d*e**2*x**2*s
qrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**3/9 - 8*b**4*d*e**2*x**
2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/27 - 8*b**4*e**2*x*as
inh(c + d*x)**2/3 - 160*b**4*e**2*x/27 + 8*b**4*e**2*sqrt(c**2 + 2*c*d*x +
d**2*x**2 + 1)*asinh(c + d*x)**3/(9*d) + 160*b**4*e**2*sqrt(c**2 + 2*c*d*x
+ d**2*x**2 + 1)*asinh(c + d*x)/(27*d), Ne(d, 0)), (c**2*e**2*x*(a + b*asin
h(c))**4, True))
```

3.149 $\int (ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^4\,dx$

Optimal. Leaf size=195

$$-\frac{3b^3e(c+dx)\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)}{2d}+\frac{3b^2e(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)^2}{2d}+\frac{3b^2e\left(a+b\sinh\right.}{4d}$$

```
[Out] 3/4*b^4*e*(d*x+c)^2/d+3/4*b^2*e*(a+b*arcsinh(d*x+c))^2/d+3/2*b^2*e*(d*x+c)^
2*(a+b*arcsinh(d*x+c))^2/d+1/4*e*(a+b*arcsinh(d*x+c))^4/d+1/2*e*(d*x+c)^2*(
a+b*arcsinh(d*x+c))^4/d-3/2*b^3*e*(d*x+c)*(a+b*arcsinh(d*x+c))*(1+(d*x+c)^2
)^(1/2)/d-b*e*(d*x+c)*(a+b*arcsinh(d*x+c))^3*(1+(d*x+c)^2)^(1/2)/d
```

Rubi [A] time = 0.32, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5865, 12, 5661, 5758, 5675, 30}

$$-\frac{3b^3e(c+dx)\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)}{2d}+\frac{3b^2e(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)^2}{2d}+\frac{3b^2e\left(a+b\sinh\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] (3*b^4*e*(c + d*x)^2)/(4*d) - (3*b^3*e*(c + d*x)*Sqrt[1 + (c + d*x)^2]*(a +
 b*ArcSinh[c + d*x]))/(2*d) + (3*b^2*e*(a + b*ArcSinh[c + d*x])^2)/(4*d) +
(3*b^2*e*(c + d*x)^2*(a + b*ArcSinh[c + d*x])^2)/(2*d) - (b*e*(c + d*x)*Sqr
t[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^3)/d + (e*(a + b*ArcSinh[c + d*
x])^4)/(4*d) + (e*(c + d*x)^2*(a + b*ArcSinh[c + d*x])^4)/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
```

```
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^4\,dx = \frac{\text{Subst}\left(\int ex\left(a+b\sinh^{-1}(x)\right)^4\,dx,x,c+dx\right)}{d}$$
$$= \frac{e\,\text{Subst}\left(\int x\left(a+b\sinh^{-1}(x)\right)^4\,dx,x,c+dx\right)}{d}$$
$$= \frac{e(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)^4}{2d} - \frac{(2be)\,\text{Subst}\left(\int \frac{x^2\left(a+b\sinh^{-1}(x)\right)^3}{\sqrt{1+x^2}}\,dx,\right.}{d}$$
$$= -\frac{be(c+dx)\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{d} + \frac{e(c+dx)^2\left(a+b\right.}{2}$$
$$= \frac{3b^2e(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)^2}{2d} - \frac{be(c+dx)\sqrt{1+(c+dx)^2}\left(a+\right.}{d}$$
$$= -\frac{3b^3e(c+dx)\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)}{2d} + \frac{3b^2e(c+dx)^2\left(\right.}{}$$
$$= \frac{3b^4e(c+dx)^2}{4d} - \frac{3b^3e(c+dx)\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)}{2d} + $$

Mathematica [A] time = 0.32, size = 300, normalized size = 1.54

$$\frac{e\left(-2ab\left(2a^2+3b^2\right)(c+dx)\sqrt{(c+dx)^2+1}+3b^2\sinh^{-1}(c+dx)^2\left(4a^2(c+dx)^2+2a^2-4ab(c+dx)\sqrt{(c+dx)^2+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] (e*((2*a^4 + 6*a^2*b^2 + 3*b^4)*(c + d*x)^2 - 2*a*b*(2*a^2 + 3*b^2)*(c + d*
x)*Sqrt[1 + (c + d*x)^2] + 2*a*b*(2*a^2 + 3*b^2)*ArcSinh[c + d*x] - 2*b*(c
+ d*x)*(-4*a^3*(c + d*x) - 6*a*b^2*(c + d*x) + 6*a^2*b*Sqrt[1 + (c + d*x)^2
] + 3*b^3*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x] + 3*b^2*(2*a^2 + b^2 + 4*
a^2*(c + d*x)^2 + 2*b^2*(c + d*x)^2 - 4*a*b*(c + d*x)*Sqrt[1 + (c + d*x)^2]
)*ArcSinh[c + d*x]^2 + 4*b^3*(a + 2*a*(c + d*x)^2 - b*(c + d*x)*Sqrt[1 + (c
 + d*x)^2])*ArcSinh[c + d*x]^3 + b^4*(1 + 2*(c + d*x)^2)*ArcSinh[c + d*x]^4
))/(4*d)
```

fricas [B] time = 0.60, size = 574, normalized size = 2.94

$$\frac{\left(2a^4+6a^2b^2+3b^4\right)d^2ex^2+2\left(2a^4+6a^2b^2+3b^4\right)cdex+\left(2b^4d^2ex^2+4b^4cdex+\left(2b^4c^2+b^4\right)e\right)\log\left(dx+c+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/4*((2*a^4 + 6*a^2*b^2 + 3*b^4)*d^2*e*x^2 + 2*(2*a^4 + 6*a^2*b^2 + 3*b^4)*
c*d*e*x + (2*b^4*d^2*e*x^2 + 4*b^4*c*d*e*x + (2*b^4*c^2 + b^4)*e)*log(d*x +
 c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^4 + 4*(2*a*b^3*d^2*e*x^2 + 4*a*b^3*
c*d*e*x + (2*a*b^3*c^2 + a*b^3)*e - (b^4*d*e*x + b^4*c*e)*sqrt(d^2*x^2 + 2*
c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3 + 3*(2
*(2*a^2*b^2 + b^4)*d^2*e*x^2 + 4*(2*a^2*b^2 + b^4)*c*d*e*x + (2*a^2*b^2 + b
^4 + 2*(2*a^2*b^2 + b^4)*c^2)*e - 4*(a*b^3*d*e*x + a*b^3*c*e)*sqrt(d^2*x^2
+ 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 +
2*(2*(2*a^3*b + 3*a*b^3)*d^2*e*x^2 + 4*(2*a^3*b + 3*a*b^3)*c*d*e*x + (2*a^3
*b + 3*a*b^3 + 2*(2*a^3*b + 3*a*b^3)*c^2)*e - 3*((2*a^2*b^2 + b^4)*d*e*x +
(2*a^2*b^2 + b^4)*c*e)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqr
t(d^2*x^2 + 2*c*d*x + c^2 + 1)) - 2*((2*a^3*b + 3*a*b^3)*d*e*x + (2*a^3*b +
 3*a*b^3)*c*e)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/d
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex + ce)(b \operatorname{arsinh}(dx + c) + a)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^4, x)
```

maple [B] time = 0.04, size = 371, normalized size = 1.90

$$\frac{(dx+c)^2 e a^4}{2} + e b^4 \left(\frac{\left(1+(dx+c)^2\right) \operatorname{arcsinh}(dx+c)^4}{2} - \operatorname{arcsinh}(dx+c)^3 (dx+c) \sqrt{1+(dx+c)^2} - \frac{\operatorname{arcsinh}(dx+c)^4}{4} + \frac{3 \operatorname{arcsinh}(d\ldots}{\ldots} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^4,x)
```

```
[Out] 1/d*(1/2*(d*x+c)^2*e*a^4+e*b^4*(1/2*(1+(d*x+c)^2)*arcsinh(d*x+c)^4-arcsinh(
d*x+c)^3*(d*x+c)*(1+(d*x+c)^2)^(1/2)-1/4*arcsinh(d*x+c)^4+3/2*arcsinh(d*x+c
)^2*(1+(d*x+c)^2)-3/2*arcsinh(d*x+c)*(1+(d*x+c)^2)^(1/2)*(d*x+c)-3/4*arcsin
h(d*x+c)^2+3/4*(d*x+c)^2+3/4)+4*e*a*b^3*(1/2*arcsinh(d*x+c)^3*(1+(d*x+c)^2)
-3/4*arcsinh(d*x+c)^2*(1+(d*x+c)^2)^(1/2)*(d*x+c)-1/4*arcsinh(d*x+c)^3+3/4*
arcsinh(d*x+c)*(1+(d*x+c)^2)-3/8*(d*x+c)*(1+(d*x+c)^2)^(1/2)-3/8*arcsinh(d*
x+c))+6*e*a^2*b^2*(1/2*arcsinh(d*x+c)^2*(1+(d*x+c)^2)-1/2*arcsinh(d*x+c)*(1
+(d*x+c)^2)^(1/2)*(d*x+c)-1/4*arcsinh(d*x+c)^2+1/4*(d*x+c)^2+1/4)+4*e*a^3*b
*(1/2*(d*x+c)^2*arcsinh(d*x+c)-1/4*(d*x+c)*(1+(d*x+c)^2)^(1/2)+1/4*arcsinh(
d*x+c)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2} a^4 dex^2 + \left(2x^2 \operatorname{arsinh}(dx+c) - d \left(\frac{3c^2 \operatorname{arsinh}\left(\frac{2\left(d^2x+cd\right)}{\sqrt{-4c^2d^2+4\left(c^2+1\right)d^2}} \right)}{d^3} + \frac{\sqrt{d^2x^2+2cdx+c^2+1}\, x}{d^2} - \frac{\left(c^2+1\right) \operatorname{ars}\ldots}{\ldots} \right. \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/2*a^4*d*e*x^2 + (2*x^2*arcsinh(d*x + c) - d*(3*c^2*arcsinh(2*(d^2*x + c*d
)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c^2 +
1)*x/d^2 - (c^2 + 1)*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*
d^2))/d^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c/d^3))*a^3*b*d*e + a^4*c*e
*x + 4*((d*x + c)*arcsinh(d*x + c) - sqrt((d*x + c)^2 + 1))*a^3*b*c*e/d + 1
/2*(b^4*d*e*x^2 + 2*b^4*c*e*x)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 +
 1))^4 + integrate(2*((2*(c^4*e + c^2*e)*a*b^3 + (2*a*b^3*d^4*e - b^4*d^4*e
)*x^4 + 4*(2*a*b^3*c*d^3*e - b^4*c*d^3*e)*x^3 + (2*(6*c^2*d^2*e + d^2*e)*a*
b^3 - (5*c^2*d^2*e + d^2*e)*b^4)*x^2 + 2*(2*(2*c^3*d*e + c*d*e)*a*b^3 - (c^
3*d*e + c*d*e)*b^4)*x + (2*(c^3*e + c*e)*a*b^3 + (2*a*b^3*d^3*e - b^4*d^3*e
)*x^3 + 3*(2*a*b^3*c*d^2*e - b^4*c*d^2*e)*x^2 - 2*(b^4*c^2*d*e - (3*c^2*d*e
 + d*e)*a*b^3)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2
*x^2 + 2*c*d*x + c^2 + 1))^3 + 3*(a^2*b^2*d^4*e*x^4 + 4*a^2*b^2*c*d^3*e*x^3
 + (6*c^2*d^2*e + d^2*e)*a^2*b^2*x^2 + 2*(2*c^3*d*e + c*d*e)*a^2*b^2*x + (c
^4*e + c^2*e)*a^2*b^2 + (a^2*b^2*d^3*e*x^3 + 3*a^2*b^2*c*d^2*e*x^2 + (3*c^2
*d*e + d*e)*a^2*b^2*x + (c^3*e + c*e)*a^2*b^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2
 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2)/(d^3*x^3 + 3*c*d
^2*x^2 + c^3 + (3*c^2*d + d)*x + (d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + c),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e+d\,e\,x)\,\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)*(a + b*asinh(c + d*x))^4,x)
```

```
[Out] int((c*e + d*e*x)*(a + b*asinh(c + d*x))^4, x)
```

sympy [A] time = 4.85, size = 1027, normalized size = 5.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*asinh(d*x+c))**4,x)
```

```
[Out] Piecewise((a**4*c*e*x + a**4*d*e*x**2/2 + 2*a**3*b*c**2*e*asinh(c + d*x)/d
+ 4*a**3*b*c*e*x*asinh(c + d*x) - a**3*b*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**
2 + 1)/d + 2*a**3*b*d*e*x**2*asinh(c + d*x) - a**3*b*e*x*sqrt(c**2 + 2*c*d*
x + d**2*x**2 + 1) + a**3*b*e*asinh(c + d*x)/d + 3*a**2*b**2*c**2*e*asinh(c
 + d*x)**2/d + 6*a**2*b**2*c*e*x*asinh(c + d*x)**2 + 3*a**2*b**2*c*e*x - 3*
a**2*b**2*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/d + 3*a**
2*b**2*d*e*x**2*asinh(c + d*x)**2 + 3*a**2*b**2*d*e*x**2/2 - 3*a**2*b**2*e*
x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x) + 3*a**2*b**2*e*asinh
(c + d*x)**2/(2*d) + 2*a*b**3*c**2*e*asinh(c + d*x)**3/d + 3*a*b**3*c**2*e*
asinh(c + d*x)/d + 4*a*b**3*c*e*x*asinh(c + d*x)**3 + 6*a*b**3*c*e*x*asinh(
c + d*x) - 3*a*b**3*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)
**2/d - 3*a*b**3*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(2*d) + 2*a*b**3*
d*e*x**2*asinh(c + d*x)**3 + 3*a*b**3*d*e*x**2*asinh(c + d*x) - 3*a*b**3*e*
x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2 - 3*a*b**3*e*x*sqr
t(c**2 + 2*c*d*x + d**2*x**2 + 1)/2 + a*b**3*e*asinh(c + d*x)**3/d + 3*a*b*
*3*e*asinh(c + d*x)/(2*d) + b**4*c**2*e*asinh(c + d*x)**4/(2*d) + 3*b**4*c*
*2*e*asinh(c + d*x)**2/(2*d) + b**4*c*e*x*asinh(c + d*x)**4 + 3*b**4*c*e*x*
asinh(c + d*x)**2 + 3*b**4*c*e*x/2 - b**4*c*e*sqrt(c**2 + 2*c*d*x + d**2*x*
*2 + 1)*asinh(c + d*x)**3/d - 3*b**4*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**2 +
1)*asinh(c + d*x)/(2*d) + b**4*d*e*x**2*asinh(c + d*x)**4/2 + 3*b**4*d*e*x*
*2*asinh(c + d*x)**2/2 + 3*b**4*d*e*x**2/4 - b**4*e*x*sqrt(c**2 + 2*c*d*x +
 d**2*x**2 + 1)*asinh(c + d*x)**3 - 3*b**4*e*x*sqrt(c**2 + 2*c*d*x + d**2*x
**2 + 1)*asinh(c + d*x)/2 + b**4*e*asinh(c + d*x)**4/(4*d) + 3*b**4*e*asinh
(c + d*x)**2/(4*d), Ne(d, 0)), (c*e*x*(a + b*asinh(c))**4, True))
```

3.150 $\int \left(a+b\sinh^{-1}(c+dx)\right)^4\,dx$

Optimal. Leaf size=115

$$-\frac{24b^3\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)}{d}+\frac{12b^2(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^2}{d}-\frac{4b\sqrt{(c+dx)^2+1}\left(a+b\text{s}\right.}{d}$$

```
[Out] 24*b^4*x+12*b^2*(d*x+c)*(a+b*arcsinh(d*x+c))^2/d+(d*x+c)*(a+b*arcsinh(d*x+c
))^4/d-24*b^3*(a+b*arcsinh(d*x+c))*(1+(d*x+c)^2)^(1/2)/d-4*b*(a+b*arcsinh(d
*x+c))^3*(1+(d*x+c)^2)^(1/2)/d
```

Rubi [A] time = 0.16, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5863, 5653, 5717, 8}

$$-\frac{24b^3\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)}{d}+\frac{12b^2(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^2}{d}-\frac{4b\sqrt{(c+dx)^2+1}\left(a+b\text{s}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] 24*b^4*x - (24*b^3*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x]))/d + (12*
b^2*(c + d*x)*(a + b*ArcSinh[c + d*x])^2)/d - (4*b*Sqrt[1 + (c + d*x)^2]*(a
 + b*ArcSinh[c + d*x])^3)/d + ((c + d*x)*(a + b*ArcSinh[c + d*x])^4)/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5863

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \left(a+b\sinh^{-1}(c+dx)\right)^4\,dx = \frac{\text{Subst}\left(\int \left(a+b\sinh^{-1}(x)\right)^4\,dx, x, c+dx\right)}{d}$$

$$= \frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^4}{d} - \frac{(4b)\,\text{Subst}\left(\int \frac{x\left(a+b\sinh^{-1}(x)\right)^3}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{4b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{d} + \frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^4}{d} +$$

$$= \frac{12b^2(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^2}{d} - \frac{4b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{d}$$

$$= -\frac{24b^3\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)}{d} + \frac{12b^2(c+dx)\left(a+b\sinh^{-1}(c+dx\right.}{d}$$

$$= 24b^4x - \frac{24b^3\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)}{d} + \frac{12b^2(c+dx)\left(a+b\sinh^{-1}\right.}{d}$$

Mathematica [A] time = 0.26, size = 226, normalized size = 1.97

$$\frac{-4ab\left(a^2+6b^2\right)\sqrt{(c+dx)^2+1} + 6b^2\sinh^{-1}(c+dx)^2\left(a^2(c+dx)-2ab\sqrt{(c+dx)^2+1}+2b^2(c+dx)\right)+\left(a^4+12\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] ((a^4 + 12*a^2*b^2 + 24*b^4)*(c + d*x) - 4*a*b*(a^2 + 6*b^2)*Sqrt[1 + (c +
d*x)^2] - 4*b*(-(a^3*(c + d*x)) - 6*a*b^2*(c + d*x) + 3*a^2*b*Sqrt[1 + (c +
 d*x)^2] + 6*b^3*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x] + 6*b^2*(a^2*(c +
d*x) + 2*b^2*(c + d*x) - 2*a*b*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x]^2 -
4*b^3*(-(a*(c + d*x)) + b*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x]^3 + b^4*(
c + d*x)*ArcSinh[c + d*x]^4)/d
```

fricas [B] time = 0.54, size = 344, normalized size = 2.99

$$\frac{\left(b^4dx+b^4c\right)\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)^4 + 4\left(ab^3dx+ab^3c-\sqrt{d^2x^2+2\,cdx+c^2+1}\,b^4\right)\log\left(dx+c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] ((b^4*d*x + b^4*c)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^4 + 4*(
a*b^3*d*x + a*b^3*c - sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*b^4)*log(d*x + c +
sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3 + (a^4 + 12*a^2*b^2 + 24*b^4)*d*x - 6*
(2*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*a*b^3 - (a^2*b^2 + 2*b^4)*d*x - (a^2*b
^2 + 2*b^4)*c)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 + 4*((a^3
*b + 6*a*b^3)*d*x + (a^3*b + 6*a*b^3)*c - 3*(a^2*b^2 + 2*b^4)*sqrt(d^2*x^2
+ 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - 4*
(a^3*b + 6*a*b^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/d
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\,\text{arsinh}\,(dx+c)+a)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^4, x)
```

maple [B] time = 0.06, size = 245, normalized size = 2.13

$$\frac{(dx+c)\,a^4 + b^4\left((dx+c)\operatorname{arcsinh}(dx+c)^4 - 4\operatorname{arcsinh}(dx+c)^3\sqrt{1+(dx+c)^2} + 12\,(dx+c)\operatorname{arcsinh}(dx+c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^4,x)
```

```
[Out] 1/d*((d*x+c)*a^4+b^4*((d*x+c)*arcsinh(d*x+c)^4-4*arcsinh(d*x+c)^3*(1+(d*x+c
)^2)^(1/2)+12*(d*x+c)*arcsinh(d*x+c)^2-24*arcsinh(d*x+c)*(1+(d*x+c)^2)^(1/2
)+24*d*x+24*c)+4*a*b^3*((d*x+c)*arcsinh(d*x+c)^3-3*arcsinh(d*x+c)^2*(1+(d*x
+c)^2)^(1/2)+6*(d*x+c)*arcsinh(d*x+c)-6*(1+(d*x+c)^2)^(1/2))+6*a^2*b^2*((d*
x+c)*arcsinh(d*x+c)^2-2*arcsinh(d*x+c)*(1+(d*x+c)^2)^(1/2)+2*d*x+2*c)+4*a^3
*b*((d*x+c)*arcsinh(d*x+c)-(1+(d*x+c)^2)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b^4 x \log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)^4 + a^4 x + \frac{4\left((dx+c)\operatorname{arsinh}(dx+c) - \sqrt{(dx+c)^2+1}\right)a^3 b}{d} + \int \frac{2\left(2\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] b^4*x*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^4 + a^4*x + 4*((d*x
+ c)*arcsinh(d*x + c) - sqrt((d*x + c)^2 + 1))*a^3*b/d + integrate(2*(2*((c
^3 + c)*a*b^3 + (a*b^3*d^3 - b^4*d^3)*x^3 + (3*a*b^3*c*d^2 - 2*b^4*c*d^2)*x
^2 + ((3*c^2*d + d)*a*b^3 - (c^2*d + d)*b^4)*x + ((c^2 + 1)*a*b^3 + (a*b^3*
d^2 - b^4*d^2)*x^2 + (2*a*b^3*c*d - b^4*c*d)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^
2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3 + 3*(a^2*b^2*d^3
*x^3 + 3*a^2*b^2*c*d^2*x^2 + (3*c^2*d + d)*a^2*b^2*x + (c^3 + c)*a^2*b^2 +
(a^2*b^2*d^2*x^2 + 2*a^2*b^2*c*d*x + (c^2 + 1)*a^2*b^2)*sqrt(d^2*x^2 + 2*c*
d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2)/(d^3*x^
3 + 3*c*d^2*x^2 + c^3 + (3*c^2*d + d)*x + (d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/
2) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\operatorname{asinh}(c+d\,x))^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^4,x)
```

```
[Out] int((a + b*asinh(c + d*x))^4, x)
```

sympy [A] time = 1.69, size = 444, normalized size = 3.86

$$\begin{cases} a^4 x + \frac{4a^3bc\operatorname{asinh}(c+dx)}{d} + 4a^3bx\operatorname{asinh}(c+dx) - \frac{4a^3b\sqrt{c^2+2cdx+d^2x^2+1}}{d} + \frac{6a^2b^2c\operatorname{asinh}^2(c+dx)}{d} + 6a^2b^2x\operatorname{asinh}^2(c+d \\ x\,(a+b\operatorname{asinh}(c))^4 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**4,x)
```

```
[Out] Piecewise((a**4*x + 4*a**3*b*c*asinh(c + d*x)/d + 4*a**3*b*x*asinh(c + d*x)
 - 4*a**3*b*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/d + 6*a**2*b**2*c*asinh(c
+ d*x)**2/d + 6*a**2*b**2*x*asinh(c + d*x)**2 + 12*a**2*b**2*x - 12*a**2*b*
*2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/d + 4*a*b**3*c*asinh
(c + d*x)**3/d + 24*a*b**3*c*asinh(c + d*x)/d + 4*a*b**3*x*asinh(c + d*x)**
3 + 24*a*b**3*x*asinh(c + d*x) - 12*a*b**3*sqrt(c**2 + 2*c*d*x + d**2*x**2
+ 1)*asinh(c + d*x)**2/d - 24*a*b**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/d
 + b**4*c*asinh(c + d*x)**4/d + 12*b**4*c*asinh(c + d*x)**2/d + b**4*x*asin
h(c + d*x)**4 + 12*b**4*x*asinh(c + d*x)**2 + 24*b**4*x - 4*b**4*sqrt(c**2
+ 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**3/d - 24*b**4*sqrt(c**2 + 2*c*d*
x + d**2*x**2 + 1)*asinh(c + d*x)/d, Ne(d, 0)), (x*(a + b*asinh(c))**4, Tru
e))
```

3.151 $\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{ce+dex}\,dx$

Optimal. Leaf size=186

$$-\frac{3b^3\text{Li}_4\left(e^{-2\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)}{de}-\frac{3b^2\text{Li}_3\left(e^{-2\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)^2}{de}-\frac{2b\text{Li}_2\left(e^{-2\sin\right.}}{}$$

```
[Out] 1/5*(a+b*arcsinh(d*x+c))^5/b/d/e+(a+b*arcsinh(d*x+c))^4*ln(1-1/(d*x+c+(1+(d
*x+c)^2)^(1/2))^2)/d/e-2*b*(a+b*arcsinh(d*x+c))^3*polylog(2,1/(d*x+c+(1+(d*
x+c)^2)^(1/2))^2)/d/e-3*b^2*(a+b*arcsinh(d*x+c))^2*polylog(3,1/(d*x+c+(1+(d
*x+c)^2)^(1/2))^2)/d/e-3*b^3*(a+b*arcsinh(d*x+c))*polylog(4,1/(d*x+c+(1+(d*
x+c)^2)^(1/2))^2)/d/e-3/2*b^4*polylog(5,1/(d*x+c+(1+(d*x+c)^2)^(1/2))^2)/d/
e
```

Rubi [A] time = 0.26, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {5865, 12, 5659, 3716, 2190, 2531, 6609, 2282, 6589}

$$-\frac{3b^2\text{PolyLog}\left(3,e^{2\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)^2}{de}+\frac{3b^3\text{PolyLog}\left(4,e^{2\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)}{de}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x),x]
```

```
[Out] -(a + b*ArcSinh[c + d*x])^5/(5*b*d*e) + ((a + b*ArcSinh[c + d*x])^4*Log[1 -
 E^(2*ArcSinh[c + d*x])])/(d*e) + (2*b*(a + b*ArcSinh[c + d*x])^3*PolyLog[2
, E^(2*ArcSinh[c + d*x])])/(d*e) - (3*b^2*(a + b*ArcSinh[c + d*x])^2*PolyLo
g[3, E^(2*ArcSinh[c + d*x])])/(d*e) + (3*b^3*(a + b*ArcSinh[c + d*x])*PolyL
og[4, E^(2*ArcSinh[c + d*x])])/(d*e) - (3*b^4*PolyLog[5, E^(2*ArcSinh[c + d
*x])])/(2*d*e)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
```

```
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{ce+dex}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^4}{ex}\,dx, x, c+dx\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^4}{x}\,dx, x, c+dx\right)}{de}$$
$$= \frac{\text{Subst}\left(\int (a+bx)^4\coth(x)\,dx, x, \sinh^{-1}(c+dx)\right)}{de}$$
$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^5}{5bde} - \frac{2\,\text{Subst}\left(\int \frac{e^{2x}(a+bx)^4}{1-e^{2x}}\,dx, x, \sinh^{-1}(c+dx)\right)}{de}$$
$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^5}{5bde} + \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4\log\left(1-e^{2\sinh^{-1}(c+dx)}\right)}{de} -$$
$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^5}{5bde} + \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4\log\left(1-e^{2\sinh^{-1}(c+dx)}\right)}{de} +$$
$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^5}{5bde} + \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4\log\left(1-e^{2\sinh^{-1}(c+dx)}\right)}{de} +$$
$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^5}{5bde} + \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4\log\left(1-e^{2\sinh^{-1}(c+dx)}\right)}{de} +$$
$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^5}{5bde} + \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4\log\left(1-e^{2\sinh^{-1}(c+dx)}\right)}{de} +$$
$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^5}{5bde} + \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4\log\left(1-e^{2\sinh^{-1}(c+dx)}\right)}{de} +$$

Mathematica [A] time = 0.07, size = 157, normalized size = 0.84

$$\frac{3b^3\text{Li}_4\left(e^{2\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right) - 3b^2\text{Li}_3\left(e^{2\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)^2 + 2b\text{Li}_2\left(e^{2\sinh^{-1}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x),x]
```

```
[Out] (-1/5*(a + b*ArcSinh[c + d*x])^5/b + (a + b*ArcSinh[c + d*x])^4*Log[1 - E^(
2*ArcSinh[c + d*x])] + 2*b*(a + b*ArcSinh[c + d*x])^3*PolyLog[2, E^(2*ArcSi
nh[c + d*x])] - 3*b^2*(a + b*ArcSinh[c + d*x])^2*PolyLog[3, E^(2*ArcSinh[c
+ d*x])] + 3*b^3*(a + b*ArcSinh[c + d*x])*PolyLog[4, E^(2*ArcSinh[c + d*x])
] - (3*b^4*PolyLog[5, E^(2*ArcSinh[c + d*x])])/2)/(d*e)
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^4\operatorname{arsinh}(dx+c)^4 + 4\,ab^3\operatorname{arsinh}(dx+c)^3 + 6\,a^2b^2\operatorname{arsinh}(dx+c)^2 + 4\,a^3b\operatorname{arsinh}(dx+c) + a^4}{dex+ce}, x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e),x, algorithm="fricas")
```

```
[Out] integral((b^4*arcsinh(d*x + c)^4 + 4*a*b^3*arcsinh(d*x + c)^3 + 6*a^2*b^2*a
rcsinh(d*x + c)^2 + 4*a^3*b*arcsinh(d*x + c) + a^4)/(d*e*x + c*e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arsinh}(dx+c)+a)^4}{dex+ce}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^4/(d*e*x + c*e), x)
```

maple [B] time = 0.12, size = 1153, normalized size = 6.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e),x)
```

```
[Out] 12/d*a^2*b^2/e*arcsinh(d*x+c)*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2))+4/d*a^3*
b/e*arcsinh(d*x+c)*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))+4/d*a^3*b/e*arcsinh(d*x+
c)*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))+4/d*a*b^3/e*arcsinh(d*x+c)^3*ln(1+d*x+c+
(1+(d*x+c)^2)^(1/2))+12/d*a*b^3/e*arcsinh(d*x+c)^2*polylog(2,-d*x-c-(1+(d*x
+c)^2)^(1/2))-24/d*a*b^3/e*arcsinh(d*x+c)*polylog(3,-d*x-c-(1+(d*x+c)^2)^(1
/2))+4/d*a*b^3/e*arcsinh(d*x+c)^3*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))+12/d*a*b^
3/e*arcsinh(d*x+c)^2*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2))-24/d*a*b^3/e*arcs
inh(d*x+c)*polylog(3,d*x+c+(1+(d*x+c)^2)^(1/2))+12/d*a^2*b^2/e*arcsinh(d*x+
c)*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))+6/d*a^2*b^2/e*arcsinh(d*x+c)^2*ln(
1+d*x+c+(1+(d*x+c)^2)^(1/2))+6/d*a^2*b^2/e*arcsinh(d*x+c)^2*ln(1-d*x-c-(1+(
d*x+c)^2)^(1/2))-24/d*b^4/e*polylog(5,d*x+c+(1+(d*x+c)^2)^(1/2))-24/d*b^4/e
*polylog(5,-d*x-c-(1+(d*x+c)^2)^(1/2))+1/d*a^4/e*ln(d*x+c)-1/5/d*b^4/e*arcs
inh(d*x+c)^5-12/d*b^4/e*arcsinh(d*x+c)^2*polylog(3,-d*x-c-(1+(d*x+c)^2)^(1/
2))+24/d*b^4/e*arcsinh(d*x+c)*polylog(4,-d*x-c-(1+(d*x+c)^2)^(1/2))+24/d*a*
b^3/e*polylog(4,-d*x-c-(1+(d*x+c)^2)^(1/2))+24/d*a*b^3/e*polylog(4,d*x+c+(1
+(d*x+c)^2)^(1/2))+4/d*b^4/e*arcsinh(d*x+c)^3*polylog(2,d*x+c+(1+(d*x+c)^2)
^(1/2))-2/d*a^2*b^2/e*arcsinh(d*x+c)^3-12/d*a^2*b^2/e*polylog(3,-d*x-c-(1+(
d*x+c)^2)^(1/2))-12/d*a^2*b^2/e*polylog(3,d*x+c+(1+(d*x+c)^2)^(1/2))-2/d*a^
3*b/e*arcsinh(d*x+c)^2+4/d*a^3*b/e*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))+4/
d*a^3*b/e*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2))+1/d*b^4/e*arcsinh(d*x+c)^4*l
n(1+d*x+c+(1+(d*x+c)^2)^(1/2))+4/d*b^4/e*arcsinh(d*x+c)^3*polylog(2,-d*x-c-
(1+(d*x+c)^2)^(1/2))+1/d*b^4/e*arcsinh(d*x+c)^4*ln(1-d*x-c-(1+(d*x+c)^2)^(1
/2))-12/d*b^4/e*arcsinh(d*x+c)^2*polylog(3,d*x+c+(1+(d*x+c)^2)^(1/2))+24/d*
b^4/e*arcsinh(d*x+c)*polylog(4,d*x+c+(1+(d*x+c)^2)^(1/2))-1/d*a*b^3/e*arcsi
nh(d*x+c)^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^4 \log(dex+ce)}{de}+\int \frac{b^4 \log\left(dx+c+\sqrt{(dx+c)^2+1}\right)^4}{dex+ce}+\frac{4\,ab^3 \log\left(dx+c+\sqrt{(dx+c)^2+1}\right)^3}{dex+ce}+\frac{6\,a^2b^2 \log\left(dx+\right.}{\,}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e),x, algorithm="maxima")
```

```
[Out] a^4*log(d*e*x + c*e)/(d*e) + integrate(b^4*log(d*x + c + sqrt((d*x + c)^2 +
 1))^4/(d*e*x + c*e) + 4*a*b^3*log(d*x + c + sqrt((d*x + c)^2 + 1))^3/(d*e*
x + c*e) + 6*a^2*b^2*log(d*x + c + sqrt((d*x + c)^2 + 1))^2/(d*e*x + c*e) +
 4*a^3*b*log(d*x + c + sqrt((d*x + c)^2 + 1))/(d*e*x + c*e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{asinh}(c+d\,x))^4}{c\,e+d\,e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^4/(c*e + d*e*x),x)
```

```
[Out] int((a + b*asinh(c + d*x))^4/(c*e + d*e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^4}{c+dx}\,dx + \int \frac{b^4 \operatorname{asinh}^4(c+dx)}{c+dx}\,dx + \int \frac{4ab^3 \operatorname{asinh}^3(c+dx)}{c+dx}\,dx + \int \frac{6a^2b^2 \operatorname{asinh}^2(c+dx)}{c+dx}\,dx + \int \frac{4a^3b \operatorname{asinh}(c+dx)}{c+dx}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**4/(d*e*x+c*e),x)
```

```
[Out] (Integral(a**4/(c + d*x), x) + Integral(b**4*asinh(c + d*x)**4/(c + d*x), x
) + Integral(4*a*b**3*asinh(c + d*x)**3/(c + d*x), x) + Integral(6*a**2*b**
2*asinh(c + d*x)**2/(c + d*x), x) + Integral(4*a**3*b*asinh(c + d*x)/(c + d
*x), x))/e
```

3.152 $\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{(ce+dex)^2}\,dx$

Optimal. Leaf size=234

$$\frac{24b^3\text{Li}_3\left(-e^{\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)}{de^2}-\frac{24b^3\text{Li}_3\left(e^{\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)}{de^2}-\frac{12b^2\text{Li}_2\left(-e^{\sinh^{-1}(c}\right.}{}$$

```
[Out] -(a+b*arcsinh(d*x+c))^4/d/e^2/(d*x+c)-8*b*(a+b*arcsinh(d*x+c))^3*arctanh(d*
x+c+(1+(d*x+c)^2)^(1/2))/d/e^2-12*b^2*(a+b*arcsinh(d*x+c))^2*polylog(2,-d*x
-c-(1+(d*x+c)^2)^(1/2))/d/e^2+12*b^2*(a+b*arcsinh(d*x+c))^2*polylog(2,d*x+c
+(1+(d*x+c)^2)^(1/2))/d/e^2+24*b^3*(a+b*arcsinh(d*x+c))*polylog(3,-d*x-c-(1
+(d*x+c)^2)^(1/2))/d/e^2-24*b^3*(a+b*arcsinh(d*x+c))*polylog(3,d*x+c+(1+(d*
x+c)^2)^(1/2))/d/e^2-24*b^4*polylog(4,-d*x-c-(1+(d*x+c)^2)^(1/2))/d/e^2+24*
b^4*polylog(4,d*x+c+(1+(d*x+c)^2)^(1/2))/d/e^2
```

Rubi [A] time = 0.32, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {5865, 12, 5661, 5760, 4182, 2531, 6609, 2282, 6589}

$$\frac{24b^3\text{PolyLog}\left(3,-e^{\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)}{de^2}-\frac{24b^3\text{PolyLog}\left(3,e^{\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)}{de^2}-12$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^2,x]
```

```
[Out] -((a + b*ArcSinh[c + d*x])^4/(d*e^2*(c + d*x))) - (8*b*(a + b*ArcSinh[c + d
*x])^3*ArcTanh[E^ArcSinh[c + d*x]])/(d*e^2) - (12*b^2*(a + b*ArcSinh[c + d*
x])^2*PolyLog[2, -E^ArcSinh[c + d*x]])/(d*e^2) + (12*b^2*(a + b*ArcSinh[c +
 d*x])^2*PolyLog[2, E^ArcSinh[c + d*x]])/(d*e^2) + (24*b^3*(a + b*ArcSinh[c
 + d*x])*PolyLog[3, -E^ArcSinh[c + d*x]])/(d*e^2) - (24*b^3*(a + b*ArcSinh[
c + d*x])*PolyLog[3, E^ArcSinh[c + d*x]])/(d*e^2) - (24*b^4*PolyLog[4, -E^A
rcSinh[c + d*x]])/(d*e^2) + (24*b^4*PolyLog[4, E^ArcSinh[c + d*x]])/(d*e^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{(ce+dex)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^4}{e^2x^2}\,dx,x,c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^4}{x^2}\,dx,x,c+dx\right)}{de^2}$$

$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{de^2(c+dx)} + \frac{(4b)\,\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^3}{x\sqrt{1+x^2}}\,dx,x,c+dx\right)}{de^2}$$

$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{de^2(c+dx)} + \frac{(4b)\,\text{Subst}\left(\int (a+bx)^3\text{csch}(x)\,dx,x,\sinh^{-1}(c+dx)\right)}{de^2}$$

$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{de^2(c+dx)} - \frac{8b\left(a+b\sinh^{-1}(c+dx)\right)^3\tanh^{-1}\left(e^{\sinh^{-1}(c+dx)}\right)}{de^2} - \frac{(1}{}$$

$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{de^2(c+dx)} - \frac{8b\left(a+b\sinh^{-1}(c+dx)\right)^3\tanh^{-1}\left(e^{\sinh^{-1}(c+dx)}\right)}{de^2} - \frac{12}{}$$

$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{de^2(c+dx)} - \frac{8b\left(a+b\sinh^{-1}(c+dx)\right)^3\tanh^{-1}\left(e^{\sinh^{-1}(c+dx)}\right)}{de^2} - \frac{12}{}$$

$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{de^2(c+dx)} - \frac{8b\left(a+b\sinh^{-1}(c+dx)\right)^3\tanh^{-1}\left(e^{\sinh^{-1}(c+dx)}\right)}{de^2} - \frac{12}{}$$

$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{de^2(c+dx)} - \frac{8b\left(a+b\sinh^{-1}(c+dx)\right)^3\tanh^{-1}\left(e^{\sinh^{-1}(c+dx)}\right)}{de^2} - \frac{12}{}$$

Mathematica [B] time = 1.60, size = 501, normalized size = 2.14

$$\frac{-\frac{2a^4}{c+dx} + 4a^3b\left(2\log\left(\frac{2\sinh^2\left(\frac{1}{2}\sinh^{-1}(c+dx)\right)}{c+dx}\right) - \frac{2\sinh^{-1}(c+dx)}{c+dx}\right) + 12a^2b^2\left(2\text{Li}_2\left(-e^{-\sinh^{-1}(c+dx)}\right) - 2\text{Li}_2\left(e^{-\sinh^{-1}(c+dx)}\right) -}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^2,x]
```

```
[Out] ((-2*a^4)/(c + d*x) + 4*a^3*b*((-2*ArcSinh[c + d*x])/(c + d*x) + 2*Log[(2*S
inh[ArcSinh[c + d*x]/2]^2)/(c + d*x)]) + 12*a^2*b^2*(ArcSinh[c + d*x]*(-(Ar
cSinh[c + d*x]/(c + d*x)) + 2*Log[1 - E^(-ArcSinh[c + d*x])] - 2*Log[1 + E^
(-ArcSinh[c + d*x])]) + 2*PolyLog[2, -E^(-ArcSinh[c + d*x])] - 2*PolyLog[2,
 E^(-ArcSinh[c + d*x])]) + 8*a*b^3*(-(ArcSinh[c + d*x]^3/(c + d*x)) + 3*Arc
Sinh[c + d*x]^2*Log[1 - E^(-ArcSinh[c + d*x])] - 3*ArcSinh[c + d*x]^2*Log[1
 + E^(-ArcSinh[c + d*x])] + 6*ArcSinh[c + d*x]*PolyLog[2, -E^(-ArcSinh[c +
d*x])] - 6*ArcSinh[c + d*x]*PolyLog[2, E^(-ArcSinh[c + d*x])] + 6*PolyLog[3
, -E^(-ArcSinh[c + d*x])] - 6*PolyLog[3, E^(-ArcSinh[c + d*x])]) + b^4*(Pi^
4 - 2*ArcSinh[c + d*x]^4 - (2*ArcSinh[c + d*x]^4)/(c + d*x) - 8*ArcSinh[c +
 d*x]^3*Log[1 + E^(-ArcSinh[c + d*x])] + 8*ArcSinh[c + d*x]^3*Log[1 - E^Arc
Sinh[c + d*x]] + 24*ArcSinh[c + d*x]^2*PolyLog[2, -E^(-ArcSinh[c + d*x])] +
 24*ArcSinh[c + d*x]^2*PolyLog[2, E^ArcSinh[c + d*x]] + 48*ArcSinh[c + d*x]
*PolyLog[3, -E^(-ArcSinh[c + d*x])] - 48*ArcSinh[c + d*x]*PolyLog[3, E^ArcS
inh[c + d*x]] + 48*PolyLog[4, -E^(-ArcSinh[c + d*x])] + 48*PolyLog[4, E^Arc
Sinh[c + d*x]]))/(2*d*e^2)
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^4 \operatorname{arsinh}(dx+c)^4 + 4\,ab^3 \operatorname{arsinh}(dx+c)^3 + 6\,a^2b^2 \operatorname{arsinh}(dx+c)^2 + 4\,a^3b \operatorname{arsinh}(dx+c) + a^4}{d^2e^2x^2 + 2\,cde^2x + c^2e^2}, x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^2,x, algorithm="fricas")
```

```
[Out] integral((b^4*arcsinh(d*x + c)^4 + 4*a*b^3*arcsinh(d*x + c)^3 + 6*a^2*b^2*a
rcsinh(d*x + c)^2 + 4*a^3*b*arcsinh(d*x + c) + a^4)/(d^2*e^2*x^2 + 2*c*d*e^
2*x + c^2*e^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arsinh}(dx+c) + a)^4}{(dex+ce)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^4/(d*e*x + c*e)^2, x)
```

maple [B] time = 0.20, size = 820, normalized size = 3.50

$$-\frac{a^4}{d\,e^2\,(dx+c)} - \frac{b^4 \operatorname{arcsinh}(dx+c)^4}{d\,e^2\,(dx+c)} - \frac{4b^4 \operatorname{arcsinh}(dx+c)^3 \ln\left(1+dx+c+\sqrt{1+(dx+c)^2}\right)}{d\,e^2} - \frac{12b^4 \operatorname{arcsinh}(d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^2,x)
```

```
[Out] -1/d*a^4/e^2/(d*x+c)-1/d*b^4/e^2*arcsinh(d*x+c)^4/(d*x+c)-4/d*b^4/e^2*arcsi
nh(d*x+c)^3*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))-12/d*b^4/e^2*arcsinh(d*x+c)^2*p
olylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))+24/d*b^4/e^2*arcsinh(d*x+c)*polylog(3,
-d*x-c-(1+(d*x+c)^2)^(1/2))-24*b^4*polylog(4,-d*x-c-(1+(d*x+c)^2)^(1/2))/d/
e^2+4/d*b^4/e^2*arcsinh(d*x+c)^3*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))+12/d*b^4/e
^2*arcsinh(d*x+c)^2*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2))-24/d*b^4/e^2*arcsi
nh(d*x+c)*polylog(3,d*x+c+(1+(d*x+c)^2)^(1/2))+24*b^4*polylog(4,d*x+c+(1+(d
*x+c)^2)^(1/2))/d/e^2-4/d*a*b^3/e^2*arcsinh(d*x+c)^3/(d*x+c)-12/d*a*b^3/e^2
*arcsinh(d*x+c)^2*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))-24/d*a*b^3/e^2*arcsinh(d*
x+c)*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))+24/d*a*b^3/e^2*polylog(3,-d*x-c-
(1+(d*x+c)^2)^(1/2))+12/d*a*b^3/e^2*arcsinh(d*x+c)^2*ln(1-d*x-c-(1+(d*x+c)^
2)^(1/2))+24/d*a*b^3/e^2*arcsinh(d*x+c)*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2)
)-24/d*a*b^3/e^2*polylog(3,d*x+c+(1+(d*x+c)^2)^(1/2))-6/d*a^2*b^2/e^2*arcsi
nh(d*x+c)^2/(d*x+c)-12/d*a^2*b^2/e^2*arcsinh(d*x+c)*ln(1+d*x+c+(1+(d*x+c)^2
)^(1/2))-12/d*a^2*b^2/e^2*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))+12/d*a^2*b^
2/e^2*arcsinh(d*x+c)*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))+12/d*a^2*b^2/e^2*polyl
og(2,d*x+c+(1+(d*x+c)^2)^(1/2))-4/d*a^3*b/e^2/(d*x+c)*arcsinh(d*x+c)-4/d*a^
3*b/e^2*arctanh(1/(1+(d*x+c)^2)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b^4 \log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)^4}{d^2e^2x+cde^2} - 4\,a^3b\left(\frac{\operatorname{arsinh}(dx+c)}{d^2e^2x+cde^2} + \frac{\operatorname{arsinh}\left(\frac{de^2}{|d^2e^2x+cde^2|}\right)}{de^2}\right) - \frac{a^4}{d^2e^2x+cde^2} + \int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^2,x, algorithm="maxima")
```

```
[Out] -b^4*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^4/(d^2*e^2*x + c*d*e^
2) - 4*a^3*b*(arcsinh(d*x + c)/(d^2*e^2*x + c*d*e^2) + arcsinh(d*e^2/abs(d^
2*e^2*x + c*d*e^2))/(d*e^2)) - a^4/(d^2*e^2*x + c*d*e^2) + integrate(2*(2*(
(c^3 + c)*a*b^3 + (c^3 + c)*b^4 + (a*b^3*d^3 + b^4*d^3)*x^3 + 3*(a*b^3*c*d^
2 + b^4*c*d^2)*x^2 + ((3*c^2*d + d)*a*b^3 + (3*c^2*d + d)*b^4)*x + (b^4*c^2
 + (c^2 + 1)*a*b^3 + (a*b^3*d^2 + b^4*d^2)*x^2 + 2*(a*b^3*c*d + b^4*c*d)*x)
*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x +
c^2 + 1))^3 + 3*(a^2*b^2*d^3*x^3 + 3*a^2*b^2*c*d^2*x^2 + (3*c^2*d + d)*a^2*
b^2*x + (c^3 + c)*a^2*b^2 + (a^2*b^2*d^2*x^2 + 2*a^2*b^2*c*d*x + (c^2 + 1)*
a^2*b^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*
c*d*x + c^2 + 1))^2)/(d^5*e^2*x^5 + 5*c*d^4*e^2*x^4 + c^5*e^2 + c^3*e^2 + (
10*c^2*d^3*e^2 + d^3*e^2)*x^3 + (10*c^3*d^2*e^2 + 3*c*d^2*e^2)*x^2 + (5*c^4
*d*e^2 + 3*c^2*d*e^2)*x + (d^4*e^2*x^4 + 4*c*d^3*e^2*x^3 + c^4*e^2 + c^2*e^
2 + (6*c^2*d^2*e^2 + d^2*e^2)*x^2 + 2*(2*c^3*d*e^2 + c*d*e^2)*x)*sqrt(d^2*x
^2 + 2*c*d*x + c^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\mathrm{asinh}\,(c + d\,x))^4}{(c\,e + d\,e\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^4/(c*e + d*e*x)^2,x)
```

```
[Out] int((a + b*asinh(c + d*x))^4/(c*e + d*e*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^4}{c^2+2cdx+d^2x^2}\,dx + \int \frac{b^4\,\mathrm{asinh}^4\,(c+dx)}{c^2+2cdx+d^2x^2}\,dx + \int \frac{4ab^3\,\mathrm{asinh}^3\,(c+dx)}{c^2+2cdx+d^2x^2}\,dx + \int \frac{6a^2b^2\,\mathrm{asinh}^2\,(c+dx)}{c^2+2cdx+d^2x^2}\,dx + \int \frac{4a^3b\,\mathrm{asinh}\,(c+dx)}{c^2+2cdx+d^2x^2}\,dx}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**4/(d*e*x+c*e)**2,x)
```

```
[Out] (Integral(a**4/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(b**4*asinh(c + d
*x)**4/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(4*a*b**3*asinh(c + d*x)*
*3/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(6*a**2*b**2*asinh(c + d*x)**
2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(4*a**3*b*asinh(c + d*x)/(c**2
 + 2*c*d*x + d**2*x**2), x))/e**2
```

3.153 $\int \frac{(a+b\sinh^{-1}(c+dx))^4}{(ce+dex)^3}\,dx$

Optimal. Leaf size=186

$$-\frac{6b^3\text{Li}_2\left(e^{-2\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)}{de^3}+\frac{6b^2\log\left(1-e^{-2\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)^2}{de^3}-\frac{2b\sqrt{(c+}}{}$$

```
[Out] 2*b*(a+b*arcsinh(d*x+c))^3/d/e^3-1/2*(a+b*arcsinh(d*x+c))^4/d/e^3/(d*x+c)^2
+6*b^2*(a+b*arcsinh(d*x+c))^2*ln(1-1/(d*x+c+(1+(d*x+c)^2)^(1/2))^2)/d/e^3-6
*b^3*(a+b*arcsinh(d*x+c))*polylog(2,1/(d*x+c+(1+(d*x+c)^2)^(1/2))^2)/d/e^3-
3*b^4*polylog(3,1/(d*x+c+(1+(d*x+c)^2)^(1/2))^2)/d/e^3-2*b*(a+b*arcsinh(d*x
+c))^3*(1+(d*x+c)^2)^(1/2)/d/e^3/(d*x+c)
```

Rubi [A] time = 0.33, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {5865, 12, 5661, 5723, 5659, 3716, 2190, 2531, 2282, 6589}

$$\frac{6b^3\text{PolyLog}\left(2,e^{2\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)}{de^3}-\frac{3b^4\text{PolyLog}\left(3,e^{2\sinh^{-1}(c+dx)}\right)}{de^3}+\frac{6b^2\log\left(1-e^{2\sinh^{-1}(c+d}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^3,x]
```

```
[Out] (-2*b*(a + b*ArcSinh[c + d*x])^3)/(d*e^3) - (2*b*Sqrt[1 + (c + d*x)^2]*(a +
 b*ArcSinh[c + d*x])^3)/(d*e^3*(c + d*x)) - (a + b*ArcSinh[c + d*x])^4/(2*d
*e^3*(c + d*x)^2) + (6*b^2*(a + b*ArcSinh[c + d*x])^2*Log[1 - E^(2*ArcSinh[
c + d*x])])/(d*e^3) + (6*b^3*(a + b*ArcSinh[c + d*x])*PolyLog[2, E^(2*ArcSi
nh[c + d*x])])/(d*e^3) - (3*b^4*PolyLog[3, E^(2*ArcSinh[c + d*x])])/(d*e^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5723

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)
^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 +
 c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
 d, e, f, m, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] &&
 NeQ[m, -1]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{(ce+dex)^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^4}{e^3x^3}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^4}{x^3}\,dx, x, c+dx\right)}{de^3}$$

$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{2de^3(c+dx)^2} + \frac{(2b)\,\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^3}{x^2\sqrt{1+x^2}}\,dx, x, c+dx\right)}{de^3}$$

$$= -\frac{2b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{de^3(c+dx)} - \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{2de^3(c+dx)^2} + \frac{(6b^2}{}$$

$$= -\frac{2b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{de^3(c+dx)} - \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{2de^3(c+dx)^2} + \frac{(6b^2}{}$$

$$= -\frac{2b\left(a+b\sinh^{-1}(c+dx)\right)^3}{de^3} - \frac{2b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{de^3(c+dx)} - \frac{(a}{}$$

$$= -\frac{2b\left(a+b\sinh^{-1}(c+dx)\right)^3}{de^3} - \frac{2b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{de^3(c+dx)} - \frac{(a}{}$$

$$= -\frac{2b\left(a+b\sinh^{-1}(c+dx)\right)^3}{de^3} - \frac{2b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{de^3(c+dx)} - \frac{(a}{}$$

$$= -\frac{2b\left(a+b\sinh^{-1}(c+dx)\right)^3}{de^3} - \frac{2b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{de^3(c+dx)} - \frac{(a}{}$$

$$= -\frac{2b\left(a+b\sinh^{-1}(c+dx)\right)^3}{de^3} - \frac{2b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{de^3(c+dx)} - \frac{(a}{}$$

Mathematica [C] time = 1.27, size = 360, normalized size = 1.94

$$\frac{-\frac{2a^4}{(c+dx)^2} - \frac{8a^3b\sqrt{(c+dx)^2+1}}{c+dx} - \frac{8a^3b\sinh^{-1}(c+dx)}{(c+dx)^2} + 24a^2b^2\left(\log(c+dx) - \frac{\sinh^{-1}(c+dx)^2}{2(c+dx)^2} - \frac{\sqrt{(c+dx)^2+1}\sinh^{-1}(c+dx)}{c+dx}\right) + 8ab^3}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^3,x]
```

```
[Out] ((-2*a^4)/(c + d*x)^2 - (8*a^3*b*Sqrt[1 + (c + d*x)^2])/(c + d*x) - (8*a^3*
b*ArcSinh[c + d*x])/(c + d*x)^2 - (2*b^4*ArcSinh[c + d*x]^4)/(c + d*x)^2 +
24*a^2*b^2*(-((Sqrt[1 + (c + d*x)^2]*ArcSinh[c + d*x])/(c + d*x)) - ArcSinh
[c + d*x]^2/(2*(c + d*x)^2) + Log[c + d*x]) + 8*a*b^3*(ArcSinh[c + d*x]*(3*
ArcSinh[c + d*x] - (3*Sqrt[1 + (c + d*x)^2]*ArcSinh[c + d*x])/(c + d*x) - A
rcSinh[c + d*x]^2/(c + d*x)^2 + 6*Log[1 - E^(-2*ArcSinh[c + d*x])]) - 3*Pol
yLog[2, E^(-2*ArcSinh[c + d*x])]) + b^4*(I*Pi^3 - 8*ArcSinh[c + d*x]^3 - (8
*Sqrt[1 + (c + d*x)^2]*ArcSinh[c + d*x]^3)/(c + d*x) + 24*ArcSinh[c + d*x]^
2*Log[1 - E^(2*ArcSinh[c + d*x])] + 24*ArcSinh[c + d*x]*PolyLog[2, E^(2*Arc
Sinh[c + d*x])] - 12*PolyLog[3, E^(2*ArcSinh[c + d*x])]))/(4*d*e^3)
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^4\operatorname{arsinh}(dx+c)^4 + 4\,ab^3\operatorname{arsinh}(dx+c)^3 + 6\,a^2b^2\operatorname{arsinh}(dx+c)^2 + 4\,a^3b\operatorname{arsinh}(dx+c) + a^4}{d^3e^3x^3 + 3\,cd^2e^3x^2 + 3\,c^2de^3x + c^3e^3}, x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^3,x, algorithm="fricas")
```

```
[Out] integral((b^4*arcsinh(d*x + c)^4 + 4*a*b^3*arcsinh(d*x + c)^3 + 6*a^2*b^2*a
rcsinh(d*x + c)^2 + 4*a^3*b*arcsinh(d*x + c) + a^4)/(d^3*e^3*x^3 + 3*c*d^2*
e^3*x^2 + 3*c^2*d*e^3*x + c^3*e^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arsinh}(dx+c)+a)^4}{(dex+ce)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^4/(d*e*x + c*e)^3, x)
```

maple [B] time = 0.22, size = 723, normalized size = 3.89

$$-\frac{a^4}{2d\,e^3\,(dx+c)^2}-\frac{2b^4 \operatorname{arcsinh}(dx+c)^3 \sqrt{1+(dx+c)^2}}{d\,e^3\,(dx+c)}-\frac{2b^4 \operatorname{arcsinh}(dx+c)^3}{d\,e^3}-\frac{b^4 \operatorname{arcsinh}(dx+c)^4}{2d\,e^3\,(dx+c)^2}+\frac{6b^4 \operatorname{arcsin}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^3,x)
```

```
[Out] -1/2/d*a^4/e^3/(d*x+c)^2-2/d*b^4/e^3*arcsinh(d*x+c)^3/(d*x+c)*(1+(d*x+c)^2)
^(1/2)-2/d*b^4/e^3*arcsinh(d*x+c)^3-1/2/d*b^4/e^3*arcsinh(d*x+c)^4/(d*x+c)^
2+6/d*b^4/e^3*arcsinh(d*x+c)^2*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))+12/d*b^4/e^3
*arcsinh(d*x+c)*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))-12/d*b^4/e^3*polylog(
3,-d*x-c-(1+(d*x+c)^2)^(1/2))+6/d*b^4/e^3*arcsinh(d*x+c)^2*ln(1-d*x-c-(1+(d
*x+c)^2)^(1/2))+12/d*b^4/e^3*arcsinh(d*x+c)*polylog(2,d*x+c+(1+(d*x+c)^2)^(
1/2))-12/d*b^4/e^3*polylog(3,d*x+c+(1+(d*x+c)^2)^(1/2))-6/d*a*b^3/e^3*arcsi
nh(d*x+c)^2/(d*x+c)*(1+(d*x+c)^2)^(1/2)-6/d*a*b^3/e^3*arcsinh(d*x+c)^2-2/d*
a*b^3/e^3*arcsinh(d*x+c)^3/(d*x+c)^2+12/d*a*b^3/e^3*arcsinh(d*x+c)*ln(1+d*x
+c+(1+(d*x+c)^2)^(1/2))+12/d*a*b^3/e^3*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2)
)+12/d*a*b^3/e^3*arcsinh(d*x+c)*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))+12/d*a*b^3/
e^3*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2))-6/d*a^2*b^2/e^3*arcsinh(d*x+c)-6/d
*a^2*b^2/e^3*arcsinh(d*x+c)/(d*x+c)*(1+(d*x+c)^2)^(1/2)-3/d*a^2*b^2/e^3*arc
sinh(d*x+c)^2/(d*x+c)^2+6/d*a^2*b^2/e^3*ln((d*x+c+(1+(d*x+c)^2)^(1/2))^2-1)
-2/d*a^3*b/e^3/(d*x+c)^2*arcsinh(d*x+c)-2/d*a^3*b/e^3/(d*x+c)*(1+(d*x+c)^2)
^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b^4 \log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)^4}{2\left(d^3e^3x^2+2\,cd^2e^3x+c^2de^3\right)}-6\left(\frac{\sqrt{d^2x^2+2\,cdx+c^2+1}\,d \operatorname{arsinh}(dx+c)}{d^3e^3x+cd^2e^3}-\frac{\log(dx+c)}{de^3}\right)a^2b^2-2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^3,x, algorithm="maxima")
```

```
[Out] -1/2*b^4*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^4/(d^3*e^3*x^2 +
2*c*d^2*e^3*x + c^2*d*e^3) - 6*(sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*d*arcsinh
(d*x + c)/(d^3*e^3*x + c*d^2*e^3) - log(d*x + c)/(d*e^3))*a^2*b^2 - 2*a^3*b
*(sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*d/(d^3*e^3*x + c*d^2*e^3) + arcsinh(d*x
 + c)/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3)) - 3*a^2*b^2*arcsinh(d*x +
```

```
c)^2/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3) - 1/2*a^4/(d^3*e^3*x^2 + 2*c
*d^2*e^3*x + c^2*d*e^3) + integrate(2*(2*(c^3 + c)*a*b^3 + (c^3 + c)*b^4 +
(2*a*b^3*d^3 + b^4*d^3)*x^3 + 3*(2*a*b^3*c*d^2 + b^4*c*d^2)*x^2 + (2*(3*c^2
*d + d)*a*b^3 + (3*c^2*d + d)*b^4)*x + (b^4*c^2 + 2*(c^2 + 1)*a*b^3 + (2*a*
b^3*d^2 + b^4*d^2)*x^2 + 2*(2*a*b^3*c*d + b^4*c*d)*x)*sqrt(d^2*x^2 + 2*c*d*
x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3/(d^6*e^3*x
^6 + 6*c*d^5*e^3*x^5 + c^6*e^3 + c^4*e^3 + (15*c^2*d^4*e^3 + d^4*e^3)*x^4 +
 4*(5*c^3*d^3*e^3 + c*d^3*e^3)*x^3 + 3*(5*c^4*d^2*e^3 + 2*c^2*d^2*e^3)*x^2
+ 2*(3*c^5*d*e^3 + 2*c^3*d*e^3)*x + (d^5*e^3*x^5 + 5*c*d^4*e^3*x^4 + c^5*e^
3 + c^3*e^3 + (10*c^2*d^3*e^3 + d^3*e^3)*x^3 + (10*c^3*d^2*e^3 + 3*c*d^2*e^
3)*x^2 + (5*c^4*d*e^3 + 3*c^2*d*e^3)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,\mathrm{asinh}\,(c + d\,x))^4}{(c\,e + d\,e\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^4/(c*e + d*e*x)^3,x)
```

```
[Out] int((a + b*asinh(c + d*x))^4/(c*e + d*e*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^4}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{b^4\,\mathrm{asinh}^4\,(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{4ab^3\,\mathrm{asinh}^3\,(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{6a^2b^2\,\mathrm{asinh}^2\,(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{}{c^3}}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**4/(d*e*x+c*e)**3,x)
```

```
[Out] (Integral(a**4/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integr
al(b**4*asinh(c + d*x)**4/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),
x) + Integral(4*a*b**3*asinh(c + d*x)**3/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2
 + d**3*x**3), x) + Integral(6*a**2*b**2*asinh(c + d*x)**2/(c**3 + 3*c**2*d
*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integral(4*a**3*b*asinh(c + d*x)/(c**
3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x))/e**3
```

3.154 $\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{(ce+dex)^4}\,dx$

Optimal. Leaf size=385

$$-\frac{4b^3\text{Li}_3\left(-e^{\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)}{de^4}+\frac{4b^3\text{Li}_3\left(e^{\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)}{de^4}-\frac{8b^3\tanh^{-1}\left(e^{\sinh^{-1}(\right.}{}$$

```
[Out] -2*b^2*(a+b*arcsinh(d*x+c))^2/d/e^4/(d*x+c)-1/3*(a+b*arcsinh(d*x+c))^4/d/e^
4/(d*x+c)^3-8*b^3*(a+b*arcsinh(d*x+c))*arctanh(d*x+c+(1+(d*x+c)^2)^(1/2))/d
/e^4+4/3*b*(a+b*arcsinh(d*x+c))^3*arctanh(d*x+c+(1+(d*x+c)^2)^(1/2))/d/e^4-
4*b^4*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))/d/e^4+2*b^2*(a+b*arcsinh(d*x+c)
)^2*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))/d/e^4+4*b^4*polylog(2,d*x+c+(1+(d
*x+c)^2)^(1/2))/d/e^4-2*b^2*(a+b*arcsinh(d*x+c))^2*polylog(2,d*x+c+(1+(d*x+
c)^2)^(1/2))/d/e^4-4*b^3*(a+b*arcsinh(d*x+c))*polylog(3,-d*x-c-(1+(d*x+c)^2
)^(1/2))/d/e^4+4*b^3*(a+b*arcsinh(d*x+c))*polylog(3,d*x+c+(1+(d*x+c)^2)^(1/
2))/d/e^4+4*b^4*polylog(4,-d*x-c-(1+(d*x+c)^2)^(1/2))/d/e^4-4*b^4*polylog(4
,d*x+c+(1+(d*x+c)^2)^(1/2))/d/e^4-2/3*b*(a+b*arcsinh(d*x+c))^3*(1+(d*x+c)^2
)^(1/2)/d/e^4/(d*x+c)^2
```

Rubi [A] time = 0.56, antiderivative size = 385, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {5865, 12, 5661, 5747, 5760, 4182, 2531, 6609, 2282, 6589, 2279, 2391}

$$-\frac{4b^3\text{PolyLog}\left(3,-e^{\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)}{de^4}+\frac{4b^3\text{PolyLog}\left(3,e^{\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)}{de^4}+\frac{2b^{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^4,x]
```

```
[Out] (-2*b^2*(a + b*ArcSinh[c + d*x])^2)/(d*e^4*(c + d*x)) - (2*b*Sqrt[1 + (c +
d*x)^2]*(a + b*ArcSinh[c + d*x])^3)/(3*d*e^4*(c + d*x)^2) - (a + b*ArcSinh[
c + d*x])^4/(3*d*e^4*(c + d*x)^3) - (8*b^3*(a + b*ArcSinh[c + d*x])*ArcTanh
[E^ArcSinh[c + d*x]])/(d*e^4) + (4*b*(a + b*ArcSinh[c + d*x])^3*ArcTanh[E^A
rcSinh[c + d*x]])/(3*d*e^4) - (4*b^4*PolyLog[2, -E^ArcSinh[c + d*x]])/(d*e^
4) + (2*b^2*(a + b*ArcSinh[c + d*x])^2*PolyLog[2, -E^ArcSinh[c + d*x]])/(d*
e^4) + (4*b^4*PolyLog[2, E^ArcSinh[c + d*x]])/(d*e^4) - (2*b^2*(a + b*ArcSi
nh[c + d*x])^2*PolyLog[2, E^ArcSinh[c + d*x]])/(d*e^4) - (4*b^3*(a + b*ArcS
inh[c + d*x])*PolyLog[3, -E^ArcSinh[c + d*x]])/(d*e^4) + (4*b^3*(a + b*ArcS
inh[c + d*x])*PolyLog[3, E^ArcSinh[c + d*x]])/(d*e^4) + (4*b^4*PolyLog[4, -
E^ArcSinh[c + d*x]])/(d*e^4) - (4*b^4*PolyLog[4, E^ArcSinh[c + d*x]])/(d*e^
4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{(ce+dex)^4}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^4}{e^4x^4}\,dx,x,c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^4}{x^4}\,dx,x,c+dx\right)}{de^4}$$

$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{3de^4(c+dx)^3} + \frac{(4b)\,\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^3}{x^3\sqrt{1+x^2}}\,dx,x,c+dx\right)}{3de^4}$$

$$= -\frac{2b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{3de^4(c+dx)^2} - \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{3de^4(c+dx)^3} - \frac{(2b)\,\text{Sul}}{}$$

$$= -\frac{2b^2\left(a+b\sinh^{-1}(c+dx)\right)^2}{de^4(c+dx)} - \frac{2b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{3de^4(c+dx)^2} - \frac{(a+}{}$$

$$= -\frac{2b^2\left(a+b\sinh^{-1}(c+dx)\right)^2}{de^4(c+dx)} - \frac{2b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{3de^4(c+dx)^2} - \frac{(a+}{}$$

$$= -\frac{2b^2\left(a+b\sinh^{-1}(c+dx)\right)^2}{de^4(c+dx)} - \frac{2b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{3de^4(c+dx)^2} - \frac{(a+}{}$$

$$= -\frac{2b^2\left(a+b\sinh^{-1}(c+dx)\right)^2}{de^4(c+dx)} - \frac{2b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{3de^4(c+dx)^2} - \frac{(a+}{}$$

$$= -\frac{2b^2\left(a+b\sinh^{-1}(c+dx)\right)^2}{de^4(c+dx)} - \frac{2b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{3de^4(c+dx)^2} - \frac{(a+}{}$$

$$= -\frac{2b^2\left(a+b\sinh^{-1}(c+dx)\right)^2}{de^4(c+dx)} - \frac{2b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{3de^4(c+dx)^2} - \frac{(a+}{}$$

Mathematica [B] time = 8.59, size = 1182, normalized size = 3.07

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^4,x]
```

```
[Out] -1/3*a^4/(d*e^4*(c + d*x)^3) + (a^2*b^2*(-8*PolyLog[2, -E^(-ArcSinh[c + d*x
])] - (2*(-2 + 4*ArcSinh[c + d*x]^2 + 2*Cosh[2*ArcSinh[c + d*x]] - 3*(c + d
*x)*ArcSinh[c + d*x]*Log[1 - E^(-ArcSinh[c + d*x])] + 3*(c + d*x)*ArcSinh[c
 + d*x]*Log[1 + E^(-ArcSinh[c + d*x])] - 4*(c + d*x)^3*PolyLog[2, E^(-ArcSi
nh[c + d*x])] + 2*ArcSinh[c + d*x]*Sinh[2*ArcSinh[c + d*x]] + ArcSinh[c + d
*x]*Log[1 - E^(-ArcSinh[c + d*x])]*Sinh[3*ArcSinh[c + d*x]] - ArcSinh[c + d
*x]*Log[1 + E^(-ArcSinh[c + d*x])]*Sinh[3*ArcSinh[c + d*x]]))/(c + d*x)^3))
/(4*d*e^4) + (a*b^3*(-24*ArcSinh[c + d*x]*Coth[ArcSinh[c + d*x]/2] + 4*ArcS
inh[c + d*x]^3*Coth[ArcSinh[c + d*x]/2] - 6*ArcSinh[c + d*x]^2*Csch[ArcSinh
[c + d*x]/2]^2 - (c + d*x)*ArcSinh[c + d*x]^3*Csch[ArcSinh[c + d*x]/2]^4 -
24*ArcSinh[c + d*x]^2*Log[1 - E^(-ArcSinh[c + d*x])] + 24*ArcSinh[c + d*x]^
2*Log[1 + E^(-ArcSinh[c + d*x])] + 48*Log[Tanh[ArcSinh[c + d*x]/2]] - 48*Ar
cSinh[c + d*x]*PolyLog[2, -E^(-ArcSinh[c + d*x])] + 48*ArcSinh[c + d*x]*Pol
yLog[2, E^(-ArcSinh[c + d*x])] - 48*PolyLog[3, -E^(-ArcSinh[c + d*x])] + 48
*PolyLog[3, E^(-ArcSinh[c + d*x])] - 6*ArcSinh[c + d*x]^2*Sech[ArcSinh[c +
d*x]/2]^2 - (16*ArcSinh[c + d*x]^3*Sinh[ArcSinh[c + d*x]/2]^4)/(c + d*x)^3
+ 24*ArcSinh[c + d*x]*Tanh[ArcSinh[c + d*x]/2] - 4*ArcSinh[c + d*x]^3*Tanh[
ArcSinh[c + d*x]/2]))/(12*d*e^4) + (b^4*(-2*Pi^4 + 4*ArcSinh[c + d*x]^4 - 2
4*ArcSinh[c + d*x]^2*Coth[ArcSinh[c + d*x]/2] + 2*ArcSinh[c + d*x]^4*Coth[A
rcSinh[c + d*x]/2] - 4*ArcSinh[c + d*x]^3*Csch[ArcSinh[c + d*x]/2]^2 - ((c
+ d*x)*ArcSinh[c + d*x]^4*Csch[ArcSinh[c + d*x]/2]^4)/2 + 96*ArcSinh[c + d*
x]*Log[1 - E^(-ArcSinh[c + d*x])] - 96*ArcSinh[c + d*x]*Log[1 + E^(-ArcSinh
[c + d*x])] + 16*ArcSinh[c + d*x]^3*Log[1 + E^(-ArcSinh[c + d*x])] - 16*Arc
Sinh[c + d*x]^3*Log[1 - E^ArcSinh[c + d*x]] - 48*(-2 + ArcSinh[c + d*x]^2)*
PolyLog[2, -E^(-ArcSinh[c + d*x])] - 96*PolyLog[2, E^(-ArcSinh[c + d*x])] -
 48*ArcSinh[c + d*x]^2*PolyLog[2, E^ArcSinh[c + d*x]] - 96*ArcSinh[c + d*x]
*PolyLog[3, -E^(-ArcSinh[c + d*x])] + 96*ArcSinh[c + d*x]*PolyLog[3, E^ArcS
inh[c + d*x]] - 96*PolyLog[4, -E^(-ArcSinh[c + d*x])] - 96*PolyLog[4, E^Arc
Sinh[c + d*x]] - 4*ArcSinh[c + d*x]^3*Sech[ArcSinh[c + d*x]/2]^2 - (8*ArcSi
nh[c + d*x]^4*Sinh[ArcSinh[c + d*x]/2]^4)/(c + d*x)^3 + 24*ArcSinh[c + d*x]
^2*Tanh[ArcSinh[c + d*x]/2] - 2*ArcSinh[c + d*x]^4*Tanh[ArcSinh[c + d*x]/2]
))/(24*d*e^4) + (4*a^3*b*((ArcSinh[c + d*x]*Coth[ArcSinh[c + d*x]/2])/12 -
Csch[ArcSinh[c + d*x]/2]^2/24 - (ArcSinh[c + d*x]*Coth[ArcSinh[c + d*x]/2]*
Csch[ArcSinh[c + d*x]/2]^2)/24 - Log[Tanh[ArcSinh[c + d*x]/2]]/6 - Sech[Arc
Sinh[c + d*x]/2]^2/24 - (ArcSinh[c + d*x]*Tanh[ArcSinh[c + d*x]/2])/12 - (A
rcSinh[c + d*x]*Sech[ArcSinh[c + d*x]/2]^2*Tanh[ArcSinh[c + d*x]/2])/24))/(
d*e^4)
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^4 \operatorname{arsinh}(dx+c)^4 + 4\,ab^3 \operatorname{arsinh}(dx+c)^3 + 6\,a^2b^2 \operatorname{arsinh}(dx+c)^2 + 4\,a^3b \operatorname{arsinh}(dx+c) + a^4}{d^4e^4x^4 + 4\,cd^3e^4x^3 + 6\,c^2d^2e^4x^2 + 4\,c^3de^4x + c^4e^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^4,x, algorithm="fricas")
```

```
[Out] integral((b^4*arcsinh(d*x + c)^4 + 4*a*b^3*arcsinh(d*x + c)^3 + 6*a^2*b^2*a
rcsinh(d*x + c)^2 + 4*a^3*b*arcsinh(d*x + c) + a^4)/(d^4*e^4*x^4 + 4*c*d^3*
e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^3*d*e^4*x + c^4*e^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arsinh}(dx+c) + a)^4}{(dex+ce)^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^4,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^4/(d*e*x + c*e)^4, x)
```

maple [B] time = 0.31, size = 1202, normalized size = 3.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^4,x)
```

```
[Out] -2/d*a^2*b^2/e^4/(d*x+c)^2*arcsinh(d*x+c)*(1+(d*x+c)^2)^(1/2)-2/d*a*b^3/e^4
/(d*x+c)^2*arcsinh(d*x+c)^2*(1+(d*x+c)^2)^(1/2)-4/d*a*b^3/e^4*arcsinh(d*x+c
)*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2))-2/d*a^2*b^2/e^4/(d*x+c)^3*arcsinh(d*
x+c)^2+2/d*a^2*b^2/e^4*arcsinh(d*x+c)*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))-4/3/d
*a*b^3/e^4/(d*x+c)^3*arcsinh(d*x+c)^3-4/d*a*b^3/e^4/(d*x+c)*arcsinh(d*x+c)+
2/d*a*b^3/e^4*arcsinh(d*x+c)^2*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))+4/d*a*b^3/e^
4*arcsinh(d*x+c)*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))-2/d*a*b^3/e^4*arcsin
h(d*x+c)^2*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))-2/d*a^2*b^2/e^4*arcsinh(d*x+c)*l
n(1-d*x-c-(1+(d*x+c)^2)^(1/2))-4/3/d*a^3*b/e^4/(d*x+c)^3*arcsinh(d*x+c)-2/3
/d*a^3*b/e^4/(d*x+c)^2*(1+(d*x+c)^2)^(1/2)-2/3/d*b^4/e^4/(d*x+c)^2*arcsinh(
d*x+c)^3*(1+(d*x+c)^2)^(1/2)+4/d*a*b^3/e^4*polylog(3,d*x+c+(1+(d*x+c)^2)^(1
/2))-8/d*a*b^3/e^4*arctanh(d*x+c+(1+(d*x+c)^2)^(1/2))+4/d*b^4/e^4*arcsinh(d
*x+c)*polylog(3,d*x+c+(1+(d*x+c)^2)^(1/2))-4/d*b^4/e^4*arcsinh(d*x+c)*ln(1+
d*x+c+(1+(d*x+c)^2)^(1/2))+4/d*b^4/e^4*arcsinh(d*x+c)*ln(1-d*x-c-(1+(d*x+c)
^2)^(1/2))-1/3/d*b^4/e^4/(d*x+c)^3*arcsinh(d*x+c)^4-2/d*b^4/e^4/(d*x+c)*arc
sinh(d*x+c)^2+2/3/d*b^4/e^4*arcsinh(d*x+c)^3*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2)
)+2/d*b^4/e^4*arcsinh(d*x+c)^2*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))-4/d*b^
4/e^4*arcsinh(d*x+c)*polylog(3,-d*x-c-(1+(d*x+c)^2)^(1/2))-2/3/d*b^4/e^4*ar
csinh(d*x+c)^3*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))-2/d*b^4/e^4*arcsinh(d*x+c)^2
*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2))+2/d*a^2*b^2/e^4*polylog(2,-d*x-c-(1+(
d*x+c)^2)^(1/2))-2/d*a^2*b^2/e^4*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2))+2/3/d
*a^3*b/e^4*arctanh(1/(1+(d*x+c)^2)^(1/2))-2/d*a^2*b^2/e^4/(d*x+c)-4*b^4*pol
ylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))/d/e^4+4*b^4*polylog(2,d*x+c+(1+(d*x+c)^2
)^(1/2))/d/e^4+4*b^4*polylog(4,-d*x-c-(1+(d*x+c)^2)^(1/2))/d/e^4-4*b^4*poly
log(4,d*x+c+(1+(d*x+c)^2)^(1/2))/d/e^4-4/d*a*b^3/e^4*polylog(3,-d*x-c-(1+(d
*x+c)^2)^(1/2))-1/3/d*a^4/e^4/(d*x+c)^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b^4\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)^4}{3\left(d^4e^4x^3+3\,cd^3e^4x^2+3\,c^2d^2e^4x+c^3de^4\right)}-\frac{a^4}{3\left(d^4e^4x^3+3\,cd^3e^4x^2+3\,c^2d^2e^4x+c^3de^4\right)}+\int\frac{2\left(2\left(3\left(c^3+c\right)ab^3+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^4,x, algorithm="maxima")
```

```
[Out] -1/3*b^4*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^4/(d^4*e^4*x^3 +
3*c*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^4) - 1/3*a^4/(d^4*e^4*x^3 + 3*c
*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^4) + integrate(2/3*(2*(3*(c^3 + c)
*a*b^3 + (c^3 + c)*b^4 + (3*a*b^3*d^3 + b^4*d^3)*x^3 + 3*(3*a*b^3*c*d^2 + b
^4*c*d^2)*x^2 + (3*(3*c^2*d + d)*a*b^3 + (3*c^2*d + d)*b^4)*x + (b^4*c^2 +
3*(c^2 + 1)*a*b^3 + (3*a*b^3*d^2 + b^4*d^2)*x^2 + 2*(3*a*b^3*c*d + b^4*c*d)
*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x
 + c^2 + 1))^3 + 9*(a^2*b^2*d^3*x^3 + 3*a^2*b^2*c*d^2*x^2 + (3*c^2*d + d)*a
^2*b^2*x + (c^3 + c)*a^2*b^2 + (a^2*b^2*d^2*x^2 + 2*a^2*b^2*c*d*x + (c^2 +
1)*a^2*b^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 +
 2*c*d*x + c^2 + 1))^2 + 6*(a^3*b*d^3*x^3 + 3*a^3*b*c*d^2*x^2 + (3*c^2*d +
d)*a^3*b*x + (c^3 + c)*a^3*b + (a^3*b*d^2*x^2 + 2*a^3*b*c*d*x + (c^2 + 1)*a
^3*b)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d
*x + c^2 + 1)))/(d^7*e^4*x^7 + 7*c*d^6*e^4*x^6 + c^7*e^4 + c^5*e^4 + (21*c^
2*d^5*e^4 + d^5*e^4)*x^5 + 5*(7*c^3*d^4*e^4 + c*d^4*e^4)*x^4 + 5*(7*c^4*d^3
```

```
*e^4 + 2*c^2*d^3*e^4)*x^3 + (21*c^5*d^2*e^4 + 10*c^3*d^2*e^4)*x^2 + (7*c^6*
d*e^4 + 5*c^4*d*e^4)*x + (d^6*e^4*x^6 + 6*c*d^5*e^4*x^5 + c^6*e^4 + c^4*e^4
 + (15*c^2*d^4*e^4 + d^4*e^4)*x^4 + 4*(5*c^3*d^3*e^4 + c*d^3*e^4)*x^3 + 3*(
5*c^4*d^2*e^4 + 2*c^2*d^2*e^4)*x^2 + 2*(3*c^5*d*e^4 + 2*c^3*d*e^4)*x)*sqrt(
d^2*x^2 + 2*c*d*x + c^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\mathrm{asinh}\,(c + d\,x))^4}{(c\,e + d\,e\,x)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^4/(c*e + d*e*x)^4,x)
```

```
[Out] int((a + b*asinh(c + d*x))^4/(c*e + d*e*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^4}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{b^4\,\mathrm{asinh}^4\,(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{4ab^3\,\mathrm{asinh}^3\,(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{6a}{c^4+4c^3d}}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**4/(d*e*x+c*e)**4,x)
```

```
[Out] (Integral(a**4/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4
*x**4), x) + Integral(b**4*asinh(c + d*x)**4/(c**4 + 4*c**3*d*x + 6*c**2*d*
*2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x) + Integral(4*a*b**3*asinh(c + d*x)
**3/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x)
+ Integral(6*a**2*b**2*asinh(c + d*x)**2/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x
**2 + 4*c*d**3*x**3 + d**4*x**4), x) + Integral(4*a**3*b*asinh(c + d*x)/(c*
*4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x))/e**4
```

3.155 $\int \frac{(ce+dex)^m}{a+b\sinh^{-1}(c+dx)}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{(e(c+dx))^m}{a+b\sinh^{-1}(c+dx)},x\right)$$

```
[Out] Unintegrable((e*(d*x+c))^m/(a+b*arcsinh(d*x+c)),x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(ce+dex)^m}{a+b\sinh^{-1}(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c*e + d*e*x)^m/(a + b*ArcSinh[c + d*x]),x]
[Out] Defer[Subst][Defer[Int][(e*x)^m/(a + b*ArcSinh[x]), x], x, c + d*x]/d
```

Rubi steps

$$\int \frac{(ce+dex)^m}{a+b\sinh^{-1}(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(ex)^m}{a+b\sinh^{-1}(x)}\,dx,x,c+dx\right)}{d}$$

Mathematica [A] time = 1.15, size = 0, normalized size = 0.00

$$\int \frac{(ce+dex)^m}{a+b\sinh^{-1}(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c*e + d*e*x)^m/(a + b*ArcSinh[c + d*x]),x]
[Out] Integrate[(c*e + d*e*x)^m/(a + b*ArcSinh[c + d*x]), x]
```

fricas [A] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(dex+ce)^m}{b\,\text{arsinh}\,(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m/(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
[Out] integral((d*e*x + c*e)^m/(b*arcsinh(d*x + c) + a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex+ce)^m}{b\,\text{arsinh}\,(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m/(a+b*arcsinh(d*x+c)),x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^m/(b*arcsinh(d*x + c) + a), x)
```

maple [A] time = 0.79, size = 0, normalized size = 0.00

$$\int \frac{(dex + ce)^m}{a + b \operatorname{arcsinh}(dx + c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^m/(a+b*arcsinh(d*x+c)),x)
[Out] int((d*e*x+c*e)^m/(a+b*arcsinh(d*x+c)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex + ce)^m}{b \operatorname{arsinh}(dx + c) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m/(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
[Out] integrate((d*e*x + c*e)^m/(b*arcsinh(d*x + c) + a), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(c\,e + d\,e\,x)^m}{a + b \operatorname{asinh}(c + d\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^m/(a + b*asinh(c + d*x)),x)
[Out] int((c*e + d*e*x)^m/(a + b*asinh(c + d*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\,(c + dx))^m}{a + b \operatorname{asinh}(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**m/(a+b*asinh(d*x+c)),x)
[Out] Integral((e*(c + d*x))**m/(a + b*asinh(c + d*x)), x)
```

3.156 $\int \frac{(ce+dex)^4}{a+b\sinh^{-1}(c+dx)}\,dx$

Optimal. Leaf size=213

$$\frac{e^4\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{8bd}-\frac{3e^4\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{16bd}+\frac{e^4\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{16bd}-\frac{e^4}{}$$

```
[Out] 1/8*e^4*Chi((a+b*arcsinh(d*x+c))/b)*cosh(a/b)/b/d-3/16*e^4*Chi(3*(a+b*arcsi
nh(d*x+c))/b)*cosh(3*a/b)/b/d+1/16*e^4*Chi(5*(a+b*arcsinh(d*x+c))/b)*cosh(5
*a/b)/b/d-1/8*e^4*Shi((a+b*arcsinh(d*x+c))/b)*sinh(a/b)/b/d+3/16*e^4*Shi(3*
(a+b*arcsinh(d*x+c))/b)*sinh(3*a/b)/b/d-1/16*e^4*Shi(5*(a+b*arcsinh(d*x+c))
/b)*sinh(5*a/b)/b/d
```

Rubi [A] time = 0.47, antiderivative size = 209, normalized size of antiderivative = 0.98, number of steps used = 14, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {5865, 12, 5669, 5448, 3303, 3298, 3301}

$$\frac{e^4\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)}{8bd}-\frac{3e^4\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(c+dx)\right)}{16bd}+\frac{e^4\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5a}{b}+5\sinh\right.}{16bd}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4/(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (e^4*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c + d*x]])/(8*b*d) - (3*e^4*Cosh[
(3*a)/b]*CoshIntegral[(3*a)/b + 3*ArcSinh[c + d*x]])/(16*b*d) + (e^4*Cosh[(
5*a)/b]*CoshIntegral[(5*a)/b + 5*ArcSinh[c + d*x]])/(16*b*d) - (e^4*Sinh[a/
b]*SinhIntegral[a/b + ArcSinh[c + d*x]])/(8*b*d) + (3*e^4*Sinh[(3*a)/b]*Sin
hIntegral[(3*a)/b + 3*ArcSinh[c + d*x]])/(16*b*d) - (e^4*Sinh[(5*a)/b]*Sinh
Integral[(5*a)/b + 5*ArcSinh[c + d*x]])/(16*b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
```

```
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^4}{a+b\sinh^{-1}(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{e^4x^4}{a+b\sinh^{-1}(x)}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^4\,\text{Subst}\left(\int \frac{x^4}{a+b\sinh^{-1}(x)}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^4\,\text{Subst}\left(\int \frac{\cosh(x)\sinh^4(x)}{a+bx}\,dx, x, \sinh^{-1}(c+dx)\right)}{d}$$

$$= \frac{e^4\,\text{Subst}\left(\int \left(\frac{\cosh(x)}{8(a+bx)} - \frac{3\cosh(3x)}{16(a+bx)} + \frac{\cosh(5x)}{16(a+bx)}\right) dx, x, \sinh^{-1}(c+dx)\right)}{d}$$

$$= \frac{e^4\,\text{Subst}\left(\int \frac{\cosh(5x)}{a+bx}\,dx, x, \sinh^{-1}(c+dx)\right)}{16d} + \frac{e^4\,\text{Subst}\left(\int \frac{\cosh(x)}{a+bx}\,dx, x, \sinh^{-1}(c+\right.}{8d}$$

$$= \frac{\left(e^4\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \sinh^{-1}(c+dx)\right)}{8d} - \frac{\left(3e^4\cosh\left(\frac{3a}{b}\right)\right)\text{Subst}}{}$$

$$= \frac{e^4\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)}{8bd} - \frac{3e^4\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(c+dx\right.}{16bd}$$

Mathematica [A] time = 0.31, size = 151, normalized size = 0.71

$$e^4\left(2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right) - 3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right) + \cosh\left(\frac{5a}{b}\right)\text{Chi}\left(5\left(\frac{a}{b}+\text{si}\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^4/(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (e^4*(2*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c + d*x]] - 3*Cosh[(3*a)/b]*Co
shIntegral[3*(a/b + ArcSinh[c + d*x])] + Cosh[(5*a)/b]*CoshIntegral[5*(a/b
+ ArcSinh[c + d*x])] - 2*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c + d*x]] + 3
*Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c + d*x])] - Sinh[(5*a)/b]*Sin
hIntegral[5*(a/b + ArcSinh[c + d*x])]))/(16*b*d)
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{d^4e^4x^4 + 4\,cd^3e^4x^3 + 6\,c^2d^2e^4x^2 + 4\,c^3de^4x + c^4e^4}{b\,\text{arsinh}\left(dx+c\right) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c)),x, algorithm="fricas")

[Out] integral((d^4*e^4*x^4 + 4*c*d^3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^3*d*e^4*x + c^4*e^4)/(b*arcsinh(d*x + c) + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex+ce)^4}{b \operatorname{arsinh}(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c)),x, algorithm="giac")

[Out] integrate((d*e*x + c*e)^4/(b*arcsinh(d*x + c) + a), x)

maple [A] time = 0.32, size = 194, normalized size = 0.91

$$\frac{-\frac{e^4 e^{\frac{5a}{b}} \operatorname{Ei}\left(1,5\operatorname{arcsinh}(dx+c)+\frac{5a}{b}\right)}{32b}+\frac{3e^4 e^{\frac{3a}{b}} \operatorname{Ei}\left(1,3\operatorname{arcsinh}(dx+c)+\frac{3a}{b}\right)}{32b}-\frac{e^4 e^{\frac{a}{b}} \operatorname{Ei}\left(1,\operatorname{arcsinh}(dx+c)+\frac{a}{b}\right)}{16b}-\frac{e^4 e^{-\frac{a}{b}} \operatorname{Ei}\left(1,-\operatorname{arcsinh}(dx+c)-\frac{a}{b}\right)}{16b}+\frac{3e^4}{}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c)),x)

[Out] 1/d*(-1/32*e^4/b*exp(5*a/b)*Ei(1,5*arcsinh(d*x+c)+5*a/b)+3/32*e^4/b*exp(3*a/b)*Ei(1,3*arcsinh(d*x+c)+3*a/b)-1/16*e^4/b*exp(a/b)*Ei(1,arcsinh(d*x+c)+a/b)-1/16*e^4/b*exp(-a/b)*Ei(1,-arcsinh(d*x+c)-a/b)+3/32*e^4/b*exp(-3*a/b)*Ei(1,-3*arcsinh(d*x+c)-3*a/b)-1/32*e^4/b*exp(-5*a/b)*Ei(1,-5*arcsinh(d*x+c)-5*a/b))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex+ce)^4}{b \operatorname{arsinh}(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c)),x, algorithm="maxima")

[Out] integrate((d*e*x + c*e)^4/(b*arcsinh(d*x + c) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,e+d\,e\,x)^4}{a+b\,\mathrm{asinh}\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*e + d*e*x)^4/(a + b*asinh(c + d*x)),x)

[Out] int((c*e + d*e*x)^4/(a + b*asinh(c + d*x)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^4\left(\int \frac{c^4}{a+b\operatorname{asinh}(c+dx)}\,dx+\int \frac{d^4x^4}{a+b\operatorname{asinh}(c+dx)}\,dx+\int \frac{4cd^3x^3}{a+b\operatorname{asinh}(c+dx)}\,dx+\int \frac{6c^2d^2x^2}{a+b\operatorname{asinh}(c+dx)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4/(a+b*asinh(d*x+c)),x)
```

```
[Out] e**4*(Integral(c**4/(a + b*asinh(c + d*x)), x) + Integral(d**4*x**4/(a + b*
asinh(c + d*x)), x) + Integral(4*c*d**3*x**3/(a + b*asinh(c + d*x)), x) + I
ntegral(6*c**2*d**2*x**2/(a + b*asinh(c + d*x)), x) + Integral(4*c**3*d*x/(
a + b*asinh(c + d*x)), x))
```

3.157 $\int \frac{(ce+dex)^3}{a+b\sinh^{-1}(c+dx)}\,dx$

Optimal. Leaf size=145

$$\frac{e^3\sinh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2(a+b\sinh^{-1}(c+dx))}{b}\right)}{4bd}-\frac{e^3\sinh\left(\frac{4a}{b}\right)\mathrm{Chi}\left(\frac{4(a+b\sinh^{-1}(c+dx))}{b}\right)}{8bd}-\frac{e^3\cosh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(\frac{2(a+b\sinh^{-1}(c+dx))}{b}\right)}{4bd}+\cdots$$

```
[Out] -1/4*e^3*cosh(2*a/b)*Shi(2*(a+b*arcsinh(d*x+c))/b)/b/d+1/8*e^3*cosh(4*a/b)*
Shi(4*(a+b*arcsinh(d*x+c))/b)/b/d+1/4*e^3*Chi(2*(a+b*arcsinh(d*x+c))/b)*sin
h(2*a/b)/b/d-1/8*e^3*Chi(4*(a+b*arcsinh(d*x+c))/b)*sinh(4*a/b)/b/d
```

Rubi [A] time = 0.34, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {5865, 12, 5669, 5448, 3303, 3298, 3301}

$$\frac{e^3\sinh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(c+dx)\right)}{4bd}-\frac{e^3\sinh\left(\frac{4a}{b}\right)\mathrm{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(c+dx)\right)}{8bd}-\frac{e^3\cosh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(\frac{2a}{b}+2\sinh^{-1}\cdots\right)}{4bd}\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3/(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (e^3*CoshIntegral[(2*a)/b + 2*ArcSinh[c + d*x]]*Sinh[(2*a)/b])/(4*b*d) - (e
^3*CoshIntegral[(4*a)/b + 4*ArcSinh[c + d*x]]*Sinh[(4*a)/b])/(8*b*d) - (e^3
*Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSinh[c + d*x]])/(4*b*d) + (e^3*C
osh[(4*a)/b]*SinhIntegral[(4*a)/b + 4*ArcSinh[c + d*x]])/(8*b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^3}{a+b\sinh^{-1}(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{e^3x^3}{a+b\sinh^{-1}(x)}\,dx,x,c+dx\right)}{d}$$
$$= \frac{e^3\,\text{Subst}\left(\int \frac{x^3}{a+b\sinh^{-1}(x)}\,dx,x,c+dx\right)}{d}$$
$$= \frac{e^3\,\text{Subst}\left(\int \frac{\cosh(x)\sinh^3(x)}{a+bx}\,dx,x,\sinh^{-1}(c+dx)\right)}{d}$$
$$= \frac{e^3\,\text{Subst}\left(\int \left(-\frac{\sinh(2x)}{4(a+bx)}+\frac{\sinh(4x)}{8(a+bx)}\right)dx,x,\sinh^{-1}(c+dx)\right)}{d}$$
$$= \frac{e^3\,\text{Subst}\left(\int \frac{\sinh(4x)}{a+bx}\,dx,x,\sinh^{-1}(c+dx)\right)}{8d} - \frac{e^3\,\text{Subst}\left(\int \frac{\sinh(2x)}{a+bx}\,dx,x,\sinh^{-1}(c+\right.}{4d}$$
$$= -\frac{\left(e^3\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx,x,\sinh^{-1}(c+dx)\right)}{4d} + \frac{\left(e^3\cosh\left(\frac{4a}{b}\right)\right)\text{Sub}}{}$$
$$= \frac{e^3\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(c+dx)\right)\sinh\left(\frac{2a}{b}\right)}{4bd} - \frac{e^3\text{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(c+dx)\right)\sinh\left(\right.}{8bd}$$

Mathematica [A] time = 0.22, size = 109, normalized size = 0.75

$$\frac{e^3\left(2\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)-\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(4\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)-2\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}\right.\right.}{8bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^3/(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (e^3*(2*CoshIntegral[2*(a/b + ArcSinh[c + d*x])]*Sinh[(2*a)/b] - CoshIntegr
al[4*(a/b + ArcSinh[c + d*x])]*Sinh[(4*a)/b] - 2*Cosh[(2*a)/b]*SinhIntegral
[2*(a/b + ArcSinh[c + d*x])] + Cosh[(4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[
c + d*x])]))/(8*b*d)
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{d^3e^3x^3+3\,cd^2e^3x^2+3\,c^2de^3x+c^3e^3}{b\,\text{arsinh}\,(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((d^3*e^3*x^3 + 3*c*d^2*e^3*x^2 + 3*c^2*d*e^3*x + c^3*e^3)/(b*arcsi
nh(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex + ce)^3}{b\,\mathrm{arsinh}\,(dx + c) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^3/(b*arcsinh(d*x + c) + a), x)
```

maple [A] time = 0.25, size = 134, normalized size = 0.92

$$\frac{\frac{e^3 \mathrm{e}^{\frac{4a}{b}}\,\mathrm{Ei}\left(1,4\,\mathrm{arcsinh}(dx+c)+\frac{4a}{b}\right)}{16b}-\frac{e^3 \mathrm{e}^{\frac{2a}{b}}\,\mathrm{Ei}\left(1,2\,\mathrm{arcsinh}(dx+c)+\frac{2a}{b}\right)}{8b}+\frac{e^3 \mathrm{e}^{-\frac{2a}{b}}\,\mathrm{Ei}\left(1,-2\,\mathrm{arcsinh}(dx+c)-\frac{2a}{b}\right)}{8b}-\frac{e^3 \mathrm{e}^{-\frac{4a}{b}}\,\mathrm{Ei}\left(1,-4\,\mathrm{arcsinh}(dx+c)-\frac{4a}{b}\right)}{16b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c)),x)
```

```
[Out] 1/d*(1/16*e^3/b*exp(4*a/b)*Ei(1,4*arcsinh(d*x+c)+4*a/b)-1/8*e^3/b*exp(2*a/b
)*Ei(1,2*arcsinh(d*x+c)+2*a/b)+1/8*e^3/b*exp(-2*a/b)*Ei(1,-2*arcsinh(d*x+c)
-2*a/b)-1/16*e^3/b*exp(-4*a/b)*Ei(1,-4*arcsinh(d*x+c)-4*a/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex + ce)^3}{b\,\mathrm{arsinh}\,(dx + c) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^3/(b*arcsinh(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,e + d\,e\,x)^3}{a + b\,\mathrm{asinh}\,(c + d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3/(a + b*asinh(c + d*x)),x)
```

```
[Out] int((c*e + d*e*x)^3/(a + b*asinh(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^3 \left(\int \frac{c^3}{a + b\,\mathrm{asinh}\,(c + dx)}\,dx + \int \frac{d^3x^3}{a + b\,\mathrm{asinh}\,(c + dx)}\,dx + \int \frac{3cd^2x^2}{a + b\,\mathrm{asinh}\,(c + dx)}\,dx + \int \frac{3c^2dx}{a + b\,\mathrm{asinh}\,(c + dx)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3/(a+b*asinh(d*x+c)),x)
```

```
[Out] e**3*(Integral(c**3/(a + b*asinh(c + d*x)), x) + Integral(d**3*x**3/(a + b*
asinh(c + d*x)), x) + Integral(3*c*d**2*x**2/(a + b*asinh(c + d*x)), x) + I
ntegral(3*c**2*d*x/(a + b*asinh(c + d*x)), x))
```

3.158 $\int \frac{(ce+dex)^2}{a+b\sinh^{-1}(c+dx)}\,dx$

Optimal. Leaf size=141

$$-\frac{e^2\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{4bd}+\frac{e^2\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{4bd}+\frac{e^2\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{4bd}-\frac{e^2\,\mathrm{s}\ldots}{}$$

```
[Out] -1/4*e^2*Chi((a+b*arcsinh(d*x+c))/b)*cosh(a/b)/b/d+1/4*e^2*Chi(3*(a+b*arcsi
nh(d*x+c))/b)*cosh(3*a/b)/b/d+1/4*e^2*Shi((a+b*arcsinh(d*x+c))/b)*sinh(a/b)
/b/d-1/4*e^2*Shi(3*(a+b*arcsinh(d*x+c))/b)*sinh(3*a/b)/b/d
```

Rubi [A] time = 0.29, antiderivative size = 137, normalized size of antiderivative = 0.97, number of steps used = 11, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {5865, 12, 5669, 5448, 3303, 3298, 3301}

$$-\frac{e^2\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)}{4bd}+\frac{e^2\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(c+dx)\right)}{4bd}+\frac{e^2\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\sinh^{-1}\ldots\right)}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2/(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] -(e^2*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c + d*x]])/(4*b*d) + (e^2*Cosh[(
3*a)/b]*CoshIntegral[(3*a)/b + 3*ArcSinh[c + d*x]])/(4*b*d) + (e^2*Sinh[a/b
]*SinhIntegral[a/b + ArcSinh[c + d*x]])/(4*b*d) - (e^2*Sinh[(3*a)/b]*SinhIn
tegral[(3*a)/b + 3*ArcSinh[c + d*x]])/(4*b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(ce+dex)^2}{a+b\sinh^{-1}(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{e^2x^2}{a+b\sinh^{-1}(x)}\,dx, x, c+dx\right)}{d} \\
&= \frac{e^2\,\text{Subst}\left(\int \frac{x^2}{a+b\sinh^{-1}(x)}\,dx, x, c+dx\right)}{d} \\
&= \frac{e^2\,\text{Subst}\left(\int \frac{\cosh(x)\sinh^2(x)}{a+bx}\,dx, x, \sinh^{-1}(c+dx)\right)}{d} \\
&= \frac{e^2\,\text{Subst}\left(\int \left(-\frac{\cosh(x)}{4(a+bx)}+\frac{\cosh(3x)}{4(a+bx)}\right)dx, x, \sinh^{-1}(c+dx)\right)}{d} \\
&= -\frac{e^2\,\text{Subst}\left(\int \frac{\cosh(x)}{a+bx}\,dx, x, \sinh^{-1}(c+dx)\right)}{4d}+\frac{e^2\,\text{Subst}\left(\int \frac{\cosh(3x)}{a+bx}\,dx, x, \sinh^{-1}(c+d\right.}{4d} \\
&= -\frac{\left(e^2\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \sinh^{-1}(c+dx)\right)}{4d}+\frac{\left(e^2\cosh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int\right.}{} \\
&= -\frac{e^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)}{4bd}+\frac{e^2\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(c+dx)\right)}{4bd}
\end{aligned}
$$

Mathematica [A] time = 0.18, size = 102, normalized size = 0.72

$$
\frac{e^2\left(-\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)+\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)+\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(c+\right.\right.}{4bd}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^2/(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (e^2*(-(Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c + d*x]]) + Cosh[(3*a)/b]*Cos
hIntegral[3*(a/b + ArcSinh[c + d*x])] + Sinh[a/b]*SinhIntegral[a/b + ArcSin
h[c + d*x]] - Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c + d*x])]))/(4*b
*d)
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$
\text{integral}\left(\frac{d^2e^2x^2+2\,cde^2x+c^2e^2}{b\,\text{arsinh}\,(dx+c)+a}, x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
[Out] integral((d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2)/(b*arcsinh(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex + ce)^2}{b \operatorname{arsinh}(dx + c) + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c)),x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^2/(b*arcsinh(d*x + c) + a), x)
```

maple [A] time = 0.16, size = 130, normalized size = 0.92

$$\frac{-\frac{e^2 e^{\frac{3a}{b}} \operatorname{Ei}\left(1, 3 \operatorname{arcsinh}(dx+c)+\frac{3a}{b}\right)}{8b} + \frac{e^2 e^{\frac{a}{b}} \operatorname{Ei}\left(1, \operatorname{arcsinh}(dx+c)+\frac{a}{b}\right)}{8b} + \frac{e^2 e^{-\frac{a}{b}} \operatorname{Ei}\left(1, -\operatorname{arcsinh}(dx+c)-\frac{a}{b}\right)}{8b} - \frac{e^2 e^{-\frac{3a}{b}} \operatorname{Ei}\left(1, -3 \operatorname{arcsinh}(dx+c)-\frac{3a}{b}\right)}{8b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c)),x)
[Out] 1/d*(-1/8*e^2/b*exp(3*a/b)*Ei(1,3*arcsinh(d*x+c)+3*a/b)+1/8*e^2/b*exp(a/b)*
Ei(1,arcsinh(d*x+c)+a/b)+1/8*e^2/b*exp(-a/b)*Ei(1,-arcsinh(d*x+c)-a/b)-1/8*
e^2/b*exp(-3*a/b)*Ei(1,-3*arcsinh(d*x+c)-3*a/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex + ce)^2}{b \operatorname{arsinh}(dx + c) + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
[Out] integrate((d*e*x + c*e)^2/(b*arcsinh(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,e + d\,e\,x)^2}{a + b \operatorname{asinh}(c + d\,x)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2/(a + b*asinh(c + d*x)),x)
[Out] int((c*e + d*e*x)^2/(a + b*asinh(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^2 \left(\int \frac{c^2}{a + b \operatorname{asinh}(c + dx)} \, dx + \int \frac{d^2 x^2}{a + b \operatorname{asinh}(c + dx)} \, dx + \int \frac{2cdx}{a + b \operatorname{asinh}(c + dx)} \, dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2/(a+b*asinh(d*x+c)),x)
[Out] e**2*(Integral(c**2/(a + b*asinh(c + d*x)), x) + Integral(d**2*x**2/(a + b*
asinh(c + d*x)), x) + Integral(2*c*d*x/(a + b*asinh(c + d*x)), x))
```

3.159 $\int \frac{ce+dex}{a+b\sinh^{-1}(c+dx)}\,dx$

Optimal. Leaf size=69

$$\frac{e\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{2bd}-\frac{e\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{2bd}$$

```
[Out] 1/2*e*cosh(2*a/b)*Shi(2*(a+b*arcsinh(d*x+c))/b)/b/d-1/2*e*Chi(2*(a+b*arcsin
h(d*x+c))/b)*sinh(2*a/b)/b/d
```

Rubi [A] time = 0.16, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5865, 12, 5669, 5448, 3303, 3298, 3301}

$$\frac{e\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\sinh^{-1}(c+dx)\right)}{2bd}-\frac{e\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(c+dx)\right)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)/(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] -(e*CoshIntegral[(2*a)/b + 2*ArcSinh[c + d*x]]*Sinh[(2*a)/b])/(2*b*d) + (e*
Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSinh[c + d*x]])/(2*b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
```

```
x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{ce+dex}{a+b\sinh^{-1}(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{ex}{a+b\sinh^{-1}(x)}\,dx, x, c+dx\right)}{d}\\
&= \frac{e\,\text{Subst}\left(\int \frac{x}{a+b\sinh^{-1}(x)}\,dx, x, c+dx\right)}{d}\\
&= \frac{e\,\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{a+bx}\,dx, x, \sinh^{-1}(c+dx)\right)}{d}\\
&= \frac{e\,\text{Subst}\left(\int \frac{\sinh(2x)}{2(a+bx)}\,dx, x, \sinh^{-1}(c+dx)\right)}{d}\\
&= \frac{e\,\text{Subst}\left(\int \frac{\sinh(2x)}{a+bx}\,dx, x, \sinh^{-1}(c+dx)\right)}{2d}\\
&= \frac{\left(e\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \sinh^{-1}(c+dx)\right)}{2d} - \frac{\left(e\sinh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\right.}{}\\
&= -\frac{e\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(c+dx)\right)\sinh\left(\frac{2a}{b}\right)}{2bd} + \frac{e\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\sinh^{-1}(c+d\right.}{2bd}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 61, normalized size = 0.88

$$-\frac{e\left(\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(c+dx)\right)-\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\sinh^{-1}(c+dx)\right)\right)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)/(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] -1/2*(e*(CoshIntegral[(2*a)/b + 2*ArcSinh[c + d*x]]*Sinh[(2*a)/b] - Cosh[(2
*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSinh[c + d*x]]))/(b*d)
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{dex+ce}{b\,\text{arsinh}\,(dx+c)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((d*e*x + c*e)/(b*arcsinh(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dex+ce}{b\,\text{arsinh}\,(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)/(b*arcsinh(d*x + c) + a), x)
```

maple [A] time = 0.04, size = 66, normalized size = 0.96

$$\frac{\frac{e\,\mathrm{e}^{\frac{2a}{b}}\operatorname{Ei}\left(1, 2\operatorname{arcsinh}(dx+c)+\frac{2a}{b}\right)}{4b}-\frac{e\,\mathrm{e}^{-\frac{2a}{b}}\operatorname{Ei}\left(1, -2\operatorname{arcsinh}(dx+c)-\frac{2a}{b}\right)}{4b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)/(a+b*arcsinh(d*x+c)),x)
```

```
[Out] 1/d*(1/4*e/b*exp(2*a/b)*Ei(1,2*arcsinh(d*x+c)+2*a/b)-1/4*e/b*exp(-2*a/b)*Ei
(1,-2*arcsinh(d*x+c)-2*a/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dex + ce}{b\operatorname{arsinh}(dx + c) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)/(b*arcsinh(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c\,e + d\,e\,x}{a + b\,\mathrm{asinh}\left(c + d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)/(a + b*asinh(c + d*x)),x)
```

```
[Out] int((c*e + d*e*x)/(a + b*asinh(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e\left(\int \frac{c}{a + b\operatorname{asinh}(c + dx)}\,dx + \int \frac{dx}{a + b\operatorname{asinh}(c + dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*asinh(d*x+c)),x)
```

```
[Out] e*(Integral(c/(a + b*asinh(c + d*x)), x) + Integral(d*x/(a + b*asinh(c + d*
x)), x))
```

3.160 $\int \frac{1}{a+b\sinh^{-1}(c+dx)}\,dx$

Optimal. Leaf size=58

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{bd}-\frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{bd}$$

```
[Out] Chi((a+b*arcsinh(d*x+c))/b)*cosh(a/b)/b/d-Shi((a+b*arcsinh(d*x+c))/b)*sinh(
a/b)/b/d
```

Rubi [A] time = 0.09, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5863, 5657, 3303, 3298, 3301}

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{bd}-\frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^(-1),x]
```

```
[Out] (Cosh[a/b]*CoshIntegral[(a + b*ArcSinh[c + d*x])/b])/(b*d) - (Sinh[a/b]*Sin
hIntegral[(a + b*ArcSinh[c + d*x])/b])/(b*d)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 5863

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \frac{1}{a+b\sinh^{-1}(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{a+b\sinh^{-1}(x)}\,dx, x, c+dx\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\sinh^{-1}(c+dx)\right)}{bd}$$
$$= \frac{\cosh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\sinh^{-1}(c+dx)\right)}{bd} - \frac{\sinh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x}\,d\right.}{b}$$
$$= \frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{bd} - \frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{bd}$$

Mathematica [A] time = 0.02, size = 49, normalized size = 0.84

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)-\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^(-1),x]
[Out] (Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c + d*x]] - Sinh[a/b]*SinhIntegral[a/
b + ArcSinh[c + d*x]])/(b*d)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{b\,\text{arsinh}\,(dx+c)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
[Out] integral(1/(b*arcsinh(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{b\,\text{arsinh}\,(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c)),x, algorithm="giac")
[Out] integrate(1/(b*arcsinh(d*x + c) + a), x)
```

maple [A] time = 0.03, size = 60, normalized size = 1.03

$$\frac{-\frac{e^{\frac{a}{b}}\,\text{Ei}\left(1,\text{arcsinh}(dx+c)+\frac{a}{b}\right)}{2b}-\frac{e^{-\frac{a}{b}}\,\text{Ei}\left(1,-\text{arcsinh}(dx+c)-\frac{a}{b}\right)}{2b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(d*x+c)),x)
[Out] 1/d*(-1/2/b*exp(a/b)*Ei(1,arcsinh(d*x+c)+a/b)-1/2/b*exp(-a/b)*Ei(1,-arcsinh
(d*x+c)-a/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{b \operatorname{arsinh}(dx + c) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*arcsinh(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{a + b \operatorname{asinh}(c + d\, x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*asinh(c + d*x)),x)
```

```
[Out] int(1/(a + b*asinh(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + b \operatorname{asinh}(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(d*x+c)),x)
```

```
[Out] Integral(1/(a + b*asinh(c + d*x)), x)
```

3.161 $\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)}\,dx$

Optimal. Leaf size=27

$$\frac{\text{Int}\left(\frac{1}{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)},x\right)}{e}$$

[Out] Unintegrable(1/(d*x+c)/(a+b*arcsinh(d*x+c)),x)/e

Rubi [A] time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)}\,dx$$

Verification is Not applicable to the result.

[In] Int[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])),x]

[Out] Defer[Subst][Defer[Int][1/(x*(a + b*ArcSinh[x])), x], x, c + d*x]/(d*e)

Rubi steps

$$\begin{aligned}\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{ex\left(a+b\sinh^{-1}(x)\right)}\,dx,x,c+dx\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{1}{x\left(a+b\sinh^{-1}(x)\right)}\,dx,x,c+dx\right)}{de}\end{aligned}$$

Mathematica [A] time = 0.84, size = 0, normalized size = 0.00

$$\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])),x]

[Out] Integrate[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])), x]

fricas [A] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{adex+ace+(bdex+bce)\,\text{arsinh}\,(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c)),x, algorithm="fricas")

[Out] integral(1/(a*d*e*x + a*c*e + (b*d*e*x + b*c*e)*arcsinh(d*x + c)), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dex+ce)(b\,\text{arsinh}\,(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c)),x, algorithm="giac")
[Out] integrate(1/((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)), x)
```

maple [A] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{1}{(dex+ce)\,(a+b\,\mathrm{arcsinh}\,(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c)),x)
[Out] int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dex+ce)(b\,\mathrm{arsinh}\,(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
[Out] integrate(1/((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{(c\,e+d\,e\,x)\,\left(a+b\,\mathrm{asinh}\,(c+d\,x)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*e + d*e*x)*(a + b*asinh(c + d*x))),x)
[Out] int(1/((c*e + d*e*x)*(a + b*asinh(c + d*x))), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{ac+adx+bc\,\mathrm{asinh}\,(c+dx)+bdx\,\mathrm{asinh}\,(c+dx)}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*asinh(d*x+c)),x)
[Out] Integral(1/(a*c + a*d*x + b*c*asinh(c + d*x) + b*d*x*asinh(c + d*x)), x)/e
```

3.162 $\int \frac{(ce+dex)^4}{\left(a+b\sinh^{-1}(c+dx)\right)^2}\,dx$

Optimal. Leaf size=256

$$-\frac{e^4\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{8b^2d}+\frac{9e^4\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{16b^2d}-\frac{5e^4\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{16b^2d}+\cdots$$

```
[Out] 1/8*e^4*cosh(a/b)*Shi((a+b*arcsinh(d*x+c))/b)/b^2/d-9/16*e^4*cosh(3*a/b)*Sh
i(3*(a+b*arcsinh(d*x+c))/b)/b^2/d+5/16*e^4*cosh(5*a/b)*Shi(5*(a+b*arcsinh(d
*x+c))/b)/b^2/d-1/8*e^4*Chi((a+b*arcsinh(d*x+c))/b)*sinh(a/b)/b^2/d+9/16*e^
4*Chi(3*(a+b*arcsinh(d*x+c))/b)*sinh(3*a/b)/b^2/d-5/16*e^4*Chi(5*(a+b*arcsi
nh(d*x+c))/b)*sinh(5*a/b)/b^2/d-e^4*(d*x+c)^4*(1+(d*x+c)^2)^(1/2)/b/d/(a+b*
arcsinh(d*x+c))
```

Rubi [A] time = 0.41, antiderivative size = 252, normalized size of antiderivative = 0.98, number of steps used = 13, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {5865, 12, 5665, 3303, 3298, 3301}

$$-\frac{e^4\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)}{8b^2d}+\frac{9e^4\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(c+dx)\right)}{16b^2d}-\frac{5e^4\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5a}{b}+5\sin\cdots\right.}{16b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4/(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] -((e^4*(c + d*x)^4*Sqrt[1 + (c + d*x)^2])/(b*d*(a + b*ArcSinh[c + d*x]))) -
 (e^4*CoshIntegral[a/b + ArcSinh[c + d*x]]*Sinh[a/b])/(8*b^2*d) + (9*e^4*Co
shIntegral[(3*a)/b + 3*ArcSinh[c + d*x]]*Sinh[(3*a)/b])/(16*b^2*d) - (5*e^4
*CoshIntegral[(5*a)/b + 5*ArcSinh[c + d*x]]*Sinh[(5*a)/b])/(16*b^2*d) + (e^
4*Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c + d*x]])/(8*b^2*d) - (9*e^4*Cosh[(
3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSinh[c + d*x]])/(16*b^2*d) + (5*e^4*Cos
h[(5*a)/b]*SinhIntegral[(5*a)/b + 5*ArcSinh[c + d*x]])/(16*b^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^4}{\left(a+b\sinh^{-1}(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{e^4x^4}{\left(a+b\sinh^{-1}(x)\right)^2}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e^4\,\text{Subst}\left(\int \frac{x^4}{\left(a+b\sinh^{-1}(x)\right)^2}\,dx,x,c+dx\right)}{d}$$

$$= -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{bd\left(a+b\sinh^{-1}(c+dx)\right)}+\frac{e^4\,\text{Subst}\left(\int \left(\frac{\sinh(x)}{8(a+bx)}-\frac{9\sinh(3x)}{16(a+bx)}+\frac{5\sinh(5x)}{16(a+bx)}\right)dx,\right.}{bd}$$

$$= -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{bd\left(a+b\sinh^{-1}(c+dx)\right)}+\frac{e^4\,\text{Subst}\left(\int \frac{\sinh(x)}{a+bx}\,dx,x,\sinh^{-1}(c+dx)\right)}{8bd}+\frac{(5e}{}$$

$$= -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{bd\left(a+b\sinh^{-1}(c+dx)\right)}+\frac{\left(e^4\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx,x,\sinh^{-1}(\right.}{8bd}$$

$$= -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{bd\left(a+b\sinh^{-1}(c+dx)\right)}-\frac{e^4\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\sinh\left(\frac{a}{b}\right)}{8b^2d}+\frac{9e^4\text{Chi}}{}$$

Mathematica [A] time = 1.10, size = 281, normalized size = 1.10

$$e^4\left(16\left(3\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)-\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)-3\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sin\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^4/(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] (e^4*((-16*b*(c + d*x)^4*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh[c + d*x]) +
16*(3*CoshIntegral[a/b + ArcSinh[c + d*x]]*Sinh[a/b] - CoshIntegral[3*(a/b
+ ArcSinh[c + d*x])]*Sinh[(3*a)/b] - 3*Cosh[a/b]*SinhIntegral[a/b + ArcSinh
[c + d*x]] + Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c + d*x])]) - 5*(1
0*CoshIntegral[a/b + ArcSinh[c + d*x]]*Sinh[a/b] - 5*CoshIntegral[3*(a/b +
ArcSinh[c + d*x])]*Sinh[(3*a)/b] + CoshIntegral[5*(a/b + ArcSinh[c + d*x])]
*Sinh[(5*a)/b] - 10*Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c + d*x]] + 5*Cosh
[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c + d*x])] - Cosh[(5*a)/b]*SinhInte
gral[5*(a/b + ArcSinh[c + d*x])])))/(16*b^2*d)
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{d^4e^4x^4 + 4\,cd^3e^4x^3 + 6\,c^2d^2e^4x^2 + 4\,c^3de^4x + c^4e^4}{b^2\,\text{arsinh}\,(dx+c)^2 + 2\,ab\,\text{arsinh}\,(dx+c) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((d^4*e^4*x^4 + 4*c*d^3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^3*d*e^4*x
 + c^4*e^4)/(b^2*arcsinh(d*x + c)^2 + 2*a*b*arcsinh(d*x + c) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex+ce)^4}{(b\,\text{arsinh}\,(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^4/(b*arcsinh(d*x + c) + a)^2, x)
```

maple [B] time = 0.38, size = 602, normalized size = 2.35

$$\frac{\left(16(dx+c)^5 - 16(dx+c)^4\sqrt{1+(dx+c)^2} + 20(dx+c)^3 - 12(dx+c)^2\sqrt{1+(dx+c)^2} + 5dx + 5c - \sqrt{1+(dx+c)^2}\right)e^4}{32b(a+b\,\text{arcsinh}(dx+c))} + \frac{5e^4 e^{\frac{5a}{b}}\,\text{Ei}\left(1, 5\,\text{arcsinh}(dx+c) + \frac{5a}{b}\right)}{32b^2} - \frac{3\left(-4(d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^2,x)
```

```
[Out] 1/d*(1/32*(16*(d*x+c)^5-16*(d*x+c)^4*(1+(d*x+c)^2)^(1/2)+20*(d*x+c)^3-12*(d
*x+c)^2*(1+(d*x+c)^2)^(1/2)+5*d*x+5*c-(1+(d*x+c)^2)^(1/2))*e^4/b/(a+b*arcsi
nh(d*x+c))+5/32*e^4/b^2*exp(5*a/b)*Ei(1,5*arcsinh(d*x+c)+5*a/b)-3/32*(-4*(d
*x+c)^2*(1+(d*x+c)^2)^(1/2)+4*(d*x+c)^3-(1+(d*x+c)^2)^(1/2)+3*d*x+3*c)*e^4/
b/(a+b*arcsinh(d*x+c))-9/32*e^4/b^2*exp(3*a/b)*Ei(1,3*arcsinh(d*x+c)+3*a/b)
+1/16*(-(1+(d*x+c)^2)^(1/2)+d*x+c)*e^4/b/(a+b*arcsinh(d*x+c))+1/16*e^4/b^2*
exp(a/b)*Ei(1,arcsinh(d*x+c)+a/b)-1/16/b*e^4*(d*x+c+(1+(d*x+c)^2)^(1/2))/(a
+b*arcsinh(d*x+c))-1/16/b^2*e^4*exp(-a/b)*Ei(1,-arcsinh(d*x+c)-a/b)+3/32/b*
e^4*(4*(d*x+c)^3+3*d*x+3*c+4*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+(1+(d*x+c)^2)^(1
/2))/(a+b*arcsinh(d*x+c))+9/32/b^2*e^4*exp(-3*a/b)*Ei(1,-3*arcsinh(d*x+c)-3
*a/b)-1/32/b*e^4*(16*(d*x+c)^5+20*(d*x+c)^3+16*(d*x+c)^4*(1+(d*x+c)^2)^(1/2
)+5*d*x+5*c+12*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+(1+(d*x+c)^2)^(1/2))/(a+b*arcs
inh(d*x+c))-5/32/b^2*e^4*exp(-5*a/b)*Ei(1,-5*arcsinh(d*x+c)-5*a/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(d^7*e^4*x^7 + 7*c*d^6*e^4*x^6 + c^7*e^4 + c^5*e^4 + (21*c^2*d^5*e^4 + d^5
*e^4)*x^5 + 5*(7*c^3*d^4*e^4 + c*d^4*e^4)*x^4 + 5*(7*c^4*d^3*e^4 + 2*c^2*d^
3*e^4)*x^3 + (21*c^5*d^2*e^4 + 10*c^3*d^2*e^4)*x^2 + (7*c^6*d*e^4 + 5*c^4*d
*e^4)*x + (d^6*e^4*x^6 + 6*c*d^5*e^4*x^5 + c^6*e^4 + c^4*e^4 + (15*c^2*d^4*
e^4 + d^4*e^4)*x^4 + 4*(5*c^3*d^3*e^4 + c*d^3*e^4)*x^3 + 3*(5*c^4*d^2*e^4 +
 2*c^2*d^2*e^4)*x^2 + 2*(3*c^5*d*e^4 + 2*c^3*d*e^4)*x)*sqrt(d^2*x^2 + 2*c*d
*x + c^2 + 1))/(a*b*d^3*x^2 + 2*a*b*c*d^2*x + (c^2*d + d)*a*b + (b^2*d^3*x^
```

```
2 + 2*b^2*c*d^2*x + (c^2*d + d)*b^2 + (b^2*d^2*x + b^2*c*d)*sqrt(d^2*x^2 +
2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + (a*b
*d^2*x + a*b*c*d)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + integrate((5*d^8*e^4
*x^8 + 40*c*d^7*e^4*x^7 + 5*c^8*e^4 + 10*c^6*e^4 + 5*c^4*e^4 + 10*(14*c^2*d
^6*e^4 + d^6*e^4)*x^6 + 20*(14*c^3*d^5*e^4 + 3*c*d^5*e^4)*x^5 + 5*(70*c^4*d
^4*e^4 + 30*c^2*d^4*e^4 + d^4*e^4)*x^4 + 20*(14*c^5*d^3*e^4 + 10*c^3*d^3*e^
4 + c*d^3*e^4)*x^3 + 10*(14*c^6*d^2*e^4 + 15*c^4*d^2*e^4 + 3*c^2*d^2*e^4)*x
^2 + (5*d^6*e^4*x^6 + 30*c*d^5*e^4*x^5 + 5*c^6*e^4 + 3*c^4*e^4 + 3*(25*c^2*
d^4*e^4 + d^4*e^4)*x^4 + 4*(25*c^3*d^3*e^4 + 3*c*d^3*e^4)*x^3 + 3*(25*c^4*d
^2*e^4 + 6*c^2*d^2*e^4)*x^2 + 6*(5*c^5*d*e^4 + 2*c^3*d*e^4)*x)*(d^2*x^2 + 2
*c*d*x + c^2 + 1) + 20*(2*c^7*d*e^4 + 3*c^5*d*e^4 + c^3*d*e^4)*x + (10*d^7*
e^4*x^7 + 70*c*d^6*e^4*x^6 + 10*c^7*e^4 + 13*c^5*e^4 + 4*c^3*e^4 + (210*c^2
*d^5*e^4 + 13*d^5*e^4)*x^5 + 5*(70*c^3*d^4*e^4 + 13*c*d^4*e^4)*x^4 + 2*(175
*c^4*d^3*e^4 + 65*c^2*d^3*e^4 + 2*d^3*e^4)*x^3 + 2*(105*c^5*d^2*e^4 + 65*c^
3*d^2*e^4 + 6*c*d^2*e^4)*x^2 + (70*c^6*d*e^4 + 65*c^4*d*e^4 + 12*c^2*d*e^4)
*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(a*b*d^4*x^4 + 4*a*b*c*d^3*x^3 + 2*(
3*c^2*d^2 + d^2)*a*b*x^2 + 4*(c^3*d + c*d)*a*b*x + (c^4 + 2*c^2 + 1)*a*b +
(a*b*d^2*x^2 + 2*a*b*c*d*x + a*b*c^2)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + (b^2*
d^4*x^4 + 4*b^2*c*d^3*x^3 + 2*(3*c^2*d^2 + d^2)*b^2*x^2 + 4*(c^3*d + c*d)*b
^2*x + (c^4 + 2*c^2 + 1)*b^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*(d^2*x
^2 + 2*c*d*x + c^2 + 1) + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + (3*c^2*d + d)*
b^2*x + (c^3 + c)*b^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqr
t(d^2*x^2 + 2*c*d*x + c^2 + 1)) + 2*(a*b*d^3*x^3 + 3*a*b*c*d^2*x^2 + (3*c^2
*d + d)*a*b*x + (c^3 + c)*a*b)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,e+d\,e\,x)^4}{(a+b\,\mathrm{asinh}\,(c+d\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^4/(a + b*asinh(c + d*x))^2,x)
```

```
[Out] int((c*e + d*e*x)^4/(a + b*asinh(c + d*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^4\left(\int \frac{c^4}{a^2+2ab\,\mathrm{asinh}\,(c+dx)+b^2\,\mathrm{asinh}^2\,(c+dx)}\,dx+\int \frac{d^4x^4}{a^2+2ab\,\mathrm{asinh}\,(c+dx)+b^2\,\mathrm{asinh}^2\,(c+dx)}\,dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4/(a+b*asinh(d*x+c))**2,x)
```

```
[Out] e**4*(Integral(c**4/(a**2 + 2*a*b*asinh(c + d*x) + b**2*asinh(c + d*x)**2),
 x) + Integral(d**4*x**4/(a**2 + 2*a*b*asinh(c + d*x) + b**2*asinh(c + d*x)
**2), x) + Integral(4*c*d**3*x**3/(a**2 + 2*a*b*asinh(c + d*x) + b**2*asinh
(c + d*x)**2), x) + Integral(6*c**2*d**2*x**2/(a**2 + 2*a*b*asinh(c + d*x)
+ b**2*asinh(c + d*x)**2), x) + Integral(4*c**3*d*x/(a**2 + 2*a*b*asinh(c +
 d*x) + b**2*asinh(c + d*x)**2), x))
```

3.163 $\int \frac{(ce+dex)^3}{\left(a+b\sinh^{-1}(c+dx)\right)^2}\,dx$

Optimal. Leaf size=188

$$-\frac{e^3\cosh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{2b^2d}+\frac{e^3\cosh\left(\frac{4a}{b}\right)\mathrm{Chi}\left(\frac{4\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{2b^2d}+\frac{e^3\sinh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{2b^2d}-$$

```
[Out] -1/2*e^3*Chi(2*(a+b*arcsinh(d*x+c))/b)*cosh(2*a/b)/b^2/d+1/2*e^3*Chi(4*(a+b
*arcsinh(d*x+c))/b)*cosh(4*a/b)/b^2/d+1/2*e^3*Shi(2*(a+b*arcsinh(d*x+c))/b)
*sinh(2*a/b)/b^2/d-1/2*e^3*Shi(4*(a+b*arcsinh(d*x+c))/b)*sinh(4*a/b)/b^2/d-
e^3*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))
```

Rubi [A] time = 0.31, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {5865, 12, 5665, 3303, 3298, 3301}

$$-\frac{e^3\cosh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(c+dx)\right)}{2b^2d}+\frac{e^3\cosh\left(\frac{4a}{b}\right)\mathrm{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(c+dx)\right)}{2b^2d}+\frac{e^3\sinh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(\frac{2a}{b}+2\,\mathrm{si}\right.}{2b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3/(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] -((e^3*(c + d*x)^3*Sqrt[1 + (c + d*x)^2])/(b*d*(a + b*ArcSinh[c + d*x]))) -
 (e^3*Cosh[(2*a)/b]*CoshIntegral[(2*a)/b + 2*ArcSinh[c + d*x]])/(2*b^2*d) +
 (e^3*Cosh[(4*a)/b]*CoshIntegral[(4*a)/b + 4*ArcSinh[c + d*x]])/(2*b^2*d) +
 (e^3*Sinh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSinh[c + d*x]])/(2*b^2*d) -
 (e^3*Sinh[(4*a)/b]*SinhIntegral[(4*a)/b + 4*ArcSinh[c + d*x]])/(2*b^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
```

```
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^3}{\left(a+b\sinh^{-1}(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{e^3x^3}{\left(a+b\sinh^{-1}(x)\right)^2}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^3\,\text{Subst}\left(\int \frac{x^3}{\left(a+b\sinh^{-1}(x)\right)^2}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{bd\left(a+b\sinh^{-1}(c+dx)\right)} + \frac{e^3\,\text{Subst}\left(\int \left(-\frac{\cosh(2x)}{2(a+bx)}+\frac{\cosh(4x)}{2(a+bx)}\right)dx, x, \sinh^{-1}\right.}{bd}$$

$$= -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{bd\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{e^3\,\text{Subst}\left(\int \frac{\cosh(2x)}{a+bx}\,dx, x, \sinh^{-1}(c+dx)\right)}{2bd} + \frac{e^3}{\ldots}$$

$$= -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{bd\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{\left(e^3\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \sinh\right.}{2bd}$$

$$= -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{bd\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{e^3\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(c+dx)\right)}{2b^2d} + \frac{e^3}{\ldots}$$

Mathematica [A] time = 0.88, size = 193, normalized size = 1.03

$$-\frac{e^3\left(\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)-\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(4\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)-4\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}\right.\right.\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^3/(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] -1/2*(e^3*((2*b*(c + d*x)^3*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh[c + d*x])
 + Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcSinh[c + d*x])] - Cosh[(4*a)/b]*C
oshIntegral[4*(a/b + ArcSinh[c + d*x])] - 3*Log[a + b*ArcSinh[c + d*x]] - 4
*Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c + d*x])] + 3*(Log[a + b*ArcS
inh[c + d*x]] + Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c + d*x])]) + S
inh[(4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[c + d*x])]))/(b^2*d)
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{d^3e^3x^3+3\,cd^2e^3x^2+3\,c^2de^3x+c^3e^3}{b^2\,\text{arsinh}\left(dx+c\right)^2+2\,ab\,\text{arsinh}\left(dx+c\right)+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((d^3*e^3*x^3 + 3*c*d^2*e^3*x^2 + 3*c^2*d*e^3*x + c^3*e^3)/(b^2*arc
sinh(d*x + c)^2 + 2*a*b*arcsinh(d*x + c) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex+ce)^3}{(b\operatorname{arsinh}(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^3/(b*arcsinh(d*x + c) + a)^2, x)
```

maple [B] time = 0.30, size = 388, normalized size = 2.06

$$\frac{\frac{\left(8(dx+c)^4-8(dx+c)^3\sqrt{1+(dx+c)^2}+8(dx+c)^2-4(dx+c)\sqrt{1+(dx+c)^2}+1\right)e^3}{16(a+b\operatorname{arcsinh}(dx+c))b}-\frac{e^3\mathrm{e}^{\frac{4a}{b}}\operatorname{Ei}\left(1,4\operatorname{arcsinh}(dx+c)+\frac{4a}{b}\right)}{4b^2}-\frac{\left(2(dx+c)^2-2(dx+c)\sqrt{1+(dx+c)^2}+1\right.}{8(a+b\operatorname{arcsinh}(dx+c))b}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^2,x)
```

```
[Out] 1/d*(1/16*(8*(d*x+c)^4-8*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)+8*(d*x+c)^2-4*(d*x+c
)*(1+(d*x+c)^2)^(1/2)+1)*e^3/(a+b*arcsinh(d*x+c))/b-1/4*e^3/b^2*exp(4*a/b)*
Ei(1,4*arcsinh(d*x+c)+4*a/b)-1/8*(2*(d*x+c)^2-2*(d*x+c)*(1+(d*x+c)^2)^(1/2)
+1)*e^3/(a+b*arcsinh(d*x+c))/b+1/4*e^3/b^2*exp(2*a/b)*Ei(1,2*arcsinh(d*x+c)
+2*a/b)+1/8/b*e^3*(2*(d*x+c)^2+1+2*(d*x+c)*(1+(d*x+c)^2)^(1/2))/(a+b*arcsin
h(d*x+c))+1/4/b^2*e^3*exp(-2*a/b)*Ei(1,-2*arcsinh(d*x+c)-2*a/b)-1/16/b*e^3*
(8*(d*x+c)^4+8*(d*x+c)^2+8*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)+4*(d*x+c)*(1+(d*x+
c)^2)^(1/2)+1)/(a+b*arcsinh(d*x+c))-1/4/b^2*e^3*exp(-4*a/b)*Ei(1,-4*arcsinh
(d*x+c)-4*a/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(d^6*e^3*x^6 + 6*c*d^5*e^3*x^5 + c^6*e^3 + c^4*e^3 + (15*c^2*d^4*e^3 + d^4
*e^3)*x^4 + 4*(5*c^3*d^3*e^3 + c*d^3*e^3)*x^3 + 3*(5*c^4*d^2*e^3 + 2*c^2*d^
2*e^3)*x^2 + 2*(3*c^5*d*e^3 + 2*c^3*d*e^3)*x + (d^5*e^3*x^5 + 5*c*d^4*e^3*x
^4 + c^5*e^3 + c^3*e^3 + (10*c^2*d^3*e^3 + d^3*e^3)*x^3 + (10*c^3*d^2*e^3 +
 3*c*d^2*e^3)*x^2 + (5*c^4*d*e^3 + 3*c^2*d*e^3)*x)*sqrt(d^2*x^2 + 2*c*d*x +
 c^2 + 1))/(a*b*d^3*x^2 + 2*a*b*c*d^2*x + (c^2*d + d)*a*b + (b^2*d^3*x^2 +
2*b^2*c*d^2*x + (c^2*d + d)*b^2 + (b^2*d^2*x + b^2*c*d)*sqrt(d^2*x^2 + 2*c*
d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + (a*b*d^2
*x + a*b*c*d)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + integrate((4*d^7*e^3*x^7
 + 28*c*d^6*e^3*x^6 + 4*c^7*e^3 + 8*c^5*e^3 + 4*c^3*e^3 + 4*(21*c^2*d^5*e^3
 + 2*d^5*e^3)*x^5 + 20*(7*c^3*d^4*e^3 + 2*c*d^4*e^3)*x^4 + 4*(35*c^4*d^3*e^
3 + 20*c^2*d^3*e^3 + d^3*e^3)*x^3 + 4*(21*c^5*d^2*e^3 + 20*c^3*d^2*e^3 + 3*
c*d^2*e^3)*x^2 + 2*(2*d^5*e^3*x^5 + 10*c*d^4*e^3*x^4 + 2*c^5*e^3 + c^3*e^3
+ (20*c^2*d^3*e^3 + d^3*e^3)*x^3 + (20*c^3*d^2*e^3 + 3*c*d^2*e^3)*x^2 + (10
*c^4*d*e^3 + 3*c^2*d*e^3)*x)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + 4*(7*c^6*d*e^3
 + 10*c^4*d*e^3 + 3*c^2*d*e^3)*x + (8*d^6*e^3*x^6 + 48*c*d^5*e^3*x^5 + 8*c^
6*e^3 + 10*c^4*e^3 + 3*c^2*e^3 + 10*(12*c^2*d^4*e^3 + d^4*e^3)*x^4 + 40*(4*
c^3*d^3*e^3 + c*d^3*e^3)*x^3 + 3*(40*c^4*d^2*e^3 + 20*c^2*d^2*e^3 + d^2*e^3
```

```
)*x^2 + 2*(24*c^5*d*e^3 + 20*c^3*d*e^3 + 3*c*d*e^3)*x)*sqrt(d^2*x^2 + 2*c*d
*x + c^2 + 1))/(a*b*d^4*x^4 + 4*a*b*c*d^3*x^3 + 2*(3*c^2*d^2 + d^2)*a*b*x^2
 + 4*(c^3*d + c*d)*a*b*x + (c^4 + 2*c^2 + 1)*a*b + (a*b*d^2*x^2 + 2*a*b*c*d
*x + a*b*c^2)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^
3 + 2*(3*c^2*d^2 + d^2)*b^2*x^2 + 4*(c^3*d + c*d)*b^2*x + (c^4 + 2*c^2 + 1)
*b^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*(d^2*x^2 + 2*c*d*x + c^2 + 1)
+ 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + (3*c^2*d + d)*b^2*x + (c^3 + c)*b^2)*s
qrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^
2 + 1)) + 2*(a*b*d^3*x^3 + 3*a*b*c*d^2*x^2 + (3*c^2*d + d)*a*b*x + (c^3 + c
)*a*b)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,e+d\,e\,x)^3}{(a+b\,\mathrm{asinh}\,(c+d\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3/(a + b*asinh(c + d*x))^2,x)
```

```
[Out] int((c*e + d*e*x)^3/(a + b*asinh(c + d*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^3\left(\int \frac{c^3}{a^2+2ab\,\mathrm{asinh}\,(c+dx)+b^2\,\mathrm{asinh}^2\,(c+dx)}\,dx+\int \frac{d^3x^3}{a^2+2ab\,\mathrm{asinh}\,(c+dx)+b^2\,\mathrm{asinh}^2\,(c+dx)}\,dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3/(a+b*asinh(d*x+c))**2,x)
```

```
[Out] e**3*(Integral(c**3/(a**2 + 2*a*b*asinh(c + d*x) + b**2*asinh(c + d*x)**2),
 x) + Integral(d**3*x**3/(a**2 + 2*a*b*asinh(c + d*x) + b**2*asinh(c + d*x)
**2), x) + Integral(3*c*d**2*x**2/(a**2 + 2*a*b*asinh(c + d*x) + b**2*asinh
(c + d*x)**2), x) + Integral(3*c**2*d*x/(a**2 + 2*a*b*asinh(c + d*x) + b**2
*asinh(c + d*x)**2), x))
```

3.164 $\int \frac{(ce+dex)^2}{\left(a+b\sinh^{-1}(c+dx)\right)^2}\,dx$

Optimal. Leaf size=184

$$\frac{e^2\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{4b^2d}-\frac{3e^2\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{4b^2d}-\frac{e^2\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{4b^2d}+\frac{3e^2\cos}{}$$

```
[Out] -1/4*e^2*cosh(a/b)*Shi((a+b*arcsinh(d*x+c))/b)/b^2/d+3/4*e^2*cosh(3*a/b)*Sh
i(3*(a+b*arcsinh(d*x+c))/b)/b^2/d+1/4*e^2*Chi((a+b*arcsinh(d*x+c))/b)*sinh(
a/b)/b^2/d-3/4*e^2*Chi(3*(a+b*arcsinh(d*x+c))/b)*sinh(3*a/b)/b^2/d-e^2*(d*x
+c)^2*(1+(d*x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))
```

Rubi [A] time = 0.29, antiderivative size = 180, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {5865, 12, 5665, 3303, 3298, 3301}

$$\frac{e^2\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)}{4b^2d}-\frac{3e^2\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(c+dx)\right)}{4b^2d}-\frac{e^2\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(c-\right.}{4b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2/(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] -((e^2*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])/(b*d*(a + b*ArcSinh[c + d*x]))) +
 (e^2*CoshIntegral[a/b + ArcSinh[c + d*x]]*Sinh[a/b])/(4*b^2*d) - (3*e^2*Co
shIntegral[(3*a)/b + 3*ArcSinh[c + d*x]]*Sinh[(3*a)/b])/(4*b^2*d) - (e^2*Co
sh[a/b]*SinhIntegral[a/b + ArcSinh[c + d*x]])/(4*b^2*d) + (3*e^2*Cosh[(3*a)
/b]*SinhIntegral[(3*a)/b + 3*ArcSinh[c + d*x]])/(4*b^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
```

```
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^2}{\left(a+b\sinh^{-1}(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{e^2x^2}{\left(a+b\sinh^{-1}(x)\right)^2}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e^2\,\text{Subst}\left(\int \frac{x^2}{\left(a+b\sinh^{-1}(x)\right)^2}\,dx,x,c+dx\right)}{d}$$

$$= -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{bd\left(a+b\sinh^{-1}(c+dx)\right)} + \frac{e^2\,\text{Subst}\left(\int \left(-\frac{\sinh(x)}{4(a+bx)}+\frac{3\sinh(3x)}{4(a+bx)}\right)dx,x,\sinh^{-1}\right.}{bd}$$

$$= -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{bd\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{e^2\,\text{Subst}\left(\int \frac{\sinh(x)}{a+bx}\,dx,x,\sinh^{-1}(c+dx)\right)}{4bd} + \frac{(3e}{}$$

$$= -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{bd\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{\left(e^2\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx,x,\sinh^{-1}(c\right.}{4bd}$$

$$= -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{bd\left(a+b\sinh^{-1}(c+dx)\right)} + \frac{e^2\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\sinh\left(\frac{a}{b}\right)}{4b^2d} - \frac{3e^2\text{Chi}}{}$$

Mathematica [A] time = 0.74, size = 138, normalized size = 0.75

$$\frac{e^2\left(\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)-3\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)-\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(c\right.\right.}{4b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^2/(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] (e^2*((-4*b*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh[c + d*x]) + C
oshIntegral[a/b + ArcSinh[c + d*x]]*Sinh[a/b] - 3*CoshIntegral[3*(a/b + Arc
Sinh[c + d*x])]*Sinh[(3*a)/b] - Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c + d*
x]] + 3*Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c + d*x])]))/(4*b^2*d)
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{d^2e^2x^2+2\,cde^2x+c^2e^2}{b^2\,\text{arsinh}\,(dx+c)^2+2\,ab\,\text{arsinh}\,(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2)/(b^2*arcsinh(d*x + c)^2 + 2*
a*b*arcsinh(d*x + c) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex + ce)^2}{(b \operatorname{arsinh}(dx + c) + a)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2/(b*arcsinh(d*x + c) + a)^2, x)
```

maple [A] time = 0.24, size = 342, normalized size = 1.86

$$\frac{\frac{\left(-4(dx+c)^2\sqrt{1+(dx+c)^2}+4(dx+c)^3-\sqrt{1+(dx+c)^2}+3dx+3c\right)e^2}{8b(a+b\operatorname{arcsinh}(dx+c))}+\frac{3e^2\mathrm{e}^{\frac{3a}{b}}\operatorname{Ei}\left(1,3\operatorname{arcsinh}(dx+c)+\frac{3a}{b}\right)}{8b^2}-\frac{\left(-\sqrt{1+(dx+c)^2}+dx+c\right)e^2}{8b(a+b\operatorname{arcsinh}(dx+c))}-\frac{e^2\mathrm{e}^{\frac{a}{b}}\operatorname{Ei}\left(1,\operatorname{arcsi}\right.}{8b}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^2,x)
```

```
[Out] 1/d*(1/8*(-4*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+4*(d*x+c)^3-(1+(d*x+c)^2)^(1/2)+
3*d*x+3*c)*e^2/b/(a+b*arcsinh(d*x+c))+3/8*e^2/b^2*exp(3*a/b)*Ei(1,3*arcsinh
(d*x+c)+3*a/b)-1/8*(-(1+(d*x+c)^2)^(1/2)+d*x+c)*e^2/b/(a+b*arcsinh(d*x+c))-
1/8*e^2/b^2*exp(a/b)*Ei(1,arcsinh(d*x+c)+a/b)+1/8/b*e^2*(d*x+c+(1+(d*x+c)^2
)^(1/2))/(a+b*arcsinh(d*x+c))+1/8/b^2*e^2*exp(-a/b)*Ei(1,-arcsinh(d*x+c)-a/
b)-1/8/b*e^2*(4*(d*x+c)^3+3*d*x+3*c+4*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+(1+(d*x
+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))-3/8/b^2*e^2*exp(-3*a/b)*Ei(1,-3*arcsinh(
d*x+c)-3*a/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(d^5*e^2*x^5 + 5*c*d^4*e^2*x^4 + c^5*e^2 + c^3*e^2 + (10*c^2*d^3*e^2 + d^3
*e^2)*x^3 + (10*c^3*d^2*e^2 + 3*c*d^2*e^2)*x^2 + (5*c^4*d*e^2 + 3*c^2*d*e^2
)*x + (d^4*e^2*x^4 + 4*c*d^3*e^2*x^3 + c^4*e^2 + c^2*e^2 + (6*c^2*d^2*e^2 +
 d^2*e^2)*x^2 + 2*(2*c^3*d*e^2 + c*d*e^2)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 +
 1))/(a*b*d^3*x^2 + 2*a*b*c*d^2*x + (c^2*d + d)*a*b + (b^2*d^3*x^2 + 2*b^2*
c*d^2*x + (c^2*d + d)*b^2 + (b^2*d^2*x + b^2*c*d)*sqrt(d^2*x^2 + 2*c*d*x +
c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + (a*b*d^2*x + a
*b*c*d)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + integrate((3*d^6*e^2*x^6 + 18*
c*d^5*e^2*x^5 + 3*c^6*e^2 + 6*c^4*e^2 + 3*(15*c^2*d^4*e^2 + 2*d^4*e^2)*x^4
+ 3*c^2*e^2 + 12*(5*c^3*d^3*e^2 + 2*c*d^3*e^2)*x^3 + 3*(15*c^4*d^2*e^2 + 12
*c^2*d^2*e^2 + d^2*e^2)*x^2 + (3*d^4*e^2*x^4 + 12*c*d^3*e^2*x^3 + 3*c^4*e^2
 + c^2*e^2 + (18*c^2*d^2*e^2 + d^2*e^2)*x^2 + 2*(6*c^3*d*e^2 + c*d*e^2)*x)*
(d^2*x^2 + 2*c*d*x + c^2 + 1) + 6*(3*c^5*d*e^2 + 4*c^3*d*e^2 + c*d*e^2)*x +
 (6*d^5*e^2*x^5 + 30*c*d^4*e^2*x^4 + 6*c^5*e^2 + 7*c^3*e^2 + (60*c^2*d^3*e^
2 + 7*d^3*e^2)*x^3 + 2*c*e^2 + 3*(20*c^3*d^2*e^2 + 7*c*d^2*e^2)*x^2 + (30*c
^4*d*e^2 + 21*c^2*d*e^2 + 2*d*e^2)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(a
*b*d^4*x^4 + 4*a*b*c*d^3*x^3 + 2*(3*c^2*d^2 + d^2)*a*b*x^2 + 4*(c^3*d + c*d
)*a*b*x + (c^4 + 2*c^2 + 1)*a*b + (a*b*d^2*x^2 + 2*a*b*c*d*x + a*b*c^2)*(d^
2*x^2 + 2*c*d*x + c^2 + 1) + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 2*(3*c^2*d^2
```

```
+ d^2)*b^2*x^2 + 4*(c^3*d + c*d)*b^2*x + (c^4 + 2*c^2 + 1)*b^2 + (b^2*d^2*x
^2 + 2*b^2*c*d*x + b^2*c^2)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + 2*(b^2*d^3*x^3
+ 3*b^2*c*d^2*x^2 + (3*c^2*d + d)*b^2*x + (c^3 + c)*b^2)*sqrt(d^2*x^2 + 2*c
*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + 2*(a*b*
d^3*x^3 + 3*a*b*c*d^2*x^2 + (3*c^2*d + d)*a*b*x + (c^3 + c)*a*b)*sqrt(d^2*x
^2 + 2*c*d*x + c^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,e+d\,e\,x)^2}{(a+b\,\mathrm{asinh}\,(c+d\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2/(a + b*asinh(c + d*x))^2,x)
```

```
[Out] int((c*e + d*e*x)^2/(a + b*asinh(c + d*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^2\left(\int \frac{c^2}{a^2+2ab\,\mathrm{asinh}\,(c+dx)+b^2\,\mathrm{asinh}^2\,(c+dx)}\,dx+\int \frac{d^2x^2}{a^2+2ab\,\mathrm{asinh}\,(c+dx)+b^2\,\mathrm{asinh}^2\,(c+dx)}\,dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2/(a+b*asinh(d*x+c))**2,x)
```

```
[Out] e**2*(Integral(c**2/(a**2 + 2*a*b*asinh(c + d*x) + b**2*asinh(c + d*x)**2),
 x) + Integral(d**2*x**2/(a**2 + 2*a*b*asinh(c + d*x) + b**2*asinh(c + d*x)
**2), x) + Integral(2*c*d*x/(a**2 + 2*a*b*asinh(c + d*x) + b**2*asinh(c + d
*x)**2), x))
```

3.165 $\int \frac{ce+dex}{\left(a+b\sinh^{-1}(c+dx)\right)^2}\,dx$

Optimal. Leaf size=103

$$\frac{e\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{b^2d}-\frac{e\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{b^2d}-\frac{e\sqrt{(c+dx)^2+1}\,(c+dx)}{bd\left(a+b\sinh^{-1}(c+dx)\right)}$$

```
[Out] e*Chi(2*(a+b*arcsinh(d*x+c))/b)*cosh(2*a/b)/b^2/d-e*Shi(2*(a+b*arcsinh(d*x+
c))/b)*sinh(2*a/b)/b^2/d-e*(d*x+c)*(1+(d*x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x
+c))
```

Rubi [A] time = 0.15, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5865, 12, 5665, 3303, 3298, 3301}

$$\frac{e\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(c+dx)\right)}{b^2d}-\frac{e\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\sinh^{-1}(c+dx)\right)}{b^2d}-\frac{e\sqrt{(c+dx)^2+1}\,(c+dx)}{bd\left(a+b\sinh^{-1}(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)/(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] -((e*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(b*d*(a + b*ArcSinh[c + d*x]))) + (e*
Cosh[(2*a)/b]*CoshIntegral[(2*a)/b + 2*ArcSinh[c + d*x]])/(b^2*d) - (e*Sinh
[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSinh[c + d*x]])/(b^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
```

```
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{ce+dex}{\left(a+b\sinh^{-1}(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{ex}{\left(a+b\sinh^{-1}(x)\right)^2}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e\,\text{Subst}\left(\int \frac{x}{\left(a+b\sinh^{-1}(x)\right)^2}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{e(c+dx)\sqrt{1+(c+dx)^2}}{bd\left(a+b\sinh^{-1}(c+dx)\right)} + \frac{e\,\text{Subst}\left(\int \frac{\cosh(2x)}{a+bx}\,dx, x, \sinh^{-1}(c+dx)\right)}{bd}$$

$$= -\frac{e(c+dx)\sqrt{1+(c+dx)^2}}{bd\left(a+b\sinh^{-1}(c+dx)\right)} + \frac{\left(e\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \sinh^{-1}\right.}{bd}$$

$$= -\frac{e(c+dx)\sqrt{1+(c+dx)^2}}{bd\left(a+b\sinh^{-1}(c+dx)\right)} + \frac{e\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(c+dx)\right)}{b^2d} - \frac{e\sin}{}$$

Mathematica [A] time = 0.30, size = 97, normalized size = 0.94

$$\frac{e\left(-\frac{b\sqrt{c^2+2cdx+d^2x^2+1}\,(c+dx)}{a+b\sinh^{-1}(c+dx)}+\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)-\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)\right)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)/(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] (e*(-((b*(c + d*x)*Sqrt[1 + c^2 + 2*c*d*x + d^2*x^2])/(a + b*ArcSinh[c + d*
x])) + Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcSinh[c + d*x])] - Sinh[(2*a)/
b]*SinhIntegral[2*(a/b + ArcSinh[c + d*x])]))/(b^2*d)
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{dex+ce}{b^2\,\text{arsinh}\,(dx+c)^2+2\,ab\,\text{arsinh}\,(dx+c)+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((d*e*x + c*e)/(b^2*arcsinh(d*x + c)^2 + 2*a*b*arcsinh(d*x + c) + a
^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dex+ce}{\left(b\,\text{arsinh}\,(dx+c)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)/(b*arcsinh(d*x + c) + a)^2, x)
```

maple [A] time = 0.06, size = 160, normalized size = 1.55

$$\frac{\frac{\left(2(dx+c)^2-2(dx+c)\sqrt{1+(dx+c)^2}+1\right)e}{4(a+b\,\mathrm{arcsinh}(dx+c))b}-\frac{e\,\mathrm{e}^{\frac{2a}{b}}\,\mathrm{Ei}\left(1,2\,\mathrm{arcsinh}(dx+c)+\frac{2a}{b}\right)}{2b^2}-\frac{e\left(2(dx+c)^2+1+2(dx+c)\sqrt{1+(dx+c)^2}\right)}{4b(a+b\,\mathrm{arcsinh}(dx+c))}-\frac{e\,\mathrm{e}^{-\frac{2a}{b}}\,\mathrm{Ei}\left(1,-2\,\mathrm{arcsinh}(dx+c)-\right.}{2b^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^2,x)
```

```
[Out] 1/d*(1/4*(2*(d*x+c)^2-2*(d*x+c)*(1+(d*x+c)^2)^(1/2)+1)*e/(a+b*arcsinh(d*x+c
))/b-1/2*e/b^2*exp(2*a/b)*Ei(1,2*arcsinh(d*x+c)+2*a/b)-1/4/b*e*(2*(d*x+c)^2
+1+2*(d*x+c)*(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))-1/2/b^2*e*exp(-2*a/b
)*Ei(1,-2*arcsinh(d*x+c)-2*a/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{d^4ex^4+4\,cd^3ex^3+c^4e+c^2e+\left(6\,c^2d^2e+d^2e\right)x^2+2\left(2\,c^3de+cde\right)x+\left(d^3ex^3+3\,cd^2e\right.}{abd^3x^2+2\,abcd^2x+\left(c^2d+d\right)ab+\left(b^2d^3x^2+2\,b^2cd^2x+\left(c^2d+d\right)b^2+\left(b^2d^2x+b^2cd\right)\sqrt{d^2x^2+2\,cdx+c^2+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(d^4*e*x^4 + 4*c*d^3*e*x^3 + c^4*e + c^2*e + (6*c^2*d^2*e + d^2*e)*x^2 + 2
*(2*c^3*d*e + c*d*e)*x + (d^3*e*x^3 + 3*c*d^2*e*x^2 + c^3*e + c*e + (3*c^2*
d*e + d*e)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(a*b*d^3*x^2 + 2*a*b*c*d^2
*x + (c^2*d + d)*a*b + (b^2*d^3*x^2 + 2*b^2*c*d^2*x + (c^2*d + d)*b^2 + (b^
2*d^2*x + b^2*c*d)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^
2*x^2 + 2*c*d*x + c^2 + 1)) + (a*b*d^2*x + a*b*c*d)*sqrt(d^2*x^2 + 2*c*d*x
+ c^2 + 1)) + integrate((2*d^5*e*x^5 + 10*c*d^4*e*x^4 + 2*c^5*e + 4*c^3*e +
 4*(5*c^2*d^3*e + d^3*e)*x^3 + 4*(5*c^3*d^2*e + 3*c*d^2*e)*x^2 + 2*(d^3*e*x
^3 + 3*c*d^2*e*x^2 + 3*c^2*d*e*x + c^3*e)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + 2
*c*e + 2*(5*c^4*d*e + 6*c^2*d*e + d*e)*x + (4*d^4*e*x^4 + 16*c*d^3*e*x^3 +
4*c^4*e + 4*c^2*e + 4*(6*c^2*d^2*e + d^2*e)*x^2 + 8*(2*c^3*d*e + c*d*e)*x +
 e)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(a*b*d^4*x^4 + 4*a*b*c*d^3*x^3 + 2*(
3*c^2*d^2 + d^2)*a*b*x^2 + 4*(c^3*d + c*d)*a*b*x + (c^4 + 2*c^2 + 1)*a*b +
(a*b*d^2*x^2 + 2*a*b*c*d*x + a*b*c^2)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + (b^2*
d^4*x^4 + 4*b^2*c*d^3*x^3 + 2*(3*c^2*d^2 + d^2)*b^2*x^2 + 4*(c^3*d + c*d)*b
^2*x + (c^4 + 2*c^2 + 1)*b^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*(d^2*x
^2 + 2*c*d*x + c^2 + 1) + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + (3*c^2*d + d)*
b^2*x + (c^3 + c)*b^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqr
t(d^2*x^2 + 2*c*d*x + c^2 + 1)) + 2*(a*b*d^3*x^3 + 3*a*b*c*d^2*x^2 + (3*c^2
*d + d)*a*b*x + (c^3 + c)*a*b)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c\,e+d\,e\,x}{{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)/(a + b*asinh(c + d*x))^2,x)
```

```
[Out] int((c*e + d*e*x)/(a + b*asinh(c + d*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e\left(\int \frac{c}{a^2 + 2ab\operatorname{asinh}(c + dx) + b^2\operatorname{asinh}^2(c + dx)}\,dx + \int \frac{dx}{a^2 + 2ab\operatorname{asinh}(c + dx) + b^2\operatorname{asinh}^2(c + dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*asinh(d*x+c))**2,x)
```

```
[Out] e*(Integral(c/(a**2 + 2*a*b*asinh(c + d*x) + b**2*asinh(c + d*x)**2), x) +
Integral(d*x/(a**2 + 2*a*b*asinh(c + d*x) + b**2*asinh(c + d*x)**2), x))
```

3.166 $\int \frac{1}{\left(a+b\sinh^{-1}(c+dx)\right)^2}\,dx$

Optimal. Leaf size=91

$$-\frac{\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{b^2d}+\frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{b^2d}-\frac{\sqrt{(c+dx)^2+1}}{bd\left(a+b\sinh^{-1}(c+dx)\right)}$$

```
[Out] cosh(a/b)*Shi((a+b*arcsinh(d*x+c))/b)/b^2/d-Chi((a+b*arcsinh(d*x+c))/b)*sin
h(a/b)/b^2/d-(1+(d*x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))
```

Rubi [A] time = 0.17, antiderivative size = 87, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5863, 5655, 5779, 3303, 3298, 3301}

$$-\frac{\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)}{b^2d}+\frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)}{b^2d}-\frac{\sqrt{(c+dx)^2+1}}{bd\left(a+b\sinh^{-1}(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^(-2),x]
```

```
[Out] -(Sqrt[1 + (c + d*x)^2]/(b*d*(a + b*ArcSinh[c + d*x]))) - (CoshIntegral[a/b
 + ArcSinh[c + d*x]]*Sinh[a/b])/(b^2*d) + (Cosh[a/b]*SinhIntegral[a/b + Arc
Sinh[c + d*x]])/(b^2*d)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
```

```
Q[p] || GtQ[d, 0])
```

Rule 5863

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sinh^{-1}(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+b\sinh^{-1}(x)\right)^2}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{\sqrt{1+(c+dx)^2}}{bd\left(a+b\sinh^{-1}(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\left(a+b\sinh^{-1}(x)\right)}\,dx, x, c+dx\right)}{bd}$$

$$= -\frac{\sqrt{1+(c+dx)^2}}{bd\left(a+b\sinh^{-1}(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{\sinh(x)}{a+bx}\,dx, x, \sinh^{-1}(c+dx)\right)}{bd}$$

$$= -\frac{\sqrt{1+(c+dx)^2}}{bd\left(a+b\sinh^{-1}(c+dx)\right)} + \frac{\cosh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \sinh^{-1}(c+dx\right.}{bd}$$

$$= -\frac{\sqrt{1+(c+dx)^2}}{bd\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\sinh\left(\frac{a}{b}\right)}{b^2d} + \frac{\cosh\left(\frac{a}{b}\right)\text{S}}{}$$

Mathematica [A] time = 0.09, size = 77, normalized size = 0.85

$$\frac{-\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)+\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)-\frac{b\sqrt{(c+dx)^2+1}}{a+b\sinh^{-1}(c+dx)}}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^(-2),x]
```

```
[Out] (-((b*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh[c + d*x])) - CoshIntegral[a/b +
 ArcSinh[c + d*x]]*Sinh[a/b] + Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c + d*x
]])/(b^2*d)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{b^2\,\text{arsinh}\,(dx+c)^2+2\,ab\,\text{arsinh}\,(dx+c)+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(b^2*arcsinh(d*x + c)^2 + 2*a*b*arcsinh(d*x + c) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,\text{arsinh}\,(dx+c)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(-2), x)
```

maple [A] time = 0.05, size = 128, normalized size = 1.41

$$\frac{\frac{-\sqrt{1+(dx+c)^2}+dx+c}{2b(a+b\operatorname{arcsinh}(dx+c))}+\frac{e^{\frac{a}{b}}\operatorname{Ei}\left(1,\operatorname{arcsinh}(dx+c)+\frac{a}{b}\right)}{2b^2}-\frac{dx+c+\sqrt{1+(dx+c)^2}}{2b(a+b\operatorname{arcsinh}(dx+c))}-\frac{e^{-\frac{a}{b}}\operatorname{Ei}\left(1,-\operatorname{arcsinh}(dx+c)-\frac{a}{b}\right)}{2b^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(d*x+c))^2,x)
```

```
[Out] 1/d*(1/2*(-(1+(d*x+c)^2)^(1/2)+d*x+c)/b/(a+b*arcsinh(d*x+c))+1/2/b^2*exp(a/
b)*Ei(1,arcsinh(d*x+c)+a/b)-1/2/b*(d*x+c+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(
d*x+c))-1/2/b^2*exp(-a/b)*Ei(1,-arcsinh(d*x+c)-a/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{d^3x^3+3\,cd^2x^2+c^3+\left(3\,c^2d+d\right)x+\left(d^2x^2+2\,c\right.}{abd^3x^2+2\,abcd^2x+\left(c^2d+d\right)ab+\left(b^2d^3x^2+2\,b^2cd^2x+\left(c^2d+d\right)b^2+\left(b^2d^2x+b^2cd\right)\sqrt{d^2x^2+2\,cdx+c^2+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (3*c^2*d + d)*x + (d^2*x^2 + 2*c*d*x + c^2
+ 1)^(3/2) + c)/(a*b*d^3*x^2 + 2*a*b*c*d^2*x + (c^2*d + d)*a*b + (b^2*d^3*x
^2 + 2*b^2*c*d^2*x + (c^2*d + d)*b^2 + (b^2*d^2*x + b^2*c*d)*sqrt(d^2*x^2 +
 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + (a*
b*d^2*x + a*b*c*d)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + integrate((d^4*x^4
+ 4*c*d^3*x^3 + c^4 + 2*(3*c^2*d^2 + d^2)*x^2 + (d^2*x^2 + 2*c*d*x + c^2 +
1)*(d^2*x^2 + 2*c*d*x + c^2 - 1) + 2*c^2 + 4*(c^3*d + c*d)*x + (2*d^3*x^3 +
 6*c*d^2*x^2 + 2*c^3 + (6*c^2*d + d)*x + c)*sqrt(d^2*x^2 + 2*c*d*x + c^2 +
1) + 1)/(a*b*d^4*x^4 + 4*a*b*c*d^3*x^3 + 2*(3*c^2*d^2 + d^2)*a*b*x^2 + 4*(c
^3*d + c*d)*a*b*x + (c^4 + 2*c^2 + 1)*a*b + (a*b*d^2*x^2 + 2*a*b*c*d*x + a*
b*c^2)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 2*(
3*c^2*d^2 + d^2)*b^2*x^2 + 4*(c^3*d + c*d)*b^2*x + (c^4 + 2*c^2 + 1)*b^2 +
(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + 2*(b^
2*d^3*x^3 + 3*b^2*c*d^2*x^2 + (3*c^2*d + d)*b^2*x + (c^3 + c)*b^2)*sqrt(d^2
*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))
 + 2*(a*b*d^3*x^3 + 3*a*b*c*d^2*x^2 + (3*c^2*d + d)*a*b*x + (c^3 + c)*a*b)*
sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*asinh(c + d*x))^2,x)
```

```
[Out] int(1/(a + b*asinh(c + d*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b \operatorname{asinh}{\left(c + dx \right)}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(d*x+c))**2,x)
```

```
[Out] Integral((a + b*asinh(c + d*x))**(-2), x)
```

3.167 $\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^2}\,dx$

Optimal. Leaf size=27

$$\frac{\text{Int}\left(\frac{1}{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^2},x\right)}{e}$$

```
[Out] Unintegrable(1/(d*x+c)/(a+b*arcsinh(d*x+c))^2,x)/e
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^2),x]
[Out] Defer[Subst][Defer[Int][1/(x*(a + b*ArcSinh[x])^2), x], x, c + d*x]/(d*e)
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{ex\left(a+b\sinh^{-1}(x)\right)^2}\,dx,x,c+dx\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{1}{x\left(a+b\sinh^{-1}(x)\right)^2}\,dx,x,c+dx\right)}{de}\end{aligned}$$

Mathematica [A] time = 1.44, size = 0, normalized size = 0.00

$$\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^2),x]
[Out] Integrate[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^2), x]
```

fricas [A] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{a^2dex+a^2ce+\left(b^2dex+b^2ce\right)\operatorname{arsinh}(dx+c)^2+2\,(abdex+abce)\operatorname{arsinh}(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
[Out] integral(1/(a^2*d*e*x + a^2*c*e + (b^2*d*e*x + b^2*c*e)*arcsinh(d*x + c)^2
+ 2*(a*b*d*e*x + a*b*c*e)*arcsinh(d*x + c)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dex+ce)(b \operatorname{arsinh}(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^2), x)
```

maple [A] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{1}{(dex+ce)\,(a+b \operatorname{arcsinh}(dx+c))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^2,x)
```

```
[Out] int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{d^3}{abd^4ex^3 + 3\,abcd^3ex^2 + \left(3\,c^2d^2e + d^2e\right)abx + \left(c^3de + cde\right)ab + \left(b^2d^4ex^3 + 3\,b^2cd^3ex^2 + \left(3\,c^2d^2e + d^2e\right)b^2x + \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (3*c^2*d + d)*x + (d^2*x^2 + 2*c*d*x + c^2
+ 1)^(3/2) + c)/(a*b*d^4*e*x^3 + 3*a*b*c*d^3*e*x^2 + (3*c^2*d^2*e + d^2*e)*
a*b*x + (c^3*d*e + c*d*e)*a*b + (b^2*d^4*e*x^3 + 3*b^2*c*d^3*e*x^2 + (3*c^2
*d^2*e + d^2*e)*b^2*x + (c^3*d*e + c*d*e)*b^2 + (b^2*d^3*e*x^2 + 2*b^2*c*d^
2*e*x + b^2*c^2*d*e)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(
d^2*x^2 + 2*c*d*x + c^2 + 1)) + (a*b*d^3*e*x^2 + 2*a*b*c*d^2*e*x + a*b*c^2*
d*e)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - integrate((2*(d^2*x^2 + 2*c*d*x +
 c^2 + 1)*(d*x + c) + (2*d^2*x^2 + 4*c*d*x + 2*c^2 + 1)*sqrt(d^2*x^2 + 2*c*
d*x + c^2 + 1))/(a*b*d^6*e*x^6 + 6*a*b*c*d^5*e*x^5 + (15*c^2*d^4*e + 2*d^4*
e)*a*b*x^4 + 4*(5*c^3*d^3*e + 2*c*d^3*e)*a*b*x^3 + (15*c^4*d^2*e + 12*c^2*d
^2*e + d^2*e)*a*b*x^2 + 2*(3*c^5*d*e + 4*c^3*d*e + c*d*e)*a*b*x + (c^6*e +
2*c^4*e + c^2*e)*a*b + (a*b*d^4*e*x^4 + 4*a*b*c*d^3*e*x^3 + 6*a*b*c^2*d^2*e
*x^2 + 4*a*b*c^3*d*e*x + a*b*c^4*e)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + (b^2*d^
6*e*x^6 + 6*b^2*c*d^5*e*x^5 + (15*c^2*d^4*e + 2*d^4*e)*b^2*x^4 + 4*(5*c^3*d
^3*e + 2*c*d^3*e)*b^2*x^3 + (15*c^4*d^2*e + 12*c^2*d^2*e + d^2*e)*b^2*x^2 +
 2*(3*c^5*d*e + 4*c^3*d*e + c*d*e)*b^2*x + (c^6*e + 2*c^4*e + c^2*e)*b^2 +
(b^2*d^4*e*x^4 + 4*b^2*c*d^3*e*x^3 + 6*b^2*c^2*d^2*e*x^2 + 4*b^2*c^3*d*e*x
+ b^2*c^4*e)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + 2*(b^2*d^5*e*x^5 + 5*b^2*c*d^4
*e*x^4 + (10*c^2*d^3*e + d^3*e)*b^2*x^3 + (10*c^3*d^2*e + 3*c*d^2*e)*b^2*x^
2 + (5*c^4*d*e + 3*c^2*d*e)*b^2*x + (c^5*e + c^3*e)*b^2)*sqrt(d^2*x^2 + 2*c
*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + 2*(a*b*
d^5*e*x^5 + 5*a*b*c*d^4*e*x^4 + (10*c^2*d^3*e + d^3*e)*a*b*x^3 + (10*c^3*d^
2*e + 3*c*d^2*e)*a*b*x^2 + (5*c^4*d*e + 3*c^2*d*e)*a*b*x + (c^5*e + c^3*e)*
a*b)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{(c\,e+d\,e\,x)\,\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*e + d*e*x)*(a + b*asinh(c + d*x))^2),x)
```

```
[Out] int(1/((c*e + d*e*x)*(a + b*asinh(c + d*x))^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^2c+a^2dx+2abc\operatorname{asinh}(c+dx)+2abdx\operatorname{asinh}(c+dx)+b^2c\operatorname{asinh}^2(c+dx)+b^2dx\operatorname{asinh}^2(c+dx)}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*asinh(d*x+c))**2,x)
```

```
[Out] Integral(1/(a**2*c + a**2*d*x + 2*a*b*c*asinh(c + d*x) + 2*a*b*d*x*asinh(c
+ d*x) + b**2*c*asinh(c + d*x)**2 + b**2*d*x*asinh(c + d*x)**2), x)/e
```

3.168 $\int \frac{(ce+dex)^4}{\left(a+b\sinh^{-1}(c+dx)\right)^3}\,dx$

Optimal. Leaf size=320

$$\frac{e^4\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{16b^3d}-\frac{27e^4\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{32b^3d}+\frac{25e^4\cosh\left(\frac{5a}{b}\right)\mathrm{Chi}\left(\frac{5\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{32b^3d}$$

```
[Out] -2*e^4*(d*x+c)^3/b^2/d/(a+b*arcsinh(d*x+c))-5/2*e^4*(d*x+c)^5/b^2/d/(a+b*ar
csinh(d*x+c))+1/16*e^4*Chi((a+b*arcsinh(d*x+c))/b)*cosh(a/b)/b^3/d-27/32*e^
4*Chi(3*(a+b*arcsinh(d*x+c))/b)*cosh(3*a/b)/b^3/d+25/32*e^4*Chi(5*(a+b*arcs
inh(d*x+c))/b)*cosh(5*a/b)/b^3/d-1/16*e^4*Shi((a+b*arcsinh(d*x+c))/b)*sinh(
a/b)/b^3/d+27/32*e^4*Shi(3*(a+b*arcsinh(d*x+c))/b)*sinh(3*a/b)/b^3/d-25/32*
e^4*Shi(5*(a+b*arcsinh(d*x+c))/b)*sinh(5*a/b)/b^3/d-1/2*e^4*(d*x+c)^4*(1+(d
*x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))^2
```

Rubi [A] time = 0.89, antiderivative size = 316, normalized size of antiderivative = 0.99, number of steps used = 26, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {5865, 12, 5667, 5774, 5669, 5448, 3303, 3298, 3301}

$$\frac{e^4\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)}{16b^3d}-\frac{27e^4\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(c+dx)\right)}{32b^3d}+\frac{25e^4\cosh\left(\frac{5a}{b}\right)\mathrm{Chi}\left(\frac{5a}{b}+\right.}{32b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4/(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] -(e^4*(c + d*x)^4*Sqrt[1 + (c + d*x)^2])/(2*b*d*(a + b*ArcSinh[c + d*x])^2)
 - (2*e^4*(c + d*x)^3)/(b^2*d*(a + b*ArcSinh[c + d*x])) - (5*e^4*(c + d*x)^
5)/(2*b^2*d*(a + b*ArcSinh[c + d*x])) + (e^4*Cosh[a/b]*CoshIntegral[a/b + A
rcSinh[c + d*x]])/(16*b^3*d) - (27*e^4*Cosh[(3*a)/b]*CoshIntegral[(3*a)/b +
 3*ArcSinh[c + d*x]])/(32*b^3*d) + (25*e^4*Cosh[(5*a)/b]*CoshIntegral[(5*a)
/b + 5*ArcSinh[c + d*x]])/(32*b^3*d) - (e^4*Sinh[a/b]*SinhIntegral[a/b + Ar
cSinh[c + d*x]])/(16*b^3*d) + (27*e^4*Sinh[(3*a)/b]*SinhIntegral[(3*a)/b +
3*ArcSinh[c + d*x]])/(32*b^3*d) - (25*e^4*Sinh[(5*a)/b]*SinhIntegral[(5*a)/
b + 5*ArcSinh[c + d*x]])/(32*b^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^4}{\left(a+b\sinh^{-1}(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{e^4x^4}{\left(a+b\sinh^{-1}(x)\right)^3}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^4\,\text{Subst}\left(\int \frac{x^4}{\left(a+b\sinh^{-1}(x)\right)^3}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} + \frac{\left(2e^4\right)\text{Subst}\left(\int \frac{x^3}{\sqrt{1+x^2}\left(a+b\sinh^{-1}(x)\right)^2}\,dx, x, c+ \right.}{bd}$$

$$= -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{2e^4(c+dx)^3}{b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{5e^4(c+}{2b^2d\left(a+b\sinh\right.}$$

$$= -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{2e^4(c+dx)^3}{b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{5e^4(c+}{2b^2d\left(a+b\sinh\right.}$$

$$= -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{2e^4(c+dx)^3}{b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{5e^4(c+}{2b^2d\left(a+b\sinh\right.}$$

$$= -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{2e^4(c+dx)^3}{b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{5e^4(c+}{2b^2d\left(a+b\sinh\right.}$$

$$= -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{2e^4(c+dx)^3}{b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{5e^4(c+}{2b^2d\left(a+b\sinh\right.}$$

$$= -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{2e^4(c+dx)^3}{b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{5e^4(c+}{2b^2d\left(a+b\sinh\right.}$$

Mathematica [A] time = 1.25, size = 316, normalized size = 0.99

$$\frac{e^4\left(-\frac{16b^2\sqrt{(c+dx)^2+1}\,(c+dx)^4}{\left(a+b\sinh^{-1}(c+dx)\right)^2} + 48\left(-\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right) + \cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^4/(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] (e^4*((-16*b^2*(c + d*x)^4*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh[c + d*x])^
2 + (16*b*(-4*(c + d*x)^3 - 5*(c + d*x)^5))/(a + b*ArcSinh[c + d*x]) + 48*(
-(Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c + d*x]]) + Cosh[(3*a)/b]*CoshInteg
ral[3*(a/b + ArcSinh[c + d*x])] + Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c +
d*x]] - Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c + d*x])]) + 25*(2*Cos
h[a/b]*CoshIntegral[a/b + ArcSinh[c + d*x]] - 3*Cosh[(3*a)/b]*CoshIntegral[
3*(a/b + ArcSinh[c + d*x])] + Cosh[(5*a)/b]*CoshIntegral[5*(a/b + ArcSinh[c
 + d*x])] - 2*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c + d*x]] + 3*Sinh[(3*a)
/b]*SinhIntegral[3*(a/b + ArcSinh[c + d*x])] - Sinh[(5*a)/b]*SinhIntegral[5
*(a/b + ArcSinh[c + d*x])])))/(32*b^3*d)
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{d^4e^4x^4 + 4\,cd^3e^4x^3 + 6\,c^2d^2e^4x^2 + 4\,c^3de^4x + c^4e^4}{b^3\,\text{arsinh}\,(dx+c)^3 + 3\,ab^2\,\text{arsinh}\,(dx+c)^2 + 3\,a^2b\,\text{arsinh}\,(dx+c) + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((d^4*e^4*x^4 + 4*c*d^3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^3*d*e^4*x
 + c^4*e^4)/(b^3*arcsinh(d*x + c)^3 + 3*a*b^2*arcsinh(d*x + c)^2 + 3*a^2*b*
arcsinh(d*x + c) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex+ce)^4}{(b\,\text{arsinh}\,(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^4/(b*arcsinh(d*x + c) + a)^3, x)
```

maple [B] time = 0.39, size = 896, normalized size = 2.80

$$-\frac{\frac{\left(16(dx+c)^5-16(dx+c)^4\sqrt{1+(dx+c)^2}+20(dx+c)^3-12(dx+c)^2\sqrt{1+(dx+c)^2}+5dx+5c-\sqrt{1+(dx+c)^2}\right)e^4(5b\,\text{arcsinh}(dx+c)+5a-b)}{64b^2\left(b^2\,\text{arcsinh}(dx+c)^2+2ab\,\text{arcsinh}(dx+c)+a^2\right)} - \frac{25e^4\mathrm{e}^{\frac{5a}{b}}\,\mathrm{Ei}\left(1,5\,\text{arc}\right.}{64b}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^3,x)
```

```
[Out] 1/d*(-1/64*(16*(d*x+c)^5-16*(d*x+c)^4*(1+(d*x+c)^2)^(1/2)+20*(d*x+c)^3-12*(
d*x+c)^2*(1+(d*x+c)^2)^(1/2)+5*d*x+5*c-(1+(d*x+c)^2)^(1/2))*e^4*(5*b*arcsin
h(d*x+c)+5*a-b)/b^2/(b^2*arcsinh(d*x+c)^2+2*a*b*arcsinh(d*x+c)+a^2)-25/64*e
^4/b^3*exp(5*a/b)*Ei(1,5*arcsinh(d*x+c)+5*a/b)+3/64*(-4*(d*x+c)^2*(1+(d*x+c
)^2)^(1/2)+4*(d*x+c)^3-(1+(d*x+c)^2)^(1/2)+3*d*x+3*c)*e^4*(3*b*arcsinh(d*x+
c)+3*a-b)/b^2/(b^2*arcsinh(d*x+c)^2+2*a*b*arcsinh(d*x+c)+a^2)+27/64*e^4/b^3
*exp(3*a/b)*Ei(1,3*arcsinh(d*x+c)+3*a/b)-1/32*(-(1+(d*x+c)^2)^(1/2)+d*x+c)*
e^4*(b*arcsinh(d*x+c)+a-b)/b^2/(b^2*arcsinh(d*x+c)^2+2*a*b*arcsinh(d*x+c)+a
^2)-1/32*e^4/b^3*exp(a/b)*Ei(1,arcsinh(d*x+c)+a/b)-1/32/b*e^4*(d*x+c+(1+(d*
x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^2-1/32/b^2*e^4*(d*x+c+(1+(d*x+c)^2)^(1/
2))/(a+b*arcsinh(d*x+c))-1/32/b^3*e^4*exp(-a/b)*Ei(1,-arcsinh(d*x+c)-a/b)+3
/64/b*e^4*(4*(d*x+c)^3+3*d*x+3*c+4*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+(1+(d*x+c)
^2)^(1/2))/(a+b*arcsinh(d*x+c))^2+9/64/b^2*e^4*(4*(d*x+c)^3+3*d*x+3*c+4*(d*
x+c)^2*(1+(d*x+c)^2)^(1/2)+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))+27/64/
b^3*e^4*exp(-3*a/b)*Ei(1,-3*arcsinh(d*x+c)-3*a/b)-1/64/b*e^4*(16*(d*x+c)^5+
20*(d*x+c)^3+16*(d*x+c)^4*(1+(d*x+c)^2)^(1/2)+5*d*x+5*c+12*(d*x+c)^2*(1+(d*
x+c)^2)^(1/2)+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^2-5/64/b^2*e^4*(16*
(d*x+c)^5+20*(d*x+c)^3+16*(d*x+c)^4*(1+(d*x+c)^2)^(1/2)+5*d*x+5*c+12*(d*x+c
)^2*(1+(d*x+c)^2)^(1/2)+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))-25/64/b^3
*e^4*exp(-5*a/b)*Ei(1,-5*arcsinh(d*x+c)-5*a/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")

[Out] -1/2*((5*a*d^11*e^4 + b*d^11*e^4)*x^11 + 11*(5*a*c*d^10*e^4 + b*c*d^10*e^4)*x^10 + (5*(55*c^2*d^9*e^4 + 3*d^9*e^4)*a + (55*c^2*d^9*e^4 + 3*d^9*e^4)*b)*x^9 + 3*(5*(55*c^3*d^8*e^4 + 9*c*d^8*e^4)*a + (55*c^3*d^8*e^4 + 9*c*d^8*e^4)*b)*x^8 + 3*(5*(110*c^4*d^7*e^4 + 36*c^2*d^7*e^4 + d^7*e^4)*a + (110*c^4*d^7*e^4 + 36*c^2*d^7*e^4 + d^7*e^4)*b)*x^7 + 21*(5*(22*c^5*d^6*e^4 + 12*c^3*d^6*e^4 + c*d^6*e^4)*a + (22*c^5*d^6*e^4 + 12*c^3*d^6*e^4 + c*d^6*e^4)*b)*x^6 + (5*(462*c^6*d^5*e^4 + 378*c^4*d^5*e^4 + 63*c^2*d^5*e^4 + d^5*e^4)*a + (462*c^6*d^5*e^4 + 378*c^4*d^5*e^4 + 63*c^2*d^5*e^4 + d^5*e^4)*b)*x^5 + (5*(330*c^7*d^4*e^4 + 378*c^5*d^4*e^4 + 105*c^3*d^4*e^4 + 5*c*d^4*e^4)*a + (330*c^7*d^4*e^4 + 378*c^5*d^4*e^4 + 105*c^3*d^4*e^4 + 5*c*d^4*e^4)*b)*x^4 + (5*(165*c^8*d^3*e^4 + 252*c^6*d^3*e^4 + 105*c^4*d^3*e^4 + 10*c^2*d^3*e^4)*a + (165*c^8*d^3*e^4 + 252*c^6*d^3*e^4 + 105*c^4*d^3*e^4 + 10*c^2*d^3*e^4)*b)*x^3 + (5*(55*c^9*d^2*e^4 + 108*c^7*d^2*e^4 + 63*c^5*d^2*e^4 + 10*c^3*d^2*e^4)*a + (55*c^9*d^2*e^4 + 108*c^7*d^2*e^4 + 63*c^5*d^2*e^4 + 10*c^3*d^2*e^4)*b)*x^2 + ((5*a*d^8*e^4 + b*d^8*e^4)*x^8 + 8*(5*a*c*d^7*e^4 + b*c*d^7*e^4)*x^7 + (4*(35*c^2*d^6*e^4 + 2*d^6*e^4)*a + (28*c^2*d^6*e^4 + d^6*e^4)*b)*x^6 + 2*(4*(35*c^3*d^5*e^4 + 6*c*d^5*e^4)*a + (28*c^3*d^5*e^4 + 3*c*d^5*e^4)*b)*x^5 + ((350*c^4*d^4*e^4 + 120*c^2*d^4*e^4 + 3*d^4*e^4)*a + 5*(14*c^4*d^4*e^4 + 3*c^2*d^4*e^4)*b)*x^4 + 4*((70*c^5*d^3*e^4 + 40*c^3*d^3*e^4 + 3*c*d^3*e^4)*a + (14*c^5*d^3*e^4 + 5*c^3*d^3*e^4)*b)*x^3 + (2*(70*c^6*d^2*e^4 + 60*c^4*d^2*e^4 + 9*c^2*d^2*e^4)*a + (28*c^6*d^2*e^4 + 15*c^4*d^2*e^4)*b)*x^2 + (5*c^8*e^4 + 8*c^6*e^4 + 3*c^4*e^4)*a + (c^8*e^4 + c^6*e^4)*b + 2*(2*(10*c^7*d*e^4 + 12*c^5*d*e^4 + 3*c^3*d*e^4)*a + (4*c^7*d*e^4 + 3*c^5*d*e^4)*b)*x)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + (3*(5*a*d^9*e^4 + b*d^9*e^4)*x^9 + 27*(5*a*c*d^8*e^4 + b*c*d^8*e^4)*x^8 + ((540*c^2*d^7*e^4 + 31*d^7*e^4)*a + (108*c^2*d^7*e^4 + 5*d^7*e^4)*b)*x^7 + 7*((180*c^3*d^6*e^4 + 31*c*d^6*e^4)*a + (36*c^3*d^6*e^4 + 5*c*d^6*e^4)*b)*x^6 + ((1890*c^4*d^5*e^4 + 651*c^2*d^5*e^4 + 20*d^5*e^4)*a + (378*c^4*d^5*e^4 + 105*c^2*d^5*e^4 + 2*d^5*e^4)*b)*x^5 + (5*(378*c^5*d^4*e^4 + 217*c^3*d^4*e^4 + 20*c*d^4*e^4)*a + (378*c^5*d^4*e^4 + 175*c^3*d^4*e^4 + 10*c*d^4*e^4)*b)*x^4 + ((1260*c^6*d^3*e^4 + 1085*c^4*d^3*e^4 + 200*c^2*d^3*e^4 + 4*d^3*e^4)*a + (252*c^6*d^3*e^4 + 175*c^4*d^3*e^4 + 20*c^2*d^3*e^4)*b)*x^3 + ((540*c^7*d^2*e^4 + 651*c^5*d^2*e^4 + 200*c^3*d^2*e^4 + 12*c*d^2*e^4)*a + (108*c^7*d^2*e^4 + 105*c^5*d^2*e^4 + 20*c^3*d^2*e^4)*b)*x^2 + (15*c^9*e^4 + 31*c^7*e^4 + 20*c^5*e^4 + 4*c^3*e^4)*a + (3*c^9*e^4 + 5*c^7*e^4 + 2*c^5*e^4)*b + ((135*c^8*d*e^4 + 217*c^6*d*e^4 + 100*c^4*d*e^4 + 12*c^2*d*e^4)*a + (27*c^8*d*e^4 + 35*c^6*d*e^4 + 10*c^4*d*e^4)*b)*x)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + 5*(c^11*e^4 + 3*c^9*e^4 + 3*c^7*e^4 + c^5*e^4)*a + (c^11*e^4 + 3*c^9*e^4 + 3*c^7*e^4 + c^5*e^4)*b + (5*(11*c^10*d*e^4 + 27*c^8*d*e^4 + 21*c^6*d*e^4 + 5*c^4*d*e^4)*a + (11*c^10*d*e^4 + 27*c^8*d*e^4 + 21*c^6*d*e^4 + 5*c^4*d*e^4)*b)*x + (5*b*d^11*e^4*x^11 + 55*b*c*d^10*e^4*x^10 + 5*(55*c^2*d^9*e^4 + 3*d^9*e^4)*b*x^9 + 15*(55*c^3*d^8*e^4 + 9*c*d^8*e^4)*b*x^8 + 15*(110*c^4*d^7*e^4 + 36*c^2*d^7*e^4 + d^7*e^4)*b*x^7 + 105*(22*c^5*d^6*e^4 + 12*c^3*d^6*e^4 + c*d^6*e^4)*b*x^6 + 5*(462*c^6*d^5*e^4 + 378*c^4*d^5*e^4 + 63*c^2*d^5*e^4 + d^5*e^4)*b*x^5 + 5*(330*c^7*d^4*e^4 + 378*c^5*d^4*e^4 + 105*c^3*d^4*e^4 + 5*c*d^4*e^4)*b*x^4 + 5*(165*c^8*d^3*e^4 + 252*c^6*d^3*e^4 + 105*c^4*d^3*e^4 + 10*c^2*d^3*e^4)*b*x^3 + 5*(55*c^9*d^2*e^4 + 108*c^7*d^2*e^4 + 63*c^5*d^2*e^4 + 10*c^3*d^2*e^4)*b*x^2 + 5*(11*c^10*d*e^4 + 27*c^8*d*e^4 + 21*c^6*d*e^4 + 5*c^4*d*e^4)*b*x + (5*b*d^8*e^4*x^8 + 40*b*c*d^7*e^4*x^7 + 4*(35*c^2*d^6*e^4 + 2*d^6*e^4)*b*x^6 + 8*(35*c^3*d^5*e^4 + 6*c*d^5*e^4)*b*x^5 + (350*c^4*d^4*e^4 + 120*c^2*d^4*e^4 + 3*d^4*e^4)*b*x^4 + 4*(70*c^5*d^3*e^4 + 40*c^3*d^3*e^4 + 3*c*d^3*e^4)*b*x^3 + 2*(70*c^6*d^2*e^4 + 60*c^4*d^2*e^4 + 9*c^2*d^2*e^4)*b*x^2 + 4*(10*c^7*d*e^4 + 12*c^5*d*e^4 + 3*c^3*d*e^4)*b*x + (5*c^8*e^4 + 8*c^6*e^4 + 3*c^4*e^4)*b)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + (15*b*d^9*e^4*x^9 + 135*b*c*d^8*e^4*x^8 + (540*c^2*d^7*e^4 + 31*d^7*e^4)*b*x^7 + 7*(180*c^3*d^6*e^4 + 31*c*d^6*e^4)*b*x^6 + (1890*c^4*d^5*e^4 + 651*c^2*d^5*e^4 + 20*d^5*e^4)*b*x^5 + 5*(378*c^5*d^4*e^4 + 217*c^3*d^4*e^4 + 20*c*d^4*e^4)*b*x^4 + (1260*c^6*d^3*e^4 + 1085*c^4*d^3*e^4 + 200*c^2*d^3*e^4 + 4*d^3*e^4)*b*x^3 + (540*c^7*d^2

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*e^4 + 651*c^5*d^2*e^4 + 200*c^3*d^2*e^4 + 12*c*d^2*e^4)*b*x^2 + (135*c^8*d
*e^4 + 217*c^6*d*e^4 + 100*c^4*d*e^4 + 12*c^2*d*e^4)*b*x + (15*c^9*e^4 + 31
*c^7*e^4 + 20*c^5*e^4 + 4*c^3*e^4)*b)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + 5*(c^
11*e^4 + 3*c^9*e^4 + 3*c^7*e^4 + c^5*e^4)*b + (15*b*d^10*e^4*x^10 + 150*b*c
*d^9*e^4*x^9 + (675*c^2*d^8*e^4 + 38*d^8*e^4)*b*x^8 + 8*(225*c^3*d^7*e^4 +
38*c*d^7*e^4)*b*x^7 + 2*(1575*c^4*d^6*e^4 + 532*c^2*d^6*e^4 + 16*d^6*e^4)*b
*x^6 + 4*(945*c^5*d^5*e^4 + 532*c^3*d^5*e^4 + 48*c*d^5*e^4)*b*x^5 + (3150*c
^6*d^4*e^4 + 2660*c^4*d^4*e^4 + 480*c^2*d^4*e^4 + 9*d^4*e^4)*b*x^4 + 4*(450
*c^7*d^3*e^4 + 532*c^5*d^3*e^4 + 160*c^3*d^3*e^4 + 9*c*d^3*e^4)*b*x^3 + (67
5*c^8*d^2*e^4 + 1064*c^6*d^2*e^4 + 480*c^4*d^2*e^4 + 54*c^2*d^2*e^4)*b*x^2
+ 2*(75*c^9*d*e^4 + 152*c^7*d*e^4 + 96*c^5*d*e^4 + 18*c^3*d*e^4)*b*x + (15*
c^10*e^4 + 38*c^8*e^4 + 32*c^6*e^4 + 9*c^4*e^4)*b)*sqrt(d^2*x^2 + 2*c*d*x +
 c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + (3*(5*a*d^10*
e^4 + b*d^10*e^4)*x^10 + 30*(5*a*c*d^9*e^4 + b*c*d^9*e^4)*x^9 + ((675*c^2*d
^8*e^4 + 38*d^8*e^4)*a + (135*c^2*d^8*e^4 + 7*d^8*e^4)*b)*x^8 + 8*((225*c^3
*d^7*e^4 + 38*c*d^7*e^4)*a + (45*c^3*d^7*e^4 + 7*c*d^7*e^4)*b)*x^7 + (2*(15
75*c^4*d^6*e^4 + 532*c^2*d^6*e^4 + 16*d^6*e^4)*a + (630*c^4*d^6*e^4 + 196*c
^2*d^6*e^4 + 5*d^6*e^4)*b)*x^6 + 2*(2*(945*c^5*d^5*e^4 + 532*c^3*d^5*e^4 +
48*c*d^5*e^4)*a + (378*c^5*d^5*e^4 + 196*c^3*d^5*e^4 + 15*c*d^5*e^4)*b)*x^5
 + ((3150*c^6*d^4*e^4 + 2660*c^4*d^4*e^4 + 480*c^2*d^4*e^4 + 9*d^4*e^4)*a +
 (630*c^6*d^4*e^4 + 490*c^4*d^4*e^4 + 75*c^2*d^4*e^4 + d^4*e^4)*b)*x^4 + 4*
((450*c^7*d^3*e^4 + 532*c^5*d^3*e^4 + 160*c^3*d^3*e^4 + 9*c*d^3*e^4)*a + (9
0*c^7*d^3*e^4 + 98*c^5*d^3*e^4 + 25*c^3*d^3*e^4 + c*d^3*e^4)*b)*x^3 + ((675
*c^8*d^2*e^4 + 1064*c^6*d^2*e^4 + 480*c^4*d^2*e^4 + 54*c^2*d^2*e^4)*a + (13
5*c^8*d^2*e^4 + 196*c^6*d^2*e^4 + 75*c^4*d^2*e^4 + 6*c^2*d^2*e^4)*b)*x^2 +
(15*c^10*e^4 + 38*c^8*e^4 + 32*c^6*e^4 + 9*c^4*e^4)*a + (3*c^10*e^4 + 7*c^8
*e^4 + 5*c^6*e^4 + c^4*e^4)*b + 2*((75*c^9*d*e^4 + 152*c^7*d*e^4 + 96*c^5*d
*e^4 + 18*c^3*d*e^4)*a + (15*c^9*d*e^4 + 28*c^7*d*e^4 + 15*c^5*d*e^4 + 2*c^
3*d*e^4)*b)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(a^2*b^2*d^7*x^6 + 6*a^2*
b^2*c*d^6*x^5 + 3*(5*c^2*d^5 + d^5)*a^2*b^2*x^4 + 4*(5*c^3*d^4 + 3*c*d^4)*a
^2*b^2*x^3 + 3*(5*c^4*d^3 + 6*c^2*d^3 + d^3)*a^2*b^2*x^2 + 6*(c^5*d^2 + 2*c
^3*d^2 + c*d^2)*a^2*b^2*x + (c^6*d + 3*c^4*d + 3*c^2*d + d)*a^2*b^2 + (b^4*
d^7*x^6 + 6*b^4*c*d^6*x^5 + 3*(5*c^2*d^5 + d^5)*b^4*x^4 + 4*(5*c^3*d^4 + 3*
c*d^4)*b^4*x^3 + 3*(5*c^4*d^3 + 6*c^2*d^3 + d^3)*b^4*x^2 + 6*(c^5*d^2 + 2*c
^3*d^2 + c*d^2)*b^4*x + (c^6*d + 3*c^4*d + 3*c^2*d + d)*b^4 + (b^4*d^4*x^3
+ 3*b^4*c*d^3*x^2 + 3*b^4*c^2*d^2*x + b^4*c^3*d)*(d^2*x^2 + 2*c*d*x + c^2 +
 1)^(3/2) + 3*(b^4*d^5*x^4 + 4*b^4*c*d^4*x^3 + (6*c^2*d^3 + d^3)*b^4*x^2 +
2*(2*c^3*d^2 + c*d^2)*b^4*x + (c^4*d + c^2*d)*b^4)*(d^2*x^2 + 2*c*d*x + c^2
 + 1) + 3*(b^4*d^6*x^5 + 5*b^4*c*d^5*x^4 + 2*(5*c^2*d^4 + d^4)*b^4*x^3 + 2*
(5*c^3*d^3 + 3*c*d^3)*b^4*x^2 + (5*c^4*d^2 + 6*c^2*d^2 + d^2)*b^4*x + (c^5*
d + 2*c^3*d + c*d)*b^4)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sq
rt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 + (a^2*b^2*d^4*x^3 + 3*a^2*b^2*c*d^3*x^2
 + 3*a^2*b^2*c^2*d^2*x + a^2*b^2*c^3*d)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2)
 + 3*(a^2*b^2*d^5*x^4 + 4*a^2*b^2*c*d^4*x^3 + (6*c^2*d^3 + d^3)*a^2*b^2*x^2
 + 2*(2*c^3*d^2 + c*d^2)*a^2*b^2*x + (c^4*d + c^2*d)*a^2*b^2)*(d^2*x^2 + 2*
c*d*x + c^2 + 1) + 2*(a*b^3*d^7*x^6 + 6*a*b^3*c*d^6*x^5 + 3*(5*c^2*d^5 + d^
5)*a*b^3*x^4 + 4*(5*c^3*d^4 + 3*c*d^4)*a*b^3*x^3 + 3*(5*c^4*d^3 + 6*c^2*d^3
 + d^3)*a*b^3*x^2 + 6*(c^5*d^2 + 2*c^3*d^2 + c*d^2)*a*b^3*x + (c^6*d + 3*c^
4*d + 3*c^2*d + d)*a*b^3 + (a*b^3*d^4*x^3 + 3*a*b^3*c*d^3*x^2 + 3*a*b^3*c^2
*d^2*x + a*b^3*c^3*d)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + 3*(a*b^3*d^5*x^
4 + 4*a*b^3*c*d^4*x^3 + (6*c^2*d^3 + d^3)*a*b^3*x^2 + 2*(2*c^3*d^2 + c*d^2)
*a*b^3*x + (c^4*d + c^2*d)*a*b^3)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + 3*(a*b^3*
d^6*x^5 + 5*a*b^3*c*d^5*x^4 + 2*(5*c^2*d^4 + d^4)*a*b^3*x^3 + 2*(5*c^3*d^3
+ 3*c*d^3)*a*b^3*x^2 + (5*c^4*d^2 + 6*c^2*d^2 + d^2)*a*b^3*x + (c^5*d + 2*c
^3*d + c*d)*a*b^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^
2*x^2 + 2*c*d*x + c^2 + 1)) + 3*(a^2*b^2*d^6*x^5 + 5*a^2*b^2*c*d^5*x^4 + 2*
(5*c^2*d^4 + d^4)*a^2*b^2*x^3 + 2*(5*c^3*d^3 + 3*c*d^3)*a^2*b^2*x^2 + (5*c^
4*d^2 + 6*c^2*d^2 + d^2)*a^2*b^2*x + (c^5*d + 2*c^3*d + c*d)*a^2*b^2)*sqrt(
d^2*x^2 + 2*c*d*x + c^2 + 1)) + integrate(1/2*(25*d^12*e^4*x^12 + 300*c*d^1
```

1*e^4*x^11 + 25*c^12*e^4 + 100*c^10*e^4 + 150*c^8*e^4 + 50*(33*c^2*d^10*e^4 + 2*d^10*e^4)*x^10 + 100*c^6*e^4 + 500*(11*c^3*d^9*e^4 + 2*c*d^9*e^4)*x^9 + 75*(165*c^4*d^8*e^4 + 60*c^2*d^8*e^4 + 2*d^8*e^4)*x^8 + 25*c^4*e^4 + 600*(33*c^5*d^7*e^4 + 20*c^3*d^7*e^4 + 2*c*d^7*e^4)*x^7 + 100*(231*c^6*d^6*e^4 + 210*c^4*d^6*e^4 + 42*c^2*d^6*e^4 + d^6*e^4)*x^6 + 600*(33*c^7*d^5*e^4 + 42*c^5*d^5*e^4 + 14*c^3*d^5*e^4 + c*d^5*e^4)*x^5 + 25*(495*c^8*d^4*e^4 + 840*c^6*d^4*e^4 + 420*c^4*d^4*e^4 + 60*c^2*d^4*e^4 + d^4*e^4)*x^4 + 100*(55*c^9*d^3*e^4 + 120*c^7*d^3*e^4 + 84*c^5*d^3*e^4 + 20*c^3*d^3*e^4 + c*d^3*e^4)*x^3 + (25*d^8*e^4*x^8 + 200*c*d^7*e^4*x^7 + 25*c^8*e^4 + 24*c^6*e^4 + 3*c^4*e^4 + 4*(175*c^2*d^6*e^4 + 6*d^6*e^4)*x^6 + 8*(175*c^3*d^5*e^4 + 18*c*d^5*e^4)*x^5 + (1750*c^4*d^4*e^4 + 360*c^2*d^4*e^4 + 3*d^4*e^4)*x^4 + 4*(350*c^5*d^3*e^4 + 120*c^3*d^3*e^4 + 3*c*d^3*e^4)*x^3 + 2*(350*c^6*d^2*e^4 + 180*c^4*d^2*e^4 + 9*c^2*d^2*e^4)*x^2 + 4*(50*c^7*d*e^4 + 36*c^5*d*e^4 + 3*c^3*d*e^4)*x)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 150*(11*c^10*d^2*e^4 + 30*c^8*d^2*e^4 + 28*c^6*d^2*e^4 + 10*c^4*d^2*e^4 + c^2*d^2*e^4)*x^2 + (100*d^9*e^4*x^9 + 900*c*d^8*e^4*x^8 + 100*c^9*e^4 + 172*c^7*e^4 + 87*c^5*e^4 + 4*(900*c^2*d^7*e^4 + 43*d^7*e^4)*x^7 + 12*c^3*e^4 + 28*(300*c^3*d^6*e^4 + 43*c*d^6*e^4)*x^6 + 3*(4200*c^4*d^5*e^4 + 1204*c^2*d^5*e^4 + 29*d^5*e^4)*x^5 + 5*(2520*c^5*d^4*e^4 + 1204*c^3*d^4*e^4 + 87*c*d^4*e^4)*x^4 + 2*(4200*c^6*d^3*e^4 + 3010*c^4*d^3*e^4 + 435*c^2*d^3*e^4 + 6*d^3*e^4)*x^3 + 6*(600*c^7*d^2*e^4 + 602*c^5*d^2*e^4 + 145*c^3*d^2*e^4 + 6*c*d^2*e^4)*x^2 + (900*c^8*d*e^4 + 1204*c^6*d*e^4 + 435*c^4*d*e^4 + 36*c^2*d*e^4)*x)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + 3*(50*d^10*e^4*x^10 + 500*c*d^9*e^4*x^9 + 50*c^10*e^4 + 124*c^8*e^4 + 105*c^6*e^4 + 2*(1125*c^2*d^8*e^4 + 62*d^8*e^4)*x^8 + 35*c^4*e^4 + 16*(375*c^3*d^7*e^4 + 62*c*d^7*e^4)*x^7 + 7*(1500*c^4*d^6*e^4 + 496*c^2*d^6*e^4 + 15*d^6*e^4)*x^6 + 4*c^2*e^4 + 14*(900*c^5*d^5*e^4 + 496*c^3*d^5*e^4 + 45*c*d^5*e^4)*x^5 + 35*(300*c^6*d^4*e^4 + 248*c^4*d^4*e^4 + 45*c^2*d^4*e^4 + d^4*e^4)*x^4 + 4*(1500*c^7*d^3*e^4 + 1736*c^5*d^3*e^4 + 525*c^3*d^3*e^4 + 35*c*d^3*e^4)*x^3 + (2250*c^8*d^2*e^4 + 3472*c^6*d^2*e^4 + 1575*c^4*d^2*e^4 + 210*c^2*d^2*e^4 + 4*d^2*e^4)*x^2 + 2*(250*c^9*d*e^4 + 496*c^7*d*e^4 + 315*c^5*d*e^4 + 70*c^3*d*e^4 + 4*c*d*e^4)*x)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + 100*(3*c^11*d*e^4 + 10*c^9*d*e^4 + 12*c^7*d*e^4 + 6*c^5*d*e^4 + c^3*d*e^4)*x + (100*d^11*e^4*x^11 + 1100*c*d^10*e^4*x^10 + 100*c^11*e^4 + 324*c^9*e^4 + 381*c^7*e^4 + 4*(1375*c^2*d^9*e^4 + 81*d^9*e^4)*x^9 + 193*c^5*e^4 + 12*(1375*c^3*d^8*e^4 + 243*c*d^8*e^4)*x^8 + 3*(11000*c^4*d^7*e^4 + 3888*c^2*d^7*e^4 + 127*d^7*e^4)*x^7 + 36*c^3*e^4 + 21*(2200*c^5*d^6*e^4 + 1296*c^3*d^6*e^4 + 127*c*d^6*e^4)*x^6 + (46200*c^6*d^5*e^4 + 40824*c^4*d^5*e^4 + 8001*c^2*d^5*e^4 + 193*d^5*e^4)*x^5 + (33000*c^7*d^4*e^4 + 40824*c^5*d^4*e^4 + 13335*c^3*d^4*e^4 + 965*c*d^4*e^4)*x^4 + (16500*c^8*d^3*e^4 + 27216*c^6*d^3*e^4 + 13335*c^4*d^3*e^4 + 1930*c^2*d^3*e^4 + 36*d^3*e^4)*x^3 + (5500*c^9*d^2*e^4 + 11664*c^7*d^2*e^4 + 8001*c^5*d^2*e^4 + 1930*c^3*d^2*e^4 + 108*c*d^2*e^4)*x^2 + (1100*c^10*d*e^4 + 2916*c^8*d*e^4 + 2667*c^6*d*e^4 + 965*c^4*d*e^4 + 108*c^2*d*e^4)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(a*b^2*d^8*x^8 + 8*a*b^2*c*d^7*x^7 + 4*(7*c^2*d^6 + d^6)*a*b^2*x^6 + 8*(7*c^3*d^5 + 3*c*d^5)*a*b^2*x^5 + 2*(35*c^4*d^4 + 30*c^2*d^4 + 3*d^4)*a*b^2*x^4 + 8*(7*c^5*d^3 + 10*c^3*d^3 + 3*c*d^3)*a*b^2*x^3 + 4*(7*c^6*d^2 + 15*c^4*d^2 + 9*c^2*d^2 + d^2)*a*b^2*x^2 + 8*(c^7*d + 3*c^5*d + 3*c^3*d + c*d)*a*b^2*x + (c^8 + 4*c^6 + 6*c^4 + 4*c^2 + 1)*a*b^2 + (a*b^2*d^4*x^4 + 4*a*b^2*c*d^3*x^3 + 6*a*b^2*c^2*d^2*x^2 + 4*a*b^2*c^3*d*x + a*b^2*c^4)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 4*(a*b^2*d^5*x^5 + 5*a*b^2*c*d^4*x^4 + (10*c^2*d^3 + d^3)*a*b^2*x^3 + (10*c^3*d^2 + 3*c*d^2)*a*b^2*x^2 + (5*c^4*d + 3*c^2*d)*a*b^2*x + (c^5 + c^3)*a*b^2)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + 6*(a*b^2*d^6*x^6 + 6*a*b^2*c*d^5*x^5 + (15*c^2*d^4 + 2*d^4)*a*b^2*x^4 + 4*(5*c^3*d^3 + 2*c*d^3)*a*b^2*x^3 + (15*c^4*d^2 + 12*c^2*d^2 + d^2)*a*b^2*x^2 + 2*(3*c^5*d + 4*c^3*d + c*d)*a*b^2*x + (c^6 + 2*c^4 + c^2)*a*b^2)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + (b^3*d^8*x^8 + 8*b^3*c*d^7*x^7 + 4*(7*c^2*d^6 + d^6)*b^3*x^6 + 8*(7*c^3*d^5 + 3*c*d^5)*b^3*x^5 + 2*(35*c^4*d^4 + 30*c^2*d^4 + 3*d^4)*b^3*x^4 + 8*(7*c^5*d^3 + 10*c^3*d^3 + 3*c*d^3)*b^3*x^3 + 4*(7*c^6*d^2 + 15*c^4*d^2 + 9*c^2*d^2 + d^2)*b^3*x^2 + 8*(c^7*d + 3*c^5*d + 3*c^3*d + c*d)*b^3*x + (c^8 + 4*c^6 + 6*c^

```
4 + 4*c^2 + 1)*b^3 + (b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*x^2 + 4
*b^3*c^3*d*x + b^3*c^4)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 4*(b^3*d^5*x^5 +
5*b^3*c*d^4*x^4 + (10*c^2*d^3 + d^3)*b^3*x^3 + (10*c^3*d^2 + 3*c*d^2)*b^3*x
^2 + (5*c^4*d + 3*c^2*d)*b^3*x + (c^5 + c^3)*b^3)*(d^2*x^2 + 2*c*d*x + c^2
+ 1)^(3/2) + 6*(b^3*d^6*x^6 + 6*b^3*c*d^5*x^5 + (15*c^2*d^4 + 2*d^4)*b^3*x^
4 + 4*(5*c^3*d^3 + 2*c*d^3)*b^3*x^3 + (15*c^4*d^2 + 12*c^2*d^2 + d^2)*b^3*x
^2 + 2*(3*c^5*d + 4*c^3*d + c*d)*b^3*x + (c^6 + 2*c^4 + c^2)*b^3)*(d^2*x^2
+ 2*c*d*x + c^2 + 1) + 4*(b^3*d^7*x^7 + 7*b^3*c*d^6*x^6 + 3*(7*c^2*d^5 + d^
5)*b^3*x^5 + 5*(7*c^3*d^4 + 3*c*d^4)*b^3*x^4 + (35*c^4*d^3 + 30*c^2*d^3 + 3
*d^3)*b^3*x^3 + 3*(7*c^5*d^2 + 10*c^3*d^2 + 3*c*d^2)*b^3*x^2 + (7*c^6*d + 1
5*c^4*d + 9*c^2*d + d)*b^3*x + (c^7 + 3*c^5 + 3*c^3 + c)*b^3)*sqrt(d^2*x^2
+ 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + 4*
(a*b^2*d^7*x^7 + 7*a*b^2*c*d^6*x^6 + 3*(7*c^2*d^5 + d^5)*a*b^2*x^5 + 5*(7*c
^3*d^4 + 3*c*d^4)*a*b^2*x^4 + (35*c^4*d^3 + 30*c^2*d^3 + 3*d^3)*a*b^2*x^3 +
 3*(7*c^5*d^2 + 10*c^3*d^2 + 3*c*d^2)*a*b^2*x^2 + (7*c^6*d + 15*c^4*d + 9*c
^2*d + d)*a*b^2*x + (c^7 + 3*c^5 + 3*c^3 + c)*a*b^2)*sqrt(d^2*x^2 + 2*c*d*x
 + c^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,e+d\,e\,x)^4}{(a+b\,\mathrm{asinh}\,(c+d\,x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^4/(a + b*asinh(c + d*x))^3,x)
```

```
[Out] int((c*e + d*e*x)^4/(a + b*asinh(c + d*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^4\left(\int \frac{c^4}{a^3+3a^2b\,\mathrm{asinh}\,(c+dx)+3ab^2\,\mathrm{asinh}^2\,(c+dx)+b^3\,\mathrm{asinh}^3\,(c+dx)}\,dx+\int \frac{}{a^3+3a^2b\,\mathrm{asinh}\,(c+dx)+3a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4/(a+b*asinh(d*x+c))**3,x)
```

```
[Out] e**4*(Integral(c**4/(a**3 + 3*a**2*b*asinh(c + d*x) + 3*a*b**2*asinh(c + d*
x)**2 + b**3*asinh(c + d*x)**3), x) + Integral(d**4*x**4/(a**3 + 3*a**2*b*a
sinh(c + d*x) + 3*a*b**2*asinh(c + d*x)**2 + b**3*asinh(c + d*x)**3), x) +
Integral(4*c*d**3*x**3/(a**3 + 3*a**2*b*asinh(c + d*x) + 3*a*b**2*asinh(c +
 d*x)**2 + b**3*asinh(c + d*x)**3), x) + Integral(6*c**2*d**2*x**2/(a**3 +
3*a**2*b*asinh(c + d*x) + 3*a*b**2*asinh(c + d*x)**2 + b**3*asinh(c + d*x)*
*3), x) + Integral(4*c**3*d*x/(a**3 + 3*a**2*b*asinh(c + d*x) + 3*a*b**2*as
inh(c + d*x)**2 + b**3*asinh(c + d*x)**3), x))
```

3.169 $\int \frac{(ce+dex)^3}{\left(a+b\sinh^{-1}(c+dx)\right)^3}\,dx$

Optimal. Leaf size=247

$$\frac{e^3\sinh\left(\frac{2a}{b}\right)\operatorname{Chi}\left(\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{2b^3d}-\frac{e^3\sinh\left(\frac{4a}{b}\right)\operatorname{Chi}\left(\frac{4\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{b^3d}-\frac{e^3\cosh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{2b^3d}$$

```
[Out] -3/2*e^3*(d*x+c)^2/b^2/d/(a+b*arcsinh(d*x+c))-2*e^3*(d*x+c)^4/b^2/d/(a+b*ar
csinh(d*x+c))-1/2*e^3*cosh(2*a/b)*Shi(2*(a+b*arcsinh(d*x+c))/b)/b^3/d+e^3*c
osh(4*a/b)*Shi(4*(a+b*arcsinh(d*x+c))/b)/b^3/d+1/2*e^3*Chi(2*(a+b*arcsinh(d
*x+c))/b)*sinh(2*a/b)/b^3/d-e^3*Chi(4*(a+b*arcsinh(d*x+c))/b)*sinh(4*a/b)/b
^3/d-1/2*e^3*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))^2
```

Rubi [A] time = 0.68, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {5865, 12, 5667, 5774, 5669, 5448, 3303, 3298, 3301}

$$\frac{e^3\sinh\left(\frac{2a}{b}\right)\operatorname{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(c+dx)\right)}{2b^3d}-\frac{e^3\sinh\left(\frac{4a}{b}\right)\operatorname{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(c+dx)\right)}{b^3d}-\frac{e^3\cosh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(\frac{2a}{b}+2\sinh^{-1}(c+dx)\right)}{2b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3/(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] -(e^3*(c + d*x)^3*Sqrt[1 + (c + d*x)^2])/(2*b*d*(a + b*ArcSinh[c + d*x])^2)
 - (3*e^3*(c + d*x)^2)/(2*b^2*d*(a + b*ArcSinh[c + d*x])) - (2*e^3*(c + d*x
)^4)/(b^2*d*(a + b*ArcSinh[c + d*x])) + (e^3*CoshIntegral[(2*a)/b + 2*ArcSi
nh[c + d*x]]*Sinh[(2*a)/b])/(2*b^3*d) - (e^3*CoshIntegral[(4*a)/b + 4*ArcSi
nh[c + d*x]]*Sinh[(4*a)/b])/(b^3*d) - (e^3*Cosh[(2*a)/b]*SinhIntegral[(2*a)
/b + 2*ArcSinh[c + d*x]])/(2*b^3*d) + (e^3*Cosh[(4*a)/b]*SinhIntegral[(4*a)
/b + 4*ArcSinh[c + d*x]])/(b^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^3}{\left(a+b\sinh^{-1}(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{e^3x^3}{\left(a+b\sinh^{-1}(x)\right)^3}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^3\,\text{Subst}\left(\int \frac{x^3}{\left(a+b\sinh^{-1}(x)\right)^3}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} + \frac{\left(3e^3\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1+x^2}\left(a+b\sinh^{-1}(x)\right)^2}\,dx, x, c+\right.}{2bd}$$

$$= -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{3e^3(c+dx)^2}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{2e^3(c+}{b^2d\left(a+b\sinh\right.}$$

$$= -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{3e^3(c+dx)^2}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{2e^3(c+}{b^2d\left(a+b\sinh\right.}$$

$$= -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{3e^3(c+dx)^2}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{2e^3(c+}{b^2d\left(a+b\sinh\right.}$$

$$= -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{3e^3(c+dx)^2}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{2e^3(c+}{b^2d\left(a+b\sinh\right.}$$

$$= -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{3e^3(c+dx)^2}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{2e^3(c+}{b^2d\left(a+b\sinh\right.}$$

$$= -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{3e^3(c+dx)^2}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{2e^3(c+}{b^2d\left(a+b\sinh\right.}$$

Mathematica [A] time = 0.66, size = 179, normalized size = 0.72

$$\frac{e^3\left(-\frac{b^2\sqrt{(c+dx)^2+1}\,(c+dx)^3}{\left(a+b\sinh^{-1}(c+dx)\right)^2} + \sinh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right) - 2\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(4\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right) - \right.}{2b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^3/(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] (e^3*(-((b^2*(c + d*x)^3*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh[c + d*x])^2)
 + (b*(-3*(c + d*x)^2 - 4*(c + d*x)^4))/(a + b*ArcSinh[c + d*x]) + CoshInte
gral[2*(a/b + ArcSinh[c + d*x])]*Sinh[(2*a)/b] - 2*CoshIntegral[4*(a/b + Ar
cSinh[c + d*x])]*Sinh[(4*a)/b] - Cosh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSin
h[c + d*x])] + 2*Cosh[(4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[c + d*x])]))/(
2*b^3*d)
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{d^3e^3x^3 + 3\,cd^2e^3x^2 + 3\,c^2de^3x + c^3e^3}{b^3\,\text{arsinh}\,(dx+c)^3 + 3\,ab^2\,\text{arsinh}\,(dx+c)^2 + 3\,a^2b\,\text{arsinh}\,(dx+c) + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((d^3*e^3*x^3 + 3*c*d^2*e^3*x^2 + 3*c^2*d*e^3*x + c^3*e^3)/(b^3*arc
sinh(d*x + c)^3 + 3*a*b^2*arcsinh(d*x + c)^2 + 3*a^2*b*arcsinh(d*x + c) + a
^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex+ce)^3}{(b\,\mathrm{arsinh}\,(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^3/(b*arcsinh(d*x + c) + a)^3, x)
```

maple [B] time = 0.31, size = 579, normalized size = 2.34

$$-\frac{\left(8(dx+c)^4-8(dx+c)^3\sqrt{1+(dx+c)^2}+8(dx+c)^2-4(dx+c)\sqrt{1+(dx+c)^2}+1\right)e^3(4b\,\mathrm{arcsinh}(dx+c)+4a-b)}{32b^2\left(b^2\,\mathrm{arcsinh}(dx+c)^2+2ab\,\mathrm{arcsinh}(dx+c)+a^2\right)}+\frac{e^3\mathrm{e}^{\frac{4a}{b}}\,\mathrm{Ei}\left(1,4\,\mathrm{arcsinh}(dx+c)+\frac{4a}{b}\right)}{2b^3}+\frac{\left(2(dx+c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^3,x)
```

```
[Out] 1/d*(-1/32*(8*(d*x+c)^4-8*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)+8*(d*x+c)^2-4*(d*x+
c)*(1+(d*x+c)^2)^(1/2)+1)*e^3*(4*b*arcsinh(d*x+c)+4*a-b)/b^2/(b^2*arcsinh(d
*x+c)^2+2*a*b*arcsinh(d*x+c)+a^2)+1/2*e^3/b^3*exp(4*a/b)*Ei(1,4*arcsinh(d*x
+c)+4*a/b)+1/16*(2*(d*x+c)^2-2*(d*x+c)*(1+(d*x+c)^2)^(1/2)+1)*e^3*(2*b*arcs
inh(d*x+c)+2*a-b)/b^2/(b^2*arcsinh(d*x+c)^2+2*a*b*arcsinh(d*x+c)+a^2)-1/4*e
^3/b^3*exp(2*a/b)*Ei(1,2*arcsinh(d*x+c)+2*a/b)+1/16/b*e^3*(2*(d*x+c)^2+1+2*
(d*x+c)*(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^2+1/8/b^2*e^3*(2*(d*x+c)^
2+1+2*(d*x+c)*(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))+1/4/b^3*e^3*exp(-2*
a/b)*Ei(1,-2*arcsinh(d*x+c)-2*a/b)-1/32/b*e^3*(8*(d*x+c)^4+8*(d*x+c)^2+8*(d
*x+c)^3*(1+(d*x+c)^2)^(1/2)+4*(d*x+c)*(1+(d*x+c)^2)^(1/2)+1)/(a+b*arcsinh(d
*x+c))^2-1/8/b^2*e^3*(8*(d*x+c)^4+8*(d*x+c)^2+8*(d*x+c)^3*(1+(d*x+c)^2)^(1/
2)+4*(d*x+c)*(1+(d*x+c)^2)^(1/2)+1)/(a+b*arcsinh(d*x+c))-1/2/b^3*e^3*exp(-4
*a/b)*Ei(1,-4*arcsinh(d*x+c)-4*a/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*((4*a*d^10*e^3 + b*d^10*e^3)*x^10 + 10*(4*a*c*d^9*e^3 + b*c*d^9*e^3)*x
^9 + 3*(4*(15*c^2*d^8*e^3 + d^8*e^3)*a + (15*c^2*d^8*e^3 + d^8*e^3)*b)*x^8
+ 24*(4*(5*c^3*d^7*e^3 + c*d^7*e^3)*a + (5*c^3*d^7*e^3 + c*d^7*e^3)*b)*x^7
+ 3*(4*(70*c^4*d^6*e^3 + 28*c^2*d^6*e^3 + d^6*e^3)*a + (70*c^4*d^6*e^3 + 28
*c^2*d^6*e^3 + d^6*e^3)*b)*x^6 + 6*(4*(42*c^5*d^5*e^3 + 28*c^3*d^5*e^3 + 3*
c*d^5*e^3)*a + (42*c^5*d^5*e^3 + 28*c^3*d^5*e^3 + 3*c*d^5*e^3)*b)*x^5 + (4*
(210*c^6*d^4*e^3 + 210*c^4*d^4*e^3 + 45*c^2*d^4*e^3 + d^4*e^3)*a + (210*c^6
*d^4*e^3 + 210*c^4*d^4*e^3 + 45*c^2*d^4*e^3 + d^4*e^3)*b)*x^4 + 4*(4*(30*c^
7*d^3*e^3 + 42*c^5*d^3*e^3 + 15*c^3*d^3*e^3 + c*d^3*e^3)*a + (30*c^7*d^3*e^
3 + 42*c^5*d^3*e^3 + 15*c^3*d^3*e^3 + c*d^3*e^3)*b)*x^3 + 3*(4*(15*c^8*d^2*
e^3 + 28*c^6*d^2*e^3 + 15*c^4*d^2*e^3 + 2*c^2*d^2*e^3)*a + (15*c^8*d^2*e^3
```

```
+ 28*c^6*d^2*e^3 + 15*c^4*d^2*e^3 + 2*c^2*d^2*e^3)*b)*x^2 + ((4*a*d^7*e^3 +
 b*d^7*e^3)*x^7 + 7*(4*a*c*d^6*e^3 + b*c*d^6*e^3)*x^6 + (6*(14*c^2*d^5*e^3
+ d^5*e^3)*a + (21*c^2*d^5*e^3 + d^5*e^3)*b)*x^5 + 5*(2*(14*c^3*d^4*e^3 + 3
*c*d^4*e^3)*a + (7*c^3*d^4*e^3 + c*d^4*e^3)*b)*x^4 + (2*(70*c^4*d^3*e^3 + 3
0*c^2*d^3*e^3 + d^3*e^3)*a + 5*(7*c^4*d^3*e^3 + 2*c^2*d^3*e^3)*b)*x^3 + (6*
(14*c^5*d^2*e^3 + 10*c^3*d^2*e^3 + c*d^2*e^3)*a + (21*c^5*d^2*e^3 + 10*c^3*
d^2*e^3)*b)*x^2 + 2*(2*c^7*e^3 + 3*c^5*e^3 + c^3*e^3)*a + (c^7*e^3 + c^5*e^
3)*b + (2*(14*c^6*d*e^3 + 15*c^4*d*e^3 + 3*c^2*d*e^3)*a + (7*c^6*d*e^3 + 5*
c^4*d*e^3)*b)*x)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + (3*(4*a*d^8*e^3 + b*
d^8*e^3)*x^8 + 24*(4*a*c*d^7*e^3 + b*c*d^7*e^3)*x^7 + (24*(14*c^2*d^6*e^3 +
 d^6*e^3)*a + (84*c^2*d^6*e^3 + 5*d^6*e^3)*b)*x^6 + 6*(8*(14*c^3*d^5*e^3 +
3*c*d^5*e^3)*a + (28*c^3*d^5*e^3 + 5*c*d^5*e^3)*b)*x^5 + (15*(56*c^4*d^4*e^
3 + 24*c^2*d^4*e^3 + d^4*e^3)*a + (210*c^4*d^4*e^3 + 75*c^2*d^4*e^3 + 2*d^4
*e^3)*b)*x^4 + 4*(3*(56*c^5*d^3*e^3 + 40*c^3*d^3*e^3 + 5*c*d^3*e^3)*a + (42
*c^5*d^3*e^3 + 25*c^3*d^3*e^3 + 2*c*d^3*e^3)*b)*x^3 + 3*((112*c^6*d^2*e^3 +
 120*c^4*d^2*e^3 + 30*c^2*d^2*e^3 + d^2*e^3)*a + (28*c^6*d^2*e^3 + 25*c^4*d
^2*e^3 + 4*c^2*d^2*e^3)*b)*x^2 + 3*(4*c^8*e^3 + 8*c^6*e^3 + 5*c^4*e^3 + c^2
*e^3)*a + (3*c^8*e^3 + 5*c^6*e^3 + 2*c^4*e^3)*b + 2*(3*(16*c^7*d*e^3 + 24*c
^5*d*e^3 + 10*c^3*d*e^3 + c*d*e^3)*a + (12*c^7*d*e^3 + 15*c^5*d*e^3 + 4*c^3
*d*e^3)*b)*x)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + 4*(c^10*e^3 + 3*c^8*e^3 + 3*c
^6*e^3 + c^4*e^3)*a + (c^10*e^3 + 3*c^8*e^3 + 3*c^6*e^3 + c^4*e^3)*b + 2*(4
*(5*c^9*d*e^3 + 12*c^7*d*e^3 + 9*c^5*d*e^3 + 2*c^3*d*e^3)*a + (5*c^9*d*e^3
+ 12*c^7*d*e^3 + 9*c^5*d*e^3 + 2*c^3*d*e^3)*b)*x + (4*b*d^10*e^3*x^10 + 40*
b*c*d^9*e^3*x^9 + 12*(15*c^2*d^8*e^3 + d^8*e^3)*b*x^8 + 96*(5*c^3*d^7*e^3 +
 c*d^7*e^3)*b*x^7 + 12*(70*c^4*d^6*e^3 + 28*c^2*d^6*e^3 + d^6*e^3)*b*x^6 +
24*(42*c^5*d^5*e^3 + 28*c^3*d^5*e^3 + 3*c*d^5*e^3)*b*x^5 + 4*(210*c^6*d^4*e
^3 + 210*c^4*d^4*e^3 + 45*c^2*d^4*e^3 + d^4*e^3)*b*x^4 + 16*(30*c^7*d^3*e^3
 + 42*c^5*d^3*e^3 + 15*c^3*d^3*e^3 + c*d^3*e^3)*b*x^3 + 12*(15*c^8*d^2*e^3
+ 28*c^6*d^2*e^3 + 15*c^4*d^2*e^3 + 2*c^2*d^2*e^3)*b*x^2 + 8*(5*c^9*d*e^3 +
 12*c^7*d*e^3 + 9*c^5*d*e^3 + 2*c^3*d*e^3)*b*x + 2*(2*b*d^7*e^3*x^7 + 14*b*
c*d^6*e^3*x^6 + 3*(14*c^2*d^5*e^3 + d^5*e^3)*b*x^5 + 5*(14*c^3*d^4*e^3 + 3*
c*d^4*e^3)*b*x^4 + (70*c^4*d^3*e^3 + 30*c^2*d^3*e^3 + d^3*e^3)*b*x^3 + 3*(1
4*c^5*d^2*e^3 + 10*c^3*d^2*e^3 + c*d^2*e^3)*b*x^2 + (14*c^6*d*e^3 + 15*c^4*
d*e^3 + 3*c^2*d*e^3)*b*x + (2*c^7*e^3 + 3*c^5*e^3 + c^3*e^3)*b)*(d^2*x^2 +
2*c*d*x + c^2 + 1)^(3/2) + 3*(4*b*d^8*e^3*x^8 + 32*b*c*d^7*e^3*x^7 + 8*(14*
c^2*d^6*e^3 + d^6*e^3)*b*x^6 + 16*(14*c^3*d^5*e^3 + 3*c*d^5*e^3)*b*x^5 + 5*
(56*c^4*d^4*e^3 + 24*c^2*d^4*e^3 + d^4*e^3)*b*x^4 + 4*(56*c^5*d^3*e^3 + 40*
c^3*d^3*e^3 + 5*c*d^3*e^3)*b*x^3 + (112*c^6*d^2*e^3 + 120*c^4*d^2*e^3 + 30*
c^2*d^2*e^3 + d^2*e^3)*b*x^2 + 2*(16*c^7*d*e^3 + 24*c^5*d*e^3 + 10*c^3*d*e^
3 + c*d*e^3)*b*x + (4*c^8*e^3 + 8*c^6*e^3 + 5*c^4*e^3 + c^2*e^3)*b)*(d^2*x^
2 + 2*c*d*x + c^2 + 1) + 4*(c^10*e^3 + 3*c^8*e^3 + 3*c^6*e^3 + c^4*e^3)*b +
 (12*b*d^9*e^3*x^9 + 108*b*c*d^8*e^3*x^8 + 6*(72*c^2*d^7*e^3 + 5*d^7*e^3)*b
*x^7 + 42*(24*c^3*d^6*e^3 + 5*c*d^6*e^3)*b*x^6 + (1512*c^4*d^5*e^3 + 630*c^
2*d^5*e^3 + 25*d^5*e^3)*b*x^5 + (1512*c^5*d^4*e^3 + 1050*c^3*d^4*e^3 + 125*
c*d^4*e^3)*b*x^4 + (1008*c^6*d^3*e^3 + 1050*c^4*d^3*e^3 + 250*c^2*d^3*e^3 +
 7*d^3*e^3)*b*x^3 + (432*c^7*d^2*e^3 + 630*c^5*d^2*e^3 + 250*c^3*d^2*e^3 +
21*c*d^2*e^3)*b*x^2 + (108*c^8*d*e^3 + 210*c^6*d*e^3 + 125*c^4*d*e^3 + 21*c
^2*d*e^3)*b*x + (12*c^9*e^3 + 30*c^7*e^3 + 25*c^5*e^3 + 7*c^3*e^3)*b)*sqrt(
d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 +
1)) + (3*(4*a*d^9*e^3 + b*d^9*e^3)*x^9 + 27*(4*a*c*d^8*e^3 + b*c*d^8*e^3)*x
^8 + (6*(72*c^2*d^7*e^3 + 5*d^7*e^3)*a + (108*c^2*d^7*e^3 + 7*d^7*e^3)*b)*x
^7 + 7*(6*(24*c^3*d^6*e^3 + 5*c*d^6*e^3)*a + (36*c^3*d^6*e^3 + 7*c*d^6*e^3)
*b)*x^6 + ((1512*c^4*d^5*e^3 + 630*c^2*d^5*e^3 + 25*d^5*e^3)*a + (378*c^4*d
^5*e^3 + 147*c^2*d^5*e^3 + 5*d^5*e^3)*b)*x^5 + ((1512*c^5*d^4*e^3 + 1050*c^
3*d^4*e^3 + 125*c*d^4*e^3)*a + (378*c^5*d^4*e^3 + 245*c^3*d^4*e^3 + 25*c*d^
4*e^3)*b)*x^4 + ((1008*c^6*d^3*e^3 + 1050*c^4*d^3*e^3 + 250*c^2*d^3*e^3 + 7
*d^3*e^3)*a + (252*c^6*d^3*e^3 + 245*c^4*d^3*e^3 + 50*c^2*d^3*e^3 + d^3*e^3
)*b)*x^3 + ((432*c^7*d^2*e^3 + 630*c^5*d^2*e^3 + 250*c^3*d^2*e^3 + 21*c*d^2
*e^3)*a + (108*c^7*d^2*e^3 + 147*c^5*d^2*e^3 + 50*c^3*d^2*e^3 + 3*c*d^2*e^3
```

```
)*b)*x^2 + (12*c^9*e^3 + 30*c^7*e^3 + 25*c^5*e^3 + 7*c^3*e^3)*a + (3*c^9*e^
3 + 7*c^7*e^3 + 5*c^5*e^3 + c^3*e^3)*b + ((108*c^8*d*e^3 + 210*c^6*d*e^3 +
125*c^4*d*e^3 + 21*c^2*d*e^3)*a + (27*c^8*d*e^3 + 49*c^6*d*e^3 + 25*c^4*d*e
^3 + 3*c^2*d*e^3)*b)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(a^2*b^2*d^7*x^6
 + 6*a^2*b^2*c*d^6*x^5 + 3*(5*c^2*d^5 + d^5)*a^2*b^2*x^4 + 4*(5*c^3*d^4 + 3
*c*d^4)*a^2*b^2*x^3 + 3*(5*c^4*d^3 + 6*c^2*d^3 + d^3)*a^2*b^2*x^2 + 6*(c^5*
d^2 + 2*c^3*d^2 + c*d^2)*a^2*b^2*x + (c^6*d + 3*c^4*d + 3*c^2*d + d)*a^2*b^
2 + (b^4*d^7*x^6 + 6*b^4*c*d^6*x^5 + 3*(5*c^2*d^5 + d^5)*b^4*x^4 + 4*(5*c^3
*d^4 + 3*c*d^4)*b^4*x^3 + 3*(5*c^4*d^3 + 6*c^2*d^3 + d^3)*b^4*x^2 + 6*(c^5*
d^2 + 2*c^3*d^2 + c*d^2)*b^4*x + (c^6*d + 3*c^4*d + 3*c^2*d + d)*b^4 + (b^4
*d^4*x^3 + 3*b^4*c*d^3*x^2 + 3*b^4*c^2*d^2*x + b^4*c^3*d)*(d^2*x^2 + 2*c*d*
x + c^2 + 1)^(3/2) + 3*(b^4*d^5*x^4 + 4*b^4*c*d^4*x^3 + (6*c^2*d^3 + d^3)*b
^4*x^2 + 2*(2*c^3*d^2 + c*d^2)*b^4*x + (c^4*d + c^2*d)*b^4)*(d^2*x^2 + 2*c*
d*x + c^2 + 1) + 3*(b^4*d^6*x^5 + 5*b^4*c*d^5*x^4 + 2*(5*c^2*d^4 + d^4)*b^4
*x^3 + 2*(5*c^3*d^3 + 3*c*d^3)*b^4*x^2 + (5*c^4*d^2 + 6*c^2*d^2 + d^2)*b^4*
x + (c^5*d + 2*c^3*d + c*d)*b^4)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x
 + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 + (a^2*b^2*d^4*x^3 + 3*a^2*b^2*
c*d^3*x^2 + 3*a^2*b^2*c^2*d^2*x + a^2*b^2*c^3*d)*(d^2*x^2 + 2*c*d*x + c^2 +
 1)^(3/2) + 3*(a^2*b^2*d^5*x^4 + 4*a^2*b^2*c*d^4*x^3 + (6*c^2*d^3 + d^3)*a^
2*b^2*x^2 + 2*(2*c^3*d^2 + c*d^2)*a^2*b^2*x + (c^4*d + c^2*d)*a^2*b^2)*(d^2
*x^2 + 2*c*d*x + c^2 + 1) + 2*(a*b^3*d^7*x^6 + 6*a*b^3*c*d^6*x^5 + 3*(5*c^2
*d^5 + d^5)*a*b^3*x^4 + 4*(5*c^3*d^4 + 3*c*d^4)*a*b^3*x^3 + 3*(5*c^4*d^3 +
6*c^2*d^3 + d^3)*a*b^3*x^2 + 6*(c^5*d^2 + 2*c^3*d^2 + c*d^2)*a*b^3*x + (c^6
*d + 3*c^4*d + 3*c^2*d + d)*a*b^3 + (a*b^3*d^4*x^3 + 3*a*b^3*c*d^3*x^2 + 3*
a*b^3*c^2*d^2*x + a*b^3*c^3*d)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + 3*(a*b
^3*d^5*x^4 + 4*a*b^3*c*d^4*x^3 + (6*c^2*d^3 + d^3)*a*b^3*x^2 + 2*(2*c^3*d^2
 + c*d^2)*a*b^3*x + (c^4*d + c^2*d)*a*b^3)*(d^2*x^2 + 2*c*d*x + c^2 + 1) +
3*(a*b^3*d^6*x^5 + 5*a*b^3*c*d^5*x^4 + 2*(5*c^2*d^4 + d^4)*a*b^3*x^3 + 2*(5
*c^3*d^3 + 3*c*d^3)*a*b^3*x^2 + (5*c^4*d^2 + 6*c^2*d^2 + d^2)*a*b^3*x + (c^
5*d + 2*c^3*d + c*d)*a*b^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c
+ sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + 3*(a^2*b^2*d^6*x^5 + 5*a^2*b^2*c*d^5
*x^4 + 2*(5*c^2*d^4 + d^4)*a^2*b^2*x^3 + 2*(5*c^3*d^3 + 3*c*d^3)*a^2*b^2*x^
2 + (5*c^4*d^2 + 6*c^2*d^2 + d^2)*a^2*b^2*x + (c^5*d + 2*c^3*d + c*d)*a^2*b
^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + integrate(1/2*(16*d^11*e^3*x^11 +
176*c*d^10*e^3*x^10 + 16*c^11*e^3 + 64*c^9*e^3 + 96*c^7*e^3 + 16*(55*c^2*d^
9*e^3 + 4*d^9*e^3)*x^9 + 48*(55*c^3*d^8*e^3 + 12*c*d^8*e^3)*x^8 + 64*c^5*e^
3 + 96*(55*c^4*d^7*e^3 + 24*c^2*d^7*e^3 + d^7*e^3)*x^7 + 672*(11*c^5*d^6*e^
3 + 8*c^3*d^6*e^3 + c*d^6*e^3)*x^6 + 16*c^3*e^3 + 32*(231*c^6*d^5*e^3 + 252
*c^4*d^5*e^3 + 63*c^2*d^5*e^3 + 2*d^5*e^3)*x^5 + 32*(165*c^7*d^4*e^3 + 252*
c^5*d^4*e^3 + 105*c^3*d^4*e^3 + 10*c*d^4*e^3)*x^4 + 16*(165*c^8*d^3*e^3 + 3
36*c^6*d^3*e^3 + 210*c^4*d^3*e^3 + 40*c^2*d^3*e^3 + d^3*e^3)*x^3 + 4*(4*d^7
*e^3*x^7 + 28*c*d^6*e^3*x^6 + 4*c^7*e^3 + 3*c^5*e^3 + 3*(28*c^2*d^5*e^3 + d
^5*e^3)*x^5 + 5*(28*c^3*d^4*e^3 + 3*c*d^4*e^3)*x^4 + 10*(14*c^4*d^3*e^3 + 3
*c^2*d^3*e^3)*x^3 + 6*(14*c^5*d^2*e^3 + 5*c^3*d^2*e^3)*x^2 + (28*c^6*d*e^3
+ 15*c^4*d*e^3)*x)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 16*(55*c^9*d^2*e^3 + 1
44*c^7*d^2*e^3 + 126*c^5*d^2*e^3 + 40*c^3*d^2*e^3 + 3*c*d^2*e^3)*x^2 + (64*
d^8*e^3*x^8 + 512*c*d^7*e^3*x^7 + 64*c^8*e^3 + 100*c^6*e^3 + 42*c^4*e^3 + 4
*(448*c^2*d^6*e^3 + 25*d^6*e^3)*x^6 + 8*(448*c^3*d^5*e^3 + 75*c*d^5*e^3)*x^
5 + 3*c^2*e^3 + 2*(2240*c^4*d^4*e^3 + 750*c^2*d^4*e^3 + 21*d^4*e^3)*x^4 + 8
*(448*c^5*d^3*e^3 + 250*c^3*d^3*e^3 + 21*c*d^3*e^3)*x^3 + (1792*c^6*d^2*e^3
 + 1500*c^4*d^2*e^3 + 252*c^2*d^2*e^3 + 3*d^2*e^3)*x^2 + 2*(256*c^7*d*e^3 +
 300*c^5*d*e^3 + 84*c^3*d*e^3 + 3*c*d*e^3)*x)*(d^2*x^2 + 2*c*d*x + c^2 + 1)
^(3/2) + 6*(16*d^9*e^3*x^9 + 144*c*d^8*e^3*x^8 + 16*c^9*e^3 + 38*c^7*e^3 +
30*c^5*e^3 + 2*(288*c^2*d^7*e^3 + 19*d^7*e^3)*x^7 + 14*(96*c^3*d^6*e^3 + 19
*c*d^6*e^3)*x^6 + 9*c^3*e^3 + 6*(336*c^4*d^5*e^3 + 133*c^2*d^5*e^3 + 5*d^5*
e^3)*x^5 + 2*(1008*c^5*d^4*e^3 + 665*c^3*d^4*e^3 + 75*c*d^4*e^3)*x^4 + c*e^
3 + (1344*c^6*d^3*e^3 + 1330*c^4*d^3*e^3 + 300*c^2*d^3*e^3 + 9*d^3*e^3)*x^3
 + 3*(192*c^7*d^2*e^3 + 266*c^5*d^2*e^3 + 100*c^3*d^2*e^3 + 9*c*d^2*e^3)*x^
2 + (144*c^8*d*e^3 + 266*c^6*d*e^3 + 150*c^4*d*e^3 + 27*c^2*d*e^3 + d*e^3)*
```

```
x)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + 16*(11*c^10*d*e^3 + 36*c^8*d*e^3 + 42*c^
6*d*e^3 + 20*c^4*d*e^3 + 3*c^2*d*e^3)*x + (64*d^10*e^3*x^10 + 640*c*d^9*e^3
*x^9 + 64*c^10*e^3 + 204*c^8*e^3 + 234*c^6*e^3 + 12*(240*c^2*d^8*e^3 + 17*d
^8*e^3)*x^8 + 96*(80*c^3*d^7*e^3 + 17*c*d^7*e^3)*x^7 + 115*c^4*e^3 + 6*(224
0*c^4*d^6*e^3 + 952*c^2*d^6*e^3 + 39*d^6*e^3)*x^6 + 12*(1344*c^5*d^5*e^3 +
952*c^3*d^5*e^3 + 117*c*d^5*e^3)*x^5 + 21*c^2*e^3 + 5*(2688*c^6*d^4*e^3 + 2
856*c^4*d^4*e^3 + 702*c^2*d^4*e^3 + 23*d^4*e^3)*x^4 + 4*(1920*c^7*d^3*e^3 +
 2856*c^5*d^3*e^3 + 1170*c^3*d^3*e^3 + 115*c*d^3*e^3)*x^3 + 3*(960*c^8*d^2*
e^3 + 1904*c^6*d^2*e^3 + 1170*c^4*d^2*e^3 + 230*c^2*d^2*e^3 + 7*d^2*e^3)*x^
2 + 2*(320*c^9*d*e^3 + 816*c^7*d*e^3 + 702*c^5*d*e^3 + 230*c^3*d*e^3 + 21*c
*d*e^3)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(a*b^2*d^8*x^8 + 8*a*b^2*c*d^
7*x^7 + 4*(7*c^2*d^6 + d^6)*a*b^2*x^6 + 8*(7*c^3*d^5 + 3*c*d^5)*a*b^2*x^5 +
 2*(35*c^4*d^4 + 30*c^2*d^4 + 3*d^4)*a*b^2*x^4 + 8*(7*c^5*d^3 + 10*c^3*d^3
+ 3*c*d^3)*a*b^2*x^3 + 4*(7*c^6*d^2 + 15*c^4*d^2 + 9*c^2*d^2 + d^2)*a*b^2*x
^2 + 8*(c^7*d + 3*c^5*d + 3*c^3*d + c*d)*a*b^2*x + (c^8 + 4*c^6 + 6*c^4 + 4
*c^2 + 1)*a*b^2 + (a*b^2*d^4*x^4 + 4*a*b^2*c*d^3*x^3 + 6*a*b^2*c^2*d^2*x^2
+ 4*a*b^2*c^3*d*x + a*b^2*c^4)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 4*(a*b^2*d
^5*x^5 + 5*a*b^2*c*d^4*x^4 + (10*c^2*d^3 + d^3)*a*b^2*x^3 + (10*c^3*d^2 + 3
*c*d^2)*a*b^2*x^2 + (5*c^4*d + 3*c^2*d)*a*b^2*x + (c^5 + c^3)*a*b^2)*(d^2*x
^2 + 2*c*d*x + c^2 + 1)^(3/2) + 6*(a*b^2*d^6*x^6 + 6*a*b^2*c*d^5*x^5 + (15*
c^2*d^4 + 2*d^4)*a*b^2*x^4 + 4*(5*c^3*d^3 + 2*c*d^3)*a*b^2*x^3 + (15*c^4*d^
2 + 12*c^2*d^2 + d^2)*a*b^2*x^2 + 2*(3*c^5*d + 4*c^3*d + c*d)*a*b^2*x + (c^
6 + 2*c^4 + c^2)*a*b^2)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + (b^3*d^8*x^8 + 8*b^
3*c*d^7*x^7 + 4*(7*c^2*d^6 + d^6)*b^3*x^6 + 8*(7*c^3*d^5 + 3*c*d^5)*b^3*x^5
 + 2*(35*c^4*d^4 + 30*c^2*d^4 + 3*d^4)*b^3*x^4 + 8*(7*c^5*d^3 + 10*c^3*d^3
+ 3*c*d^3)*b^3*x^3 + 4*(7*c^6*d^2 + 15*c^4*d^2 + 9*c^2*d^2 + d^2)*b^3*x^2 +
 8*(c^7*d + 3*c^5*d + 3*c^3*d + c*d)*b^3*x + (c^8 + 4*c^6 + 6*c^4 + 4*c^2 +
 1)*b^3 + (b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*
x + b^3*c^4)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 4*(b^3*d^5*x^5 + 5*b^3*c*d^4
*x^4 + (10*c^2*d^3 + d^3)*b^3*x^3 + (10*c^3*d^2 + 3*c*d^2)*b^3*x^2 + (5*c^4
*d + 3*c^2*d)*b^3*x + (c^5 + c^3)*b^3)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2)
+ 6*(b^3*d^6*x^6 + 6*b^3*c*d^5*x^5 + (15*c^2*d^4 + 2*d^4)*b^3*x^4 + 4*(5*c^
3*d^3 + 2*c*d^3)*b^3*x^3 + (15*c^4*d^2 + 12*c^2*d^2 + d^2)*b^3*x^2 + 2*(3*c
^5*d + 4*c^3*d + c*d)*b^3*x + (c^6 + 2*c^4 + c^2)*b^3)*(d^2*x^2 + 2*c*d*x +
 c^2 + 1) + 4*(b^3*d^7*x^7 + 7*b^3*c*d^6*x^6 + 3*(7*c^2*d^5 + d^5)*b^3*x^5
+ 5*(7*c^3*d^4 + 3*c*d^4)*b^3*x^4 + (35*c^4*d^3 + 30*c^2*d^3 + 3*d^3)*b^3*x
^3 + 3*(7*c^5*d^2 + 10*c^3*d^2 + 3*c*d^2)*b^3*x^2 + (7*c^6*d + 15*c^4*d + 9
*c^2*d + d)*b^3*x + (c^7 + 3*c^5 + 3*c^3 + c)*b^3)*sqrt(d^2*x^2 + 2*c*d*x +
 c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + 4*(a*b^2*d^7*
x^7 + 7*a*b^2*c*d^6*x^6 + 3*(7*c^2*d^5 + d^5)*a*b^2*x^5 + 5*(7*c^3*d^4 + 3*
c*d^4)*a*b^2*x^4 + (35*c^4*d^3 + 30*c^2*d^3 + 3*d^3)*a*b^2*x^3 + 3*(7*c^5*d
^2 + 10*c^3*d^2 + 3*c*d^2)*a*b^2*x^2 + (7*c^6*d + 15*c^4*d + 9*c^2*d + d)*a
*b^2*x + (c^7 + 3*c^5 + 3*c^3 + c)*a*b^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,e+d\,e\,x)^3}{(a+b\,\mathrm{asinh}\,(c+d\,x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3/(a + b*asinh(c + d*x))^3,x)
```

```
[Out] int((c*e + d*e*x)^3/(a + b*asinh(c + d*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^3\left(\int \frac{c^3}{a^3+3a^2b\,\mathrm{asinh}\,(c+dx)+3ab^2\,\mathrm{asinh}^2\,(c+dx)+b^3\,\mathrm{asinh}^3\,(c+dx)}\,dx+\int \frac{}{a^3+3a^2b\,\mathrm{asinh}\,(c+dx)+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3/(a+b*asinh(d*x+c))**3,x)
```

```
[Out] e**3*(Integral(c**3/(a**3 + 3*a**2*b*asinh(c + d*x) + 3*a*b**2*asinh(c + d*
x)**2 + b**3*asinh(c + d*x)**3), x) + Integral(d**3*x**3/(a**3 + 3*a**2*b*a
sinh(c + d*x) + 3*a*b**2*asinh(c + d*x)**2 + b**3*asinh(c + d*x)**3), x) +
Integral(3*c*d**2*x**2/(a**3 + 3*a**2*b*asinh(c + d*x) + 3*a*b**2*asinh(c +
 d*x)**2 + b**3*asinh(c + d*x)**3), x) + Integral(3*c**2*d*x/(a**3 + 3*a**2
*b*asinh(c + d*x) + 3*a*b**2*asinh(c + d*x)**2 + b**3*asinh(c + d*x)**3), x
))
```

3.170 $\int \frac{(ce+dex)^2}{\left(a+b\sinh^{-1}(c+dx)\right)^3}\,dx$

Optimal. Leaf size=246

$$-\frac{e^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{8b^3d}+\frac{9e^2\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{8b^3d}+\frac{e^2\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{8b^3d}-\frac{9e}{}$$

```
[Out] -e^2*(d*x+c)/b^2/d/(a+b*arcsinh(d*x+c))-3/2*e^2*(d*x+c)^3/b^2/d/(a+b*arcsin
h(d*x+c))-1/8*e^2*Chi((a+b*arcsinh(d*x+c))/b)*cosh(a/b)/b^3/d+9/8*e^2*Chi(3
*(a+b*arcsinh(d*x+c))/b)*cosh(3*a/b)/b^3/d+1/8*e^2*Shi((a+b*arcsinh(d*x+c))
/b)*sinh(a/b)/b^3/d-9/8*e^2*Shi(3*(a+b*arcsinh(d*x+c))/b)*sinh(3*a/b)/b^3/d
-1/2*e^2*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))^2
```

Rubi [A] time = 0.61, antiderivative size = 305, normalized size of antiderivative = 1.24, number of steps used = 18, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {5865, 12, 5667, 5774, 5669, 5448, 3303, 3298, 3301, 5657}

$$-\frac{9e^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)}{8b^3d}+\frac{9e^2\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(c+dx)\right)}{8b^3d}+\frac{e^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh}{b}\right)}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2/(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] -(e^2*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])/(2*b*d*(a + b*ArcSinh[c + d*x])^2)
 - (e^2*(c + d*x))/(b^2*d*(a + b*ArcSinh[c + d*x])) - (3*e^2*(c + d*x)^3)/(
2*b^2*d*(a + b*ArcSinh[c + d*x])) - (9*e^2*Cosh[a/b]*CoshIntegral[a/b + Arc
Sinh[c + d*x]])/(8*b^3*d) + (9*e^2*Cosh[(3*a)/b]*CoshIntegral[(3*a)/b + 3*A
rcSinh[c + d*x]])/(8*b^3*d) + (e^2*Cosh[a/b]*CoshIntegral[(a + b*ArcSinh[c
+ d*x])/b])/(b^3*d) + (9*e^2*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c + d*x]]
)/(8*b^3*d) - (9*e^2*Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSinh[c + d*x
]])/(8*b^3*d) - (e^2*Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[c + d*x])/b])/(b
^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^2}{\left(a+b\sinh^{-1}(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{e^2x^2}{\left(a+b\sinh^{-1}(x)\right)^3}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^2\,\text{Subst}\left(\int \frac{x^2}{\left(a+b\sinh^{-1}(x)\right)^3}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} + \frac{e^2\,\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\left(a+b\sinh^{-1}(x)\right)^2}\,dx, x, c+dx\right)}{bd}$$

$$= -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{e^2(c+dx)}{b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{3e^2(c+}{2b^2d\left(a+b\sinh\right.}$$

$$= -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{e^2(c+dx)}{b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{3e^2(c+}{2b^2d\left(a+b\sinh\right.}$$

$$= -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{e^2(c+dx)}{b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{3e^2(c+}{2b^2d\left(a+b\sinh\right.}$$

$$= -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{e^2(c+dx)}{b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{3e^2(c+}{2b^2d\left(a+b\sinh\right.}$$

$$= -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{e^2(c+dx)}{b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{3e^2(c+}{2b^2d\left(a+b\sinh\right.}$$

$$= -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{e^2(c+dx)}{b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{3e^2(c+}{2b^2d\left(a+b\sinh\right.}$$

Mathematica [A] time = 0.73, size = 216, normalized size = 0.88

$$\frac{e^2\left(-\frac{4b^2\sqrt{(c+dx)^2+1}\,(c+dx)^2}{\left(a+b\sinh^{-1}(c+dx)\right)^2} + 9\left(-\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right) + \cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right) + \right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^2/(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] (e^2*((-4*b^2*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh[c + d*x])^2
 + (4*b*(-2*(c + d*x) - 3*(c + d*x)^3))/(a + b*ArcSinh[c + d*x]) + 8*Cosh[a
/b]*CoshIntegral[a/b + ArcSinh[c + d*x]] - 8*Sinh[a/b]*SinhIntegral[a/b + A
rcSinh[c + d*x]] + 9*(-(Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c + d*x]]) + C
osh[(3*a)/b]*CoshIntegral[3*(a/b + ArcSinh[c + d*x])] + Sinh[a/b]*SinhInteg
ral[a/b + ArcSinh[c + d*x]] - Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c
 + d*x])])))/(8*b^3*d)
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{d^2e^2x^2 + 2\,cde^2x + c^2e^2}{b^3\,\text{arsinh}\,(dx+c)^3 + 3\,ab^2\,\text{arsinh}\,(dx+c)^2 + 3\,a^2b\,\text{arsinh}\,(dx+c) + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2)/(b^3*arcsinh(d*x + c)^3 + 3*
a*b^2*arcsinh(d*x + c)^2 + 3*a^2*b*arcsinh(d*x + c) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex+ce)^2}{(b\,\mathrm{arsinh}\,(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2/(b*arcsinh(d*x + c) + a)^3, x)
```

maple [B] time = 0.24, size = 507, normalized size = 2.06

$$-\frac{\left(-4(dx+c)^2\sqrt{1+(dx+c)^2}+4(dx+c)^3-\sqrt{1+(dx+c)^2}+3dx+3c\right)e^2(3b\,\mathrm{arcsinh}(dx+c)+3a-b)}{16b^2\left(b^2\,\mathrm{arcsinh}(dx+c)^2+2ab\,\mathrm{arcsinh}(dx+c)+a^2\right)}-\frac{9e^2\mathrm{e}^{\frac{3a}{b}}\,\mathrm{Ei}\left(1,3\,\mathrm{arcsinh}(dx+c)+\frac{3a}{b}\right)}{16b^3}+\frac{\left(-\sqrt{1+(dx+c)^2}+\cdots\right.}{16b^2\left(b^2\,\mathrm{arcsinh}(\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^3,x)
```

```
[Out] 1/d*(-1/16*(-4*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+4*(d*x+c)^3-(1+(d*x+c)^2)^(1/2
)+3*d*x+3*c)*e^2*(3*b*arcsinh(d*x+c)+3*a-b)/b^2/(b^2*arcsinh(d*x+c)^2+2*a*b
*arcsinh(d*x+c)+a^2)-9/16*e^2/b^3*exp(3*a/b)*Ei(1,3*arcsinh(d*x+c)+3*a/b)+1
/16*(-(1+(d*x+c)^2)^(1/2)+d*x+c)*e^2*(b*arcsinh(d*x+c)+a-b)/b^2/(b^2*arcsin
h(d*x+c)^2+2*a*b*arcsinh(d*x+c)+a^2)+1/16*e^2/b^3*exp(a/b)*Ei(1,arcsinh(d*x
+c)+a/b)+1/16/b*e^2*(d*x+c+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^2+1/16
/b^2*e^2*(d*x+c+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))+1/16/b^3*e^2*exp(
-a/b)*Ei(1,-arcsinh(d*x+c)-a/b)-1/16/b*e^2*(4*(d*x+c)^3+3*d*x+3*c+4*(d*x+c)
^2*(1+(d*x+c)^2)^(1/2)+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^2-3/16/b^2
*e^2*(4*(d*x+c)^3+3*d*x+3*c+4*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+(1+(d*x+c)^2)^(
1/2))/(a+b*arcsinh(d*x+c))-9/16/b^3*e^2*exp(-3*a/b)*Ei(1,-3*arcsinh(d*x+c)-
3*a/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*((3*a*d^9*e^2 + b*d^9*e^2)*x^9 + 9*(3*a*c*d^8*e^2 + b*c*d^8*e^2)*x^8 +
 3*(3*(12*c^2*d^7*e^2 + d^7*e^2)*a + (12*c^2*d^7*e^2 + d^7*e^2)*b)*x^7 + 21
*(3*(4*c^3*d^6*e^2 + c*d^6*e^2)*a + (4*c^3*d^6*e^2 + c*d^6*e^2)*b)*x^6 + 3*
(3*(42*c^4*d^5*e^2 + 21*c^2*d^5*e^2 + d^5*e^2)*a + (42*c^4*d^5*e^2 + 21*c^2
*d^5*e^2 + d^5*e^2)*b)*x^5 + 3*(3*(42*c^5*d^4*e^2 + 35*c^3*d^4*e^2 + 5*c*d^
4*e^2)*a + (42*c^5*d^4*e^2 + 35*c^3*d^4*e^2 + 5*c*d^4*e^2)*b)*x^4 + (3*(84*
c^6*d^3*e^2 + 105*c^4*d^3*e^2 + 30*c^2*d^3*e^2 + d^3*e^2)*a + (84*c^6*d^3*e
^2 + 105*c^4*d^3*e^2 + 30*c^2*d^3*e^2 + d^3*e^2)*b)*x^3 + 3*(3*(12*c^7*d^2*
e^2 + 21*c^5*d^2*e^2 + 10*c^3*d^2*e^2 + c*d^2*e^2)*a + (12*c^7*d^2*e^2 + 21
*c^5*d^2*e^2 + 10*c^3*d^2*e^2 + c*d^2*e^2)*b)*x^2 + ((3*a*d^6*e^2 + b*d^6*e
^2)*x^6 + 6*(3*a*c*d^5*e^2 + b*c*d^5*e^2)*x^5 + ((45*c^2*d^4*e^2 + 4*d^4*e^
2)*a + (15*c^2*d^4*e^2 + d^4*e^2)*b)*x^4 + 4*((15*c^3*d^3*e^2 + 4*c*d^3*e^2
)*a + (5*c^3*d^3*e^2 + c*d^3*e^2)*b)*x^3 + ((45*c^4*d^2*e^2 + 24*c^2*d^2*e^
```

```
2 + d^2*e^2)*a + 3*(5*c^4*d^2*e^2 + 2*c^2*d^2*e^2)*b)*x^2 + (3*c^6*e^2 + 4*
c^4*e^2 + c^2*e^2)*a + (c^6*e^2 + c^4*e^2)*b + 2*((9*c^5*d*e^2 + 8*c^3*d*e^
2 + c*d*e^2)*a + (3*c^5*d*e^2 + 2*c^3*d*e^2)*b)*x)*(d^2*x^2 + 2*c*d*x + c^2
 + 1)^(3/2) + (3*(3*a*d^7*e^2 + b*d^7*e^2)*x^7 + 21*(3*a*c*d^6*e^2 + b*c*d^
6*e^2)*x^6 + ((189*c^2*d^5*e^2 + 17*d^5*e^2)*a + (63*c^2*d^5*e^2 + 5*d^5*e^
2)*b)*x^5 + 5*((63*c^3*d^4*e^2 + 17*c*d^4*e^2)*a + (21*c^3*d^4*e^2 + 5*c*d^
4*e^2)*b)*x^4 + (5*(63*c^4*d^3*e^2 + 34*c^2*d^3*e^2 + 2*d^3*e^2)*a + (105*c
^4*d^3*e^2 + 50*c^2*d^3*e^2 + 2*d^3*e^2)*b)*x^3 + ((189*c^5*d^2*e^2 + 170*c
^3*d^2*e^2 + 30*c*d^2*e^2)*a + (63*c^5*d^2*e^2 + 50*c^3*d^2*e^2 + 6*c*d^2*e
^2)*b)*x^2 + (9*c^7*e^2 + 17*c^5*e^2 + 10*c^3*e^2 + 2*c*e^2)*a + (3*c^7*e^2
 + 5*c^5*e^2 + 2*c^3*e^2)*b + ((63*c^6*d*e^2 + 85*c^4*d*e^2 + 30*c^2*d*e^2
+ 2*d*e^2)*a + (21*c^6*d*e^2 + 25*c^4*d*e^2 + 6*c^2*d*e^2)*b)*x)*(d^2*x^2 +
 2*c*d*x + c^2 + 1) + 3*(c^9*e^2 + 3*c^7*e^2 + 3*c^5*e^2 + c^3*e^2)*a + (c^
9*e^2 + 3*c^7*e^2 + 3*c^5*e^2 + c^3*e^2)*b + 3*(3*(3*c^8*d*e^2 + 7*c^6*d*e^
2 + 5*c^4*d*e^2 + c^2*d*e^2)*a + (3*c^8*d*e^2 + 7*c^6*d*e^2 + 5*c^4*d*e^2 +
 c^2*d*e^2)*b)*x + (3*b*d^9*e^2*x^9 + 27*b*c*d^8*e^2*x^8 + 9*(12*c^2*d^7*e^
2 + d^7*e^2)*b*x^7 + 63*(4*c^3*d^6*e^2 + c*d^6*e^2)*b*x^6 + 9*(42*c^4*d^5*e
^2 + 21*c^2*d^5*e^2 + d^5*e^2)*b*x^5 + 9*(42*c^5*d^4*e^2 + 35*c^3*d^4*e^2 +
 5*c*d^4*e^2)*b*x^4 + 3*(84*c^6*d^3*e^2 + 105*c^4*d^3*e^2 + 30*c^2*d^3*e^2
+ d^3*e^2)*b*x^3 + 9*(12*c^7*d^2*e^2 + 21*c^5*d^2*e^2 + 10*c^3*d^2*e^2 + c*
d^2*e^2)*b*x^2 + 9*(3*c^8*d*e^2 + 7*c^6*d*e^2 + 5*c^4*d*e^2 + c^2*d*e^2)*b*
x + (3*b*d^6*e^2*x^6 + 18*b*c*d^5*e^2*x^5 + (45*c^2*d^4*e^2 + 4*d^4*e^2)*b*
x^4 + 4*(15*c^3*d^3*e^2 + 4*c*d^3*e^2)*b*x^3 + (45*c^4*d^2*e^2 + 24*c^2*d^2
*e^2 + d^2*e^2)*b*x^2 + 2*(9*c^5*d*e^2 + 8*c^3*d*e^2 + c*d*e^2)*b*x + (3*c^
6*e^2 + 4*c^4*e^2 + c^2*e^2)*b)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + (9*b*
d^7*e^2*x^7 + 63*b*c*d^6*e^2*x^6 + (189*c^2*d^5*e^2 + 17*d^5*e^2)*b*x^5 + 5
*(63*c^3*d^4*e^2 + 17*c*d^4*e^2)*b*x^4 + 5*(63*c^4*d^3*e^2 + 34*c^2*d^3*e^2
 + 2*d^3*e^2)*b*x^3 + (189*c^5*d^2*e^2 + 170*c^3*d^2*e^2 + 30*c*d^2*e^2)*b*
x^2 + (63*c^6*d*e^2 + 85*c^4*d*e^2 + 30*c^2*d*e^2 + 2*d*e^2)*b*x + (9*c^7*e
^2 + 17*c^5*e^2 + 10*c^3*e^2 + 2*c*e^2)*b)*(d^2*x^2 + 2*c*d*x + c^2 + 1) +
3*(c^9*e^2 + 3*c^7*e^2 + 3*c^5*e^2 + c^3*e^2)*b + (9*b*d^8*e^2*x^8 + 72*b*c
*d^7*e^2*x^7 + 2*(126*c^2*d^6*e^2 + 11*d^6*e^2)*b*x^6 + 12*(42*c^3*d^5*e^2
+ 11*c*d^5*e^2)*b*x^5 + 6*(105*c^4*d^4*e^2 + 55*c^2*d^4*e^2 + 3*d^4*e^2)*b*
x^4 + 8*(63*c^5*d^3*e^2 + 55*c^3*d^3*e^2 + 9*c*d^3*e^2)*b*x^3 + (252*c^6*d^
2*e^2 + 330*c^4*d^2*e^2 + 108*c^2*d^2*e^2 + 5*d^2*e^2)*b*x^2 + 2*(36*c^7*d*
e^2 + 66*c^5*d*e^2 + 36*c^3*d*e^2 + 5*c*d*e^2)*b*x + (9*c^8*e^2 + 22*c^6*e^
2 + 18*c^4*e^2 + 5*c^2*e^2)*b)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x +
 c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + (3*(3*a*d^8*e^2 + b*d^8*e^2)*x^8
+ 24*(3*a*c*d^7*e^2 + b*c*d^7*e^2)*x^7 + (2*(126*c^2*d^6*e^2 + 11*d^6*e^2)*
a + 7*(12*c^2*d^6*e^2 + d^6*e^2)*b)*x^6 + 6*(2*(42*c^3*d^5*e^2 + 11*c*d^5*e
^2)*a + 7*(4*c^3*d^5*e^2 + c*d^5*e^2)*b)*x^5 + (6*(105*c^4*d^4*e^2 + 55*c^2
*d^4*e^2 + 3*d^4*e^2)*a + 5*(42*c^4*d^4*e^2 + 21*c^2*d^4*e^2 + d^4*e^2)*b)*
x^4 + 4*(2*(63*c^5*d^3*e^2 + 55*c^3*d^3*e^2 + 9*c*d^3*e^2)*a + (42*c^5*d^3*
e^2 + 35*c^3*d^3*e^2 + 5*c*d^3*e^2)*b)*x^3 + ((252*c^6*d^2*e^2 + 330*c^4*d^
2*e^2 + 108*c^2*d^2*e^2 + 5*d^2*e^2)*a + (84*c^6*d^2*e^2 + 105*c^4*d^2*e^2
+ 30*c^2*d^2*e^2 + d^2*e^2)*b)*x^2 + (9*c^8*e^2 + 22*c^6*e^2 + 18*c^4*e^2 +
 5*c^2*e^2)*a + (3*c^8*e^2 + 7*c^6*e^2 + 5*c^4*e^2 + c^2*e^2)*b + 2*((36*c^
7*d*e^2 + 66*c^5*d*e^2 + 36*c^3*d*e^2 + 5*c*d*e^2)*a + (12*c^7*d*e^2 + 21*c
^5*d*e^2 + 10*c^3*d*e^2 + c*d*e^2)*b)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))
/(a^2*b^2*d^7*x^6 + 6*a^2*b^2*c*d^6*x^5 + 3*(5*c^2*d^5 + d^5)*a^2*b^2*x^4 +
 4*(5*c^3*d^4 + 3*c*d^4)*a^2*b^2*x^3 + 3*(5*c^4*d^3 + 6*c^2*d^3 + d^3)*a^2*
b^2*x^2 + 6*(c^5*d^2 + 2*c^3*d^2 + c*d^2)*a^2*b^2*x + (c^6*d + 3*c^4*d + 3*
c^2*d + d)*a^2*b^2 + (b^4*d^7*x^6 + 6*b^4*c*d^6*x^5 + 3*(5*c^2*d^5 + d^5)*b
^4*x^4 + 4*(5*c^3*d^4 + 3*c*d^4)*b^4*x^3 + 3*(5*c^4*d^3 + 6*c^2*d^3 + d^3)*
b^4*x^2 + 6*(c^5*d^2 + 2*c^3*d^2 + c*d^2)*b^4*x + (c^6*d + 3*c^4*d + 3*c^2*
d + d)*b^4 + (b^4*d^4*x^3 + 3*b^4*c*d^3*x^2 + 3*b^4*c^2*d^2*x + b^4*c^3*d)*
(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + 3*(b^4*d^5*x^4 + 4*b^4*c*d^4*x^3 + (6
*c^2*d^3 + d^3)*b^4*x^2 + 2*(2*c^3*d^2 + c*d^2)*b^4*x + (c^4*d + c^2*d)*b^4
)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + 3*(b^4*d^6*x^5 + 5*b^4*c*d^5*x^4 + 2*(5*c
```

```
^2*d^4 + d^4)*b^4*x^3 + 2*(5*c^3*d^3 + 3*c*d^3)*b^4*x^2 + (5*c^4*d^2 + 6*c^
2*d^2 + d^2)*b^4*x + (c^5*d + 2*c^3*d + c*d)*b^4)*sqrt(d^2*x^2 + 2*c*d*x +
c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 + (a^2*b^2*d^4
*x^3 + 3*a^2*b^2*c*d^3*x^2 + 3*a^2*b^2*c^2*d^2*x + a^2*b^2*c^3*d)*(d^2*x^2
+ 2*c*d*x + c^2 + 1)^(3/2) + 3*(a^2*b^2*d^5*x^4 + 4*a^2*b^2*c*d^4*x^3 + (6*
c^2*d^3 + d^3)*a^2*b^2*x^2 + 2*(2*c^3*d^2 + c*d^2)*a^2*b^2*x + (c^4*d + c^2
*d)*a^2*b^2)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + 2*(a*b^3*d^7*x^6 + 6*a*b^3*c*d
^6*x^5 + 3*(5*c^2*d^5 + d^5)*a*b^3*x^4 + 4*(5*c^3*d^4 + 3*c*d^4)*a*b^3*x^3
+ 3*(5*c^4*d^3 + 6*c^2*d^3 + d^3)*a*b^3*x^2 + 6*(c^5*d^2 + 2*c^3*d^2 + c*d^
2)*a*b^3*x + (c^6*d + 3*c^4*d + 3*c^2*d + d)*a*b^3 + (a*b^3*d^4*x^3 + 3*a*b
^3*c*d^3*x^2 + 3*a*b^3*c^2*d^2*x + a*b^3*c^3*d)*(d^2*x^2 + 2*c*d*x + c^2 +
1)^(3/2) + 3*(a*b^3*d^5*x^4 + 4*a*b^3*c*d^4*x^3 + (6*c^2*d^3 + d^3)*a*b^3*x
^2 + 2*(2*c^3*d^2 + c*d^2)*a*b^3*x + (c^4*d + c^2*d)*a*b^3)*(d^2*x^2 + 2*c*
d*x + c^2 + 1) + 3*(a*b^3*d^6*x^5 + 5*a*b^3*c*d^5*x^4 + 2*(5*c^2*d^4 + d^4)
*a*b^3*x^3 + 2*(5*c^3*d^3 + 3*c*d^3)*a*b^3*x^2 + (5*c^4*d^2 + 6*c^2*d^2 + d
^2)*a*b^3*x + (c^5*d + 2*c^3*d + c*d)*a*b^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 +
 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + 3*(a^2*b^2*d^6*x^5
+ 5*a^2*b^2*c*d^5*x^4 + 2*(5*c^2*d^4 + d^4)*a^2*b^2*x^3 + 2*(5*c^3*d^3 + 3*
c*d^3)*a^2*b^2*x^2 + (5*c^4*d^2 + 6*c^2*d^2 + d^2)*a^2*b^2*x + (c^5*d + 2*c
^3*d + c*d)*a^2*b^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + integrate(1/2*(9*
d^10*e^2*x^10 + 90*c*d^9*e^2*x^9 + 9*c^10*e^2 + 36*c^8*e^2 + 9*(45*c^2*d^8*
e^2 + 4*d^8*e^2)*x^8 + 54*c^6*e^2 + 72*(15*c^3*d^7*e^2 + 4*c*d^7*e^2)*x^7 +
 18*(105*c^4*d^6*e^2 + 56*c^2*d^6*e^2 + 3*d^6*e^2)*x^6 + 36*c^4*e^2 + 36*(6
3*c^5*d^5*e^2 + 56*c^3*d^5*e^2 + 9*c*d^5*e^2)*x^5 + 18*(105*c^6*d^4*e^2 + 1
40*c^4*d^4*e^2 + 45*c^2*d^4*e^2 + 2*d^4*e^2)*x^4 + 9*c^2*e^2 + 72*(15*c^7*d
^3*e^2 + 28*c^5*d^3*e^2 + 15*c^3*d^3*e^2 + 2*c*d^3*e^2)*x^3 + (9*d^6*e^2*x^
6 + 54*c*d^5*e^2*x^5 + 9*c^6*e^2 + 4*c^4*e^2 + (135*c^2*d^4*e^2 + 4*d^4*e^2
)*x^4 - c^2*e^2 + 4*(45*c^3*d^3*e^2 + 4*c*d^3*e^2)*x^3 + (135*c^4*d^2*e^2 +
 24*c^2*d^2*e^2 - d^2*e^2)*x^2 + 2*(27*c^5*d*e^2 + 8*c^3*d*e^2 - c*d*e^2)*x
)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 9*(45*c^8*d^2*e^2 + 112*c^6*d^2*e^2 + 9
0*c^4*d^2*e^2 + 24*c^2*d^2*e^2 + d^2*e^2)*x^2 + (36*d^7*e^2*x^7 + 252*c*d^6
*e^2*x^6 + 36*c^7*e^2 + 48*c^5*e^2 + 12*(63*c^2*d^5*e^2 + 4*d^5*e^2)*x^5 +
13*c^3*e^2 + 60*(21*c^3*d^4*e^2 + 4*c*d^4*e^2)*x^4 + (1260*c^4*d^3*e^2 + 48
0*c^2*d^3*e^2 + 13*d^3*e^2)*x^3 - 2*c*e^2 + 3*(252*c^5*d^2*e^2 + 160*c^3*d^
2*e^2 + 13*c*d^2*e^2)*x^2 + (252*c^6*d*e^2 + 240*c^4*d*e^2 + 39*c^2*d*e^2 -
 2*d*e^2)*x)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + (54*d^8*e^2*x^8 + 432*c*
d^7*e^2*x^7 + 54*c^8*e^2 + 120*c^6*e^2 + 24*(63*c^2*d^6*e^2 + 5*d^6*e^2)*x^
6 + 83*c^4*e^2 + 144*(21*c^3*d^5*e^2 + 5*c*d^5*e^2)*x^5 + (3780*c^4*d^4*e^2
 + 1800*c^2*d^4*e^2 + 83*d^4*e^2)*x^4 + 19*c^2*e^2 + 4*(756*c^5*d^3*e^2 + 6
00*c^3*d^3*e^2 + 83*c*d^3*e^2)*x^3 + (1512*c^6*d^2*e^2 + 1800*c^4*d^2*e^2 +
 498*c^2*d^2*e^2 + 19*d^2*e^2)*x^2 + 2*e^2 + 2*(216*c^7*d*e^2 + 360*c^5*d*e
^2 + 166*c^3*d*e^2 + 19*c*d*e^2)*x)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + 18*(5*c
^9*d*e^2 + 16*c^7*d*e^2 + 18*c^5*d*e^2 + 8*c^3*d*e^2 + c*d*e^2)*x + (36*d^9
*e^2*x^9 + 324*c*d^8*e^2*x^8 + 36*c^9*e^2 + 112*c^7*e^2 + 16*(81*c^2*d^7*e^
2 + 7*d^7*e^2)*x^7 + 123*c^5*e^2 + 112*(27*c^3*d^6*e^2 + 7*c*d^6*e^2)*x^6 +
 3*(1512*c^4*d^5*e^2 + 784*c^2*d^5*e^2 + 41*d^5*e^2)*x^5 + 57*c^3*e^2 + (45
36*c^5*d^4*e^2 + 3920*c^3*d^4*e^2 + 615*c*d^4*e^2)*x^4 + (3024*c^6*d^3*e^2
+ 3920*c^4*d^3*e^2 + 1230*c^2*d^3*e^2 + 57*d^3*e^2)*x^3 + 10*c*e^2 + 3*(432
*c^7*d^2*e^2 + 784*c^5*d^2*e^2 + 410*c^3*d^2*e^2 + 57*c*d^2*e^2)*x^2 + (324
*c^8*d*e^2 + 784*c^6*d*e^2 + 615*c^4*d*e^2 + 171*c^2*d*e^2 + 10*d*e^2)*x)*s
qrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(a*b^2*d^8*x^8 + 8*a*b^2*c*d^7*x^7 + 4*(7
*c^2*d^6 + d^6)*a*b^2*x^6 + 8*(7*c^3*d^5 + 3*c*d^5)*a*b^2*x^5 + 2*(35*c^4*d
^4 + 30*c^2*d^4 + 3*d^4)*a*b^2*x^4 + 8*(7*c^5*d^3 + 10*c^3*d^3 + 3*c*d^3)*a
*b^2*x^3 + 4*(7*c^6*d^2 + 15*c^4*d^2 + 9*c^2*d^2 + d^2)*a*b^2*x^2 + 8*(c^7*
d + 3*c^5*d + 3*c^3*d + c*d)*a*b^2*x + (c^8 + 4*c^6 + 6*c^4 + 4*c^2 + 1)*a*
b^2 + (a*b^2*d^4*x^4 + 4*a*b^2*c*d^3*x^3 + 6*a*b^2*c^2*d^2*x^2 + 4*a*b^2*c^
3*d*x + a*b^2*c^4)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 4*(a*b^2*d^5*x^5 + 5*a
*b^2*c*d^4*x^4 + (10*c^2*d^3 + d^3)*a*b^2*x^3 + (10*c^3*d^2 + 3*c*d^2)*a*b^
2*x^2 + (5*c^4*d + 3*c^2*d)*a*b^2*x + (c^5 + c^3)*a*b^2)*(d^2*x^2 + 2*c*d*x
```

```
 + c^2 + 1)^(3/2) + 6*(a*b^2*d^6*x^6 + 6*a*b^2*c*d^5*x^5 + (15*c^2*d^4 + 2*
d^4)*a*b^2*x^4 + 4*(5*c^3*d^3 + 2*c*d^3)*a*b^2*x^3 + (15*c^4*d^2 + 12*c^2*d
^2 + d^2)*a*b^2*x^2 + 2*(3*c^5*d + 4*c^3*d + c*d)*a*b^2*x + (c^6 + 2*c^4 +
c^2)*a*b^2)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + (b^3*d^8*x^8 + 8*b^3*c*d^7*x^7
+ 4*(7*c^2*d^6 + d^6)*b^3*x^6 + 8*(7*c^3*d^5 + 3*c*d^5)*b^3*x^5 + 2*(35*c^4
*d^4 + 30*c^2*d^4 + 3*d^4)*b^3*x^4 + 8*(7*c^5*d^3 + 10*c^3*d^3 + 3*c*d^3)*b
^3*x^3 + 4*(7*c^6*d^2 + 15*c^4*d^2 + 9*c^2*d^2 + d^2)*b^3*x^2 + 8*(c^7*d +
3*c^5*d + 3*c^3*d + c*d)*b^3*x + (c^8 + 4*c^6 + 6*c^4 + 4*c^2 + 1)*b^3 + (b
^3*d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*x + b^3*c^4)
*(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 4*(b^3*d^5*x^5 + 5*b^3*c*d^4*x^4 + (10*c
^2*d^3 + d^3)*b^3*x^3 + (10*c^3*d^2 + 3*c*d^2)*b^3*x^2 + (5*c^4*d + 3*c^2*d
)*b^3*x + (c^5 + c^3)*b^3)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + 6*(b^3*d^6
*x^6 + 6*b^3*c*d^5*x^5 + (15*c^2*d^4 + 2*d^4)*b^3*x^4 + 4*(5*c^3*d^3 + 2*c*
d^3)*b^3*x^3 + (15*c^4*d^2 + 12*c^2*d^2 + d^2)*b^3*x^2 + 2*(3*c^5*d + 4*c^3
*d + c*d)*b^3*x + (c^6 + 2*c^4 + c^2)*b^3)*(d^2*x^2 + 2*c*d*x + c^2 + 1) +
4*(b^3*d^7*x^7 + 7*b^3*c*d^6*x^6 + 3*(7*c^2*d^5 + d^5)*b^3*x^5 + 5*(7*c^3*d
^4 + 3*c*d^4)*b^3*x^4 + (35*c^4*d^3 + 30*c^2*d^3 + 3*d^3)*b^3*x^3 + 3*(7*c^
5*d^2 + 10*c^3*d^2 + 3*c*d^2)*b^3*x^2 + (7*c^6*d + 15*c^4*d + 9*c^2*d + d)*
b^3*x + (c^7 + 3*c^5 + 3*c^3 + c)*b^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*l
og(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + 4*(a*b^2*d^7*x^7 + 7*a*b^
2*c*d^6*x^6 + 3*(7*c^2*d^5 + d^5)*a*b^2*x^5 + 5*(7*c^3*d^4 + 3*c*d^4)*a*b^2
*x^4 + (35*c^4*d^3 + 30*c^2*d^3 + 3*d^3)*a*b^2*x^3 + 3*(7*c^5*d^2 + 10*c^3*
d^2 + 3*c*d^2)*a*b^2*x^2 + (7*c^6*d + 15*c^4*d + 9*c^2*d + d)*a*b^2*x + (c^
7 + 3*c^5 + 3*c^3 + c)*a*b^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,e+d\,e\,x)^2}{(a+b\,\mathrm{asinh}\,(c+d\,x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2/(a + b*asinh(c + d*x))^3,x)
```

```
[Out] int((c*e + d*e*x)^2/(a + b*asinh(c + d*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^2\left(\int \frac{c^2}{a^3+3a^2b\operatorname{asinh}(c+dx)+3ab^2\operatorname{asinh}^2(c+dx)+b^3\operatorname{asinh}^3(c+dx)}\,dx+\int \frac{}{a^3+3a^2b\operatorname{asinh}(c+dx)+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2/(a+b*asinh(d*x+c))**3,x)
```

```
[Out] e**2*(Integral(c**2/(a**3 + 3*a**2*b*asinh(c + d*x) + 3*a*b**2*asinh(c + d*
x)**2 + b**3*asinh(c + d*x)**3), x) + Integral(d**2*x**2/(a**3 + 3*a**2*b*a
sinh(c + d*x) + 3*a*b**2*asinh(c + d*x)**2 + b**3*asinh(c + d*x)**3), x) +
Integral(2*c*d*x/(a**3 + 3*a**2*b*asinh(c + d*x) + 3*a*b**2*asinh(c + d*x)*
*2 + b**3*asinh(c + d*x)**3), x))
```

3.171 $\int \frac{ce+dex}{\left(a+b\sinh^{-1}(c+dx)\right)^3}\,dx$

Optimal. Leaf size=156

$$-\frac{e\sinh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{b^3d}+\frac{e\cosh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{b^3d}-\frac{e(c+dx)^2}{b^2d\left(a+b\sinh^{-1}(c+dx)\right)}-\frac{}{2b^2d\left(a+b\right.}$$

```
[Out] -1/2*e/b^2/d/(a+b*arcsinh(d*x+c))-e*(d*x+c)^2/b^2/d/(a+b*arcsinh(d*x+c))+e*
cosh(2*a/b)*Shi(2*(a+b*arcsinh(d*x+c))/b)/b^3/d-e*Chi(2*(a+b*arcsinh(d*x+c)
)/b)*sinh(2*a/b)/b^3/d-1/2*e*(d*x+c)*(1+(d*x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d
*x+c))^2
```

Rubi [A] time = 0.33, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {5865, 12, 5667, 5774, 5669, 5448, 3303, 3298, 3301, 5675}

$$-\frac{e\sinh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(c+dx)\right)}{b^3d}+\frac{e\cosh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(\frac{2a}{b}+2\sinh^{-1}(c+dx)\right)}{b^3d}-\frac{e(c+dx)^2}{b^2d\left(a+b\sinh^{-1}(c+dx)\right)}-$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)/(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] -(e*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(2*b*d*(a + b*ArcSinh[c + d*x])^2) - e
/(2*b^2*d*(a + b*ArcSinh[c + d*x])) - (e*(c + d*x)^2)/(b^2*d*(a + b*ArcSinh
[c + d*x])) - (e*CoshIntegral[(2*a)/b + 2*ArcSinh[c + d*x]]*Sinh[(2*a)/b])/
(b^3*d) + (e*Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSinh[c + d*x]])/(b^3
*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{ce+dex}{\left(a+b\sinh^{-1}(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{ex}{\left(a+b\sinh^{-1}(x)\right)^3}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e\,\text{Subst}\left(\int \frac{x}{\left(a+b\sinh^{-1}(x)\right)^3}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{e(c+dx)\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} + \frac{e\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}\left(a+b\sinh^{-1}(x)\right)^2}\,dx, x, c+dx\right)}{2bd} + -e$$

$$= -\frac{e(c+dx)\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{e}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{e(c+dx)^2}{b^2d\left(a+b\sinh^{-1}(\right.}$$

$$= -\frac{e(c+dx)\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{e}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{e(c+dx)^2}{b^2d\left(a+b\sinh^{-1}(\right.}$$

$$= -\frac{e(c+dx)\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{e}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{e(c+dx)^2}{b^2d\left(a+b\sinh^{-1}(\right.}$$

$$= -\frac{e(c+dx)\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{e}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{e(c+dx)^2}{b^2d\left(a+b\sinh^{-1}(\right.}$$

$$= -\frac{e(c+dx)\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{e}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{e(c+dx)^2}{b^2d\left(a+b\sinh^{-1}(\right.}$$

$$= -\frac{e(c+dx)\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{e}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{e(c+dx)^2}{b^2d\left(a+b\sinh^{-1}(\right.}$$

Mathematica [A] time = 0.31, size = 120, normalized size = 0.77

$$\frac{e\left(-\frac{b^2(c+dx)\sqrt{(c+dx)^2+1}}{\left(a+b\sinh^{-1}(c+dx)\right)^2} - 2\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right) + 2\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right) + \frac{b(\cdot}{a+b}\right.}{2b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)/(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] (e*(-((b^2*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh[c + d*x])^2) + (
b*(-1 - 2*(c + d*x)^2))/(a + b*ArcSinh[c + d*x]) - 2*CoshIntegral[2*(a/b +
ArcSinh[c + d*x])]*Sinh[(2*a)/b] + 2*Cosh[(2*a)/b]*SinhIntegral[2*(a/b + Ar
cSinh[c + d*x])]))/(2*b^3*d)
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{dex+ce}{b^3\,\text{arsinh}\,(dx+c)^3 + 3\,ab^2\,\text{arsinh}\,(dx+c)^2 + 3\,a^2b\,\text{arsinh}\,(dx+c) + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((d*e*x + c*e)/(b^3*arcsinh(d*x + c)^3 + 3*a*b^2*arcsinh(d*x + c)^2
 + 3*a^2*b*arcsinh(d*x + c) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dex + ce}{\left(b\operatorname{arsinh}\left(dx + c\right) + a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)/(b*arcsinh(d*x + c) + a)^3, x)
```

maple [A] time = 0.07, size = 239, normalized size = 1.53

$$\frac{-\frac{\left(2(dx+c)^2-2(dx+c)\sqrt{1+(dx+c)^2}+1\right)e(2b\operatorname{arcsinh}(dx+c)+2a-b)}{8b^2\left(b^2\operatorname{arcsinh}(dx+c)^2+2ab\operatorname{arcsinh}(dx+c)+a^2\right)}+\frac{e\,e^{\frac{2a}{b}}\operatorname{Ei}\left(1,2\operatorname{arcsinh}(dx+c)+\frac{2a}{b}\right)}{2b^3}-\frac{e\left(2(dx+c)^2+1+2(dx+c)\sqrt{1+(dx+c)^2}\right)}{8b(a+b\operatorname{arcsinh}(dx+c))^2}-}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^3,x)
```

```
[Out] 1/d*(-1/8*(2*(d*x+c)^2-2*(d*x+c)*(1+(d*x+c)^2)^(1/2)+1)*e*(2*b*arcsinh(d*x+
c)+2*a-b)/b^2/(b^2*arcsinh(d*x+c)^2+2*a*b*arcsinh(d*x+c)+a^2)+1/2*e/b^3*exp
(2*a/b)*Ei(1,2*arcsinh(d*x+c)+2*a/b)-1/8/b*e*(2*(d*x+c)^2+1+2*(d*x+c)*(1+(d
*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^2-1/4/b^2*e*(2*(d*x+c)^2+1+2*(d*x+c)*(
1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))-1/2/b^3*e*exp(-2*a/b)*Ei(1,-2*arcs
inh(d*x+c)-2*a/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*((2*a*d^8*e + b*d^8*e)*x^8 + 8*(2*a*c*d^7*e + b*c*d^7*e)*x^7 + (2*(28*
c^2*d^6*e + 3*d^6*e)*a + (28*c^2*d^6*e + 3*d^6*e)*b)*x^6 + 2*(2*(28*c^3*d^5
*e + 9*c*d^5*e)*a + (28*c^3*d^5*e + 9*c*d^5*e)*b)*x^5 + (2*(70*c^4*d^4*e +
45*c^2*d^4*e + 3*d^4*e)*a + (70*c^4*d^4*e + 45*c^2*d^4*e + 3*d^4*e)*b)*x^4
+ 4*(2*(14*c^5*d^3*e + 15*c^3*d^3*e + 3*c*d^3*e)*a + (14*c^5*d^3*e + 15*c^3
*d^3*e + 3*c*d^3*e)*b)*x^3 + (2*(28*c^6*d^2*e + 45*c^4*d^2*e + 18*c^2*d^2*e
 + d^2*e)*a + (28*c^6*d^2*e + 45*c^4*d^2*e + 18*c^2*d^2*e + d^2*e)*b)*x^2 +
 ((2*a*d^5*e + b*d^5*e)*x^5 + 5*(2*a*c*d^4*e + b*c*d^4*e)*x^4 + (2*(10*c^2*
d^3*e + d^3*e)*a + (10*c^2*d^3*e + d^3*e)*b)*x^3 + (2*(10*c^3*d^2*e + 3*c*d
^2*e)*a + (10*c^3*d^2*e + 3*c*d^2*e)*b)*x^2 + 2*(c^5*e + c^3*e)*a + (c^5*e
+ c^3*e)*b + (2*(5*c^4*d*e + 3*c^2*d*e)*a + (5*c^4*d*e + 3*c^2*d*e)*b)*x)*(
d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + (3*(2*a*d^6*e + b*d^6*e)*x^6 + 18*(2*a
*c*d^5*e + b*c*d^5*e)*x^5 + 5*(2*(9*c^2*d^4*e + d^4*e)*a + (9*c^2*d^4*e + d
^4*e)*b)*x^4 + 20*(2*(3*c^3*d^3*e + c*d^3*e)*a + (3*c^3*d^3*e + c*d^3*e)*b)
*x^3 + (5*(18*c^4*d^2*e + 12*c^2*d^2*e + d^2*e)*a + (45*c^4*d^2*e + 30*c^2*
d^2*e + 2*d^2*e)*b)*x^2 + (6*c^6*e + 10*c^4*e + 5*c^2*e + e)*a + (3*c^6*e +
 5*c^4*e + 2*c^2*e)*b + 2*((18*c^5*d*e + 20*c^3*d*e + 5*c*d*e)*a + (9*c^5*d
*e + 10*c^3*d*e + 2*c*d*e)*b)*x)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + 2*(c^8*e +
 3*c^6*e + 3*c^4*e + c^2*e)*a + (c^8*e + 3*c^6*e + 3*c^4*e + c^2*e)*b + 2*(
2*(4*c^7*d*e + 9*c^5*d*e + 6*c^3*d*e + c*d*e)*a + (4*c^7*d*e + 9*c^5*d*e +
6*c^3*d*e + c*d*e)*b)*x + (2*b*d^8*e*x^8 + 16*b*c*d^7*e*x^7 + 2*(28*c^2*d^6
```

*e + 3*d^6*e)*b*x^6 + 4*(28*c^3*d^5*e + 9*c*d^5*e)*b*x^5 + 2*(70*c^4*d^4*e + 45*c^2*d^4*e + 3*d^4*e)*b*x^4 + 8*(14*c^5*d^3*e + 15*c^3*d^3*e + 3*c*d^3*e)*b*x^3 + 2*(28*c^6*d^2*e + 45*c^4*d^2*e + 18*c^2*d^2*e + d^2*e)*b*x^2 + 4*(4*c^7*d*e + 9*c^5*d*e + 6*c^3*d*e + c*d*e)*b*x + 2*(b*d^5*e*x^5 + 5*b*c*d^4*e*x^4 + (10*c^2*d^3*e + d^3*e)*b*x^3 + (10*c^3*d^2*e + 3*c*d^2*e)*b*x^2 + (5*c^4*d*e + 3*c^2*d*e)*b*x + (c^5*e + c^3*e)*b)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + (6*b*d^6*e*x^6 + 36*b*c*d^5*e*x^5 + 10*(9*c^2*d^4*e + d^4*e)*b*x^4 + 40*(3*c^3*d^3*e + c*d^3*e)*b*x^3 + 5*(18*c^4*d^2*e + 12*c^2*d^2*e + d^2*e)*b*x^2 + 2*(18*c^5*d*e + 20*c^3*d*e + 5*c*d*e)*b*x + (6*c^6*e + 10*c^4*e + 5*c^2*e + e)*b)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + 2*(c^8*e + 3*c^6*e + 3*c^4*e + c^2*e)*b + (6*b*d^7*e*x^7 + 42*b*c*d^6*e*x^6 + 14*(9*c^2*d^5*e + d^5*e)*b*x^5 + 70*(3*c^3*d^4*e + c*d^4*e)*b*x^4 + (210*c^4*d^3*e + 140*c^2*d^3*e + 11*d^3*e)*b*x^3 + (126*c^5*d^2*e + 140*c^3*d^2*e + 33*c*d^2*e)*b*x^2 + (42*c^6*d*e + 70*c^4*d*e + 33*c^2*d*e + 3*d*e)*b*x + (6*c^7*e + 14*c^5*e + 11*c^3*e + 3*c*e)*b)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + (3*(2*a*d^7*e + b*d^7*e)*x^7 + 21*(2*a*c*d^6*e + b*c*d^6*e)*x^6 + 7*(2*(9*c^2*d^5*e + d^5*e)*a + (9*c^2*d^5*e + d^5*e)*b)*x^5 + 35*(2*(3*c^3*d^4*e + c*d^4*e)*a + (3*c^3*d^4*e + c*d^4*e)*b)*x^4 + ((210*c^4*d^3*e + 140*c^2*d^3*e + 11*d^3*e)*a + 5*(21*c^4*d^3*e + 14*c^2*d^3*e + d^3*e)*b)*x^3 + ((126*c^5*d^2*e + 140*c^3*d^2*e + 33*c*d^2*e)*a + (63*c^5*d^2*e + 70*c^3*d^2*e + 15*c*d^2*e)*b)*x^2 + (6*c^7*e + 14*c^5*e + 11*c^3*e + 3*c*e)*a + (3*c^7*e + 7*c^5*e + 5*c^3*e + c*e)*b + ((42*c^6*d*e + 70*c^4*d*e + 33*c^2*d*e + 3*d*e)*a + (21*c^6*d*e + 35*c^4*d*e + 15*c^2*d*e + d*e)*b)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(a^2*b^2*d^7*x^6 + 6*a^2*b^2*c*d^6*x^5 + 3*(5*c^2*d^5 + d^5)*a^2*b^2*x^4 + 4*(5*c^3*d^4 + 3*c*d^4)*a^2*b^2*x^3 + 3*(5*c^4*d^3 + 6*c^2*d^3 + d^3)*a^2*b^2*x^2 + 6*(c^5*d^2 + 2*c^3*d^2 + c*d^2)*a^2*b^2*x + (c^6*d + 3*c^4*d + 3*c^2*d + d)*a^2*b^2 + (b^4*d^7*x^6 + 6*b^4*c*d^6*x^5 + 3*(5*c^2*d^5 + d^5)*b^4*x^4 + 4*(5*c^3*d^4 + 3*c*d^4)*b^4*x^3 + 3*(5*c^4*d^3 + 6*c^2*d^3 + d^3)*b^4*x^2 + 6*(c^5*d^2 + 2*c^3*d^2 + c*d^2)*b^4*x + (c^6*d + 3*c^4*d + 3*c^2*d + d)*b^4 + (b^4*d^4*x^3 + 3*b^4*c*d^3*x^2 + 3*b^4*c^2*d^2*x + b^4*c^3*d)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + 3*(b^4*d^5*x^4 + 4*b^4*c*d^4*x^3 + (6*c^2*d^3 + d^3)*b^4*x^2 + 2*(2*c^3*d^2 + c*d^2)*b^4*x + (c^4*d + c^2*d)*b^4)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + 3*(b^4*d^6*x^5 + 5*b^4*c*d^5*x^4 + 2*(5*c^2*d^4 + d^4)*b^4*x^3 + 2*(5*c^3*d^3 + 3*c*d^3)*b^4*x^2 + (5*c^4*d^2 + 6*c^2*d^2 + d^2)*b^4*x + (c^5*d + 2*c^3*d + c*d)*b^4)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 + (a^2*b^2*d^4*x^3 + 3*a^2*b^2*c*d^3*x^2 + 3*a^2*b^2*c^2*d^2*x + a^2*b^2*c^3*d)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + 3*(a^2*b^2*d^5*x^4 + 4*a^2*b^2*c*d^4*x^3 + (6*c^2*d^3 + d^3)*a^2*b^2*x^2 + 2*(2*c^3*d^2 + c*d^2)*a^2*b^2*x + (c^4*d + c^2*d)*a^2*b^2)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + 2*(a*b^3*d^7*x^6 + 6*a*b^3*c*d^6*x^5 + 3*(5*c^2*d^5 + d^5)*a*b^3*x^4 + 4*(5*c^3*d^4 + 3*c*d^4)*a*b^3*x^3 + 3*(5*c^4*d^3 + 6*c^2*d^3 + d^3)*a*b^3*x^2 + 6*(c^5*d^2 + 2*c^3*d^2 + c*d^2)*a*b^3*x + (c^6*d + 3*c^4*d + 3*c^2*d + d)*a*b^3 + (a*b^3*d^4*x^3 + 3*a*b^3*c*d^3*x^2 + 3*a*b^3*c^2*d^2*x + a*b^3*c^3*d)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + 3*(a*b^3*d^5*x^4 + 4*a*b^3*c*d^4*x^3 + (6*c^2*d^3 + d^3)*a*b^3*x^2 + 2*(2*c^3*d^2 + c*d^2)*a*b^3*x + (c^4*d + c^2*d)*a*b^3)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + 3*(a*b^3*d^6*x^5 + 5*a*b^3*c*d^5*x^4 + 2*(5*c^2*d^4 + d^4)*a*b^3*x^3 + 2*(5*c^3*d^3 + 3*c*d^3)*a*b^3*x^2 + (5*c^4*d^2 + 6*c^2*d^2 + d^2)*a*b^3*x + (c^5*d + 2*c^3*d + c*d)*a*b^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + 3*(a^2*b^2*d^6*x^5 + 5*a^2*b^2*c*d^5*x^4 + 2*(5*c^2*d^4 + d^4)*a^2*b^2*x^3 + 2*(5*c^3*d^3 + 3*c*d^3)*a^2*b^2*x^2 + (5*c^4*d^2 + 6*c^2*d^2 + d^2)*a^2*b^2*x + (c^5*d + 2*c^3*d + c*d)*a^2*b^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + integrate(1/2*(4*d^9*e*x^9 + 36*c*d^8*e*x^8 + 4*c^9*e + 16*c^7*e + 16*(9*c^2*d^7*e + d^7*e)*x^7 + 112*(3*c^3*d^6*e + c*d^6*e)*x^6 + 24*c^5*e + 24*(21*c^4*d^5*e + 14*c^2*d^5*e + d^5*e)*x^5 + 8*(63*c^5*d^4*e + 70*c^3*d^4*e + 15*c*d^4*e)*x^4 + 16*c^3*e + 16*(21*c^6*d^3*e + 35*c^4*d^3*e + 15*c^2*d^3*e + d^3*e)*x^3 + 4*(d^5*e*x^5 + 5*c*d^4*e*x^4 + 10*c^2*d^3*e*x^3 + 10*c^3*d^2*e*x^2 + 5*c^4*d*e*x + c^5*e)*(d^2*x^2 +

```
2*c*d*x + c^2 + 1)^2 + 48*(3*c^7*d^2*e + 7*c^5*d^2*e + 5*c^3*d^2*e + c*d^2*
e)*x^2 + (16*d^6*e*x^6 + 96*c*d^5*e*x^5 + 16*c^6*e + 16*c^4*e + 16*(15*c^2*
d^4*e + d^4*e)*x^4 + 64*(5*c^3*d^3*e + c*d^3*e)*x^3 + 48*(5*c^4*d^2*e + 2*c
^2*d^2*e)*x^2 + 32*(3*c^5*d*e + 2*c^3*d*e)*x - 3*e)*(d^2*x^2 + 2*c*d*x + c^
2 + 1)^(3/2) + 24*(d^7*e*x^7 + 7*c*d^6*e*x^6 + c^7*e + 2*c^5*e + (21*c^2*d^
5*e + 2*d^5*e)*x^5 + 5*(7*c^3*d^4*e + 2*c*d^4*e)*x^4 + c^3*e + (35*c^4*d^3*
e + 20*c^2*d^3*e + d^3*e)*x^3 + (21*c^5*d^2*e + 20*c^3*d^2*e + 3*c*d^2*e)*x
^2 + (7*c^6*d*e + 10*c^4*d*e + 3*c^2*d*e)*x)*(d^2*x^2 + 2*c*d*x + c^2 + 1)
+ 4*c*e + 4*(9*c^8*d*e + 28*c^6*d*e + 30*c^4*d*e + 12*c^2*d*e + d*e)*x + (1
6*d^8*e*x^8 + 128*c*d^7*e*x^7 + 16*c^8*e + 48*c^6*e + 16*(28*c^2*d^6*e + 3*
d^6*e)*x^6 + 32*(28*c^3*d^5*e + 9*c*d^5*e)*x^5 + 48*c^4*e + 16*(70*c^4*d^4*
e + 45*c^2*d^4*e + 3*d^4*e)*x^4 + 64*(14*c^5*d^3*e + 15*c^3*d^3*e + 3*c*d^3
*e)*x^3 + 19*c^2*e + (448*c^6*d^2*e + 720*c^4*d^2*e + 288*c^2*d^2*e + 19*d^
2*e)*x^2 + 2*(64*c^7*d*e + 144*c^5*d*e + 96*c^3*d*e + 19*c*d*e)*x + 3*e)*sq
rt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(a*b^2*d^8*x^8 + 8*a*b^2*c*d^7*x^7 + 4*(7*
c^2*d^6 + d^6)*a*b^2*x^6 + 8*(7*c^3*d^5 + 3*c*d^5)*a*b^2*x^5 + 2*(35*c^4*d^
4 + 30*c^2*d^4 + 3*d^4)*a*b^2*x^4 + 8*(7*c^5*d^3 + 10*c^3*d^3 + 3*c*d^3)*a*
b^2*x^3 + 4*(7*c^6*d^2 + 15*c^4*d^2 + 9*c^2*d^2 + d^2)*a*b^2*x^2 + 8*(c^7*d
 + 3*c^5*d + 3*c^3*d + c*d)*a*b^2*x + (c^8 + 4*c^6 + 6*c^4 + 4*c^2 + 1)*a*b
^2 + (a*b^2*d^4*x^4 + 4*a*b^2*c*d^3*x^3 + 6*a*b^2*c^2*d^2*x^2 + 4*a*b^2*c^3
*d*x + a*b^2*c^4)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 4*(a*b^2*d^5*x^5 + 5*a*
b^2*c*d^4*x^4 + (10*c^2*d^3 + d^3)*a*b^2*x^3 + (10*c^3*d^2 + 3*c*d^2)*a*b^2
*x^2 + (5*c^4*d + 3*c^2*d)*a*b^2*x + (c^5 + c^3)*a*b^2)*(d^2*x^2 + 2*c*d*x
+ c^2 + 1)^(3/2) + 6*(a*b^2*d^6*x^6 + 6*a*b^2*c*d^5*x^5 + (15*c^2*d^4 + 2*d
^4)*a*b^2*x^4 + 4*(5*c^3*d^3 + 2*c*d^3)*a*b^2*x^3 + (15*c^4*d^2 + 12*c^2*d^
2 + d^2)*a*b^2*x^2 + 2*(3*c^5*d + 4*c^3*d + c*d)*a*b^2*x + (c^6 + 2*c^4 + c
^2)*a*b^2)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + (b^3*d^8*x^8 + 8*b^3*c*d^7*x^7 +
 4*(7*c^2*d^6 + d^6)*b^3*x^6 + 8*(7*c^3*d^5 + 3*c*d^5)*b^3*x^5 + 2*(35*c^4*
d^4 + 30*c^2*d^4 + 3*d^4)*b^3*x^4 + 8*(7*c^5*d^3 + 10*c^3*d^3 + 3*c*d^3)*b^
3*x^3 + 4*(7*c^6*d^2 + 15*c^4*d^2 + 9*c^2*d^2 + d^2)*b^3*x^2 + 8*(c^7*d + 3
*c^5*d + 3*c^3*d + c*d)*b^3*x + (c^8 + 4*c^6 + 6*c^4 + 4*c^2 + 1)*b^3 + (b^
3*d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*x + b^3*c^4)*
(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 4*(b^3*d^5*x^5 + 5*b^3*c*d^4*x^4 + (10*c^
2*d^3 + d^3)*b^3*x^3 + (10*c^3*d^2 + 3*c*d^2)*b^3*x^2 + (5*c^4*d + 3*c^2*d)
*b^3*x + (c^5 + c^3)*b^3)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + 6*(b^3*d^6*
x^6 + 6*b^3*c*d^5*x^5 + (15*c^2*d^4 + 2*d^4)*b^3*x^4 + 4*(5*c^3*d^3 + 2*c*d
^3)*b^3*x^3 + (15*c^4*d^2 + 12*c^2*d^2 + d^2)*b^3*x^2 + 2*(3*c^5*d + 4*c^3*
d + c*d)*b^3*x + (c^6 + 2*c^4 + c^2)*b^3)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + 4
*(b^3*d^7*x^7 + 7*b^3*c*d^6*x^6 + 3*(7*c^2*d^5 + d^5)*b^3*x^5 + 5*(7*c^3*d^
4 + 3*c*d^4)*b^3*x^4 + (35*c^4*d^3 + 30*c^2*d^3 + 3*d^3)*b^3*x^3 + 3*(7*c^5
*d^2 + 10*c^3*d^2 + 3*c*d^2)*b^3*x^2 + (7*c^6*d + 15*c^4*d + 9*c^2*d + d)*b
^3*x + (c^7 + 3*c^5 + 3*c^3 + c)*b^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*lo
g(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + 4*(a*b^2*d^7*x^7 + 7*a*b^2
*c*d^6*x^6 + 3*(7*c^2*d^5 + d^5)*a*b^2*x^5 + 5*(7*c^3*d^4 + 3*c*d^4)*a*b^2*
x^4 + (35*c^4*d^3 + 30*c^2*d^3 + 3*d^3)*a*b^2*x^3 + 3*(7*c^5*d^2 + 10*c^3*d
^2 + 3*c*d^2)*a*b^2*x^2 + (7*c^6*d + 15*c^4*d + 9*c^2*d + d)*a*b^2*x + (c^7
 + 3*c^5 + 3*c^3 + c)*a*b^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c\,e+d\,e\,x}{{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)/(a + b*asinh(c + d*x))^3,x)
```

```
[Out] int((c*e + d*e*x)/(a + b*asinh(c + d*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e\left(\int \frac{c}{a^3 + 3a^2b\operatorname{asinh}(c + dx) + 3ab^2\operatorname{asinh}^2(c + dx) + b^3\operatorname{asinh}^3(c + dx)}\,dx + \int \frac{}{a^3 + 3a^2b\operatorname{asinh}(c + dx) + 3ab\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*asinh(d*x+c))**3,x)
```

```
[Out] e*(Integral(c/(a**3 + 3*a**2*b*asinh(c + d*x) + 3*a*b**2*asinh(c + d*x)**2
+ b**3*asinh(c + d*x)**3), x) + Integral(d*x/(a**3 + 3*a**2*b*asinh(c + d*x
) + 3*a*b**2*asinh(c + d*x)**2 + b**3*asinh(c + d*x)**3), x))
```

3.172 $\int \frac{1}{\left(a+b\sinh^{-1}(c+dx)\right)^3}\,dx$

Optimal. Leaf size=125

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{2b^3d}-\frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{2b^3d}-\frac{c+dx}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)}-\frac{\sqrt{(c+dx)^2\cdots}}{2bd\left(a+b\sinh^{-1}(\cdots\right.}$$

```
[Out] 1/2*(-d*x-c)/b^2/d/(a+b*arcsinh(d*x+c))+1/2*Chi((a+b*arcsinh(d*x+c))/b)*cos
h(a/b)/b^3/d-1/2*Shi((a+b*arcsinh(d*x+c))/b)*sinh(a/b)/b^3/d-1/2*(1+(d*x+c)
^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))^2
```

Rubi [A] time = 0.18, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {5863, 5655, 5774, 5657, 3303, 3298, 3301}

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{2b^3d}-\frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{2b^3d}-\frac{c+dx}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)}-\frac{\sqrt{(c+dx)^2\cdots}}{2bd\left(a+b\sinh^{-1}(\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^(-3),x]
```

```
[Out] -Sqrt[1 + (c + d*x)^2]/(2*b*d*(a + b*ArcSinh[c + d*x])^2) - (c + d*x)/(2*b^
2*d*(a + b*ArcSinh[c + d*x])) + (Cosh[a/b]*CoshIntegral[(a + b*ArcSinh[c +
d*x])/b])/(2*b^3*d) - (Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[c + d*x])/b])/
(2*b^3*d)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
```

```
c, n}, x]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5863

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sinh^{-1}(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+b\sinh^{-1}(x)\right)^3}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} + \frac{\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\left(a+b\sinh^{-1}(x)\right)^2}\,dx, x, c+dx\right)}{2bd}$$

$$= -\frac{\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{c+dx}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{a+b\sinh^{-1}\ldots}\right.}{2b\ldots}$$

$$= -\frac{\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{c+dx}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\ldots\right)}{x}\right.}{\ldots}$$

$$= -\frac{\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{c+dx}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)} + \frac{\cosh\left(\frac{a}{b}\right)\text{Subst}\left(\ldots\right.}{\ldots}$$

$$= -\frac{\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{c+dx}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)} + \frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+\ldots}{\ldots}\right.}{2b^3d\ldots}$$

Mathematica [A] time = 0.28, size = 100, normalized size = 0.80

$$-\frac{\frac{b^2\sqrt{(c+dx)^2+1}}{\left(a+b\sinh^{-1}(c+dx)\right)^2} - \cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right) + \sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right) + \frac{b(c+dx)}{a+b\sinh^{-1}(c+dx)}}{2b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^(-3),x]
```

```
[Out] -1/2*((b^2*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh[c + d*x])^2 + (b*(c + d*x)
)/(a + b*ArcSinh[c + d*x]) - Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c + d*x]]
 + Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c + d*x]])/(b^3*d)
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{1}{b^3\,\mathrm{arsinh}\,(dx+c)^3+3\,ab^2\,\mathrm{arsinh}\,(dx+c)^2+3\,a^2b\,\mathrm{arsinh}\,(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(1/(b^3*arcsinh(d*x + c)^3 + 3*a*b^2*arcsinh(d*x + c)^2 + 3*a^2*b*a
rcsinh(d*x + c) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,\mathrm{arsinh}\,(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(-3), x)
```

maple [A] time = 0.06, size = 190, normalized size = 1.52

$$\frac{-\frac{\left(-\sqrt{1+(dx+c)^2}+dx+c\right)(b\,\mathrm{arcsinh}(dx+c)+a-b)}{4b^2\left(b^2\,\mathrm{arcsinh}(dx+c)^2+2ab\,\mathrm{arcsinh}(dx+c)+a^2\right)}-\frac{e^{\frac{a}{b}}\,\mathrm{Ei}\left(1,\mathrm{arcsinh}(dx+c)+\frac{a}{b}\right)}{4b^3}-\frac{dx+c+\sqrt{1+(dx+c)^2}}{4b(a+b\,\mathrm{arcsinh}(dx+c))^2}-\frac{dx+c+\sqrt{1+(dx+c)^2}}{4b^2(a+b\,\mathrm{arcsinh}(dx+c))}-\frac{e^{-\frac{a}{b}}\,\mathrm{I}}{}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(d*x+c))^3,x)
```

```
[Out] 1/d*(-1/4*(-(1+(d*x+c)^2)^(1/2)+d*x+c)*(b*arcsinh(d*x+c)+a-b)/b^2/(b^2*arcs
inh(d*x+c)^2+2*a*b*arcsinh(d*x+c)+a^2)-1/4/b^3*exp(a/b)*Ei(1,arcsinh(d*x+c)
+a/b)-1/4/b*(d*x+c+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^2-1/4/b^2*(d*x
+c+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))-1/4/b^3*exp(-a/b)*Ei(1,-arcsin
h(d*x+c)-a/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*((a*d^7 + b*d^7)*x^7 + 7*(a*c*d^6 + b*c*d^6)*x^6 + 3*((7*c^2*d^5 + d^5
)*a + (7*c^2*d^5 + d^5)*b)*x^5 + 5*((7*c^3*d^4 + 3*c*d^4)*a + (7*c^3*d^4 +
3*c*d^4)*b)*x^4 + ((35*c^4*d^3 + 30*c^2*d^3 + 3*d^3)*a + (35*c^4*d^3 + 30*c
^2*d^3 + 3*d^3)*b)*x^3 + 3*((7*c^5*d^2 + 10*c^3*d^2 + 3*c*d^2)*a + (7*c^5*d
^2 + 10*c^3*d^2 + 3*c*d^2)*b)*x^2 + ((a*d^4 + b*d^4)*x^4 + 4*(a*c*d^3 + b*c
*d^3)*x^3 + (6*a*c^2*d^2 + (6*c^2*d^2 + d^2)*b)*x^2 + (c^4 - 1)*a + (c^4 +
c^2)*b + 2*(2*a*c^3*d + (2*c^3*d + c*d)*b)*x)*(d^2*x^2 + 2*c*d*x + c^2 + 1)
^(3/2) + (3*(a*d^5 + b*d^5)*x^5 + 15*(a*c*d^4 + b*c*d^4)*x^4 + (3*(10*c^2*d
^3 + d^3)*a + 5*(6*c^2*d^3 + d^3)*b)*x^3 + 3*((10*c^3*d^2 + 3*c*d^2)*a + 5*
(2*c^3*d^2 + c*d^2)*b)*x^2 + 3*(c^5 + c^3)*a + (3*c^5 + 5*c^3 + 2*c)*b + (3
*(5*c^4*d + 3*c^2*d)*a + (15*c^4*d + 15*c^2*d + 2*d)*b)*x)*(d^2*x^2 + 2*c*d
*x + c^2 + 1) + (c^7 + 3*c^5 + 3*c^3 + c)*a + (c^7 + 3*c^5 + 3*c^3 + c)*b +
 ((7*c^6*d + 15*c^4*d + 9*c^2*d + d)*a + (7*c^6*d + 15*c^4*d + 9*c^2*d + d)
*b)*x + (b*d^7*x^7 + 7*b*c*d^6*x^6 + 3*(7*c^2*d^5 + d^5)*b*x^5 + 5*(7*c^3*d
^4 + 3*c*d^4)*b*x^4 + (35*c^4*d^3 + 30*c^2*d^3 + 3*d^3)*b*x^3 + 3*(7*c^5*d^
```

```
2 + 10*c^3*d^2 + 3*c*d^2)*b*x^2 + (7*c^6*d + 15*c^4*d + 9*c^2*d + d)*b*x +
(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x + (c^4 - 1)*b)*(
d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + 3*(b*d^5*x^5 + 5*b*c*d^4*x^4 + (10*c^2
*d^3 + d^3)*b*x^3 + (10*c^3*d^2 + 3*c*d^2)*b*x^2 + (5*c^4*d + 3*c^2*d)*b*x
+ (c^5 + c^3)*b)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + (c^7 + 3*c^5 + 3*c^3 + c)*
b + (3*b*d^6*x^6 + 18*b*c*d^5*x^5 + 3*(15*c^2*d^4 + 2*d^4)*b*x^4 + 12*(5*c^
3*d^3 + 2*c*d^3)*b*x^3 + (45*c^4*d^2 + 36*c^2*d^2 + 4*d^2)*b*x^2 + 2*(9*c^5
*d + 12*c^3*d + 4*c*d)*b*x + (3*c^6 + 6*c^4 + 4*c^2 + 1)*b)*sqrt(d^2*x^2 +
2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + (3*(
a*d^6 + b*d^6)*x^6 + 18*(a*c*d^5 + b*c*d^5)*x^5 + (3*(15*c^2*d^4 + 2*d^4)*a
 + (45*c^2*d^4 + 7*d^4)*b)*x^4 + 4*(3*(5*c^3*d^3 + 2*c*d^3)*a + (15*c^3*d^3
 + 7*c*d^3)*b)*x^3 + ((45*c^4*d^2 + 36*c^2*d^2 + 4*d^2)*a + (45*c^4*d^2 + 4
2*c^2*d^2 + 5*d^2)*b)*x^2 + (3*c^6 + 6*c^4 + 4*c^2 + 1)*a + (3*c^6 + 7*c^4
+ 5*c^2 + 1)*b + 2*((9*c^5*d + 12*c^3*d + 4*c*d)*a + (9*c^5*d + 14*c^3*d +
5*c*d)*b)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(a^2*b^2*d^7*x^6 + 6*a^2*b^
2*c*d^6*x^5 + 3*(5*c^2*d^5 + d^5)*a^2*b^2*x^4 + 4*(5*c^3*d^4 + 3*c*d^4)*a^2
*b^2*x^3 + 3*(5*c^4*d^3 + 6*c^2*d^3 + d^3)*a^2*b^2*x^2 + 6*(c^5*d^2 + 2*c^3
*d^2 + c*d^2)*a^2*b^2*x + (c^6*d + 3*c^4*d + 3*c^2*d + d)*a^2*b^2 + (b^4*d^
7*x^6 + 6*b^4*c*d^6*x^5 + 3*(5*c^2*d^5 + d^5)*b^4*x^4 + 4*(5*c^3*d^4 + 3*c*
d^4)*b^4*x^3 + 3*(5*c^4*d^3 + 6*c^2*d^3 + d^3)*b^4*x^2 + 6*(c^5*d^2 + 2*c^3
*d^2 + c*d^2)*b^4*x + (c^6*d + 3*c^4*d + 3*c^2*d + d)*b^4 + (b^4*d^4*x^3 +
3*b^4*c*d^3*x^2 + 3*b^4*c^2*d^2*x + b^4*c^3*d)*(d^2*x^2 + 2*c*d*x + c^2 + 1
)^(3/2) + 3*(b^4*d^5*x^4 + 4*b^4*c*d^4*x^3 + (6*c^2*d^3 + d^3)*b^4*x^2 + 2*
(2*c^3*d^2 + c*d^2)*b^4*x + (c^4*d + c^2*d)*b^4)*(d^2*x^2 + 2*c*d*x + c^2 +
 1) + 3*(b^4*d^6*x^5 + 5*b^4*c*d^5*x^4 + 2*(5*c^2*d^4 + d^4)*b^4*x^3 + 2*(5
*c^3*d^3 + 3*c*d^3)*b^4*x^2 + (5*c^4*d^2 + 6*c^2*d^2 + d^2)*b^4*x + (c^5*d
+ 2*c^3*d + c*d)*b^4)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt
(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 + (a^2*b^2*d^4*x^3 + 3*a^2*b^2*c*d^3*x^2 +
 3*a^2*b^2*c^2*d^2*x + a^2*b^2*c^3*d)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) +
 3*(a^2*b^2*d^5*x^4 + 4*a^2*b^2*c*d^4*x^3 + (6*c^2*d^3 + d^3)*a^2*b^2*x^2 +
 2*(2*c^3*d^2 + c*d^2)*a^2*b^2*x + (c^4*d + c^2*d)*a^2*b^2)*(d^2*x^2 + 2*c*
d*x + c^2 + 1) + 2*(a*b^3*d^7*x^6 + 6*a*b^3*c*d^6*x^5 + 3*(5*c^2*d^5 + d^5)
*a*b^3*x^4 + 4*(5*c^3*d^4 + 3*c*d^4)*a*b^3*x^3 + 3*(5*c^4*d^3 + 6*c^2*d^3 +
 d^3)*a*b^3*x^2 + 6*(c^5*d^2 + 2*c^3*d^2 + c*d^2)*a*b^3*x + (c^6*d + 3*c^4*
d + 3*c^2*d + d)*a*b^3 + (a*b^3*d^4*x^3 + 3*a*b^3*c*d^3*x^2 + 3*a*b^3*c^2*d
^2*x + a*b^3*c^3*d)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + 3*(a*b^3*d^5*x^4
+ 4*a*b^3*c*d^4*x^3 + (6*c^2*d^3 + d^3)*a*b^3*x^2 + 2*(2*c^3*d^2 + c*d^2)*a
*b^3*x + (c^4*d + c^2*d)*a*b^3)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + 3*(a*b^3*d^
6*x^5 + 5*a*b^3*c*d^5*x^4 + 2*(5*c^2*d^4 + d^4)*a*b^3*x^3 + 2*(5*c^3*d^3 +
3*c*d^3)*a*b^3*x^2 + (5*c^4*d^2 + 6*c^2*d^2 + d^2)*a*b^3*x + (c^5*d + 2*c^3
*d + c*d)*a*b^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*
x^2 + 2*c*d*x + c^2 + 1)) + 3*(a^2*b^2*d^6*x^5 + 5*a^2*b^2*c*d^5*x^4 + 2*(5
*c^2*d^4 + d^4)*a^2*b^2*x^3 + 2*(5*c^3*d^3 + 3*c*d^3)*a^2*b^2*x^2 + (5*c^4*
d^2 + 6*c^2*d^2 + d^2)*a^2*b^2*x + (c^5*d + 2*c^3*d + c*d)*a^2*b^2)*sqrt(d^
2*x^2 + 2*c*d*x + c^2 + 1)) + integrate(1/2*(d^8*x^8 + 8*c*d^7*x^7 + c^8 +
4*(7*c^2*d^6 + d^6)*x^6 + 4*c^6 + 8*(7*c^3*d^5 + 3*c*d^5)*x^5 + 2*(35*c^4*d
^4 + 30*c^2*d^4 + 3*d^4)*x^4 + 6*c^4 + 8*(7*c^5*d^3 + 10*c^3*d^3 + 3*c*d^3)
*x^3 + (d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x + c^4 + 3)*(d^2*x
^2 + 2*c*d*x + c^2 + 1)^2 + 4*(7*c^6*d^2 + 15*c^4*d^2 + 9*c^2*d^2 + d^2)*x^
2 + (4*d^5*x^5 + 20*c*d^4*x^4 + 4*c^5 + 4*(10*c^2*d^3 + d^3)*x^3 + 4*c^3 +
4*(10*c^3*d^2 + 3*c*d^2)*x^2 + (20*c^4*d + 12*c^2*d + 3*d)*x + 3*c)*(d^2*x^
2 + 2*c*d*x + c^2 + 1)^(3/2) + 3*(2*d^6*x^6 + 12*c*d^5*x^5 + 2*c^6 + 2*(15*
c^2*d^4 + 2*d^4)*x^4 + 4*c^4 + 8*(5*c^3*d^3 + 2*c*d^3)*x^3 + (30*c^4*d^2 +
24*c^2*d^2 + d^2)*x^2 + c^2 + 2*(6*c^5*d + 8*c^3*d + c*d)*x - 1)*(d^2*x^2 +
 2*c*d*x + c^2 + 1) + 4*c^2 + 8*(c^7*d + 3*c^5*d + 3*c^3*d + c*d)*x + (4*d^
7*x^7 + 28*c*d^6*x^6 + 4*c^7 + 12*(7*c^2*d^5 + d^5)*x^5 + 12*c^5 + 20*(7*c^
3*d^4 + 3*c*d^4)*x^4 + (140*c^4*d^3 + 120*c^2*d^3 + 9*d^3)*x^3 + 9*c^3 + 3*
(28*c^5*d^2 + 40*c^3*d^2 + 9*c*d^2)*x^2 + (28*c^6*d + 60*c^4*d + 27*c^2*d +
 d)*x + c)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1) + 1)/(a*b^2*d^8*x^8 + 8*a*b^2*
```

```
c*d^7*x^7 + 4*(7*c^2*d^6 + d^6)*a*b^2*x^6 + 8*(7*c^3*d^5 + 3*c*d^5)*a*b^2*x
^5 + 2*(35*c^4*d^4 + 30*c^2*d^4 + 3*d^4)*a*b^2*x^4 + 8*(7*c^5*d^3 + 10*c^3*
d^3 + 3*c*d^3)*a*b^2*x^3 + 4*(7*c^6*d^2 + 15*c^4*d^2 + 9*c^2*d^2 + d^2)*a*b
^2*x^2 + 8*(c^7*d + 3*c^5*d + 3*c^3*d + c*d)*a*b^2*x + (c^8 + 4*c^6 + 6*c^4
 + 4*c^2 + 1)*a*b^2 + (a*b^2*d^4*x^4 + 4*a*b^2*c*d^3*x^3 + 6*a*b^2*c^2*d^2*
x^2 + 4*a*b^2*c^3*d*x + a*b^2*c^4)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 4*(a*b
^2*d^5*x^5 + 5*a*b^2*c*d^4*x^4 + (10*c^2*d^3 + d^3)*a*b^2*x^3 + (10*c^3*d^2
 + 3*c*d^2)*a*b^2*x^2 + (5*c^4*d + 3*c^2*d)*a*b^2*x + (c^5 + c^3)*a*b^2)*(d
^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + 6*(a*b^2*d^6*x^6 + 6*a*b^2*c*d^5*x^5 +
(15*c^2*d^4 + 2*d^4)*a*b^2*x^4 + 4*(5*c^3*d^3 + 2*c*d^3)*a*b^2*x^3 + (15*c^
4*d^2 + 12*c^2*d^2 + d^2)*a*b^2*x^2 + 2*(3*c^5*d + 4*c^3*d + c*d)*a*b^2*x +
 (c^6 + 2*c^4 + c^2)*a*b^2)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + (b^3*d^8*x^8 +
8*b^3*c*d^7*x^7 + 4*(7*c^2*d^6 + d^6)*b^3*x^6 + 8*(7*c^3*d^5 + 3*c*d^5)*b^3
*x^5 + 2*(35*c^4*d^4 + 30*c^2*d^4 + 3*d^4)*b^3*x^4 + 8*(7*c^5*d^3 + 10*c^3*
d^3 + 3*c*d^3)*b^3*x^3 + 4*(7*c^6*d^2 + 15*c^4*d^2 + 9*c^2*d^2 + d^2)*b^3*x
^2 + 8*(c^7*d + 3*c^5*d + 3*c^3*d + c*d)*b^3*x + (c^8 + 4*c^6 + 6*c^4 + 4*c
^2 + 1)*b^3 + (b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*x^2 + 4*b^3*c^
3*d*x + b^3*c^4)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 4*(b^3*d^5*x^5 + 5*b^3*c
*d^4*x^4 + (10*c^2*d^3 + d^3)*b^3*x^3 + (10*c^3*d^2 + 3*c*d^2)*b^3*x^2 + (5
*c^4*d + 3*c^2*d)*b^3*x + (c^5 + c^3)*b^3)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3
/2) + 6*(b^3*d^6*x^6 + 6*b^3*c*d^5*x^5 + (15*c^2*d^4 + 2*d^4)*b^3*x^4 + 4*(
5*c^3*d^3 + 2*c*d^3)*b^3*x^3 + (15*c^4*d^2 + 12*c^2*d^2 + d^2)*b^3*x^2 + 2*
(3*c^5*d + 4*c^3*d + c*d)*b^3*x + (c^6 + 2*c^4 + c^2)*b^3)*(d^2*x^2 + 2*c*d
*x + c^2 + 1) + 4*(b^3*d^7*x^7 + 7*b^3*c*d^6*x^6 + 3*(7*c^2*d^5 + d^5)*b^3*
x^5 + 5*(7*c^3*d^4 + 3*c*d^4)*b^3*x^4 + (35*c^4*d^3 + 30*c^2*d^3 + 3*d^3)*b
^3*x^3 + 3*(7*c^5*d^2 + 10*c^3*d^2 + 3*c*d^2)*b^3*x^2 + (7*c^6*d + 15*c^4*d
 + 9*c^2*d + d)*b^3*x + (c^7 + 3*c^5 + 3*c^3 + c)*b^3)*sqrt(d^2*x^2 + 2*c*d
*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + 4*(a*b^2*
d^7*x^7 + 7*a*b^2*c*d^6*x^6 + 3*(7*c^2*d^5 + d^5)*a*b^2*x^5 + 5*(7*c^3*d^4
+ 3*c*d^4)*a*b^2*x^4 + (35*c^4*d^3 + 30*c^2*d^3 + 3*d^3)*a*b^2*x^3 + 3*(7*c
^5*d^2 + 10*c^3*d^2 + 3*c*d^2)*a*b^2*x^2 + (7*c^6*d + 15*c^4*d + 9*c^2*d +
d)*a*b^2*x + (c^7 + 3*c^5 + 3*c^3 + c)*a*b^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2
+ 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*asinh(c + d*x))^3,x)
```

```
[Out] int(1/(a + b*asinh(c + d*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\,\mathrm{asinh}\left(c+dx\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(d*x+c))**3,x)
```

```
[Out] Integral((a + b*asinh(c + d*x))**(-3), x)
```

3.173 $\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^3}\,dx$

Optimal. Leaf size=27

$$\frac{\text{Int}\left(\frac{1}{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^3},x\right)}{e}$$

[Out] Unintegrable(1/(d*x+c)/(a+b*arcsinh(d*x+c))^3,x)/e

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^3}\,dx$$

Verification is Not applicable to the result.

[In] Int[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^3),x]

[Out] Defer[Subst][Defer[Int][1/(x*(a + b*ArcSinh[x])^3), x], x, c + d*x]/(d*e)

Rubi steps

$$\begin{aligned}\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^3}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{ex\left(a+b\sinh^{-1}(x)\right)^3}\,dx,x,c+dx\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{1}{x\left(a+b\sinh^{-1}(x)\right)^3}\,dx,x,c+dx\right)}{de}\end{aligned}$$

Mathematica [A] time = 1.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^3}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^3),x]

[Out] Integrate[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^3), x]

fricas [A] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{a^3dex + a^3ce + \left(b^3dex + b^3ce\right)\text{arsinh}\,(dx+c)^3 + 3\left(ab^2dex + ab^2ce\right)\text{arsinh}\,(dx+c)^2 + 3\left(a^2bdex + a^2b\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")

[Out] integral(1/(a^3*d*e*x + a^3*c*e + (b^3*d*e*x + b^3*c*e)*arcsinh(d*x + c)^3 + 3*(a*b^2*d*e*x + a*b^2*c*e)*arcsinh(d*x + c)^2 + 3*(a^2*b*d*e*x + a^2*b*c *e)*arcsinh(d*x + c)), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dex + ce)(b \operatorname{arsinh}(dx + c) + a)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
[Out] integrate(1/((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^3), x)
```

maple [A] time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{1}{(dex + ce)(a + b \operatorname{arcsinh}(dx + c))^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^3,x)
[Out] int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
[Out] -1/2*(b*d^8*x^8 + 8*b*c*d^7*x^7 + (28*c^2*d^6 + 3*d^6)*b*x^6 + 2*(28*c^3*d^
5 + 9*c*d^5)*b*x^5 + (70*c^4*d^4 + 45*c^2*d^4 + 3*d^4)*b*x^4 + 4*(14*c^5*d^
3 + 15*c^3*d^3 + 3*c*d^3)*b*x^3 + (28*c^6*d^2 + 45*c^4*d^2 + 18*c^2*d^2 + d
^2)*b*x^2 + 2*(4*c^7*d + 9*c^5*d + 6*c^3*d + c*d)*b*x + (b*d^5*x^5 + 5*b*c*
d^4*x^4 - (2*a*d^3 - (10*c^2*d^3 + d^3)*b)*x^3 - (6*a*c*d^2 - (10*c^3*d^2 +
 3*c*d^2)*b)*x^2 - 2*(c^3 + c)*a + (c^5 + c^3)*b - (2*(3*c^2*d + d)*a - (5*
c^4*d + 3*c^2*d)*b)*x)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + (3*b*d^6*x^6 +
 18*b*c*d^5*x^5 - (4*a*d^4 - 5*(9*c^2*d^4 + d^4)*b)*x^4 - 4*(4*a*c*d^3 - 5*
(3*c^3*d^3 + c*d^3)*b)*x^3 - ((24*c^2*d^2 + 5*d^2)*a - (45*c^4*d^2 + 30*c^2
*d^2 + 2*d^2)*b)*x^2 - (4*c^4 + 5*c^2 + 1)*a + (3*c^6 + 5*c^4 + 2*c^2)*b -
2*((8*c^3*d + 5*c*d)*a - (9*c^5*d + 10*c^3*d + 2*c*d)*b)*x)*(d^2*x^2 + 2*c*
d*x + c^2 + 1) + (c^8 + 3*c^6 + 3*c^4 + c^2)*b - (2*(b*d^3*x^3 + 3*b*c*d^2*
x^2 + (3*c^2*d + d)*b*x + (c^3 + c)*b)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2)
+ (4*b*d^4*x^4 + 16*b*c*d^3*x^3 + (24*c^2*d^2 + 5*d^2)*b*x^2 + 2*(8*c^3*d +
 5*c*d)*b*x + (4*c^4 + 5*c^2 + 1)*b)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + (2*b*d
^5*x^5 + 10*b*c*d^4*x^4 + (20*c^2*d^3 + 3*d^3)*b*x^3 + (20*c^3*d^2 + 9*c*d^
2)*b*x^2 + (10*c^4*d + 9*c^2*d + d)*b*x + (2*c^5 + 3*c^3 + c)*b)*sqrt(d^2*x
^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) +
 (3*b*d^7*x^7 + 21*b*c*d^6*x^6 - (2*a*d^5 - 7*(9*c^2*d^5 + d^5)*b)*x^5 - 5*
(2*a*c*d^4 - 7*(3*c^3*d^4 + c*d^4)*b)*x^4 - ((20*c^2*d^3 + 3*d^3)*a - 5*(21
*c^4*d^3 + 14*c^2*d^3 + d^3)*b)*x^3 - ((20*c^3*d^2 + 9*c*d^2)*a - (63*c^5*d
^2 + 70*c^3*d^2 + 15*c*d^2)*b)*x^2 - (2*c^5 + 3*c^3 + c)*a + (3*c^7 + 7*c^5
 + 5*c^3 + c)*b - ((10*c^4*d + 9*c^2*d + d)*a - (21*c^6*d + 35*c^4*d + 15*c
^2*d + d)*b)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(a^2*b^2*d^9*e*x^8 + 8*a
^2*b^2*c*d^8*e*x^7 + (28*c^2*d^7*e + 3*d^7*e)*a^2*b^2*x^6 + 2*(28*c^3*d^6*e
 + 9*c*d^6*e)*a^2*b^2*x^5 + (70*c^4*d^5*e + 45*c^2*d^5*e + 3*d^5*e)*a^2*b^2
*x^4 + 4*(14*c^5*d^4*e + 15*c^3*d^4*e + 3*c*d^4*e)*a^2*b^2*x^3 + (28*c^6*d^
3*e + 45*c^4*d^3*e + 18*c^2*d^3*e + d^3*e)*a^2*b^2*x^2 + 2*(4*c^7*d^2*e + 9
*c^5*d^2*e + 6*c^3*d^2*e + c*d^2*e)*a^2*b^2*x + (c^8*d*e + 3*c^6*d*e + 3*c^
4*d*e + c^2*d*e)*a^2*b^2 + (b^4*d^9*e*x^8 + 8*b^4*c*d^8*e*x^7 + (28*c^2*d^7
*e + 3*d^7*e)*b^4*x^6 + 2*(28*c^3*d^6*e + 9*c*d^6*e)*b^4*x^5 + (70*c^4*d^5*
```

```
e + 45*c^2*d^5*e + 3*d^5*e)*b^4*x^4 + 4*(14*c^5*d^4*e + 15*c^3*d^4*e + 3*c*
d^4*e)*b^4*x^3 + (28*c^6*d^3*e + 45*c^4*d^3*e + 18*c^2*d^3*e + d^3*e)*b^4*x
^2 + 2*(4*c^7*d^2*e + 9*c^5*d^2*e + 6*c^3*d^2*e + c*d^2*e)*b^4*x + (c^8*d*e
 + 3*c^6*d*e + 3*c^4*d*e + c^2*d*e)*b^4 + (b^4*d^6*e*x^5 + 5*b^4*c*d^5*e*x^
4 + 10*b^4*c^2*d^4*e*x^3 + 10*b^4*c^3*d^3*e*x^2 + 5*b^4*c^4*d^2*e*x + b^4*c
^5*d*e)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + 3*(b^4*d^7*e*x^6 + 6*b^4*c*d^
6*e*x^5 + (15*c^2*d^5*e + d^5*e)*b^4*x^4 + 4*(5*c^3*d^4*e + c*d^4*e)*b^4*x^
3 + 3*(5*c^4*d^3*e + 2*c^2*d^3*e)*b^4*x^2 + 2*(3*c^5*d^2*e + 2*c^3*d^2*e)*b
^4*x + (c^6*d*e + c^4*d*e)*b^4)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + 3*(b^4*d^8*
e*x^7 + 7*b^4*c*d^7*e*x^6 + (21*c^2*d^6*e + 2*d^6*e)*b^4*x^5 + 5*(7*c^3*d^5
*e + 2*c*d^5*e)*b^4*x^4 + (35*c^4*d^4*e + 20*c^2*d^4*e + d^4*e)*b^4*x^3 + (
21*c^5*d^3*e + 20*c^3*d^3*e + 3*c*d^3*e)*b^4*x^2 + (7*c^6*d^2*e + 10*c^4*d^
2*e + 3*c^2*d^2*e)*b^4*x + (c^7*d*e + 2*c^5*d*e + c^3*d*e)*b^4)*sqrt(d^2*x^
2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2
+ (a^2*b^2*d^6*e*x^5 + 5*a^2*b^2*c*d^5*e*x^4 + 10*a^2*b^2*c^2*d^4*e*x^3 + 1
0*a^2*b^2*c^3*d^3*e*x^2 + 5*a^2*b^2*c^4*d^2*e*x + a^2*b^2*c^5*d*e)*(d^2*x^2
 + 2*c*d*x + c^2 + 1)^(3/2) + 3*(a^2*b^2*d^7*e*x^6 + 6*a^2*b^2*c*d^6*e*x^5
+ (15*c^2*d^5*e + d^5*e)*a^2*b^2*x^4 + 4*(5*c^3*d^4*e + c*d^4*e)*a^2*b^2*x^
3 + 3*(5*c^4*d^3*e + 2*c^2*d^3*e)*a^2*b^2*x^2 + 2*(3*c^5*d^2*e + 2*c^3*d^2*
e)*a^2*b^2*x + (c^6*d*e + c^4*d*e)*a^2*b^2)*(d^2*x^2 + 2*c*d*x + c^2 + 1) +
 2*(a*b^3*d^9*e*x^8 + 8*a*b^3*c*d^8*e*x^7 + (28*c^2*d^7*e + 3*d^7*e)*a*b^3*
x^6 + 2*(28*c^3*d^6*e + 9*c*d^6*e)*a*b^3*x^5 + (70*c^4*d^5*e + 45*c^2*d^5*e
 + 3*d^5*e)*a*b^3*x^4 + 4*(14*c^5*d^4*e + 15*c^3*d^4*e + 3*c*d^4*e)*a*b^3*x
^3 + (28*c^6*d^3*e + 45*c^4*d^3*e + 18*c^2*d^3*e + d^3*e)*a*b^3*x^2 + 2*(4*
c^7*d^2*e + 9*c^5*d^2*e + 6*c^3*d^2*e + c*d^2*e)*a*b^3*x + (c^8*d*e + 3*c^6
*d*e + 3*c^4*d*e + c^2*d*e)*a*b^3 + (a*b^3*d^6*e*x^5 + 5*a*b^3*c*d^5*e*x^4
+ 10*a*b^3*c^2*d^4*e*x^3 + 10*a*b^3*c^3*d^3*e*x^2 + 5*a*b^3*c^4*d^2*e*x + a
*b^3*c^5*d*e)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + 3*(a*b^3*d^7*e*x^6 + 6*
a*b^3*c*d^6*e*x^5 + (15*c^2*d^5*e + d^5*e)*a*b^3*x^4 + 4*(5*c^3*d^4*e + c*d
^4*e)*a*b^3*x^3 + 3*(5*c^4*d^3*e + 2*c^2*d^3*e)*a*b^3*x^2 + 2*(3*c^5*d^2*e
+ 2*c^3*d^2*e)*a*b^3*x + (c^6*d*e + c^4*d*e)*a*b^3)*(d^2*x^2 + 2*c*d*x + c^
2 + 1) + 3*(a*b^3*d^8*e*x^7 + 7*a*b^3*c*d^7*e*x^6 + (21*c^2*d^6*e + 2*d^6*e
)*a*b^3*x^5 + 5*(7*c^3*d^5*e + 2*c*d^5*e)*a*b^3*x^4 + (35*c^4*d^4*e + 20*c^
2*d^4*e + d^4*e)*a*b^3*x^3 + (21*c^5*d^3*e + 20*c^3*d^3*e + 3*c*d^3*e)*a*b^
3*x^2 + (7*c^6*d^2*e + 10*c^4*d^2*e + 3*c^2*d^2*e)*a*b^3*x + (c^7*d*e + 2*c
^5*d*e + c^3*d*e)*a*b^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + s
qrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + 3*(a^2*b^2*d^8*e*x^7 + 7*a^2*b^2*c*d^7*
e*x^6 + (21*c^2*d^6*e + 2*d^6*e)*a^2*b^2*x^5 + 5*(7*c^3*d^5*e + 2*c*d^5*e)*
a^2*b^2*x^4 + (35*c^4*d^4*e + 20*c^2*d^4*e + d^4*e)*a^2*b^2*x^3 + (21*c^5*d
^3*e + 20*c^3*d^3*e + 3*c*d^3*e)*a^2*b^2*x^2 + (7*c^6*d^2*e + 10*c^4*d^2*e
+ 3*c^2*d^2*e)*a^2*b^2*x + (c^7*d*e + 2*c^5*d*e + c^3*d*e)*a^2*b^2)*sqrt(d^
2*x^2 + 2*c*d*x + c^2 + 1)) + integrate(1/2*(4*(d^4*x^4 + 4*c*d^3*x^3 + c^4
 + 2*(3*c^2*d^2 + d^2)*x^2 + 2*c^2 + 4*(c^3*d + c*d)*x)*(d^2*x^2 + 2*c*d*x
+ c^2 + 1)^2 + (12*d^5*x^5 + 60*c*d^4*x^4 + 12*c^5 + 2*(60*c^2*d^3 + 11*d^3
)*x^3 + 22*c^3 + 6*(20*c^3*d^2 + 11*c*d^2)*x^2 + (60*c^4*d + 66*c^2*d + 7*d
)*x + 7*c)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + 2*(6*d^6*x^6 + 36*c*d^5*x^
5 + 6*c^6 + 10*(9*c^2*d^4 + d^4)*x^4 + 10*c^4 + 40*(3*c^3*d^3 + c*d^3)*x^3
+ 5*(18*c^4*d^2 + 12*c^2*d^2 + d^2)*x^2 + 5*c^2 + 2*(18*c^5*d + 20*c^3*d +
5*c*d)*x + 1)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + (4*d^7*x^7 + 28*c*d^6*x^6 + 4
*c^7 + 6*(14*c^2*d^5 + d^5)*x^5 + 6*c^5 + 10*(14*c^3*d^4 + 3*c*d^4)*x^4 + (
140*c^4*d^3 + 60*c^2*d^3 + 3*d^3)*x^3 + 3*c^3 + 3*(28*c^5*d^2 + 20*c^3*d^2
+ 3*c*d^2)*x^2 + (28*c^6*d + 30*c^4*d + 9*c^2*d + d)*x + c)*sqrt(d^2*x^2 +
2*c*d*x + c^2 + 1))/(a*b^2*d^11*e*x^11 + 11*a*b^2*c*d^10*e*x^10 + (55*c^2*d
^9*e + 4*d^9*e)*a*b^2*x^9 + 3*(55*c^3*d^8*e + 12*c*d^8*e)*a*b^2*x^8 + 6*(55
*c^4*d^7*e + 24*c^2*d^7*e + d^7*e)*a*b^2*x^7 + 42*(11*c^5*d^6*e + 8*c^3*d^6
*e + c*d^6*e)*a*b^2*x^6 + 2*(231*c^6*d^5*e + 252*c^4*d^5*e + 63*c^2*d^5*e +
 2*d^5*e)*a*b^2*x^5 + 2*(165*c^7*d^4*e + 252*c^5*d^4*e + 105*c^3*d^4*e + 10
*c*d^4*e)*a*b^2*x^4 + (165*c^8*d^3*e + 336*c^6*d^3*e + 210*c^4*d^3*e + 40*c
^2*d^3*e + d^3*e)*a*b^2*x^3 + (55*c^9*d^2*e + 144*c^7*d^2*e + 126*c^5*d^2*e
```

```
 + 40*c^3*d^2*e + 3*c*d^2*e)*a*b^2*x^2 + (11*c^10*d*e + 36*c^8*d*e + 42*c^6
*d*e + 20*c^4*d*e + 3*c^2*d*e)*a*b^2*x + (c^11*e + 4*c^9*e + 6*c^7*e + 4*c^
5*e + c^3*e)*a*b^2 + (a*b^2*d^7*e*x^7 + 7*a*b^2*c*d^6*e*x^6 + 21*a*b^2*c^2*
d^5*e*x^5 + 35*a*b^2*c^3*d^4*e*x^4 + 35*a*b^2*c^4*d^3*e*x^3 + 21*a*b^2*c^5*
d^2*e*x^2 + 7*a*b^2*c^6*d*e*x + a*b^2*c^7*e)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^
2 + 4*(a*b^2*d^8*e*x^8 + 8*a*b^2*c*d^7*e*x^7 + (28*c^2*d^6*e + d^6*e)*a*b^2
*x^6 + 2*(28*c^3*d^5*e + 3*c*d^5*e)*a*b^2*x^5 + 5*(14*c^4*d^4*e + 3*c^2*d^4
*e)*a*b^2*x^4 + 4*(14*c^5*d^3*e + 5*c^3*d^3*e)*a*b^2*x^3 + (28*c^6*d^2*e +
15*c^4*d^2*e)*a*b^2*x^2 + 2*(4*c^7*d*e + 3*c^5*d*e)*a*b^2*x + (c^8*e + c^6*
e)*a*b^2)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + 6*(a*b^2*d^9*e*x^9 + 9*a*b^
2*c*d^8*e*x^8 + 2*(18*c^2*d^7*e + d^7*e)*a*b^2*x^7 + 14*(6*c^3*d^6*e + c*d^
6*e)*a*b^2*x^6 + (126*c^4*d^5*e + 42*c^2*d^5*e + d^5*e)*a*b^2*x^5 + (126*c^
5*d^4*e + 70*c^3*d^4*e + 5*c*d^4*e)*a*b^2*x^4 + 2*(42*c^6*d^3*e + 35*c^4*d^
3*e + 5*c^2*d^3*e)*a*b^2*x^3 + 2*(18*c^7*d^2*e + 21*c^5*d^2*e + 5*c^3*d^2*e
)*a*b^2*x^2 + (9*c^8*d*e + 14*c^6*d*e + 5*c^4*d*e)*a*b^2*x + (c^9*e + 2*c^7
*e + c^5*e)*a*b^2)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + (b^3*d^11*e*x^11 + 11*b^
3*c*d^10*e*x^10 + (55*c^2*d^9*e + 4*d^9*e)*b^3*x^9 + 3*(55*c^3*d^8*e + 12*c
*d^8*e)*b^3*x^8 + 6*(55*c^4*d^7*e + 24*c^2*d^7*e + d^7*e)*b^3*x^7 + 42*(11*
c^5*d^6*e + 8*c^3*d^6*e + c*d^6*e)*b^3*x^6 + 2*(231*c^6*d^5*e + 252*c^4*d^5
*e + 63*c^2*d^5*e + 2*d^5*e)*b^3*x^5 + 2*(165*c^7*d^4*e + 252*c^5*d^4*e + 1
05*c^3*d^4*e + 10*c*d^4*e)*b^3*x^4 + (165*c^8*d^3*e + 336*c^6*d^3*e + 210*c
^4*d^3*e + 40*c^2*d^3*e + d^3*e)*b^3*x^3 + (55*c^9*d^2*e + 144*c^7*d^2*e +
126*c^5*d^2*e + 40*c^3*d^2*e + 3*c*d^2*e)*b^3*x^2 + (11*c^10*d*e + 36*c^8*d
*e + 42*c^6*d*e + 20*c^4*d*e + 3*c^2*d*e)*b^3*x + (c^11*e + 4*c^9*e + 6*c^7
*e + 4*c^5*e + c^3*e)*b^3 + (b^3*d^7*e*x^7 + 7*b^3*c*d^6*e*x^6 + 21*b^3*c^2
*d^5*e*x^5 + 35*b^3*c^3*d^4*e*x^4 + 35*b^3*c^4*d^3*e*x^3 + 21*b^3*c^5*d^2*e
*x^2 + 7*b^3*c^6*d*e*x + b^3*c^7*e)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 4*(b^
3*d^8*e*x^8 + 8*b^3*c*d^7*e*x^7 + (28*c^2*d^6*e + d^6*e)*b^3*x^6 + 2*(28*c^
3*d^5*e + 3*c*d^5*e)*b^3*x^5 + 5*(14*c^4*d^4*e + 3*c^2*d^4*e)*b^3*x^4 + 4*(
14*c^5*d^3*e + 5*c^3*d^3*e)*b^3*x^3 + (28*c^6*d^2*e + 15*c^4*d^2*e)*b^3*x^2
 + 2*(4*c^7*d*e + 3*c^5*d*e)*b^3*x + (c^8*e + c^6*e)*b^3)*(d^2*x^2 + 2*c*d*
x + c^2 + 1)^(3/2) + 6*(b^3*d^9*e*x^9 + 9*b^3*c*d^8*e*x^8 + 2*(18*c^2*d^7*e
 + d^7*e)*b^3*x^7 + 14*(6*c^3*d^6*e + c*d^6*e)*b^3*x^6 + (126*c^4*d^5*e + 4
2*c^2*d^5*e + d^5*e)*b^3*x^5 + (126*c^5*d^4*e + 70*c^3*d^4*e + 5*c*d^4*e)*b
^3*x^4 + 2*(42*c^6*d^3*e + 35*c^4*d^3*e + 5*c^2*d^3*e)*b^3*x^3 + 2*(18*c^7*
d^2*e + 21*c^5*d^2*e + 5*c^3*d^2*e)*b^3*x^2 + (9*c^8*d*e + 14*c^6*d*e + 5*c
^4*d*e)*b^3*x + (c^9*e + 2*c^7*e + c^5*e)*b^3)*(d^2*x^2 + 2*c*d*x + c^2 + 1
) + 4*(b^3*d^10*e*x^10 + 10*b^3*c*d^9*e*x^9 + 3*(15*c^2*d^8*e + d^8*e)*b^3*
x^8 + 24*(5*c^3*d^7*e + c*d^7*e)*b^3*x^7 + 3*(70*c^4*d^6*e + 28*c^2*d^6*e +
 d^6*e)*b^3*x^6 + 6*(42*c^5*d^5*e + 28*c^3*d^5*e + 3*c*d^5*e)*b^3*x^5 + (21
0*c^6*d^4*e + 210*c^4*d^4*e + 45*c^2*d^4*e + d^4*e)*b^3*x^4 + 4*(30*c^7*d^3
*e + 42*c^5*d^3*e + 15*c^3*d^3*e + c*d^3*e)*b^3*x^3 + 3*(15*c^8*d^2*e + 28*
c^6*d^2*e + 15*c^4*d^2*e + 2*c^2*d^2*e)*b^3*x^2 + 2*(5*c^9*d*e + 12*c^7*d*e
 + 9*c^5*d*e + 2*c^3*d*e)*b^3*x + (c^10*e + 3*c^8*e + 3*c^6*e + c^4*e)*b^3)
*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x +
c^2 + 1)) + 4*(a*b^2*d^10*e*x^10 + 10*a*b^2*c*d^9*e*x^9 + 3*(15*c^2*d^8*e +
 d^8*e)*a*b^2*x^8 + 24*(5*c^3*d^7*e + c*d^7*e)*a*b^2*x^7 + 3*(70*c^4*d^6*e
+ 28*c^2*d^6*e + d^6*e)*a*b^2*x^6 + 6*(42*c^5*d^5*e + 28*c^3*d^5*e + 3*c*d^
5*e)*a*b^2*x^5 + (210*c^6*d^4*e + 210*c^4*d^4*e + 45*c^2*d^4*e + d^4*e)*a*b
^2*x^4 + 4*(30*c^7*d^3*e + 42*c^5*d^3*e + 15*c^3*d^3*e + c*d^3*e)*a*b^2*x^3
 + 3*(15*c^8*d^2*e + 28*c^6*d^2*e + 15*c^4*d^2*e + 2*c^2*d^2*e)*a*b^2*x^2 +
 2*(5*c^9*d*e + 12*c^7*d*e + 9*c^5*d*e + 2*c^3*d*e)*a*b^2*x + (c^10*e + 3*c
^8*e + 3*c^6*e + c^4*e)*a*b^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{(c\,e + d\,e\,x)\,\left(a + b\,\mathrm{asinh}\left(c + d\,x\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*e + d*e*x)*(a + b*asinh(c + d*x))^3),x)
```

```
[Out] int(1/((c*e + d*e*x)*(a + b*asinh(c + d*x))^3), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^3c+a^3dx+3a^2bc\operatorname{asinh}(c+dx)+3a^2bdx\operatorname{asinh}(c+dx)+3ab^2c\operatorname{asinh}^2(c+dx)+3ab^2dx\operatorname{asinh}^2(c+dx)+b^3c\operatorname{asinh}^3(c+dx)+b^3dx\operatorname{asinh}^3(c+dx)}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*asinh(d*x+c))**3,x)
```

```
[Out] Integral(1/(a**3*c + a**3*d*x + 3*a**2*b*c*asinh(c + d*x) + 3*a**2*b*d*x*as
inh(c + d*x) + 3*a*b**2*c*asinh(c + d*x)**2 + 3*a*b**2*d*x*asinh(c + d*x)**
2 + b**3*c*asinh(c + d*x)**3 + b**3*d*x*asinh(c + d*x)**3), x)/e
```

3.174 $\int \frac{(ce+dex)^4}{\left(a+b\sinh^{-1}(c+dx)\right)^4}\,dx$

Optimal. Leaf size=410

$$-\frac{e^4\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{48b^4d}+\frac{27e^4\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{32b^4d}-\frac{125e^4\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5\left(a+b\sinh^{-1}(c+\right.}{b}\right.}{96b^4d}$$

```
[Out] -2/3*e^4*(d*x+c)^3/b^2/d/(a+b*arcsinh(d*x+c))^2-5/6*e^4*(d*x+c)^5/b^2/d/(a+
b*arcsinh(d*x+c))^2+1/48*e^4*cosh(a/b)*Shi((a+b*arcsinh(d*x+c))/b)/b^4/d-27
/32*e^4*cosh(3*a/b)*Shi(3*(a+b*arcsinh(d*x+c))/b)/b^4/d+125/96*e^4*cosh(5*a
/b)*Shi(5*(a+b*arcsinh(d*x+c))/b)/b^4/d-1/48*e^4*Chi((a+b*arcsinh(d*x+c))/b
)*sinh(a/b)/b^4/d+27/32*e^4*Chi(3*(a+b*arcsinh(d*x+c))/b)*sinh(3*a/b)/b^4/d
-125/96*e^4*Chi(5*(a+b*arcsinh(d*x+c))/b)*sinh(5*a/b)/b^4/d-1/3*e^4*(d*x+c)
^4*(1+(d*x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))^3-2*e^4*(d*x+c)^2*(1+(d*x+c
)^2)^(1/2)/b^3/d/(a+b*arcsinh(d*x+c))-25/6*e^4*(d*x+c)^4*(1+(d*x+c)^2)^(1/2
)/b^3/d/(a+b*arcsinh(d*x+c))
```

Rubi [A] time = 0.88, antiderivative size = 406, normalized size of antiderivative = 0.99, number of steps used = 24, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {5865, 12, 5667, 5774, 5665, 3303, 3298, 3301}

$$-\frac{e^4\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)}{48b^4d}+\frac{27e^4\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(c+dx)\right)}{32b^4d}-\frac{125e^4\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5a}{b}\right.}{96b^4d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4/(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] -(e^4*(c + d*x)^4*Sqrt[1 + (c + d*x)^2])/(3*b*d*(a + b*ArcSinh[c + d*x])^3)
 - (2*e^4*(c + d*x)^3)/(3*b^2*d*(a + b*ArcSinh[c + d*x])^2) - (5*e^4*(c + d
*x)^5)/(6*b^2*d*(a + b*ArcSinh[c + d*x])^2) - (2*e^4*(c + d*x)^2*Sqrt[1 + (
c + d*x)^2])/(b^3*d*(a + b*ArcSinh[c + d*x])) - (25*e^4*(c + d*x)^4*Sqrt[1
+ (c + d*x)^2])/(6*b^3*d*(a + b*ArcSinh[c + d*x])) - (e^4*CoshIntegral[a/b
+ ArcSinh[c + d*x]]*Sinh[a/b])/(48*b^4*d) + (27*e^4*CoshIntegral[(3*a)/b +
3*ArcSinh[c + d*x]]*Sinh[(3*a)/b])/(32*b^4*d) - (125*e^4*CoshIntegral[(5*a)
/b + 5*ArcSinh[c + d*x]]*Sinh[(5*a)/b])/(96*b^4*d) + (e^4*Cosh[a/b]*SinhInt
egral[a/b + ArcSinh[c + d*x]])/(48*b^4*d) - (27*e^4*Cosh[(3*a)/b]*SinhInteg
ral[(3*a)/b + 3*ArcSinh[c + d*x]])/(32*b^4*d) + (125*e^4*Cosh[(5*a)/b]*Sinh
Integral[(5*a)/b + 5*ArcSinh[c + d*x]])/(96*b^4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^4}{\left(a+b\sinh^{-1}(c+dx)\right)^4}\,dx = \frac{\text{Subst}\left(\int \frac{e^4x^4}{\left(a+b\sinh^{-1}(x)\right)^4}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^4\,\text{Subst}\left(\int \frac{x^4}{\left(a+b\sinh^{-1}(x)\right)^4}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} + \frac{\left(4e^4\right)\text{Subst}\left(\int \frac{x^3}{\sqrt{1+x^2}\left(a+b\sinh^{-1}(x)\right)^3}\,dx, x, c+ \right.}{3bd}$$

$$= -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{2e^4(c+dx)^3}{3b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{5e^4(c}{6b^2d\left(a+b\,\text{si}\right.}$$

$$= -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{2e^4(c+dx)^3}{3b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{5e^4(c}{6b^2d\left(a+b\,\text{si}\right.}$$

$$= -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{2e^4(c+dx)^3}{3b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{5e^4(c}{6b^2d\left(a+b\,\text{si}\right.}$$

$$= -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{2e^4(c+dx)^3}{3b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{5e^4(c}{6b^2d\left(a+b\,\text{si}\right.}$$

$$= -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{2e^4(c+dx)^3}{3b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{5e^4(c}{6b^2d\left(a+b\,\text{si}\right.}$$

Mathematica [A] time = 1.90, size = 410, normalized size = 1.00

$$-\frac{e^4\left(\frac{32b^3\sqrt{(c+dx)^2+1}\,(c+dx)^4}{\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{16b^2\left(-5(c+dx)^5-4(c+dx)^3\right)}{\left(a+b\sinh^{-1}(c+dx)\right)^2} + 384\left(\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right) - \cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^4/(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] -1/96*(e^4*((32*b^3*(c + d*x)^4*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh[c + d
*x])^3 - (16*b^2*(-4*(c + d*x)^3 - 5*(c + d*x)^5))/(a + b*ArcSinh[c + d*x])
^2 + (16*b*Sqrt[1 + (c + d*x)^2]*(12*(c + d*x)^2 + 25*(c + d*x)^4))/(a + b*
ArcSinh[c + d*x]) + 384*(CoshIntegral[a/b + ArcSinh[c + d*x]]*Sinh[a/b] - C
osh[a/b]*SinhIntegral[a/b + ArcSinh[c + d*x]]) + 544*(-3*CoshIntegral[a/b +
 ArcSinh[c + d*x]]*Sinh[a/b] + CoshIntegral[3*(a/b + ArcSinh[c + d*x])]*Sin
h[(3*a)/b] + 3*Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c + d*x]] - Cosh[(3*a)/
b]*SinhIntegral[3*(a/b + ArcSinh[c + d*x])]) + 125*(10*CoshIntegral[a/b + A
rcSinh[c + d*x]]*Sinh[a/b] - 5*CoshIntegral[3*(a/b + ArcSinh[c + d*x])]*Sin
h[(3*a)/b] + CoshIntegral[5*(a/b + ArcSinh[c + d*x])]*Sinh[(5*a)/b] - 10*Co
sh[a/b]*SinhIntegral[a/b + ArcSinh[c + d*x]] + 5*Cosh[(3*a)/b]*SinhIntegral
[3*(a/b + ArcSinh[c + d*x])] - Cosh[(5*a)/b]*SinhIntegral[5*(a/b + ArcSinh[
c + d*x])])))/(b^4*d)
```

fricas [F] time = 0.99, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{d^4e^4x^4 + 4\,cd^3e^4x^3 + 6\,c^2d^2e^4x^2 + 4\,c^3de^4x + c^4e^4}{b^4\operatorname{arsinh}\left(dx+c\right)^4 + 4\,ab^3\operatorname{arsinh}\left(dx+c\right)^3 + 6\,a^2b^2\operatorname{arsinh}\left(dx+c\right)^2 + 4\,a^3b\operatorname{arsinh}\left(dx+c\right) + a^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral((d^4*e^4*x^4 + 4*c*d^3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^3*d*e^4*x
 + c^4*e^4)/(b^4*arcsinh(d*x + c)^4 + 4*a*b^3*arcsinh(d*x + c)^3 + 6*a^2*b^
2*arcsinh(d*x + c)^2 + 4*a^3*b*arcsinh(d*x + c) + a^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dex+ce\right)^4}{\left(b\operatorname{arsinh}\left(dx+c\right)+a\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^4/(b*arcsinh(d*x + c) + a)^4, x)
```

maple [B] time = 0.43, size = 1244, normalized size = 3.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^4,x)
```

```
[Out] 1/d*(1/192*(16*(d*x+c)^5-16*(d*x+c)^4*(1+(d*x+c)^2)^(1/2)+20*(d*x+c)^3-12*(
d*x+c)^2*(1+(d*x+c)^2)^(1/2)+5*d*x+5*c-(1+(d*x+c)^2)^(1/2))*e^4*(25*b^2*arc
sinh(d*x+c)^2+50*a*b*arcsinh(d*x+c)-5*arcsinh(d*x+c)*b^2+25*a^2-5*a*b+2*b^2
)/b^3/(b^3*arcsinh(d*x+c)^3+3*a*b^2*arcsinh(d*x+c)^2+3*a^2*b*arcsinh(d*x+c)
+a^3)+125/192*e^4/b^4*exp(5*a/b)*Ei(1,5*arcsinh(d*x+c)+5*a/b)-1/64*(-4*(d*x
+c)^2*(1+(d*x+c)^2)^(1/2)+4*(d*x+c)^3-(1+(d*x+c)^2)^(1/2)+3*d*x+3*c)*e^4*(9
*b^2*arcsinh(d*x+c)^2+18*a*b*arcsinh(d*x+c)-3*arcsinh(d*x+c)*b^2+9*a^2-3*a*
b+2*b^2)/b^3/(b^3*arcsinh(d*x+c)^3+3*a*b^2*arcsinh(d*x+c)^2+3*a^2*b*arcsinh
(d*x+c)+a^3)-27/64*e^4/b^4*exp(3*a/b)*Ei(1,3*arcsinh(d*x+c)+3*a/b)+1/96*(-(
1+(d*x+c)^2)^(1/2)+d*x+c)*e^4*(b^2*arcsinh(d*x+c)^2+2*a*b*arcsinh(d*x+c)-ar
csinh(d*x+c)*b^2+a^2-a*b+2*b^2)/b^3/(b^3*arcsinh(d*x+c)^3+3*a*b^2*arcsinh(d
*x+c)^2+3*a^2*b*arcsinh(d*x+c)+a^3)+1/96*e^4/b^4*exp(a/b)*Ei(1,arcsinh(d*x+
c)+a/b)-1/48/b*e^4*(d*x+c+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^3-1/96/
b^2*e^4*(d*x+c+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^2-1/96/b^3*e^4*(d*
x+c+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))-1/96/b^4*e^4*exp(-a/b)*Ei(1,-
arcsinh(d*x+c)-a/b)+1/32/b*e^4*(4*(d*x+c)^3+3*d*x+3*c+4*(d*x+c)^2*(1+(d*x+c
)^2)^(1/2)+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^3+3/64/b^2*e^4*(4*(d*x
+c)^3+3*d*x+3*c+4*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+(1+(d*x+c)^2)^(1/2))/(a+b*a
rcsinh(d*x+c))^2+9/64/b^3*e^4*(4*(d*x+c)^3+3*d*x+3*c+4*(d*x+c)^2*(1+(d*x+c)
^2)^(1/2)+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))+27/64/b^4*e^4*exp(-3*a/
b)*Ei(1,-3*arcsinh(d*x+c)-3*a/b)-1/96/b*e^4*(16*(d*x+c)^5+20*(d*x+c)^3+16*(
d*x+c)^4*(1+(d*x+c)^2)^(1/2)+5*d*x+5*c+12*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+(1+
(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^3-5/192/b^2*e^4*(16*(d*x+c)^5+20*(d*
x+c)^3+16*(d*x+c)^4*(1+(d*x+c)^2)^(1/2)+5*d*x+5*c+12*(d*x+c)^2*(1+(d*x+c)^2
)^(1/2)+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^2-25/192/b^3*e^4*(16*(d*x
+c)^5+20*(d*x+c)^3+16*(d*x+c)^4*(1+(d*x+c)^2)^(1/2)+5*d*x+5*c+12*(d*x+c)^2*
(1+(d*x+c)^2)^(1/2)+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))-125/192/b^4*e
^4*exp(-5*a/b)*Ei(1,-5*arcsinh(d*x+c)-5*a/b))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,e+d\,e\,x)^4}{(a+b\,\mathrm{asinh}\,(c+d\,x))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^4/(a + b*asinh(c + d*x))^4,x)
```

```
[Out] int((c*e + d*e*x)^4/(a + b*asinh(c + d*x))^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^4\left(\int \frac{c^4}{a^4+4a^3b\,\mathrm{asinh}\,(c+dx)+6a^2b^2\,\mathrm{asinh}^2\,(c+dx)+4ab^3\,\mathrm{asinh}^3\,(c+dx)+b^4\,\mathrm{asinh}^4\,(c+dx)}\,dx+\int \frac{}{a^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4/(a+b*asinh(d*x+c))**4,x)
```

```
[Out] e**4*(Integral(c**4/(a**4 + 4*a**3*b*asinh(c + d*x) + 6*a**2*b**2*asinh(c +
 d*x)**2 + 4*a*b**3*asinh(c + d*x)**3 + b**4*asinh(c + d*x)**4), x) + Integ
ral(d**4*x**4/(a**4 + 4*a**3*b*asinh(c + d*x) + 6*a**2*b**2*asinh(c + d*x)*
*2 + 4*a*b**3*asinh(c + d*x)**3 + b**4*asinh(c + d*x)**4), x) + Integral(4*
c*d**3*x**3/(a**4 + 4*a**3*b*asinh(c + d*x) + 6*a**2*b**2*asinh(c + d*x)**2
 + 4*a*b**3*asinh(c + d*x)**3 + b**4*asinh(c + d*x)**4), x) + Integral(6*c*
*2*d**2*x**2/(a**4 + 4*a**3*b*asinh(c + d*x) + 6*a**2*b**2*asinh(c + d*x)**
2 + 4*a*b**3*asinh(c + d*x)**3 + b**4*asinh(c + d*x)**4), x) + Integral(4*c
**3*d*x/(a**4 + 4*a**3*b*asinh(c + d*x) + 6*a**2*b**2*asinh(c + d*x)**2 + 4
*a*b**3*asinh(c + d*x)**3 + b**4*asinh(c + d*x)**4), x))
```

3.175 $\int \frac{(ce+dex)^3}{\left(a+b\sinh^{-1}(c+dx)\right)^4}\,dx$

Optimal. Leaf size=340

$$-\frac{e^3\cosh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{3b^4d}+\frac{4e^3\cosh\left(\frac{4a}{b}\right)\mathrm{Chi}\left(\frac{4\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{3b^4d}+\frac{e^3\sinh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{3b^4d}$$

```
[Out] -1/2*e^3*(d*x+c)^2/b^2/d/(a+b*arcsinh(d*x+c))^2-2/3*e^3*(d*x+c)^4/b^2/d/(a+
b*arcsinh(d*x+c))^2-1/3*e^3*Chi(2*(a+b*arcsinh(d*x+c))/b)*cosh(2*a/b)/b^4/d
+4/3*e^3*Chi(4*(a+b*arcsinh(d*x+c))/b)*cosh(4*a/b)/b^4/d+1/3*e^3*Shi(2*(a+b
*arcsinh(d*x+c))/b)*sinh(2*a/b)/b^4/d-4/3*e^3*Shi(4*(a+b*arcsinh(d*x+c))/b)
*sinh(4*a/b)/b^4/d-1/3*e^3*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d
*x+c))^3-e^3*(d*x+c)*(1+(d*x+c)^2)^(1/2)/b^3/d/(a+b*arcsinh(d*x+c))-8/3*e^3
*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)/b^3/d/(a+b*arcsinh(d*x+c))
```

Rubi [A] time = 0.70, antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {5865, 12, 5667, 5774, 5665, 3303, 3298, 3301}

$$-\frac{e^3\cosh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(c+dx)\right)}{3b^4d}+\frac{4e^3\cosh\left(\frac{4a}{b}\right)\mathrm{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(c+dx)\right)}{3b^4d}+\frac{e^3\sinh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(\frac{2a}{b}+2\ldots\right)}{3b^4d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3/(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] -(e^3*(c + d*x)^3*Sqrt[1 + (c + d*x)^2])/(3*b*d*(a + b*ArcSinh[c + d*x])^3)
 - (e^3*(c + d*x)^2)/(2*b^2*d*(a + b*ArcSinh[c + d*x])^2) - (2*e^3*(c + d*x
)^4)/(3*b^2*d*(a + b*ArcSinh[c + d*x])^2) - (e^3*(c + d*x)*Sqrt[1 + (c + d*
x)^2])/(b^3*d*(a + b*ArcSinh[c + d*x])) - (8*e^3*(c + d*x)^3*Sqrt[1 + (c +
d*x)^2])/(3*b^3*d*(a + b*ArcSinh[c + d*x])) - (e^3*Cosh[(2*a)/b]*CoshIntegr
al[(2*a)/b + 2*ArcSinh[c + d*x]])/(3*b^4*d) + (4*e^3*Cosh[(4*a)/b]*CoshInte
gral[(4*a)/b + 4*ArcSinh[c + d*x]])/(3*b^4*d) + (e^3*Sinh[(2*a)/b]*SinhInte
gral[(2*a)/b + 2*ArcSinh[c + d*x]])/(3*b^4*d) - (4*e^3*Sinh[(4*a)/b]*SinhIn
tegral[(4*a)/b + 4*ArcSinh[c + d*x]])/(3*b^4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^3}{\left(a+b\sinh^{-1}(c+dx)\right)^4}\,dx = \frac{\text{Subst}\left(\int \frac{e^3x^3}{\left(a+b\sinh^{-1}(x)\right)^4}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^3\,\text{Subst}\left(\int \frac{x^3}{\left(a+b\sinh^{-1}(x)\right)^4}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} + \frac{e^3\,\text{Subst}\left(\int \frac{x^2}{\sqrt{1+x^2}\left(a+b\sinh^{-1}(x)\right)^3}\,dx, x, c+dx\right)}{bd} +$$

$$= -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{e^3(c+dx)^2}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{2e^3(c+}{3b^2d\left(a+b\sinh\right.}$$

$$= -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{e^3(c+dx)^2}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{2e^3(c+}{3b^2d\left(a+b\sinh\right.}$$

$$= -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{e^3(c+dx)^2}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{2e^3(c+}{3b^2d\left(a+b\sinh\right.}$$

$$= -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{e^3(c+dx)^2}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{2e^3(c+}{3b^2d\left(a+b\sinh\right.}$$

$$= -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{e^3(c+dx)^2}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{2e^3(c+}{3b^2d\left(a+b\sinh\right.}$$

Mathematica [A] time = 1.11, size = 318, normalized size = 0.94

$$e^3\left(-\frac{2b^3\sqrt{(c+dx)^2+1}\,(c+dx)^3}{\left(a+b\sinh^{-1}(c+dx)\right)^3} + \frac{b^2\left(-4(c+dx)^4-3(c+dx)^2\right)}{\left(a+b\sinh^{-1}(c+dx)\right)^2} + 30\left(\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right) - \sinh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}\right.\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^3/(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] (e^3*((-2*b^3*(c + d*x)^3*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh[c + d*x])^3
 + (b^2*(-3*(c + d*x)^2 - 4*(c + d*x)^4))/(a + b*ArcSinh[c + d*x])^2 - (2*b
*Sqrt[1 + (c + d*x)^2]*(3*(c + d*x) + 8*(c + d*x)^3))/(a + b*ArcSinh[c + d*
x]) + 6*Log[a + b*ArcSinh[c + d*x]] + 30*(Cosh[(2*a)/b]*CoshIntegral[2*(a/b
 + ArcSinh[c + d*x])] - Log[a + b*ArcSinh[c + d*x]] - Sinh[(2*a)/b]*SinhInt
egral[2*(a/b + ArcSinh[c + d*x])]) + 8*(-4*Cosh[(2*a)/b]*CoshIntegral[2*(a/
b + ArcSinh[c + d*x])] + Cosh[(4*a)/b]*CoshIntegral[4*(a/b + ArcSinh[c + d*
x])] + 3*Log[a + b*ArcSinh[c + d*x]] + 4*Sinh[(2*a)/b]*SinhIntegral[2*(a/b
+ ArcSinh[c + d*x])] - Sinh[(4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[c + d*x]
)])))/(6*b^4*d)
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{d^3e^3x^3 + 3\,cd^2e^3x^2 + 3\,c^2de^3x + c^3e^3}{b^4\,\text{arsinh}\,(dx+c)^4 + 4\,ab^3\,\text{arsinh}\,(dx+c)^3 + 6\,a^2b^2\,\text{arsinh}\,(dx+c)^2 + 4\,a^3b\,\text{arsinh}\,(dx+c) + a^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral((d^3*e^3*x^3 + 3*c*d^2*e^3*x^2 + 3*c^2*d*e^3*x + c^3*e^3)/(b^4*arc
sinh(d*x + c)^4 + 4*a*b^3*arcsinh(d*x + c)^3 + 6*a^2*b^2*arcsinh(d*x + c)^2
 + 4*a^3*b*arcsinh(d*x + c) + a^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex+ce)^3}{(b\operatorname{arsinh}(dx+c)+a)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^3/(b*arcsinh(d*x + c) + a)^4, x)
```

maple [B] time = 0.35, size = 800, normalized size = 2.35

$$\frac{\left(8(dx+c)^4-8(dx+c)^3\sqrt{1+(dx+c)^2}+8(dx+c)^2-4(dx+c)\sqrt{1+(dx+c)^2}+1\right)e^3\left(8b^2\operatorname{arcsinh}(dx+c)^2+16ab\operatorname{arcsinh}(dx+c)-2\operatorname{arcsinh}(dx+c)b^2+8a^2-2ab+\right.}{48b^3\left(b^3\operatorname{arcsinh}(dx+c)^3+3a\,b^2\operatorname{arcsinh}(dx+c)^2+3a^2b\operatorname{arcsinh}(dx+c)+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^4,x)
```

```
[Out] 1/d*(1/48*(8*(d*x+c)^4-8*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)+8*(d*x+c)^2-4*(d*x+c
)*(1+(d*x+c)^2)^(1/2)+1)*e^3*(8*b^2*arcsinh(d*x+c)^2+16*a*b*arcsinh(d*x+c)-
2*arcsinh(d*x+c)*b^2+8*a^2-2*a*b+b^2)/b^3/(b^3*arcsinh(d*x+c)^3+3*a*b^2*arc
sinh(d*x+c)^2+3*a^2*b*arcsinh(d*x+c)+a^3)-2/3*e^3/b^4*exp(4*a/b)*Ei(1,4*arc
sinh(d*x+c)+4*a/b)-1/24*(2*(d*x+c)^2-2*(d*x+c)*(1+(d*x+c)^2)^(1/2)+1)*e^3*(
2*b^2*arcsinh(d*x+c)^2+4*a*b*arcsinh(d*x+c)-arcsinh(d*x+c)*b^2+2*a^2-a*b+b^
2)/b^3/(b^3*arcsinh(d*x+c)^3+3*a*b^2*arcsinh(d*x+c)^2+3*a^2*b*arcsinh(d*x+c
)+a^3)+1/6*e^3/b^4*exp(2*a/b)*Ei(1,2*arcsinh(d*x+c)+2*a/b)+1/24/b*e^3*(2*(d
*x+c)^2+1+2*(d*x+c)*(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^3+1/24/b^2*e^
3*(2*(d*x+c)^2+1+2*(d*x+c)*(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^2+1/12
/b^3*e^3*(2*(d*x+c)^2+1+2*(d*x+c)*(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))
+1/6/b^4*e^3*exp(-2*a/b)*Ei(1,-2*arcsinh(d*x+c)-2*a/b)-1/48/b*e^3*(8*(d*x+c
)^4+8*(d*x+c)^2+8*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)+4*(d*x+c)*(1+(d*x+c)^2)^(1/
2)+1)/(a+b*arcsinh(d*x+c))^3-1/24/b^2*e^3*(8*(d*x+c)^4+8*(d*x+c)^2+8*(d*x+c
)^3*(1+(d*x+c)^2)^(1/2)+4*(d*x+c)*(1+(d*x+c)^2)^(1/2)+1)/(a+b*arcsinh(d*x+c
))^2-1/6/b^3*e^3*(8*(d*x+c)^4+8*(d*x+c)^2+8*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)+4
*(d*x+c)*(1+(d*x+c)^2)^(1/2)+1)/(a+b*arcsinh(d*x+c))-2/3/b^4*e^3*exp(-4*a/b
)*Ei(1,-4*arcsinh(d*x+c)-4*a/b))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,e+d\,e\,x)^3}{(a+b\,\mathrm{asinh}\,(c+d\,x))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3/(a + b*asinh(c + d*x))^4,x)
```

```
[Out] int((c*e + d*e*x)^3/(a + b*asinh(c + d*x))^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^3 \left(\int \frac{c^3}{a^4 + 4a^3 b \operatorname{asinh}(c + dx) + 6a^2 b^2 \operatorname{asinh}^2(c + dx) + 4ab^3 \operatorname{asinh}^3(c + dx) + b^4 \operatorname{asinh}^4(c + dx)}\, dx + \int \frac{}{a^4 +}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3/(a+b*asinh(d*x+c))**4,x)
```

```
[Out] e**3*(Integral(c**3/(a**4 + 4*a**3*b*asinh(c + d*x) + 6*a**2*b**2*asinh(c +
 d*x)**2 + 4*a*b**3*asinh(c + d*x)**3 + b**4*asinh(c + d*x)**4), x) + Integ
ral(d**3*x**3/(a**4 + 4*a**3*b*asinh(c + d*x) + 6*a**2*b**2*asinh(c + d*x)*
*2 + 4*a*b**3*asinh(c + d*x)**3 + b**4*asinh(c + d*x)**4), x) + Integral(3*
c*d**2*x**2/(a**4 + 4*a**3*b*asinh(c + d*x) + 6*a**2*b**2*asinh(c + d*x)**2
 + 4*a*b**3*asinh(c + d*x)**3 + b**4*asinh(c + d*x)**4), x) + Integral(3*c*
*2*d*x/(a**4 + 4*a**3*b*asinh(c + d*x) + 6*a**2*b**2*asinh(c + d*x)**2 + 4*
a*b**3*asinh(c + d*x)**3 + b**4*asinh(c + d*x)**4), x))
```

3.176 $\int \frac{(ce+dex)^2}{\left(a+b\sinh^{-1}(c+dx)\right)^4}\,dx$

Optimal. Leaf size=331

$$\frac{e^2\sinh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{24b^4d}-\frac{9e^2\sinh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{8b^4d}-\frac{e^2\cosh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{24b^4d}+\frac{9e^2 \cdots}{}$$

```
[Out] -1/3*e^2*(d*x+c)/b^2/d/(a+b*arcsinh(d*x+c))^2-1/2*e^2*(d*x+c)^3/b^2/d/(a+b*
arcsinh(d*x+c))^2-1/24*e^2*cosh(a/b)*Shi((a+b*arcsinh(d*x+c))/b)/b^4/d+9/8*
e^2*cosh(3*a/b)*Shi(3*(a+b*arcsinh(d*x+c))/b)/b^4/d+1/24*e^2*Chi((a+b*arcsi
nh(d*x+c))/b)*sinh(a/b)/b^4/d-9/8*e^2*Chi(3*(a+b*arcsinh(d*x+c))/b)*sinh(3*
a/b)/b^4/d-1/3*e^2*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))^3
-1/3*e^2*(1+(d*x+c)^2)^(1/2)/b^3/d/(a+b*arcsinh(d*x+c))-3/2*e^2*(d*x+c)^2*(
1+(d*x+c)^2)^(1/2)/b^3/d/(a+b*arcsinh(d*x+c))
```

Rubi [A] time = 0.67, antiderivative size = 327, normalized size of antiderivative = 0.99, number of steps used = 18, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {5865, 12, 5667, 5774, 5665, 3303, 3298, 3301, 5655, 5779}

$$\frac{e^2\sinh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)}{24b^4d}-\frac{9e^2\sinh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(c+dx)\right)}{8b^4d}-\frac{e^2\cosh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\sinh^{-1} \cdots\right)}{24b^4d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2/(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] -(e^2*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])/(3*b*d*(a + b*ArcSinh[c + d*x])^3)
 - (e^2*(c + d*x))/(3*b^2*d*(a + b*ArcSinh[c + d*x])^2) - (e^2*(c + d*x)^3)
/(2*b^2*d*(a + b*ArcSinh[c + d*x])^2) - (e^2*Sqrt[1 + (c + d*x)^2])/(3*b^3*
d*(a + b*ArcSinh[c + d*x])) - (3*e^2*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])/(2*
b^3*d*(a + b*ArcSinh[c + d*x])) + (e^2*CoshIntegral[a/b + ArcSinh[c + d*x]]
*Sinh[a/b])/(24*b^4*d) - (9*e^2*CoshIntegral[(3*a)/b + 3*ArcSinh[c + d*x]]*
Sinh[(3*a)/b])/(8*b^4*d) - (e^2*Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c + d*
x]])/(24*b^4*d) + (9*e^2*Cosh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSinh[c +
 d*x]])/(8*b^4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[(((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^2}{\left(a+b\sinh^{-1}(c+dx)\right)^4}\,dx = \frac{\text{Subst}\left(\int \frac{e^2x^2}{\left(a+b\sinh^{-1}(x)\right)^4}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^2\,\text{Subst}\left(\int \frac{x^2}{\left(a+b\sinh^{-1}(x)\right)^4}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} + \frac{\left(2e^2\right)\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\left(a+b\sinh^{-1}(x)\right)^3}\,dx, x, c+d\right.}{3bd}$$

$$= -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{e^2(c+dx)}{3b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{e^2(c}{2b^2d\left(a+b\,\text{si}\right.}$$

$$= -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{e^2(c+dx)}{3b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{e^2(c}{2b^2d\left(a+b\,\text{si}\right.}$$

$$= -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{e^2(c+dx)}{3b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{e^2(c}{2b^2d\left(a+b\,\text{si}\right.}$$

$$= -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{e^2(c+dx)}{3b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{e^2(c}{2b^2d\left(a+b\,\text{si}\right.}$$

$$= -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{e^2(c+dx)}{3b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{e^2(c}{2b^2d\left(a+b\,\text{si}\right.}$$

Mathematica [A] time = 0.84, size = 258, normalized size = 0.78

$$\frac{e^2\left(-\frac{8b^3(c+dx)^2\sqrt{(c+dx)^2+1}}{\left(a+b\sinh^{-1}(c+dx)\right)^3} + \frac{4b^2\left(-3(c+dx)^3-2(c+dx)\right)}{\left(a+b\sinh^{-1}(c+dx)\right)^2} + 27\left(3\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right) - \sinh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^2/(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] (e^2*((-8*b^3*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh[c + d*x])^3
 + (4*b^2*(-2*(c + d*x) - 3*(c + d*x)^3))/(a + b*ArcSinh[c + d*x])^2 - (4*b
*Sqrt[1 + (c + d*x)^2]*(2 + 9*(c + d*x)^2))/(a + b*ArcSinh[c + d*x]) - 80*C
oshIntegral[a/b + ArcSinh[c + d*x]]*Sinh[a/b] + 80*Cosh[a/b]*SinhIntegral[a
/b + ArcSinh[c + d*x]] + 27*(3*CoshIntegral[a/b + ArcSinh[c + d*x]]*Sinh[a/
b] - CoshIntegral[3*(a/b + ArcSinh[c + d*x])]*Sinh[(3*a)/b] - 3*Cosh[a/b]*S
inhIntegral[a/b + ArcSinh[c + d*x]] + Cosh[(3*a)/b]*SinhIntegral[3*(a/b + A
rcSinh[c + d*x])])))/(24*b^4*d)
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{d^2e^2x^2 + 2\,cde^2x + c^2e^2}{b^4\,\text{arsinh}\,(dx+c)^4 + 4\,ab^3\,\text{arsinh}\,(dx+c)^3 + 6\,a^2b^2\,\text{arsinh}\,(dx+c)^2 + 4\,a^3b\,\text{arsinh}\,(dx+c) + a^4}, x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral((d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2)/(b^4*arcsinh(d*x + c)^4 + 4*
a*b^3*arcsinh(d*x + c)^3 + 6*a^2*b^2*arcsinh(d*x + c)^2 + 4*a^3*b*arcsinh(d
*x + c) + a^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex + ce)^2}{(b \operatorname{arsinh}(dx + c) + a)^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2/(b*arcsinh(d*x + c) + a)^4, x)
```

maple [B] time = 0.28, size = 709, normalized size = 2.14

$$\frac{\frac{\left(-4(dx+c)^2\sqrt{1+(dx+c)^2}+4(dx+c)^3-\sqrt{1+(dx+c)^2}+3dx+3c\right)e^2\left(9b^2\operatorname{arcsinh}(dx+c)^2+18ab\operatorname{arcsinh}(dx+c)-3\operatorname{arcsinh}(dx+c)b^2+9a^2-3ab+2b^2\right)}{48b^3\left(b^3\operatorname{arcsinh}(dx+c)^3+3ab^2\operatorname{arcsinh}(dx+c)^2+3a^2b\operatorname{arcsinh}(dx+c)+a^3\right)}+\frac{9e^2\mathrm{e}^{\frac{3}{b}\cdots}}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^4,x)
```

```
[Out] 1/d*(1/48*(-4*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+4*(d*x+c)^3-(1+(d*x+c)^2)^(1/2)
+3*d*x+3*c)*e^2*(9*b^2*arcsinh(d*x+c)^2+18*a*b*arcsinh(d*x+c)-3*arcsinh(d*x
+c)*b^2+9*a^2-3*a*b+2*b^2)/b^3/(b^3*arcsinh(d*x+c)^3+3*a*b^2*arcsinh(d*x+c)
^2+3*a^2*b*arcsinh(d*x+c)+a^3)+9/16*e^2/b^4*exp(3*a/b)*Ei(1,3*arcsinh(d*x+c
)+3*a/b)-1/48*(-(1+(d*x+c)^2)^(1/2)+d*x+c)*e^2*(b^2*arcsinh(d*x+c)^2+2*a*b*
arcsinh(d*x+c)-arcsinh(d*x+c)*b^2+a^2-a*b+2*b^2)/b^3/(b^3*arcsinh(d*x+c)^3+
3*a*b^2*arcsinh(d*x+c)^2+3*a^2*b*arcsinh(d*x+c)+a^3)-1/48*e^2/b^4*exp(a/b)*
Ei(1,arcsinh(d*x+c)+a/b)+1/24/b*e^2*(d*x+c+(1+(d*x+c)^2)^(1/2))/(a+b*arcsin
h(d*x+c))^3+1/48/b^2*e^2*(d*x+c+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^2
+1/48/b^3*e^2*(d*x+c+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))+1/48/b^4*e^2
*exp(-a/b)*Ei(1,-arcsinh(d*x+c)-a/b)-1/24/b*e^2*(4*(d*x+c)^3+3*d*x+3*c+4*(d
*x+c)^2*(1+(d*x+c)^2)^(1/2)+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^3-1/1
6/b^2*e^2*(4*(d*x+c)^3+3*d*x+3*c+4*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+(1+(d*x+c)
^2)^(1/2))/(a+b*arcsinh(d*x+c))^2-3/16/b^3*e^2*(4*(d*x+c)^3+3*d*x+3*c+4*(d*
x+c)^2*(1+(d*x+c)^2)^(1/2)+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))-9/16/b
^4*e^2*exp(-3*a/b)*Ei(1,-3*arcsinh(d*x+c)-3*a/b))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,e + d\,e\,x)^2}{(a + b\,\mathrm{asinh}(c + d\,x))^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2/(a + b*asinh(c + d*x))^4,x)
```

```
[Out] int((c*e + d*e*x)^2/(a + b*asinh(c + d*x))^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^2\left(\int \frac{c^2}{a^4 + 4a^3b\operatorname{asinh}(c+dx) + 6a^2b^2\operatorname{asinh}^2(c+dx) + 4ab^3\operatorname{asinh}^3(c+dx) + b^4\operatorname{asinh}^4(c+dx)}\,dx + \int \frac{}{a^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2/(a+b*asinh(d*x+c))**4,x)
```

```
[Out] e**2*(Integral(c**2/(a**4 + 4*a**3*b*asinh(c + d*x) + 6*a**2*b**2*asinh(c +
 d*x)**2 + 4*a*b**3*asinh(c + d*x)**3 + b**4*asinh(c + d*x)**4), x) + Integ
ral(d**2*x**2/(a**4 + 4*a**3*b*asinh(c + d*x) + 6*a**2*b**2*asinh(c + d*x)*
*2 + 4*a*b**3*asinh(c + d*x)**3 + b**4*asinh(c + d*x)**4), x) + Integral(2*
c*d*x/(a**4 + 4*a**3*b*asinh(c + d*x) + 6*a**2*b**2*asinh(c + d*x)**2 + 4*a
*b**3*asinh(c + d*x)**3 + b**4*asinh(c + d*x)**4), x))
```

3.177 $\int \frac{ce+dex}{\left(a+b\sinh^{-1}(c+dx)\right)^4}\,dx$

Optimal. Leaf size=204

$$\frac{2e\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{3b^4d}-\frac{2e\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{3b^4d}-\frac{2e\sqrt{(c+dx)^2+1}\,(c+dx)}{3b^3d\left(a+b\sinh^{-1}(c+dx)\right)}-\frac{}{3b^2d\left(a+\right.}$$

```
[Out] -1/6*e/b^2/d/(a+b*arcsinh(d*x+c))^2-1/3*e*(d*x+c)^2/b^2/d/(a+b*arcsinh(d*x+
c))^2+2/3*e*Chi(2*(a+b*arcsinh(d*x+c))/b)*cosh(2*a/b)/b^4/d-2/3*e*Shi(2*(a+
b*arcsinh(d*x+c))/b)*sinh(2*a/b)/b^4/d-1/3*e*(d*x+c)*(1+(d*x+c)^2)^(1/2)/b/
d/(a+b*arcsinh(d*x+c))^3-2/3*e*(d*x+c)*(1+(d*x+c)^2)^(1/2)/b^3/d/(a+b*arcsi
nh(d*x+c))
```

Rubi [A] time = 0.34, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5865, 12, 5667, 5774, 5665, 3303, 3298, 3301, 5675}

$$\frac{2e\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(c+dx)\right)}{3b^4d}-\frac{2e\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\sinh^{-1}(c+dx)\right)}{3b^4d}-\frac{e(c+dx)^2}{3b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)/(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] -(e*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(3*b*d*(a + b*ArcSinh[c + d*x])^3) - e
/(6*b^2*d*(a + b*ArcSinh[c + d*x])^2) - (e*(c + d*x)^2)/(3*b^2*d*(a + b*Arc
Sinh[c + d*x])^2) - (2*e*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(3*b^3*d*(a + b*A
rcSinh[c + d*x])) + (2*e*Cosh[(2*a)/b]*CoshIntegral[(2*a)/b + 2*ArcSinh[c +
 d*x]])/(3*b^4*d) - (2*e*Sinh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSinh[c +
 d*x]])/(3*b^4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{ce+dex}{\left(a+b\sinh^{-1}(c+dx)\right)^4}\,dx = \frac{\text{Subst}\left(\int \frac{ex}{\left(a+b\sinh^{-1}(x)\right)^4}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e\,\text{Subst}\left(\int \frac{x}{\left(a+b\sinh^{-1}(x)\right)^4}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{e(c+dx)\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} + \frac{e\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}\left(a+b\sinh^{-1}(x)\right)^3}\,dx, x, c+dx\right)}{3bd} + \cdots$$

$$= -\frac{e(c+dx)\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{e}{6b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{e(c+d\cdots}{3b^2d\left(a+b\sinh\cdots\right.}$$

$$= -\frac{e(c+dx)\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{e}{6b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{e(c+d\cdots}{3b^2d\left(a+b\sinh\cdots\right.}$$

$$= -\frac{e(c+dx)\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{e}{6b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{e(c+d\cdots}{3b^2d\left(a+b\sinh\cdots\right.}$$

$$= -\frac{e(c+dx)\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{e}{6b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{e(c+d\cdots}{3b^2d\left(a+b\sinh\cdots\right.}$$

Mathematica [A] time = 0.81, size = 181, normalized size = 0.89

$$\frac{e\left(-\frac{2b^3(c+dx)\sqrt{(c+dx)^2+1}}{\left(a+b\sinh^{-1}(c+dx)\right)^3} + \frac{b^2\left(-2(c+dx)^2-1\right)}{\left(a+b\sinh^{-1}(c+dx)\right)^2} + 4\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right) - 4\left(\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\text{s}\cdots\right.\right.\right.\right.}{6b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)/(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] (e*((-2*b^3*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh[c + d*x])^3 + (
b^2*(-1 - 2*(c + d*x)^2))/(a + b*ArcSinh[c + d*x])^2 - (4*b*(c + d*x)*Sqrt[
1 + (c + d*x)^2])/(a + b*ArcSinh[c + d*x]) + 4*Cosh[(2*a)/b]*CoshIntegral[2
*(a/b + ArcSinh[c + d*x])] + 4*Log[a + b*ArcSinh[c + d*x]] - 4*(Log[a + b*A
rcSinh[c + d*x]] + Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c + d*x])]))
)/(6*b^4*d)
```

fricas [F] time = 1.29, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{dex+ce}{b^4\operatorname{arsinh}(dx+c)^4 + 4ab^3\operatorname{arsinh}(dx+c)^3 + 6a^2b^2\operatorname{arsinh}(dx+c)^2 + 4a^3b\operatorname{arsinh}(dx+c) + a^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral((d*e*x + c*e)/(b^4*arcsinh(d*x + c)^4 + 4*a*b^3*arcsinh(d*x + c)^3
 + 6*a^2*b^2*arcsinh(d*x + c)^2 + 4*a^3*b*arcsinh(d*x + c) + a^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dex + ce}{\left(b \operatorname{arsinh}\left(dx + c\right) + a\right)^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)/(b*arcsinh(d*x + c) + a)^4, x)
```

maple [A] time = 0.08, size = 333, normalized size = 1.63

$$\frac{\frac{\left(2(dx+c)^2-2(dx+c)\sqrt{1+(dx+c)^2}+1\right)e\left(2b^2\operatorname{arcsinh}(dx+c)^2+4ab\operatorname{arcsinh}(dx+c)-\operatorname{arcsinh}(dx+c)b^2+2a^2-ab+b^2\right)}{12b^3\left(b^3\operatorname{arcsinh}(dx+c)^3+3a\,b^2\operatorname{arcsinh}(dx+c)^2+3a^2b\operatorname{arcsinh}(dx+c)+a^3\right)}-\frac{e\,\mathrm{e}^{\frac{2a}{b}}\operatorname{Ei}\left(1,2\operatorname{arcsinh}(dx+c)+\frac{2a}{b}\right)}{3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^4,x)
```

```
[Out] 1/d*(1/12*(2*(d*x+c)^2-2*(d*x+c)*(1+(d*x+c)^2)^(1/2)+1)*e*(2*b^2*arcsinh(d*
x+c)^2+4*a*b*arcsinh(d*x+c)-arcsinh(d*x+c)*b^2+2*a^2-a*b+b^2)/b^3/(b^3*arcs
inh(d*x+c)^3+3*a*b^2*arcsinh(d*x+c)^2+3*a^2*b*arcsinh(d*x+c)+a^3)-1/3*e/b^4
*exp(2*a/b)*Ei(1,2*arcsinh(d*x+c)+2*a/b)-1/12/b*e*(2*(d*x+c)^2+1+2*(d*x+c)*
(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^3-1/12/b^2*e*(2*(d*x+c)^2+1+2*(d*
x+c)*(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^2-1/6/b^3*e*(2*(d*x+c)^2+1+2
*(d*x+c)*(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))-1/3/b^4*e*exp(-2*a/b)*Ei
(1,-2*arcsinh(d*x+c)-2*a/b))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{c\,e+d\,e\,x}{{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)/(a + b*asinh(c + d*x))^4,x)
```

```
[Out] int((c*e + d*e*x)/(a + b*asinh(c + d*x))^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e\left(\int \frac{c}{a^4 + 4a^3b\operatorname{asinh}(c+dx) + 6a^2b^2\operatorname{asinh}^2(c+dx) + 4ab^3\operatorname{asinh}^3(c+dx) + b^4\operatorname{asinh}^4(c+dx)}\, dx + \int \frac{}{a^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*asinh(d*x+c))**4,x)
```

```
[Out] e*(Integral(c/(a**4 + 4*a**3*b*asinh(c + d*x) + 6*a**2*b**2*asinh(c + d*x)*
*2 + 4*a*b**3*asinh(c + d*x)**3 + b**4*asinh(c + d*x)**4), x) + Integral(d*
x/(a**4 + 4*a**3*b*asinh(c + d*x) + 6*a**2*b**2*asinh(c + d*x)**2 + 4*a*b**
3*asinh(c + d*x)**3 + b**4*asinh(c + d*x)**4), x))
```

3.178 $\int \frac{1}{\left(a+b\sinh^{-1}(c+dx)\right)^4}\,dx$

Optimal. Leaf size=160

$$-\frac{\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{6b^4d}+\frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{6b^4d}-\frac{\sqrt{(c+dx)^2+1}}{6b^3d\left(a+b\sinh^{-1}(c+dx)\right)}-\frac{c+dx}{6b^2d\left(a+b\sinh^{-1}(\right.}$$

```
[Out] 1/6*(-d*x-c)/b^2/d/(a+b*arcsinh(d*x+c))^2+1/6*cosh(a/b)*Shi((a+b*arcsinh(d*
x+c))/b)/b^4/d-1/6*Chi((a+b*arcsinh(d*x+c))/b)*sinh(a/b)/b^4/d-1/3*(1+(d*x+
c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))^3-1/6*(1+(d*x+c)^2)^(1/2)/b^3/d/(a+b*a
rcsinh(d*x+c))
```

Rubi [A] time = 0.27, antiderivative size = 156, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5863, 5655, 5774, 5779, 3303, 3298, 3301}

$$-\frac{\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)}{6b^4d}+\frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)}{6b^4d}-\frac{c+dx}{6b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2}-\frac{\sqrt{(}}{6b^3d\left(a-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^(-4),x]
```

```
[Out] -Sqrt[1 + (c + d*x)^2]/(3*b*d*(a + b*ArcSinh[c + d*x])^3) - (c + d*x)/(6*b^
2*d*(a + b*ArcSinh[c + d*x])^2) - Sqrt[1 + (c + d*x)^2]/(6*b^3*d*(a + b*Arc
Sinh[c + d*x])) - (CoshIntegral[a/b + ArcSinh[c + d*x]]*Sinh[a/b])/(6*b^4*d
) + (Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c + d*x]])/(6*b^4*d)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
```

```
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 5863

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sinh^{-1}(c+dx)\right)^4}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+b\sinh^{-1}(x)\right)^4}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} + \frac{\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\left(a+b\sinh^{-1}(x)\right)^3}\,dx, x, c+dx\right)}{3bd}$$

$$= -\frac{\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{c+dx}{6b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} + \frac{\text{Subst}\left(\int \frac{}{(a+b}\right.}{}$$

$$= -\frac{\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{c+dx}{6b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{\sqrt{1+(c}}{6b^3d\left(a+b\,\text{si}\right.}$$

$$= -\frac{\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{c+dx}{6b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{\sqrt{1+(c}}{6b^3d\left(a+b\,\text{si}\right.}$$

$$= -\frac{\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{c+dx}{6b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{\sqrt{1+(c}}{6b^3d\left(a+b\,\text{si}\right.}$$

$$= -\frac{\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{c+dx}{6b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{\sqrt{1+(c}}{6b^3d\left(a+b\,\text{si}\right.}$$

Mathematica [A] time = 0.44, size = 130, normalized size = 0.81

$$-\frac{\frac{2b^3\sqrt{(c+dx)^2+1}}{\left(a+b\sinh^{-1}(c+dx)\right)^3} + \frac{b^2(c+dx)}{\left(a+b\sinh^{-1}(c+dx)\right)^2} + \sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right) - \cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)}{6b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^(-4),x]
```

```
[Out] -1/6*((2*b^3*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh[c + d*x])^3 + (b^2*(c +
d*x))/(a + b*ArcSinh[c + d*x])^2 + (b*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh
[c + d*x]) + CoshIntegral[a/b + ArcSinh[c + d*x]]*Sinh[a/b] - Cosh[a/b]*Sin
hIntegral[a/b + ArcSinh[c + d*x]])/(b^4*d)
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{b^4 \operatorname{arsinh}(dx+c)^4 + 4\,ab^3 \operatorname{arsinh}(dx+c)^3 + 6\,a^2b^2 \operatorname{arsinh}(dx+c)^2 + 4\,a^3b \operatorname{arsinh}(dx+c) + a^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral(1/(b^4*arcsinh(d*x + c)^4 + 4*a*b^3*arcsinh(d*x + c)^3 + 6*a^2*b^2
*arcsinh(d*x + c)^2 + 4*a^3*b*arcsinh(d*x + c) + a^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \operatorname{arsinh}(dx+c) + a)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(-4), x)
```

maple [A] time = 0.07, size = 272, normalized size = 1.70

$$\frac{\frac{\left(-\sqrt{1+(dx+c)^2}+dx+c\right)\left(b^2 \operatorname{arcsinh}(dx+c)^2+2ab \operatorname{arcsinh}(dx+c)-\operatorname{arcsinh}(dx+c)b^2+a^2-ab+2b^2\right)}{12b^3\left(b^3 \operatorname{arcsinh}(dx+c)^3+3a\,b^2 \operatorname{arcsinh}(dx+c)^2+3a^2b \operatorname{arcsinh}(dx+c)+a^3\right)} + \frac{e^{\frac{a}{b}} \operatorname{Ei}\left(1, \operatorname{arcsinh}(dx+c)+\frac{a}{b}\right)}{12b^4} - \frac{dx+c+\sqrt{1+(dx+c}}{6b(a+b \operatorname{arcsinh}(dx+}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(d*x+c))^4,x)
```

```
[Out] 1/d*(1/12*(-(1+(d*x+c)^2)^(1/2)+d*x+c)*(b^2*arcsinh(d*x+c)^2+2*a*b*arcsinh(
d*x+c)-arcsinh(d*x+c)*b^2+a^2-a*b+2*b^2)/b^3/(b^3*arcsinh(d*x+c)^3+3*a*b^2*
arcsinh(d*x+c)^2+3*a^2*b*arcsinh(d*x+c)+a^3)+1/12/b^4*exp(a/b)*Ei(1,arcsinh
(d*x+c)+a/b)-1/6/b*(d*x+c+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^3-1/12/
b^2*(d*x+c+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^2-1/12/b^3*(d*x+c+(1+(
d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))-1/12/b^4*exp(-a/b)*Ei(1,-arcsinh(d*x+
c)-a/b))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a + b \operatorname{asinh}(c + d\,x))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*asinh(c + d*x))^4,x)
```

```
[Out] int(1/(a + b*asinh(c + d*x))^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b\operatorname{asinh}(c + dx))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(d*x+c))**4,x)
```

```
[Out] Integral((a + b*asinh(c + d*x))**(-4), x)
```

3.179 $\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^4}\,dx$

Optimal. Leaf size=27

$$\frac{\text{Int}\left(\frac{1}{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^4},x\right)}{e}$$

[Out] Unintegrable(1/(d*x+c)/(a+b*arcsinh(d*x+c))^4,x)/e

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^4}\,dx$$

Verification is Not applicable to the result.

[In] Int[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^4),x]

[Out] Defer[Subst][Defer[Int][1/(x*(a + b*ArcSinh[x])^4), x], x, c + d*x]/(d*e)

Rubi steps

$$\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^4}\,dx = \frac{\text{Subst}\left(\int \frac{1}{ex\left(a+b\sinh^{-1}(x)\right)^4}\,dx,x,c+dx\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x\left(a+b\sinh^{-1}(x)\right)^4}\,dx,x,c+dx\right)}{de}$$

Mathematica [A] time = 3.63, size = 0, normalized size = 0.00

$$\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^4}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^4),x]

[Out] Integrate[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^4), x]

fricas [A] time = 1.08, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{a^4dex + a^4ce + \left(b^4dex + b^4ce\right)\operatorname{arsinh}(dx+c)^4 + 4\left(ab^3dex + ab^3ce\right)\operatorname{arsinh}(dx+c)^3 + 6\left(a^2b^2dex + a^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^4,x, algorithm="fricas")

[Out] integral(1/(a^4*d*e*x + a^4*c*e + (b^4*d*e*x + b^4*c*e)*arcsinh(d*x + c)^4 + 4*(a*b^3*d*e*x + a*b^3*c*e)*arcsinh(d*x + c)^3 + 6*(a^2*b^2*d*e*x + a^2*b

```
^2*c*e)*arcsinh(d*x + c)^2 + 4*(a^3*b*d*e*x + a^3*b*c*e)*arcsinh(d*x + c)),
 x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dex+ce)(b\operatorname{arsinh}(dx+c)+a)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate(1/((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^4), x)
```

maple [A] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{1}{(dex+ce)\left(a+b\operatorname{arcsinh}(dx+c)\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^4,x)
```

```
[Out] int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^4,x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{(c\,e+d\,e\,x)\,\left(a+b\operatorname{asinh}(c+d\,x)\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*e + d*e*x)*(a + b*asinh(c + d*x))^4),x)
```

```
[Out] int(1/((c*e + d*e*x)*(a + b*asinh(c + d*x))^4), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^4c+a^4dx+4a^3bc\operatorname{asinh}(c+dx)+4a^3bdx\operatorname{asinh}(c+dx)+6a^2b^2c\operatorname{asinh}^2(c+dx)+6a^2b^2dx\operatorname{asinh}^2(c+dx)+4ab^3c\operatorname{asinh}^3(c+dx)+4ab^3dx\operatorname{asinh}^3(c+dx)}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*asinh(d*x+c))**4,x)
```

```
[Out] Integral(1/(a**4*c + a**4*d*x + 4*a**3*b*c*asinh(c + d*x) + 4*a**3*b*d*x*as
inh(c + d*x) + 6*a**2*b**2*c*asinh(c + d*x)**2 + 6*a**2*b**2*d*x*asinh(c +
d*x)**2 + 4*a*b**3*c*asinh(c + d*x)**3 + 4*a*b**3*d*x*asinh(c + d*x)**3 + b
**4*c*asinh(c + d*x)**4 + b**4*d*x*asinh(c + d*x)**4), x)/e
```

3.180 $\int (ce+dex)^4\sqrt{a+b\sinh^{-1}(c+dx)}\,dx$

Optimal. Leaf size=361

$$\frac{\sqrt{\pi}\sqrt{b}e^4e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32d}-\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}e^4e^{\frac{3a}{b}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{64d}+\frac{\sqrt{\frac{\pi}{5}}\sqrt{b}e^4e^{\frac{5a}{b}}\operatorname{erf}\left(\frac{\sqrt{5}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{320d}$$

```
[Out] 1/1600*e^4*exp(5*a/b)*erf(5^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/
2)*5^(1/2)*Pi^(1/2)/d-1/1600*e^4*erfi(5^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^
(1/2))*b^(1/2)*5^(1/2)*Pi^(1/2)/d/exp(5*a/b)-1/192*e^4*exp(3*a/b)*erf(3^(1/
2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*3^(1/2)*Pi^(1/2)/d+1/192*e^4
*erfi(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*3^(1/2)*Pi^(1/2)/
d/exp(3*a/b)+1/32*e^4*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1
/2)*Pi^(1/2)/d-1/32*e^4*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*Pi
^(1/2)/d/exp(a/b)+1/5*e^4*(d*x+c)^5*(a+b*arcsinh(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.88, antiderivative size = 361, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {5865, 12, 5663, 5779, 3312, 3308, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}\sqrt{b}e^4e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32d}-\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}e^4e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{64d}+\frac{\sqrt{\frac{\pi}{5}}\sqrt{b}e^4e^{\frac{5a}{b}}\text{Erf}\left(\frac{\sqrt{5}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{320d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4*Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (e^4*(c + d*x)^5*Sqrt[a + b*ArcSinh[c + d*x]])/(5*d) + (Sqrt[b]*e^4*E^(a/b)
*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(32*d) - (Sqrt[b]*e^4*
E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])
/(64*d) + (Sqrt[b]*e^4*E^((5*a)/b)*Sqrt[Pi/5]*Erf[(Sqrt[5]*Sqrt[a + b*ArcSi
nh[c + d*x]])/Sqrt[b]])/(320*d) - (Sqrt[b]*e^4*Sqrt[Pi]*Erfi[Sqrt[a + b*Arc
Sinh[c + d*x]]/Sqrt[b]])/(32*d*E^(a/b)) + (Sqrt[b]*e^4*Sqrt[Pi/3]*Erfi[(Sqr
t[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(64*d*E^((3*a)/b)) - (Sqrt[b]*
e^4*Sqrt[Pi/5]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(320*d
*E^((5*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^4\sqrt{a+b\sinh^{-1}(c+dx)}\,dx = \frac{\text{Subst}\left(\int e^4x^4\sqrt{a+b\sinh^{-1}(x)}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e^4\,\text{Subst}\left(\int x^4\sqrt{a+b\sinh^{-1}(x)}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e^4(c+dx)^5\sqrt{a+b\sinh^{-1}(c+dx)}}{5d} - \frac{\left(be^4\right)\text{Subst}\left(\int \frac{x^5}{\sqrt{1+x^2}\sqrt{a+b\sinh^{-1}(x)}}\right.}{10d}$$

$$= \frac{e^4(c+dx)^5\sqrt{a+b\sinh^{-1}(c+dx)}}{5d} - \frac{\left(be^4\right)\text{Subst}\left(\int \frac{\sinh^5(x)}{\sqrt{a+bx}}\,dx,x,\sinh\right.}{10d}$$

$$= \frac{e^4(c+dx)^5\sqrt{a+b\sinh^{-1}(c+dx)}}{5d} + \frac{\left(ibe^4\right)\text{Subst}\left(\int \left(\frac{5i\sinh(x)}{8\sqrt{a+bx}} - \frac{5i\sinh(}{16\sqrt{a+}}\right.\right.}{}$$

$$= \frac{e^4(c+dx)^5\sqrt{a+b\sinh^{-1}(c+dx)}}{5d} - \frac{\left(be^4\right)\text{Subst}\left(\int \frac{\sinh(5x)}{\sqrt{a+bx}}\,dx,x,\sinh\right.}{160d}$$

$$= \frac{e^4(c+dx)^5\sqrt{a+b\sinh^{-1}(c+dx)}}{5d} + \frac{\left(be^4\right)\text{Subst}\left(\int \frac{e^{-5x}}{\sqrt{a+bx}}\,dx,x,\sinh^{-}\right.}{320d}$$

$$= \frac{e^4(c+dx)^5\sqrt{a+b\sinh^{-1}(c+dx)}}{5d} + \frac{e^4\,\text{Subst}\left(\int e^{\frac{5a}{b}-\frac{5x^2}{b}}\,dx,x,\sqrt{a+b\right.}{160d}$$

$$= \frac{e^4(c+dx)^5\sqrt{a+b\sinh^{-1}(c+dx)}}{5d} + \frac{\sqrt{b}\,e^4e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right.}{32d}$$

Mathematica [A] time = 0.65, size = 342, normalized size = 0.95

$$\frac{e^4e^{-\frac{5a}{b}}\sqrt{a+b\sinh^{-1}(c+dx)}\left(-150e^{\frac{6a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{3}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)+3\sqrt{5}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\Gamma\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^4*Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (e^4*Sqrt[a + b*ArcSinh[c + d*x]]*(-150*E^((6*a)/b)*Sqrt[-((a + b*ArcSinh[c
 + d*x])/b)]*Gamma[3/2, a/b + ArcSinh[c + d*x]] + 3*Sqrt[5]*Sqrt[a/b + ArcS
inh[c + d*x]]*Gamma[3/2, (-5*(a + b*ArcSinh[c + d*x]))/b] - 25*Sqrt[3]*E^((
2*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[3/2, (-3*(a + b*ArcSinh[c + d*x]
))/b] + 150*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[3/2, -((a + b*Ar
cSinh[c + d*x])/b)] + 25*Sqrt[3]*E^((8*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x]
)/b)]*Gamma[3/2, (3*(a + b*ArcSinh[c + d*x]))/b] - 3*Sqrt[5]*E^((10*a)/b)*S
qrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[3/2, (5*(a + b*ArcSinh[c + d*x]))/
b]))/(2400*d*E^((5*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])^2/b^2)])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex + ce)^4 \sqrt{b \operatorname{arsinh}(dx + c) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^4*sqrt(b*arcsinh(d*x + c) + a), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int (dex + ce)^4 \sqrt{a + b \operatorname{arcsinh}(dx + c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(1/2),x)
```

```
[Out] int((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex + ce)^4 \sqrt{b \operatorname{arsinh}(dx + c) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^4*sqrt(b*arcsinh(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e + d\,e\,x)^4 \sqrt{a + b \operatorname{asinh}(c + d\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^4*(a + b*asinh(c + d*x))^(1/2),x)
```

```
[Out] int((c*e + d*e*x)^4*(a + b*asinh(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^4 \left(\int c^4 \sqrt{a + b \operatorname{asinh}(c + dx)}\, dx + \int d^4x^4 \sqrt{a + b \operatorname{asinh}(c + dx)}\, dx + \int 4cd^3x^3 \sqrt{a + b \operatorname{asinh}(c + dx)}\, dx + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4*(a+b*asinh(d*x+c))**(1/2),x)
```

```
[Out] e**4*(Integral(c**4*sqrt(a + b*asinh(c + d*x)), x) + Integral(d**4*x**4*sqr
t(a + b*asinh(c + d*x)), x) + Integral(4*c*d**3*x**3*sqrt(a + b*asinh(c + d
*x)), x) + Integral(6*c**2*d**2*x**2*sqrt(a + b*asinh(c + d*x)), x) + Integ
ral(4*c**3*d*x*sqrt(a + b*asinh(c + d*x)), x))
```

3.181 $\int (ce+dex)^3\sqrt{a+b\sinh^{-1}(c+dx)}\,dx$

Optimal. Leaf size=272

$$-\frac{\sqrt{\pi}\sqrt{b}\,e^3e^{\frac{4a}{b}}\operatorname{erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{256d}+\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}\,e^3e^{\frac{2a}{b}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32d}-\frac{\sqrt{\pi}\sqrt{b}\,e^3e^{-\frac{4a}{b}}\operatorname{erfi}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{256d}$$

```
[Out] 1/64*e^3*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)
*2^(1/2)*Pi^(1/2)/d+1/64*e^3*erfi(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2
))*b^(1/2)*2^(1/2)*Pi^(1/2)/d/exp(2*a/b)-1/256*e^3*exp(4*a/b)*erf(2*(a+b*ar
csinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/d-1/256*e^3*erfi(2*(a+b*arcsi
nh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/d/exp(4*a/b)-3/32*e^3*(a+b*arcsi
nh(d*x+c))^(1/2)/d+1/4*e^3*(d*x+c)^4*(a+b*arcsinh(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.67, antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {5865, 12, 5663, 5779, 3312, 3307, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}\sqrt{b}\,e^3e^{\frac{4a}{b}}\operatorname{Erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{256d}+\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}\,e^3e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32d}-\frac{\sqrt{\pi}\sqrt{b}\,e^3e^{-\frac{4a}{b}}\operatorname{Erfi}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx}}{\sqrt{b}}\right)}{256d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3*Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (-3*e^3*Sqrt[a + b*ArcSinh[c + d*x]])/(32*d) + (e^3*(c + d*x)^4*Sqrt[a + b*
ArcSinh[c + d*x]])/(4*d) - (Sqrt[b]*e^3*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a
+ b*ArcSinh[c + d*x]])/Sqrt[b]])/(256*d) + (Sqrt[b]*e^3*E^((2*a)/b)*Sqrt[Pi
/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(32*d) - (Sqrt[b]*
e^3*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(256*d*E^((4*a
)/b)) + (Sqrt[b]*e^3*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])
/Sqrt[b]])/(32*d*E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^3\sqrt{a+b\sinh^{-1}(c+dx)}\,dx = \frac{\text{Subst}\left(\int e^3x^3\sqrt{a+b\sinh^{-1}(x)}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e^3\,\text{Subst}\left(\int x^3\sqrt{a+b\sinh^{-1}(x)}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e^3(c+dx)^4\sqrt{a+b\sinh^{-1}(c+dx)}}{4d} - \frac{(be^3)\,\text{Subst}\left(\int \frac{x^4}{\sqrt{1+x^2}\sqrt{a+b\sinh^{-1}(x)}}\right.}{8d}$$

$$= \frac{e^3(c+dx)^4\sqrt{a+b\sinh^{-1}(c+dx)}}{4d} - \frac{(be^3)\,\text{Subst}\left(\int \frac{\sinh^4(x)}{\sqrt{a+bx}}\,dx,x,\sinh\right.}{8d}$$

$$= \frac{e^3(c+dx)^4\sqrt{a+b\sinh^{-1}(c+dx)}}{4d} - \frac{(be^3)\,\text{Subst}\left(\int\left(\frac{3}{8\sqrt{a+bx}} - \frac{\cosh(2x)}{2\sqrt{a+bx}}\right.\right.}{8}$$

$$= -\frac{3e^3\sqrt{a+b\sinh^{-1}(c+dx)}}{32d} + \frac{e^3(c+dx)^4\sqrt{a+b\sinh^{-1}(c+dx)}}{4d} - \frac{(be}{}$$

$$= -\frac{3e^3\sqrt{a+b\sinh^{-1}(c+dx)}}{32d} + \frac{e^3(c+dx)^4\sqrt{a+b\sinh^{-1}(c+dx)}}{4d} - \frac{(be}{}$$

$$= -\frac{3e^3\sqrt{a+b\sinh^{-1}(c+dx)}}{32d} + \frac{e^3(c+dx)^4\sqrt{a+b\sinh^{-1}(c+dx)}}{4d} - \frac{e^3}{}$$

$$= -\frac{3e^3\sqrt{a+b\sinh^{-1}(c+dx)}}{32d} + \frac{e^3(c+dx)^4\sqrt{a+b\sinh^{-1}(c+dx)}}{4d} - \frac{\sqrt{b}}{}$$

Mathematica [A] time = 0.29, size = 223, normalized size = 0.82

$$\frac{e^3e^{-\frac{4a}{b}}\sqrt{a+b\sinh^{-1}(c+dx)}\left(\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\Gamma\left(\frac{3}{2},-\frac{4\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right) - 4\sqrt{2}\,e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\Gamma\left(\frac{3}{2}\right.\right.}{128d\sqrt{-\frac{\left(a+b\sinh^{-}\right.}{b^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^3*Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (e^3*Sqrt[a + b*ArcSinh[c + d*x]]*(Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[3/2,
(-4*(a + b*ArcSinh[c + d*x]))/b] - 4*Sqrt[2]*E^((2*a)/b)*Sqrt[a/b + ArcSinh
[c + d*x]]*Gamma[3/2, (-2*(a + b*ArcSinh[c + d*x]))/b] + E^((6*a)/b)*Sqrt[-
((a + b*ArcSinh[c + d*x])/b)]*(-4*Sqrt[2]*Gamma[3/2, (2*(a + b*ArcSinh[c +
d*x]))/b] + E^((2*a)/b)*Gamma[3/2, (4*(a + b*ArcSinh[c + d*x]))/b])))/(128*
d*E^((4*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])^2/b^2)])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex+ce)^3\sqrt{b\,\mathrm{arsinh}\,(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^3*sqrt(b*arcsinh(d*x + c) + a), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int (dex+ce)^3\sqrt{a+b\,\mathrm{arcsinh}\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(1/2),x)
[Out] int((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex+ce)^3\sqrt{b\,\mathrm{arsinh}\,(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate((d*e*x + c*e)^3*sqrt(b*arcsinh(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e+d\,e\,x)^3\,\sqrt{a+b\,\mathrm{asinh}\,(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3*(a + b*asinh(c + d*x))^(1/2),x)
[Out] int((c*e + d*e*x)^3*(a + b*asinh(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^3\left(\int c^3\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\,dx+\int d^3x^3\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\,dx+\int 3cd^2x^2\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\,dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3*(a+b*asinh(d*x+c))**(1/2),x)
[Out] e**3*(Integral(c**3*sqrt(a + b*asinh(c + d*x)), x) + Integral(d**3*x**3*sqr
t(a + b*asinh(c + d*x)), x) + Integral(3*c*d**2*x**2*sqrt(a + b*asinh(c + d
*x)), x) + Integral(3*c**2*d*x*sqrt(a + b*asinh(c + d*x)), x))
```

3.182 $\int (ce+dex)^2 \sqrt{a+b\sinh^{-1}(c+dx)}\,dx$

Optimal. Leaf size=245

$$-\frac{\sqrt{\pi}\sqrt{b}e^2e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}+\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}e^2e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{48d}+\frac{\sqrt{\pi}\sqrt{b}e^2e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}-$$

```
[Out] 1/144*e^2*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2
)*3^(1/2)*Pi^(1/2)/d-1/144*e^2*erfi(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1
/2))*b^(1/2)*3^(1/2)*Pi^(1/2)/d/exp(3*a/b)-1/16*e^2*exp(a/b)*erf((a+b*arcsi
nh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/d+1/16*e^2*erfi((a+b*arcsinh(d*x
+c))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/d/exp(a/b)+1/3*e^2*(d*x+c)^3*(a+b*arcs
inh(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.65, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {5865, 12, 5663, 5779, 3312, 3308, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}\sqrt{b}e^2e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}+\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}e^2e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{48d}+\frac{\sqrt{\pi}\sqrt{b}e^2e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2*Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (e^2*(c + d*x)^3*Sqrt[a + b*ArcSinh[c + d*x]])/(3*d) - (Sqrt[b]*e^2*E^(a/b)
*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(16*d) + (Sqrt[b]*e^2*
E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])
/(48*d) + (Sqrt[b]*e^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])
/(16*d*E^(a/b)) - (Sqrt[b]*e^2*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[
c + d*x]])/Sqrt[b]])/(48*d*E^((3*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^2\sqrt{a+b\sinh^{-1}(c+dx)}\,dx = \frac{\text{Subst}\left(\int e^2x^2\sqrt{a+b\sinh^{-1}(x)}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e^2\,\text{Subst}\left(\int x^2\sqrt{a+b\sinh^{-1}(x)}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e^2(c+dx)^3\sqrt{a+b\sinh^{-1}(c+dx)}}{3d} - \frac{(be^2)\,\text{Subst}\left(\int \frac{x^3}{\sqrt{1+x^2}\sqrt{a+b\sinh^{-1}(x)}}\right.}{6d}$$

$$= \frac{e^2(c+dx)^3\sqrt{a+b\sinh^{-1}(c+dx)}}{3d} - \frac{(be^2)\,\text{Subst}\left(\int \frac{\sinh^3(x)}{\sqrt{a+bx}}\,dx,x,\sinh\right.}{6d}$$

$$= \frac{e^2(c+dx)^3\sqrt{a+b\sinh^{-1}(c+dx)}}{3d} - \frac{(ibe^2)\,\text{Subst}\left(\int \left(\frac{3i\sinh(x)}{4\sqrt{a+bx}} - \frac{i\sinh(3}{4\sqrt{a+b}}\right.\right.}{6d}$$

$$= \frac{e^2(c+dx)^3\sqrt{a+b\sinh^{-1}(c+dx)}}{3d} - \frac{(be^2)\,\text{Subst}\left(\int \frac{\sinh(3x)}{\sqrt{a+bx}}\,dx,x,\sinh\right.}{24d}$$

$$= \frac{e^2(c+dx)^3\sqrt{a+b\sinh^{-1}(c+dx)}}{3d} + \frac{(be^2)\,\text{Subst}\left(\int \frac{e^{-3x}}{\sqrt{a+bx}}\,dx,x,\sinh\right.}{48d}$$

$$= \frac{e^2(c+dx)^3\sqrt{a+b\sinh^{-1}(c+dx)}}{3d} + \frac{e^2\,\text{Subst}\left(\int e^{\frac{3a}{b}-\frac{3x^2}{b}}\,dx,x,\sqrt{a+b}\right.}{24d}$$

$$= \frac{e^2(c+dx)^3\sqrt{a+b\sinh^{-1}(c+dx)}}{3d} - \frac{\sqrt{b}\,e^2e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right.}{16d}$$

Mathematica [A] time = 0.41, size = 238, normalized size = 0.97

$$\frac{e^2e^{-\frac{3a}{b}}\sqrt{a+b\sinh^{-1}(c+dx)}\left(9e^{\frac{4a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{3}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)+\sqrt{3}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\Gamma\left(\frac{3}{2},-\right.\right.}{72d\sqrt{-\frac{(}{}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^2*Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (e^2*Sqrt[a + b*ArcSinh[c + d*x]]*(9*E^((4*a)/b)*Sqrt[-((a + b*ArcSinh[c +
d*x])/b)]*Gamma[3/2, a/b + ArcSinh[c + d*x]] + Sqrt[3]*Sqrt[a/b + ArcSinh[c
 + d*x]]*Gamma[3/2, (-3*(a + b*ArcSinh[c + d*x]))/b] - 9*E^((2*a)/b)*Sqrt[a
/b + ArcSinh[c + d*x]]*Gamma[3/2, -((a + b*ArcSinh[c + d*x])/b)] - Sqrt[3]*
E^((6*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[3/2, (3*(a + b*ArcSin
h[c + d*x]))/b]))/(72*d*E^((3*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])^2/b^2)]
)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int (dex+ce)^2\sqrt{b\,\mathrm{arsinh}\,(dx+c)+a}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2*sqrt(b*arcsinh(d*x + c) + a), x)
```

maple [F(-2)]   time = 180.00, size = 0, normalized size = 0.00

$$\int (dex+ce)^2\,\sqrt{a+b\,\mathrm{arcsinh}\,(dx+c)}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(1/2),x)
```

```
[Out] int((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(1/2),x)
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int (dex+ce)^2\sqrt{b\,\mathrm{arsinh}\,(dx+c)+a}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^2*sqrt(b*arcsinh(d*x + c) + a), x)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e+d\,e\,x)^2\,\sqrt{a+b\,\mathrm{asinh}\,(c+d\,x)}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2*(a + b*asinh(c + d*x))^(1/2),x)
```

```
[Out] int((c*e + d*e*x)^2*(a + b*asinh(c + d*x))^(1/2), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$e^2\left(\int c^2\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\;dx+\int d^2x^2\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\;dx+\int 2cdx\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\;dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2*(a+b*asinh(d*x+c))**(1/2),x)
```

```
[Out] e**2*(Integral(c**2*sqrt(a + b*asinh(c + d*x)), x) + Integral(d**2*x**2*sqr
t(a + b*asinh(c + d*x)), x) + Integral(2*c*d*x*sqrt(a + b*asinh(c + d*x)),
x))
```

3.183 $\int (ce+dex)\sqrt{a+b\sinh^{-1}(c+dx)}\,dx$

Optimal. Leaf size=164

$$-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}\,ee^{\frac{2a}{b}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}\,ee^{-\frac{2a}{b}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}+\frac{e(c+dx)^2\sqrt{a+b\sinh^{-1}(c+dx)}}{2d}$$

```
[Out] -1/32*e*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*
2^(1/2)*Pi^(1/2)/d-1/32*e*erfi(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*
b^(1/2)*2^(1/2)*Pi^(1/2)/d/exp(2*a/b)+1/4*e*(a+b*arcsinh(d*x+c))^(1/2)/d+1/
2*e*(d*x+c)^2*(a+b*arcsinh(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.43, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {5865, 12, 5663, 5779, 3312, 3307, 2180, 2204, 2205}

$$-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}\,ee^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}\,ee^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}+\frac{e(c+dx)^2\sqrt{a+b\sinh^{-1}(c+dx)}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)*Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (e*Sqrt[a + b*ArcSinh[c + d*x]])/(4*d) + (e*(c + d*x)^2*Sqrt[a + b*ArcSinh[
c + d*x]])/(2*d) - (Sqrt[b]*e*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a +
b*ArcSinh[c + d*x]])/Sqrt[b]])/(16*d) - (Sqrt[b]*e*Sqrt[Pi/2]*Erfi[(Sqrt[2]
*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(16*d*E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
```

```
f, m}, x] && IntegerQ[2*k]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)\sqrt{a+b\sinh^{-1}(c+dx)}\,dx = \frac{\text{Subst}\left(\int ex\sqrt{a+b\sinh^{-1}(x)}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e\,\text{Subst}\left(\int x\sqrt{a+b\sinh^{-1}(x)}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e(c+dx)^2\sqrt{a+b\sinh^{-1}(c+dx)}}{2d} - \frac{(be)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{1+x^2}\sqrt{a+b\sinh^{-1}(x)}}\,dx\right.}{4d}$$

$$= \frac{e(c+dx)^2\sqrt{a+b\sinh^{-1}(c+dx)}}{2d} - \frac{(be)\,\text{Subst}\left(\int \frac{\sinh^2(x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c\right.}{4d}$$

$$= \frac{e(c+dx)^2\sqrt{a+b\sinh^{-1}(c+dx)}}{2d} + \frac{(be)\,\text{Subst}\left(\int \left(\frac{1}{2\sqrt{a+bx}} - \frac{\cosh(2x)}{2\sqrt{a+bx}}\right) d\right.}{4d}$$

$$= \frac{e\sqrt{a+b\sinh^{-1}(c+dx)}}{4d} + \frac{e(c+dx)^2\sqrt{a+b\sinh^{-1}(c+dx)}}{2d} - \frac{(be)\,\text{Sub}}{}$$

$$= \frac{e\sqrt{a+b\sinh^{-1}(c+dx)}}{4d} + \frac{e(c+dx)^2\sqrt{a+b\sinh^{-1}(c+dx)}}{2d} - \frac{(be)\,\text{Sub}}{}$$

$$= \frac{e\sqrt{a+b\sinh^{-1}(c+dx)}}{4d} + \frac{e(c+dx)^2\sqrt{a+b\sinh^{-1}(c+dx)}}{2d} - \frac{e\,\text{Subst}}{}$$

$$= \frac{e\sqrt{a+b\sinh^{-1}(c+dx)}}{4d} + \frac{e(c+dx)^2\sqrt{a+b\sinh^{-1}(c+dx)}}{2d} - \frac{\sqrt{b}\,ee^{\frac{2a}{b}}}{}$$

Mathematica [A] time = 0.10, size = 140, normalized size = 0.85

$$\frac{ee^{-\frac{2a}{b}}\sqrt{a+b\sinh^{-1}(c+dx)}\left(\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\Gamma\left(\frac{3}{2}, -\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right) + e^{\frac{4a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{3}{2}, \frac{2\left(a+b\sinh\right.}{b}\right.\right.}{8\sqrt{2}\,d\sqrt{-\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{b^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)*Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (e*Sqrt[a + b*ArcSinh[c + d*x]]*(Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[3/2, (-
2*(a + b*ArcSinh[c + d*x]))/b] + E^((4*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x]
)/b)]*Gamma[3/2, (2*(a + b*ArcSinh[c + d*x]))/b]))/(8*Sqrt[2]*d*E^((2*a)/b)
*Sqrt[-((a + b*ArcSinh[c + d*x])^2/b^2)])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex + ce)\sqrt{b \operatorname{arsinh}(dx + c) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((d*e*x + c*e)*sqrt(b*arcsinh(d*x + c) + a), x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int (dex + ce)\sqrt{a + b \operatorname{arcsinh}(dx + c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(1/2),x)
[Out] int((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex + ce)\sqrt{b \operatorname{arsinh}(dx + c) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate((d*e*x + c*e)*sqrt(b*arcsinh(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e + d\,e\,x)\sqrt{a + b \operatorname{asinh}(c + d\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)*(a + b*asinh(c + d*x))^(1/2),x)
[Out] int((c*e + d*e*x)*(a + b*asinh(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e\left(\int c\sqrt{a + b \operatorname{asinh}(c + dx)}\, dx + \int dx\sqrt{a + b \operatorname{asinh}(c + dx)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*asinh(d*x+c))**(1/2),x)
[Out] e*(Integral(c*sqrt(a + b*asinh(c + d*x)), x) + Integral(d*x*sqrt(a + b*asin
h(c + d*x)), x))
```

3.184 $\int \sqrt{a+b\sinh^{-1}(c+dx)}\,dx$

Optimal. Leaf size=115

$$\frac{\sqrt{\pi}\sqrt{b}\,e^{a/b}\mathrm{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4d}-\frac{\sqrt{\pi}\sqrt{b}\,e^{-\frac{a}{b}}\mathrm{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4d}+\frac{(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d}$$

```
[Out] 1/4*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/d-1/4
*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/d/exp(a/b)+(d*x+
c)*(a+b*arcsinh(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.25, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5863, 5653, 5779, 3308, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}\sqrt{b}\,e^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4d}-\frac{\sqrt{\pi}\sqrt{b}\,e^{-\frac{a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4d}+\frac{(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] ((c + d*x)*Sqrt[a + b*ArcSinh[c + d*x]])/d + (Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[
Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(4*d) - (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[
a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(4*d*E^(a/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 5863

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+b\sinh^{-1}(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \sqrt{a+b\sinh^{-1}(x)}\,dx,x,c+dx\right)}{d}\\ &= \frac{(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d}-\frac{b\,\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\sqrt{a+b\sinh^{-1}(x)}}\,dx,x,c+dx\right)}{2d}\\ &= \frac{(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d}-\frac{b\,\text{Subst}\left(\int \frac{\sinh(x)}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(c+dx)\right)}{2d}\\ &= \frac{(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d}+\frac{b\,\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(c+dx)\right)}{4d}-\ b\\ &= \frac{(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d}+\frac{\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx,x,\sqrt{a+b\sinh^{-1}(c+dx)}\right)}{2d}\\ &= \frac{(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d}+\frac{\sqrt{b}\,e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4d}-\frac{\sqrt{b}\,e^{-\frac{a}{b}}\sqrt{}}{}\end{aligned}$$

Mathematica [A] time = 0.12, size = 111, normalized size = 0.97

$$\frac{e^{-\frac{a}{b}}\sqrt{a+b\sinh^{-1}(c+dx)}\left(\frac{\Gamma\left(\frac{3}{2},-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}}-\frac{e^{\frac{2a}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)}{\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}}\right)}{2d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (Sqrt[a + b*ArcSinh[c + d*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcSinh[c +
d*x]])/Sqrt[a/b + ArcSinh[c + d*x]]) + Gamma[3/2, -((a + b*ArcSinh[c + d*x]
)/b)]/Sqrt[-((a + b*ArcSinh[c + d*x])/b)]))/(2*d*E^(a/b))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \operatorname{arsinh}(dx + c) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*arcsinh(d*x + c) + a), x)
```

maple [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \operatorname{arcsinh}(dx + c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^(1/2),x)
[Out] int((a+b*arcsinh(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \operatorname{arsinh}(dx + c) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*arcsinh(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a + b \operatorname{asinh}(c + d\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^(1/2),x)
[Out] int((a + b*asinh(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \operatorname{asinh}(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a + b*asinh(c + d*x)), x)
```

3.185 $\int \frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{ce+dex}\,dx$

Optimal. Leaf size=29

$$\frac{\text{Int}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{c+dx},x\right)}{e}$$

```
[Out] Unintegrable((a+b*arcsinh(d*x+c))^(1/2)/(d*x+c),x)/e
```

Rubi [A] time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{ce+dex}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[a + b*ArcSinh[c + d*x]]/(c*e + d*e*x),x]
```

```
[Out] Defer[Subst][Defer[Int][Sqrt[a + b*ArcSinh[x]]/x, x], x, c + d*x]/(d*e)
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{ce+dex}\,dx &= \frac{\text{Subst}\left(\int \frac{\sqrt{a+b\sinh^{-1}(x)}}{ex}\,dx,x,c+dx\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{\sqrt{a+b\sinh^{-1}(x)}}{x}\,dx,x,c+dx\right)}{de}\end{aligned}$$

Mathematica [A] time = 2.16, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{ce+dex}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + b*ArcSinh[c + d*x]]/(c*e + d*e*x),x]
```

```
[Out] Integrate[Sqrt[a + b*ArcSinh[c + d*x]]/(c*e + d*e*x), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(1/2)/(d*e*x+c*e),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\,\text{arsinh}\left(dx+c\right)+a}}{dex+ce}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(1/2)/(d*e*x+c*e),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*arcsinh(d*x + c) + a)/(d*e*x + c*e), x)
```

maple [A] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\,\mathrm{arcsinh}\,(dx+c)}}{dex+ce}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^(1/2)/(d*e*x+c*e),x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^(1/2)/(d*e*x+c*e),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\,\mathrm{arsinh}\,(dx+c)+a}}{dex+ce}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(1/2)/(d*e*x+c*e),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*arcsinh(d*x + c) + a)/(d*e*x + c*e), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{a+b\,\mathrm{asinh}\,(c+d\,x)}}{c\,e+d\,e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^(1/2)/(c*e + d*e*x),x)
```

```
[Out] int((a + b*asinh(c + d*x))^(1/2)/(c*e + d*e*x), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}}{c+dx}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**(1/2)/(d*e*x+c*e),x)
```

```
[Out] Integral(sqrt(a + b*asinh(c + d*x))/(c + d*x), x)/e
```

3.186 $\int (ce+dex)^4 \left(a+b\sinh^{-1}(c+dx)\right)^{3/2}\,dx$

Optimal. Leaf size=601

$$\frac{3\sqrt{\pi}\,b^{3/2}e^4e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{64d}-\frac{3\sqrt{3\pi}\,b^{3/2}e^4e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3200d}-\frac{\sqrt{\frac{\pi}{3}}\,b^{3/2}e^4e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(}{\sqrt{b}}\right.}{200d}$$

```
[Out] 1/5*e^4*(d*x+c)^5*(a+b*arcsinh(d*x+c))^(3/2)/d+3/16000*b^(3/2)*e^4*exp(5*a/
b)*erf(5^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*5^(1/2)*Pi^(1/2)/d+3/160
00*b^(3/2)*e^4*erfi(5^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*5^(1/2)*Pi^
(1/2)/d/exp(5*a/b)-1/384*b^(3/2)*e^4*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(d*
x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/d-1/384*b^(3/2)*e^4*erfi(3^(1/2)*(a+b
*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/d/exp(3*a/b)+3/64*b^(3/2)*
e^4*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d+3/64*b^(3/2
)*e^4*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/exp(a/b)-4/25*b*e
^4*(1+(d*x+c)^2)^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/d+2/25*b*e^4*(d*x+c)^2*(1
+(d*x+c)^2)^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/d-3/50*b*e^4*(d*x+c)^4*(1+(d*x
+c)^2)^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/d
```

Rubi [A] time = 1.66, antiderivative size = 601, normalized size of antiderivative = 1.00, number of steps used = 43, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {5865, 12, 5663, 5758, 5717, 5657, 3307, 2180, 2205, 2204, 5669, 5448}

$$\frac{3\sqrt{\pi}\,b^{3/2}e^4e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{64d}-\frac{3\sqrt{3\pi}\,b^{3/2}e^4e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3200d}-\frac{\sqrt{\frac{\pi}{3}}\,b^{3/2}e^4e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}}}{\sqrt{b}}\right.}{200d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4*(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (-4*b*e^4*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/(25*d) + (2*b
*e^4*(c + d*x)^2*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/(25*d)
 - (3*b*e^4*(c + d*x)^4*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])
/(50*d) + (e^4*(c + d*x)^5*(a + b*ArcSinh[c + d*x])^(3/2))/(5*d) + (3*b^(3/
2)*e^4*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(64*d) -
 (b^(3/2)*e^4*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*
x]])/Sqrt[b]])/(200*d) - (3*b^(3/2)*e^4*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]
*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(3200*d) + (3*b^(3/2)*e^4*E^((5*a)
/b)*Sqrt[Pi/5]*Erf[(Sqrt[5]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(3200*d
) + (3*b^(3/2)*e^4*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(64
*d*E^(a/b)) - (b^(3/2)*e^4*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c +
d*x]])/Sqrt[b]])/(200*d*E^((3*a)/b)) - (3*b^(3/2)*e^4*Sqrt[3*Pi]*Erfi[(Sqrt
[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(3200*d*E^((3*a)/b)) + (3*b^(3/
2)*e^4*Sqrt[Pi/5]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(32
00*d*E^((5*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
```

```
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^4 \left(a+b\sinh^{-1}(c+dx)\right)^{3/2} dx = \frac{\text{Subst}\left(\int e^4x^4 \left(a+b\sinh^{-1}(x)\right)^{3/2} dx, x, c+dx\right)}{d}$$

$$= \frac{e^4\,\text{Subst}\left(\int x^4 \left(a+b\sinh^{-1}(x)\right)^{3/2} dx, x, c+dx\right)}{d}$$

$$= \frac{e^4(c+dx)^5 \left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{5d} - \frac{\left(3be^4\right)\text{Subst}\left(\int \frac{x^5\sqrt{a+b\sin}}{\sqrt{1+x}}\right.}{10d}$$

$$= -\frac{3be^4(c+dx)^4\sqrt{1+(c+dx)^2}\,\sqrt{a+b\sinh^{-1}(c+dx)}}{50d} + \frac{e^4(c+dx}{}$$

$$= \frac{2be^4(c+dx)^2\sqrt{1+(c+dx)^2}\,\sqrt{a+b\sinh^{-1}(c+dx)}}{25d} - \frac{3be^4(c+d}{}$$

$$= -\frac{4be^4\sqrt{1+(c+dx)^2}\,\sqrt{a+b\sinh^{-1}(c+dx)}}{25d} + \frac{2be^4(c+dx)^2\sqrt{1-}}{}$$

$$= -\frac{4be^4\sqrt{1+(c+dx)^2}\,\sqrt{a+b\sinh^{-1}(c+dx)}}{25d} + \frac{2be^4(c+dx)^2\sqrt{1-}}{}$$

$$= -\frac{4be^4\sqrt{1+(c+dx)^2}\,\sqrt{a+b\sinh^{-1}(c+dx)}}{25d} + \frac{2be^4(c+dx)^2\sqrt{1-}}{}$$

$$= -\frac{4be^4\sqrt{1+(c+dx)^2}\,\sqrt{a+b\sinh^{-1}(c+dx)}}{25d} + \frac{2be^4(c+dx)^2\sqrt{1-}}{}$$

$$= -\frac{4be^4\sqrt{1+(c+dx)^2}\,\sqrt{a+b\sinh^{-1}(c+dx)}}{25d} + \frac{2be^4(c+dx)^2\sqrt{1-}}{}$$

$$= -\frac{4be^4\sqrt{1+(c+dx)^2}\,\sqrt{a+b\sinh^{-1}(c+dx)}}{25d} + \frac{2be^4(c+dx)^2\sqrt{1-}}{}$$

Mathematica [A] time = 0.48, size = 343, normalized size = 0.57

$$-\frac{be^4e^{-\frac{5a}{b}}\sqrt{a+b\sinh^{-1}(c+dx)}\left(2250e^{\frac{6a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\Gamma\left(\frac{5}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)+9\sqrt{5}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^4*(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] -1/36000*(b*e^4*Sqrt[a + b*ArcSinh[c + d*x]]*(2250*E^((6*a)/b)*Sqrt[-((a +
b*ArcSinh[c + d*x])/b)]*Gamma[5/2, a/b + ArcSinh[c + d*x]] + 9*Sqrt[5]*Sqrt
[a/b + ArcSinh[c + d*x]]*Gamma[5/2, (-5*(a + b*ArcSinh[c + d*x]))/b] - 125*
Sqrt[3]*E^((2*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[5/2, (-3*(a + b*ArcS
inh[c + d*x]))/b] + 2250*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[5/2
, -((a + b*ArcSinh[c + d*x])/b)] - 125*Sqrt[3]*E^((8*a)/b)*Sqrt[-((a + b*Ar
cSinh[c + d*x])/b)]*Gamma[5/2, (3*(a + b*ArcSinh[c + d*x]))/b] + 9*Sqrt[5]*
E^((10*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[5/2, (5*(a + b*ArcSi
nh[c + d*x]))/b]))/(d*E^((5*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])^2/b^2)])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex+ce)^4(b\,\mathrm{arsinh}\,(dx+c)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^4*(b*arcsinh(d*x + c) + a)^(3/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int (dex+ce)^4\,(a+b\,\mathrm{arcsinh}\,(dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(3/2),x)
```

```
[Out] int((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex+ce)^4(b\,\mathrm{arsinh}\,(dx+c)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^4*(b*arcsinh(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e+d\,e\,x)^4\,(a+b\,\mathrm{asinh}\,(c+d\,x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^4*(a + b*asinh(c + d*x))^(3/2),x)
```

```
[Out] int((c*e + d*e*x)^4*(a + b*asinh(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^4\left(\int ac^4\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\;dx+\int ad^4x^4\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\;dx+\int bc^4\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\;\mathrm{asinh}\,(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4*(a+b*asinh(d*x+c))**(3/2),x)
```

```
[Out] e**4*(Integral(a*c**4*sqrt(a + b*asinh(c + d*x)), x) + Integral(a*d**4*x**4
*sqrt(a + b*asinh(c + d*x)), x) + Integral(b*c**4*sqrt(a + b*asinh(c + d*x)
)*asinh(c + d*x), x) + Integral(4*a*c*d**3*x**3*sqrt(a + b*asinh(c + d*x)),
 x) + Integral(6*a*c**2*d**2*x**2*sqrt(a + b*asinh(c + d*x)), x) + Integral
(4*a*c**3*d*x*sqrt(a + b*asinh(c + d*x)), x) + Integral(b*d**4*x**4*sqrt(a
+ b*asinh(c + d*x))*asinh(c + d*x), x) + Integral(4*b*c*d**3*x**3*sqrt(a +
b*asinh(c + d*x))*asinh(c + d*x), x) + Integral(6*b*c**2*d**2*x**2*sqrt(a +
 b*asinh(c + d*x))*asinh(c + d*x), x) + Integral(4*b*c**3*d*x*sqrt(a + b*as
inh(c + d*x))*asinh(c + d*x), x))
```

3.187 $\int (ce+dex)^3 \left(a+b\sinh^{-1}(c+dx)\right)^{3/2}\,dx$

Optimal. Leaf size=360

$$-\frac{3\sqrt{\pi}\,b^{3/2}e^3e^{\frac{4a}{b}}\operatorname{erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2048d}+\frac{3\sqrt{\frac{\pi}{2}}\,b^{3/2}e^3e^{\frac{2a}{b}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{128d}+\frac{3\sqrt{\pi}\,b^{3/2}e^3e^{-\frac{4a}{b}}\operatorname{erfi}\left(\frac{2\sqrt{a+b\sinh^{-1}(c}{\sqrt{b}}\right.}{2048d}$$

```
[Out] -3/32*e^3*(a+b*arcsinh(d*x+c))^(3/2)/d+1/4*e^3*(d*x+c)^4*(a+b*arcsinh(d*x+c
))^(3/2)/d+3/256*b^(3/2)*e^3*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^(1
/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d-3/256*b^(3/2)*e^3*erfi(2^(1/2)*(a+b*arcsinh
(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d/exp(2*a/b)-3/2048*b^(3/2)*e^3*ex
p(4*a/b)*erf(2*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d+3/2048*b^(3/2
)*e^3*erfi(2*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/exp(4*a/b)+9/64
*b*e^3*(d*x+c)*(1+(d*x+c)^2)^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/d-3/32*b*e^3*
(d*x+c)^3*(1+(d*x+c)^2)^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/d
```

Rubi [A] time = 1.06, antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {5865, 12, 5663, 5758, 5675, 5669, 5448, 3308, 2180, 2204, 2205}

$$-\frac{3\sqrt{\pi}\,b^{3/2}e^3e^{\frac{4a}{b}}\operatorname{Erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2048d}+\frac{3\sqrt{\frac{\pi}{2}}\,b^{3/2}e^3e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{128d}+\frac{3\sqrt{\pi}\,b^{3/2}e^3e^{-\frac{4a}{b}}\operatorname{Erfi}\left(\frac{2\sqrt{a+b\sinh^{-1}}}{\sqrt{b}}\right.}{2048d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3*(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (9*b*e^3*(c + d*x)*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/(64*
d) - (3*b*e^3*(c + d*x)^3*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]
])/(32*d) - (3*e^3*(a + b*ArcSinh[c + d*x])^(3/2))/(32*d) + (e^3*(c + d*x)^
4*(a + b*ArcSinh[c + d*x])^(3/2))/(4*d) - (3*b^(3/2)*e^3*E^((4*a)/b)*Sqrt[P
i]*Erf[(2*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(2048*d) + (3*b^(3/2)*e^3
*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]
)/(128*d) + (3*b^(3/2)*e^3*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcSinh[c + d*x]])/S
qrt[b]])/(2048*d*E^((4*a)/b)) - (3*b^(3/2)*e^3*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqr
t[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(128*d*E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^3 \left(a+b\sinh^{-1}(c+dx)\right)^{3/2} dx = \frac{\text{Subst}\left(\int e^3x^3 \left(a+b\sinh^{-1}(x)\right)^{3/2} dx, x, c+dx\right)}{d}$$
$$= \frac{e^3\,\text{Subst}\left(\int x^3 \left(a+b\sinh^{-1}(x)\right)^{3/2} dx, x, c+dx\right)}{d}$$
$$= \frac{e^3(c+dx)^4 \left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{4d} - \frac{\left(3be^3\right)\text{Subst}\left(\int \frac{x^4\sqrt{a+b\sinh^{-1}}}{\sqrt{1+x^2}}\right.}{8d}$$
$$= -\frac{3be^3(c+dx)^3\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{32d} + \frac{e^3(c+dx)^4}{}$$
$$= \frac{9be^3(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{64d} - \frac{3be^3(c+dx)^3}{}$$
$$= \frac{9be^3(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{64d} - \frac{3be^3(c+dx)^3}{}$$
$$= \frac{9be^3(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{64d} - \frac{3be^3(c+dx)^3}{}$$
$$= \frac{9be^3(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{64d} - \frac{3be^3(c+dx)^3}{}$$
$$= \frac{9be^3(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{64d} - \frac{3be^3(c+dx)^3}{}$$
$$= \frac{9be^3(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{64d} - \frac{3be^3(c+dx)^3}{}$$
$$= \frac{9be^3(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{64d} - \frac{3be^3(c+dx)^3}{}$$

Mathematica [A] time = 0.29, size = 225, normalized size = 0.62

$$\frac{be^3e^{-\frac{4a}{b}}\sqrt{a+b\sinh^{-1}(c+dx)}\left(-\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\Gamma\left(\frac{5}{2},-\frac{4\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)+8\sqrt{2}\,e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\Gamma\right.}{512d\sqrt{-\frac{\left(a+b\sinh\right.}{}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^3*(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (b*e^3*Sqrt[a + b*ArcSinh[c + d*x]]*(-(Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[5
/2, (-4*(a + b*ArcSinh[c + d*x]))/b]) + 8*Sqrt[2]*E^((2*a)/b)*Sqrt[a/b + Ar
cSinh[c + d*x]]*Gamma[5/2, (-2*(a + b*ArcSinh[c + d*x]))/b] + E^((6*a)/b)*S
qrt[-((a + b*ArcSinh[c + d*x])/b)]*(-8*Sqrt[2]*Gamma[5/2, (2*(a + b*ArcSinh
[c + d*x]))/b] + E^((2*a)/b)*Gamma[5/2, (4*(a + b*ArcSinh[c + d*x]))/b])))/
(512*d*E^((4*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])^2/b^2)])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex+ce)^3 (b\operatorname{arsinh}(dx+c)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^3*(b*arcsinh(d*x + c) + a)^(3/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int (dex+ce)^3 \left(a+b\operatorname{arcsinh}(dx+c)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(3/2),x)
```

```
[Out] int((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex+ce)^3 (b\operatorname{arsinh}(dx+c)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^3*(b*arcsinh(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(c\,e+d\,e\,x\right)}^3\,{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3*(a + b*asinh(c + d*x))^(3/2),x)
```

```
[Out] int((c*e + d*e*x)^3*(a + b*asinh(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^{3} \left(\int ac^{3} \sqrt{a + b \operatorname{asinh}{\left(c + dx \right)}}\, dx + \int ad^{3} x^{3} \sqrt{a + b \operatorname{asinh}{\left(c + dx \right)}}\, dx + \int bc^{3} \sqrt{a + b \operatorname{asinh}{\left(c + dx \right)}} \operatorname{asinh}{(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3*(a+b*asinh(d*x+c))**(3/2),x)
```

```
[Out] e**3*(Integral(a*c**3*sqrt(a + b*asinh(c + d*x)), x) + Integral(a*d**3*x**3
*sqrt(a + b*asinh(c + d*x)), x) + Integral(b*c**3*sqrt(a + b*asinh(c + d*x)
)*asinh(c + d*x), x) + Integral(3*a*c*d**2*x**2*sqrt(a + b*asinh(c + d*x)),
 x) + Integral(3*a*c**2*d*x*sqrt(a + b*asinh(c + d*x)), x) + Integral(b*d**
3*x**3*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x), x) + Integral(3*b*c*d**2*
x**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x), x) + Integral(3*b*c**2*d*x*
sqrt(a + b*asinh(c + d*x))*asinh(c + d*x), x))
```

3.188 $\int (ce+dex)^2 \left(a+b\sinh^{-1}(c+dx)\right)^{3/2}\,dx$

Optimal. Leaf size=328

$$-\frac{3\sqrt{\pi}\,b^{3/2}e^2e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32d}+\frac{\sqrt{\frac{\pi}{3}}\,b^{3/2}e^2e^{\frac{3a}{b}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{96d}-\frac{3\sqrt{\pi}\,b^{3/2}e^2e^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32d}$$

```
[Out] 1/3*e^2*(d*x+c)^3*(a+b*arcsinh(d*x+c))^(3/2)/d+1/288*b^(3/2)*e^2*exp(3*a/b)
*erf(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/d+1/288*b
^(3/2)*e^2*erfi(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2
)/d/exp(3*a/b)-3/32*b^(3/2)*e^2*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(
1/2))*Pi^(1/2)/d-3/32*b^(3/2)*e^2*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*
Pi^(1/2)/d/exp(a/b)+1/3*b*e^2*(1+(d*x+c)^2)^(1/2)*(a+b*arcsinh(d*x+c))^(1/2
)/d-1/6*b*e^2*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.88, antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {5865, 12, 5663, 5758, 5717, 5657, 3307, 2180, 2205, 2204, 5669, 5448}

$$-\frac{3\sqrt{\pi}\,b^{3/2}e^2e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32d}+\frac{\sqrt{\frac{\pi}{3}}\,b^{3/2}e^2e^{\frac{3a}{b}}\operatorname{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{96d}-\frac{3\sqrt{\pi}\,b^{3/2}e^2e^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2*(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (b*e^2*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/(3*d) - (b*e^2*(
c + d*x)^2*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/(6*d) + (e^2
*(c + d*x)^3*(a + b*ArcSinh[c + d*x])^(3/2))/(3*d) - (3*b^(3/2)*e^2*E^(a/b)
*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(32*d) + (b^(3/2)*e^2*
E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])
/(96*d) - (3*b^(3/2)*e^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]
])/(32*d*E^(a/b)) + (b^(3/2)*e^2*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSin
h[c + d*x]])/Sqrt[b]])/(96*d*E^((3*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
```

```
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^2 \left(a+b\sinh^{-1}(c+dx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int e^2x^2\left(a+b\sinh^{-1}(x)\right)^{3/2}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^2\,\text{Subst}\left(\int x^2\left(a+b\sinh^{-1}(x)\right)^{3/2}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^2(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{3d} - \frac{\left(be^2\right)\text{Subst}\left(\int \frac{x^3\sqrt{a+b\sinh^{-1}(x}}{\sqrt{1+x^2}}\right.}{2d}$$

$$= -\frac{be^2(c+dx)^2\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{6d} + \frac{e^2(c+dx)^3\left(\right.}{}$$

$$= \frac{be^2\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{3d} - \frac{be^2(c+dx)^2\sqrt{1+(c+d}}{}$$

$$= \frac{be^2\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{3d} - \frac{be^2(c+dx)^2\sqrt{1+(c+d}}{}$$

$$= \frac{be^2\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{3d} - \frac{be^2(c+dx)^2\sqrt{1+(c+d}}{}$$

$$= \frac{be^2\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{3d} - \frac{be^2(c+dx)^2\sqrt{1+(c+d}}{}$$

$$= \frac{be^2\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{3d} - \frac{be^2(c+dx)^2\sqrt{1+(c+d}}{}$$

$$= \frac{be^2\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{3d} - \frac{be^2(c+dx)^2\sqrt{1+(c+d}}{}$$

Mathematica [A] time = 0.30, size = 238, normalized size = 0.73

$$-\frac{be^2e^{-\frac{3a}{b}}\sqrt{a+b\sinh^{-1}(c+dx)}\left(-27e^{\frac{4a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{5}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)+\sqrt{3}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\Gamma\right.}{216d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^2*(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] -1/216*(b*e^2*Sqrt[a + b*ArcSinh[c + d*x]]*(-27*E^((4*a)/b)*Sqrt[-((a + b*A
rcSinh[c + d*x])/b)]*Gamma[5/2, a/b + ArcSinh[c + d*x]] + Sqrt[3]*Sqrt[a/b
+ ArcSinh[c + d*x]]*Gamma[5/2, (-3*(a + b*ArcSinh[c + d*x]))/b] - 27*E^((2*
a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[5/2, -((a + b*ArcSinh[c + d*x])/b)
] + Sqrt[3]*E^((6*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[5/2, (3*(
```

```
a + b*ArcSinh[c + d*x]))/b]))/(d*E^((3*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x]
)^2/b^2)])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex + ce)^2 (b \operatorname{arsinh}(dx + c) + a)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2*(b*arcsinh(d*x + c) + a)^(3/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int (dex + ce)^2 \left(a + b \operatorname{arcsinh}(dx + c)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(3/2),x)
```

```
[Out] int((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex + ce)^2 (b \operatorname{arsinh}(dx + c) + a)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^2*(b*arcsinh(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(c\,e + d\,e\,x\right)}^2\,{\left(a + b\,\mathrm{asinh}\left(c + d\,x\right)\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2*(a + b*asinh(c + d*x))^(3/2),x)
```

```
[Out] int((c*e + d*e*x)^2*(a + b*asinh(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^{2} \left(\int ac^{2} \sqrt{a + b \operatorname{asinh}{\left(c + dx \right)}}\, dx + \int ad^{2}x^{2} \sqrt{a + b \operatorname{asinh}{\left(c + dx \right)}}\, dx + \int bc^{2} \sqrt{a + b \operatorname{asinh}{\left(c + dx \right)}} \operatorname{asinh}(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2*(a+b*asinh(d*x+c))**(3/2),x)
```

```
[Out] e**2*(Integral(a*c**2*sqrt(a + b*asinh(c + d*x)), x) + Integral(a*d**2*x**2
*sqrt(a + b*asinh(c + d*x)), x) + Integral(b*c**2*sqrt(a + b*asinh(c + d*x)
)*asinh(c + d*x), x) + Integral(2*a*c*d*x*sqrt(a + b*asinh(c + d*x)), x) +
Integral(b*d**2*x**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x), x) + Integr
al(2*b*c*d*x*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x), x))
```

3.189 $\int (ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}\,dx$

Optimal. Leaf size=205

$$-\frac{3\sqrt{\frac{\pi}{2}}\,b^{3/2}ee^{\frac{2a}{b}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{64d}+\frac{3\sqrt{\frac{\pi}{2}}\,b^{3/2}ee^{-\frac{2a}{b}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{64d}+\frac{e(c+dx)^2\left(a+b\sinh^{-1}(c+\right.}{2d}$$

```
[Out] 1/4*e*(a+b*arcsinh(d*x+c))^(3/2)/d+1/2*e*(d*x+c)^2*(a+b*arcsinh(d*x+c))^(3/
2)/d-3/128*b^(3/2)*e*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1
/2))*2^(1/2)*Pi^(1/2)/d+3/128*b^(3/2)*e*erfi(2^(1/2)*(a+b*arcsinh(d*x+c))^(
1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d/exp(2*a/b)-3/8*b*e*(d*x+c)*(1+(d*x+c)^2)^(
1/2)*(a+b*arcsinh(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.48, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {5865, 12, 5663, 5758, 5675, 5669, 5448, 3308, 2180, 2204, 2205}

$$-\frac{3\sqrt{\frac{\pi}{2}}\,b^{3/2}ee^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{64d}+\frac{3\sqrt{\frac{\pi}{2}}\,b^{3/2}ee^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{64d}+\frac{e(c+dx)^2\left(a+b\sinh^{-1}(c-\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (-3*b*e*(c + d*x)*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/(8*d)
 + (e*(a + b*ArcSinh[c + d*x])^(3/2))/(4*d) + (e*(c + d*x)^2*(a + b*ArcSinh
[c + d*x])^(3/2))/(2*d) - (3*b^(3/2)*e*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*
Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(64*d) + (3*b^(3/2)*e*Sqrt[Pi/2]*Er
fi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(64*d*E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
```

```
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int ex\left(a+b\sinh^{-1}(x)\right)^{3/2}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e\,\text{Subst}\left(\int x\left(a+b\sinh^{-1}(x)\right)^{3/2}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{2d} - \frac{(3be)\,\text{Subst}\left(\int \frac{x^2\sqrt{a+b\sinh^{-1}(}}{\sqrt{1+x^2}}\right.}{4d}$$

$$= -\frac{3be(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{8d} + \frac{e(c+dx)^2\,(}{}$$

$$= -\frac{3be(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{8d} + \frac{e\left(a+b\sinh\right.}{}$$

$$= -\frac{3be(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{8d} + \frac{e\left(a+b\sinh\right.}{}$$

$$= -\frac{3be(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{8d} + \frac{e\left(a+b\sinh\right.}{}$$

$$= -\frac{3be(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{8d} + \frac{e\left(a+b\sinh\right.}{}$$

$$= -\frac{3be(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{8d} + \frac{e\left(a+b\sinh\right.}{}$$

$$= -\frac{3be(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{8d} + \frac{e\left(a+b\sinh\right.}{}$$

Mathematica [A] time = 0.11, size = 142, normalized size = 0.69

$$\frac{bee^{-\frac{2a}{b}}\sqrt{a+b\sinh^{-1}(c+dx)}\left(e^{\frac{4a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{5}{2},\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)-\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\Gamma\left(\frac{5}{2},-\frac{2(a+}{}\right.\right.}{16\sqrt{2}\,d\sqrt{-\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{b^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (b*e*Sqrt[a + b*ArcSinh[c + d*x]]*(-(Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[5/2
, (-2*(a + b*ArcSinh[c + d*x]))/b]) + E^((4*a)/b)*Sqrt[-((a + b*ArcSinh[c +
 d*x])/b)]*Gamma[5/2, (2*(a + b*ArcSinh[c + d*x]))/b]))/(16*Sqrt[2]*d*E^((2
*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])^2/b^2)])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex + ce)(b \operatorname{arsinh}(dx + c) + a)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^(3/2), x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int (dex + ce)\left(a + b \operatorname{arcsinh}(dx + c)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(3/2),x)
[Out] int((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex + ce)(b \operatorname{arsinh}(dx + c) + a)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e + d\,e\,x)\,\left(a + b\,\mathrm{asinh}\,(c + d\,x)\right)^{3/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)*(a + b*asinh(c + d*x))^(3/2),x)
[Out] int((c*e + d*e*x)*(a + b*asinh(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e\left(\int ac\sqrt{a + b \operatorname{asinh}(c + dx)}\, dx + \int adx\sqrt{a + b \operatorname{asinh}(c + dx)}\, dx + \int bc\sqrt{a + b \operatorname{asinh}(c + dx)} \operatorname{asinh}(c + dx)\, d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*asinh(d*x+c))**(3/2),x)
[Out] e*(Integral(a*c*sqrt(a + b*asinh(c + d*x)), x) + Integral(a*d*x*sqrt(a + b*
asinh(c + d*x)), x) + Integral(b*c*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x
), x) + Integral(b*d*x*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x), x))
```

3.190 $\int \left(a+b\sinh^{-1}(c+dx)\right)^{3/2}\,dx$

Optimal. Leaf size=150

$$\frac{3\sqrt{\pi}\,b^{3/2}e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8d}+\frac{3\sqrt{\pi}\,b^{3/2}e^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8d}-\frac{3b\sqrt{(c+dx)^2+1}\,\sqrt{a+b\sinh^{-1}(c+d}}{2d}$$

```
[Out] (d*x+c)*(a+b*arcsinh(d*x+c))^(3/2)/d+3/8*b^(3/2)*exp(a/b)*erf((a+b*arcsinh(
d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d+3/8*b^(3/2)*erfi((a+b*arcsinh(d*x+c))^(1/
2)/b^(1/2))*Pi^(1/2)/d/exp(a/b)-3/2*b*(1+(d*x+c)^2)^(1/2)*(a+b*arcsinh(d*x+
c))^(1/2)/d
```

Rubi [A] time = 0.25, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5863, 5653, 5717, 5657, 3307, 2180, 2205, 2204}

$$\frac{3\sqrt{\pi}\,b^{3/2}e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8d}+\frac{3\sqrt{\pi}\,b^{3/2}e^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8d}-\frac{3b\sqrt{(c+dx)^2+1}\,\sqrt{a+b\sinh^{-1}(c+}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (-3*b*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/(2*d) + ((c + d*x
)*(a + b*ArcSinh[c + d*x])^(3/2))/d + (3*b^(3/2)*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[
a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(8*d) + (3*b^(3/2)*Sqrt[Pi]*Erfi[Sqrt[a +
 b*ArcSinh[c + d*x]]/Sqrt[b]])/(8*d*E^(a/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
```

```
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5863

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \left(a+b\sinh^{-1}(c+dx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \left(a+b\sinh^{-1}(x)\right)^{3/2}\,dx,x,c+dx\right)}{d}$$

$$= \frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{d} - \frac{(3b)\,\text{Subst}\left(\int \frac{x\sqrt{a+b\sinh^{-1}(x)}}{\sqrt{1+x^2}}\,dx,x,c+dx\right)}{2d}$$

$$= -\frac{3b\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{2d} + \frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{d}$$

$$= -\frac{3b\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{2d} + \frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{d}$$

$$= -\frac{3b\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{2d} + \frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{d}$$

$$= -\frac{3b\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{2d} + \frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{d}$$

$$= -\frac{3b\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{2d} + \frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{d}$$

Mathematica [A] time = 0.23, size = 272, normalized size = 1.81

$$\frac{\sqrt{b}\left(\sqrt{\pi}\,(3b-2a)\left(\sinh\left(\frac{a}{b}\right)+\cosh\left(\frac{a}{b}\right)\right)\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)+\sqrt{\pi}\,(2a+3b)\left(\cosh\left(\frac{a}{b}\right)-\sinh\left(\frac{a}{b}\right)\right)\text{erfi}\left(\frac{\sqrt{a+b\,\text{si}}}{}\right.\right.}{8d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (a*Sqrt[a + b*ArcSinh[c + d*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcSinh[c
+ d*x]])/Sqrt[a/b + ArcSinh[c + d*x]]) + Gamma[3/2, -((a + b*ArcSinh[c + d*
x])/b)]/Sqrt[-((a + b*ArcSinh[c + d*x])/b)]))/(2*d*E^(a/b)) + (Sqrt[b]*(4*S
qrt[b]*Sqrt[a + b*ArcSinh[c + d*x]]*(-3*Sqrt[1 + (c + d*x)^2] + 2*(c + d*x)
*ArcSinh[c + d*x]) + (2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]
/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + (-2*a + 3*b)*Sqrt[Pi]*Erf[Sqrt[a + b*Ar
cSinh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])))/(8*d)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arsinh}(dx + c) + a)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(3/2), x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{arcsinh}(dx + c))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^(3/2),x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arsinh}(dx + c) + a)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b \operatorname{asinh}(c + d\,x))^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^(3/2),x)
```

```
[Out] int((a + b*asinh(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{asinh}(c + dx))^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))**(3/2), x)
```

3.191 $\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{ce+dex}\,dx$

Optimal. Leaf size=29

$$\frac{\text{Int}\left(\frac{\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{c+dx},x\right)}{e}$$

```
[Out] Unintegrable((a+b*arcsinh(d*x+c))^(3/2)/(d*x+c),x)/e
```

Rubi [A] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{ce+dex}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcSinh[c + d*x])^(3/2)/(c*e + d*e*x),x]
```

```
[Out] Defer[Subst][Defer[Int][(a + b*ArcSinh[x])^(3/2)/x, x], x, c + d*x]/(d*e)
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{ce+dex}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^{3/2}}{ex}\,dx,x,c+dx\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^{3/2}}{x}\,dx,x,c+dx\right)}{de}\end{aligned}$$

Mathematica [A] time = 1.50, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{ce+dex}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^(3/2)/(c*e + d*e*x),x]
```

```
[Out] Integrate[(a + b*ArcSinh[c + d*x])^(3/2)/(c*e + d*e*x), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(3/2)/(d*e*x+c*e),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,\text{arsinh}\left(dx+c\right)+a\right)^{\frac{3}{2}}}{dex+ce}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(3/2)/(d*e*x+c*e),x, algorithm="giac")
[Out] integrate((b*arcsinh(d*x + c) + a)^(3/2)/(d*e*x + c*e), x)
```

maple [A] time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(dx+c))^{\frac{3}{2}}}{dex+ce}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^(3/2)/(d*e*x+c*e),x)
[Out] int((a+b*arcsinh(d*x+c))^(3/2)/(d*e*x+c*e),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\mathrm{arsinh}\,(dx+c)+a)^{\frac{3}{2}}}{dex+ce}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(3/2)/(d*e*x+c*e),x, algorithm="maxima")
[Out] integrate((b*arcsinh(d*x + c) + a)^(3/2)/(d*e*x + c*e), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(a+b\,\mathrm{asinh}\,(c+d\,x))^{3/2}}{c\,e+d\,e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^(3/2)/(c*e + d*e*x),x)
[Out] int((a + b*asinh(c + d*x))^(3/2)/(c*e + d*e*x), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}}{c+dx}\,dx+\int \frac{b\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\,\mathrm{asinh}\,(c+dx)}{c+dx}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**(3/2)/(d*e*x+c*e),x)
[Out] (Integral(a*sqrt(a + b*asinh(c + d*x))/(c + d*x), x) + Integral(b*sqrt(a +
b*asinh(c + d*x))*asinh(c + d*x)/(c + d*x), x))/e
```

3.192 $\int (ce+dex)^4 \left(a+b\sinh^{-1}(c+dx)\right)^{5/2} dx$

Optimal. Leaf size=701

$$\frac{15\sqrt{\pi}\, b^{5/2}e^4 e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{128d}-\frac{\sqrt{3\pi}\, b^{5/2}e^4 e^{\frac{3a}{b}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{1280d}-\frac{\sqrt{\frac{\pi}{3}}\, b^{5/2}e^4 e^{\frac{3a}{b}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(}{\sqrt{b}}\right.}{240d}$$

```
[Out] 1/5*e^4*(d*x+c)^5*(a+b*arcsinh(d*x+c))^(5/2)/d+3/32000*b^(5/2)*e^4*exp(5*a/
b)*erf(5^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*5^(1/2)*Pi^(1/2)/d-3/320
00*b^(5/2)*e^4*erfi(5^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*5^(1/2)*Pi^
(1/2)/d/exp(5*a/b)-5/2304*b^(5/2)*e^4*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(d
*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/d+5/2304*b^(5/2)*e^4*erfi(3^(1/2)*(a
+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/d/exp(3*a/b)+15/128*b^(5
/2)*e^4*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d-15/128*
b^(5/2)*e^4*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/exp(a/b)-4/
15*b*e^4*(a+b*arcsinh(d*x+c))^(3/2)*(1+(d*x+c)^2)^(1/2)/d+2/15*b*e^4*(d*x+c
)^2*(a+b*arcsinh(d*x+c))^(3/2)*(1+(d*x+c)^2)^(1/2)/d-1/10*b*e^4*(d*x+c)^4*(
a+b*arcsinh(d*x+c))^(3/2)*(1+(d*x+c)^2)^(1/2)/d+2/5*b^2*e^4*(d*x+c)*(a+b*ar
csinh(d*x+c))^(1/2)/d-1/15*b^2*e^4*(d*x+c)^3*(a+b*arcsinh(d*x+c))^(1/2)/d+3
/100*b^2*e^4*(d*x+c)^5*(a+b*arcsinh(d*x+c))^(1/2)/d
```

Rubi [A] time = 2.20, antiderivative size = 701, normalized size of antiderivative = 1.00, number of steps used = 46, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {5865, 12, 5663, 5758, 5717, 5653, 5779, 3308, 2180, 2204, 2205, 3312}

$$\frac{15\sqrt{\pi}\, b^{5/2}e^4 e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{128d}-\frac{\sqrt{3\pi}\, b^{5/2}e^4 e^{\frac{3a}{b}}\operatorname{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{1280d}-\frac{\sqrt{\frac{\pi}{3}}\, b^{5/2}e^4 e^{\frac{3a}{b}}\operatorname{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}}{\sqrt{b}}\right.}{240d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4*(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] (2*b^2*e^4*(c + d*x)*Sqrt[a + b*ArcSinh[c + d*x]])/(5*d) - (b^2*e^4*(c + d*
x)^3*Sqrt[a + b*ArcSinh[c + d*x]])/(15*d) + (3*b^2*e^4*(c + d*x)^5*Sqrt[a +
 b*ArcSinh[c + d*x]])/(100*d) - (4*b*e^4*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSi
nh[c + d*x])^(3/2))/(15*d) + (2*b*e^4*(c + d*x)^2*Sqrt[1 + (c + d*x)^2]*(a
+ b*ArcSinh[c + d*x])^(3/2))/(15*d) - (b*e^4*(c + d*x)^4*Sqrt[1 + (c + d*x)
^2]*(a + b*ArcSinh[c + d*x])^(3/2))/(10*d) + (e^4*(c + d*x)^5*(a + b*ArcSin
h[c + d*x])^(5/2))/(5*d) + (15*b^(5/2)*e^4*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*
ArcSinh[c + d*x]]/Sqrt[b]])/(128*d) - (b^(5/2)*e^4*E^((3*a)/b)*Sqrt[Pi/3]*E
rf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(240*d) - (b^(5/2)*e^4*
E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])
/(1280*d) + (3*b^(5/2)*e^4*E^((5*a)/b)*Sqrt[Pi/5]*Erf[(Sqrt[5]*Sqrt[a + b*A
rcSinh[c + d*x]])/Sqrt[b]])/(6400*d) - (15*b^(5/2)*e^4*Sqrt[Pi]*Erfi[Sqrt[a
 + b*ArcSinh[c + d*x]]/Sqrt[b]])/(128*d*E^(a/b)) + (b^(5/2)*e^4*Sqrt[Pi/3]*
Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(240*d*E^((3*a)/b)) +
 (b^(5/2)*e^4*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b
]])/(1280*d*E^((3*a)/b)) - (3*b^(5/2)*e^4*Sqrt[Pi/5]*Erfi[(Sqrt[5]*Sqrt[a +
 b*ArcSinh[c + d*x]])/Sqrt[b]])/(6400*d*E^((5*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

```
&& GtQ[m, 1] && IntegerQ[m]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^4 \left(a+b\sinh^{-1}(c+dx)\right)^{5/2} dx = \frac{\text{Subst}\left(\int e^4x^4\left(a+b\sinh^{-1}(x)\right)^{5/2} dx, x, c+dx\right)}{d}$$

$$= \frac{e^4\,\text{Subst}\left(\int x^4\left(a+b\sinh^{-1}(x)\right)^{5/2} dx, x, c+dx\right)}{d}$$

$$= \frac{e^4(c+dx)^5\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}{5d} - \frac{\left(be^4\right)\text{Subst}\left(\int \frac{x^5\left(a+b\sinh^{-1}(x)\right.}{\sqrt{1+x^2}}\right.}{2d}$$

$$= -\frac{be^4(c+dx)^4\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{10d} + \frac{e^4(c+dx)^5}{}$$

$$= \frac{3b^2e^4(c+dx)^5\sqrt{a+b\sinh^{-1}(c+dx)}}{100d} + \frac{2be^4(c+dx)^2\sqrt{1+(c+dx)}}{15}$$

$$= -\frac{b^2e^4(c+dx)^3\sqrt{a+b\sinh^{-1}(c+dx)}}{15d} + \frac{3b^2e^4(c+dx)^5\sqrt{a+b\sinh}}{100d}$$

$$= \frac{2b^2e^4(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{5d} - \frac{b^2e^4(c+dx)^3\sqrt{a+b\sinh^{-1}(}}{15d}$$

$$= \frac{2b^2e^4(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{5d} - \frac{b^2e^4(c+dx)^3\sqrt{a+b\sinh^{-1}(}}{15d}$$

$$= \frac{2b^2e^4(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{5d} - \frac{b^2e^4(c+dx)^3\sqrt{a+b\sinh^{-1}(}}{15d}$$

$$= \frac{2b^2e^4(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{5d} - \frac{b^2e^4(c+dx)^3\sqrt{a+b\sinh^{-1}(}}{15d}$$

$$= \frac{2b^2e^4(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{5d} - \frac{b^2e^4(c+dx)^3\sqrt{a+b\sinh^{-1}(}}{15d}$$

$$= \frac{2b^2e^4(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{5d} - \frac{b^2e^4(c+dx)^3\sqrt{a+b\sinh^{-1}(}}{15d}$$

Mathematica [A] time = 0.70, size = 342, normalized size = 0.49

$$-\frac{e^4e^{-\frac{5a}{b}}\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}\left(-33750e^{\frac{6a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{7}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)+27\sqrt{5}\sqrt{\frac{a}{b}+\sinh^{-1}(c\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^4*(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] -1/540000*(e^4*(a + b*ArcSinh[c + d*x])^(5/2)*(-33750*E^((6*a)/b)*Sqrt[-((a
 + b*ArcSinh[c + d*x])/b)]*Gamma[7/2, a/b + ArcSinh[c + d*x]] + 27*Sqrt[5]*
```

```
Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[7/2, (-5*(a + b*ArcSinh[c + d*x]))/b] -
625*Sqrt[3]*E^((2*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[7/2, (-3*(a + b*
ArcSinh[c + d*x]))/b] + 33750*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamm
a[7/2, -((a + b*ArcSinh[c + d*x])/b)] + 625*Sqrt[3]*E^((8*a)/b)*Sqrt[-((a +
 b*ArcSinh[c + d*x])/b)]*Gamma[7/2, (3*(a + b*ArcSinh[c + d*x]))/b] - 27*Sq
rt[5]*E^((10*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[7/2, (5*(a + b
*ArcSinh[c + d*x]))/b]))/(d*E^((5*a)/b)*(-((a + b*ArcSinh[c + d*x])^2/b^2))
^(3/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex + ce)^4 (b \operatorname{arsinh}(dx + c) + a)^{\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^4*(b*arcsinh(d*x + c) + a)^(5/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int (dex + ce)^4 \left(a + b \operatorname{arcsinh}(dx + c)\right)^{\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(5/2),x)
```

```
[Out] int((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex + ce)^4 (b \operatorname{arsinh}(dx + c) + a)^{\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^4*(b*arcsinh(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e + d\,e\,x)^4 \, \left(a + b \operatorname{asinh}(c + d\,x)\right)^{5/2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^4*(a + b*asinh(c + d*x))^(5/2),x)
```

```
[Out] int((c*e + d*e*x)^4*(a + b*asinh(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4*(a+b*asinh(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.193 $\int (ce+dex)^3 \left(a+b\sinh^{-1}(c+dx)\right)^{5/2} dx$

Optimal. Leaf size=455

$$-\frac{15\sqrt{\pi}\, b^{5/2}e^3e^{\frac{4a}{b}}\operatorname{erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16384d}+\frac{15\sqrt{\frac{\pi}{2}}\, b^{5/2}e^3e^{\frac{2a}{b}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{512d}-\frac{15\sqrt{\pi}\, b^{5/2}e^3e^{-\frac{4a}{b}}\operatorname{erfi}\left(\frac{2\sqrt{a+b}}{}\right.}{16384d}$$

```
[Out] -3/32*e^3*(a+b*arcsinh(d*x+c))^(5/2)/d+1/4*e^3*(d*x+c)^4*(a+b*arcsinh(d*x+c
))^(5/2)/d+15/1024*b^(5/2)*e^3*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^
(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d+15/1024*b^(5/2)*e^3*erfi(2^(1/2)*(a+b*arc
sinh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d/exp(2*a/b)-15/16384*b^(5/2)*
e^3*exp(4*a/b)*erf(2*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d-15/1638
4*b^(5/2)*e^3*erfi(2*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/exp(4*a
/b)+15/64*b*e^3*(d*x+c)*(a+b*arcsinh(d*x+c))^(3/2)*(1+(d*x+c)^2)^(1/2)/d-5/
32*b*e^3*(d*x+c)^3*(a+b*arcsinh(d*x+c))^(3/2)*(1+(d*x+c)^2)^(1/2)/d-225/204
8*b^2*e^3*(a+b*arcsinh(d*x+c))^(1/2)/d-45/256*b^2*e^3*(d*x+c)^2*(a+b*arcsin
h(d*x+c))^(1/2)/d+15/256*b^2*e^3*(d*x+c)^4*(a+b*arcsinh(d*x+c))^(1/2)/d
```

Rubi [A] time = 1.52, antiderivative size = 455, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {5865, 12, 5663, 5758, 5675, 5779, 3312, 3307, 2180, 2204, 2205}

$$-\frac{15\sqrt{\pi}\, b^{5/2}e^3e^{\frac{4a}{b}}\operatorname{Erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16384d}+\frac{15\sqrt{\frac{\pi}{2}}\, b^{5/2}e^3e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{512d}-\frac{15\sqrt{\pi}\, b^{5/2}e^3e^{-\frac{4a}{b}}\operatorname{Erfi}\left(\frac{2\sqrt{a+}}{}\right.}{16384d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3*(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] (-225*b^2*e^3*Sqrt[a + b*ArcSinh[c + d*x]])/(2048*d) - (45*b^2*e^3*(c + d*x
)^2*Sqrt[a + b*ArcSinh[c + d*x]])/(256*d) + (15*b^2*e^3*(c + d*x)^4*Sqrt[a
+ b*ArcSinh[c + d*x]])/(256*d) + (15*b*e^3*(c + d*x)*Sqrt[1 + (c + d*x)^2]*
(a + b*ArcSinh[c + d*x])^(3/2))/(64*d) - (5*b*e^3*(c + d*x)^3*Sqrt[1 + (c +
 d*x)^2]*(a + b*ArcSinh[c + d*x])^(3/2))/(32*d) - (3*e^3*(a + b*ArcSinh[c +
 d*x])^(5/2))/(32*d) + (e^3*(c + d*x)^4*(a + b*ArcSinh[c + d*x])^(5/2))/(4*
d) - (15*b^(5/2)*e^3*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcSinh[c + d*x
]])/Sqrt[b]])/(16384*d) + (15*b^(5/2)*e^3*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[
2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(512*d) - (15*b^(5/2)*e^3*Sqrt[P
i]*Erfi[(2*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(16384*d*E^((4*a)/b)) +
(15*b^(5/2)*e^3*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt
[b]])/(512*d*E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^3 \left(a+b\sinh^{-1}(c+dx)\right)^{5/2} dx = \frac{\text{Subst}\left(\int e^3x^3 \left(a+b\sinh^{-1}(x)\right)^{5/2} dx, x, c+dx\right)}{d}$$

$$= \frac{e^3 \text{Subst}\left(\int x^3 \left(a+b\sinh^{-1}(x)\right)^{5/2} dx, x, c+dx\right)}{d}$$

$$= \frac{e^3(c+dx)^4 \left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}{4d} - \frac{\left(5be^3\right) \text{Subst}\left(\int \frac{x^4\left(a+b\sinh\right.}{\sqrt{1+}}\right.}{8d}$$

$$= -\frac{5be^3(c+dx)^3\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{32d} + \frac{e^3(c+}{}$$

$$= \frac{15b^2e^3(c+dx)^4\sqrt{a+b\sinh^{-1}(c+dx)}}{256d} + \frac{15be^3(c+dx)\sqrt{1+(c+}}{}$$

$$= -\frac{45b^2e^3(c+dx)^2\sqrt{a+b\sinh^{-1}(c+dx)}}{256d} + \frac{15b^2e^3(c+dx)^4\sqrt{a+}}{256d}$$

$$= -\frac{45b^2e^3(c+dx)^2\sqrt{a+b\sinh^{-1}(c+dx)}}{256d} + \frac{15b^2e^3(c+dx)^4\sqrt{a+}}{256d}$$

$$= -\frac{45b^2e^3\sqrt{a+b\sinh^{-1}(c+dx)}}{2048d} - \frac{45b^2e^3(c+dx)^2\sqrt{a+b\sinh^{-1}(}}{256d}$$

$$= -\frac{225b^2e^3\sqrt{a+b\sinh^{-1}(c+dx)}}{2048d} - \frac{45b^2e^3(c+dx)^2\sqrt{a+b\sinh^{-}}}{256d}$$

$$= -\frac{225b^2e^3\sqrt{a+b\sinh^{-1}(c+dx)}}{2048d} - \frac{45b^2e^3(c+dx)^2\sqrt{a+b\sinh^{-}}}{256d}$$

$$= -\frac{225b^2e^3\sqrt{a+b\sinh^{-1}(c+dx)}}{2048d} - \frac{45b^2e^3(c+dx)^2\sqrt{a+b\sinh^{-}}}{256d}$$

$$= -\frac{225b^2e^3\sqrt{a+b\sinh^{-1}(c+dx)}}{2048d} - \frac{45b^2e^3(c+dx)^2\sqrt{a+b\sinh^{-}}}{256d}$$

Mathematica [A] time = 0.32, size = 223, normalized size = 0.49

$$-\frac{e^3e^{-\frac{4a}{b}}\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}\left(\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\Gamma\left(\frac{7}{2},-\frac{4\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)-16\sqrt{2}\,e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(c+}\right.}{2048d\left(-\frac{(a+}{}\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^3*(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] -1/2048*(e^3*(a + b*ArcSinh[c + d*x])^(5/2)*(Sqrt[a/b + ArcSinh[c + d*x]]*G
amma[7/2, (-4*(a + b*ArcSinh[c + d*x]))/b] - 16*Sqrt[2]*E^((2*a)/b)*Sqrt[a/
b + ArcSinh[c + d*x]]*Gamma[7/2, (-2*(a + b*ArcSinh[c + d*x]))/b] + E^((6*a
```

```
)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*(-16*Sqrt[2]*Gamma[7/2, (2*(a + b*
ArcSinh[c + d*x]))/b] + E^((2*a)/b)*Gamma[7/2, (4*(a + b*ArcSinh[c + d*x]))
/b])))/(d*E^((4*a)/b)*(-((a + b*ArcSinh[c + d*x])^2/b^2))^(3/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex + ce)^3 (b \operatorname{arsinh}(dx + c) + a)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^3*(b*arcsinh(d*x + c) + a)^(5/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int (dex + ce)^3 \left(a + b \operatorname{arcsinh}(dx + c)\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(5/2),x)
```

```
[Out] int((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex + ce)^3 (b \operatorname{arsinh}(dx + c) + a)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^3*(b*arcsinh(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(c\,e+d\,e\,x\right)}^3\,{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3*(a + b*asinh(c + d*x))^(5/2),x)
```

```
[Out] int((c*e + d*e*x)^3*(a + b*asinh(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3*(a+b*asinh(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.194 $\int (ce+dex)^2 \left(a+b\sinh^{-1}(c+dx)\right)^{5/2} dx$

Optimal. Leaf size=394

$$-\frac{15\sqrt{\pi}\,b^{5/2}e^2e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{64d}+\frac{5\sqrt{\frac{\pi}{3}}\,b^{5/2}e^2e^{\frac{3a}{b}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{576d}+\frac{15\sqrt{\pi}\,b^{5/2}e^2e^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{64d}$$

```
[Out] 1/3*e^2*(d*x+c)^3*(a+b*arcsinh(d*x+c))^(5/2)/d+5/1728*b^(5/2)*e^2*exp(3*a/b
)*erf(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/d-5/1728
*b^(5/2)*e^2*erfi(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1
/2)/d/exp(3*a/b)-15/64*b^(5/2)*e^2*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/
b^(1/2))*Pi^(1/2)/d+15/64*b^(5/2)*e^2*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/
2))*Pi^(1/2)/d/exp(a/b)+5/9*b*e^2*(a+b*arcsinh(d*x+c))^(3/2)*(1+(d*x+c)^2)^
(1/2)/d-5/18*b*e^2*(d*x+c)^2*(a+b*arcsinh(d*x+c))^(3/2)*(1+(d*x+c)^2)^(1/2)
/d-5/6*b^2*e^2*(d*x+c)*(a+b*arcsinh(d*x+c))^(1/2)/d+5/36*b^2*e^2*(d*x+c)^3*
(a+b*arcsinh(d*x+c))^(1/2)/d
```

Rubi [A] time = 1.24, antiderivative size = 394, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {5865, 12, 5663, 5758, 5717, 5653, 5779, 3308, 2180, 2204, 2205, 3312}

$$-\frac{15\sqrt{\pi}\,b^{5/2}e^2e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{64d}+\frac{5\sqrt{\frac{\pi}{3}}\,b^{5/2}e^2e^{\frac{3a}{b}}\operatorname{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{576d}+\frac{15\sqrt{\pi}\,b^{5/2}e^2e^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{64d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2*(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] (-5*b^2*e^2*(c + d*x)*Sqrt[a + b*ArcSinh[c + d*x]])/(6*d) + (5*b^2*e^2*(c +
 d*x)^3*Sqrt[a + b*ArcSinh[c + d*x]])/(36*d) + (5*b*e^2*Sqrt[1 + (c + d*x)^
2]*(a + b*ArcSinh[c + d*x])^(3/2))/(9*d) - (5*b*e^2*(c + d*x)^2*Sqrt[1 + (c
 + d*x)^2]*(a + b*ArcSinh[c + d*x])^(3/2))/(18*d) + (e^2*(c + d*x)^3*(a + b
*ArcSinh[c + d*x])^(5/2))/(3*d) - (15*b^(5/2)*e^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt
[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(64*d) + (5*b^(5/2)*e^2*E^((3*a)/b)*Sqrt
[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(576*d) + (15*b
^(5/2)*e^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(64*d*E^(a/
b)) - (5*b^(5/2)*e^2*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])
/Sqrt[b]])/(576*d*E^((3*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^2 \left(a+b\sinh^{-1}(c+dx)\right)^{5/2} dx = \frac{\text{Subst}\left(\int e^2x^2 \left(a+b\sinh^{-1}(x)\right)^{5/2} dx, x, c+dx\right)}{d}$$

$$= \frac{e^2\,\text{Subst}\left(\int x^2 \left(a+b\sinh^{-1}(x)\right)^{5/2} dx, x, c+dx\right)}{d}$$

$$= \frac{e^2(c+dx)^3 \left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}{3d} - \frac{\left(5be^2\right)\text{Subst}\left(\int \frac{x^3(a+b\sinh}{\sqrt{1+}} \right.}{6d}$$

$$= -\frac{5be^2(c+dx)^2\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{18d} + \frac{e^2(c+}{}$$

$$= \frac{5b^2e^2(c+dx)^3\sqrt{a+b\sinh^{-1}(c+dx)}}{36d} + \frac{5be^2\sqrt{1+(c+dx)^2}\left(a+\right.}{9d}$$

$$= -\frac{5b^2e^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{6d} + \frac{5b^2e^2(c+dx)^3\sqrt{a+b\,\text{si}}}{36d}$$

$$= -\frac{5b^2e^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{6d} + \frac{5b^2e^2(c+dx)^3\sqrt{a+b\,\text{si}}}{36d}$$

$$= -\frac{5b^2e^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{6d} + \frac{5b^2e^2(c+dx)^3\sqrt{a+b\,\text{si}}}{36d}$$

$$= -\frac{5b^2e^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{6d} + \frac{5b^2e^2(c+dx)^3\sqrt{a+b\,\text{si}}}{36d}$$

$$= -\frac{5b^2e^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{6d} + \frac{5b^2e^2(c+dx)^3\sqrt{a+b\,\text{si}}}{36d}$$

$$= -\frac{5b^2e^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{6d} + \frac{5b^2e^2(c+dx)^3\sqrt{a+b\,\text{si}}}{36d}$$

Mathematica [A] time = 0.42, size = 238, normalized size = 0.60

$$-\frac{e^2e^{-\frac{3a}{b}}\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}\left(81e^{\frac{4a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{7}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)+\sqrt{3}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx}\right.}{648}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^2*(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] -1/648*(e^2*(a + b*ArcSinh[c + d*x])^(5/2)*(81*E^((4*a)/b)*Sqrt[-((a + b*Ar
cSinh[c + d*x])/b)]*Gamma[7/2, a/b + ArcSinh[c + d*x]] + Sqrt[3]*Sqrt[a/b +
 ArcSinh[c + d*x]]*Gamma[7/2, (-3*(a + b*ArcSinh[c + d*x]))/b] - 81*E^((2*a
)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[7/2, -((a + b*ArcSinh[c + d*x])/b)]
 - Sqrt[3]*E^((6*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[7/2, (3*(a
 + b*ArcSinh[c + d*x]))/b]))/(d*E^((3*a)/b)*(-((a + b*ArcSinh[c + d*x])^2/b
^2))^(3/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex + ce)^2 (b \operatorname{arsinh}(dx + c) + a)^{\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2*(b*arcsinh(d*x + c) + a)^(5/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int (dex + ce)^2 (a + b \operatorname{arcsinh}(dx + c))^{\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(5/2),x)
```

```
[Out] int((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex + ce)^2 (b \operatorname{arsinh}(dx + c) + a)^{\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^2*(b*arcsinh(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e + d\,e\,x)^2 \, (a + b \operatorname{asinh}(c + d\,x))^{5/2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2*(a + b*asinh(c + d*x))^(5/2),x)
```

```
[Out] int((c*e + d*e*x)^2*(a + b*asinh(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^2 \left(\int a^2 c^2 \sqrt{a + b \operatorname{asinh}(c + dx)} \, dx + \int a^2 d^2 x^2 \sqrt{a + b \operatorname{asinh}(c + dx)} \, dx + \int b^2 c^2 \sqrt{a + b \operatorname{asinh}(c + dx)} \operatorname{asinh}^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2*(a+b*asinh(d*x+c))**(5/2),x)
```

```
[Out] e**2*(Integral(a**2*c**2*sqrt(a + b*asinh(c + d*x)), x) + Integral(a**2*d**
2*x**2*sqrt(a + b*asinh(c + d*x)), x) + Integral(b**2*c**2*sqrt(a + b*asinh
(c + d*x))*asinh(c + d*x)**2, x) + Integral(2*a*b*c**2*sqrt(a + b*asinh(c +
 d*x))*asinh(c + d*x), x) + Integral(2*a**2*c*d*x*sqrt(a + b*asinh(c + d*x)
), x) + Integral(b**2*d**2*x**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**
2, x) + Integral(2*a*b*d**2*x**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x),
 x) + Integral(2*b**2*c*d*x*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**2, x
) + Integral(4*a*b*c*d*x*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x), x))
```

3.195 $\int (ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}\,dx$

Optimal. Leaf size=262

$$-\frac{15\sqrt{\frac{\pi}{2}}\,b^{5/2}ee^{\frac{2a}{b}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{256d}-\frac{15\sqrt{\frac{\pi}{2}}\,b^{5/2}ee^{-\frac{2a}{b}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{256d}+\frac{15b^2e(c+dx)^2\sqrt{a+b\sinh^{-1}}}{32d}$$

```
[Out] 1/4*e*(a+b*arcsinh(d*x+c))^(5/2)/d+1/2*e*(d*x+c)^2*(a+b*arcsinh(d*x+c))^(5/
2)/d-15/512*b^(5/2)*e*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(
1/2))*2^(1/2)*Pi^(1/2)/d-15/512*b^(5/2)*e*erfi(2^(1/2)*(a+b*arcsinh(d*x+c))
^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d/exp(2*a/b)-5/8*b*e*(d*x+c)*(a+b*arcsinh(
d*x+c))^(3/2)*(1+(d*x+c)^2)^(1/2)/d+15/64*b^2*e*(a+b*arcsinh(d*x+c))^(1/2)/
d+15/32*b^2*e*(d*x+c)^2*(a+b*arcsinh(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.73, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {5865, 12, 5663, 5758, 5675, 5779, 3312, 3307, 2180, 2204, 2205}

$$-\frac{15\sqrt{\frac{\pi}{2}}\,b^{5/2}ee^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{256d}-\frac{15\sqrt{\frac{\pi}{2}}\,b^{5/2}ee^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{256d}+\frac{15b^2e(c+dx)^2\sqrt{a+b\sinh^{-1}}}{32d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] (15*b^2*e*Sqrt[a + b*ArcSinh[c + d*x]])/(64*d) + (15*b^2*e*(c + d*x)^2*Sqrt
[a + b*ArcSinh[c + d*x]])/(32*d) - (5*b*e*(c + d*x)*Sqrt[1 + (c + d*x)^2]*(
a + b*ArcSinh[c + d*x])^(3/2))/(8*d) + (e*(a + b*ArcSinh[c + d*x])^(5/2))/(
4*d) + (e*(c + d*x)^2*(a + b*ArcSinh[c + d*x])^(5/2))/(2*d) - (15*b^(5/2)*e
*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]
)/(256*d) - (15*b^(5/2)*e*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d
*x]])/Sqrt[b]])/(256*d*E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}dx = \frac{\text{Subst}\left(\int ex\left(a+b\sinh^{-1}(x)\right)^{5/2}dx,x,c+dx\right)}{d}$$

$$= \frac{e\,\text{Subst}\left(\int x\left(a+b\sinh^{-1}(x)\right)^{5/2}dx,x,c+dx\right)}{d}$$

$$= \frac{e(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}{2d} - \frac{(5be)\,\text{Subst}\left(\int \frac{x^2\left(a+b\sinh^{-1}(x)\right)^{3/2}}{\sqrt{1+x^2}}\right.}{4d}$$

$$= -\frac{5be(c+dx)\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{8d} + \frac{e(c+dx)^2\left(a\right.}{}$$

$$= \frac{15b^2e(c+dx)^2\sqrt{a+b\sinh^{-1}(c+dx)}}{32d} - \frac{5be(c+dx)\sqrt{1+(c+dx)^2}\left(\right.}{8d}$$

$$= \frac{15b^2e(c+dx)^2\sqrt{a+b\sinh^{-1}(c+dx)}}{32d} - \frac{5be(c+dx)\sqrt{1+(c+dx)^2}\left(\right.}{8d}$$

$$= \frac{15b^2e(c+dx)^2\sqrt{a+b\sinh^{-1}(c+dx)}}{32d} - \frac{5be(c+dx)\sqrt{1+(c+dx)^2}\left(\right.}{8d}$$

$$= \frac{15b^2e\sqrt{a+b\sinh^{-1}(c+dx)}}{64d} + \frac{15b^2e(c+dx)^2\sqrt{a+b\sinh^{-1}(c+dx)}}{32d}$$

$$= \frac{15b^2e\sqrt{a+b\sinh^{-1}(c+dx)}}{64d} + \frac{15b^2e(c+dx)^2\sqrt{a+b\sinh^{-1}(c+dx)}}{32d}$$

$$= \frac{15b^2e\sqrt{a+b\sinh^{-1}(c+dx)}}{64d} + \frac{15b^2e(c+dx)^2\sqrt{a+b\sinh^{-1}(c+dx)}}{32d}$$

$$= \frac{15b^2e\sqrt{a+b\sinh^{-1}(c+dx)}}{64d} + \frac{15b^2e(c+dx)^2\sqrt{a+b\sinh^{-1}(c+dx)}}{32d}$$

Mathematica [A] time = 0.08, size = 126, normalized size = 0.48

$$\frac{ee^{-\frac{2a}{b}}\left(b^3e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\Gamma\left(\frac{7}{2},\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)-b^3\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{7}{2},-\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)\right)}{32\sqrt{2}\,d\sqrt{a+b\sinh^{-1}(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] (e*(-(b^3*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[7/2, (-2*(a + b*ArcSinh
[c + d*x]))/b]) + b^3*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[7/2, (
2*(a + b*ArcSinh[c + d*x]))/b]))/(32*Sqrt[2]*d*E^((2*a)/b)*Sqrt[a + b*ArcSi
nh[c + d*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex + ce)(b \operatorname{arsinh}(dx + c) + a)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^(5/2), x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int (dex + ce)\,(a + b \operatorname{arcsinh}(dx + c))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(5/2),x)
```

```
[Out] int((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex + ce)(b \operatorname{arsinh}(dx + c) + a)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e + d\,e\,x)\,\left(a + b\operatorname{asinh}(c + d\,x)\right)^{5/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)*(a + b*asinh(c + d*x))^(5/2),x)
```

```
[Out] int((c*e + d*e*x)*(a + b*asinh(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e\left(\int a^2c\sqrt{a + b\operatorname{asinh}(c + dx)}\, dx + \int a^2dx\sqrt{a + b\operatorname{asinh}(c + dx)}\, dx + \int b^2c\sqrt{a + b\operatorname{asinh}(c + dx)}\operatorname{asinh}^2(c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*asinh(d*x+c))**(5/2),x)
```

```
[Out] e*(Integral(a**2*c*sqrt(a + b*asinh(c + d*x)), x) + Integral(a**2*d*x*sqrt(
a + b*asinh(c + d*x)), x) + Integral(b**2*c*sqrt(a + b*asinh(c + d*x))*asin
h(c + d*x)**2, x) + Integral(2*a*b*c*sqrt(a + b*asinh(c + d*x))*asinh(c + d
*x), x) + Integral(b**2*d*x*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**2, x
) + Integral(2*a*b*d*x*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x), x))
```

3.196 $\int \left(a+b\sinh^{-1}(c+dx)\right)^{5/2}\,dx$

Optimal. Leaf size=179

$$\frac{15\sqrt{\pi}\,b^{5/2}e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}-\frac{15\sqrt{\pi}\,b^{5/2}e^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}+\frac{15b^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{4d}-5$$

```
[Out] (d*x+c)*(a+b*arcsinh(d*x+c))^(5/2)/d+15/16*b^(5/2)*exp(a/b)*erf((a+b*arcsin
h(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d-15/16*b^(5/2)*erfi((a+b*arcsinh(d*x+c))
^(1/2)/b^(1/2))*Pi^(1/2)/d/exp(a/b)-5/2*b*(a+b*arcsinh(d*x+c))^(3/2)*(1+(d*
x+c)^2)^(1/2)/d+15/4*b^2*(d*x+c)*(a+b*arcsinh(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.40, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5863, 5653, 5717, 5779, 3308, 2180, 2204, 2205}

$$\frac{15\sqrt{\pi}\,b^{5/2}e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}-\frac{15\sqrt{\pi}\,b^{5/2}e^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}+\frac{15b^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{4d}-5$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] (15*b^2*(c + d*x)*Sqrt[a + b*ArcSinh[c + d*x]])/(4*d) - (5*b*Sqrt[1 + (c +
d*x)^2]*(a + b*ArcSinh[c + d*x])^(3/2))/(2*d) + ((c + d*x)*(a + b*ArcSinh[c
 + d*x])^(5/2))/d + (15*b^(5/2)*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c +
 d*x]]/Sqrt[b]])/(16*d) - (15*b^(5/2)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c +
d*x]]/Sqrt[b]])/(16*d*E^(a/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
```

```
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 5863

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \left(a+b\sinh^{-1}(c+dx)\right)^{5/2}\,dx = \frac{\text{Subst}\left(\int \left(a+b\sinh^{-1}(x)\right)^{5/2}\,dx,x,c+dx\right)}{d}$$

$$= \frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}{d} - \frac{(5b)\,\text{Subst}\left(\int \frac{x\left(a+b\sinh^{-1}(x)\right)^{3/2}}{\sqrt{1+x^2}}\,dx,x,c+\right.}{2d}$$

$$= -\frac{5b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{2d} + \frac{(c+dx)\left(a+b\sinh^{-1}(c+d\right.}{d}$$

$$= \frac{15b^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{4d} - \frac{5b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+d\right.}{2d}$$

$$= \frac{15b^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{4d} - \frac{5b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+d\right.}{2d}$$

$$= \frac{15b^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{4d} - \frac{5b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+d\right.}{2d}$$

$$= \frac{15b^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{4d} - \frac{5b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+d\right.}{2d}$$

$$= \frac{15b^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{4d} - \frac{5b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+d\right.}{2d}$$

Mathematica [B] time = 1.50, size = 458, normalized size = 2.56

$$\sqrt{b}\left(\sqrt{\pi}\left(4a^2-12ab+15b^2\right)\left(\sinh\left(\frac{a}{b}\right)+\cosh\left(\frac{a}{b}\right)\right)\operatorname{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)+\sqrt{\pi}\left(4a^2+12ab+15b^2\right)\left(\sinh\left(\frac{a}{b}\right)-\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] ((8*a^2*Sqrt[a + b*ArcSinh[c + d*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcSi
nh[c + d*x]])/Sqrt[a/b + ArcSinh[c + d*x]]) + Gamma[3/2, -((a + b*ArcSinh[c
 + d*x])/b)]/Sqrt[-((a + b*ArcSinh[c + d*x])/b)]))/E^(a/b) + 4*a*Sqrt[b]*(4
*Sqrt[b]*Sqrt[a + b*ArcSinh[c + d*x]]*(-3*Sqrt[1 + (c + d*x)^2] + 2*(c + d*
x)*ArcSinh[c + d*x]) + (2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x
]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + (-2*a + 3*b)*Sqrt[Pi]*Erf[Sqrt[a + b*
ArcSinh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])) + Sqrt[b]*(4*Sqrt[b]*Sq
rt[a + b*ArcSinh[c + d*x]]*(2*Sqrt[1 + (c + d*x)^2]*(a - 5*b*ArcSinh[c + d*
x]) + b*(c + d*x)*(15 + 4*ArcSinh[c + d*x]^2)) + (4*a^2 + 12*a*b + 15*b^2)*
Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]*(-Cosh[a/b] + Sinh[a/b]
) + (4*a^2 - 12*a*b + 15*b^2)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqr
t[b]]*(Cosh[a/b] + Sinh[a/b])))/(16*d)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\operatorname{arsinh}(dx+c)+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(5/2), x)
```

maple [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\operatorname{arcsinh}(dx+c))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^(5/2),x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\operatorname{arsinh}(dx+c)+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b\,\mathrm{asinh}\,(c + d\,x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^(5/2),x)
```

```
[Out] int((a + b*asinh(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\,\mathrm{asinh}\,(c + dx))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**(5/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))**(5/2), x)
```

3.197 $\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}{ce+dex}\,dx$

Optimal. Leaf size=29

$$\frac{\text{Int}\left(\frac{\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}{c+dx},x\right)}{e}$$

[Out] Unintegrable((a+b*arcsinh(d*x+c))^(5/2)/(d*x+c),x)/e

Rubi [A] time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}{ce+dex}\,dx$$

Verification is Not applicable to the result.

[In] Int[(a + b*ArcSinh[c + d*x])^(5/2)/(c*e + d*e*x),x]

[Out] Defer[Subst][Defer[Int][(a + b*ArcSinh[x])^(5/2)/x, x], x, c + d*x]/(d*e)

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}{ce+dex}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^{5/2}}{ex}\,dx,x,c+dx\right)}{d} = \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^{5/2}}{x}\,dx,x,c+dx\right)}{de}$$

Mathematica [A] time = 1.13, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}{ce+dex}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b*ArcSinh[c + d*x])^(5/2)/(c*e + d*e*x),x]

[Out] Integrate[(a + b*ArcSinh[c + d*x])^(5/2)/(c*e + d*e*x), x]

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsinh(d*x+c))^(5/2)/(d*e*x+c*e),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integ rate: implementation incomplete (constant residues)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{arsinh}\,(dx+c)+a)^{\frac{5}{2}}}{dex+ce}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(5/2)/(d*e*x+c*e),x, algorithm="giac")
[Out] integrate((b*arcsinh(d*x + c) + a)^(5/2)/(d*e*x + c*e), x)
```

maple [A] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{arcsinh}(dx + c))^{\frac{5}{2}}}{dex + ce}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^(5/2)/(d*e*x+c*e),x)
[Out] int((a+b*arcsinh(d*x+c))^(5/2)/(d*e*x+c*e),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arsinh}(dx + c) + a)^{\frac{5}{2}}}{dex + ce}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(5/2)/(d*e*x+c*e),x, algorithm="maxima")
[Out] integrate((b*arcsinh(d*x + c) + a)^(5/2)/(d*e*x + c*e), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(a + b \operatorname{asinh}(c + d\,x))^{5/2}}{c\,e + d\,e\,x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^(5/2)/(c*e + d*e*x),x)
[Out] int((a + b*asinh(c + d*x))^(5/2)/(c*e + d*e*x), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2\sqrt{a+b \operatorname{asinh}(c+dx)}}{c+dx}\, dx + \int \frac{b^2\sqrt{a+b \operatorname{asinh}(c+dx)} \operatorname{asinh}^2(c+dx)}{c+dx}\, dx + \int \frac{2ab\sqrt{a+b \operatorname{asinh}(c+dx)} \operatorname{asinh}(c+dx)}{c+dx}\, dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**(5/2)/(d*e*x+c*e),x)
[Out] (Integral(a**2*sqrt(a + b*asinh(c + d*x))/(c + d*x), x) + Integral(b**2*sqr
t(a + b*asinh(c + d*x))*asinh(c + d*x)**2/(c + d*x), x) + Integral(2*a*b*sq
rt(a + b*asinh(c + d*x))*asinh(c + d*x)/(c + d*x), x))/e
```

3.198 $\int (ce+dex)^4 \left(a+b\sinh^{-1}(c+dx)\right)^{7/2} dx$

Optimal. Leaf size=835

$$\frac{e^4\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}(c+dx)^5}{5d}+\frac{7b^2e^4\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}(c+dx)^5}{100d}-\frac{7be^4\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}\right.}{50d}$$

```
[Out] 14/15*b^2*e^4*(d*x+c)*(a+b*arcsinh(d*x+c))^(3/2)/d-7/45*b^2*e^4*(d*x+c)^3*(
a+b*arcsinh(d*x+c))^(3/2)/d+7/100*b^2*e^4*(d*x+c)^5*(a+b*arcsinh(d*x+c))^(3
/2)/d+1/5*e^4*(d*x+c)^5*(a+b*arcsinh(d*x+c))^(7/2)/d+21/320000*b^(7/2)*e^4*
exp(5*a/b)*erf(5^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*5^(1/2)*Pi^(1/2)
/d+21/320000*b^(7/2)*e^4*erfi(5^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*5
^(1/2)*Pi^(1/2)/d/exp(5*a/b)-35/13824*b^(7/2)*e^4*exp(3*a/b)*erf(3^(1/2)*(a
+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/d-35/13824*b^(7/2)*e^4*e
rfi(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/d/exp(3*a/
b)+105/256*b^(7/2)*e^4*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^
(1/2)/d+105/256*b^(7/2)*e^4*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/
2)/d/exp(a/b)-28/75*b*e^4*(a+b*arcsinh(d*x+c))^(5/2)*(1+(d*x+c)^2)^(1/2)/d+
14/75*b*e^4*(d*x+c)^2*(a+b*arcsinh(d*x+c))^(5/2)*(1+(d*x+c)^2)^(1/2)/d-7/50
*b*e^4*(d*x+c)^4*(a+b*arcsinh(d*x+c))^(5/2)*(1+(d*x+c)^2)^(1/2)/d-1813/1125
*b^3*e^4*(1+(d*x+c)^2)^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/d+119/1125*b^3*e^4*
(d*x+c)^2*(1+(d*x+c)^2)^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/d-21/1000*b^3*e^4*
(d*x+c)^4*(1+(d*x+c)^2)^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/d
```

Rubi [A] time = 3.29, antiderivative size = 835, normalized size of antiderivative = 1.00, number of steps used = 77, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {5865, 12, 5663, 5758, 5717, 5653, 5657, 3307, 2180, 2205, 2204, 5669, 5448}

$$\frac{e^4\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}(c+dx)^5}{5d}+\frac{7b^2e^4\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}(c+dx)^5}{100d}-\frac{7be^4\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}\right.}{50d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4*(a + b*ArcSinh[c + d*x])^(7/2),x]
```

```
[Out] (-1813*b^3*e^4*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/(1125*d)
 + (119*b^3*e^4*(c + d*x)^2*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*
x]])/(1125*d) - (21*b^3*e^4*(c + d*x)^4*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*Ar
cSinh[c + d*x]])/(1000*d) + (14*b^2*e^4*(c + d*x)*(a + b*ArcSinh[c + d*x])^
(3/2))/(15*d) - (7*b^2*e^4*(c + d*x)^3*(a + b*ArcSinh[c + d*x])^(3/2))/(45*
d) + (7*b^2*e^4*(c + d*x)^5*(a + b*ArcSinh[c + d*x])^(3/2))/(100*d) - (28*b
*e^4*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^(5/2))/(75*d) + (14*b*e
^4*(c + d*x)^2*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^(5/2))/(75*d)
 - (7*b*e^4*(c + d*x)^4*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^(5/2
))/(50*d) + (e^4*(c + d*x)^5*(a + b*ArcSinh[c + d*x])^(7/2))/(5*d) + (105*b
^(7/2)*e^4*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(256
*d) - (119*b^(7/2)*e^4*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSi
nh[c + d*x]])/Sqrt[b]])/(18000*d) - (21*b^(7/2)*e^4*E^((3*a)/b)*Sqrt[3*Pi]*
Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(64000*d) + (21*b^(7/2
)*e^4*E^((5*a)/b)*Sqrt[Pi/5]*Erf[(Sqrt[5]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqr
t[b]])/(64000*d) + (105*b^(7/2)*e^4*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*
x]]/Sqrt[b]])/(256*d*E^(a/b)) - (119*b^(7/2)*e^4*Sqrt[Pi/3]*Erfi[(Sqrt[3]*S
qrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(18000*d*E^((3*a)/b)) - (21*b^(7/2)*
e^4*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(64000
*d*E^((3*a)/b)) + (21*b^(7/2)*e^4*Sqrt[Pi/5]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcSi
nh[c + d*x]])/Sqrt[b]])/(64000*d*E^((5*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
```

```
x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^4 \left(a+b\sinh^{-1}(c+dx)\right)^{7/2} dx = \frac{\text{Subst}\left(\int e^4x^4\left(a+b\sinh^{-1}(x)\right)^{7/2} dx, x, c+dx\right)}{d}$$

$$= \frac{e^4\,\text{Subst}\left(\int x^4\left(a+b\sinh^{-1}(x)\right)^{7/2} dx, x, c+dx\right)}{d}$$

$$= \frac{e^4(c+dx)^5\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}{5d} - \frac{\left(7be^4\right)\text{Subst}\left(\int \frac{x^5(a+b\sinh}{\sqrt{1+}}\right.}{10d}$$

$$= -\frac{7be^4(c+dx)^4\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}{50d} + \frac{e^4(c+}{}$$

$$= \frac{7b^2e^4(c+dx)^5\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{100d} + \frac{14be^4(c+dx)^2\sqrt{1+(c}}{}$$

$$= -\frac{21b^3e^4(c+dx)^4\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{1000d} - \frac{7b^2e^4(}{}$$

$$= \frac{119b^3e^4(c+dx)^2\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{1125d} - \frac{21b^3e^4(}{}$$

$$= -\frac{1813b^3e^4\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{1125d} + \frac{119b^3e^4(c+d}{}$$

$$= -\frac{1813b^3e^4\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{1125d} + \frac{119b^3e^4(c+d}{}$$

$$= -\frac{1813b^3e^4\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{1125d} + \frac{119b^3e^4(c+d}{}$$

$$= -\frac{1813b^3e^4\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{1125d} + \frac{119b^3e^4(c+d}{}$$

$$= -\frac{1813b^3e^4\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{1125d} + \frac{119b^3e^4(c+d}{}$$

$$= -\frac{1813b^3e^4\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{1125d} + \frac{119b^3e^4(c+d}{}$$

Mathematica [A] time = 0.52, size = 343, normalized size = 0.41

$$\frac{be^4e^{-\frac{5a}{b}}\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}\left(506250e^{\frac{6a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{9}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)+81\sqrt{5}\sqrt{\frac{a}{b}+\sinh^{-1}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^4*(a + b*ArcSinh[c + d*x])^(7/2),x]
```

```
[Out] (b*e^4*(a + b*ArcSinh[c + d*x])^(5/2)*(506250*E^((6*a)/b)*Sqrt[-((a + b*Arc
Sinh[c + d*x])/b)]*Gamma[9/2, a/b + ArcSinh[c + d*x]] + 81*Sqrt[5]*Sqrt[a/b
 + ArcSinh[c + d*x]]*Gamma[9/2, (-5*(a + b*ArcSinh[c + d*x]))/b] - 3125*Sqr
t[3]*E^((2*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[9/2, (-3*(a + b*ArcSinh
[c + d*x]))/b] + 506250*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[9/2,
 -((a + b*ArcSinh[c + d*x])/b)] - 3125*Sqrt[3]*E^((8*a)/b)*Sqrt[-((a + b*Ar
cSinh[c + d*x])/b)]*Gamma[9/2, (3*(a + b*ArcSinh[c + d*x]))/b] + 81*Sqrt[5]
*E^((10*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[9/2, (5*(a + b*ArcS
inh[c + d*x]))/b]))/(8100000*d*E^((5*a)/b)*(-((a + b*ArcSinh[c + d*x])^2/b^
2))^(3/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int (dex + ce)^4 (a + b \operatorname{arcsinh}(dx + c))^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(7/2),x)
```

```
[Out] int((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(7/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex + ce)^4 (b \operatorname{arsinh}(dx + c) + a)^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^4*(b*arcsinh(d*x + c) + a)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e + d\,e\,x)^4\,(a + b\,\operatorname{asinh}(c + d\,x))^{7/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^4*(a + b*asinh(c + d*x))^(7/2),x)
```

```
[Out] int((c*e + d*e*x)^4*(a + b*asinh(c + d*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4*(a+b*asinh(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

3.199 $\int (ce+dex)^3\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}\,dx$

Optimal. Leaf size=547

$$-\frac{105\sqrt{\pi}\,b^{7/2}e^3e^{\frac{4a}{b}}\operatorname{erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{131072d}+\frac{105\sqrt{\frac{\pi}{2}}\,b^{7/2}e^3e^{\frac{2a}{b}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2048d}+\frac{105\sqrt{\pi}\,b^{7/2}e^3e^{-\frac{4a}{b}}\operatorname{erfi}\left(\frac{2\sqrt{a+b}\cdots}{}\right)}{131072d}$$

```
[Out] -525/2048*b^2*e^3*(a+b*arcsinh(d*x+c))^(3/2)/d-105/256*b^2*e^3*(d*x+c)^2*(a
+b*arcsinh(d*x+c))^(3/2)/d+35/256*b^2*e^3*(d*x+c)^4*(a+b*arcsinh(d*x+c))^(3
/2)/d-3/32*e^3*(a+b*arcsinh(d*x+c))^(7/2)/d+1/4*e^3*(d*x+c)^4*(a+b*arcsinh(
d*x+c))^(7/2)/d+105/4096*b^(7/2)*e^3*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*
x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d-105/4096*b^(7/2)*e^3*erfi(2^(1/2)*(
a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d/exp(2*a/b)-105/131072
*b^(7/2)*e^3*exp(4*a/b)*erf(2*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/
d+105/131072*b^(7/2)*e^3*erfi(2*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2
)/d/exp(4*a/b)+21/64*b*e^3*(d*x+c)*(a+b*arcsinh(d*x+c))^(5/2)*(1+(d*x+c)^2)
^(1/2)/d-7/32*b*e^3*(d*x+c)^3*(a+b*arcsinh(d*x+c))^(5/2)*(1+(d*x+c)^2)^(1/2
)/d+1575/4096*b^3*e^3*(d*x+c)*(1+(d*x+c)^2)^(1/2)*(a+b*arcsinh(d*x+c))^(1/2
)/d-105/2048*b^3*e^3*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)*(a+b*arcsinh(d*x+c))^(1/
2)/d
```

Rubi [A] time = 2.14, antiderivative size = 547, normalized size of antiderivative = 1.00, number of steps used = 42, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {5865, 12, 5663, 5758, 5675, 5669, 5448, 3308, 2180, 2204, 2205}

$$-\frac{105\sqrt{\pi}\,b^{7/2}e^3e^{\frac{4a}{b}}\operatorname{Erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{131072d}+\frac{105\sqrt{\frac{\pi}{2}}\,b^{7/2}e^3e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2048d}+\frac{105\sqrt{\pi}\,b^{7/2}e^3e^{-\frac{4a}{b}}\operatorname{Erfi}\left(\frac{2\sqrt{a}\cdots}{}\right)}{131072d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3*(a + b*ArcSinh[c + d*x])^(7/2),x]
```

```
[Out] (1575*b^3*e^3*(c + d*x)*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])
/(4096*d) - (105*b^3*e^3*(c + d*x)^3*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSi
nh[c + d*x]])/(2048*d) - (525*b^2*e^3*(a + b*ArcSinh[c + d*x])^(3/2))/(2048
*d) - (105*b^2*e^3*(c + d*x)^2*(a + b*ArcSinh[c + d*x])^(3/2))/(256*d) + (3
5*b^2*e^3*(c + d*x)^4*(a + b*ArcSinh[c + d*x])^(3/2))/(256*d) + (21*b*e^3*(
c + d*x)*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^(5/2))/(64*d) - (7*
b*e^3*(c + d*x)^3*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^(5/2))/(32
*d) - (3*e^3*(a + b*ArcSinh[c + d*x])^(7/2))/(32*d) + (e^3*(c + d*x)^4*(a +
 b*ArcSinh[c + d*x])^(7/2))/(4*d) - (105*b^(7/2)*e^3*E^((4*a)/b)*Sqrt[Pi]*E
rf[(2*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(131072*d) + (105*b^(7/2)*e^3
*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]
)/(2048*d) + (105*b^(7/2)*e^3*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcSinh[c + d*x]]
)/Sqrt[b]])/(131072*d*E^((4*a)/b)) - (105*b^(7/2)*e^3*Sqrt[Pi/2]*Erfi[(Sqrt
[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(2048*d*E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
```

```
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^3 \left(a+b\sinh^{-1}(c+dx)\right)^{7/2} dx = \frac{\text{Subst}\left(\int e^3x^3 \left(a+b\sinh^{-1}(x)\right)^{7/2} dx, x, c+dx\right)}{d}$$

$$= \frac{e^3\,\text{Subst}\left(\int x^3 \left(a+b\sinh^{-1}(x)\right)^{7/2} dx, x, c+dx\right)}{d}$$

$$= \frac{e^3(c+dx)^4 \left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}{4d} - \frac{\left(7be^3\right)\text{Subst}\left(\int \frac{x^4\left(a+b\sinh^{-1}(\cdots\right.}{\sqrt{1+x^2}}\cdots\right.}{8d}$$

$$= -\frac{7be^3(c+dx)^3\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}{32d} + \frac{e^3(c+dx\cdots}{\cdots}$$

$$= \frac{35b^2e^3(c+dx)^4\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{256d} + \frac{21be^3(c+dx)\sqrt{1+(c+\cdots}}{\cdots}$$

$$= -\frac{105b^3e^3(c+dx)^3\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{2048d} - \frac{105b^2e^3(\cdots}{\cdots}$$

$$= \frac{1575b^3e^3(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{4096d} - \frac{105b^3e^3(c\cdots}{\cdots}$$

$$= \frac{1575b^3e^3(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{4096d} - \frac{105b^3e^3(c\cdots}{\cdots}$$

$$= \frac{1575b^3e^3(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{4096d} - \frac{105b^3e^3(c\cdots}{\cdots}$$

$$= \frac{1575b^3e^3(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{4096d} - \frac{105b^3e^3(c\cdots}{\cdots}$$

$$= \frac{1575b^3e^3(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{4096d} - \frac{105b^3e^3(c\cdots}{\cdots}$$

$$= \frac{1575b^3e^3(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{4096d} - \frac{105b^3e^3(c\cdots}{\cdots}$$

$$= \frac{1575b^3e^3(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{4096d} - \frac{105b^3e^3(c\cdots}{\cdots}$$

Mathematica [A] time = 0.32, size = 225, normalized size = 0.41

$$-\frac{be^3e^{-\frac{4a}{b}}\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}\left(-\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\Gamma\left(\frac{9}{2},-\frac{4\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)+32\sqrt{2}\,e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(c}\right.}{8192d\left(-\frac{(a}{}\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^3*(a + b*ArcSinh[c + d*x])^(7/2),x]
```

```
[Out] -1/8192*(b*e^3*(a + b*ArcSinh[c + d*x])^(5/2)*(-(Sqrt[a/b + ArcSinh[c + d*x
]]*Gamma[9/2, (-4*(a + b*ArcSinh[c + d*x]))/b]) + 32*Sqrt[2]*E^((2*a)/b)*Sq
rt[a/b + ArcSinh[c + d*x]]*Gamma[9/2, (-2*(a + b*ArcSinh[c + d*x]))/b] + E^
((6*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*(-32*Sqrt[2]*Gamma[9/2, (2*(a
 + b*ArcSinh[c + d*x]))/b] + E^((2*a)/b)*Gamma[9/2, (4*(a + b*ArcSinh[c + d
*x]))/b])))/(d*E^((4*a)/b)*(-((a + b*ArcSinh[c + d*x])^2/b^2))^(3/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int (dex+ce)^3\left(a+b\,\mathrm{arcsinh}\,(dx+c)\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(7/2),x)
```

```
[Out] int((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(7/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex+ce)^3(b\,\mathrm{arsinh}\,(dx+c)+a)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^3*(b*arcsinh(d*x + c) + a)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e+d\,e\,x)^3\,\left(a+b\,\mathrm{asinh}\,(c+d\,x)\right)^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3*(a + b*asinh(c + d*x))^(7/2),x)
[Out] int((c*e + d*e*x)^3*(a + b*asinh(c + d*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3*(a+b*asinh(d*x+c))**(7/2),x)
[Out] Timed out
```

3.200 $\int (ce+dex)^2 \left(a+b\sinh^{-1}(c+dx)\right)^{7/2}\, dx$

Optimal. Leaf size=481

$$-\frac{105\sqrt{\pi}\, b^{7/2}e^2e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{128d}+\frac{35\sqrt{\frac{\pi}{3}}\, b^{7/2}e^2e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3456d}-\frac{105\sqrt{\pi}\, b^{7/2}e^2e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b}}{}\right.}{128d}$$

```
[Out] -35/18*b^2*e^2*(d*x+c)*(a+b*arcsinh(d*x+c))^(3/2)/d+35/108*b^2*e^2*(d*x+c)^
3*(a+b*arcsinh(d*x+c))^(3/2)/d+1/3*e^2*(d*x+c)^3*(a+b*arcsinh(d*x+c))^(7/2)
/d+35/10368*b^(7/2)*e^2*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b
^(1/2))*3^(1/2)*Pi^(1/2)/d+35/10368*b^(7/2)*e^2*erfi(3^(1/2)*(a+b*arcsinh(d
*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/d/exp(3*a/b)-105/128*b^(7/2)*e^2*exp
(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d-105/128*b^(7/2)*e^
2*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/exp(a/b)+7/9*b*e^2*(a
+b*arcsinh(d*x+c))^(5/2)*(1+(d*x+c)^2)^(1/2)/d-7/18*b*e^2*(d*x+c)^2*(a+b*ar
csinh(d*x+c))^(5/2)*(1+(d*x+c)^2)^(1/2)/d+175/54*b^3*e^2*(1+(d*x+c)^2)^(1/2
)*(a+b*arcsinh(d*x+c))^(1/2)/d-35/216*b^3*e^2*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)
*(a+b*arcsinh(d*x+c))^(1/2)/d
```

Rubi [A] time = 1.68, antiderivative size = 481, normalized size of antiderivative = 1.00, number of steps used = 35, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {5865, 12, 5663, 5758, 5717, 5653, 5657, 3307, 2180, 2205, 2204, 5669, 5448}

$$-\frac{105\sqrt{\pi}\, b^{7/2}e^2e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{128d}+\frac{35\sqrt{\frac{\pi}{3}}\, b^{7/2}e^2e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3456d}-\frac{105\sqrt{\pi}\, b^{7/2}e^2e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+}}{}\right.}{128d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2*(a + b*ArcSinh[c + d*x])^(7/2),x]
```

```
[Out] (175*b^3*e^2*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/(54*d) - (
35*b^3*e^2*(c + d*x)^2*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/
(216*d) - (35*b^2*e^2*(c + d*x)*(a + b*ArcSinh[c + d*x])^(3/2))/(18*d) + (3
5*b^2*e^2*(c + d*x)^3*(a + b*ArcSinh[c + d*x])^(3/2))/(108*d) + (7*b*e^2*Sq
rt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^(5/2))/(9*d) - (7*b*e^2*(c + d
*x)^2*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^(5/2))/(18*d) + (e^2*(
c + d*x)^3*(a + b*ArcSinh[c + d*x])^(7/2))/(3*d) - (105*b^(7/2)*e^2*E^(a/b)
*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(128*d) + (35*b^(7/2)*
e^2*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[
b]])/(3456*d) - (105*b^(7/2)*e^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]
/Sqrt[b]])/(128*d*E^(a/b)) + (35*b^(7/2)*e^2*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[
a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(3456*d*E^((3*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^2 \left(a+b\sinh^{-1}(c+dx)\right)^{7/2} dx = \frac{\text{Subst}\left(\int e^2x^2 \left(a+b\sinh^{-1}(x)\right)^{7/2} dx, x, c+dx\right)}{d}$$

$$= \frac{e^2\,\text{Subst}\left(\int x^2 \left(a+b\sinh^{-1}(x)\right)^{7/2} dx, x, c+dx\right)}{d}$$

$$= \frac{e^2(c+dx)^3 \left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}{3d} - \frac{\left(7be^2\right)\text{Subst}\left(\int \frac{x^3\left(a+b\sinh^{-1}(\right.}{\sqrt{1+x^2}}\right.}{6d}$$

$$= -\frac{7be^2(c+dx)^2\sqrt{1+(c+dx)^2} \left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}{18d} + \frac{e^2(c+dx}{}$$

$$= \frac{35b^2e^2(c+dx)^3 \left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{108d} + \frac{7be^2\sqrt{1+(c+dx)^2} \left(a+\right.}{9d}$$

$$= -\frac{35b^3e^2(c+dx)^2\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{216d} - \frac{35b^2e^2(c}{}$$

$$= \frac{175b^3e^2\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{54d} - \frac{35b^3e^2(c+dx)^2\sqrt{1}}{}$$

$$= \frac{175b^3e^2\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{54d} - \frac{35b^3e^2(c+dx)^2\sqrt{1}}{}$$

$$= \frac{175b^3e^2\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{54d} - \frac{35b^3e^2(c+dx)^2\sqrt{1}}{}$$

$$= \frac{175b^3e^2\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{54d} - \frac{35b^3e^2(c+dx)^2\sqrt{1}}{}$$

$$= \frac{175b^3e^2\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{54d} - \frac{35b^3e^2(c+dx)^2\sqrt{1}}{}$$

$$= \frac{175b^3e^2\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{54d} - \frac{35b^3e^2(c+dx)^2\sqrt{1}}{}$$

Mathematica [A] time = 0.34, size = 238, normalized size = 0.49

$$\frac{be^2e^{-\frac{3a}{b}} \left(a+b\sinh^{-1}(c+dx)\right)^{5/2} \left(-243e^{\frac{4a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{9}{2}, \frac{a}{b}+\sinh^{-1}(c+dx)\right) + \sqrt{3}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx\right.}{1944d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^2*(a + b*ArcSinh[c + d*x])^(7/2),x]
```

```
[Out] (b*e^2*(a + b*ArcSinh[c + d*x])^(5/2)*(-243*E^((4*a)/b)*Sqrt[-((a + b*ArcSi
nh[c + d*x])/b)]*Gamma[9/2, a/b + ArcSinh[c + d*x]] + Sqrt[3]*Sqrt[a/b + Ar
```

```
cSinh[c + d*x]]*Gamma[9/2, (-3*(a + b*ArcSinh[c + d*x]))/b] - 243*E^((2*a)/
b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[9/2, -((a + b*ArcSinh[c + d*x])/b)] +
 Sqrt[3]*E^((6*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[9/2, (3*(a +
 b*ArcSinh[c + d*x]))/b]))/(1944*d*E^((3*a)/b)*(-((a + b*ArcSinh[c + d*x])^
2/b^2))^(3/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int (dex + ce)^2 (a + b \operatorname{arcsinh}(dx + c))^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(7/2),x)
```

```
[Out] int((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(7/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex + ce)^2 (b \operatorname{arsinh}(dx + c) + a)^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^2*(b*arcsinh(d*x + c) + a)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e + d\,e\,x)^2\, (a + b \operatorname{asinh}(c + d\,x))^{7/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2*(a + b*asinh(c + d*x))^(7/2),x)
```

```
[Out] int((c*e + d*e*x)^2*(a + b*asinh(c + d*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2*(a+b*asinh(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

3.201 $\int (ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}\,dx$

Optimal. Leaf size=305

$$-\frac{105\sqrt{\frac{\pi}{2}}\,b^{7/2}e e^{\frac{2a}{b}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{1024d}+\frac{105\sqrt{\frac{\pi}{2}}\,b^{7/2}e e^{-\frac{2a}{b}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{1024d}-\frac{105b^3e(c+dx)\sqrt{(c+dx)^2}}{12}$$

```
[Out] 35/64*b^2*e*(a+b*arcsinh(d*x+c))^(3/2)/d+35/32*b^2*e*(d*x+c)^2*(a+b*arcsinh
(d*x+c))^(3/2)/d+1/4*e*(a+b*arcsinh(d*x+c))^(7/2)/d+1/2*e*(d*x+c)^2*(a+b*ar
csinh(d*x+c))^(7/2)/d-105/2048*b^(7/2)*e*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsin
h(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d+105/2048*b^(7/2)*e*erfi(2^(1/2)
*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d/exp(2*a/b)-7/8*b*e*
(d*x+c)*(a+b*arcsinh(d*x+c))^(5/2)*(1+(d*x+c)^2)^(1/2)/d-105/128*b^3*e*(d*x
+c)*(1+(d*x+c)^2)^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.80, antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {5865, 12, 5663, 5758, 5675, 5669, 5448, 3308, 2180, 2204, 2205}

$$-\frac{105\sqrt{\frac{\pi}{2}}\,b^{7/2}e e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{1024d}+\frac{105\sqrt{\frac{\pi}{2}}\,b^{7/2}e e^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{1024d}-\frac{105b^3e(c+dx)\sqrt{(c+dx)^2}}{12}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(7/2),x]
```

```
[Out] (-105*b^3*e*(c + d*x)*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/(
128*d) + (35*b^2*e*(a + b*ArcSinh[c + d*x])^(3/2))/(64*d) + (35*b^2*e*(c +
d*x)^2*(a + b*ArcSinh[c + d*x])^(3/2))/(32*d) - (7*b*e*(c + d*x)*Sqrt[1 + (
c + d*x)^2]*(a + b*ArcSinh[c + d*x])^(5/2))/(8*d) + (e*(a + b*ArcSinh[c + d
*x])^(7/2))/(4*d) + (e*(c + d*x)^2*(a + b*ArcSinh[c + d*x])^(7/2))/(2*d) -
(105*b^(7/2)*e*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d
*x]])/Sqrt[b]])/(1024*d) + (105*b^(7/2)*e*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a +
 b*ArcSinh[c + d*x]])/Sqrt[b]])/(1024*d*E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}dx = \frac{\text{Subst}\left(\int ex\left(a+b\sinh^{-1}(x)\right)^{7/2}dx,x,c+dx\right)}{d}$$

$$= \frac{e\,\text{Subst}\left(\int x\left(a+b\sinh^{-1}(x)\right)^{7/2}dx,x,c+dx\right)}{d}$$

$$= \frac{e(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}{2d} - \frac{(7be)\,\text{Subst}\left(\int \frac{x^2\left(a+b\sinh^{-1}(x)\right)^{5/2}}{\sqrt{1+x^2}}\right.}{4d}$$

$$= -\frac{7be(c+dx)\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}{8d} + \frac{e(c+dx)^2\left(a\right.}{}$$

$$= \frac{35b^2e(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{32d} - \frac{7be(c+dx)\sqrt{1+(c+dx)^2}}{8}$$

$$= -\frac{105b^3e(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{128d} + \frac{35b^2e(c+d}{}$$

$$= -\frac{105b^3e(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{128d} + \frac{35b^2e\left(a+\right.}{}$$

$$= -\frac{105b^3e(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{128d} + \frac{35b^2e\left(a+\right.}{}$$

$$= -\frac{105b^3e(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{128d} + \frac{35b^2e\left(a+\right.}{}$$

$$= -\frac{105b^3e(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{128d} + \frac{35b^2e\left(a+\right.}{}$$

$$= -\frac{105b^3e(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{128d} + \frac{35b^2e\left(a+\right.}{}$$

$$= -\frac{105b^3e(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{128d} + \frac{35b^2e\left(a+\right.}{}$$

<u>**Mathematica**</u> **[A]** time = 0.08, size = 125, normalized size = 0.41

$$\frac{ee^{-\frac{2a}{b}}\left(b^4\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{9}{2},-\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)+b^4e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\Gamma\left(\frac{9}{2},\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)\right)}{64\sqrt{2}\,d\sqrt{a+b\sinh^{-1}(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(7/2),x]
```

```
[Out] (e*(b^4*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[9/2, (-2*(a + b*ArcSinh[c
 + d*x]))/b] + b^4*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[9/2, (2*(
a + b*ArcSinh[c + d*x]))/b]))/(64*Sqrt[2]*d*E^((2*a)/b)*Sqrt[a + b*ArcSinh[
c + d*x]])
```

<u>**fricas**</u> **[F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex + ce)(b \operatorname{arsinh}(dx + c) + a)^{\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^(7/2), x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int (dex + ce)(a + b \operatorname{arcsinh}(dx + c))^{\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(7/2),x)
```

```
[Out] int((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(7/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex + ce)(b \operatorname{arsinh}(dx + c) + a)^{\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e + d\,e\,x)\,\left(a + b\,\mathrm{asinh}\left(c + d\,x\right)\right)^{7/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)*(a + b*asinh(c + d*x))^(7/2),x)
```

```
[Out] int((c*e + d*e*x)*(a + b*asinh(c + d*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*asinh(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

3.202 $\int \left(a+b\sinh^{-1}(c+dx)\right)^{7/2}\,dx$

Optimal. Leaf size=216

$$\frac{105\sqrt{\pi}\,b^{7/2}e^{a/b}\mathrm{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32d}+\frac{105\sqrt{\pi}\,b^{7/2}e^{-\frac{a}{b}}\mathrm{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32d}-\frac{105b^3\sqrt{(c+dx)^2+1}\,\sqrt{a+b\sinh^{-1}(}}{8d}$$

```
[Out] 35/4*b^2*(d*x+c)*(a+b*arcsinh(d*x+c))^(3/2)/d+(d*x+c)*(a+b*arcsinh(d*x+c))^
(7/2)/d+105/32*b^(7/2)*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^
(1/2)/d+105/32*b^(7/2)*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/
exp(a/b)-7/2*b*(a+b*arcsinh(d*x+c))^(5/2)*(1+(d*x+c)^2)^(1/2)/d-105/8*b^3*(
1+(d*x+c)^2)^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.42, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5863, 5653, 5717, 5657, 3307, 2180, 2205, 2204}

$$\frac{105\sqrt{\pi}\,b^{7/2}e^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32d}+\frac{105\sqrt{\pi}\,b^{7/2}e^{-\frac{a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32d}-\frac{105b^3\sqrt{(c+dx)^2+1}\,\sqrt{a+b\sinh^{-1}}}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^(7/2),x]
```

```
[Out] (-105*b^3*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/(8*d) + (35*b
^2*(c + d*x)*(a + b*ArcSinh[c + d*x])^(3/2))/(4*d) - (7*b*Sqrt[1 + (c + d*x
)^2]*(a + b*ArcSinh[c + d*x])^(5/2))/(2*d) + ((c + d*x)*(a + b*ArcSinh[c +
d*x])^(7/2))/d + (105*b^(7/2)*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d
*x]]/Sqrt[b]])/(32*d) + (105*b^(7/2)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d
*x]]/Sqrt[b]])/(32*d*E^(a/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5863

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \left(a+b\sinh^{-1}(c+dx)\right)^{7/2}\,dx = \frac{\mathrm{Subst}\left(\int \left(a+b\sinh^{-1}(x)\right)^{7/2}\,dx, x, c+dx\right)}{d}$$

$$= \frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}{d} - \frac{(7b)\,\mathrm{Subst}\left(\int \frac{x\left(a+b\sinh^{-1}(x)\right)^{5/2}}{\sqrt{1+x^2}}\,dx, x, c+dx\right.}{2d}$$

$$= -\frac{7b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}{2d} + \frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{7}}{d}$$

$$= \frac{35b^2(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{4d} - \frac{7b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx\right.}{2d}$$

$$= -\frac{105b^3\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{8d} + \frac{35b^2(c+dx)\left(a+b\sinh^{-1}(c+\right.}{4d}$$

$$= -\frac{105b^3\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{8d} + \frac{35b^2(c+dx)\left(a+b\sinh^{-1}(c+\right.}{4d}$$

$$= -\frac{105b^3\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{8d} + \frac{35b^2(c+dx)\left(a+b\sinh^{-1}(c+\right.}{4d}$$

$$= -\frac{105b^3\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{8d} + \frac{35b^2(c+dx)\left(a+b\sinh^{-1}(c+\right.}{4d}$$

$$= -\frac{105b^3\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{8d} + \frac{35b^2(c+dx)\left(a+b\sinh^{-1}(c+\right.}{4d}$$

Mathematica [B] time = 4.68, size = 698, normalized size = 3.23

$$16a^3e^{-\frac{a}{b}}\sqrt{a+b\sinh^{-1}(c+dx)}\left(\frac{\Gamma\left(\frac{3}{2},-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}} - \frac{e^{\frac{2a}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)}{\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}}\right) + 6a\sqrt{b}\left(\sqrt{\pi}\left(4a^2-12ab+15b^2\right)\left(\sin\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^(7/2),x]
```

```
[Out] ((16*a^3*Sqrt[a + b*ArcSinh[c + d*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcS
inh[c + d*x]])/Sqrt[a/b + ArcSinh[c + d*x]]) + Gamma[3/2, -((a + b*ArcSinh[
c + d*x])/b)]/Sqrt[-((a + b*ArcSinh[c + d*x])/b)]))/E^(a/b) + 12*a^2*Sqrt[b
]*(4*Sqrt[b]*Sqrt[a + b*ArcSinh[c + d*x]]*(-3*Sqrt[1 + (c + d*x)^2] + 2*(c
+ d*x)*ArcSinh[c + d*x]) + (2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c +
 d*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + (-2*a + 3*b)*Sqrt[Pi]*Erf[Sqrt[a
+ b*ArcSinh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])) + 6*a*Sqrt[b]*(4*Sq
rt[b]*Sqrt[a + b*ArcSinh[c + d*x]]*(2*Sqrt[1 + (c + d*x)^2]*(a - 5*b*ArcSin
h[c + d*x]) + b*(c + d*x)*(15 + 4*ArcSinh[c + d*x]^2)) + (4*a^2 + 12*a*b +
15*b^2)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]*(-Cosh[a/b] + S
inh[a/b]) + (4*a^2 - 12*a*b + 15*b^2)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d
*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])) + Sqrt[b]*(4*Sqrt[b]*Sqrt[a + b*ArcS
inh[c + d*x]]*(2*b*(c + d*x)*(-10*a + 35*b*ArcSinh[c + d*x] + 4*b*ArcSinh[c
```

```
 + d*x]^3) + Sqrt[1 + (c + d*x)^2]*(-4*a^2 + 4*a*b*ArcSinh[c + d*x] - 7*b^2
*(15 + 4*ArcSinh[c + d*x]^2))) + (8*a^3 + 36*a^2*b + 90*a*b^2 + 105*b^3)*Sq
rt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) +
 (-8*a^3 + 36*a^2*b - 90*a*b^2 + 105*b^3)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c
 + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])))/(32*d)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arsinh}(dx + c) + a)^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(7/2), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{arcsinh}(dx + c))^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^(7/2),x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^(7/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arsinh}(dx + c) + a)^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b \operatorname{asinh}(c + d\,x))^{7/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^(7/2),x)
```

```
[Out] int((a + b*asinh(c + d*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

3.203 $\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}{ce+dex}\,dx$

Optimal. Leaf size=29

$$\frac{\text{Int}\left(\frac{\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}{c+dx},x\right)}{e}$$

```
[Out] Unintegrable((a+b*arcsinh(d*x+c))^(7/2)/(d*x+c),x)/e
```

Rubi [A] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}{ce+dex}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcSinh[c + d*x])^(7/2)/(c*e + d*e*x),x]
```

```
[Out] Defer[Subst][Defer[Int][(a + b*ArcSinh[x])^(7/2)/x, x], x, c + d*x]/(d*e)
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}{ce+dex}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^{7/2}}{ex}\,dx, x, c+dx\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^{7/2}}{x}\,dx, x, c+dx\right)}{de}\end{aligned}$$

Mathematica [A] time = 1.14, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}{ce+dex}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^(7/2)/(c*e + d*e*x),x]
```

```
[Out] Integrate[(a + b*ArcSinh[c + d*x])^(7/2)/(c*e + d*e*x), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(7/2)/(d*e*x+c*e),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,\text{arsinh}\left(dx+c\right)+a\right)^{\frac{7}{2}}}{dex+ce}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(7/2)/(d*e*x+c*e),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(7/2)/(d*e*x + c*e), x)
```

maple [A] time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{(a+b \operatorname{arcsinh}(dx+c))^{\frac{7}{2}}}{dex+ce}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^(7/2)/(d*e*x+c*e),x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^(7/2)/(d*e*x+c*e),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arsinh}(dx+c)+a)^{\frac{7}{2}}}{dex+ce}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(7/2)/(d*e*x+c*e),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(7/2)/(d*e*x + c*e), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(a+b \operatorname{asinh}(c+d\,x))^{7/2}}{c\,e+d\,e\,x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^(7/2)/(c*e + d*e*x),x)
```

```
[Out] int((a + b*asinh(c + d*x))^(7/2)/(c*e + d*e*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**(7/2)/(d*e*x+c*e),x)
```

```
[Out] Timed out
```

3.204 $\int \frac{(ce+dex)^4}{\sqrt{a+b\sinh^{-1}(c+dx)}}\,dx$

Optimal. Leaf size=326

$$\frac{\sqrt{\pi}\,e^4e^{a/b}\mathrm{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16\sqrt{b}\,d}-\frac{\sqrt{3\pi}\,e^4e^{\frac{3a}{b}}\mathrm{erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}\,d}+\frac{\sqrt{\frac{\pi}{5}}\,e^4e^{\frac{5a}{b}}\mathrm{erf}\left(\frac{\sqrt{5}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}\,d}+\frac{\sqrt{\pi}\,e^4e^{-}}{}$$

```
[Out] 1/160*e^4*exp(5*a/b)*erf(5^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*5^(1/2
)*Pi^(1/2)/d/b^(1/2)+1/160*e^4*erfi(5^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1
/2))*5^(1/2)*Pi^(1/2)/d/exp(5*a/b)/b^(1/2)+1/16*e^4*exp(a/b)*erf((a+b*arcsi
nh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/b^(1/2)+1/16*e^4*erfi((a+b*arcsinh(d*x
+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/exp(a/b)/b^(1/2)-1/32*e^4*exp(3*a/b)*erf(3^(
1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/d/b^(1/2)-1/32*e^
4*erfi(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/d/exp(3
*a/b)/b^(1/2)
```

Rubi [A] time = 0.64, antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {5865, 12, 5669, 5448, 3307, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}\,e^4e^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16\sqrt{b}\,d}-\frac{\sqrt{3\pi}\,e^4e^{\frac{3a}{b}}\mathrm{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}\,d}+\frac{\sqrt{\frac{\pi}{5}}\,e^4e^{\frac{5a}{b}}\mathrm{Erf}\left(\frac{\sqrt{5}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}\,d}+\frac{\sqrt{\pi}\,e^4e}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (e^4*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(16*Sqrt[b
]*d) - (e^4*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]
])/Sqrt[b]])/(32*Sqrt[b]*d) + (e^4*E^((5*a)/b)*Sqrt[Pi/5]*Erf[(Sqrt[5]*Sqrt
[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(32*Sqrt[b]*d) + (e^4*Sqrt[Pi]*Erfi[Sqr
t[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(16*Sqrt[b]*d*E^(a/b)) - (e^4*Sqrt[3*Pi
]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(32*Sqrt[b]*d*E^((3
*a)/b)) + (e^4*Sqrt[Pi/5]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[
b]])/(32*Sqrt[b]*d*E^((5*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^4}{\sqrt{a+b\sinh^{-1}(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{e^4x^4}{\sqrt{a+b\sinh^{-1}(x)}}\,dx,x,c+dx\right)}{d}$$
$$= \frac{e^4\,\text{Subst}\left(\int \frac{x^4}{\sqrt{a+b\sinh^{-1}(x)}}\,dx,x,c+dx\right)}{d}$$
$$= \frac{e^4\,\text{Subst}\left(\int \frac{\cosh(x)\sinh^4(x)}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(c+dx)\right)}{d}$$
$$= \frac{e^4\,\text{Subst}\left(\int \left(\frac{\cosh(x)}{8\sqrt{a+bx}}-\frac{3\cosh(3x)}{16\sqrt{a+bx}}+\frac{\cosh(5x)}{16\sqrt{a+bx}}\right)dx,x,\sinh^{-1}(c+dx)\right)}{d}$$
$$= \frac{e^4\,\text{Subst}\left(\int \frac{\cosh(5x)}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(c+dx)\right)}{16d}+\frac{e^4\,\text{Subst}\left(\int \frac{\cosh(x)}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(c+\right.}{8d}$$
$$= \frac{e^4\,\text{Subst}\left(\int \frac{e^{-5x}}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(c+dx)\right)}{32d}+\frac{e^4\,\text{Subst}\left(\int \frac{e^{5x}}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(c+d\right.}{32d}$$
$$= \frac{e^4\,\text{Subst}\left(\int e^{\frac{5a}{b}-\frac{5x^2}{b}}\,dx,x,\sqrt{a+b\sinh^{-1}(c+dx)}\right)}{16bd}+\frac{e^4\,\text{Subst}\left(\int e^{-\frac{5a}{b}+\frac{5x^2}{b}}\,dx,x,\right.}{16bd}$$
$$= \frac{e^4e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16\sqrt{b}\,d}-\frac{e^4e^{\frac{3a}{b}}\sqrt{3\pi}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}\,d}+\frac{e^4e^{\frac{5a}{b}}\sqrt{\frac{\pi}{5}}}{}$$

Mathematica [A] time = 0.37, size = 320, normalized size = 0.98

$$\frac{e^4e^{-\frac{5a}{b}}\left(-10e^{\frac{6a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)+\sqrt{5}\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{5\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)-\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^4/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (e^4*(-10*E^((6*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2, a/b + ArcSinh
[c + d*x]] + Sqrt[5]*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/2, (-5*(a
+ b*ArcSinh[c + d*x]))/b] - 5*Sqrt[3]*E^((2*a)/b)*Sqrt[-((a + b*ArcSinh[c +
 d*x])/b)]*Gamma[1/2, (-3*(a + b*ArcSinh[c + d*x]))/b] + 10*E^((4*a)/b)*Sqr
t[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c + d*x])/b)]
+ 5*Sqrt[3]*E^((8*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2, (3*(a + b*A
rcSinh[c + d*x]))/b] - Sqrt[5]*E^((10*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Ga
mma[1/2, (5*(a + b*ArcSinh[c + d*x]))/b]))/(160*d*E^((5*a)/b)*Sqrt[a + b*Ar
cSinh[c + d*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex+ce)^4}{\sqrt{b\,\mathrm{arsinh}\,(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^4/sqrt(b*arcsinh(d*x + c) + a), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(dex+ce)^4}{\sqrt{a+b\,\mathrm{arcsinh}\,(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(1/2),x)
```

```
[Out] int((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex+ce)^4}{\sqrt{b\,\mathrm{arsinh}\,(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^4/sqrt(b*arcsinh(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,e+d\,e\,x)^4}{\sqrt{a+b\,\mathrm{asinh}\,(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^4/(a + b*asinh(c + d*x))^(1/2),x)
```

```
[Out] int((c*e + d*e*x)^4/(a + b*asinh(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^4\left(\int \frac{c^4}{\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}}\,dx+\int \frac{d^4x^4}{\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}}\,dx+\int \frac{4cd^3x^3}{\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}}\,dx+\int \frac{6c}{\sqrt{a+b\,\mathrm{as}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4/(a+b*asinh(d*x+c))**(1/2),x)
```

```
[Out] e**4*(Integral(c**4/sqrt(a + b*asinh(c + d*x)), x) + Integral(d**4*x**4/sqr
t(a + b*asinh(c + d*x)), x) + Integral(4*c*d**3*x**3/sqrt(a + b*asinh(c + d
*x)), x) + Integral(6*c**2*d**2*x**2/sqrt(a + b*asinh(c + d*x)), x) + Integ
ral(4*c**3*d*x/sqrt(a + b*asinh(c + d*x)), x))
```

3.205 $\int \frac{(ce+dex)^3}{\sqrt{a+b\sinh^{-1}(c+dx)}}\,dx$

Optimal. Leaf size=217

$$-\frac{\sqrt{\pi}\,e^3e^{\frac{4a}{b}}\,\text{erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}\,d}+\frac{\sqrt{\frac{\pi}{2}}\,e^3e^{\frac{2a}{b}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,d}+\frac{\sqrt{\pi}\,e^3e^{-\frac{4a}{b}}\,\text{erfi}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}\,d}-\frac{\sqrt{\frac{\pi}{2}}\,e^3e}{}$$

```
[Out] 1/16*e^3*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)
*Pi^(1/2)/d/b^(1/2)-1/16*e^3*erfi(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2
))*2^(1/2)*Pi^(1/2)/d/exp(2*a/b)/b^(1/2)-1/32*e^3*exp(4*a/b)*erf(2*(a+b*arc
sinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/b^(1/2)+1/32*e^3*erfi(2*(a+b*arcsinh
(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/exp(4*a/b)/b^(1/2)
```

Rubi [A] time = 0.46, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {5865, 12, 5669, 5448, 3308, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}\,e^3e^{\frac{4a}{b}}\,\text{Erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}\,d}+\frac{\sqrt{\frac{\pi}{2}}\,e^3e^{\frac{2a}{b}}\,\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,d}+\frac{\sqrt{\pi}\,e^3e^{-\frac{4a}{b}}\,\text{Erfi}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}\,d}-\frac{\sqrt{\frac{\pi}{2}}\,e^3}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] -(e^3*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(
32*Sqrt[b]*d) + (e^3*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh
[c + d*x]])/Sqrt[b]])/(8*Sqrt[b]*d) + (e^3*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcS
inh[c + d*x]])/Sqrt[b]])/(32*Sqrt[b]*d*E^((4*a)/b)) - (e^3*Sqrt[Pi/2]*Erfi[
(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(8*Sqrt[b]*d*E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^3}{\sqrt{a+b\sinh^{-1}(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{e^3x^3}{\sqrt{a+b\sinh^{-1}(x)}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^3\,\text{Subst}\left(\int \frac{x^3}{\sqrt{a+b\sinh^{-1}(x)}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^3\,\text{Subst}\left(\int \frac{\cosh(x)\sinh^3(x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx)\right)}{d}$$

$$= \frac{e^3\,\text{Subst}\left(\int \left(-\frac{\sinh(2x)}{4\sqrt{a+bx}}+\frac{\sinh(4x)}{8\sqrt{a+bx}}\right)dx, x, \sinh^{-1}(c+dx)\right)}{d}$$

$$= \frac{e^3\,\text{Subst}\left(\int \frac{\sinh(4x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx)\right)}{8d} - \frac{e^3\,\text{Subst}\left(\int \frac{\sinh(2x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(\right.}{4d}$$

$$= -\frac{e^3\,\text{Subst}\left(\int \frac{e^{-4x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx)\right)}{16d} + \frac{e^3\,\text{Subst}\left(\int \frac{e^{4x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(\right.}{16d}$$

$$= -\frac{e^3\,\text{Subst}\left(\int e^{\frac{4a}{b}-\frac{4x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(c+dx)}\right)}{8bd} + \frac{e^3\,\text{Subst}\left(\int e^{-\frac{4a}{b}+\frac{4x^2}{b}}\,dx\right.}{}$$

$$= -\frac{e^3e^{\frac{4a}{b}}\sqrt{\pi}\,\text{erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}\,d} + \frac{e^3e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,d} + \frac{e^3e^{-}}{}$$

Mathematica [A] time = 0.23, size = 205, normalized size = 0.94

$$\frac{e^3e^{-\frac{4a}{b}}\left(\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{4\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)-2\sqrt{2}\,e^{\frac{2a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)+e^{\frac{6a}{b}}\sqrt{\frac{a}{b}}\right.}{32d\sqrt{a+b\sinh^{-1}(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^3/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (e^3*(Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/2, (-4*(a + b*ArcSinh[c +
 d*x]))/b] - 2*Sqrt[2]*E^((2*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamm
a[1/2, (-2*(a + b*ArcSinh[c + d*x]))/b] + E^((6*a)/b)*Sqrt[a/b + ArcSinh[c
+ d*x]]*(-2*Sqrt[2]*Gamma[1/2, (2*(a + b*ArcSinh[c + d*x]))/b] + E^((2*a)/b
)*Gamma[1/2, (4*(a + b*ArcSinh[c + d*x]))/b])))/(32*d*E^((4*a)/b)*Sqrt[a +
b*ArcSinh[c + d*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex+ce)^3}{\sqrt{b\,\mathrm{arsinh}\,(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^3/sqrt(b*arcsinh(d*x + c) + a), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(dex+ce)^3}{\sqrt{a+b\,\mathrm{arcsinh}\,(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(1/2),x)
```

```
[Out] int((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex+ce)^3}{\sqrt{b\,\mathrm{arsinh}\,(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^3/sqrt(b*arcsinh(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,e+d\,e\,x)^3}{\sqrt{a+b\,\mathrm{asinh}\,(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3/(a + b*asinh(c + d*x))^(1/2),x)
```

```
[Out] int((c*e + d*e*x)^3/(a + b*asinh(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^3\left(\int \frac{c^3}{\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}}\,dx+\int \frac{d^3x^3}{\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}}\,dx+\int \frac{3cd^2x^2}{\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}}\,dx+\int \frac{3}{\sqrt{a+b\,\mathrm{as}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3/(a+b*asinh(d*x+c))**(1/2),x)
```

```
[Out] e**3*(Integral(c**3/sqrt(a + b*asinh(c + d*x)), x) + Integral(d**3*x**3/sqr
t(a + b*asinh(c + d*x)), x) + Integral(3*c*d**2*x**2/sqrt(a + b*asinh(c + d
*x)), x) + Integral(3*c**2*d*x/sqrt(a + b*asinh(c + d*x)), x))
```

3.206 $\int \frac{(ce+dex)^2}{\sqrt{a+b\sinh^{-1}(c+dx)}}\,dx$

Optimal. Leaf size=214

$$-\frac{\sqrt{\pi}\,e^2e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,d}+\frac{\sqrt{\frac{\pi}{3}}\,e^2e^{\frac{3a}{b}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,d}-\frac{\sqrt{\pi}\,e^2e^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,d}+\sqrt{\frac{\pi}{3}}\,e^2e^{-\frac{3}{}}$$

```
[Out] 1/24*e^2*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)
*Pi^(1/2)/d/b^(1/2)+1/24*e^2*erfi(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2
))*3^(1/2)*Pi^(1/2)/d/exp(3*a/b)/b^(1/2)-1/8*e^2*exp(a/b)*erf((a+b*arcsinh(
d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/b^(1/2)-1/8*e^2*erfi((a+b*arcsinh(d*x+c))
^(1/2)/b^(1/2))*Pi^(1/2)/d/exp(a/b)/b^(1/2)
```

Rubi [A] time = 0.46, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {5865, 12, 5669, 5448, 3307, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}\,e^2e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,d}+\frac{\sqrt{\frac{\pi}{3}}\,e^2e^{\frac{3a}{b}}\operatorname{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,d}-\frac{\sqrt{\pi}\,e^2e^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,d}+\sqrt{\frac{\pi}{3}}\,e^2e^{-}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] -(e^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(8*Sqrt[b
]*d) + (e^2*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]
])/Sqrt[b]])/(8*Sqrt[b]*d) - (e^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]
]/Sqrt[b]])/(8*Sqrt[b]*d*E^(a/b)) + (e^2*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a +
b*ArcSinh[c + d*x]])/Sqrt[b]])/(8*Sqrt[b]*d*E^((3*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^2}{\sqrt{a+b\sinh^{-1}(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{e^2x^2}{\sqrt{a+b\sinh^{-1}(x)}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^2\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+b\sinh^{-1}(x)}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^2\,\text{Subst}\left(\int \frac{\cosh(x)\sinh^2(x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx)\right)}{d}$$

$$= \frac{e^2\,\text{Subst}\left(\int \left(-\frac{\cosh(x)}{4\sqrt{a+bx}}+\frac{\cosh(3x)}{4\sqrt{a+bx}}\right)dx, x, \sinh^{-1}(c+dx)\right)}{d}$$

$$= -\frac{e^2\,\text{Subst}\left(\int \frac{\cosh(x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx)\right)}{4d} + \frac{e^2\,\text{Subst}\left(\int \frac{\cosh(3x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}\right.}{4d}$$

$$= \frac{e^2\,\text{Subst}\left(\int \frac{e^{-3x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx)\right)}{8d} - \frac{e^2\,\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c\right.}{8d}$$

$$= \frac{e^2\,\text{Subst}\left(\int e^{\frac{3a}{b}-\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(c+dx)}\right)}{4bd} - \frac{e^2\,\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \right.}{4bd}$$

$$= -\frac{e^2e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,d} + \frac{e^2e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,d} - \frac{e^2e^{-\frac{a}{b}}}{}$$

Mathematica [A] time = 0.24, size = 217, normalized size = 1.01

$$\frac{e^2e^{-\frac{3a}{b}}\left(3e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)+\sqrt{3}\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{3\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)-3e^{\cdots}\right.}{24d\sqrt{a+b\sinh^{-1}(c+\cdots}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^2/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (e^2*(3*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2, a/b + ArcSinh[c
 + d*x]] + Sqrt[3]*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/2, (-3*(a +
b*ArcSinh[c + d*x]))/b] - 3*E^((2*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]
*Gamma[1/2, -((a + b*ArcSinh[c + d*x])/b)] - Sqrt[3]*E^((6*a)/b)*Sqrt[a/b +
 ArcSinh[c + d*x]]*Gamma[1/2, (3*(a + b*ArcSinh[c + d*x]))/b]))/(24*d*E^((3
*a)/b)*Sqrt[a + b*ArcSinh[c + d*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex+ce)^2}{\sqrt{b\,\mathrm{arsinh}\,(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2/sqrt(b*arcsinh(d*x + c) + a), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(dex+ce)^2}{\sqrt{a+b\,\mathrm{arcsinh}\,(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(1/2),x)
```

```
[Out] int((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex+ce)^2}{\sqrt{b\,\mathrm{arsinh}\,(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^2/sqrt(b*arcsinh(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,e+d\,e\,x)^2}{\sqrt{a+b\,\mathrm{asinh}\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2/(a + b*asinh(c + d*x))^(1/2),x)
```

```
[Out] int((c*e + d*e*x)^2/(a + b*asinh(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^2\left(\int \frac{c^2}{\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}}\,dx+\int \frac{d^2x^2}{\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}}\,dx+\int \frac{2cdx}{\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2/(a+b*asinh(d*x+c))**(1/2),x)
```

```
[Out] e**2*(Integral(c**2/sqrt(a + b*asinh(c + d*x)), x) + Integral(d**2*x**2/sqr
t(a + b*asinh(c + d*x)), x) + Integral(2*c*d*x/sqrt(a + b*asinh(c + d*x)),
x))
```

3.207 $\int \frac{ce+dex}{\sqrt{a+b\sinh^{-1}(c+dx)}}\,dx$

Optimal. Leaf size=113

$$\frac{\sqrt{\frac{\pi}{2}}\,ee^{-\frac{2a}{b}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4\sqrt{b}\,d}-\frac{\sqrt{\frac{\pi}{2}}\,ee^{\frac{2a}{b}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4\sqrt{b}\,d}$$

```
[Out] -1/8*e*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*P
i^(1/2)/d/b^(1/2)+1/8*e*erfi(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^
(1/2)*Pi^(1/2)/d/exp(2*a/b)/b^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {5865, 12, 5669, 5448, 3308, 2180, 2204, 2205}

$$\frac{\sqrt{\frac{\pi}{2}}\,ee^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4\sqrt{b}\,d}-\frac{\sqrt{\frac{\pi}{2}}\,ee^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4\sqrt{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] -(e*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[
b]])/(4*Sqrt[b]*d) + (e*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x
]])/Sqrt[b]])/(4*Sqrt[b]*d*E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{ce+dex}{\sqrt{a+b\sinh^{-1}(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{ex}{\sqrt{a+b\sinh^{-1}(x)}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e\,\text{Subst}\left(\int \frac{x}{\sqrt{a+b\sinh^{-1}(x)}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e\,\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx)\right)}{d}$$

$$= \frac{e\,\text{Subst}\left(\int \frac{\sinh(2x)}{2\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx)\right)}{d}$$

$$= \frac{e\,\text{Subst}\left(\int \frac{\sinh(2x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx)\right)}{2d}$$

$$= -\frac{e\,\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx)\right)}{4d} + \frac{e\,\text{Subst}\left(\int \frac{e^{2x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c\ \ldots\right.}{4d}$$

$$= -\frac{e\,\text{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(c+dx)}\right)}{2bd} + \frac{e\,\text{Subst}\left(\int e^{-\frac{2a}{b}+\frac{2x^2}{b}}\,dx, x\ \ldots\right.}{2\ldots}$$

$$= -\frac{ee^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4\sqrt{b}\,d} + \frac{ee^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4\sqrt{b}\,d}$$

Mathematica [A] time = 0.07, size = 119, normalized size = 1.05

$$\frac{ee^{-\frac{2a}{b}}\left(\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)+e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\Gamma\left(\frac{1}{2},\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)\right)}{4\sqrt{2}\,d\sqrt{a+b\sinh^{-1}(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (e*(Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/2, (-2*(a + b*ArcSinh[c + d
*x]))/b] + E^((4*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2, (2*(a + b*Ar
cSinh[c + d*x]))/b]))/(4*Sqrt[2]*d*E^((2*a)/b)*Sqrt[a + b*ArcSinh[c + d*x]]
)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dex + ce}{\sqrt{b \operatorname{arsinh}(dx + c) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)/sqrt(b*arcsinh(d*x + c) + a), x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{dex + ce}{\sqrt{a + b \operatorname{arcsinh}(dx + c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(1/2),x)
```

```
[Out] int((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dex + ce}{\sqrt{b \operatorname{arsinh}(dx + c) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)/sqrt(b*arcsinh(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c\,e + d\,e\,x}{\sqrt{a + b \operatorname{asinh}(c + d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)/(a + b*asinh(c + d*x))^(1/2),x)
```

```
[Out] int((c*e + d*e*x)/(a + b*asinh(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e \left(\int \frac{c}{\sqrt{a + b \operatorname{asinh}(c + dx)}}\,dx + \int \frac{dx}{\sqrt{a + b \operatorname{asinh}(c + dx)}}\,dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*asinh(d*x+c))**(1/2),x)
```

```
[Out] e*(Integral(c/sqrt(a + b*asinh(c + d*x)), x) + Integral(d*x/sqrt(a + b*asin
h(c + d*x)), x))
```

3.208 $\int \frac{1}{\sqrt{a+b\sinh^{-1}(c+dx)}}\,dx$

Optimal. Leaf size=92

$$\frac{\sqrt{\pi}e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d}+\frac{\sqrt{\pi}e^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d}$$

```
[Out] 1/2*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/b^(1/2)+1/2
*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/exp(a/b)/b^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5863, 5657, 3307, 2180, 2205, 2204}

$$\frac{\sqrt{\pi}e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d}+\frac{\sqrt{\pi}e^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(2*Sqrt[b]*d)
+ (Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(2*Sqrt[b]*d*E^(a/b
))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 5863

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\sinh^{-1}(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+b\sinh^{-1}(x)}}\,dx, x, c+dx\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\sinh^{-1}(c+dx)\right)}{bd}$$
$$= \frac{\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\sinh^{-1}(c+dx)\right)}{2bd} + \frac{\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\right.}{2bd}$$
$$= \frac{\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(c+dx)}\right)}{bd} + \frac{\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\right.}{bd}$$
$$= \frac{e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d} + \frac{e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d}$$

Mathematica [A] time = 0.07, size = 111, normalized size = 1.21

$$\frac{e^{-\frac{a}{b}}\left(\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)-e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)}{2d\sqrt{a+b\sinh^{-1}(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (-(E^((2*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2, a/b + ArcSinh[c + d*
x]]) + Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c +
d*x])/b)])/(2*d*E^(a/b)*Sqrt[a + b*ArcSinh[c + d*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\,\text{arsinh}\,(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*arcsinh(d*x + c) + a), x)
```

maple [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\operatorname{arcsinh}\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(d*x+c))^(1/2),x)
```

```
[Out] int(1/(a+b*arcsinh(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\operatorname{arsinh}\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*arcsinh(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+b\operatorname{asinh}\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*asinh(c + d*x))^(1/2),x)
```

```
[Out] int(1/(a + b*asinh(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\operatorname{asinh}\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*asinh(c + d*x)), x)
```

3.209 $\int \frac{1}{(ce+dex)\sqrt{a+b\sinh^{-1}(c+dx)}}\,dx$

Optimal. Leaf size=29

$$\frac{\text{Int}\left(\frac{1}{(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}},x\right)}{e}$$

[Out] Unintegrable(1/(d*x+c)/(a+b*arcsinh(d*x+c))^(1/2),x)/e

Rubi [A] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(ce+dex)\sqrt{a+b\sinh^{-1}(c+dx)}}\,dx$$

Verification is Not applicable to the result.

[In] Int[1/((c*e + d*e*x)*Sqrt[a + b*ArcSinh[c + d*x]]),x]

[Out] Defer[Subst][Defer[Int][1/(x*Sqrt[a + b*ArcSinh[x]]), x], x, c + d*x]/(d*e)

Rubi steps

$$\begin{aligned}\int \frac{1}{(ce+dex)\sqrt{a+b\sinh^{-1}(c+dx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{ex\sqrt{a+b\sinh^{-1}(x)}}\,dx,x,c+dx\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+b\sinh^{-1}(x)}}\,dx,x,c+dx\right)}{de}\end{aligned}$$

Mathematica [A] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{(ce+dex)\sqrt{a+b\sinh^{-1}(c+dx)}}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((c*e + d*e*x)*Sqrt[a + b*ArcSinh[c + d*x]]),x]

[Out] Integrate[1/((c*e + d*e*x)*Sqrt[a + b*ArcSinh[c + d*x]]), x]

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dex + ce)\sqrt{b \operatorname{arsinh}(dx + c) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*e*x + c*e)*sqrt(b*arcsinh(d*x + c) + a)), x)
```

maple [A] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{1}{(dex + ce)\sqrt{a + b \operatorname{arcsinh}(dx + c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(1/2),x)
```

```
[Out] int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dex + ce)\sqrt{b \operatorname{arsinh}(dx + c) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*e*x + c*e)*sqrt(b*arcsinh(d*x + c) + a)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{(c\,e + d\,e\,x)\,\sqrt{a + b\,\operatorname{asinh}(c + d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*e + d*e*x)*(a + b*asinh(c + d*x))^(1/2)),x)
```

```
[Out] int(1/((c*e + d*e*x)*(a + b*asinh(c + d*x))^(1/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{c\sqrt{a+b\operatorname{asinh}(c+dx)}+dx\sqrt{a+b\operatorname{asinh}(c+dx)}}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*asinh(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(c*sqrt(a + b*asinh(c + d*x)) + d*x*sqrt(a + b*asinh(c + d*x))),
 x)/e
```

3.210 $\int \frac{(ce+dex)^4}{\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}\,dx$

Optimal. Leaf size=367

$$-\frac{\sqrt{\pi}\,e^4e^{a/b}\mathrm{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8b^{3/2}d}+\frac{3\sqrt{3\pi}\,e^4e^{\frac{3a}{b}}\mathrm{erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16b^{3/2}d}-\frac{\sqrt{5\pi}\,e^4e^{\frac{5a}{b}}\mathrm{erf}\left(\frac{\sqrt{5}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16b^{3/2}d}+-$$

```
[Out] -1/8*e^4*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/
d+1/8*e^4*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/d/exp(a
/b)+3/16*e^4*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(
1/2)*Pi^(1/2)/b^(3/2)/d-3/16*e^4*erfi(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^
(1/2))*3^(1/2)*Pi^(1/2)/b^(3/2)/d/exp(3*a/b)-1/16*e^4*exp(5*a/b)*erf(5^(1/2
)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*5^(1/2)*Pi^(1/2)/b^(3/2)/d+1/16*e^4*e
rfi(5^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*5^(1/2)*Pi^(1/2)/b^(3/2)/d/
exp(5*a/b)-2*e^4*(d*x+c)^4*(1+(d*x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))^(1/
2)
```

Rubi [A] time = 0.65, antiderivative size = 367, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {5865, 12, 5665, 3308, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}\,e^4e^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8b^{3/2}d}+\frac{3\sqrt{3\pi}\,e^4e^{\frac{3a}{b}}\mathrm{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16b^{3/2}d}-\frac{\sqrt{5\pi}\,e^4e^{\frac{5a}{b}}\mathrm{Erf}\left(\frac{\sqrt{5}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16b^{3/2}d}+$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4/(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (-2*e^4*(c + d*x)^4*Sqrt[1 + (c + d*x)^2])/(b*d*Sqrt[a + b*ArcSinh[c + d*x]
]) - (e^4*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(8*b^
(3/2)*d) + (3*e^4*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c
+ d*x]])/Sqrt[b]])/(16*b^(3/2)*d) - (e^4*E^((5*a)/b)*Sqrt[5*Pi]*Erf[(Sqrt[5
]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(16*b^(3/2)*d) + (e^4*Sqrt[Pi]*Er
fi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(8*b^(3/2)*d*E^(a/b)) - (3*e^4*Sq
rt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(16*b^(3/2)*
d*E^((3*a)/b)) + (e^4*Sqrt[5*Pi]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcSinh[c + d*x]]
)/Sqrt[b]])/(16*b^(3/2)*d*E^((5*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^4}{\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{e^4x^4}{\left(a+b\sinh^{-1}(x)\right)^{3/2}}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e^4\,\text{Subst}\left(\int \frac{x^4}{\left(a+b\sinh^{-1}(x)\right)^{3/2}}\,dx,x,c+dx\right)}{d}$$

$$= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{\left(2e^4\right)\text{Subst}\left(\int\left(\frac{\sinh(x)}{8\sqrt{a+bx}}-\frac{9\sinh(3x)}{16\sqrt{a+bx}}+\frac{5\sinh(5x)}{16\sqrt{a+bx}}\right)\right.}{bd}$$

$$= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{e^4\,\text{Subst}\left(\int \frac{\sinh(x)}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(c+dx)\right)}{4bd} + \frac{(5e}{}$$

$$= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{e^4\,\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(c+dx)\right)}{8bd} + \frac{e^4\,\text{S}}{}$$

$$= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{e^4\,\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx,x,\sqrt{a+b\sinh^{-1}(c+dx)}\right.}{4b^2d}$$

$$= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{e^4e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8b^{3/2}d} + \frac{3e^4e^{\frac{3a}{b}}\sqrt{3\pi}\,\text{e}}{}$$

Mathematica [A] time = 0.67, size = 490, normalized size = 1.34

$$e^4 e^{-5\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)}\left(3e^{\frac{5a}{b}+2\sinh^{-1}(c+dx)} - 2e^{\frac{5a}{b}+4\sinh^{-1}(c+dx)} - 2e^{\frac{5a}{b}+6\sinh^{-1}(c+dx)} + 3e^{\frac{5a}{b}+8\sinh^{-1}(c+dx)} - e^{\frac{5a}{b}+10\sinh^{-1}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^4/(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (e^4*(-E^((5*a)/b) + 3*E^((5*a)/b + 2*ArcSinh[c + d*x]) - 2*E^((5*a)/b + 4*
ArcSinh[c + d*x]) - 2*E^((5*a)/b + 6*ArcSinh[c + d*x]) + 3*E^((5*a)/b + 8*A
rcSinh[c + d*x]) - E^((5*a)/b + 10*ArcSinh[c + d*x]) + 2*E^((6*a)/b + 5*Arc
Sinh[c + d*x])*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2, a/b + ArcSinh[c + d*
x]] + Sqrt[5]*E^(5*ArcSinh[c + d*x])*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Ga
mma[1/2, (-5*(a + b*ArcSinh[c + d*x]))/b] - 3*Sqrt[3]*E^((2*a)/b + 5*ArcSin
h[c + d*x])*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/2, (-3*(a + b*ArcSi
nh[c + d*x]))/b] + 2*E^((4*a)/b + 5*ArcSinh[c + d*x])*Sqrt[-((a + b*ArcSinh
[c + d*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c + d*x])/b)] - 3*Sqrt[3]*E^((8*
a)/b + 5*ArcSinh[c + d*x])*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2, (3*(a +
b*ArcSinh[c + d*x]))/b] + Sqrt[5]*E^(5*((2*a)/b + ArcSinh[c + d*x]))*Sqrt[a
/b + ArcSinh[c + d*x]]*Gamma[1/2, (5*(a + b*ArcSinh[c + d*x]))/b]))/(16*b*d
*E^(5*(a/b + ArcSinh[c + d*x]))*Sqrt[a + b*ArcSinh[c + d*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex+ce)^4}{(b\,\mathrm{arsinh}\,(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^4/(b*arcsinh(d*x + c) + a)^(3/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(dex+ce)^4}{(a+b\,\mathrm{arcsinh}\,(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(3/2),x)
```

```
[Out] int((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex+ce)^4}{(b\,\mathrm{arsinh}\,(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^4/(b*arcsinh(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,e+d\,e\,x)^4}{(a+b\,\mathrm{asinh}\,(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^4/(a + b*asinh(c + d*x))^(3/2),x)
```

```
[Out] int((c*e + d*e*x)^4/(a + b*asinh(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^4\left(\int \frac{c^4}{a\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}+b\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\,\mathrm{asinh}\,(c+dx)}\,dx+\int \frac{d^4x}{a\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}+b\sqrt{a+}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4/(a+b*asinh(d*x+c))**(3/2),x)
```

```
[Out] e**4*(Integral(c**4/(a*sqrt(a + b*asinh(c + d*x)) + b*sqrt(a + b*asinh(c +
d*x))*asinh(c + d*x)), x) + Integral(d**4*x**4/(a*sqrt(a + b*asinh(c + d*x)
) + b*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)), x) + Integral(4*c*d**3*x*
*3/(a*sqrt(a + b*asinh(c + d*x)) + b*sqrt(a + b*asinh(c + d*x))*asinh(c + d
*x)), x) + Integral(6*c**2*d**2*x**2/(a*sqrt(a + b*asinh(c + d*x)) + b*sqrt
(a + b*asinh(c + d*x))*asinh(c + d*x)), x) + Integral(4*c**3*d*x/(a*sqrt(a
+ b*asinh(c + d*x)) + b*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)), x))
```

3.211 $\int \frac{(ce+dex)^3}{\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}\,dx$

Optimal. Leaf size=262

$$\frac{\sqrt{\pi}e^3e^{\frac{4a}{b}}\operatorname{erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d}-\frac{\sqrt{\frac{\pi}{2}}e^3e^{\frac{2a}{b}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2b^{3/2}d}+\frac{\sqrt{\pi}e^3e^{-\frac{4a}{b}}\operatorname{erfi}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d}-\frac{\sqrt{\frac{\pi}{2}}e^3}{}$$

```
[Out] -1/4*e^3*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)
*Pi^(1/2)/b^(3/2)/d-1/4*e^3*erfi(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2)
)*2^(1/2)*Pi^(1/2)/b^(3/2)/d/exp(2*a/b)+1/4*e^3*exp(4*a/b)*erf(2*(a+b*arcsi
nh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/d+1/4*e^3*erfi(2*(a+b*arcsinh(d*
x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/d/exp(4*a/b)-2*e^3*(d*x+c)^3*(1+(d*x+
c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))^(1/2)
```

Rubi [A] time = 0.45, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {5865, 12, 5665, 3307, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}e^3e^{\frac{4a}{b}}\operatorname{Erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d}-\frac{\sqrt{\frac{\pi}{2}}e^3e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2b^{3/2}d}+\frac{\sqrt{\pi}e^3e^{-\frac{4a}{b}}\operatorname{Erfi}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d}-\frac{\sqrt{\frac{\pi}{2}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3/(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (-2*e^3*(c + d*x)^3*Sqrt[1 + (c + d*x)^2])/(b*d*Sqrt[a + b*ArcSinh[c + d*x]
]) + (e^3*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]
])/(4*b^(3/2)*d) - (e^3*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcS
inh[c + d*x]])/Sqrt[b]])/(2*b^(3/2)*d) + (e^3*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*A
rcSinh[c + d*x]])/Sqrt[b]])/(4*b^(3/2)*d*E^((4*a)/b)) - (e^3*Sqrt[Pi/2]*Erf
i[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(2*b^(3/2)*d*E^((2*a)/b)
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^3}{\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{e^3x^3}{\left(a+b\sinh^{-1}(x)\right)^{3/2}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^3\,\text{Subst}\left(\int \frac{x^3}{\left(a+b\sinh^{-1}(x)\right)^{3/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{\left(2e^3\right)\text{Subst}\left(\int \left(-\frac{\cosh(2x)}{2\sqrt{a+bx}} + \frac{\cosh(4x)}{2\sqrt{a+bx}}\right) dx, x, \sinh\right.}{bd}$$

$$= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{e^3\,\text{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx)\right)}{bd} + \frac{e^3}{}$$

$$= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{e^3\,\text{Subst}\left(\int \frac{e^{-4x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx)\right)}{2bd} - \frac{e^3}{}$$

$$= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{e^3\,\text{Subst}\left(\int e^{\frac{4a}{b}-\frac{4x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(c+dx}\right.}{b^2d}$$

$$= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{e^3e^{\frac{4a}{b}}\sqrt{\pi}\,\text{erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d} - \frac{e^3e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}}{}$$

Mathematica [A] time = 0.45, size = 253, normalized size = 0.97

$$e^3e^{-\frac{4a}{b}}\left(\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{4\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)-\sqrt{2}\,e^{\frac{2a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)-e^{\frac{4a}{b}}\left(-\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^3/(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (e^3*(Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/2, (-4*(a + b*ArcSinh[c +
 d*x]))/b] - Sqrt[2]*E^((2*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[
1/2, (-2*(a + b*ArcSinh[c + d*x]))/b] - E^((4*a)/b)*(-(Sqrt[2]*E^((2*a)/b)*
Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2, (2*(a + b*ArcSinh[c + d*x]))/b]) +
E^((4*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2, (4*(a + b*ArcSinh[c + d
*x]))/b] - 2*Sinh[2*ArcSinh[c + d*x]] + Sinh[4*ArcSinh[c + d*x]])))/(4*b*d*
E^((4*a)/b)*Sqrt[a + b*ArcSinh[c + d*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex+ce)^3}{(b\operatorname{arsinh}(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^3/(b*arcsinh(d*x + c) + a)^(3/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(dex+ce)^3}{(a+b\operatorname{arcsinh}(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(3/2),x)
```

```
[Out] int((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex+ce)^3}{(b\operatorname{arsinh}(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^3/(b*arcsinh(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,e+d\,e\,x)^3}{(a+b\,\mathrm{asinh}\,(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3/(a + b*asinh(c + d*x))^(3/2),x)
```

```
[Out] int((c*e + d*e*x)^3/(a + b*asinh(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^3\left(\int \frac{c^3}{a\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}+b\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\,\mathrm{asinh}\,(c+dx)}\,dx+\int \frac{d^3x}{a\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}+b\sqrt{a+}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3/(a+b*asinh(d*x+c))**(3/2),x)
```

```
[Out] e**3*(Integral(c**3/(a*sqrt(a + b*asinh(c + d*x)) + b*sqrt(a + b*asinh(c +
d*x))*asinh(c + d*x)), x) + Integral(d**3*x**3/(a*sqrt(a + b*asinh(c + d*x)
) + b*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)), x) + Integral(3*c*d**2*x*
*2/(a*sqrt(a + b*asinh(c + d*x)) + b*sqrt(a + b*asinh(c + d*x))*asinh(c + d
*x)), x) + Integral(3*c**2*d*x/(a*sqrt(a + b*asinh(c + d*x)) + b*sqrt(a + b
*asinh(c + d*x))*asinh(c + d*x)), x))
```

3.212 $\int \frac{(ce+dex)^2}{\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}\,dx$

Optimal. Leaf size=255

$$\frac{\sqrt{\pi}\, e^2 e^{a/b} \operatorname{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d}-\frac{\sqrt{3\pi}\, e^2 e^{\frac{3a}{b}} \operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d}-\frac{\sqrt{\pi}\, e^2 e^{-\frac{a}{b}} \operatorname{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d}+\sqrt{3\pi}\, e^2$$

```
[Out] 1/4*e^2*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/d
-1/4*e^2*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/d/exp(a/
b)-1/4*e^2*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(1/
2)*Pi^(1/2)/b^(3/2)/d+1/4*e^2*erfi(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/
2))*3^(1/2)*Pi^(1/2)/b^(3/2)/d/exp(3*a/b)-2*e^2*(d*x+c)^2*(1+(d*x+c)^2)^(1/
2)/b/d/(a+b*arcsinh(d*x+c))^(1/2)
```

Rubi [A] time = 0.47, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {5865, 12, 5665, 3308, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}\, e^2 e^{a/b} \operatorname{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d}-\frac{\sqrt{3\pi}\, e^2 e^{\frac{3a}{b}} \operatorname{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d}-\frac{\sqrt{\pi}\, e^2 e^{-\frac{a}{b}} \operatorname{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d}+\sqrt{3\pi}\, e$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2/(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (-2*e^2*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])/(b*d*Sqrt[a + b*ArcSinh[c + d*x]
]) + (e^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(4*b^
(3/2)*d) - (e^2*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c +
d*x]])/Sqrt[b]])/(4*b^(3/2)*d) - (e^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c +
d*x]]/Sqrt[b]])/(4*b^(3/2)*d*E^(a/b)) + (e^2*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[
a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(4*b^(3/2)*d*E^((3*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^2}{\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{e^2x^2}{\left(a+b\sinh^{-1}(x)\right)^{3/2}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^2\,\text{Subst}\left(\int \frac{x^2}{\left(a+b\sinh^{-1}(x)\right)^{3/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{\left(2e^2\right)\text{Subst}\left(\int \left(-\frac{\sinh(x)}{4\sqrt{a+bx}}+\frac{3\sinh(3x)}{4\sqrt{a+bx}}\right)dx, x, \text{sin}\right.}{bd}$$

$$= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{e^2\,\text{Subst}\left(\int \frac{\sinh(x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx)\right)}{2bd} + \frac{(3e}{}$$

$$= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{e^2\,\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx)\right)}{4bd} - \frac{e^2\,\text{S}}{}$$

$$= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{e^2\,\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(c+dx)}\right.}{2b^2d}$$

$$= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{e^2e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d} - \frac{e^2e^{\frac{3a}{b}}\sqrt{3\pi}\,\text{er}}{}$$

Mathematica [A] time = 0.37, size = 327, normalized size = 1.28

$$\frac{e^2e^{-3\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)}\left(e^{\frac{3a}{b}+2\sinh^{-1}(c+dx)}+e^{\frac{3a}{b}+4\sinh^{-1}(c+dx)}-e^{\frac{3a}{b}+6\sinh^{-1}(c+dx)}-e^{\frac{4a}{b}+3\sinh^{-1}(c+dx)}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^2/(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (e^2*(-E^((3*a)/b) + E^((3*a)/b + 2*ArcSinh[c + d*x]) + E^((3*a)/b + 4*ArcS
inh[c + d*x]) - E^((3*a)/b + 6*ArcSinh[c + d*x]) - E^((4*a)/b + 3*ArcSinh[c
 + d*x])*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2, a/b + ArcSinh[c + d*x]] +
Sqrt[3]*E^(3*ArcSinh[c + d*x])*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/
2, (-3*(a + b*ArcSinh[c + d*x]))/b] - E^((2*a)/b + 3*ArcSinh[c + d*x])*Sqrt
[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c + d*x])/b)] +
 Sqrt[3]*E^((6*a)/b + 3*ArcSinh[c + d*x])*Sqrt[a/b + ArcSinh[c + d*x]]*Gamm
a[1/2, (3*(a + b*ArcSinh[c + d*x]))/b]))/(4*b*d*E^(3*(a/b + ArcSinh[c + d*x
]))*Sqrt[a + b*ArcSinh[c + d*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex + ce)^2}{(b \operatorname{arsinh}(dx + c) + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2/(b*arcsinh(d*x + c) + a)^(3/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(dex + ce)^2}{(a + b \operatorname{arcsinh}(dx + c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(3/2),x)
```

```
[Out] int((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex + ce)^2}{(b \operatorname{arsinh}(dx + c) + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^2/(b*arcsinh(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,e + d\,e\,x)^2}{(a + b\,\mathrm{asinh}\,(c + d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2/(a + b*asinh(c + d*x))^(3/2),x)
```

```
[Out] int((c*e + d*e*x)^2/(a + b*asinh(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^2 \left(\int \frac{c^2}{a\sqrt{a + b\operatorname{asinh}(c + dx)} + b\sqrt{a + b\operatorname{asinh}(c + dx)}\operatorname{asinh}(c + dx)}\,dx + \int \frac{d^2x}{a\sqrt{a + b\operatorname{asinh}(c + dx)} + b\sqrt{a +}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2/(a+b*asinh(d*x+c))**(3/2),x)
```

```
[Out] e**2*(Integral(c**2/(a*sqrt(a + b*asinh(c + d*x)) + b*sqrt(a + b*asinh(c +
d*x))*asinh(c + d*x)), x) + Integral(d**2*x**2/(a*sqrt(a + b*asinh(c + d*x)
) + b*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)), x) + Integral(2*c*d*x/(a*
sqrt(a + b*asinh(c + d*x)) + b*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)),
x))
```

3.213 $\int \frac{ce+dex}{\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}\,dx$

Optimal. Leaf size=148

$$\frac{\sqrt{\frac{\pi}{2}}\,ee^{\frac{2a}{b}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}+\frac{\sqrt{\frac{\pi}{2}}\,ee^{-\frac{2a}{b}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}-\frac{2e\sqrt{(c+dx)^2+1}\,(c+dx)}{bd\sqrt{a+b\sinh^{-1}(c+dx)}}$$

```
[Out] 1/2*e*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi
^(1/2)/b^(3/2)/d+1/2*e*erfi(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(
1/2)*Pi^(1/2)/b^(3/2)/d/exp(2*a/b)-2*e*(d*x+c)*(1+(d*x+c)^2)^(1/2)/b/d/(a+b
*arcsinh(d*x+c))^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {5865, 12, 5665, 3307, 2180, 2204, 2205}

$$\frac{\sqrt{\frac{\pi}{2}}\,ee^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}+\frac{\sqrt{\frac{\pi}{2}}\,ee^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}-\frac{2e\sqrt{(c+dx)^2+1}\,(c+dx)}{bd\sqrt{a+b\sinh^{-1}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)/(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (-2*e*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(b*d*Sqrt[a + b*ArcSinh[c + d*x]]) +
 (e*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[
b]])/(b^(3/2)*d) + (e*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]]
)/Sqrt[b]])/(b^(3/2)*d*E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
```

```
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{ce+dex}{\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{ex}{\left(a+b\sinh^{-1}(x)\right)^{3/2}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e\,\text{Subst}\left(\int \frac{x}{\left(a+b\sinh^{-1}(x)\right)^{3/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{(2e)\,\text{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx)\right)}{bd}$$

$$= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{e\,\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx)\right)}{bd} + \frac{e\,\text{Subs}}{}$$

$$= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{(2e)\,\text{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(c+dx}\right.}{b^2d}$$

$$= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{ee^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d} + \frac{ee^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}}{}$$

Mathematica [A] time = 0.12, size = 147, normalized size = 0.99

$$\frac{ee^{-\frac{2a}{b}}\left(-2e^{\frac{2a}{b}}\sinh\left(2\sinh^{-1}(c+dx)\right)+\sqrt{2}\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)-\sqrt{2}\,e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(c+}\right.}{2bd\sqrt{a+b\sinh^{-1}(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)/(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (e*(Sqrt[2]*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/2, (-2*(a + b*ArcSi
nh[c + d*x]))/b] - Sqrt[2]*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[1
```

```
/2, (2*(a + b*ArcSinh[c + d*x]))/b] - 2*E^((2*a)/b)*Sinh[2*ArcSinh[c + d*x]
]))/(2*b*d*E^((2*a)/b)*Sqrt[a + b*ArcSinh[c + d*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dex + ce}{(b \operatorname{arsinh}(dx + c) + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((d*e*x + c*e)/(b*arcsinh(d*x + c) + a)^(3/2), x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{dex + ce}{(a + b \operatorname{arcsinh}(dx + c))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(3/2),x)
[Out] int((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dex + ce}{(b \operatorname{arsinh}(dx + c) + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((d*e*x + c*e)/(b*arcsinh(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c\,e + d\,e\,x}{(a + b \operatorname{asinh}(c + d\,x))^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)/(a + b*asinh(c + d*x))^(3/2),x)
[Out] int((c*e + d*e*x)/(a + b*asinh(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e\left(\int \frac{c}{a\sqrt{a + b \operatorname{asinh}(c + dx)} + b\sqrt{a + b \operatorname{asinh}(c + dx)} \operatorname{asinh}(c + dx)}\, dx + \int \frac{}{a\sqrt{a + b \operatorname{asinh}(c + dx)} + b\sqrt{a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*asinh(d*x+c))**(3/2),x)
```

```
[Out] e*(Integral(c/(a*sqrt(a + b*asinh(c + d*x)) + b*sqrt(a + b*asinh(c + d*x))*
asinh(c + d*x)), x) + Integral(d*x/(a*sqrt(a + b*asinh(c + d*x)) + b*sqrt(a
 + b*asinh(c + d*x))*asinh(c + d*x)), x))
```

3.214 $\int \frac{1}{\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}\,dx$

Optimal. Leaf size=122

$$-\frac{\sqrt{\pi}\,e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}+\frac{\sqrt{\pi}\,e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}-\frac{2\sqrt{(c+dx)^2+1}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}}$$

```
[Out] -exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/d+erfi((
a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/d/exp(a/b)-2*(1+(d*x+c)
^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5863, 5655, 5779, 3308, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}\,e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}+\frac{\sqrt{\pi}\,e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}-\frac{2\sqrt{(c+dx)^2+1}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^(-3/2),x]
```

```
[Out] (-2*Sqrt[1 + (c + d*x)^2])/(b*d*Sqrt[a + b*ArcSinh[c + d*x]]) - (E^(a/b)*Sq
rt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(b^(3/2)*d) + (Sqrt[Pi]*E
rfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(b^(3/2)*d*E^(a/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
```

```
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 5863

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+b\sinh^{-1}(x)\right)^{3/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{2\,\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\sqrt{a+b\sinh^{-1}(x)}}\,dx, x, c+dx\right)}{bd}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{2\,\text{Subst}\left(\int \frac{\sinh(x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx)\right)}{bd}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx)\right)}{bd} + \frac{\text{Subst}\left(\right.}{}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{2\,\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(c+dx)}\right)}{b^2d} -$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d} + \frac{e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+}}{}\right.}{b^{3/2}d}$$

Mathematica [A] time = 0.11, size = 155, normalized size = 1.27

$$\frac{e^{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\left(-e^{a/b}\left(e^{2\sinh^{-1}(c+dx)}+1\right)+e^{\frac{2a}{b}+\sinh^{-1}(c+dx)}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)+e^{\sinh^{-1}(}\right.}{bd\sqrt{a+b\sinh^{-1}(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^(-3/2),x]
```

```
[Out] (-(E^(a/b)*(1 + E^(2*ArcSinh[c + d*x]))) + E^((2*a)/b + ArcSinh[c + d*x])*S
qrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2, a/b + ArcSinh[c + d*x]] + E^ArcSinh[
```

```
c + d*x]*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c
+ d*x])/b)])/(b*d*E^((a + b*ArcSinh[c + d*x])/b)*Sqrt[a + b*ArcSinh[c + d*x
]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \operatorname{arsinh}(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(d*x + c) + a)^(-3/2), x)
```

maple [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b \operatorname{arcsinh}(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(d*x+c))^(3/2),x)
[Out] int(1/(a+b*arcsinh(d*x+c))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \operatorname{arsinh}(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((b*arcsinh(d*x + c) + a)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+b \operatorname{asinh}(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*asinh(c + d*x))^(3/2),x)
[Out] int(1/(a + b*asinh(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b \operatorname{asinh}(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(d*x+c))**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))**(-3/2), x)
```

3.215 $\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}\,dx$

Optimal. Leaf size=29

$$\frac{\text{Int}\left(\frac{1}{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}},x\right)}{e}$$

[Out] Unintegrable(1/(d*x+c)/(a+b*arcsinh(d*x+c))^(3/2),x)/e

Rubi [A] time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

[In] Int[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(3/2)),x]

[Out] Defer[Subst][Defer[Int][1/(x*(a + b*ArcSinh[x])^(3/2)), x], x, c + d*x]/(d*e)

Rubi steps

$$\begin{aligned}\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{ex\left(a+b\sinh^{-1}(x)\right)^{3/2}}\,dx,x,c+dx\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{1}{x\left(a+b\sinh^{-1}(x)\right)^{3/2}}\,dx,x,c+dx\right)}{de}\end{aligned}$$

Mathematica [A] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(3/2)),x]

[Out] Integrate[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(3/2)), x]

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dex + ce)(b \operatorname{arsinh}(dx + c) + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(1/((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^(3/2)), x)
```

maple [A] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{1}{(dex + ce)\,(a + b \operatorname{arcsinh}(dx + c))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(3/2),x)
[Out] int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dex + ce)(b \operatorname{arsinh}(dx + c) + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(1/((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^(3/2)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{(c\,e + d\,e\,x)\,\left(a + b\,\operatorname{asinh}(c + d\,x)\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*e + d*e*x)*(a + b*asinh(c + d*x))^(3/2)),x)
[Out] int(1/((c*e + d*e*x)*(a + b*asinh(c + d*x))^(3/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{ac\sqrt{a+b\operatorname{asinh}(c+dx)} + adx\sqrt{a+b\operatorname{asinh}(c+dx)} + bc\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx) + bdx\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)}\, dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*asinh(d*x+c))**(3/2),x)
[Out] Integral(1/(a*c*sqrt(a + b*asinh(c + d*x)) + a*d*x*sqrt(a + b*asinh(c + d*x
)) + b*c*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x) + b*d*x*sqrt(a + b*asinh
(c + d*x))*asinh(c + d*x)), x)/e
```

3.216 $\int \frac{(ce+dex)^4}{\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}\,dx$

Optimal. Leaf size=437

$$\frac{\sqrt{\pi}\,e^4 e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{12b^{5/2}d}-\frac{3\sqrt{3\pi}\,e^4 e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8b^{5/2}d}+\frac{5\sqrt{5\pi}\,e^4 e^{\frac{5a}{b}}\text{erf}\left(\frac{\sqrt{5}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{24b^{5/2}d}+\ldots$$

```
[Out] 1/12*e^4*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(5/2)/
d+1/12*e^4*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(5/2)/d/exp(
a/b)-3/8*e^4*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(
1/2)*Pi^(1/2)/b^(5/2)/d-3/8*e^4*erfi(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(
1/2))*3^(1/2)*Pi^(1/2)/b^(5/2)/d/exp(3*a/b)+5/24*e^4*exp(5*a/b)*erf(5^(1/2)
*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*5^(1/2)*Pi^(1/2)/b^(5/2)/d+5/24*e^4*er
fi(5^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*5^(1/2)*Pi^(1/2)/b^(5/2)/d/e
xp(5*a/b)-2/3*e^4*(d*x+c)^4*(1+(d*x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))^(3
/2)-16/3*e^4*(d*x+c)^3/b^2/d/(a+b*arcsinh(d*x+c))^(1/2)-20/3*e^4*(d*x+c)^5/
b^2/d/(a+b*arcsinh(d*x+c))^(1/2)
```

Rubi [A] time = 1.55, antiderivative size = 437, normalized size of antiderivative = 1.00, number of steps used = 36, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5865, 12, 5667, 5774, 5669, 5448, 3307, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}\,e^4 e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{12b^{5/2}d}-\frac{3\sqrt{3\pi}\,e^4 e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8b^{5/2}d}+\frac{5\sqrt{5\pi}\,e^4 e^{\frac{5a}{b}}\text{Erf}\left(\frac{\sqrt{5}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{24b^{5/2}d}+\ldots$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4/(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] (-2*e^4*(c + d*x)^4*Sqrt[1 + (c + d*x)^2])/(3*b*d*(a + b*ArcSinh[c + d*x])^
(3/2)) - (16*e^4*(c + d*x)^3)/(3*b^2*d*Sqrt[a + b*ArcSinh[c + d*x]]) - (20*
e^4*(c + d*x)^5)/(3*b^2*d*Sqrt[a + b*ArcSinh[c + d*x]]) + (e^4*E^(a/b)*Sqrt
[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(12*b^(5/2)*d) - (3*e^4*E^(
(3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(8
*b^(5/2)*d) + (5*e^4*E^((5*a)/b)*Sqrt[5*Pi]*Erf[(Sqrt[5]*Sqrt[a + b*ArcSinh
[c + d*x]])/Sqrt[b]])/(24*b^(5/2)*d) + (e^4*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSin
h[c + d*x]]/Sqrt[b]])/(12*b^(5/2)*d*E^(a/b)) - (3*e^4*Sqrt[3*Pi]*Erfi[(Sqrt
[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(8*b^(5/2)*d*E^((3*a)/b)) + (5*
e^4*Sqrt[5*Pi]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(24*b^
(5/2)*d*E^((5*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[(((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^4}{\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{e^4x^4}{\left(a+b\sinh^{-1}(x)\right)^{5/2}}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e^4\,\text{Subst}\left(\int \frac{x^4}{\left(a+b\sinh^{-1}(x)\right)^{5/2}}\,dx,x,c+dx\right)}{d}$$

$$= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} + \frac{(8e^4)\,\text{Subst}\left(\int \frac{x^3}{\sqrt{1+x^2}\left(a+b\sinh^{-1}(x)\right)^{3/2}}\,dx,x,\right.}{3bd}$$

$$= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{16e^4(c+dx)^3}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{20e^4}{3b^2d\sqrt{a+b}}$$

$$= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{16e^4(c+dx)^3}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{20e^4}{3b^2d\sqrt{a+b}}$$

$$= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{16e^4(c+dx)^3}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{20e^4}{3b^2d\sqrt{a+b}}$$

$$= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{16e^4(c+dx)^3}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{20e^4}{3b^2d\sqrt{a+b}}$$

$$= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{16e^4(c+dx)^3}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{20e^4}{3b^2d\sqrt{a+b}}$$

$$= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{16e^4(c+dx)^3}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{20e^4}{3b^2d\sqrt{a+b}}$$

$$= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{16e^4(c+dx)^3}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{20e^4}{3b^2d\sqrt{a+b}}$$

Mathematica [A] time = 3.45, size = 551, normalized size = 1.26

$$e^4\left(-2e^{\sinh^{-1}(c+dx)}\left(2a+2b\sinh^{-1}(c+dx)+b\right)-4be^{-\frac{a}{b}}\left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{3/2}\Gamma\left(\frac{1}{2},-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)+e^{-\sinh^{-1}(c\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^4/(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] (e^4*(-2*E^ArcSinh[c + d*x]*(2*a + b + 2*b*ArcSinh[c + d*x]) + (4*a - 2*b +
 4*b*ArcSinh[c + d*x] - 4*E^(a/b + ArcSinh[c + d*x])*Sqrt[a/b + ArcSinh[c +
 d*x]]*(a + b*ArcSinh[c + d*x])*Gamma[1/2, a/b + ArcSinh[c + d*x]])/E^ArcSi
nh[c + d*x] + (-(E^(5*(a/b + ArcSinh[c + d*x]))*(10*a + b + 10*b*ArcSinh[c
```

```
+ d*x])) - 10*Sqrt[5]*b*(-((a + b*ArcSinh[c + d*x])/b))^(3/2)*Gamma[1/2, (-
5*(a + b*ArcSinh[c + d*x]))/b])/E^((5*a)/b) + (3*E^(3*(a/b + ArcSinh[c + d*
x]))*(6*a + b + 6*b*ArcSinh[c + d*x]) + 18*Sqrt[3]*b*(-((a + b*ArcSinh[c +
d*x])/b))^(3/2)*Gamma[1/2, (-3*(a + b*ArcSinh[c + d*x]))/b])/E^((3*a)/b) -
(4*b*(-((a + b*ArcSinh[c + d*x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcSinh[c +
d*x])/b)])/E^(a/b) + (3*(-6*a + b - 6*b*ArcSinh[c + d*x] + 6*Sqrt[3]*E^(3*(
a/b + ArcSinh[c + d*x]))*Sqrt[a/b + ArcSinh[c + d*x]]*(a + b*ArcSinh[c + d*
x])*Gamma[1/2, (3*(a + b*ArcSinh[c + d*x]))/b]))/E^(3*ArcSinh[c + d*x]) + (
10*a - b + 10*b*ArcSinh[c + d*x] - 10*Sqrt[5]*E^(5*(a/b + ArcSinh[c + d*x])
)*Sqrt[a/b + ArcSinh[c + d*x]]*(a + b*ArcSinh[c + d*x])*Gamma[1/2, (5*(a +
b*ArcSinh[c + d*x]))/b])/E^(5*ArcSinh[c + d*x])))/(48*b^2*d*(a + b*ArcSinh[
c + d*x])^(3/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex + ce)^4}{(b \operatorname{arsinh}(dx + c) + a)^{\frac{5}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^4/(b*arcsinh(d*x + c) + a)^(5/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(dex + ce)^4}{(a + b \operatorname{arcsinh}(dx + c))^{\frac{5}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(5/2),x)
```

```
[Out] int((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex + ce)^4}{(b \operatorname{arsinh}(dx + c) + a)^{\frac{5}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^4/(b*arcsinh(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,e + d\,e\,x)^4}{(a + b\,\mathrm{asinh}(c + d\,x))^{5/2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^4/(a + b*asinh(c + d*x))^(5/2),x)
```

```
[Out] int((c*e + d*e*x)^4/(a + b*asinh(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^4\left(\int \frac{c^4}{a^2\sqrt{a+b\operatorname{asinh}(c+dx)}+2ab\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)+b^2\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}^2(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4/(a+b*asinh(d*x+c))**(5/2),x)
```

```
[Out] e**4*(Integral(c**4/(a**2*sqrt(a + b*asinh(c + d*x)) + 2*a*b*sqrt(a + b*asi
nh(c + d*x))*asinh(c + d*x) + b**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x
)**2), x) + Integral(d**4*x**4/(a**2*sqrt(a + b*asinh(c + d*x)) + 2*a*b*sqr
t(a + b*asinh(c + d*x))*asinh(c + d*x) + b**2*sqrt(a + b*asinh(c + d*x))*as
inh(c + d*x)**2), x) + Integral(4*c*d**3*x**3/(a**2*sqrt(a + b*asinh(c + d*
x)) + 2*a*b*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x) + b**2*sqrt(a + b*asi
nh(c + d*x))*asinh(c + d*x)**2), x) + Integral(6*c**2*d**2*x**2/(a**2*sqrt(
a + b*asinh(c + d*x)) + 2*a*b*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x) + b
**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**2), x) + Integral(4*c**3*d*x
/(a**2*sqrt(a + b*asinh(c + d*x)) + 2*a*b*sqrt(a + b*asinh(c + d*x))*asinh(
c + d*x) + b**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**2), x))
```

3.217 $\int \frac{(ce+dex)^3}{\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}\,dx$

Optimal. Leaf size=326

$$-\frac{2\sqrt{\pi}\,e^3 e^{\frac{4a}{b}}\operatorname{erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}+\frac{\sqrt{2\pi}\,e^3 e^{\frac{2a}{b}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}+\frac{2\sqrt{\pi}\,e^3 e^{-\frac{4a}{b}}\operatorname{erfi}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}-\ldots$$

```
[Out] -2/3*e^3*exp(4*a/b)*erf(2*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(5
/2)/d+2/3*e^3*erfi(2*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(5/2)/d
/exp(4*a/b)+1/3*e^3*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/
2))*2^(1/2)*Pi^(1/2)/b^(5/2)/d-1/3*e^3*erfi(2^(1/2)*(a+b*arcsinh(d*x+c))^(1
/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/b^(5/2)/d/exp(2*a/b)-2/3*e^3*(d*x+c)^3*(1+(d*
x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))^(3/2)-4*e^3*(d*x+c)^2/b^2/d/(a+b*arc
sinh(d*x+c))^(1/2)-16/3*e^3*(d*x+c)^4/b^2/d/(a+b*arcsinh(d*x+c))^(1/2)
```

Rubi [A] time = 1.10, antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5865, 12, 5667, 5774, 5669, 5448, 3308, 2180, 2204, 2205}

$$-\frac{2\sqrt{\pi}\,e^3 e^{\frac{4a}{b}}\operatorname{Erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}+\frac{\sqrt{2\pi}\,e^3 e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}+\frac{2\sqrt{\pi}\,e^3 e^{-\frac{4a}{b}}\operatorname{Erfi}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}-\ldots$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3/(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] (-2*e^3*(c + d*x)^3*Sqrt[1 + (c + d*x)^2])/(3*b*d*(a + b*ArcSinh[c + d*x])^
(3/2)) - (4*e^3*(c + d*x)^2)/(b^2*d*Sqrt[a + b*ArcSinh[c + d*x]]) - (16*e^3
*(c + d*x)^4)/(3*b^2*d*Sqrt[a + b*ArcSinh[c + d*x]]) - (2*e^3*E^((4*a)/b)*S
qrt[Pi]*Erf[(2*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(3*b^(5/2)*d) + (e^3
*E^((2*a)/b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]
)/(3*b^(5/2)*d) + (2*e^3*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcSinh[c + d*x]])/Sqr
t[b]])/(3*b^(5/2)*d*E^((4*a)/b)) - (e^3*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b
*ArcSinh[c + d*x]])/Sqrt[b]])/(3*b^(5/2)*d*E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^3}{\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{e^3x^3}{\left(a+b\sinh^{-1}(x)\right)^{5/2}}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e^3\,\text{Subst}\left(\int \frac{x^3}{\left(a+b\sinh^{-1}(x)\right)^{5/2}}\,dx,x,c+dx\right)}{d}$$

$$= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} + \frac{\left(2e^3\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1+x^2}\left(a+b\sinh^{-1}(x)\right)^{3/2}}\,dx,x,c+\right.}{bd}$$

$$= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4e^3(c+dx)^2}{b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{16e^3(c+}{3b^2d\sqrt{a+b\sin}}$$

$$= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4e^3(c+dx)^2}{b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{16e^3(c+}{3b^2d\sqrt{a+b\sin}}$$

$$= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4e^3(c+dx)^2}{b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{16e^3(c+}{3b^2d\sqrt{a+b\sin}}$$

$$= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4e^3(c+dx)^2}{b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{16e^3(c+}{3b^2d\sqrt{a+b\sin}}$$

$$= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4e^3(c+dx)^2}{b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{16e^3(c+}{3b^2d\sqrt{a+b\sin}}$$

$$= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4e^3(c+dx)^2}{b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{16e^3(c+}{3b^2d\sqrt{a+b\sin}}$$

$$= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4e^3(c+dx)^2}{b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{16e^3(c+}{3b^2d\sqrt{a+b\sin}}$$

Mathematica [A] time = 1.78, size = 390, normalized size = 1.20

$$\frac{e^3e^{-4\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)}\left(-8be^{4\sinh^{-1}(c+dx)}\left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{3/2}\Gamma\left(\frac{1}{2},-\frac{4\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)+4\sqrt{2}\,be^{\frac{2a}{b}+4\sinh^{-1}(c+dx)}\left(-\frac{a+b}{}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^3/(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] (e^3*(-8*b*E^(4*ArcSinh[c + d*x])*(-((a + b*ArcSinh[c + d*x])/b))^(3/2)*Gam
ma[1/2, (-4*(a + b*ArcSinh[c + d*x]))/b] + 4*Sqrt[2]*b*E^((2*a)/b + 4*ArcSi
nh[c + d*x])*(-((a + b*ArcSinh[c + d*x])/b))^(3/2)*Gamma[1/2, (-2*(a + b*Ar
cSinh[c + d*x]))/b] + (E^((4*a)/b)*(-((-1 + E^(2*ArcSinh[c + d*x]))^2*(b*(-
```

```
1 + E^(4*ArcSinh[c + d*x])) + 8*a*(1 + E^(2*ArcSinh[c + d*x]) + E^(4*ArcSin
h[c + d*x])) + 8*b*(1 + E^(2*ArcSinh[c + d*x]) + E^(4*ArcSinh[c + d*x]))*Ar
cSinh[c + d*x])) - 8*Sqrt[2]*E^((2*a)/b + 4*ArcSinh[c + d*x])*Sqrt[a/b + Ar
cSinh[c + d*x]]*(a + b*ArcSinh[c + d*x])*Gamma[1/2, (2*(a + b*ArcSinh[c + d
*x]))/b] + 16*E^(4*(a/b + ArcSinh[c + d*x]))*Sqrt[a/b + ArcSinh[c + d*x]]*(
a + b*ArcSinh[c + d*x])*Gamma[1/2, (4*(a + b*ArcSinh[c + d*x]))/b]))/2))/(1
2*b^2*d*E^(4*(a/b + ArcSinh[c + d*x]))*(a + b*ArcSinh[c + d*x])^(3/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex+ce)^3}{(b\,\mathrm{arsinh}\,(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^3/(b*arcsinh(d*x + c) + a)^(5/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(dex+ce)^3}{(a+b\,\mathrm{arcsinh}\,(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(5/2),x)
```

```
[Out] int((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex+ce)^3}{(b\,\mathrm{arsinh}\,(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^3/(b*arcsinh(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,e+d\,e\,x)^3}{(a+b\,\mathrm{asinh}\,(c+d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3/(a + b*asinh(c + d*x))^(5/2),x)
```

```
[Out] int((c*e + d*e*x)^3/(a + b*asinh(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^3\left(\int \frac{c^3}{a^2\sqrt{a+b\operatorname{asinh}(c+dx)}+2ab\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)+b^2\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}^2(c+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3/(a+b*asinh(d*x+c))**(5/2),x)
```

```
[Out] e**3*(Integral(c**3/(a**2*sqrt(a + b*asinh(c + d*x)) + 2*a*b*sqrt(a + b*asi
nh(c + d*x))*asinh(c + d*x) + b**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x
)**2), x) + Integral(d**3*x**3/(a**2*sqrt(a + b*asinh(c + d*x)) + 2*a*b*sqr
t(a + b*asinh(c + d*x))*asinh(c + d*x) + b**2*sqrt(a + b*asinh(c + d*x))*as
inh(c + d*x)**2), x) + Integral(3*c*d**2*x**2/(a**2*sqrt(a + b*asinh(c + d*
x)) + 2*a*b*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x) + b**2*sqrt(a + b*asi
nh(c + d*x))*asinh(c + d*x)**2), x) + Integral(3*c**2*d*x/(a**2*sqrt(a + b*
asinh(c + d*x)) + 2*a*b*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x) + b**2*sq
rt(a + b*asinh(c + d*x))*asinh(c + d*x)**2), x))
```

3.218 $\int \frac{(ce+dex)^2}{\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}\,dx$

Optimal. Leaf size=321

$$-\frac{\sqrt{\pi}\,e^2e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{6b^{5/2}d}+\frac{\sqrt{3\pi}\,e^2e^{\frac{3a}{b}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2b^{5/2}d}-\frac{\sqrt{\pi}\,e^2e^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{6b^{5/2}d}+\frac{\sqrt{3\pi}\ldots}{\ldots}$$

```
[Out] -1/6*e^2*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(5/2)/
d-1/6*e^2*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(5/2)/d/exp(a
/b)+1/2*e^2*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(1
/2)*Pi^(1/2)/b^(5/2)/d+1/2*e^2*erfi(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1
/2))*3^(1/2)*Pi^(1/2)/b^(5/2)/d/exp(3*a/b)-2/3*e^2*(d*x+c)^2*(1+(d*x+c)^2)^
(1/2)/b/d/(a+b*arcsinh(d*x+c))^(3/2)-8/3*e^2*(d*x+c)/b^2/d/(a+b*arcsinh(d*x
+c))^(1/2)-4*e^2*(d*x+c)^3/b^2/d/(a+b*arcsinh(d*x+c))^(1/2)
```

Rubi [A] time = 0.96, antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {5865, 12, 5667, 5774, 5669, 5448, 3307, 2180, 2204, 2205, 5657}

$$-\frac{\sqrt{\pi}\,e^2e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{6b^{5/2}d}+\frac{\sqrt{3\pi}\,e^2e^{\frac{3a}{b}}\operatorname{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2b^{5/2}d}-\frac{\sqrt{\pi}\,e^2e^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{6b^{5/2}d}+\frac{\sqrt{3\pi}\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2/(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] (-2*e^2*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])/(3*b*d*(a + b*ArcSinh[c + d*x])^
(3/2)) - (8*e^2*(c + d*x))/(3*b^2*d*Sqrt[a + b*ArcSinh[c + d*x]]) - (4*e^2*
(c + d*x)^3)/(b^2*d*Sqrt[a + b*ArcSinh[c + d*x]]) - (e^2*E^(a/b)*Sqrt[Pi]*E
rf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(6*b^(5/2)*d) + (e^2*E^((3*a)/b)*
Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(2*b^(5/2)*
d) - (e^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(6*b^(5/2)*d
*E^(a/b)) + (e^2*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqr
t[b]])/(2*b^(5/2)*d*E^((3*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[(((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^2}{\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{e^2x^2}{\left(a+b\sinh^{-1}(x)\right)^{5/2}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^2\,\text{Subst}\left(\int \frac{x^2}{\left(a+b\sinh^{-1}(x)\right)^{5/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} + \frac{(4e^2)\,\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\left(a+b\sinh^{-1}(x)\right)^{3/2}}\,dx, x,\right.}{3bd}$$

$$= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{8e^2(c+dx)}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{4e^2(}{b^2d\sqrt{a+b}}$$

$$= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{8e^2(c+dx)}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{4e^2(}{b^2d\sqrt{a+b}}$$

$$= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{8e^2(c+dx)}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{4e^2(}{b^2d\sqrt{a+b}}$$

$$= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{8e^2(c+dx)}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{4e^2(}{b^2d\sqrt{a+b}}$$

$$= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{8e^2(c+dx)}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{4e^2(}{b^2d\sqrt{a+b}}$$

$$= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{8e^2(c+dx)}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{4e^2(}{b^2d\sqrt{a+b}}$$

$$= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{8e^2(c+dx)}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{4e^2(}{b^2d\sqrt{a+b}}$$

Mathematica [A] time = 1.55, size = 389, normalized size = 1.21

$$\frac{e^2e^{-3\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)}\left(-6\sqrt{3}\,be^{3\sinh^{-1}(c+dx)}\left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{3/2}\Gamma\left(\frac{1}{2},-\frac{3\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)+2be^{\frac{2a}{b}+3\sinh^{-1}(c+dx)}\left(-\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^2/(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] (e^2*(2*E^((4*a)/b + 3*ArcSinh[c + d*x])*Sqrt[a/b + ArcSinh[c + d*x]]*(a +
b*ArcSinh[c + d*x])*Gamma[1/2, a/b + ArcSinh[c + d*x]] - 6*Sqrt[3]*b*E^(3*A
rcSinh[c + d*x])*(-((a + b*ArcSinh[c + d*x])/b))^(3/2)*Gamma[1/2, (-3*(a +
b*ArcSinh[c + d*x]))/b] + 2*b*E^((2*a)/b + 3*ArcSinh[c + d*x])*(-((a + b*Ar
cSinh[c + d*x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcSinh[c + d*x])/b)] - E^((3
*a)/b)*((-1 + E^(2*ArcSinh[c + d*x]))*(b*(-1 + E^(4*ArcSinh[c + d*x])) + a*
(6 + 4*E^(2*ArcSinh[c + d*x]) + 6*E^(4*ArcSinh[c + d*x])) + 2*b*(3 + 2*E^(2
*ArcSinh[c + d*x]) + 3*E^(4*ArcSinh[c + d*x]))*ArcSinh[c + d*x]) + 6*Sqrt[3
]*E^(3*(a/b + ArcSinh[c + d*x]))*Sqrt[a/b + ArcSinh[c + d*x]]*(a + b*ArcSin
h[c + d*x])*Gamma[1/2, (3*(a + b*ArcSinh[c + d*x]))/b])))/(12*b^2*d*E^(3*(a
/b + ArcSinh[c + d*x]))*(a + b*ArcSinh[c + d*x])^(3/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex + ce)^2}{(b \operatorname{arsinh}(dx + c) + a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2/(b*arcsinh(d*x + c) + a)^(5/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(dex + ce)^2}{(a + b \operatorname{arcsinh}(dx + c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(5/2),x)
```

```
[Out] int((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex + ce)^2}{(b \operatorname{arsinh}(dx + c) + a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^2/(b*arcsinh(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,e + d\,e\,x)^2}{(a + b \operatorname{asinh}(c + d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2/(a + b*asinh(c + d*x))^(5/2),x)
```

```
[Out] int((c*e + d*e*x)^2/(a + b*asinh(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^2\left(\int \frac{c^2}{a^2\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}+2ab\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\,\mathrm{asinh}\,(c+dx)+b^2\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\,\mathrm{asinh}^2\,(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2/(a+b*asinh(d*x+c))**(5/2),x)
```

```
[Out] e**2*(Integral(c**2/(a**2*sqrt(a + b*asinh(c + d*x)) + 2*a*b*sqrt(a + b*asi
nh(c + d*x))*asinh(c + d*x) + b**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x
)**2), x) + Integral(d**2*x**2/(a**2*sqrt(a + b*asinh(c + d*x)) + 2*a*b*sqr
t(a + b*asinh(c + d*x))*asinh(c + d*x) + b**2*sqrt(a + b*asinh(c + d*x))*as
inh(c + d*x)**2), x) + Integral(2*c*d*x/(a**2*sqrt(a + b*asinh(c + d*x)) +
2*a*b*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x) + b**2*sqrt(a + b*asinh(c +
 d*x))*asinh(c + d*x)**2), x))
```

3.219 $\int \frac{ce+dex}{\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}\,dx$

Optimal. Leaf size=209

$$-\frac{2\sqrt{2\pi}\,ee^{\frac{2a}{b}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}+\frac{2\sqrt{2\pi}\,ee^{-\frac{2a}{b}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}-\frac{8e(c+dx)^2}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}}-\frac{}{3b^2d\sqrt{}}$$

```
[Out] -2/3*e*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*P
i^(1/2)/b^(5/2)/d+2/3*e*erfi(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^
(1/2)*Pi^(1/2)/b^(5/2)/d/exp(2*a/b)-2/3*e*(d*x+c)*(1+(d*x+c)^2)^(1/2)/b/d/(
a+b*arcsinh(d*x+c))^(3/2)-4/3*e/b^2/d/(a+b*arcsinh(d*x+c))^(1/2)-8/3*e*(d*x
+c)^2/b^2/d/(a+b*arcsinh(d*x+c))^(1/2)
```

Rubi [A] time = 0.54, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {5865, 12, 5667, 5774, 5669, 5448, 3308, 2180, 2204, 2205, 5675}

$$-\frac{2\sqrt{2\pi}\,ee^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}+\frac{2\sqrt{2\pi}\,ee^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}-\frac{8e(c+dx)^2}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}}-\frac{}{3b^2d\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)/(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] (-2*e*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(3*b*d*(a + b*ArcSinh[c + d*x])^(3/2
)) - (4*e)/(3*b^2*d*Sqrt[a + b*ArcSinh[c + d*x]]) - (8*e*(c + d*x)^2)/(3*b^
2*d*Sqrt[a + b*ArcSinh[c + d*x]]) - (2*e*E^((2*a)/b)*Sqrt[2*Pi]*Erf[(Sqrt[2
]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(3*b^(5/2)*d) + (2*e*Sqrt[2*Pi]*E
rfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(3*b^(5/2)*d*E^((2*a)/
b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{ce+dex}{\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{ex}{\left(a+b\sinh^{-1}(x)\right)^{5/2}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e\,\text{Subst}\left(\int \frac{x}{\left(a+b\sinh^{-1}(x)\right)^{5/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} + \frac{(2e)\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}\left(a+b\sinh^{-1}(x)\right)^{3/2}}\,dx, x, c+a\right.}{3bd}$$

$$= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4e}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{8e(c+}{3b^2d\sqrt{a+b\,\text{si}}}$$

$$= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4e}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{8e(c+}{3b^2d\sqrt{a+b\,\text{si}}}$$

$$= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4e}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{8e(c+}{3b^2d\sqrt{a+b\,\text{si}}}$$

$$= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4e}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{8e(c+}{3b^2d\sqrt{a+b\,\text{si}}}$$

$$= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4e}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{8e(c+}{3b^2d\sqrt{a+b\,\text{si}}}$$

$$= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4e}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{8e(c+}{3b^2d\sqrt{a+b\,\text{si}}}$$

$$= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4e}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{8e(c+}{3b^2d\sqrt{a+b\,\text{si}}}$$

Mathematica [A] time = 0.70, size = 227, normalized size = 1.09

$$\frac{ee^{-2\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)}\left(e^{\frac{2a}{b}}\left(4\sqrt{2}\,e^{2\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\left(a+b\sinh^{-1}(c+dx)\right)\Gamma\left(\frac{1}{2},\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)/(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] (e*(-4*Sqrt[2]*b*E^(2*ArcSinh[c + d*x])*(-((a + b*ArcSinh[c + d*x])/b))^(3/
2)*Gamma[1/2, (-2*(a + b*ArcSinh[c + d*x]))/b] + E^((2*a)/b)*(-4*a + b - 4*
a*E^(4*ArcSinh[c + d*x]) - b*E^(4*ArcSinh[c + d*x]) - 4*b*(1 + E^(4*ArcSinh
[c + d*x]))*ArcSinh[c + d*x] + 4*Sqrt[2]*E^(2*(a/b + ArcSinh[c + d*x]))*Sqr
```

```
t[a/b + ArcSinh[c + d*x]]*(a + b*ArcSinh[c + d*x])*Gamma[1/2, (2*(a + b*Arc
Sinh[c + d*x]))/b])))/(6*b^2*d*E^(2*(a/b + ArcSinh[c + d*x]))*(a + b*ArcSin
h[c + d*x])^(3/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dex + ce}{(b \operatorname{arsinh}(dx + c) + a)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)/(b*arcsinh(d*x + c) + a)^(5/2), x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{dex + ce}{(a + b \operatorname{arcsinh}(dx + c))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(5/2),x)
```

```
[Out] int((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dex + ce}{(b \operatorname{arsinh}(dx + c) + a)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)/(b*arcsinh(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{c\,e + d\,e\,x}{(a + b\,\mathrm{asinh}(c + d\,x))^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)/(a + b*asinh(c + d*x))^(5/2),x)
```

```
[Out] int((c*e + d*e*x)/(a + b*asinh(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e\left(\int \frac{c}{a^2\sqrt{a + b \operatorname{asinh}(c + dx)} + 2ab\sqrt{a + b \operatorname{asinh}(c + dx)} \operatorname{asinh}(c + dx) + b^2\sqrt{a + b \operatorname{asinh}(c + dx)} \operatorname{asinh}^2(c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*asinh(d*x+c))**(5/2),x)
```

```
[Out] e*(Integral(c/(a**2*sqrt(a + b*asinh(c + d*x)) + 2*a*b*sqrt(a + b*asinh(c +
 d*x))*asinh(c + d*x) + b**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**2),
 x) + Integral(d*x/(a**2*sqrt(a + b*asinh(c + d*x)) + 2*a*b*sqrt(a + b*asin
h(c + d*x))*asinh(c + d*x) + b**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)
**2), x))
```

3.220 $\int \frac{1}{\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}\,dx$

Optimal. Leaf size=158

$$\frac{2\sqrt{\pi}\,e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}+\frac{2\sqrt{\pi}\,e^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}-\frac{4(c+dx)}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}}-\frac{2\sqrt{(c+d}}{3bd\left(a+b\sinh\right.}$$

```
[Out] 2/3*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(5/2)/d+2/3
*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(5/2)/d/exp(a/b)-2/3*(
1+(d*x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))^(3/2)-4/3*(d*x+c)/b^2/d/(a+b*ar
csinh(d*x+c))^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5863, 5655, 5774, 5657, 3307, 2180, 2205, 2204}

$$\frac{2\sqrt{\pi}\,e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}+\frac{2\sqrt{\pi}\,e^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}-\frac{4(c+dx)}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}}-\frac{2\sqrt{(c+}}{3bd\left(a+b\sinh\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^(-5/2),x]
```

```
[Out] (-2*Sqrt[1 + (c + d*x)^2])/(3*b*d*(a + b*ArcSinh[c + d*x])^(3/2)) - (4*(c +
 d*x))/(3*b^2*d*Sqrt[a + b*ArcSinh[c + d*x]]) + (2*E^(a/b)*Sqrt[Pi]*Erf[Sqr
t[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(3*b^(5/2)*d) + (2*Sqrt[Pi]*Erfi[Sqrt[a
 + b*ArcSinh[c + d*x]]/Sqrt[b]])/(3*b^(5/2)*d*E^(a/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5863

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+b\sinh^{-1}(x)\right)^{5/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} + \frac{2\,\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\left(a+b\sinh^{-1}(x)\right)^{3/2}}\,dx, x, c+ d\right.}{3bd}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4(c+dx)}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{4\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4(c+dx)}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{4\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4(c+dx)}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{2\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4(c+dx)}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{4\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4(c+dx)}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{2e^{a/b}\sqrt{\pi}\,\text{erf}}{}$$

Mathematica [A] time = 0.29, size = 207, normalized size = 1.31

$$e^{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\left(-e^{a/b}\left(2a\left(e^{2\sinh^{-1}(c+dx)}-1\right)-2b\sinh^{-1}(c+dx)+be^{2\sinh^{-1}(c+dx)}\left(2\sinh^{-1}(c+dx)+1\right)+b\right)-\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^(-5/2),x]
```

```
[Out] (-(E^(a/b)*(b + 2*a*(-1 + E^(2*ArcSinh[c + d*x])) - 2*b*ArcSinh[c + d*x] +
b*E^(2*ArcSinh[c + d*x])*(1 + 2*ArcSinh[c + d*x]))) - 2*E^((2*a)/b + ArcSin
h[c + d*x])*Sqrt[a/b + ArcSinh[c + d*x]]*(a + b*ArcSinh[c + d*x])*Gamma[1/2
, a/b + ArcSinh[c + d*x]] - 2*b*E^ArcSinh[c + d*x]*(-((a + b*ArcSinh[c + d*
x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcSinh[c + d*x])/b)])/(3*b^2*d*E^((a + b
*ArcSinh[c + d*x])/b)*(a + b*ArcSinh[c + d*x])^(3/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\operatorname{arsinh}(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(d*x + c) + a)^(-5/2), x)
```

maple [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\operatorname{arcsinh}(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(d*x+c))^(5/2),x)
[Out] int(1/(a+b*arcsinh(d*x+c))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\operatorname{arsinh}(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*arcsinh(d*x + c) + a)^(-5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+b\operatorname{asinh}(c+d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*asinh(c + d*x))^(5/2),x)
[Out] int(1/(a + b*asinh(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\operatorname{asinh}(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(d*x+c))**(5/2),x)
[Out] Integral((a + b*asinh(c + d*x))**(-5/2), x)
```

3.221 $\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}\,dx$

Optimal. Leaf size=29

$$\frac{\text{Int}\left(\frac{1}{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}},x\right)}{e}$$

[Out] Unintegrable(1/(d*x+c)/(a+b*arcsinh(d*x+c))^(5/2),x)/e

Rubi [A] time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

[In] Int[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(5/2)),x]

[Out] Defer[Subst][Defer[Int][1/(x*(a + b*ArcSinh[x])^(5/2)), x], x, c + d*x]/(d*e)

Rubi steps

$$\begin{aligned}\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{ex\left(a+b\sinh^{-1}(x)\right)^{5/2}}\,dx,x,c+dx\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{1}{x\left(a+b\sinh^{-1}(x)\right)^{5/2}}\,dx,x,c+dx\right)}{de}\end{aligned}$$

Mathematica [A] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(5/2)),x]

[Out] Integrate[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(5/2)), x]

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

giac [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dex+ce)(b \operatorname{arsinh}(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^(5/2)), x)
```

maple [**A**] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{1}{(dex+ce)\,(a+b \operatorname{arcsinh}(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(5/2),x)
```

```
[Out] int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(5/2),x)
```

maxima [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dex+ce)(b \operatorname{arsinh}(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^(5/2)), x)
```

mupad [**A**] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{(c\,e+d\,e\,x)\,(a+b \operatorname{asinh}(c+d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*e + d*e*x)*(a + b*asinh(c + d*x))^(5/2)),x)
```

```
[Out] int(1/((c*e + d*e*x)*(a + b*asinh(c + d*x))^(5/2)), x)
```

sympy [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^2c\sqrt{a+b \operatorname{asinh}(c+dx)}+a^2dx\sqrt{a+b \operatorname{asinh}(c+dx)}+2abc\sqrt{a+b \operatorname{asinh}(c+dx)} \operatorname{asinh}(c+dx)+2abdx\sqrt{a+b \operatorname{asinh}(c+dx)} \operatorname{asinh}(c+dx)+b^2c\sqrt{a+b \operatorname{asinh}(c}}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*asinh(d*x+c))**(5/2),x)
```

```
[Out] Integral(1/(a**2*c*sqrt(a + b*asinh(c + d*x)) + a**2*d*x*sqrt(a + b*asinh(c
 + d*x)) + 2*a*b*c*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x) + 2*a*b*d*x*sq
rt(a + b*asinh(c + d*x))*asinh(c + d*x) + b**2*c*sqrt(a + b*asinh(c + d*x))
*asinh(c + d*x)**2 + b**2*d*x*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**2)
, x)/e
```

3.222 $\int \frac{(ce+dex)^4}{\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}\,dx$

Optimal. Leaf size=531

$$-\frac{\sqrt{\pi}\,e^4e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{30b^{7/2}d}+\frac{9\sqrt{3\pi}\,e^4e^{\frac{3a}{b}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{20b^{7/2}d}-\frac{5\sqrt{5\pi}\,e^4e^{\frac{5a}{b}}\operatorname{erf}\left(\frac{\sqrt{5}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{12b^{7/2}d}+$$

```
[Out] -16/15*e^4*(d*x+c)^3/b^2/d/(a+b*arcsinh(d*x+c))^(3/2)-4/3*e^4*(d*x+c)^5/b^2
/d/(a+b*arcsinh(d*x+c))^(3/2)-1/30*e^4*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1
/2)/b^(1/2))*Pi^(1/2)/b^(7/2)/d+1/30*e^4*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^
(1/2))*Pi^(1/2)/b^(7/2)/d/exp(a/b)+9/20*e^4*exp(3*a/b)*erf(3^(1/2)*(a+b*arc
sinh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/b^(7/2)/d-9/20*e^4*erfi(3^(1/2
)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/b^(7/2)/d/exp(3*a/b)
-5/12*e^4*exp(5*a/b)*erf(5^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*5^(1/2
)*Pi^(1/2)/b^(7/2)/d+5/12*e^4*erfi(5^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/
2))*5^(1/2)*Pi^(1/2)/b^(7/2)/d/exp(5*a/b)-2/5*e^4*(d*x+c)^4*(1+(d*x+c)^2)^(
1/2)/b/d/(a+b*arcsinh(d*x+c))^(5/2)-32/5*e^4*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)/
b^3/d/(a+b*arcsinh(d*x+c))^(1/2)-40/3*e^4*(d*x+c)^4*(1+(d*x+c)^2)^(1/2)/b^3
/d/(a+b*arcsinh(d*x+c))^(1/2)
```

Rubi [A] time = 1.47, antiderivative size = 531, normalized size of antiderivative = 1.00, number of steps used = 34, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {5865, 12, 5667, 5774, 5665, 3308, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}\,e^4e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{30b^{7/2}d}+\frac{9\sqrt{3\pi}\,e^4e^{\frac{3a}{b}}\operatorname{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{20b^{7/2}d}-\frac{5\sqrt{5\pi}\,e^4e^{\frac{5a}{b}}\operatorname{Erf}\left(\frac{\sqrt{5}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{12b^{7/2}d}+$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4/(a + b*ArcSinh[c + d*x])^(7/2),x]
```

```
[Out] (-2*e^4*(c + d*x)^4*Sqrt[1 + (c + d*x)^2])/(5*b*d*(a + b*ArcSinh[c + d*x])^
(5/2)) - (16*e^4*(c + d*x)^3)/(15*b^2*d*(a + b*ArcSinh[c + d*x])^(3/2)) - (
4*e^4*(c + d*x)^5)/(3*b^2*d*(a + b*ArcSinh[c + d*x])^(3/2)) - (32*e^4*(c +
d*x)^2*Sqrt[1 + (c + d*x)^2])/(5*b^3*d*Sqrt[a + b*ArcSinh[c + d*x]]) - (40*
e^4*(c + d*x)^4*Sqrt[1 + (c + d*x)^2])/(3*b^3*d*Sqrt[a + b*ArcSinh[c + d*x]
]) - (e^4*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(30*b
^(7/2)*d) + (9*e^4*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c
 + d*x]])/Sqrt[b]])/(20*b^(7/2)*d) - (5*e^4*E^((5*a)/b)*Sqrt[5*Pi]*Erf[(Sqr
t[5]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(12*b^(7/2)*d) + (e^4*Sqrt[Pi]
*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(30*b^(7/2)*d*E^(a/b)) - (9*e^
4*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(20*b^(7
/2)*d*E^((3*a)/b)) + (5*e^4*Sqrt[5*Pi]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcSinh[c +
 d*x]])/Sqrt[b]])/(12*b^(7/2)*d*E^((5*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
```

```
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^4}{\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}\,dx = \frac{\text{Subst}\left(\int \frac{e^4x^4}{\left(a+b\sinh^{-1}(x)\right)^{7/2}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^4\,\text{Subst}\left(\int \frac{x^4}{\left(a+b\sinh^{-1}(x)\right)^{7/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} + \frac{\left(8e^4\right)\text{Subst}\left(\int \frac{x^3}{\sqrt{1+x^2}\left(a+b\sinh^{-1}(x)\right)^{5/2}}\,dx, x, c\right.}{5bd}$$

$$= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{16e^4(c+dx)^3}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{}{3b^2d\left(a\right.}$$

$$= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{16e^4(c+dx)^3}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{}{3b^2d\left(a\right.}$$

$$= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{16e^4(c+dx)^3}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{}{3b^2d\left(a\right.}$$

$$= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{16e^4(c+dx)^3}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{}{3b^2d\left(a\right.}$$

$$= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{16e^4(c+dx)^3}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{}{3b^2d\left(a\right.}$$

$$= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{16e^4(c+dx)^3}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{}{3b^2d\left(a\right.}$$

Mathematica [A] time = 2.72, size = 701, normalized size = 1.32

$$e^4\left(e^{-\sinh^{-1}(c+dx)}\left(-8a^2-4b(4a-b)\sinh^{-1}(c+dx)+8e^{\frac{a}{b}+\sinh^{-1}(c+dx)}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\left(a+b\sinh^{-1}(c+d\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^4/(a + b*ArcSinh[c + d*x])^(7/2),x]
```

```
[Out] (e^4*(-6*b^2*E^ArcSinh[c + d*x] - 3*b^2*E^(5*ArcSinh[c + d*x]) + (-8*a^2 +
4*a*b - 6*b^2 - 4*(4*a - b)*b*ArcSinh[c + d*x] - 8*b^2*ArcSinh[c + d*x]^2 +
 8*E^(a/b + ArcSinh[c + d*x])*Sqrt[a/b + ArcSinh[c + d*x]]*(a + b*ArcSinh[c
 + d*x])^2*Gamma[1/2, a/b + ArcSinh[c + d*x]])/E^ArcSinh[c + d*x] - (10*(a
+ b*ArcSinh[c + d*x])*(E^(5*(a/b + ArcSinh[c + d*x]))*(10*a + b + 10*b*ArcS
inh[c + d*x]) + 10*Sqrt[5]*b*(-((a + b*ArcSinh[c + d*x])/b))^(3/2)*Gamma[1/
2, (-5*(a + b*ArcSinh[c + d*x]))/b]))/E^((5*a)/b) + 9*(b^2*E^(3*ArcSinh[c +
 d*x]) + (2*(a + b*ArcSinh[c + d*x])*(E^(3*(a/b + ArcSinh[c + d*x]))*(6*a +
```

```
 b + 6*b*ArcSinh[c + d*x]) + 6*Sqrt[3]*b*(-((a + b*ArcSinh[c + d*x])/b))^(3
/2)*Gamma[1/2, (-3*(a + b*ArcSinh[c + d*x]))/b]))/E^((3*a)/b)) - (4*(a + b*
ArcSinh[c + d*x])*(E^(a/b + ArcSinh[c + d*x])*(2*a + b + 2*b*ArcSinh[c + d*
x]) + 2*b*(-((a + b*ArcSinh[c + d*x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcSinh
[c + d*x])/b)]))/E^(a/b) + (9*(b^2 + 2*(a + b*ArcSinh[c + d*x])*(6*a - b +
6*b*ArcSinh[c + d*x] - 6*Sqrt[3]*E^(3*(a/b + ArcSinh[c + d*x]))*Sqrt[a/b +
ArcSinh[c + d*x]]*(a + b*ArcSinh[c + d*x])*Gamma[1/2, (3*(a + b*ArcSinh[c +
 d*x]))/b])))/E^(3*ArcSinh[c + d*x]) - (3*b^2 + 10*(a + b*ArcSinh[c + d*x])
*(10*a - b + 10*b*ArcSinh[c + d*x] - 10*Sqrt[5]*E^(5*(a/b + ArcSinh[c + d*x
]))*Sqrt[a/b + ArcSinh[c + d*x]]*(a + b*ArcSinh[c + d*x])*Gamma[1/2, (5*(a
+ b*ArcSinh[c + d*x]))/b]))/E^(5*ArcSinh[c + d*x])))/(240*b^3*d*(a + b*ArcS
inh[c + d*x])^(5/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex + ce)^4}{(b \operatorname{arsinh}(dx + c) + a)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^4/(b*arcsinh(d*x + c) + a)^(7/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(dex + ce)^4}{(a + b \operatorname{arcsinh}(dx + c))^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(7/2),x)
```

```
[Out] int((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(7/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex + ce)^4}{(b \operatorname{arsinh}(dx + c) + a)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^4/(b*arcsinh(d*x + c) + a)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,e + d\,e\,x)^4}{(a + b\,\mathrm{asinh}\,(c + d\,x))^{7/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^4/(a + b*asinh(c + d*x))^(7/2),x)
```

```
[Out] int((c*e + d*e*x)^4/(a + b*asinh(c + d*x))^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^4\left(\int \frac{c^4}{a^3\sqrt{a+b\operatorname{asinh}(c+dx)}+3a^2b\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)+3ab^2\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asin}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4/(a+b*asinh(d*x+c))**(7/2),x)
```

```
[Out] e**4*(Integral(c**4/(a**3*sqrt(a + b*asinh(c + d*x)) + 3*a**2*b*sqrt(a + b*
asinh(c + d*x))*asinh(c + d*x) + 3*a*b**2*sqrt(a + b*asinh(c + d*x))*asinh(
c + d*x)**2 + b**3*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**3), x) + Inte
gral(d**4*x**4/(a**3*sqrt(a + b*asinh(c + d*x)) + 3*a**2*b*sqrt(a + b*asinh
(c + d*x))*asinh(c + d*x) + 3*a*b**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d
*x)**2 + b**3*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**3), x) + Integral(
4*c*d**3*x**3/(a**3*sqrt(a + b*asinh(c + d*x)) + 3*a**2*b*sqrt(a + b*asinh(
c + d*x))*asinh(c + d*x) + 3*a*b**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*
x)**2 + b**3*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**3), x) + Integral(6
*c**2*d**2*x**2/(a**3*sqrt(a + b*asinh(c + d*x)) + 3*a**2*b*sqrt(a + b*asin
h(c + d*x))*asinh(c + d*x) + 3*a*b**2*sqrt(a + b*asinh(c + d*x))*asinh(c +
d*x)**2 + b**3*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**3), x) + Integral
(4*c**3*d*x/(a**3*sqrt(a + b*asinh(c + d*x)) + 3*a**2*b*sqrt(a + b*asinh(c
+ d*x))*asinh(c + d*x) + 3*a*b**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)
**2 + b**3*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**3), x))
```

3.223 $\int \frac{(ce+dex)^3}{\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}\,dx$

Optimal. Leaf size=420

$$\frac{16\sqrt{\pi}e^3e^{\frac{4a}{b}}\operatorname{erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}-\frac{4\sqrt{2\pi}e^3e^{\frac{2a}{b}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}+\frac{16\sqrt{\pi}e^3e^{-\frac{4a}{b}}\operatorname{erfi}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}-4$$

```
[Out] -4/5*e^3*(d*x+c)^2/b^2/d/(a+b*arcsinh(d*x+c))^(3/2)-16/15*e^3*(d*x+c)^4/b^2
/d/(a+b*arcsinh(d*x+c))^(3/2)+16/15*e^3*exp(4*a/b)*erf(2*(a+b*arcsinh(d*x+c
))^(1/2)/b^(1/2))*Pi^(1/2)/b^(7/2)/d+16/15*e^3*erfi(2*(a+b*arcsinh(d*x+c))^
(1/2)/b^(1/2))*Pi^(1/2)/b^(7/2)/d/exp(4*a/b)-4/15*e^3*exp(2*a/b)*erf(2^(1/2
)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)/d-4/15*e^3*e
rfi(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)/d/
exp(2*a/b)-2/5*e^3*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))^(
5/2)-16/5*e^3*(d*x+c)*(1+(d*x+c)^2)^(1/2)/b^3/d/(a+b*arcsinh(d*x+c))^(1/2)-
128/15*e^3*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)/b^3/d/(a+b*arcsinh(d*x+c))^(1/2)
```

Rubi [A] time = 1.10, antiderivative size = 420, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {5865, 12, 5667, 5774, 5665, 3307, 2180, 2204, 2205}

$$\frac{16\sqrt{\pi}e^3e^{\frac{4a}{b}}\operatorname{Erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}-\frac{4\sqrt{2\pi}e^3e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}+\frac{16\sqrt{\pi}e^3e^{-\frac{4a}{b}}\operatorname{Erfi}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}-4$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3/(a + b*ArcSinh[c + d*x])^(7/2),x]
```

```
[Out] (-2*e^3*(c + d*x)^3*Sqrt[1 + (c + d*x)^2])/(5*b*d*(a + b*ArcSinh[c + d*x])^
(5/2)) - (4*e^3*(c + d*x)^2)/(5*b^2*d*(a + b*ArcSinh[c + d*x])^(3/2)) - (16
*e^3*(c + d*x)^4)/(15*b^2*d*(a + b*ArcSinh[c + d*x])^(3/2)) - (16*e^3*(c +
d*x)*Sqrt[1 + (c + d*x)^2])/(5*b^3*d*Sqrt[a + b*ArcSinh[c + d*x]]) - (128*e
^3*(c + d*x)^3*Sqrt[1 + (c + d*x)^2])/(15*b^3*d*Sqrt[a + b*ArcSinh[c + d*x]
]) + (16*e^3*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt
[b]])/(15*b^(7/2)*d) - (4*e^3*E^((2*a)/b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a +
b*ArcSinh[c + d*x]])/Sqrt[b]])/(15*b^(7/2)*d) + (16*e^3*Sqrt[Pi]*Erfi[(2*Sq
rt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(15*b^(7/2)*d*E^((4*a)/b)) - (4*e^3*S
qrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(15*b^(7/2)
*d*E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^3}{\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}\,dx = \frac{\text{Subst}\left(\int \frac{e^3x^3}{\left(a+b\sinh^{-1}(x)\right)^{7/2}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^3\,\text{Subst}\left(\int \frac{x^3}{\left(a+b\sinh^{-1}(x)\right)^{7/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} + \frac{\left(6e^3\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1+x^2}\left(a+b\sinh^{-1}(x)\right)^{5/2}}\,dx, x, c+\right.}{5bd}$$

$$= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4e^3(c+dx)^2}{5b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{16e}{15b^2d\left(a+b\right.}$$

$$= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4e^3(c+dx)^2}{5b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{16e}{15b^2d\left(a+b\right.}$$

$$= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4e^3(c+dx)^2}{5b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{16e}{15b^2d\left(a+b\right.}$$

$$= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4e^3(c+dx)^2}{5b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{16e}{15b^2d\left(a+b\right.}$$

$$= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4e^3(c+dx)^2}{5b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{16e}{15b^2d\left(a+b\right.}$$

$$= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4e^3(c+dx)^2}{5b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{16e}{15b^2d\left(a+b\right.}$$

Mathematica [A] time = 2.11, size = 429, normalized size = 1.02

$$\frac{e^3\left(4\left(a+b\sinh^{-1}(c+dx)\right)\left(e^{2\sinh^{-1}(c+dx)}\left(4a+4b\sinh^{-1}(c+dx)+b\right)+4\sqrt{2}\,be^{-\frac{2a}{b}}\left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{3/2}\Gamma\left(\frac{1}{2},-\frac{2(}{}\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^3/(a + b*ArcSinh[c + d*x])^(7/2),x]
```

```
[Out] (e^3*(4*(a + b*ArcSinh[c + d*x])*((-4*a)/E^(2*ArcSinh[c + d*x]) + b/E^(2*Ar
cSinh[c + d*x]) - (4*b*ArcSinh[c + d*x])/E^(2*ArcSinh[c + d*x]) + E^(2*ArcS
inh[c + d*x])*(4*a + b + 4*b*ArcSinh[c + d*x]) + (4*Sqrt[2]*b*(-((a + b*Arc
Sinh[c + d*x])/b))^(3/2)*Gamma[1/2, (-2*(a + b*ArcSinh[c + d*x]))/b])/E^((2
*a)/b) + 4*Sqrt[2]*E^((2*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*(a + b*ArcSinh[
c + d*x])*Gamma[1/2, (2*(a + b*ArcSinh[c + d*x]))/b]) - 4*(a + b*ArcSinh[c
+ d*x])*((-8*a)/E^(4*ArcSinh[c + d*x]) + (b*(1 - 8*ArcSinh[c + d*x]))/E^(4*
```

```
ArcSinh[c + d*x]) + E^(4*ArcSinh[c + d*x])*(8*a + b + 8*b*ArcSinh[c + d*x])
 + (16*b*(-((a + b*ArcSinh[c + d*x])/b))^(3/2)*Gamma[1/2, (-4*(a + b*ArcSin
h[c + d*x]))/b])/E^((4*a)/b) + 16*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*
(a + b*ArcSinh[c + d*x])*Gamma[1/2, (4*(a + b*ArcSinh[c + d*x]))/b]) + 6*b^
2*Sinh[2*ArcSinh[c + d*x]] - 3*b^2*Sinh[4*ArcSinh[c + d*x]]))/(60*b^3*d*(a
+ b*ArcSinh[c + d*x])^(5/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex + ce)^3}{(b \operatorname{arsinh}(dx + c) + a)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^3/(b*arcsinh(d*x + c) + a)^(7/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(dex + ce)^3}{(a + b \operatorname{arcsinh}(dx + c))^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(7/2),x)
```

```
[Out] int((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(7/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex + ce)^3}{(b \operatorname{arsinh}(dx + c) + a)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^3/(b*arcsinh(d*x + c) + a)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,e + d\,e\,x)^3}{(a + b\,\mathrm{asinh}(c + d\,x))^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3/(a + b*asinh(c + d*x))^(7/2),x)
```

```
[Out] int((c*e + d*e*x)^3/(a + b*asinh(c + d*x))^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^3\left(\int \frac{c^3}{a^3\sqrt{a+b\operatorname{asinh}(c+dx)}+3a^2b\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)+3ab^2\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}^2(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3/(a+b*asinh(d*x+c))**(7/2),x)
```

```
[Out] e**3*(Integral(c**3/(a**3*sqrt(a + b*asinh(c + d*x)) + 3*a**2*b*sqrt(a + b*
asinh(c + d*x))*asinh(c + d*x) + 3*a*b**2*sqrt(a + b*asinh(c + d*x))*asinh(
c + d*x)**2 + b**3*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**3), x) + Inte
gral(d**3*x**3/(a**3*sqrt(a + b*asinh(c + d*x)) + 3*a**2*b*sqrt(a + b*asinh
(c + d*x))*asinh(c + d*x) + 3*a*b**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d
*x)**2 + b**3*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**3), x) + Integral(
3*c*d**2*x**2/(a**3*sqrt(a + b*asinh(c + d*x)) + 3*a**2*b*sqrt(a + b*asinh(
c + d*x))*asinh(c + d*x) + 3*a*b**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*
x)**2 + b**3*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**3), x) + Integral(3
*c**2*d*x/(a**3*sqrt(a + b*asinh(c + d*x)) + 3*a**2*b*sqrt(a + b*asinh(c +
d*x))*asinh(c + d*x) + 3*a*b**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**
2 + b**3*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**3), x))
```

3.224 $\int \frac{(ce+dex)^2}{\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}\,dx$

Optimal. Leaf size=410

$$\frac{\sqrt{\pi}e^2e^{a/b}\mathrm{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}-\frac{3\sqrt{3\pi}e^2e^{\frac{3a}{b}}\mathrm{erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{5b^{7/2}d}-\frac{\sqrt{\pi}e^2e^{-\frac{a}{b}}\mathrm{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}+3\sqrt{3\pi}$$

```
[Out] -8/15*e^2*(d*x+c)/b^2/d/(a+b*arcsinh(d*x+c))^(3/2)-4/5*e^2*(d*x+c)^3/b^2/d/
(a+b*arcsinh(d*x+c))^(3/2)+1/15*e^2*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)
/b^(1/2))*Pi^(1/2)/b^(7/2)/d-1/15*e^2*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/
2))*Pi^(1/2)/b^(7/2)/d/exp(a/b)-3/5*e^2*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh
(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/b^(7/2)/d+3/5*e^2*erfi(3^(1/2)*(a+
b*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/b^(7/2)/d/exp(3*a/b)-2/5*
e^2*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))^(5/2)-16/15*e^2*
(1+(d*x+c)^2)^(1/2)/b^3/d/(a+b*arcsinh(d*x+c))^(1/2)-24/5*e^2*(d*x+c)^2*(1+
(d*x+c)^2)^(1/2)/b^3/d/(a+b*arcsinh(d*x+c))^(1/2)
```

Rubi [A] time = 1.11, antiderivative size = 410, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {5865, 12, 5667, 5774, 5665, 3308, 2180, 2204, 2205, 5655, 5779}

$$\frac{\sqrt{\pi}e^2e^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}-\frac{3\sqrt{3\pi}e^2e^{\frac{3a}{b}}\mathrm{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{5b^{7/2}d}-\frac{\sqrt{\pi}e^2e^{-\frac{a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}+3\sqrt{3}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2/(a + b*ArcSinh[c + d*x])^(7/2),x]
```

```
[Out] (-2*e^2*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])/(5*b*d*(a + b*ArcSinh[c + d*x])^
(5/2)) - (8*e^2*(c + d*x))/(15*b^2*d*(a + b*ArcSinh[c + d*x])^(3/2)) - (4*e
^2*(c + d*x)^3)/(5*b^2*d*(a + b*ArcSinh[c + d*x])^(3/2)) - (16*e^2*Sqrt[1 +
 (c + d*x)^2])/(15*b^3*d*Sqrt[a + b*ArcSinh[c + d*x]]) - (24*e^2*(c + d*x)^
2*Sqrt[1 + (c + d*x)^2])/(5*b^3*d*Sqrt[a + b*ArcSinh[c + d*x]]) + (e^2*E^(a
/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(15*b^(7/2)*d) - (3
*e^2*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt
[b]])/(5*b^(7/2)*d) - (e^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[
b]])/(15*b^(7/2)*d*E^(a/b)) + (3*e^2*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*Ar
cSinh[c + d*x]])/Sqrt[b]])/(5*b^(7/2)*d*E^((3*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
```

```
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^2}{\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}\,dx = \frac{\text{Subst}\left(\int \frac{e^2x^2}{\left(a+b\sinh^{-1}(x)\right)^{7/2}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^2\,\text{Subst}\left(\int \frac{x^2}{\left(a+b\sinh^{-1}(x)\right)^{7/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} + \frac{\left(4e^2\right)\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\left(a+b\sinh^{-1}(x)\right)^{5/2}}\,dx, x, \right.}{5bd}$$

$$= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{8e^2(c+dx)}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{}{5b^2d\left(a\right.}$$

$$= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{8e^2(c+dx)}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{}{5b^2d\left(a\right.}$$

$$= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{8e^2(c+dx)}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{}{5b^2d\left(a\right.}$$

$$= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{8e^2(c+dx)}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{}{5b^2d\left(a\right.}$$

$$= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{8e^2(c+dx)}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{}{5b^2d\left(a\right.}$$

$$= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{8e^2(c+dx)}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{}{5b^2d\left(a\right.}$$

Mathematica [A] time = 1.49, size = 474, normalized size = 1.16

$$\frac{e^2\left(e^{-\sinh^{-1}(c+dx)}\left(4a^2+2b(4a-b)\sinh^{-1}(c+dx)-4e^{\frac{a}{b}+\sinh^{-1}(c+dx)}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\left(a+b\sinh^{-1}(c+dx\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^2/(a + b*ArcSinh[c + d*x])^(7/2),x]
```

```
[Out] (e^2*(3*b^2*E^ArcSinh[c + d*x] + (4*a^2 - 2*a*b + 3*b^2 + 2*(4*a - b)*b*Arc
Sinh[c + d*x] + 4*b^2*ArcSinh[c + d*x]^2 - 4*E^(a/b + ArcSinh[c + d*x])*Sqr
t[a/b + ArcSinh[c + d*x]]*(a + b*ArcSinh[c + d*x])^2*Gamma[1/2, a/b + ArcSi
```

```
nh[c + d*x]])/E^ArcSinh[c + d*x] - 3*(b^2*E^(3*ArcSinh[c + d*x]) + (2*(a +
b*ArcSinh[c + d*x])*(E^(3*(a/b + ArcSinh[c + d*x]))*(6*a + b + 6*b*ArcSinh[
c + d*x]) + 6*Sqrt[3]*b*(-((a + b*ArcSinh[c + d*x])/b))^(3/2)*Gamma[1/2, (-
3*(a + b*ArcSinh[c + d*x]))/b]))/E^((3*a)/b)) + (2*(a + b*ArcSinh[c + d*x])
*(E^(a/b + ArcSinh[c + d*x])*(2*a + b + 2*b*ArcSinh[c + d*x]) + 2*b*(-((a +
 b*ArcSinh[c + d*x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcSinh[c + d*x])/b)]))/
E^(a/b) - (3*(b^2 + 2*(a + b*ArcSinh[c + d*x])*(6*a - b + 6*b*ArcSinh[c + d
*x] - 6*Sqrt[3]*E^(3*(a/b + ArcSinh[c + d*x]))*Sqrt[a/b + ArcSinh[c + d*x]]
*(a + b*ArcSinh[c + d*x])*Gamma[1/2, (3*(a + b*ArcSinh[c + d*x]))/b])))/E^(
3*ArcSinh[c + d*x])))/(60*b^3*d*(a + b*ArcSinh[c + d*x])^(5/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex + ce)^2}{(b \operatorname{arsinh}(dx + c) + a)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2/(b*arcsinh(d*x + c) + a)^(7/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(dex + ce)^2}{(a + b \operatorname{arcsinh}(dx + c))^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(7/2),x)
```

```
[Out] int((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(7/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex + ce)^2}{(b \operatorname{arsinh}(dx + c) + a)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^2/(b*arcsinh(d*x + c) + a)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,e + d\,e\,x)^2}{(a + b\,\operatorname{asinh}(c + d\,x))^{7/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2/(a + b*asinh(c + d*x))^(7/2),x)
```

```
[Out] int((c*e + d*e*x)^2/(a + b*asinh(c + d*x))^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^2 \left(\int \frac{c^2}{a^3 \sqrt{a + b \operatorname{asinh}(c + dx)} + 3a^2 b \sqrt{a + b \operatorname{asinh}(c + dx)} \operatorname{asinh}(c + dx) + 3ab^2 \sqrt{a + b \operatorname{asinh}(c + dx)} \operatorname{asin}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2/(a+b*asinh(d*x+c))**(7/2),x)
```

```
[Out] e**2*(Integral(c**2/(a**3*sqrt(a + b*asinh(c + d*x)) + 3*a**2*b*sqrt(a + b*
asinh(c + d*x))*asinh(c + d*x) + 3*a*b**2*sqrt(a + b*asinh(c + d*x))*asinh(
c + d*x)**2 + b**3*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**3), x) + Inte
gral(d**2*x**2/(a**3*sqrt(a + b*asinh(c + d*x)) + 3*a**2*b*sqrt(a + b*asinh
(c + d*x))*asinh(c + d*x) + 3*a*b**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d
*x)**2 + b**3*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**3), x) + Integral(
2*c*d*x/(a**3*sqrt(a + b*asinh(c + d*x)) + 3*a**2*b*sqrt(a + b*asinh(c + d*
x))*asinh(c + d*x) + 3*a*b**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**2
+ b**3*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**3), x))
```

3.225 $\int \frac{ce+dex}{\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}} dx$

Optimal. Leaf size=252

$$\frac{8\sqrt{2\pi}\, ee^{\frac{2a}{b}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}+\frac{8\sqrt{2\pi}\, ee^{-\frac{2a}{b}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}-\frac{32e\sqrt{(c+dx)^2+1}\,(c+dx)}{15b^3d\sqrt{a+b\sinh^{-1}(c+dx)}}-\frac{}{15b^2d\,(}$$

```
[Out] -4/15*e/b^2/d/(a+b*arcsinh(d*x+c))^(3/2)-8/15*e*(d*x+c)^2/b^2/d/(a+b*arcsin
h(d*x+c))^(3/2)+8/15*e*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^
(1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)/d+8/15*e*erfi(2^(1/2)*(a+b*arcsinh(d*x+c))^
(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)/d/exp(2*a/b)-2/5*e*(d*x+c)*(1+(d*x+
c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))^(5/2)-32/15*e*(d*x+c)*(1+(d*x+c)^2)^(1
/2)/b^3/d/(a+b*arcsinh(d*x+c))^(1/2)
```

Rubi [A] time = 0.56, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {5865, 12, 5667, 5774, 5665, 3307, 2180, 2204, 2205, 5675}

$$\frac{8\sqrt{2\pi}\, ee^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}+\frac{8\sqrt{2\pi}\, ee^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}-\frac{8e(c+dx)^2}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}-\frac{32}{15b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)/(a + b*ArcSinh[c + d*x])^(7/2),x]
```

```
[Out] (-2*e*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(5*b*d*(a + b*ArcSinh[c + d*x])^(5/2
)) - (4*e)/(15*b^2*d*(a + b*ArcSinh[c + d*x])^(3/2)) - (8*e*(c + d*x)^2)/(1
5*b^2*d*(a + b*ArcSinh[c + d*x])^(3/2)) - (32*e*(c + d*x)*Sqrt[1 + (c + d*x
)^2])/(15*b^3*d*Sqrt[a + b*ArcSinh[c + d*x]]) + (8*e*E^((2*a)/b)*Sqrt[2*Pi]
*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(15*b^(7/2)*d) + (8*e
*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(15*b^(7/
2)*d*E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{ce+dex}{\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}\,dx = \frac{\text{Subst}\left(\int \frac{ex}{\left(a+b\sinh^{-1}(x)\right)^{7/2}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e\,\text{Subst}\left(\int \frac{x}{\left(a+b\sinh^{-1}(x)\right)^{7/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} + \frac{(2e)\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}\left(a+b\sinh^{-1}(x)\right)^{5/2}}\,dx, x, c+a\right.}{5bd}$$

$$= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4e}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{8}{15b^2d\left(a+\right.}$$

$$= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4e}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{8}{15b^2d\left(a+\right.}$$

$$= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4e}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{8}{15b^2d\left(a+\right.}$$

$$= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4e}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{8}{15b^2d\left(a+\right.}$$

$$= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4e}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{8}{15b^2d\left(a+\right.}$$

Mathematica [A] time = 0.88, size = 235, normalized size = 0.93

$$-\frac{e\left(\left(a+b\sinh^{-1}(c+dx)\right)\left(e^{-\frac{2a}{b}}\left(2e^{2\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)}\left(4a+4b\sinh^{-1}(c+dx)+b\right)+8\sqrt{2}\,b\left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{3/2}\Gamma\left(\right.\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)/(a + b*ArcSinh[c + d*x])^(7/2),x]
```

```
[Out] -1/15*(e*((a + b*ArcSinh[c + d*x])*((2*E^(2*(a/b + ArcSinh[c + d*x]))*(4*a
+ b + 4*b*ArcSinh[c + d*x]) + 8*Sqrt[2]*b*(-((a + b*ArcSinh[c + d*x])/b))^(
3/2)*Gamma[1/2, (-2*(a + b*ArcSinh[c + d*x]))/b])/E^((2*a)/b) + (-8*a + 2*b
 - 8*b*ArcSinh[c + d*x] + 8*Sqrt[2]*E^(2*(a/b + ArcSinh[c + d*x]))*Sqrt[a/b
 + ArcSinh[c + d*x]]*(a + b*ArcSinh[c + d*x])*Gamma[1/2, (2*(a + b*ArcSinh[
c + d*x]))/b])/E^(2*ArcSinh[c + d*x])) + 3*b^2*Sinh[2*ArcSinh[c + d*x]]))/(
b^3*d*(a + b*ArcSinh[c + d*x])^(5/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dex + ce}{(b \operatorname{arsinh}(dx + c) + a)^{\frac{7}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)/(b*arcsinh(d*x + c) + a)^(7/2), x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{dex + ce}{(a + b \operatorname{arcsinh}(dx + c))^{\frac{7}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(7/2),x)
```

```
[Out] int((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(7/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dex + ce}{(b \operatorname{arsinh}(dx + c) + a)^{\frac{7}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)/(b*arcsinh(d*x + c) + a)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{c\,e + d\,e\,x}{(a + b\,\operatorname{asinh}(c + d\,x))^{7/2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)/(a + b*asinh(c + d*x))^(7/2),x)
```

```
[Out] int((c*e + d*e*x)/(a + b*asinh(c + d*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*asinh(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

3.226 $\int \frac{1}{\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}\,dx$

Optimal. Leaf size=195

$$-\frac{4\sqrt{\pi}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}+\frac{4\sqrt{\pi}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}-\frac{8\sqrt{(c+dx)^2+1}}{15b^3d\sqrt{a+b\sinh^{-1}(c+dx)}}-\frac{4(c+\cdots}{15b^2d\left(a+b\sin\cdots\right.}$$

```
[Out] -4/15*(d*x+c)/b^2/d/(a+b*arcsinh(d*x+c))^(3/2)-4/15*exp(a/b)*erf((a+b*arcsi
nh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(7/2)/d+4/15*erfi((a+b*arcsinh(d*x+c))
^(1/2)/b^(1/2))*Pi^(1/2)/b^(7/2)/d/exp(a/b)-2/5*(1+(d*x+c)^2)^(1/2)/b/d/(a+
b*arcsinh(d*x+c))^(5/2)-8/15*(1+(d*x+c)^2)^(1/2)/b^3/d/(a+b*arcsinh(d*x+c))
^(1/2)
```

Rubi [A] time = 0.47, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5863, 5655, 5774, 5779, 3308, 2180, 2204, 2205}

$$-\frac{4\sqrt{\pi}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}+\frac{4\sqrt{\pi}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}-\frac{4(c+dx)}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}-\frac{8\sqrt{(c+\cdots}}{15b^3d\sqrt{a+b\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^(-7/2),x]
```

```
[Out] (-2*Sqrt[1 + (c + d*x)^2])/(5*b*d*(a + b*ArcSinh[c + d*x])^(5/2)) - (4*(c +
 d*x))/(15*b^2*d*(a + b*ArcSinh[c + d*x])^(3/2)) - (8*Sqrt[1 + (c + d*x)^2]
)/(15*b^3*d*Sqrt[a + b*ArcSinh[c + d*x]]) - (4*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a
+ b*ArcSinh[c + d*x]]/Sqrt[b]])/(15*b^(7/2)*d) + (4*Sqrt[Pi]*Erfi[Sqrt[a +
b*ArcSinh[c + d*x]]/Sqrt[b]])/(15*b^(7/2)*d*E^(a/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 5863

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+b\sinh^{-1}(x)\right)^{7/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} + \frac{2\,\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\left(a+b\sinh^{-1}(x)\right)^{5/2}}\,dx, x, c+dx\right)}{5bd}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4(c+dx)}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} + \frac{4\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4(c+dx)}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{8\sqrt{1}}{15b^3d\sqrt{a+}}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4(c+dx)}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{8\sqrt{1}}{15b^3d\sqrt{a+}}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4(c+dx)}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{8\sqrt{1}}{15b^3d\sqrt{a+}}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4(c+dx)}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{8\sqrt{1}}{15b^3d\sqrt{a+}}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4(c+dx)}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{8\sqrt{1}}{15b^3d\sqrt{a+}}$$

Mathematica [A] time = 0.19, size = 238, normalized size = 1.22

$$-2e^{-\sinh^{-1}(c+dx)}\left(4a^2+2ab\left(4\sinh^{-1}(c+dx)-1\right)+b^2\left(4\sinh^{-1}(c+dx)^2-2\sinh^{-1}(c+dx)+3\right)\right)+8e^{a/b}\sqrt{\frac{a}{b}+\text{s}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^(-7/2),x]
```

```
[Out] (-6*b^2*E^ArcSinh[c + d*x] - (2*(4*a^2 + 2*a*b*(-1 + 4*ArcSinh[c + d*x]) +
b^2*(3 - 2*ArcSinh[c + d*x] + 4*ArcSinh[c + d*x]^2)))/E^ArcSinh[c + d*x] +
8*E^(a/b)*Sqrt[a/b + ArcSinh[c + d*x]]*(a + b*ArcSinh[c + d*x])^2*Gamma[1/2
, a/b + ArcSinh[c + d*x]] - (4*(a + b*ArcSinh[c + d*x])*(E^(a/b + ArcSinh[c
 + d*x])*(2*a + b + 2*b*ArcSinh[c + d*x]) + 2*b*(-((a + b*ArcSinh[c + d*x])
/b))^(3/2)*Gamma[1/2, -((a + b*ArcSinh[c + d*x])/b)]))/E^(a/b))/(30*b^3*d*(
a + b*ArcSinh[c + d*x])^(5/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \operatorname{arsinh}(dx + c) + a)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(-7/2), x)
```

maple [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b \operatorname{arcsinh}(dx + c))^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(d*x+c))^(7/2),x)
```

```
[Out] int(1/(a+b*arcsinh(d*x+c))^(7/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \operatorname{arsinh}(dx + c) + a)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(-7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a + b \operatorname{asinh}(c + d\,x))^{7/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*asinh(c + d*x))^(7/2),x)
```

```
[Out] int(1/(a + b*asinh(c + d*x))^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b \operatorname{asinh}(c + dx))^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(d*x+c))**(7/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))**(-7/2), x)
```

3.227 $\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}\,dx$

Optimal. Leaf size=29

$$\frac{\text{Int}\left(\frac{1}{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}},x\right)}{e}$$

```
[Out] Unintegrable(1/(d*x+c)/(a+b*arcsinh(d*x+c))^(7/2),x)/e
```

Rubi [A] time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(7/2)),x]
[Out] Defer[Subst][Defer[Int][1/(x*(a + b*ArcSinh[x])^(7/2)), x], x, c + d*x]/(d*
e)
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{ex\left(a+b\sinh^{-1}(x)\right)^{7/2}}\,dx,x,c+dx\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{1}{x\left(a+b\sinh^{-1}(x)\right)^{7/2}}\,dx,x,c+dx\right)}{de}\end{aligned}$$

Mathematica [A] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(7/2)),x]
[Out] Integrate[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(7/2)), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dex+ce)(b\,\mathrm{arsinh}\,(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate(1/((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^(7/2)), x)
```

maple [A] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{1}{(dex+ce)\,(a+b\,\mathrm{arcsinh}\,(dx+c))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(7/2),x)
[Out] int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(7/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dex+ce)(b\,\mathrm{arsinh}\,(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="maxima")
[Out] integrate(1/((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^(7/2)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{(c\,e+d\,e\,x)\,\left(a+b\,\mathrm{asinh}\,(c+d\,x)\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*e + d*e*x)*(a + b*asinh(c + d*x))^(7/2)),x)
[Out] int(1/((c*e + d*e*x)*(a + b*asinh(c + d*x))^(7/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*asinh(d*x+c))**(7/2),x)
[Out] Timed out
```

3.228 $\int (ce + dex)^{7/2} \left(a + b\sinh^{-1}(c + dx)\right) dx$

Optimal. Leaf size=298

$$\frac{2(e(c+dx))^{9/2}\left(a+b\sinh^{-1}(c+dx)\right)}{9de}-\frac{14be^{7/2}(c+dx+1)\sqrt{\frac{(c+dx)^2+1}{(c+dx+1)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right)|\frac{1}{2}\right)}{135d\sqrt{(c+dx)^2+1}}+\frac{28be^{7/2}(c+dx+$$

```
[Out] 2/9*(e*(d*x+c))^(9/2)*(a+b*arcsinh(d*x+c))/d/e+28/405*b*e^2*(e*(d*x+c))^(3/
2)*(1+(d*x+c)^2)^(1/2)/d-4/81*b*(e*(d*x+c))^(7/2)*(1+(d*x+c)^2)^(1/2)/d-28/
135*b*e^3*(e*(d*x+c))^(1/2)*(1+(d*x+c)^2)^(1/2)/d/(d*x+c+1)+28/135*b*e^(7/2
)*(d*x+c+1)*(cos(2*arctan((e*(d*x+c))^(1/2)/e^(1/2)))^2)^(1/2)/cos(2*arctan
((e*(d*x+c))^(1/2)/e^(1/2)))*EllipticE(sin(2*arctan((e*(d*x+c))^(1/2)/e^(1/
2))),1/2*2^(1/2))*((1+(d*x+c)^2)/(d*x+c+1)^2)^(1/2)/d/(1+(d*x+c)^2)^(1/2)-1
4/135*b*e^(7/2)*(d*x+c+1)*(cos(2*arctan((e*(d*x+c))^(1/2)/e^(1/2)))^2)^(1/2
)/cos(2*arctan((e*(d*x+c))^(1/2)/e^(1/2)))*EllipticF(sin(2*arctan((e*(d*x+c
))^(1/2)/e^(1/2))),1/2*2^(1/2))*((1+(d*x+c)^2)/(d*x+c+1)^2)^(1/2)/d/(1+(d*x
+c)^2)^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {5865, 5661, 321, 329, 305, 220, 1196}

$$\frac{2(e(c+dx))^{9/2}\left(a+b\sinh^{-1}(c+dx)\right)}{9de}+\frac{28be^2\sqrt{(c+dx)^2+1}\,(e(c+dx))^{3/2}}{405d}-\frac{28be^3\sqrt{(c+dx)^2+1}\,\sqrt{e(c+dx)}}{135d(c+dx+1)}-\frac{14b$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^(7/2)*(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (28*b*e^2*(e*(c + d*x))^(3/2)*Sqrt[1 + (c + d*x)^2])/(405*d) - (4*b*(e*(c +
 d*x))^(7/2)*Sqrt[1 + (c + d*x)^2])/(81*d) - (28*b*e^3*Sqrt[e*(c + d*x)]*Sq
rt[1 + (c + d*x)^2])/(135*d*(1 + c + d*x)) + (2*(e*(c + d*x))^(9/2)*(a + b*
ArcSinh[c + d*x]))/(9*d*e) + (28*b*e^(7/2)*(1 + c + d*x)*Sqrt[(1 + (c + d*x
)^2)/(1 + c + d*x)^2]*EllipticE[2*ArcTan[Sqrt[e*(c + d*x)]/Sqrt[e]], 1/2])/
(135*d*Sqrt[1 + (c + d*x)^2]) - (14*b*e^(7/2)*(1 + c + d*x)*Sqrt[(1 + (c +
d*x)^2)/(1 + c + d*x)^2]*EllipticF[2*ArcTan[Sqrt[e*(c + d*x)]/Sqrt[e]], 1/2
])/(135*d*Sqrt[1 + (c + d*x)^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^{7/2}\left(a+b\sinh^{-1}(c+dx)\right)dx = \frac{\text{Subst}\left(\int (ex)^{7/2}\left(a+b\sinh^{-1}(x)\right)dx, x, c+dx\right)}{d}$$

$$= \frac{2(e(c+dx))^{9/2}\left(a+b\sinh^{-1}(c+dx)\right)}{9de} - \frac{(2b)\,\text{Subst}\left(\int \frac{(ex)^{9/2}}{\sqrt{1+x^2}}\,dx, x\right.}{9de}$$

$$= -\frac{4b(e(c+dx))^{7/2}\sqrt{1+(c+dx)^2}}{81d} + \frac{2(e(c+dx))^{9/2}\left(a+b\sinh^{-1}(c\right.}{9de}$$

$$= \frac{28be^2(e(c+dx))^{3/2}\sqrt{1+(c+dx)^2}}{405d} - \frac{4b(e(c+dx))^{7/2}\sqrt{1+(c+dx}}{81d}$$

$$= \frac{28be^2(e(c+dx))^{3/2}\sqrt{1+(c+dx)^2}}{405d} - \frac{4b(e(c+dx))^{7/2}\sqrt{1+(c+dx}}{81d}$$

$$= \frac{28be^2(e(c+dx))^{3/2}\sqrt{1+(c+dx)^2}}{405d} - \frac{4b(e(c+dx))^{7/2}\sqrt{1+(c+dx}}{81d}$$

$$= \frac{28be^2(e(c+dx))^{3/2}\sqrt{1+(c+dx)^2}}{405d} - \frac{4b(e(c+dx))^{7/2}\sqrt{1+(c+dx}}{81d}$$

Mathematica [C] time = 0.21, size = 113, normalized size = 0.38

$$\frac{2(e(c+dx))^{7/2}\left(45a(c+dx)^3-14b\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-(c+dx)^2\right)-10b\sqrt{(c+dx)^2+1}\,(c+dx)^2+14b\sqrt{(c+dx)^2+1}\,+\right.}{405d(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^(7/2)*(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (2*(e*(c + d*x))^(7/2)*(45*a*(c + d*x)^3 + 14*b*Sqrt[1 + (c + d*x)^2] - 10*
b*(c + d*x)^2*Sqrt[1 + (c + d*x)^2] + 45*b*(c + d*x)^3*ArcSinh[c + d*x] - 1
4*b*Hypergeometric2F1[1/2, 3/4, 7/4, -(c + d*x)^2]))/(405*d*(c + d*x)^2)
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(ad^3e^3x^3+3\,acd^2e^3x^2+3\,ac^2de^3x+ac^3e^3+\left(bd^3e^3x^3+3\,bcd^2e^3x^2+3\,bc^2de^3x+bc^3e^3\right)\text{arsinh}\,(dx+c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((a*d^3*e^3*x^3 + 3*a*c*d^2*e^3*x^2 + 3*a*c^2*d*e^3*x + a*c^3*e^3 +
 (b*d^3*e^3*x^3 + 3*b*c*d^2*e^3*x^2 + 3*b*c^2*d*e^3*x + b*c^3*e^3)*arcsinh(
d*x + c))*sqrt(d*e*x + c*e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex+ce)^{\frac{7}{2}}(b\,\text{arsinh}\,(dx+c)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^(7/2)*(b*arcsinh(d*x + c) + a), x)
```

maple [C] time = 0.03, size = 238, normalized size = 0.80

$$\frac{\frac{2(dex+ce)^{\frac{9}{2}}a}{9}+2b\left(\frac{(dex+ce)^{\frac{9}{2}}\,\text{arcsinh}\left(\frac{dex+ce}{e}\right)}{9}-\frac{2\left(\frac{e^2(dex+ce)^{\frac{7}{2}}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}{9}-\frac{7e^4(dex+ce)^{\frac{3}{2}}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}{45}+\frac{7ie^5\sqrt{1-\frac{i(dex+ce)}{e}}\sqrt{1+\frac{i(dex+ce)}{e}}\left(\text{EllipticF}\left(\right.\right.}{15\sqrt{\frac{i}{e}}\sqrt{\frac{(de}{}}}\right.}{9e}\right.}{de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c)),x)
```

```
[Out] 2/d/e*(1/9*(d*e*x+c*e)^(9/2)*a+b*(1/9*(d*e*x+c*e)^(9/2)*arcsinh((d*e*x+c*e)
/e)-2/9/e*(1/9*e^2*(d*e*x+c*e)^(7/2)*((d*e*x+c*e)^2/e^2+1)^(1/2)-7/45*e^4*(
d*e*x+c*e)^(3/2)*((d*e*x+c*e)^2/e^2+1)^(1/2)+7/15*I*e^5/(I/e)^(1/2)*(1-I/e*
(d*e*x+c*e))^(1/2)*(1+I/e*(d*e*x+c*e))^(1/2)/((d*e*x+c*e)^2/e^2+1)^(1/2)*(E
llipticF((d*e*x+c*e)^(1/2)*(I/e)^(1/2),I)-EllipticE((d*e*x+c*e)^(1/2)*(I/e)
^(1/2),I)))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\,(dex+ce)^{\frac{9}{2}}a}{9\,de}+\frac{1}{810}\left(\frac{180\left(d^4e^{\frac{7}{2}}x^4+4\,cd^3e^{\frac{7}{2}}x^3+6\,c^2d^2e^{\frac{7}{2}}x^2+4\,c^3de^{\frac{7}{2}}x+c^4e^{\frac{7}{2}}\right)\sqrt{dx+c}\,\log\left(dx+c+\sqrt{d^2x^2+2\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2/9*(d*e*x + c*e)^(9/2)*a/(d*e) + 1/810*(180*(d^4*e^(7/2)*x^4 + 4*c*d^3*e^(
7/2)*x^3 + 6*c^2*d^2*e^(7/2)*x^2 + 4*c^3*d*e^(7/2)*x + c^4*e^(7/2))*sqrt(d*
x + c)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/d - (40*(d*x + c)^(
9/2)*e^(7/2) - 72*(d*x + c)^(5/2)*e^(7/2) + 360*sqrt(d*x + c)*e^(7/2) + 45*
(I*sqrt(2)*e^3*(log(1/2*I*sqrt(2)*(sqrt(2) + 2*sqrt(d*x + c)) + 1) - log(-1
/2*I*sqrt(2)*(sqrt(2) + 2*sqrt(d*x + c)) + 1)) - I*sqrt(2)*e^3*(log(1/2*I*s
qrt(2)*(sqrt(2) - 2*sqrt(d*x + c)) + 1) - log(-1/2*I*sqrt(2)*(sqrt(2) - 2*s
qrt(d*x + c)) + 1)) - sqrt(2)*e^3*log(d*x + sqrt(2)*sqrt(d*x + c) + c + 1)
+ sqrt(2)*e^3*log(d*x - sqrt(2)*sqrt(d*x + c) + c + 1))*sqrt(e))/d - 810*in
tegrate(2/9*(d^4*e^(7/2)*x^4 + 4*c*d^3*e^(7/2)*x^3 + 6*c^2*d^2*e^(7/2)*x^2
+ 4*c^3*d*e^(7/2)*x + c^4*e^(7/2))*sqrt(d*x + c)/(d^3*x^3 + 3*c*d^2*x^2 + c
^3 + (3*c^2*d + d)*x + (d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + c), x))*b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e+d\,e\,x)^{7/2}\,\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^(7/2)*(a + b*asinh(c + d*x)),x)
```

```
[Out] int((c*e + d*e*x)^(7/2)*(a + b*asinh(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(7/2)*(a+b*asinh(d*x+c)),x)
```

```
[Out] Timed out
```

3.229 $\int (ce + dex)^{5/2} \left(a + b \sinh^{-1}(c + dx)\right) dx$

Optimal. Leaf size=177

$$\frac{2(e(c+dx))^{7/2}\left(a+b\sinh^{-1}(c+dx)\right)}{7de}-\frac{10be^{5/2}(c+dx+1)\sqrt{\frac{(c+dx)^2+1}{(c+dx+1)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right)|\frac{1}{2}\right)}{147d\sqrt{(c+dx)^2+1}}+\frac{20be^2\sqrt{(c+dx)^2}}{147}$$

```
[Out] 2/7*(e*(d*x+c))^(7/2)*(a+b*arcsinh(d*x+c))/d/e-4/49*b*(e*(d*x+c))^(5/2)*(1+
(d*x+c)^2)^(1/2)/d+20/147*b*e^2*(e*(d*x+c))^(1/2)*(1+(d*x+c)^2)^(1/2)/d-10/
147*b*e^(5/2)*(d*x+c+1)*(cos(2*arctan((e*(d*x+c))^(1/2)/e^(1/2)))^2)^(1/2)/
cos(2*arctan((e*(d*x+c))^(1/2)/e^(1/2)))*EllipticF(sin(2*arctan((e*(d*x+c))
^(1/2)/e^(1/2))),1/2*2^(1/2))*((1+(d*x+c)^2)/(d*x+c+1)^2)^(1/2)/d/(1+(d*x+c
)^2)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {5865, 5661, 321, 329, 220}

$$\frac{2(e(c+dx))^{7/2}\left(a+b\sinh^{-1}(c+dx)\right)}{7de}+\frac{20be^2\sqrt{(c+dx)^2+1}\sqrt{e(c+dx)}}{147d}-\frac{10be^{5/2}(c+dx+1)\sqrt{\frac{(c+dx)^2+1}{(c+dx+1)^2}}F\left(2\tan^{-1}\right.}{147d\sqrt{(c+dx)^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^(5/2)*(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (20*b*e^2*Sqrt[e*(c + d*x)]*Sqrt[1 + (c + d*x)^2])/(147*d) - (4*b*(e*(c + d
*x))^(5/2)*Sqrt[1 + (c + d*x)^2])/(49*d) + (2*(e*(c + d*x))^(7/2)*(a + b*Ar
cSinh[c + d*x]))/(7*d*e) - (10*b*e^(5/2)*(1 + c + d*x)*Sqrt[(1 + (c + d*x)^
2)/(1 + c + d*x)^2]*EllipticF[2*ArcTan[Sqrt[e*(c + d*x)]/Sqrt[e]], 1/2])/(1
47*d*Sqrt[1 + (c + d*x)^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
```

```
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^{5/2}\left(a+b\sinh^{-1}(c+dx)\right)dx = \frac{\text{Subst}\left(\int (ex)^{5/2}\left(a+b\sinh^{-1}(x)\right)dx, x, c+dx\right)}{d}$$

$$= \frac{2(e(c+dx))^{7/2}\left(a+b\sinh^{-1}(c+dx)\right)}{7de} - \frac{(2b)\,\text{Subst}\left(\int \frac{(ex)^{7/2}}{\sqrt{1+x^2}}\,dx, x\right.}{7de}$$

$$= -\frac{4b(e(c+dx))^{5/2}\sqrt{1+(c+dx)^2}}{49d} + \frac{2(e(c+dx))^{7/2}\left(a+b\sinh^{-1}(c\right.}{7de}$$

$$= \frac{20be^2\sqrt{e(c+dx)}\sqrt{1+(c+dx)^2}}{147d} - \frac{4b(e(c+dx))^{5/2}\sqrt{1+(c+dx)^2}}{49d}$$

$$= \frac{20be^2\sqrt{e(c+dx)}\sqrt{1+(c+dx)^2}}{147d} - \frac{4b(e(c+dx))^{5/2}\sqrt{1+(c+dx)^2}}{49d}$$

$$= \frac{20be^2\sqrt{e(c+dx)}\sqrt{1+(c+dx)^2}}{147d} - \frac{4b(e(c+dx))^{5/2}\sqrt{1+(c+dx)^2}}{49d}$$

Mathematica [C] time = 0.19, size = 113, normalized size = 0.64

$$\frac{2(e(c+dx))^{5/2}\left(21a(c+dx)^3 - 10b\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-(c+dx)^2\right) - 6b\sqrt{(c+dx)^2+1}\,(c+dx)^2 + 10b\sqrt{(c+dx)^2+1}\right.}{147d(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^(5/2)*(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (2*(e*(c + d*x))^(5/2)*(21*a*(c + d*x)^3 + 10*b*Sqrt[1 + (c + d*x)^2] - 6*b
*(c + d*x)^2*Sqrt[1 + (c + d*x)^2] + 21*b*(c + d*x)^3*ArcSinh[c + d*x] - 10
*b*Hypergeometric2F1[1/4, 1/2, 5/4, -(c + d*x)^2]))/(147*d*(c + d*x)^2)
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(ad^2e^2x^2 + 2\,acde^2x + ac^2e^2 + \left(bd^2e^2x^2 + 2\,bcde^2x + bc^2e^2\right)\text{arsinh}\,(dx+c)\right)\sqrt{dex+ce}\,, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((a*d^2*e^2*x^2 + 2*a*c*d*e^2*x + a*c^2*e^2 + (b*d^2*e^2*x^2 + 2*b*
c*d*e^2*x + b*c^2*e^2)*arcsinh(d*x + c))*sqrt(d*e*x + c*e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex+ce)^{\frac{5}{2}}(b\,\text{arsinh}\,(dx+c)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^(5/2)*(b*arcsinh(d*x + c) + a), x)
```

maple [C] time = 0.01, size = 212, normalized size = 1.20

$$\frac{\frac{2(dex+ce)^{\frac{7}{2}}a}{7}+2b\left(\frac{(dex+ce)^{\frac{7}{2}}\operatorname{arcsinh}\left(\frac{dex+ce}{e}\right)}{7}-\frac{2\left(\frac{e^2(dex+ce)^{\frac{5}{2}}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}{7}-\frac{5e^4\sqrt{dex+ce}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}{21}+\frac{5e^4\sqrt{1-\frac{i(dex+ce)}{e}}\sqrt{1+\frac{i(dex+ce)}{e}}\operatorname{EllipticF}\left(\sqrt{a}\right.}{21\sqrt{\frac{i}{e}}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}\right.}{7e}\right.}{de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c)),x)
```

```
[Out] 2/d/e*(1/7*(d*e*x+c*e)^(7/2)*a+b*(1/7*(d*e*x+c*e)^(7/2)*arcsinh((d*e*x+c*e)
/e)-2/7/e*(1/7*e^2*(d*e*x+c*e)^(5/2)*((d*e*x+c*e)^2/e^2+1)^(1/2)-5/21*e^4*(
d*e*x+c*e)^(1/2)*((d*e*x+c*e)^2/e^2+1)^(1/2)+5/21*e^4/(I/e)^(1/2)*(1-I/e*(d
*e*x+c*e))^(1/2)*(1+I/e*(d*e*x+c*e))^(1/2)/((d*e*x+c*e)^2/e^2+1)^(1/2)*Elli
pticF((d*e*x+c*e)^(1/2)*(I/e)^(1/2),I))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\,(dex+ce)^{\frac{7}{2}}a}{7\,de}+\frac{1}{294}\left(\frac{84\left(d^3e^{\frac{5}{2}}x^3+3\,cd^2e^{\frac{5}{2}}x^2+3\,c^2de^{\frac{5}{2}}x+c^3e^{\frac{5}{2}}\right)\sqrt{dx+c}\,\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2/7*(d*e*x + c*e)^(7/2)*a/(d*e) + 1/294*(84*(d^3*e^(5/2)*x^3 + 3*c*d^2*e^(5
/2)*x^2 + 3*c^2*d*e^(5/2)*x + c^3*e^(5/2))*sqrt(d*x + c)*log(d*x + c + sqrt
(d^2*x^2 + 2*c*d*x + c^2 + 1))/d - (24*(d*x + c)^(7/2)*e^(5/2) - 56*(d*x +
c)^(3/2)*e^(5/2) - 21*(I*sqrt(2)*(log(1/2*I*sqrt(2)*(sqrt(2) + 2*sqrt(d*x +
 c)) + 1) - log(-1/2*I*sqrt(2)*(sqrt(2) + 2*sqrt(d*x + c)) + 1)) - I*sqrt(2
)*(log(1/2*I*sqrt(2)*(sqrt(2) - 2*sqrt(d*x + c)) + 1) - log(-1/2*I*sqrt(2)*
(sqrt(2) - 2*sqrt(d*x + c)) + 1)) + sqrt(2)*log(d*x + sqrt(2)*sqrt(d*x + c)
 + c + 1) - sqrt(2)*log(d*x - sqrt(2)*sqrt(d*x + c) + c + 1))*e^(5/2))/d -
294*integrate(2/7*(d^3*e^(5/2)*x^3 + 3*c*d^2*e^(5/2)*x^2 + 3*c^2*d*e^(5/2)*
x + c^3*e^(5/2))*sqrt(d*x + c)/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (3*c^2*d + d)
*x + (d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + c), x))*b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e+d\,e\,x)^{5/2}\,\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^(5/2)*(a + b*asinh(c + d*x)),x)
```

```
[Out] int((c*e + d*e*x)^(5/2)*(a + b*asinh(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(5/2)*(a+b*asinh(d*x+c)),x)
```

```
[Out] Timed out
```

3.230 $\int (ce+dex)^{3/2} \left(a+b\sinh^{-1}(c+dx)\right) dx$

Optimal. Leaf size=261

$$\frac{2(e(c+dx))^{5/2}\left(a+b\sinh^{-1}(c+dx)\right)}{5de}+\frac{6be^{3/2}(c+dx+1)\sqrt{\frac{(c+dx)^2+1}{(c+dx+1)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right)|\frac{1}{2}\right)}{25d\sqrt{(c+dx)^2+1}}-\frac{12be^{3/2}(c+dx+1)}{2}$$

```
[Out] 2/5*(e*(d*x+c))^(5/2)*(a+b*arcsinh(d*x+c))/d/e-4/25*b*(e*(d*x+c))^(3/2)*(1+
(d*x+c)^2)^(1/2)/d+12/25*b*e*(e*(d*x+c))^(1/2)*(1+(d*x+c)^2)^(1/2)/d/(d*x+c
+1)-12/25*b*e^(3/2)*(d*x+c+1)*(cos(2*arctan((e*(d*x+c))^(1/2)/e^(1/2)))^2)^
(1/2)/cos(2*arctan((e*(d*x+c))^(1/2)/e^(1/2)))*EllipticE(sin(2*arctan((e*(d
*x+c))^(1/2)/e^(1/2))),1/2*2^(1/2))*((1+(d*x+c)^2)/(d*x+c+1)^2)^(1/2)/d/(1+
(d*x+c)^2)^(1/2)+6/25*b*e^(3/2)*(d*x+c+1)*(cos(2*arctan((e*(d*x+c))^(1/2)/e
^(1/2)))^2)^(1/2)/cos(2*arctan((e*(d*x+c))^(1/2)/e^(1/2)))*EllipticF(sin(2*
arctan((e*(d*x+c))^(1/2)/e^(1/2))),1/2*2^(1/2))*((1+(d*x+c)^2)/(d*x+c+1)^2)
^(1/2)/d/(1+(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {5865, 5661, 321, 329, 305, 220, 1196}

$$\frac{2(e(c+dx))^{5/2}\left(a+b\sinh^{-1}(c+dx)\right)}{5de}+\frac{6be^{3/2}(c+dx+1)\sqrt{\frac{(c+dx)^2+1}{(c+dx+1)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right)|\frac{1}{2}\right)}{25d\sqrt{(c+dx)^2+1}}-\frac{12be^{3/2}(c+dx+1)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^(3/2)*(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (-4*b*(e*(c + d*x))^(3/2)*Sqrt[1 + (c + d*x)^2])/(25*d) + (12*b*e*Sqrt[e*(c
 + d*x)]*Sqrt[1 + (c + d*x)^2])/(25*d*(1 + c + d*x)) + (2*(e*(c + d*x))^(5/
2)*(a + b*ArcSinh[c + d*x]))/(5*d*e) - (12*b*e^(3/2)*(1 + c + d*x)*Sqrt[(1
+ (c + d*x)^2)/(1 + c + d*x)^2]*EllipticE[2*ArcTan[Sqrt[e*(c + d*x)]/Sqrt[e
]], 1/2])/(25*d*Sqrt[1 + (c + d*x)^2]) + (6*b*e^(3/2)*(1 + c + d*x)*Sqrt[(1
 + (c + d*x)^2)/(1 + c + d*x)^2]*EllipticF[2*ArcTan[Sqrt[e*(c + d*x)]/Sqrt[
e]], 1/2])/(25*d*Sqrt[1 + (c + d*x)^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^{3/2}\left(a+b\sinh^{-1}(c+dx)\right)dx=\frac{\text{Subst}\left(\int (ex)^{3/2}\left(a+b\sinh^{-1}(x)\right)dx,x,c+dx\right)}{d}$$

$$=\frac{2(e(c+dx))^{5/2}\left(a+b\sinh^{-1}(c+dx)\right)}{5de}-\frac{(2b)\,\text{Subst}\left(\int \frac{(ex)^{5/2}}{\sqrt{1+x^2}}\,dx,x\right.}{5de}$$

$$=-\frac{4b(e(c+dx))^{3/2}\sqrt{1+(c+dx)^2}}{25d}+\frac{2(e(c+dx))^{5/2}\left(a+b\sinh^{-1}(c\right.}{5de}$$

$$=-\frac{4b(e(c+dx))^{3/2}\sqrt{1+(c+dx)^2}}{25d}+\frac{2(e(c+dx))^{5/2}\left(a+b\sinh^{-1}(c\right.}{5de}$$

$$=-\frac{4b(e(c+dx))^{3/2}\sqrt{1+(c+dx)^2}}{25d}+\frac{2(e(c+dx))^{5/2}\left(a+b\sinh^{-1}(c\right.}{5de}$$

$$=-\frac{4b(e(c+dx))^{3/2}\sqrt{1+(c+dx)^2}}{25d}+\frac{12be\sqrt{e(c+dx)}\sqrt{1+(c+dx)^2}}{25d(1+c+dx)}$$

Mathematica [C] time = 0.05, size = 87, normalized size = 0.33

$$\frac{2(e(c+dx))^{3/2}\left(5ac+5adx+2b\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-(c+dx)^2\right)-2b\sqrt{(c+dx)^2+1}+5bc\sinh^{-1}(c+dx)+5bdx\,\text{sinl}\right.}{25d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^(3/2)*(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (2*(e*(c + d*x))^(3/2)*(5*a*c + 5*a*d*x - 2*b*Sqrt[1 + (c + d*x)^2] + 5*b*c
*ArcSinh[c + d*x] + 5*b*d*x*ArcSinh[c + d*x] + 2*b*Hypergeometric2F1[1/2, 3
/4, 7/4, -(c + d*x)^2]))/(25*d)
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\mathrm{integral}\left((adex + ace + (bdex + bce)\operatorname{arsinh}(dx + c))\sqrt{dex + ce}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((a*d*e*x + a*c*e + (b*d*e*x + b*c*e)*arcsinh(d*x + c))*sqrt(d*e*x
+ c*e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex + ce)^{\frac{3}{2}}(b\operatorname{arsinh}(dx + c) + a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^(3/2)*(b*arcsinh(d*x + c) + a), x)
```

maple [C] time = 0.01, size = 205, normalized size = 0.79

$$\frac{\frac{2(dex+ce)^{\frac{5}{2}}a}{5} + 2b\left(\frac{(dex+ce)^{\frac{5}{2}}\operatorname{arcsinh}\left(\frac{dex+ce}{e}\right)}{5} - \frac{2\left(\frac{e^2(dex+ce)^{\frac{3}{2}}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}{5} - \frac{3ie^3\sqrt{1-\frac{i(dex+ce)}{e}}\sqrt{1+\frac{i(dex+ce)}{e}}\left(\mathrm{EllipticF}\left(\sqrt{dex+ce}\sqrt{\frac{i}{e}}, i\right) - \mathrm{EllipticE}\left(\sqrt{dex}\right.\right.}{5\sqrt{\frac{i}{e}}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}\right.}{5e}\right.}{de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c)),x)
```

```
[Out] 2/d/e*(1/5*(d*e*x+c*e)^(5/2)*a+b*(1/5*(d*e*x+c*e)^(5/2)*arcsinh((d*e*x+c*e)
/e)-2/5/e*(1/5*e^2*(d*e*x+c*e)^(3/2)*((d*e*x+c*e)^2/e^2+1)^(1/2)-3/5*I*e^3/
(I/e)^(1/2)*(1-I/e*(d*e*x+c*e))^(1/2)*(1+I/e*(d*e*x+c*e))^(1/2)/((d*e*x+c*e
)^2/e^2+1)^(1/2)*(EllipticF((d*e*x+c*e)^(1/2)*(I/e)^(1/2),I)-EllipticE((d*e
*x+c*e)^(1/2)*(I/e)^(1/2),I)))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{50}\left(\frac{20\left(d^2e^{\frac{3}{2}}x^2 + 2\,cde^{\frac{3}{2}}x + c^2e^{\frac{3}{2}}\right)\sqrt{dx+c}\,\log\left(dx + c + \sqrt{d^2x^2 + 2\,cdx + c^2 + 1}\right)}{d} - \frac{8\,(dx+c)^{\frac{5}{2}}e^{\frac{3}{2}} - 40\sqrt{dx+c}\,e\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/50*(20*(d^2*e^(3/2)*x^2 + 2*c*d*e^(3/2)*x + c^2*e^(3/2))*sqrt(d*x + c)*lo
g(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/d - (8*(d*x + c)^(5/2)*e^(3/
2) - 40*sqrt(d*x + c)*e^(3/2) - 5*(I*sqrt(2)*e*(log(1/2*I*sqrt(2)*(sqrt(2)
```

```
+ 2*sqrt(d*x + c)) + 1) - log(-1/2*I*sqrt(2)*(sqrt(2) + 2*sqrt(d*x + c)) +
1)) - I*sqrt(2)*e*(log(1/2*I*sqrt(2)*(sqrt(2) - 2*sqrt(d*x + c)) + 1) - log
(-1/2*I*sqrt(2)*(sqrt(2) - 2*sqrt(d*x + c)) + 1)) - sqrt(2)*e*log(d*x + sqr
t(2)*sqrt(d*x + c) + c + 1) + sqrt(2)*e*log(d*x - sqrt(2)*sqrt(d*x + c) + c
 + 1))*sqrt(e))/d - 50*integrate(2/5*(d^2*e^(3/2)*x^2 + 2*c*d*e^(3/2)*x + c
^2*e^(3/2))*sqrt(d*x + c)/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (3*c^2*d + d)*x +
(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + c), x))*b + 2/5*(d*e*x + c*e)^(5/2)*a
/(d*e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e+d\,e\,x)^{3/2}\,\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^(3/2)*(a + b*asinh(c + d*x)),x)
[Out] int((c*e + d*e*x)^(3/2)*(a + b*asinh(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e(c+dx))^{\frac{3}{2}}\,(a+b\,\mathrm{asinh}\,(c+dx))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(3/2)*(a+b*asinh(d*x+c)),x)
[Out] Integral((e*(c + d*x))**(3/2)*(a + b*asinh(c + d*x)), x)
```

3.231 $\int \sqrt{ce+dex}\left(a+b\sinh^{-1}(c+dx)\right)dx$

Optimal. Leaf size=142

$$\frac{2(e(c+dx))^{3/2}\left(a+b\sinh^{-1}(c+dx)\right)}{3de}-\frac{4b\sqrt{(c+dx)^2+1}\sqrt{e(c+dx)}}{9d}+\frac{2b\sqrt{e}(c+dx+1)\sqrt{\frac{(c+dx)^2+1}{(c+dx+1)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt{e}}{}\right.\right.}{9d\sqrt{(c+dx)^2+1}}$$

```
[Out] 2/3*(e*(d*x+c))^(3/2)*(a+b*arcsinh(d*x+c))/d/e-4/9*b*(e*(d*x+c))^(1/2)*(1+(
d*x+c)^2)^(1/2)/d+2/9*b*(d*x+c+1)*(cos(2*arctan((e*(d*x+c))^(1/2)/e^(1/2)))
^2)^(1/2)/cos(2*arctan((e*(d*x+c))^(1/2)/e^(1/2)))*EllipticF(sin(2*arctan((
e*(d*x+c))^(1/2)/e^(1/2))),1/2*2^(1/2))*e^(1/2)*((1+(d*x+c)^2)/(d*x+c+1)^2)
^(1/2)/d/(1+(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {5865, 5661, 321, 329, 220}

$$\frac{2(e(c+dx))^{3/2}\left(a+b\sinh^{-1}(c+dx)\right)}{3de}-\frac{4b\sqrt{(c+dx)^2+1}\sqrt{e(c+dx)}}{9d}+\frac{2b\sqrt{e}(c+dx+1)\sqrt{\frac{(c+dx)^2+1}{(c+dx+1)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt{e}}{}\right.\right.}{9d\sqrt{(c+dx)^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*e + d*e*x]*(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (-4*b*Sqrt[e*(c + d*x)]*Sqrt[1 + (c + d*x)^2])/(9*d) + (2*(e*(c + d*x))^(3/
2)*(a + b*ArcSinh[c + d*x]))/(3*d*e) + (2*b*Sqrt[e]*(1 + c + d*x)*Sqrt[(1 +
 (c + d*x)^2)/(1 + c + d*x)^2]*EllipticF[2*ArcTan[Sqrt[e*(c + d*x)]/Sqrt[e]
], 1/2])/(9*d*Sqrt[1 + (c + d*x)^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \sqrt{ce+dex}\left(a+b\sinh^{-1}(c+dx)\right)dx = \frac{\text{Subst}\left(\int \sqrt{ex}\left(a+b\sinh^{-1}(x)\right)dx, x, c+dx\right)}{d}$$

$$= \frac{2(e(c+dx))^{3/2}\left(a+b\sinh^{-1}(c+dx)\right)}{3de} - \frac{(2b)\,\text{Subst}\left(\int \frac{(ex)^{3/2}}{\sqrt{1+x^2}}\,dx, x, c\right.}{3de}$$

$$= -\frac{4b\sqrt{e(c+dx)}\sqrt{1+(c+dx)^2}}{9d} + \frac{2(e(c+dx))^{3/2}\left(a+b\sinh^{-1}(c+a\right.}{3de}$$

$$= -\frac{4b\sqrt{e(c+dx)}\sqrt{1+(c+dx)^2}}{9d} + \frac{2(e(c+dx))^{3/2}\left(a+b\sinh^{-1}(c+a\right.}{3de}$$

$$= -\frac{4b\sqrt{e(c+dx)}\sqrt{1+(c+dx)^2}}{9d} + \frac{2(e(c+dx))^{3/2}\left(a+b\sinh^{-1}(c+a\right.}{3de}$$

Mathematica [C] time = 0.03, size = 87, normalized size = 0.61

$$\frac{2\sqrt{e(c+dx)}\left(3ac+3adx+2b\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-(c+dx)^2\right)-2b\sqrt{(c+dx)^2+1}+3bc\sinh^{-1}(c+dx)+3bdx\sinh^{-}\right.}{9d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*e + d*e*x]*(a + b*ArcSinh[c + d*x]),x]
[Out] (2*Sqrt[e*(c + d*x)]*(3*a*c + 3*a*d*x - 2*b*Sqrt[1 + (c + d*x)^2] + 3*b*c*A
rcSinh[c + d*x] + 3*b*d*x*ArcSinh[c + d*x] + 2*b*Hypergeometric2F1[1/4, 1/2
, 5/4, -(c + d*x)^2]))/(9*d)
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{dex+ce}\,(b\,\text{arsinh}\,(dx+c)+a), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))*(d*e*x+c*e)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(d*e*x + c*e)*(b*arcsinh(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{dex+ce}\,(b\,\text{arsinh}\,(dx+c)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))*(d*e*x+c*e)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(d*e*x + c*e)*(b*arcsinh(d*x + c) + a), x)
```

maple [C] time = 0.01, size = 179, normalized size = 1.26

$$\frac{\frac{2(dex+ce)^{\frac{3}{2}}a}{3}+2b\left(\frac{(dex+ce)^{\frac{3}{2}}\operatorname{arcsinh}\left(\frac{dex+ce}{e}\right)}{3}-\frac{2\left(\frac{e^2\sqrt{dex+ce}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}{3}-\frac{e^2\sqrt{1-\frac{i(dex+ce)}{e}}\sqrt{1+\frac{i(dex+ce)}{e}}\operatorname{EllipticF}\left(\sqrt{dex+ce}\sqrt{\frac{i}{e}},i\right)}{3\sqrt{\frac{i}{e}}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}\right)}{3e}\right)}{de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))*(d*e*x+c*e)^(1/2),x)
```

```
[Out] 2/d/e*(1/3*(d*e*x+c*e)^(3/2)*a+b*(1/3*(d*e*x+c*e)^(3/2)*arcsinh((d*e*x+c*e)
/e)-2/3/e*(1/3*e^2*(d*e*x+c*e)^(1/2)*((d*e*x+c*e)^2/e^2+1)^(1/2)-1/3*e^2/(I
/e)^(1/2)*(1-I/e*(d*e*x+c*e))^(1/2)*(1+I/e*(d*e*x+c*e))^(1/2)/((d*e*x+c*e)^
2/e^2+1)^(1/2)*EllipticF((d*e*x+c*e)^(1/2)*(I/e)^(1/2),I))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{18}\left(\frac{12\left(d\sqrt{e}\,x+c\sqrt{e}\right)\sqrt{dx+c}\,\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)}{d}-\frac{8\,(dx+c)^{\frac{3}{2}}\sqrt{e}\,+3\left(i\sqrt{2}\left(\log\left(\frac{1}{2}i\sqrt{2}\left(\sqrt{2}\right.\right.\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))*(d*e*x+c*e)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/18*(12*(d*sqrt(e)*x + c*sqrt(e))*sqrt(d*x + c)*log(d*x + c + sqrt(d^2*x^2
 + 2*c*d*x + c^2 + 1))/d - (8*(d*x + c)^(3/2)*sqrt(e) + 3*(I*sqrt(2)*(log(1
/2*I*sqrt(2)*(sqrt(2) + 2*sqrt(d*x + c)) + 1) - log(-1/2*I*sqrt(2)*(sqrt(2)
 + 2*sqrt(d*x + c)) + 1)) - I*sqrt(2)*(log(1/2*I*sqrt(2)*(sqrt(2) - 2*sqrt(
d*x + c)) + 1) - log(-1/2*I*sqrt(2)*(sqrt(2) - 2*sqrt(d*x + c)) + 1)) + sqr
t(2)*log(d*x + sqrt(2)*sqrt(d*x + c) + c + 1) - sqrt(2)*log(d*x - sqrt(2)*s
qrt(d*x + c) + c + 1))*sqrt(e))/d - 18*integrate(2/3*(d*sqrt(e)*x + c*sqrt(
e))*sqrt(d*x + c)/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (3*c^2*d + d)*x + (d^2*x^2
 + 2*c*d*x + c^2 + 1)^(3/2) + c), x))*b + 2/3*(d*e*x + c*e)^(3/2)*a/(d*e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{c\,e+d\,e\,x}\,\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^(1/2)*(a + b*asinh(c + d*x)),x)
```

```
[Out] int((c*e + d*e*x)^(1/2)*(a + b*asinh(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{e\left(c+dx\right)}\,\left(a+b\operatorname{asinh}\left(c+dx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))*(d*e*x+c*e)**(1/2),x)
```

```
[Out] Integral(sqrt(e*(c + d*x))*(a + b*asinh(c + d*x)), x)
```

3.232 $\int \frac{a+b\sinh^{-1}(c+dx)}{\sqrt{ce+dex}}\,dx$

Optimal. Leaf size=223

$$\frac{2\sqrt{e(c+dx)}\left(a+b\sinh^{-1}(c+dx)\right)}{de}-\frac{4b\sqrt{(c+dx)^2+1}\,\sqrt{e(c+dx)}}{de(c+dx+1)}-\frac{2b(c+dx+1)\sqrt{\frac{(c+dx)^2+1}{(c+dx+1)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt{e(c-}}{\sqrt{}}\right.\right.}{d\sqrt{e}\,\sqrt{(c+dx)^2+1}}$$

```
[Out] 2*(a+b*arcsinh(d*x+c))*(e*(d*x+c))^(1/2)/d/e-4*b*(e*(d*x+c))^(1/2)*(1+(d*x+
c)^2)^(1/2)/d/e/(d*x+c+1)+4*b*(d*x+c+1)*(cos(2*arctan((e*(d*x+c))^(1/2)/e^(
1/2)))^2)^(1/2)/cos(2*arctan((e*(d*x+c))^(1/2)/e^(1/2)))*EllipticE(sin(2*ar
ctan((e*(d*x+c))^(1/2)/e^(1/2))),1/2*2^(1/2))*((1+(d*x+c)^2)/(d*x+c+1)^2)^(
1/2)/d/e^(1/2)/(1+(d*x+c)^2)^(1/2)-2*b*(d*x+c+1)*(cos(2*arctan((e*(d*x+c))^
(1/2)/e^(1/2)))^2)^(1/2)/cos(2*arctan((e*(d*x+c))^(1/2)/e^(1/2)))*EllipticF
(sin(2*arctan((e*(d*x+c))^(1/2)/e^(1/2))),1/2*2^(1/2))*((1+(d*x+c)^2)/(d*x+
c+1)^2)^(1/2)/d/e^(1/2)/(1+(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {5865, 5661, 329, 305, 220, 1196}

$$\frac{2\sqrt{e(c+dx)}\left(a+b\sinh^{-1}(c+dx)\right)}{de}-\frac{4b\sqrt{(c+dx)^2+1}\,\sqrt{e(c+dx)}}{de(c+dx+1)}-\frac{2b(c+dx+1)\sqrt{\frac{(c+dx)^2+1}{(c+dx+1)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt{e(c-}}{\sqrt{}}\right.\right.}{d\sqrt{e}\,\sqrt{(c+dx)^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])/Sqrt[c*e + d*e*x],x]
```

```
[Out] (-4*b*Sqrt[e*(c + d*x)]*Sqrt[1 + (c + d*x)^2])/(d*e*(1 + c + d*x)) + (2*Sqr
t[e*(c + d*x)]*(a + b*ArcSinh[c + d*x]))/(d*e) + (4*b*(1 + c + d*x)*Sqrt[(1
 + (c + d*x)^2)/(1 + c + d*x)^2]*EllipticE[2*ArcTan[Sqrt[e*(c + d*x)]/Sqrt[
e]], 1/2])/(d*Sqrt[e]*Sqrt[1 + (c + d*x)^2]) - (2*b*(1 + c + d*x)*Sqrt[(1 +
 (c + d*x)^2)/(1 + c + d*x)^2]*EllipticF[2*ArcTan[Sqrt[e*(c + d*x)]/Sqrt[e]
], 1/2])/(d*Sqrt[e]*Sqrt[1 + (c + d*x)^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
```

```
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(c+dx)}{\sqrt{ce+dex}}\,dx = \frac{\text{Subst}\left(\int \frac{a+b\sinh^{-1}(x)}{\sqrt{ex}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{2\sqrt{e(c+dx)}\,\left(a+b\sinh^{-1}(c+dx)\right)}{de} - \frac{(2b)\,\text{Subst}\left(\int \frac{\sqrt{ex}}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{de}$$

$$= \frac{2\sqrt{e(c+dx)}\,\left(a+b\sinh^{-1}(c+dx)\right)}{de} - \frac{(4b)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^4}{e^2}}}\,dx, x, \sqrt{e(c+dx)}\right)}{de^2}$$

$$= \frac{2\sqrt{e(c+dx)}\,\left(a+b\sinh^{-1}(c+dx)\right)}{de} - \frac{(4b)\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{e^2}}}\,dx, x, \sqrt{e(c+dx)}\right)}{de} + \frac{(4}{}$$

$$= -\frac{4b\sqrt{e(c+dx)}\,\sqrt{1+(c+dx)^2}}{de(1+c+dx)} + \frac{2\sqrt{e(c+dx)}\,\left(a+b\sinh^{-1}(c+dx)\right)}{de} + \frac{4b(1+c+}{}$$

Mathematica [C] time = 0.03, size = 61, normalized size = 0.27

$$-\frac{2\sqrt{e(c+dx)}\,\left(2b(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-(c+dx)^2\right)-3\left(a+b\sinh^{-1}(c+dx)\right)\right)}{3de}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])/Sqrt[c*e + d*e*x],x]
```

```
[Out] (-2*Sqrt[e*(c + d*x)]*(-3*(a + b*ArcSinh[c + d*x]) + 2*b*(c + d*x)*Hypergeo
metric2F1[1/2, 3/4, 7/4, -(c + d*x)^2]))/(3*d*e)
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\,\text{arsinh}\,(dx+c)+a}{\sqrt{dex+ce}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(d*x + c) + a)/sqrt(d*e*x + c*e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arsinh}(dx+c)+a}{\sqrt{dex+ce}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)/sqrt(d*e*x + c*e), x)
```

maple [C] time = 0.01, size = 161, normalized size = 0.72

$$\frac{2a\sqrt{dex+ce}+2b\left(\sqrt{dex+ce}\,\operatorname{arcsinh}\left(\frac{dex+ce}{e}\right)-\frac{2i\sqrt{1-\frac{i(dex+ce)}{e}}\,\sqrt{1+\frac{i(dex+ce)}{e}}\,\left(\operatorname{EllipticF}\left(\sqrt{dex+ce}\,\sqrt{\frac{i}{e}},i\right)-\operatorname{EllipticE}\left(\sqrt{dex+ce}\right.\right.}{\sqrt{\frac{i}{e}}\,\sqrt{\frac{(dex+ce)^2}{e^2}+1}}}{de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(1/2),x)
```

```
[Out] 2/d/e*(a*(d*e*x+c*e)^(1/2)+b*((d*e*x+c*e)^(1/2)*arcsinh((d*e*x+c*e)/e)-2*I/
(I/e)^(1/2)*(1-I/e*(d*e*x+c*e))^(1/2)*(1+I/e*(d*e*x+c*e))^(1/2)/((d*e*x+c*e
)^2/e^2+1)^(1/2)*(EllipticF((d*e*x+c*e)^(1/2)*(I/e)^(1/2),I)-EllipticE((d*e
*x+c*e)^(1/2)*(I/e)^(1/2),I))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}b\left(\frac{\frac{i\sqrt{2}\sqrt{e}\left(\log\left(\frac{1}{2}i\sqrt{2}\left(\sqrt{2}+2\sqrt{dx+c}\right)+1\right)-\log\left(-\frac{1}{2}i\sqrt{2}\left(\sqrt{2}+2\sqrt{dx+c}\right)+1\right)\right)-i\sqrt{2}\sqrt{e}\left(\log\left(\frac{1}{2}i\sqrt{2}\left(\sqrt{2}-2\sqrt{dx+c}\right)+1\right)-\log\left(-\frac{1}{2}i\sqrt{2}\left(\sqrt{2}-2\right.\right.\right.}{e}}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*b*(((I*sqrt(2)*sqrt(e)*(log(1/2*I*sqrt(2)*(sqrt(2) + 2*sqrt(d*x + c))
+ 1) - log(-1/2*I*sqrt(2)*(sqrt(2) + 2*sqrt(d*x + c)) + 1)) - I*sqrt(2)*sqr
t(e)*(log(1/2*I*sqrt(2)*(sqrt(2) - 2*sqrt(d*x + c)) + 1) - log(-1/2*I*sqrt(
2)*(sqrt(2) - 2*sqrt(d*x + c)) + 1)) - sqrt(2)*sqrt(e)*log(d*x + sqrt(2)*sq
rt(d*x + c) + c + 1) + sqrt(2)*sqrt(e)*log(d*x - sqrt(2)*sqrt(d*x + c) + c
+ 1))/e + 8*sqrt(d*x + c)/sqrt(e))/d - 4*(d*sqrt(e)*x + c*sqrt(e))*log(d*x
+ c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(sqrt(d*x + c)*d*e) + 2*integrate(
2*(d*sqrt(e)*x + c*sqrt(e))/((d^3*e*x^3 + 3*c*d^2*e*x^2 + c^3*e + c*e + (3*
c^2*d*e + d*e)*x + (d^2*e*x^2 + 2*c*d*e*x + c^2*e + e)*sqrt(d^2*x^2 + 2*c*d
*x + c^2 + 1))*sqrt(d*x + c)), x)) + 2*sqrt(d*e*x + c*e)*a/(d*e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\operatorname{asinh}(c+d\,x)}{\sqrt{c\,e+d\,e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))/(c*e + d*e*x)^(1/2),x)
```

```
[Out] int((a + b*asinh(c + d*x))/(c*e + d*e*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{asinh}(c + dx)}{\sqrt{e(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))/(d*e*x+c*e)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))/sqrt(e*(c + d*x)), x)
```

3.233 $\int \frac{a+b\sinh^{-1}(c+dx)}{(ce+dex)^{3/2}}\,dx$

Optimal. Leaf size=106

$$\frac{2b(c+dx+1)\sqrt{\frac{(c+dx)^2+1}{(c+dx+1)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right)|\frac{1}{2}\right)}{de^{3/2}\sqrt{(c+dx)^2+1}}-\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{de\sqrt{e(c+dx)}}$$

```
[Out] -2*(a+b*arcsinh(d*x+c))/d/e/(e*(d*x+c))^(1/2)+2*b*(d*x+c+1)*(cos(2*arctan((
e*(d*x+c))^(1/2)/e^(1/2)))^2)^(1/2)/cos(2*arctan((e*(d*x+c))^(1/2)/e^(1/2))
)*EllipticF(sin(2*arctan((e*(d*x+c))^(1/2)/e^(1/2))),1/2*2^(1/2))*((1+(d*x+
c)^2)/(d*x+c+1)^2)^(1/2)/d/e^(3/2)/(1+(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {5865, 5661, 329, 220}

$$\frac{2b(c+dx+1)\sqrt{\frac{(c+dx)^2+1}{(c+dx+1)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right)|\frac{1}{2}\right)}{de^{3/2}\sqrt{(c+dx)^2+1}}-\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{de\sqrt{e(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^(3/2),x]
```

```
[Out] (-2*(a + b*ArcSinh[c + d*x]))/(d*e*Sqrt[e*(c + d*x)]) + (2*b*(1 + c + d*x)*
Sqrt[(1 + (c + d*x)^2)/(1 + c + d*x)^2]*EllipticF[2*ArcTan[Sqrt[e*(c + d*x)
]/Sqrt[e]], 1/2])/(d*e^(3/2)*Sqrt[1 + (c + d*x)^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(c+dx)}{(ce+dex)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{a+b\sinh^{-1}(x)}{(ex)^{3/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{de\sqrt{e(c+dx)}} + \frac{(2b)\,\text{Subst}\left(\int \frac{1}{\sqrt{ex}\sqrt{1+x^2}}\,dx, x, c+dx\right)}{de}$$

$$= -\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{de\sqrt{e(c+dx)}} + \frac{(4b)\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{e^2}}}\,dx, x, \sqrt{e(c+dx)}\right)}{de^2}$$

$$= -\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{de\sqrt{e(c+dx)}} + \frac{2b(1+c+dx)\sqrt{\frac{1+(c+dx)^2}{(1+c+dx)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right)|\frac{1}{2}\right)}{de^{3/2}\sqrt{1+(c+dx)^2}}$$

Mathematica [C] time = 0.03, size = 56, normalized size = 0.53

$$-\frac{2\left(a-2b(c+dx)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-(c+dx)^2\right)+b\sinh^{-1}(c+dx)\right)}{de\sqrt{e(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^(3/2),x]
```

```
[Out] (-2*(a + b*ArcSinh[c + d*x] - 2*b*(c + d*x)*Hypergeometric2F1[1/4, 1/2, 5/4
, -(c + d*x)^2]))/(d*e*Sqrt[e*(c + d*x)])
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{dex+ce}\,(b\,\text{arsinh}\,(dx+c)+a)}{d^2e^2x^2+2\,cde^2x+c^2e^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*e*x + c*e)*(b*arcsinh(d*x + c) + a)/(d^2*e^2*x^2 + 2*c*d*e^
2*x + c^2*e^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{arsinh}\,(dx+c)+a}{(dex+ce)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)/(d*e*x + c*e)^(3/2), x)
```

maple [C] time = 0.01, size = 140, normalized size = 1.32

$$\frac{-\frac{2a}{\sqrt{dex+ce}}+2b\left(-\frac{\text{arcsinh}\left(\frac{dex+ce}{e}\right)}{\sqrt{dex+ce}}+\frac{2\sqrt{1-\frac{i(dex+ce)}{e}}\,\sqrt{1+\frac{i(dex+ce)}{e}}\,\text{EllipticF}\left(\sqrt{dex+ce}\,\sqrt{\frac{i}{e}},i\right)}{e\sqrt{\frac{i}{e}}\,\sqrt{\frac{(dex+ce)^2}{e^2}+1}}\right)}{de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(3/2),x)
```

```
[Out] 2/d/e*(-a/(d*e*x+c*e)^(1/2)+b*(-1/(d*e*x+c*e)^(1/2)*arcsinh((d*e*x+c*e)/e)+
2/e/(I/e)^(1/2)*(1-I/e*(d*e*x+c*e))^(1/2)*(1+I/e*(d*e*x+c*e))^(1/2)/((d*e*x
+c*e)^2/e^2+1)^(1/2)*EllipticF((d*e*x+c*e)^(1/2)*(I/e)^(1/2),I)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\,b\left(\frac{\frac{i\sqrt{2}\left(\log\left(\frac{1}{2}i\sqrt{2}\left(\sqrt{2}+2\sqrt{dx+c}\right)+1\right)-\log\left(-\frac{1}{2}i\sqrt{2}\left(\sqrt{2}+2\sqrt{dx+c}\right)+1\right)\right)}{e^{\frac{3}{2}}}-\frac{i\sqrt{2}\left(\log\left(\frac{1}{2}i\sqrt{2}\left(\sqrt{2}-2\sqrt{dx+c}\right)+1\right)-\log\left(-\frac{1}{2}i\sqrt{2}\left(\sqrt{2}-2\sqrt{dx\right.\right.\right.}{e^{\frac{3}{2}}}}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*b*((I*sqrt(2)*(log(1/2*I*sqrt(2)*(sqrt(2) + 2*sqrt(d*x + c)) + 1) - lo
g(-1/2*I*sqrt(2)*(sqrt(2) + 2*sqrt(d*x + c)) + 1))/e^(3/2) - I*sqrt(2)*(log
(1/2*I*sqrt(2)*(sqrt(2) - 2*sqrt(d*x + c)) + 1) - log(-1/2*I*sqrt(2)*(sqrt(
2) - 2*sqrt(d*x + c)) + 1))/e^(3/2) + sqrt(2)*log(d*x + sqrt(2)*sqrt(d*x +
c) + c + 1)/e^(3/2) - sqrt(2)*log(d*x - sqrt(2)*sqrt(d*x + c) + c + 1)/e^(3
/2))/d + 4*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(sqrt(d*x + c)*
d*e^(3/2)) - 4*integrate(1/((d^2*e^(3/2)*x^2 + 2*c*d*e^(3/2)*x + c^2*e^(3/2
) + e^(3/2))*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*sqrt(d*x + c) + (d^3*e^(3/2)
*x^3 + 3*c*d^2*e^(3/2)*x^2 + c^3*e^(3/2) + c*e^(3/2) + (3*c^2*d*e^(3/2) + d
*e^(3/2))*x)*sqrt(d*x + c)), x)) - 2*a/(sqrt(d*e*x + c*e)*d*e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{asinh}\left(c+d\,x\right)}{{\left(c\,e+d\,e\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))/(c*e + d*e*x)^(3/2),x)
```

```
[Out] int((a + b*asinh(c + d*x))/(c*e + d*e*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{asinh}{\left(c + dx \right)}}{\left(e \left(c + dx\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))/(d*e*x+c*e)**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))/(e*(c + d*x))**(3/2), x)
```

3.234 $\int \frac{a+b\sinh^{-1}(c+dx)}{(ce+dex)^{5/2}}\,dx$

Optimal. Leaf size=266

$$-\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{3de(e(c+dx))^{3/2}}+\frac{2b(c+dx+1)\sqrt{\frac{(c+dx)^2+1}{(c+dx+1)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right)|\frac{1}{2}\right)}{3de^{5/2}\sqrt{(c+dx)^2+1}}-\frac{4b(c+dx+1)\sqrt{\frac{(c+dx)^2+1}{(c+dx+1)^2}}E\left(2\tan\right.}{3de^{5/2}\sqrt{(c+dx)^2+}}$$

```
[Out] -2/3*(a+b*arcsinh(d*x+c))/d/e/(e*(d*x+c))^(3/2)-4/3*b*(1+(d*x+c)^2)^(1/2)/d
/e^2/(e*(d*x+c))^(1/2)+4/3*b*(e*(d*x+c))^(1/2)*(1+(d*x+c)^2)^(1/2)/d/e^3/(d
*x+c+1)-4/3*b*(d*x+c+1)*(cos(2*arctan((e*(d*x+c))^(1/2)/e^(1/2)))^2)^(1/2)/
cos(2*arctan((e*(d*x+c))^(1/2)/e^(1/2)))*EllipticE(sin(2*arctan((e*(d*x+c))
^(1/2)/e^(1/2))),1/2*2^(1/2))*((1+(d*x+c)^2)/(d*x+c+1)^2)^(1/2)/d/e^(5/2)/(
1+(d*x+c)^2)^(1/2)+2/3*b*(d*x+c+1)*(cos(2*arctan((e*(d*x+c))^(1/2)/e^(1/2))
)^2)^(1/2)/cos(2*arctan((e*(d*x+c))^(1/2)/e^(1/2)))*EllipticF(sin(2*arctan(
(e*(d*x+c))^(1/2)/e^(1/2))),1/2*2^(1/2))*((1+(d*x+c)^2)/(d*x+c+1)^2)^(1/2)/
d/e^(5/2)/(1+(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {5865, 5661, 325, 329, 305, 220, 1196}

$$-\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{3de(e(c+dx))^{3/2}}-\frac{4b\sqrt{(c+dx)^2+1}}{3de^2\sqrt{e(c+dx)}}+\frac{4b\sqrt{(c+dx)^2+1}\sqrt{e(c+dx)}}{3de^3(c+dx+1)}+\frac{2b(c+dx+1)\sqrt{\frac{(c+dx)^2+1}{(c+dx+1)^2}}F\left(2\tan^{-}\right.}{3de^{5/2}\sqrt{(c+dx)^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^(5/2),x]
```

```
[Out] (-4*b*Sqrt[1 + (c + d*x)^2])/(3*d*e^2*Sqrt[e*(c + d*x)]) + (4*b*Sqrt[e*(c +
 d*x)]*Sqrt[1 + (c + d*x)^2])/(3*d*e^3*(1 + c + d*x)) - (2*(a + b*ArcSinh[c
 + d*x]))/(3*d*e*(e*(c + d*x))^(3/2)) - (4*b*(1 + c + d*x)*Sqrt[(1 + (c + d
*x)^2)/(1 + c + d*x)^2]*EllipticE[2*ArcTan[Sqrt[e*(c + d*x)]/Sqrt[e]], 1/2]
)/(3*d*e^(5/2)*Sqrt[1 + (c + d*x)^2]) + (2*b*(1 + c + d*x)*Sqrt[(1 + (c + d
*x)^2)/(1 + c + d*x)^2]*EllipticF[2*ArcTan[Sqrt[e*(c + d*x)]/Sqrt[e]], 1/2]
)/(3*d*e^(5/2)*Sqrt[1 + (c + d*x)^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\sinh^{-1}(c+dx)}{(ce+dex)^{5/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{a+b\sinh^{-1}(x)}{(ex)^{5/2}}\,dx, x, c+dx\right)}{d}\\
&= -\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{3de(e(c+dx))^{3/2}}+\frac{(2b)\,\text{Subst}\left(\int \frac{1}{(ex)^{3/2}\sqrt{1+x^2}}\,dx, x, c+dx\right)}{3de}\\
&= -\frac{4b\sqrt{1+(c+dx)^2}}{3de^2\sqrt{e(c+dx)}}-\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{3de(e(c+dx))^{3/2}}+\frac{(2b)\,\text{Subst}\left(\int \frac{\sqrt{ex}}{\sqrt{1+x^2}}\,dx, x, c+\ldots\right)}{3de^3}\\
&= -\frac{4b\sqrt{1+(c+dx)^2}}{3de^2\sqrt{e(c+dx)}}-\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{3de(e(c+dx))^{3/2}}+\frac{(4b)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^4}{e^2}}}\,dx, x, \sqrt{e\ldots}\right)}{3de^4}\\
&= -\frac{4b\sqrt{1+(c+dx)^2}}{3de^2\sqrt{e(c+dx)}}-\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{3de(e(c+dx))^{3/2}}+\frac{(4b)\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{e^2}}}\,dx, x, \sqrt{e\ldots}\right)}{3de^3}\\
&= -\frac{4b\sqrt{1+(c+dx)^2}}{3de^2\sqrt{e(c+dx)}}+\frac{4b\sqrt{e(c+dx)}\sqrt{1+(c+dx)^2}}{3de^3(1+c+dx)}-\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{3de(e(c+dx))^{3/2}}-\ldots
\end{aligned}
$$

Mathematica [C] time = 0.03, size = 58, normalized size = 0.22

$$
-\frac{2\left(a+2b(c+dx)\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-(c+dx)^2\right)+b\sinh^{-1}(c+dx)\right)}{3de(e(c+dx))^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^(5/2),x]
```

```
[Out] (-2*(a + b*ArcSinh[c + d*x] + 2*b*(c + d*x)*Hypergeometric2F1[-1/4, 1/2, 3/
4, -(c + d*x)^2]))/(3*d*e*(e*(c + d*x))^(3/2))
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{dex+ce}\,(b\operatorname{arsinh}(dx+c)+a)}{d^3e^3x^3+3\,cd^2e^3x^2+3\,c^2de^3x+c^3e^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*e*x + c*e)*(b*arcsinh(d*x + c) + a)/(d^3*e^3*x^3 + 3*c*d^2*
e^3*x^2 + 3*c^2*d*e^3*x + c^3*e^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\operatorname{arsinh}(dx+c)+a}{(dex+ce)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)/(d*e*x + c*e)^(5/2), x)
```

maple [C] time = 0.01, size = 202, normalized size = 0.76

$$\frac{-\frac{2a}{3(dex+ce)^{\frac{3}{2}}}+2b\left(-\frac{\operatorname{arcsinh}\left(\frac{dex+ce}{e}\right)}{3(dex+ce)^{\frac{3}{2}}}+\frac{-\frac{2\sqrt{\frac{(dex+ce)^2}{e^2}+1}}{3\sqrt{dex+ce}}+\frac{2i\sqrt{1-\frac{i(dex+ce)}{e}}\,\sqrt{1+\frac{i(dex+ce)}{e}}\,\left(\operatorname{EllipticF}\left(\sqrt{dex+ce}\,\sqrt{\frac{i}{e}},i\right)-\operatorname{EllipticE}\left(\sqrt{dex+ce}\,\sqrt{\frac{i}{e}},i\right)\right)}{3e\sqrt{\frac{i}{e}}\,\sqrt{\frac{(dex+ce)^2}{e^2}+1}}}{e}\right)}{de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(5/2),x)
```

```
[Out] 2/d/e*(-1/3*a/(d*e*x+c*e)^(3/2)+b*(-1/3/(d*e*x+c*e)^(3/2)*arcsinh((d*e*x+c*
e)/e)+2/3/e*(-((d*e*x+c*e)^2/e^2+1)^(1/2)/(d*e*x+c*e)^(1/2)+I/e/(I/e)^(1/2)
*(1-I/e*(d*e*x+c*e))^(1/2)*(1+I/e*(d*e*x+c*e))^(1/2)/((d*e*x+c*e)^2/e^2+1)^
(1/2)*(EllipticF((d*e*x+c*e)^(1/2)*(I/e)^(1/2),I)-EllipticE((d*e*x+c*e)^(1/
2)*(I/e)^(1/2),I)))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{6}\left(12\sqrt{e}\int \frac{1}{3\left(d^4e^3x^4+4\,cd^3e^3x^3+c^4e^3+c^2e^3+\left(6\,c^2d^2e^3+d^2e^3\right)x^2+2\left(2\,c^3de^3+cde^3\right)x+\left(d^3e^3x^3+3\,cd^2e^3x^2\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*(12*sqrt(e)*integrate(1/3/((d^4*e^3*x^4 + 4*c*d^3*e^3*x^3 + c^4*e^3 + c
^2*e^3 + (6*c^2*d^2*e^3 + d^2*e^3)*x^2 + 2*(2*c^3*d*e^3 + c*d*e^3)*x + (d^3
*e^3*x^3 + 3*c*d^2*e^3*x^2 + c^3*e^3 + c*e^3 + (3*c^2*d*e^3 + d*e^3)*x)*sqr
t(d^2*x^2 + 2*c*d*x + c^2 + 1))*sqrt(d*x + c)), x) - sqrt(e)*(I*sqrt(2)*(lo
g(1/2*I*sqrt(2)*(sqrt(2) + 2*sqrt(d*x + c)) + 1) - log(-1/2*I*sqrt(2)*(sqrt
(2) + 2*sqrt(d*x + c)) + 1))/e^3 - I*sqrt(2)*(log(1/2*I*sqrt(2)*(sqrt(2) -
2*sqrt(d*x + c)) + 1) - log(-1/2*I*sqrt(2)*(sqrt(2) - 2*sqrt(d*x + c)) + 1)
)/e^3 - sqrt(2)*log(d*x + sqrt(2)*sqrt(d*x + c) + c + 1)/e^3 + sqrt(2)*log(
d*x - sqrt(2)*sqrt(d*x + c) + c + 1)/e^3)/d - 4*sqrt(e)*log(d*x + c + sqrt(
d^2*x^2 + 2*c*d*x + c^2 + 1))/((d^2*e^3*x + c*d*e^3)*sqrt(d*x + c)))*b - 2/
3*a/((d*e*x + c*e)^(3/2)*d*e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \operatorname{asinh}(c + d\,x)}{(c\,e + d\,e\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))/(c*e + d*e*x)^(5/2),x)
```

```
[Out] int((a + b*asinh(c + d*x))/(c*e + d*e*x)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{asinh}(c + dx)}{(e(c + dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))/(d*e*x+c*e)**(5/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))/(e*(c + d*x))**(5/2), x)
```

3.235 $\int \frac{a+b\sinh^{-1}(c+dx)}{(ce+dex)^{7/2}}\,dx$

Optimal. Leaf size=145

$$-\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{5de(e(c+dx))^{5/2}}-\frac{2b(c+dx+1)\sqrt{\frac{(c+dx)^2+1}{(c+dx+1)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right)|\frac{1}{2}\right)}{15de^{7/2}\sqrt{(c+dx)^2+1}}-\frac{4b\sqrt{(c+dx)^2+1}}{15de^2(e(c+dx))^{3/2}}$$

```
[Out] -2/5*(a+b*arcsinh(d*x+c))/d/e/(e*(d*x+c))^(5/2)-4/15*b*(1+(d*x+c)^2)^(1/2)/
d/e^2/(e*(d*x+c))^(3/2)-2/15*b*(d*x+c+1)*(cos(2*arctan((e*(d*x+c))^(1/2)/e^
(1/2)))^2)^(1/2)/cos(2*arctan((e*(d*x+c))^(1/2)/e^(1/2)))*EllipticF(sin(2*a
rctan((e*(d*x+c))^(1/2)/e^(1/2))),1/2*2^(1/2))*((1+(d*x+c)^2)/(d*x+c+1)^2)^
(1/2)/d/e^(7/2)/(1+(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {5865, 5661, 325, 329, 220}

$$-\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{5de(e(c+dx))^{5/2}}-\frac{4b\sqrt{(c+dx)^2+1}}{15de^2(e(c+dx))^{3/2}}-\frac{2b(c+dx+1)\sqrt{\frac{(c+dx)^2+1}{(c+dx+1)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right)|\frac{1}{2}\right)}{15de^{7/2}\sqrt{(c+dx)^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^(7/2),x]
```

```
[Out] (-4*b*Sqrt[1 + (c + d*x)^2])/(15*d*e^2*(e*(c + d*x))^(3/2)) - (2*(a + b*Arc
Sinh[c + d*x]))/(5*d*e*(e*(c + d*x))^(5/2)) - (2*b*(1 + c + d*x)*Sqrt[(1 +
(c + d*x)^2)/(1 + c + d*x)^2]*EllipticF[2*ArcTan[Sqrt[e*(c + d*x)]/Sqrt[e]]
, 1/2])/(15*d*e^(7/2)*Sqrt[1 + (c + d*x)^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(c+dx)}{(ce+dex)^{7/2}}\,dx = \frac{\text{Subst}\left(\int \frac{a+b\sinh^{-1}(x)}{(ex)^{7/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{5de(e(c+dx))^{5/2}} + \frac{(2b)\,\text{Subst}\left(\int \frac{1}{(ex)^{5/2}\sqrt{1+x^2}}\,dx, x, c+dx\right)}{5de}$$

$$= -\frac{4b\sqrt{1+(c+dx)^2}}{15de^2(e(c+dx))^{3/2}} - \frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{5de(e(c+dx))^{5/2}} - \frac{(2b)\,\text{Subst}\left(\int \frac{1}{\sqrt{ex}\sqrt{1+x^2}}\,dx, x,\right.}{15de^3}$$

$$= -\frac{4b\sqrt{1+(c+dx)^2}}{15de^2(e(c+dx))^{3/2}} - \frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{5de(e(c+dx))^{5/2}} - \frac{(4b)\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{e^2}}}\,dx, x, \sqrt{e\right.}}{15de^4}$$

$$= -\frac{4b\sqrt{1+(c+dx)^2}}{15de^2(e(c+dx))^{3/2}} - \frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{5de(e(c+dx))^{5/2}} - \frac{2b(1+c+dx)\sqrt{\frac{1+(c+dx)^2}{(1+c+dx)^2}}\,F\left(2\right.}{15de^{7/2}\sqrt{1+(c}$$

Mathematica [C] time = 0.04, size = 61, normalized size = 0.42

$$\frac{-6\left(a+b\sinh^{-1}(c+dx)\right) - 4b(c+dx)\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{1}{4};-(c+dx)^2\right)}{15de(e(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^(7/2),x]
```

```
[Out] (-6*(a + b*ArcSinh[c + d*x]) - 4*b*(c + d*x)*Hypergeometric2F1[-3/4, 1/2, 1
/4, -(c + d*x)^2])/(15*d*e*(e*(c + d*x))^(5/2))
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{dex+ce}\,(b\,\text{arsinh}\,(dx+c)+a)}{d^4e^4x^4+4\,cd^3e^4x^3+6\,c^2d^2e^4x^2+4\,c^3de^4x+c^4e^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*e*x + c*e)*(b*arcsinh(d*x + c) + a)/(d^4*e^4*x^4 + 4*c*d^3*
e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^3*d*e^4*x + c^4*e^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{arsinh}\,(dx+c)+a}{(dex+ce)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)/(d*e*x + c*e)^(7/2), x)
```

maple [C] time = 0.02, size = 176, normalized size = 1.21

$$\frac{-\frac{2a}{5(dex+ce)^{\frac{5}{2}}}+2b\left(-\frac{\operatorname{arcsinh}\left(\frac{dex+ce}{e}\right)}{5(dex+ce)^{\frac{5}{2}}}+\frac{-\frac{2\sqrt{\frac{(dex+ce)^2}{e^2}+1}}{15(dex+ce)^{\frac{3}{2}}}-\frac{2\sqrt{1-\frac{i(dex+ce)}{e}}\,\sqrt{1+\frac{i(dex+ce)}{e}}\,\operatorname{EllipticF}\left(\sqrt{dex+ce}\,\sqrt{\frac{i}{e}},i\right)}{15e^2\sqrt{\frac{i}{e}}\,\sqrt{\frac{(dex+ce)^2}{e^2}+1}}}{e}\right)}{de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(7/2),x)
```

```
[Out] 2/d/e*(-1/5*a/(d*e*x+c*e)^(5/2)+b*(-1/5/(d*e*x+c*e)^(5/2)*arcsinh((d*e*x+c*
e)/e)+2/5/e*(-1/3*((d*e*x+c*e)^2/e^2+1)^(1/2)/(d*e*x+c*e)^(3/2)-1/3/e^2/(I/
e)^(1/2)*(1-I/e*(d*e*x+c*e))^(1/2)*(1+I/e*(d*e*x+c*e))^(1/2)/((d*e*x+c*e)^2
/e^2+1)^(1/2)*EllipticF((d*e*x+c*e)^(1/2)*(I/e)^(1/2),I))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{10}\left(20\sqrt{e}\int\frac{}{5\left(d^5e^4x^5+5\,cd^4e^4x^4+c^5e^4+c^3e^4+\left(10\,c^2d^3e^4+d^3e^4\right)x^3+\left(10\,c^3d^2e^4+3\,cd^2e^4\right)x^2+\left(5\,c^4de^4+3\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/10*(20*sqrt(e)*integrate(1/5/((d^5*e^4*x^5 + 5*c*d^4*e^4*x^4 + c^5*e^4 +
c^3*e^4 + (10*c^2*d^3*e^4 + d^3*e^4)*x^3 + (10*c^3*d^2*e^4 + 3*c*d^2*e^4)*x
^2 + (5*c^4*d*e^4 + 3*c^2*d*e^4)*x + (d^4*e^4*x^4 + 4*c*d^3*e^4*x^3 + c^4*e
^4 + c^2*e^4 + (6*c^2*d^2*e^4 + d^2*e^4)*x^2 + 2*(2*c^3*d*e^4 + c*d*e^4)*x)
*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*sqrt(d*x + c)), x) + sqrt(e)*((I*sqrt(2
)*(log(1/2*I*sqrt(2)*(sqrt(2) + 2*sqrt(d*x + c)) + 1) - log(-1/2*I*sqrt(2)*
(sqrt(2) + 2*sqrt(d*x + c)) + 1)) - I*sqrt(2)*(log(1/2*I*sqrt(2)*(sqrt(2) -
 2*sqrt(d*x + c)) + 1) - log(-1/2*I*sqrt(2)*(sqrt(2) - 2*sqrt(d*x + c)) + 1
)) + sqrt(2)*log(d*x + sqrt(2)*sqrt(d*x + c) + c + 1) - sqrt(2)*log(d*x - s
qrt(2)*sqrt(d*x + c) + c + 1))/e^4 - 8/(sqrt(d*x + c)*e^4))/d - 4*sqrt(e)*l
og(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/((d^3*e^4*x^2 + 2*c*d^2*e^4
*x + c^2*d*e^4)*sqrt(d*x + c)))*b - 2/5*a/((d*e*x + c*e)^(5/2)*d*e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{asinh}\left(c+d\,x\right)}{{\left(c\,e+d\,e\,x\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))/(c*e + d*e*x)^(7/2),x)
```

```
[Out] int((a + b*asinh(c + d*x))/(c*e + d*e*x)^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))/(d*e*x+c*e)**(7/2),x)
```

```
[Out] Timed out
```

3.236 $\int (ce+dex)^{7/2}\left(a+b\sinh^{-1}(c+dx)\right)^2\,dx$

Optimal. Leaf size=134

$$\frac{16b^2(e(c+dx))^{13/2}\,{}_3F_2\left(1,\frac{13}{4},\frac{13}{4};\frac{15}{4},\frac{17}{4};-(c+dx)^2\right)}{1287de^3}-\frac{8b(e(c+dx))^{11/2}\,{}_2F_1\left(\frac{1}{2},\frac{11}{4};\frac{15}{4};-(c+dx)^2\right)\left(a+b\sinh^{-1}(c+\right.}{99de^2}$$

```
[Out] 2/9*(e*(d*x+c))^(9/2)*(a+b*arcsinh(d*x+c))^2/d/e-8/99*b*(e*(d*x+c))^(11/2)*
(a+b*arcsinh(d*x+c))*hypergeom([1/2, 11/4],[15/4],-(d*x+c)^2)/d/e^2+16/1287
*b^2*(e*(d*x+c))^(13/2)*HypergeometricPFQ([1, 13/4, 13/4],[15/4, 17/4],-(d*
x+c)^2)/d/e^3
```

Rubi [A] time = 0.21, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {5865, 5661, 5762}

$$\frac{16b^2(e(c+dx))^{13/2}\,{}_3F_2\left(1,\frac{13}{4},\frac{13}{4};\frac{15}{4},\frac{17}{4};-(c+dx)^2\right)}{1287de^3}-\frac{8b(e(c+dx))^{11/2}\,{}_2F_1\left(\frac{1}{2},\frac{11}{4};\frac{15}{4};-(c+dx)^2\right)\left(a+b\sinh^{-1}(c+\right.}{99de^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^(7/2)*(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] (2*(e*(c + d*x))^(9/2)*(a + b*ArcSinh[c + d*x])^2)/(9*d*e) - (8*b*(e*(c + d
*x))^(11/2)*(a + b*ArcSinh[c + d*x])*Hypergeometric2F1[1/2, 11/4, 15/4, -(c
 + d*x)^2])/(99*d*e^2) + (16*b^2*(e*(c + d*x))^(13/2)*HypergeometricPFQ[{1,
 13/4, 13/4}, {15/4, 17/4}, -(c + d*x)^2])/(1287*d*e^3)
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5762

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*ArcSinh[c*x])*Hypergeom
etric2F1[1/2, (1 + m)/2, (3 + m)/2, -(c^2*x^2)])/(Sqrt[d]*f*(m + 1)), x] -
Simp[(b*c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2
, 2 + m/2}, -(c^2*x^2)])/(Sqrt[d]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b,
c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[d, 0] &&  !IntegerQ[m]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^{7/2}\left(a+b\sinh^{-1}(c+dx)\right)^2\,dx=\frac{\text{Subst}\left(\int (ex)^{7/2}\left(a+b\sinh^{-1}(x)\right)^2\,dx,x,c+dx\right)}{d}$$

$$=\frac{2(e(c+dx))^{9/2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{9de}-\frac{(4b)\,\text{Subst}\left(\int \frac{(ex)^{9/2}(a+b\text{s}}{\sqrt{1+}}\right.}{9d}$$

$$=\frac{2(e(c+dx))^{9/2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{9de}-\frac{8b(e(c+dx))^{11/2}\left(a+b\text{s}\right.}{}$$

Mathematica [A] time = 0.13, size = 110, normalized size = 0.82

$$\frac{2(e(c+dx))^{9/2}\left(8b^2(c+dx)^2\,{}_3F_2\left(1,\frac{13}{4},\frac{13}{4};\frac{15}{4},\frac{17}{4};-(c+dx)^2\right)-52b(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{11}{4};\frac{15}{4};-(c+dx)^2\right)\left(a+b\,\text{si}\right.\right.}{1287de}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^(7/2)*(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] (2*(e*(c + d*x))^(9/2)*(143*(a + b*ArcSinh[c + d*x])^2 - 52*b*(c + d*x)*(a
+ b*ArcSinh[c + d*x])*Hypergeometric2F1[1/2, 11/4, 15/4, -(c + d*x)^2] + 8*
b^2*(c + d*x)^2*HypergeometricPFQ[{1, 13/4, 13/4}, {15/4, 17/4}, -(c + d*x)
^2]))/(1287*d*e)
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^2d^3e^3x^3+3\,a^2cd^2e^3x^2+3\,a^2c^2de^3x+a^2c^3e^3+\left(b^2d^3e^3x^3+3\,b^2cd^2e^3x^2+3\,b^2c^2de^3x+b^2c^3e^3\right)\text{arsi}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*d^3*e^3*x^3 + 3*a^2*c*d^2*e^3*x^2 + 3*a^2*c^2*d*e^3*x + a^2*c
^3*e^3 + (b^2*d^3*e^3*x^3 + 3*b^2*c*d^2*e^3*x^2 + 3*b^2*c^2*d*e^3*x + b^2*c
^3*e^3)*arcsinh(d*x + c)^2 + 2*(a*b*d^3*e^3*x^3 + 3*a*b*c*d^2*e^3*x^2 + 3*a
*b*c^2*d*e^3*x + a*b*c^3*e^3)*arcsinh(d*x + c))*sqrt(d*e*x + c*e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex+ce)^{\frac{7}{2}}(b\,\text{arsinh}\,(dx+c)+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^(7/2)*(b*arcsinh(d*x + c) + a)^2, x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int (dex+ce)^{\frac{7}{2}}\left(a+b\,\text{arcsinh}\,(dx+c)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^2,x)
```

```
[Out] int((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\,(dex+ce)^{\frac{9}{2}}a^2}{9\,de}+\frac{2\left(b^2d^4e^{\frac{7}{2}}x^4+4\,b^2cd^3e^{\frac{7}{2}}x^3+6\,b^2c^2d^2e^{\frac{7}{2}}x^2+4\,b^2c^3de^{\frac{7}{2}}x+b^2c^4e^{\frac{7}{2}}\right)\sqrt{dx+c}\,\log\left(dx+c+\sqrt{d^2x^2}\right.}{9\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 2/9*(d*e*x + c*e)^(9/2)*a^2/(d*e) + 2/9*(b^2*d^4*e^(7/2)*x^4 + 4*b^2*c*d^3*
e^(7/2)*x^3 + 6*b^2*c^2*d^2*e^(7/2)*x^2 + 4*b^2*c^3*d*e^(7/2)*x + b^2*c^4*e
^(7/2))*sqrt(d*x + c)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2/d
+ integrate(-2/9*((2*b^2*c^5*e^(7/2) - (9*a*b*d^5*e^(7/2) - 2*b^2*d^5*e^(7/
2))*x^5 - 5*(9*a*b*c*d^4*e^(7/2) - 2*b^2*c*d^4*e^(7/2))*x^4 + (20*b^2*c^2*d
^3*e^(7/2) - 9*(10*c^2*d^3*e^(7/2) + d^3*e^(7/2))*a*b)*x^3 - 9*(c^5*e^(7/2)
 + c^3*e^(7/2))*a*b + (20*b^2*c^3*d^2*e^(7/2) - 9*(10*c^3*d^2*e^(7/2) + 3*c
*d^2*e^(7/2))*a*b)*x^2 + (10*b^2*c^4*d*e^(7/2) - 9*(5*c^4*d*e^(7/2) + 3*c^2
*d*e^(7/2))*a*b)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*sqrt(d*x + c) - ((9*a
*b*d^6*e^(7/2) - 2*b^2*d^6*e^(7/2))*x^6 + 6*(9*a*b*c*d^5*e^(7/2) - 2*b^2*c*
d^5*e^(7/2))*x^5 + (9*(15*c^2*d^4*e^(7/2) + d^4*e^(7/2))*a*b - 2*(15*c^2*d^
4*e^(7/2) + d^4*e^(7/2))*b^2)*x^4 + 4*(9*(5*c^3*d^3*e^(7/2) + c*d^3*e^(7/2)
)*a*b - 2*(5*c^3*d^3*e^(7/2) + c*d^3*e^(7/2))*b^2)*x^3 + 9*(c^6*e^(7/2) + c
^4*e^(7/2))*a*b - 2*(c^6*e^(7/2) + c^4*e^(7/2))*b^2 + 3*(9*(5*c^4*d^2*e^(7/
2) + 2*c^2*d^2*e^(7/2))*a*b - 2*(5*c^4*d^2*e^(7/2) + 2*c^2*d^2*e^(7/2))*b^2
)*x^2 + 2*(9*(3*c^5*d*e^(7/2) + 2*c^3*d*e^(7/2))*a*b - 2*(3*c^5*d*e^(7/2) +
 2*c^3*d*e^(7/2))*b^2)*x)*sqrt(d*x + c))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d
*x + c^2 + 1))/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (3*c^2*d + d)*x + (d^2*x^2 +
2*c*d*x + c^2 + 1)^(3/2) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e+d\,e\,x)^{7/2}\,(a+b\,\mathrm{asinh}\,(c+d\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^(7/2)*(a + b*asinh(c + d*x))^2,x)
```

```
[Out] int((c*e + d*e*x)^(7/2)*(a + b*asinh(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(7/2)*(a+b*asinh(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.237 $\int (ce+dex)^{5/2} \left(a+b\sinh^{-1}(c+dx)\right)^2\,dx$

Optimal. Leaf size=134

$$\frac{16b^2(e(c+dx))^{11/2}\,{}_3F_2\left(1,\frac{11}{4},\frac{11}{4};\frac{13}{4},\frac{15}{4};-(c+dx)^2\right)}{693de^3}-\frac{8b(e(c+dx))^{9/2}\,{}_2F_1\left(\frac{1}{2},\frac{9}{4};\frac{13}{4};-(c+dx)^2\right)\left(a+b\sinh^{-1}(c\right.}{63de^2}$$

```
[Out] 2/7*(e*(d*x+c))^(7/2)*(a+b*arcsinh(d*x+c))^2/d/e-8/63*b*(e*(d*x+c))^(9/2)*(
a+b*arcsinh(d*x+c))*hypergeom([1/2, 9/4],[13/4],-(d*x+c)^2)/d/e^2+16/693*b^
2*(e*(d*x+c))^(11/2)*HypergeometricPFQ([1, 11/4, 11/4],[13/4, 15/4],-(d*x+c
)^2)/d/e^3
```

Rubi [A] time = 0.22, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {5865, 5661, 5762}

$$\frac{16b^2(e(c+dx))^{11/2}\,{}_3F_2\left(1,\frac{11}{4},\frac{11}{4};\frac{13}{4},\frac{15}{4};-(c+dx)^2\right)}{693de^3}-\frac{8b(e(c+dx))^{9/2}\,{}_2F_1\left(\frac{1}{2},\frac{9}{4};\frac{13}{4};-(c+dx)^2\right)\left(a+b\sinh^{-1}(c\right.}{63de^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^(5/2)*(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] (2*(e*(c + d*x))^(7/2)*(a + b*ArcSinh[c + d*x])^2)/(7*d*e) - (8*b*(e*(c + d
*x))^(9/2)*(a + b*ArcSinh[c + d*x])*Hypergeometric2F1[1/2, 9/4, 13/4, -(c +
 d*x)^2])/(63*d*e^2) + (16*b^2*(e*(c + d*x))^(11/2)*HypergeometricPFQ[{1, 1
1/4, 11/4}, {13/4, 15/4}, -(c + d*x)^2])/(693*d*e^3)
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5762

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*ArcSinh[c*x])*Hypergeom
etric2F1[1/2, (1 + m)/2, (3 + m)/2, -(c^2*x^2)])/(Sqrt[d]*f*(m + 1)), x] -
Simp[(b*c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2
, 2 + m/2}, -(c^2*x^2)])/(Sqrt[d]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b,
c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[d, 0] &&  !IntegerQ[m]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^{5/2}\left(a+b\sinh^{-1}(c+dx)\right)^2\,dx=\frac{\text{Subst}\left(\int (ex)^{5/2}\left(a+b\sinh^{-1}(x)\right)^2\,dx,x,c+dx\right)}{d}$$

$$=\frac{2(e(c+dx))^{7/2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{7de}-\frac{(4b)\,\text{Subst}\left(\int \frac{(ex)^{7/2}(a+b\sinh}{\sqrt{1+x^2}}\right.}{7de}$$

$$=\frac{2(e(c+dx))^{7/2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{7de}-\frac{8b(e(c+dx))^{9/2}\left(a+b\sinh\right.}{}$$

Mathematica [A] time = 0.12, size = 110, normalized size = 0.82

$$\frac{2(e(c+dx))^{7/2}\left(8b^2(c+dx)^2\,{}_3F_2\left(1,\frac{11}{4},\frac{11}{4};\frac{13}{4},\frac{15}{4};-(c+dx)^2\right)-44b(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{9}{4};\frac{13}{4};-(c+dx)^2\right)\left(a+b\sinh^{-}\right.\right.}{693de}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^(5/2)*(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] (2*(e*(c + d*x))^(7/2)*(99*(a + b*ArcSinh[c + d*x])^2 - 44*b*(c + d*x)*(a +
 b*ArcSinh[c + d*x])*Hypergeometric2F1[1/2, 9/4, 13/4, -(c + d*x)^2] + 8*b^
2*(c + d*x)^2*HypergeometricPFQ[{1, 11/4, 11/4}, {13/4, 15/4}, -(c + d*x)^2
]))/(693*d*e)
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^2d^2e^2x^2+2\,a^2cde^2x+a^2c^2e^2+\left(b^2d^2e^2x^2+2\,b^2cde^2x+b^2c^2e^2\right)\text{arsinh}\,(dx+c)^2+2\left(abd^2e^2x^2+2\,ab\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*d^2*e^2*x^2 + 2*a^2*c*d*e^2*x + a^2*c^2*e^2 + (b^2*d^2*e^2*x^
2 + 2*b^2*c*d*e^2*x + b^2*c^2*e^2)*arcsinh(d*x + c)^2 + 2*(a*b*d^2*e^2*x^2
+ 2*a*b*c*d*e^2*x + a*b*c^2*e^2)*arcsinh(d*x + c))*sqrt(d*e*x + c*e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex+ce)^{\frac{5}{2}}(b\,\text{arsinh}\,(dx+c)+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^(5/2)*(b*arcsinh(d*x + c) + a)^2, x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int (dex+ce)^{\frac{5}{2}}\,(a+b\,\text{arcsinh}\,(dx+c))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^2,x)
```

```
[Out] int((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\,(dex+ce)^{\frac{7}{2}}a^2}{7\,de}+\frac{2\left(b^2d^3e^{\frac{5}{2}}x^3+3\,b^2cd^2e^{\frac{5}{2}}x^2+3\,b^2c^2de^{\frac{5}{2}}x+b^2c^3e^{\frac{5}{2}}\right)\sqrt{dx+c}\,\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2}\right.}{7\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 2/7*(d*e*x + c*e)^(7/2)*a^2/(d*e) + 2/7*(b^2*d^3*e^(5/2)*x^3 + 3*b^2*c*d^2*
e^(5/2)*x^2 + 3*b^2*c^2*d*e^(5/2)*x + b^2*c^3*e^(5/2))*sqrt(d*x + c)*log(d*
x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2/d + integrate(-2/7*((2*b^2*c^4
*e^(5/2) - (7*a*b*d^4*e^(5/2) - 2*b^2*d^4*e^(5/2))*x^4 - 4*(7*a*b*c*d^3*e^(
5/2) - 2*b^2*c*d^3*e^(5/2))*x^3 - 7*(c^4*e^(5/2) + c^2*e^(5/2))*a*b + (12*b
^2*c^2*d^2*e^(5/2) - 7*(6*c^2*d^2*e^(5/2) + d^2*e^(5/2))*a*b)*x^2 + 2*(4*b^
2*c^3*d*e^(5/2) - 7*(2*c^3*d*e^(5/2) + c*d*e^(5/2))*a*b)*x)*sqrt(d^2*x^2 +
2*c*d*x + c^2 + 1)*sqrt(d*x + c) - ((7*a*b*d^5*e^(5/2) - 2*b^2*d^5*e^(5/2))
*x^5 + 5*(7*a*b*c*d^4*e^(5/2) - 2*b^2*c*d^4*e^(5/2))*x^4 + (7*(10*c^2*d^3*e
^(5/2) + d^3*e^(5/2))*a*b - 2*(10*c^2*d^3*e^(5/2) + d^3*e^(5/2))*b^2)*x^3 +
 7*(c^5*e^(5/2) + c^3*e^(5/2))*a*b - 2*(c^5*e^(5/2) + c^3*e^(5/2))*b^2 + (7
*(10*c^3*d^2*e^(5/2) + 3*c*d^2*e^(5/2))*a*b - 2*(10*c^3*d^2*e^(5/2) + 3*c*d
^2*e^(5/2))*b^2)*x^2 + (7*(5*c^4*d*e^(5/2) + 3*c^2*d*e^(5/2))*a*b - 2*(5*c^
4*d*e^(5/2) + 3*c^2*d*e^(5/2))*b^2)*x)*sqrt(d*x + c))*log(d*x + c + sqrt(d^
2*x^2 + 2*c*d*x + c^2 + 1))/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (3*c^2*d + d)*x
+ (d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e+d\,e\,x)^{5/2}\,(a+b\,\mathrm{asinh}\,(c+d\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^(5/2)*(a + b*asinh(c + d*x))^2,x)
```

```
[Out] int((c*e + d*e*x)^(5/2)*(a + b*asinh(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(5/2)*(a+b*asinh(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.238 $\int (ce+dex)^{3/2} \left(a+b\sinh^{-1}(c+dx)\right)^2 \, dx$

Optimal. Leaf size=134

$$\frac{16b^2(e(c+dx))^{9/2}\,{}_3F_2\left(1,\frac{9}{4},\frac{9}{4};\frac{11}{4},\frac{13}{4};-(c+dx)^2\right)}{315de^3}-\frac{8b(e(c+dx))^{7/2}\,{}_2F_1\left(\frac{1}{2},\frac{7}{4};\frac{11}{4};-(c+dx)^2\right)\left(a+b\sinh^{-1}(c+dx)\right)}{35de^2}$$

```
[Out] 2/5*(e*(d*x+c))^(5/2)*(a+b*arcsinh(d*x+c))^2/d/e-8/35*b*(e*(d*x+c))^(7/2)*(
a+b*arcsinh(d*x+c))*hypergeom([1/2, 7/4],[11/4],-(d*x+c)^2)/d/e^2+16/315*b^
2*(e*(d*x+c))^(9/2)*HypergeometricPFQ([1, 9/4, 9/4],[11/4, 13/4],-(d*x+c)^2
)/d/e^3
```

Rubi [A] time = 0.22, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {5865, 5661, 5762}

$$\frac{16b^2(e(c+dx))^{9/2}\,{}_3F_2\left(1,\frac{9}{4},\frac{9}{4};\frac{11}{4},\frac{13}{4};-(c+dx)^2\right)}{315de^3}-\frac{8b(e(c+dx))^{7/2}\,{}_2F_1\left(\frac{1}{2},\frac{7}{4};\frac{11}{4};-(c+dx)^2\right)\left(a+b\sinh^{-1}(c+dx)\right)}{35de^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^(3/2)*(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] (2*(e*(c + d*x))^(5/2)*(a + b*ArcSinh[c + d*x])^2)/(5*d*e) - (8*b*(e*(c + d
*x))^(7/2)*(a + b*ArcSinh[c + d*x])*Hypergeometric2F1[1/2, 7/4, 11/4, -(c +
 d*x)^2])/(35*d*e^2) + (16*b^2*(e*(c + d*x))^(9/2)*HypergeometricPFQ[{1, 9/
4, 9/4}, {11/4, 13/4}, -(c + d*x)^2])/(315*d*e^3)
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5762

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*ArcSinh[c*x])*Hypergeom
etric2F1[1/2, (1 + m)/2, (3 + m)/2, -(c^2*x^2)])/(Sqrt[d]*f*(m + 1)), x] -
Simp[(b*c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2
, 2 + m/2}, -(c^2*x^2)])/(Sqrt[d]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b,
c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[d, 0] &&  !IntegerQ[m]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^{3/2}\left(a+b\sinh^{-1}(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int (ex)^{3/2}\left(a+b\sinh^{-1}(x)\right)^2\,dx, x, c+dx\right)}{d}$$

$$= \frac{2(e(c+dx))^{5/2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{5de} - \frac{(4b)\,\text{Subst}\left(\int \frac{(ex)^{5/2}(a+b\,\text{s}}{\sqrt{1+}}\right.}{5d}$$

$$= \frac{2(e(c+dx))^{5/2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{5de} - \frac{8b(e(c+dx))^{7/2}\left(a+b\,\text{s}\right.}{}$$

Mathematica [A] time = 0.10, size = 110, normalized size = 0.82

$$\frac{2(e(c+dx))^{5/2}\left(8b^2(c+dx)^2\,{}_3F_2\left(1,\frac{9}{4},\frac{9}{4};\frac{11}{4},\frac{13}{4};-(c+dx)^2\right)-36b(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{7}{4};\frac{11}{4};-(c+dx)^2\right)\left(a+b\,\text{sinh}\right.\right.}{315de}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^(3/2)*(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] (2*(e*(c + d*x))^(5/2)*(63*(a + b*ArcSinh[c + d*x])^2 - 36*b*(c + d*x)*(a +
 b*ArcSinh[c + d*x])*Hypergeometric2F1[1/2, 7/4, 11/4, -(c + d*x)^2] + 8*b^
2*(c + d*x)^2*HypergeometricPFQ[{1, 9/4, 9/4}, {11/4, 13/4}, -(c + d*x)^2])
)/(315*d*e)
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^2dex+a^2ce+\left(b^2dex+b^2ce\right)\text{arsinh}\left(dx+c\right)^2+2\left(abdex+abce\right)\text{arsinh}\left(dx+c\right)\right)\sqrt{dex+ce}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*d*e*x + a^2*c*e + (b^2*d*e*x + b^2*c*e)*arcsinh(d*x + c)^2 +
2*(a*b*d*e*x + a*b*c*e)*arcsinh(d*x + c))*sqrt(d*e*x + c*e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex+ce)^{\frac{3}{2}}(b\,\text{arsinh}\left(dx+c\right)+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^(3/2)*(b*arcsinh(d*x + c) + a)^2, x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int (dex+ce)^{\frac{3}{2}}\left(a+b\,\text{arcsinh}\left(dx+c\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^2,x)
```

```
[Out] int((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\,(dex+ce)^{\frac{5}{2}}a^2}{5\,de}+\frac{2\left(b^2d^2e^{\frac{3}{2}}x^2+2\,b^2cde^{\frac{3}{2}}x+b^2c^2e^{\frac{3}{2}}\right)\sqrt{dx+c}\,\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)^2}{5\,d}+\int-\frac{2\left(\left(2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 2/5*(d*e*x + c*e)^(5/2)*a^2/(d*e) + 2/5*(b^2*d^2*e^(3/2)*x^2 + 2*b^2*c*d*e^
(3/2)*x + b^2*c^2*e^(3/2))*sqrt(d*x + c)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d
*x + c^2 + 1))^2/d + integrate(-2/5*((2*b^2*c^3*e^(3/2) - (5*a*b*d^3*e^(3/2
) - 2*b^2*d^3*e^(3/2))*x^3 - 5*(c^3*e^(3/2) + c*e^(3/2))*a*b - 3*(5*a*b*c*d
^2*e^(3/2) - 2*b^2*c*d^2*e^(3/2))*x^2 + (6*b^2*c^2*d*e^(3/2) - 5*(3*c^2*d*e
^(3/2) + d*e^(3/2))*a*b)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*sqrt(d*x + c)
 - ((5*a*b*d^4*e^(3/2) - 2*b^2*d^4*e^(3/2))*x^4 + 4*(5*a*b*c*d^3*e^(3/2) -
2*b^2*c*d^3*e^(3/2))*x^3 + 5*(c^4*e^(3/2) + c^2*e^(3/2))*a*b - 2*(c^4*e^(3/
2) + c^2*e^(3/2))*b^2 + (5*(6*c^2*d^2*e^(3/2) + d^2*e^(3/2))*a*b - 2*(6*c^2
*d^2*e^(3/2) + d^2*e^(3/2))*b^2)*x^2 + 2*(5*(2*c^3*d*e^(3/2) + c*d*e^(3/2))
*a*b - 2*(2*c^3*d*e^(3/2) + c*d*e^(3/2))*b^2)*x)*sqrt(d*x + c))*log(d*x + c
 + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (3*c^2
*d + d)*x + (d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e+d\,e\,x)^{3/2}\,(a+b\,\mathrm{asinh}\,(c+d\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^(3/2)*(a + b*asinh(c + d*x))^2,x)
```

```
[Out] int((c*e + d*e*x)^(3/2)*(a + b*asinh(c + d*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\,(c+dx))^{\frac{3}{2}}\,(a+b\,\mathrm{asinh}\,(c+dx))^2\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(3/2)*(a+b*asinh(d*x+c))**2,x)
```

```
[Out] Integral((e*(c + d*x))**(3/2)*(a + b*asinh(c + d*x))**2, x)
```

3.239 $\int \sqrt{ce+dex}\left(a+b\sinh^{-1}(c+dx)\right)^2\,dx$

Optimal. Leaf size=134

$$\frac{16b^2(e(c+dx))^{7/2}\,{}_3F_2\left(1,\frac{7}{4},\frac{7}{4};\frac{9}{4},\frac{11}{4};-(c+dx)^2\right)}{105de^3}-\frac{8b(e(c+dx))^{5/2}\,{}_2F_1\left(\frac{1}{2},\frac{5}{4};\frac{9}{4};-(c+dx)^2\right)\left(a+b\sinh^{-1}(c+dx\right.}{15de^2}$$

```
[Out] 2/3*(e*(d*x+c))^(3/2)*(a+b*arcsinh(d*x+c))^2/d/e-8/15*b*(e*(d*x+c))^(5/2)*(
a+b*arcsinh(d*x+c))*hypergeom([1/2, 5/4],[9/4],-(d*x+c)^2)/d/e^2+16/105*b^2
*(e*(d*x+c))^(7/2)*HypergeometricPFQ([1, 7/4, 7/4],[9/4, 11/4],-(d*x+c)^2)/
d/e^3
```

Rubi [A] time = 0.21, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {5865, 5661, 5762}

$$\frac{16b^2(e(c+dx))^{7/2}\,{}_3F_2\left(1,\frac{7}{4},\frac{7}{4};\frac{9}{4},\frac{11}{4};-(c+dx)^2\right)}{105de^3}-\frac{8b(e(c+dx))^{5/2}\,{}_2F_1\left(\frac{1}{2},\frac{5}{4};\frac{9}{4};-(c+dx)^2\right)\left(a+b\sinh^{-1}(c+dx\right.}{15de^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*e + d*e*x]*(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] (2*(e*(c + d*x))^(3/2)*(a + b*ArcSinh[c + d*x])^2)/(3*d*e) - (8*b*(e*(c + d
*x))^(5/2)*(a + b*ArcSinh[c + d*x])*Hypergeometric2F1[1/2, 5/4, 9/4, -(c +
d*x)^2])/(15*d*e^2) + (16*b^2*(e*(c + d*x))^(7/2)*HypergeometricPFQ[{1, 7/4
, 7/4}, {9/4, 11/4}, -(c + d*x)^2])/(105*d*e^3)
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5762

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*ArcSinh[c*x])*Hypergeom
etric2F1[1/2, (1 + m)/2, (3 + m)/2, -(c^2*x^2)])/(Sqrt[d]*f*(m + 1)), x] -
Simp[(b*c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2
, 2 + m/2}, -(c^2*x^2)])/(Sqrt[d]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b,
c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[d, 0] &&  !IntegerQ[m]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \sqrt{ce+dex}\left(a+b\sinh^{-1}(c+dx)\right)^2 dx = \frac{\text{Subst}\left(\int \sqrt{ex}\left(a+b\sinh^{-1}(x)\right)^2 dx, x, c+dx\right)}{d}$$

$$= \frac{2(e(c+dx))^{3/2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{3de} - \frac{(4b)\,\text{Subst}\left(\int \frac{(ex)^{3/2}\left(a+b\sinh^{-1}\right.}{\sqrt{1+x^2}}\right.}{3de}$$

$$= \frac{2(e(c+dx))^{3/2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{3de} - \frac{8b(e(c+dx))^{5/2}\left(a+b\sinh^{-}\right.}{}$$

Mathematica [A] time = 0.08, size = 110, normalized size = 0.82

$$\frac{2(e(c+dx))^{3/2}\left(8b^2(c+dx)^2\,{}_3F_2\left(1,\frac{7}{4},\frac{7}{4};\frac{9}{4},\frac{11}{4};-(c+dx)^2\right)-28b(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{4};\frac{9}{4};-(c+dx)^2\right)\left(a+b\sinh^{-1}(c\right.\right.}{105de}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*e + d*e*x]*(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] (2*(e*(c + d*x))^(3/2)*(35*(a + b*ArcSinh[c + d*x])^2 - 28*b*(c + d*x)*(a +
 b*ArcSinh[c + d*x])*Hypergeometric2F1[1/2, 5/4, 9/4, -(c + d*x)^2] + 8*b^2
*(c + d*x)^2*HypergeometricPFQ[{1, 7/4, 7/4}, {9/4, 11/4}, -(c + d*x)^2]))/
(105*d*e)
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2 \operatorname{arsinh}(dx+c)^2 + 2\,ab \operatorname{arsinh}(dx+c) + a^2\right)\sqrt{dex+ce}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2*(d*e*x+c*e)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(d*x + c)^2 + 2*a*b*arcsinh(d*x + c) + a^2)*sqrt(d*e*x
 + c*e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{dex+ce}\,(b \operatorname{arsinh}(dx+c)+a)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2*(d*e*x+c*e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^2, x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int (a+b \operatorname{arcsinh}(dx+c))^2 \sqrt{dex+ce}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^2*(d*e*x+c*e)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^2*(d*e*x+c*e)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\left(b^2d\sqrt{e}\,x + b^2c\sqrt{e}\right)\sqrt{dx+c}\,\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)^2}{3\,d} + \frac{2\,(dex+ce)^{\frac{3}{2}}a^2}{3\,de} + \int -\frac{2\left(\left(2\,b^2c^2\sqrt{e} - 3\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2*(d*e*x+c*e)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b^2*d*sqrt(e)*x + b^2*c*sqrt(e))*sqrt(d*x + c)*log(d*x + c + sqrt(d^2*
x^2 + 2*c*d*x + c^2 + 1))^2/d + 2/3*(d*e*x + c*e)^(3/2)*a^2/(d*e) + integra
te(-2/3*((2*b^2*c^2*sqrt(e) - 3*(c^2*sqrt(e) + sqrt(e))*a*b - (3*a*b*d^2*sq
rt(e) - 2*b^2*d^2*sqrt(e))*x^2 - 2*(3*a*b*c*d*sqrt(e) - 2*b^2*c*d*sqrt(e))*
x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*sqrt(d*x + c) - ((3*a*b*d^3*sqrt(e) -
2*b^2*d^3*sqrt(e))*x^3 + 3*(c^3*sqrt(e) + c*sqrt(e))*a*b - 2*(c^3*sqrt(e) +
 c*sqrt(e))*b^2 + 3*(3*a*b*c*d^2*sqrt(e) - 2*b^2*c*d^2*sqrt(e))*x^2 + (3*(3
*c^2*d*sqrt(e) + d*sqrt(e))*a*b - 2*(3*c^2*d*sqrt(e) + d*sqrt(e))*b^2)*x)*s
qrt(d*x + c))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(d^3*x^3 + 3
*c*d^2*x^2 + c^3 + (3*c^2*d + d)*x + (d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) +
c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{c\,e+d\,e\,x}\,\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^(1/2)*(a + b*asinh(c + d*x))^2,x)
```

```
[Out] int((c*e + d*e*x)^(1/2)*(a + b*asinh(c + d*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{e\,(c+dx)}\,\left(a+b\,\mathrm{asinh}\,(c+dx)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**2*(d*e*x+c*e)**(1/2),x)
```

```
[Out] Integral(sqrt(e*(c + d*x))*(a + b*asinh(c + d*x))**2, x)
```

3.240 $\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{\sqrt{ce+dex}}\,dx$

Optimal. Leaf size=132

$$\frac{16b^2(e(c+dx))^{5/2}\,{}_3F_2\left(1,\frac{5}{4},\frac{5}{4};\frac{7}{4},\frac{9}{4};-(c+dx)^2\right)}{15de^3}-\frac{8b(e(c+dx))^{3/2}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-(c+dx)^2\right)\left(a+b\sinh^{-1}(c+dx)\right)}{3de^2}+$$

```
[Out] -8/3*b*(e*(d*x+c))^(3/2)*(a+b*arcsinh(d*x+c))*hypergeom([1/2, 3/4],[7/4],-(
d*x+c)^2)/d/e^2+16/15*b^2*(e*(d*x+c))^(5/2)*HypergeometricPFQ([1, 5/4, 5/4]
,[7/4, 9/4],-(d*x+c)^2)/d/e^3+2*(a+b*arcsinh(d*x+c))^2*(e*(d*x+c))^(1/2)/d/
e
```

Rubi [A] time = 0.20, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {5865, 5661, 5762}

$$\frac{16b^2(e(c+dx))^{5/2}\,{}_3F_2\left(1,\frac{5}{4},\frac{5}{4};\frac{7}{4},\frac{9}{4};-(c+dx)^2\right)}{15de^3}-\frac{8b(e(c+dx))^{3/2}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-(c+dx)^2\right)\left(a+b\sinh^{-1}(c+dx)\right)}{3de^2}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^2/Sqrt[c*e + d*e*x],x]
```

```
[Out] (2*Sqrt[e*(c + d*x)]*(a + b*ArcSinh[c + d*x])^2)/(d*e) - (8*b*(e*(c + d*x))
^(3/2)*(a + b*ArcSinh[c + d*x])*Hypergeometric2F1[1/2, 3/4, 7/4, -(c + d*x)
^2])/(3*d*e^2) + (16*b^2*(e*(c + d*x))^(5/2)*HypergeometricPFQ[{1, 5/4, 5/4
}, {7/4, 9/4}, -(c + d*x)^2])/(15*d*e^3)
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5762

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*ArcSinh[c*x])*Hypergeom
etric2F1[1/2, (1 + m)/2, (3 + m)/2, -(c^2*x^2)])/(Sqrt[d]*f*(m + 1)), x] -
Simp[(b*c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2
, 2 + m/2}, -(c^2*x^2)])/(Sqrt[d]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b,
c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[d, 0] &&  !IntegerQ[m]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{\sqrt{ce+dex}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^2}{\sqrt{ex}}\,dx,x,c+dx\right)}{d}$$

$$= \frac{2\sqrt{e(c+dx)}\left(a+b\sinh^{-1}(c+dx)\right)^2}{de} - \frac{(4b)\,\text{Subst}\left(\int \frac{\sqrt{ex}\left(a+b\sinh^{-1}(x)\right)}{\sqrt{1+x^2}}\,dx,x,c\right.}{de}$$

$$= \frac{2\sqrt{e(c+dx)}\left(a+b\sinh^{-1}(c+dx)\right)^2}{de} - \frac{8b(e(c+dx))^{3/2}\left(a+b\sinh^{-1}(c+dx)\right)}{3de^2}$$

Mathematica [A] time = 0.06, size = 110, normalized size = 0.83

$$\frac{2\sqrt{e(c+dx)}\left(8b^2(c+dx)^2\,{}_3F_2\left(1,\frac{5}{4},\frac{5}{4};\frac{7}{4},\frac{9}{4};-(c+dx)^2\right)-20b(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-(c+dx)^2\right)\left(a+b\sinh^{-1}(c\right.\right.}{15de}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^2/Sqrt[c*e + d*e*x],x]
```

```
[Out] (2*Sqrt[e*(c + d*x)]*(15*(a + b*ArcSinh[c + d*x])^2 - 20*b*(c + d*x)*(a + b
*ArcSinh[c + d*x])*Hypergeometric2F1[1/2, 3/4, 7/4, -(c + d*x)^2] + 8*b^2*(
c + d*x)^2*HypergeometricPFQ[{1, 5/4, 5/4}, {7/4, 9/4}, -(c + d*x)^2]))/(15
*d*e)
```

fricas [F] time = 1.12, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\operatorname{arsinh}(dx+c)^2+2\,ab\operatorname{arsinh}(dx+c)+a^2}{\sqrt{dex+ce}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(d*x + c)^2 + 2*a*b*arcsinh(d*x + c) + a^2)/sqrt(d*e*x
 + c*e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\operatorname{arsinh}(dx+c)+a)^2}{\sqrt{dex+ce}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^2/sqrt(d*e*x + c*e), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\operatorname{arcsinh}(dx+c))^2}{\sqrt{dex+ce}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\sqrt{dx+c}\,b^2\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)^2}{d\sqrt{e}}+\frac{2\sqrt{dex+ce}\,a^2}{de}+\int-\frac{2\left(\left(2\,b^2c^2\sqrt{e}-\left(c^2\sqrt{e}+\sqrt{e}\right)ab-\left(ab\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(d*x + c)*b^2*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2/(d*s
qrt(e)) + 2*sqrt(d*e*x + c*e)*a^2/(d*e) + integrate(-2*((2*b^2*c^2*sqrt(e)
- (c^2*sqrt(e) + sqrt(e))*a*b - (a*b*d^2*sqrt(e) - 2*b^2*d^2*sqrt(e))*x^2 -
 2*(a*b*c*d*sqrt(e) - 2*b^2*c*d*sqrt(e))*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 +
1)*sqrt(d*x + c) - ((a*b*d^3*sqrt(e) - 2*b^2*d^3*sqrt(e))*x^3 + (c^3*sqrt(e
) + c*sqrt(e))*a*b - 2*(c^3*sqrt(e) + c*sqrt(e))*b^2 + 3*(a*b*c*d^2*sqrt(e)
 - 2*b^2*c*d^2*sqrt(e))*x^2 + ((3*c^2*d*sqrt(e) + d*sqrt(e))*a*b - 2*(3*c^2
*d*sqrt(e) + d*sqrt(e))*b^2)*x)*sqrt(d*x + c))*log(d*x + c + sqrt(d^2*x^2 +
 2*c*d*x + c^2 + 1))/(d^4*e*x^4 + 4*c*d^3*e*x^3 + c^4*e + c^2*e + (6*c^2*d^
2*e + d^2*e)*x^2 + 2*(2*c^3*d*e + c*d*e)*x + (d^3*e*x^3 + 3*c*d^2*e*x^2 + c
^3*e + c*e + (3*c^2*d*e + d*e)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^2}{\sqrt{c\,e+d\,e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^2/(c*e + d*e*x)^(1/2),x)
```

```
[Out] int((a + b*asinh(c + d*x))^2/(c*e + d*e*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{asinh}\left(c+dx\right)\right)^2}{\sqrt{e\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**2/(d*e*x+c*e)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))**2/sqrt(e*(c + d*x)), x)
```

3.241 $\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{(ce+dex)^{3/2}}\,dx$

Optimal. Leaf size=130

$$-\frac{16b^2(e(c+dx))^{3/2}\,{}_3F_2\left(\frac{3}{4},\frac{3}{4},1;\frac{5}{4},\frac{7}{4};-(c+dx)^2\right)}{3de^3}+\frac{8b\sqrt{e(c+dx)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-(c+dx)^2\right)\left(a+b\sinh^{-1}(c+dx)\right.}{de^2}$$

```
[Out] -16/3*b^2*(e*(d*x+c))^(3/2)*HypergeometricPFQ([3/4, 3/4, 1],[5/4, 7/4],-(d*
x+c)^2)/d/e^3-2*(a+b*arcsinh(d*x+c))^2/d/e/(e*(d*x+c))^(1/2)+8*b*(a+b*arcsi
nh(d*x+c))*hypergeom([1/4, 1/2],[5/4],-(d*x+c)^2)*(e*(d*x+c))^(1/2)/d/e^2
```

Rubi [A] time = 0.22, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {5865, 5661, 5762}

$$-\frac{16b^2(e(c+dx))^{3/2}\,{}_3F_2\left(\frac{3}{4},\frac{3}{4},1;\frac{5}{4},\frac{7}{4};-(c+dx)^2\right)}{3de^3}+\frac{8b\sqrt{e(c+dx)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-(c+dx)^2\right)\left(a+b\sinh^{-1}(c+dx)\right.}{de^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^2/(c*e + d*e*x)^(3/2),x]
```

```
[Out] (-2*(a + b*ArcSinh[c + d*x])^2)/(d*e*Sqrt[e*(c + d*x)]) + (8*b*Sqrt[e*(c +
d*x)]*(a + b*ArcSinh[c + d*x])*Hypergeometric2F1[1/4, 1/2, 5/4, -(c + d*x)^
2])/(d*e^2) - (16*b^2*(e*(c + d*x))^(3/2)*HypergeometricPFQ[{3/4, 3/4, 1},
{5/4, 7/4}, -(c + d*x)^2])/(3*d*e^3)
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5762

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*ArcSinh[c*x])*Hypergeom
etric2F1[1/2, (1 + m)/2, (3 + m)/2, -(c^2*x^2)])/(Sqrt[d]*f*(m + 1)), x] -
Simp[(b*c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2
, 2 + m/2}, -(c^2*x^2)])/(Sqrt[d]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b,
c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[d, 0] &&  !IntegerQ[m]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{(ce+dex)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^2}{(ex)^{3/2}}\,dx,x,c+dx\right)}{d}$$

$$= -\frac{2\left(a+b\sinh^{-1}(c+dx)\right)^2}{de\sqrt{e(c+dx)}} + \frac{(4b)\,\text{Subst}\left(\int \frac{a+b\sinh^{-1}(x)}{\sqrt{ex}\sqrt{1+x^2}}\,dx,x,c+dx\right)}{de}$$

$$= -\frac{2\left(a+b\sinh^{-1}(c+dx)\right)^2}{de\sqrt{e(c+dx)}} + \frac{8b\sqrt{e(c+dx)}\left(a+b\sinh^{-1}(c+dx)\right){}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\right.}{de^2}$$

Mathematica [A] time = 0.10, size = 109, normalized size = 0.84

$$\frac{2\left(-4b(c+dx)\left(2b(c+dx)\,{}_3F_2\left(\frac{3}{4},\frac{3}{4},1;\frac{5}{4},\frac{7}{4};-(c+dx)^2\right)-3\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-(c+dx)^2\right)\left(a+b\sinh^{-1}(c+dx)\right)\right)-3\left(\right.}{3de\sqrt{e(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^2/(c*e + d*e*x)^(3/2),x]
```

```
[Out] (2*(-3*(a + b*ArcSinh[c + d*x])^2 - 4*b*(c + d*x)*(-3*(a + b*ArcSinh[c + d*
x])*Hypergeometric2F1[1/4, 1/2, 5/4, -(c + d*x)^2] + 2*b*(c + d*x)*Hypergeo
metricPFQ[{3/4, 3/4, 1}, {5/4, 7/4}, -(c + d*x)^2])))/(3*d*e*Sqrt[e*(c + d*
x)])
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2\,\text{arsinh}\,(dx+c)^2+2\,ab\,\text{arsinh}\,(dx+c)+a^2\right)\sqrt{dex+ce}}{d^2e^2x^2+2\,cde^2x+c^2e^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(d*x + c)^2 + 2*a*b*arcsinh(d*x + c) + a^2)*sqrt(d*e*x
 + c*e)/(d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{arsinh}\,(dx+c)+a)^2}{(dex+ce)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^2/(d*e*x + c*e)^(3/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\text{arcsinh}\,(dx+c))^2}{(dex+ce)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(3/2),x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\sqrt{dx+c}\,b^2\sqrt{e}\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)^2}{d^2e^2x+cde^2}-\frac{2\,a^2}{\sqrt{dex+ce}\,de}+\int\frac{2\left(\left(2\,b^2c^2+\left(c^2+1\right)ab+\left(abd^2+\right.\right.\right.}{d^5e^{\frac{3}{2}}x^5+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(3/2),x, algorithm="maxima")
```

```
[Out] -2*sqrt(d*x + c)*b^2*sqrt(e)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1
))^2/(d^2*e^2*x + c*d*e^2) - 2*a^2/(sqrt(d*e*x + c*e)*d*e) + integrate(2*((
2*b^2*c^2 + (c^2 + 1)*a*b + (a*b*d^2 + 2*b^2*d^2)*x^2 + 2*(a*b*c*d + 2*b^2*
c*d)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*sqrt(d*x + c) + ((a*b*d^3 + 2*b^2
*d^3)*x^3 + (c^3 + c)*a*b + 2*(c^3 + c)*b^2 + 3*(a*b*c*d^2 + 2*b^2*c*d^2)*x
^2 + ((3*c^2*d + d)*a*b + 2*(3*c^2*d + d)*b^2)*x)*sqrt(d*x + c))*log(d*x +
c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(d^5*e^(3/2)*x^5 + 5*c*d^4*e^(3/2)*x
^4 + c^5*e^(3/2) + c^3*e^(3/2) + (10*c^2*d^3*e^(3/2) + d^3*e^(3/2))*x^3 + (
10*c^3*d^2*e^(3/2) + 3*c*d^2*e^(3/2))*x^2 + (5*c^4*d*e^(3/2) + 3*c^2*d*e^(3
/2))*x + (d^4*e^(3/2)*x^4 + 4*c*d^3*e^(3/2)*x^3 + c^4*e^(3/2) + c^2*e^(3/2)
 + (6*c^2*d^2*e^(3/2) + d^2*e^(3/2))*x^2 + 2*(2*c^3*d*e^(3/2) + c*d*e^(3/2)
)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^2}{{\left(c\,e+d\,e\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^2/(c*e + d*e*x)^(3/2),x)
```

```
[Out] int((a + b*asinh(c + d*x))^2/(c*e + d*e*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{asinh}{\left(c + dx \right)}\right)^{2}}{\left(e \left(c + dx\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**2/(d*e*x+c*e)**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))**2/(e*(c + d*x))**(3/2), x)
```

3.242 $\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{(ce+dex)^{5/2}}\,dx$

Optimal. Leaf size=134

$$\frac{16b^2\sqrt{e(c+dx)}\;{}_3F_2\left(\frac{1}{4},\frac{1}{4},1;\frac{3}{4},\frac{5}{4};-(c+dx)^2\right)}{3de^3}-\frac{8b\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-(c+dx)^2\right)\left(a+b\sinh^{-1}(c+dx)\right)}{3de^2\sqrt{e(c+dx)}}-\frac{2\left(a+b\sinh\right.}{3de(e(c+}$$

```
[Out] -2/3*(a+b*arcsinh(d*x+c))^2/d/e/(e*(d*x+c))^(3/2)-8/3*b*(a+b*arcsinh(d*x+c)
)*hypergeom([-1/4, 1/2],[3/4],-(d*x+c)^2)/d/e^2/(e*(d*x+c))^(1/2)+16/3*b^2*
HypergeometricPFQ([1/4, 1/4, 1],[3/4, 5/4],-(d*x+c)^2)*(e*(d*x+c))^(1/2)/d/
e^3
```

Rubi [A] time = 0.23, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {5865, 5661, 5762}

$$\frac{16b^2\sqrt{e(c+dx)}\;{}_3F_2\left(\frac{1}{4},\frac{1}{4},1;\frac{3}{4},\frac{5}{4};-(c+dx)^2\right)}{3de^3}-\frac{8b\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-(c+dx)^2\right)\left(a+b\sinh^{-1}(c+dx)\right)}{3de^2\sqrt{e(c+dx)}}-\frac{2\left(a+b\sinh\right.}{3de(e(c+}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^2/(c*e + d*e*x)^(5/2),x]
```

```
[Out] (-2*(a + b*ArcSinh[c + d*x])^2)/(3*d*e*(e*(c + d*x))^(3/2)) - (8*b*(a + b*A
rcSinh[c + d*x])*Hypergeometric2F1[-1/4, 1/2, 3/4, -(c + d*x)^2])/(3*d*e^2*
Sqrt[e*(c + d*x)]) + (16*b^2*Sqrt[e*(c + d*x)]*HypergeometricPFQ[{1/4, 1/4,
 1}, {3/4, 5/4}, -(c + d*x)^2])/(3*d*e^3)
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5762

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*ArcSinh[c*x])*Hypergeom
etric2F1[1/2, (1 + m)/2, (3 + m)/2, -(c^2*x^2)])/(Sqrt[d]*f*(m + 1)), x] -
Simp[(b*c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2
, 2 + m/2}, -(c^2*x^2)])/(Sqrt[d]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b,
c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[d, 0] &&  !IntegerQ[m]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{(ce+dex)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^2}{(ex)^{5/2}}\,dx,x,c+dx\right)}{d}$$
$$= -\frac{2\left(a+b\sinh^{-1}(c+dx)\right)^2}{3de(e(c+dx))^{3/2}} + \frac{(4b)\,\text{Subst}\left(\int \frac{a+b\sinh^{-1}(x)}{(ex)^{3/2}\sqrt{1+x^2}}\,dx,x,c+dx\right)}{3de}$$
$$= -\frac{2\left(a+b\sinh^{-1}(c+dx)\right)^2}{3de(e(c+dx))^{3/2}} - \frac{8b\left(a+b\sinh^{-1}(c+dx)\right)\,_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-(c+d\right.}{3de^2\sqrt{e(c+dx)}}$$

Mathematica [A] time = 0.08, size = 106, normalized size = 0.79

$$-\frac{2\left(4b(c+dx)\left(\,_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-(c+dx)^2\right)\left(a+b\sinh^{-1}(c+dx)\right)-2b(c+dx)\,_3F_2\left(\frac{1}{4},\frac{1}{4},1;\frac{3}{4},\frac{5}{4};-(c+dx)^2\right)\right)+\right.}{3de(e(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^2/(c*e + d*e*x)^(5/2),x]
```

```
[Out] (-2*((a + b*ArcSinh[c + d*x])^2 + 4*b*(c + d*x)*((a + b*ArcSinh[c + d*x])*H
ypergeometric2F1[-1/4, 1/2, 3/4, -(c + d*x)^2] - 2*b*(c + d*x)*Hypergeometr
icPFQ[{1/4, 1/4, 1}, {3/4, 5/4}, -(c + d*x)^2])))/(3*d*e*(e*(c + d*x))^(3/2
))
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2\operatorname{arsinh}(dx+c)^2+2\,ab\operatorname{arsinh}(dx+c)+a^2\right)\sqrt{dex+ce}}{d^3e^3x^3+3\,cd^2e^3x^2+3\,c^2de^3x+c^3e^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(d*x + c)^2 + 2*a*b*arcsinh(d*x + c) + a^2)*sqrt(d*e*x
 + c*e)/(d^3*e^3*x^3 + 3*c*d^2*e^3*x^2 + 3*c^2*d*e^3*x + c^3*e^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\operatorname{arsinh}(dx+c)+a)^2}{(dex+ce)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^2/(d*e*x + c*e)^(5/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\operatorname{arcsinh}(dx+c))^2}{(dex+ce)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(5/2),x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\sqrt{dx+c}\,b^2\sqrt{e}\,\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)^2}{3\left(d^3e^3x^2+2\,cd^2e^3x+c^2de^3\right)}-\frac{2\,a^2}{3\,(dex+ce)^{\frac{3}{2}}de}+\int\frac{2\left(\left(2\,b^2c^2+3\left(c^2+1\right)ab+\left(3\right.\right.\right.}{3\left(d^6e^{\frac{5}{2}}x^6+6\,cd^5e^{\frac{5}{2}}x^5+c^6e^{\frac{5}{2}}+c^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*sqrt(d*x + c)*b^2*sqrt(e)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 +
 1))^2/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3) - 2/3*a^2/((d*e*x + c*e)^(
3/2)*d*e) + integrate(2/3*((2*b^2*c^2 + 3*(c^2 + 1)*a*b + (3*a*b*d^2 + 2*b^
2*d^2)*x^2 + 2*(3*a*b*c*d + 2*b^2*c*d)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)
*sqrt(d*x + c) + ((3*a*b*d^3 + 2*b^2*d^3)*x^3 + 3*(c^3 + c)*a*b + 2*(c^3 +
c)*b^2 + 3*(3*a*b*c*d^2 + 2*b^2*c*d^2)*x^2 + (3*(3*c^2*d + d)*a*b + 2*(3*c^
2*d + d)*b^2)*x)*sqrt(d*x + c))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2
+ 1))/(d^6*e^(5/2)*x^6 + 6*c*d^5*e^(5/2)*x^5 + c^6*e^(5/2) + c^4*e^(5/2) +
(15*c^2*d^4*e^(5/2) + d^4*e^(5/2))*x^4 + 4*(5*c^3*d^3*e^(5/2) + c*d^3*e^(5/
2))*x^3 + 3*(5*c^4*d^2*e^(5/2) + 2*c^2*d^2*e^(5/2))*x^2 + 2*(3*c^5*d*e^(5/2
) + 2*c^3*d*e^(5/2))*x + (d^5*e^(5/2)*x^5 + 5*c*d^4*e^(5/2)*x^4 + c^5*e^(5/
2) + c^3*e^(5/2) + (10*c^2*d^3*e^(5/2) + d^3*e^(5/2))*x^3 + (10*c^3*d^2*e^(
5/2) + 3*c*d^2*e^(5/2))*x^2 + (5*c^4*d*e^(5/2) + 3*c^2*d*e^(5/2))*x)*sqrt(d
^2*x^2 + 2*c*d*x + c^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^2}{{\left(c\,e+d\,e\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^2/(c*e + d*e*x)^(5/2),x)
```

```
[Out] int((a + b*asinh(c + d*x))^2/(c*e + d*e*x)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{asinh}{\left(c + dx \right)}\right)^{2}}{\left(e \left(c + dx\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**2/(d*e*x+c*e)**(5/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))**2/(e*(c + d*x))**(5/2), x)
```

3.243 $\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{(ce+dex)^{7/2}}\,dx$

Optimal. Leaf size=134

$$-\frac{16b^2\,{}_3F_2\left(-\frac{1}{4},-\frac{1}{4},1;\frac{1}{4},\frac{3}{4};-(c+dx)^2\right)}{15de^3\sqrt{e(c+dx)}}-\frac{8b\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{1}{4};-(c+dx)^2\right)\left(a+b\sinh^{-1}(c+dx)\right)}{15de^2(e(c+dx))^{3/2}}-\frac{2\left(a+b\sinh^{-1}(c\right.}{5de(e(c+dx}$$

```
[Out] -2/5*(a+b*arcsinh(d*x+c))^2/d/e/(e*(d*x+c))^(5/2)-8/15*b*(a+b*arcsinh(d*x+c
))*hypergeom([-3/4, 1/2],[1/4],-(d*x+c)^2)/d/e^2/(e*(d*x+c))^(3/2)-16/15*b^
2*HypergeometricPFQ([-1/4, -1/4, 1],[1/4, 3/4],-(d*x+c)^2)/d/e^3/(e*(d*x+c)
)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {5865, 5661, 5762}

$$-\frac{16b^2\,{}_3F_2\left(-\frac{1}{4},-\frac{1}{4},1;\frac{1}{4},\frac{3}{4};-(c+dx)^2\right)}{15de^3\sqrt{e(c+dx)}}-\frac{8b\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{1}{4};-(c+dx)^2\right)\left(a+b\sinh^{-1}(c+dx)\right)}{15de^2(e(c+dx))^{3/2}}-\frac{2\left(a+b\sinh^{-1}(c\right.}{5de(e(c+dx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^2/(c*e + d*e*x)^(7/2),x]
```

```
[Out] (-2*(a + b*ArcSinh[c + d*x])^2)/(5*d*e*(e*(c + d*x))^(5/2)) - (8*b*(a + b*A
rcSinh[c + d*x])*Hypergeometric2F1[-3/4, 1/2, 1/4, -(c + d*x)^2])/(15*d*e^2
*(e*(c + d*x))^(3/2)) - (16*b^2*HypergeometricPFQ[{-1/4, -1/4, 1}, {1/4, 3/
4}, -(c + d*x)^2])/(15*d*e^3*Sqrt[e*(c + d*x)])
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5762

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*ArcSinh[c*x])*Hypergeom
etric2F1[1/2, (1 + m)/2, (3 + m)/2, -(c^2*x^2)])/(Sqrt[d]*f*(m + 1)), x] -
Simp[(b*c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2
, 2 + m/2}, -(c^2*x^2)])/(Sqrt[d]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b,
c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[d, 0] &&  !IntegerQ[m]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{(ce+dex)^{7/2}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^2}{(ex)^{7/2}}\,dx,x,c+dx\right)}{d}$$

$$= -\frac{2\left(a+b\sinh^{-1}(c+dx)\right)^2}{5de(e(c+dx))^{5/2}} + \frac{(4b)\,\text{Subst}\left(\int \frac{a+b\sinh^{-1}(x)}{(ex)^{5/2}\sqrt{1+x^2}}\,dx,x,c+dx\right)}{5de}$$

$$= -\frac{2\left(a+b\sinh^{-1}(c+dx)\right)^2}{5de(e(c+dx))^{5/2}} - \frac{8b\left(a+b\sinh^{-1}(c+dx)\right)\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{1}{4};-(c+dx)^2\right.}{15de^2(e(c+dx))^{3/2}}$$

Mathematica [A] time = 0.09, size = 110, normalized size = 0.82

$$-\frac{2\left(8b^2(c+dx)^2\,{}_3F_2\left(-\frac{1}{4},-\frac{1}{4},1;\frac{1}{4},\frac{3}{4};-(c+dx)^2\right)+\left(a+b\sinh^{-1}(c+dx)\right)\left(3\left(a+b\sinh^{-1}(c+dx)\right)+4b(c+dx)\right.\right.}{15de(e(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^2/(c*e + d*e*x)^(7/2),x]
```

```
[Out] (-2*((a + b*ArcSinh[c + d*x])*(3*(a + b*ArcSinh[c + d*x]) + 4*b*(c + d*x)*H
ypergeometric2F1[-3/4, 1/2, 1/4, -(c + d*x)^2]) + 8*b^2*(c + d*x)^2*Hyperge
ometricPFQ[{-1/4, -1/4, 1}, {1/4, 3/4}, -(c + d*x)^2]))/(15*d*e*(e*(c + d*x
))^(5/2))
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2\,\text{arsinh}\,(dx+c)^2+2\,ab\,\text{arsinh}\,(dx+c)+a^2\right)\sqrt{dex+ce}}{d^4e^4x^4+4\,cd^3e^4x^3+6\,c^2d^2e^4x^2+4\,c^3de^4x+c^4e^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(d*x + c)^2 + 2*a*b*arcsinh(d*x + c) + a^2)*sqrt(d*e*x
 + c*e)/(d^4*e^4*x^4 + 4*c*d^3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^3*d*e^4*x
+ c^4*e^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{arsinh}\,(dx+c)+a)^2}{(dex+ce)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^2/(d*e*x + c*e)^(7/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\text{arcsinh}\,(dx+c))^2}{(dex+ce)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(7/2),x)
```

[Out] int((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(7/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\sqrt{dx+c}\,b^2\sqrt{e}\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)^2}{5\left(d^4e^4x^3+3\,cd^3e^4x^2+3\,c^2d^2e^4x+c^3de^4\right)}-\frac{2\,a^2}{5\,(dex+ce)^{\frac{5}{2}}de}+\int\frac{}{5\left(d^7e^{\frac{7}{2}}x^7+7\,cd^6e^{\frac{7}{2}}x^6+c^7e^{\frac{7}{2}}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(7/2),x, algorithm="maxima")

[Out] -2/5*sqrt(d*x + c)*b^2*sqrt(e)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2/(d^4*e^4*x^3 + 3*c*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^4) - 2/5*a^2/((d*e*x + c*e)^(5/2)*d*e) + integrate(2/5*((2*b^2*c^2 + 5*(c^2 + 1)*a*b + (5*a*b*d^2 + 2*b^2*d^2)*x^2 + 2*(5*a*b*c*d + 2*b^2*c*d)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*sqrt(d*x + c) + ((5*a*b*d^3 + 2*b^2*d^3)*x^3 + 5*(c^3 + c)*a*b + 2*(c^3 + c)*b^2 + 3*(5*a*b*c*d^2 + 2*b^2*c*d^2)*x^2 + (5*(3*c^2*d + d)*a*b + 2*(3*c^2*d + d)*b^2)*x)*sqrt(d*x + c))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(d^7*e^(7/2)*x^7 + 7*c*d^6*e^(7/2)*x^6 + c^7*e^(7/2) + c^5*e^(7/2) + (21*c^2*d^5*e^(7/2) + d^5*e^(7/2))*x^5 + 5*(7*c^3*d^4*e^(7/2) + c*d^4*e^(7/2))*x^4 + 5*(7*c^4*d^3*e^(7/2) + 2*c^2*d^3*e^(7/2))*x^3 + (21*c^5*d^2*e^(7/2) + 10*c^3*d^2*e^(7/2))*x^2 + (7*c^6*d*e^(7/2) + 5*c^4*d*e^(7/2))*x + (d^6*e^(7/2)*x^6 + 6*c*d^5*e^(7/2)*x^5 + c^6*e^(7/2) + c^4*e^(7/2) + (15*c^2*d^4*e^(7/2) + d^4*e^(7/2))*x^4 + 4*(5*c^3*d^3*e^(7/2) + c*d^3*e^(7/2))*x^3 + 3*(5*c^4*d^2*e^(7/2) + 2*c^2*d^2*e^(7/2))*x^2 + 2*(3*c^5*d*e^(7/2) + 2*c^3*d*e^(7/2))*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{asinh}\,(c+d\,x))^2}{(c\,e+d\,e\,x)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*asinh(c + d*x))^2/(c*e + d*e*x)^(7/2),x)

[Out] int((a + b*asinh(c + d*x))^2/(c*e + d*e*x)^(7/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*asinh(d*x+c))**2/(d*e*x+c*e)**(7/2),x)

[Out] Timed out

3.244 $\int (ce+dex)^{7/2} \left(a+b\sinh^{-1}(c+dx)\right)^3 \, dx$

Optimal. Leaf size=82

$$\frac{2(e(c+dx))^{9/2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{9de}-\frac{2b\text{Int}\left(\frac{(e(c+dx))^{9/2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{\sqrt{(c+dx)^2+1}},x\right)}{3e}$$

```
[Out] 2/9*(e*(d*x+c))^(9/2)*(a+b*arcsinh(d*x+c))^3/d/e-2/3*b*Unintegrable((e*(d*x
+c))^(9/2)*(a+b*arcsinh(d*x+c))^2/(1+(d*x+c)^2)^(1/2),x)/e
```

Rubi [A] time = 0.21, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (ce+dex)^{7/2}\left(a+b\sinh^{-1}(c+dx)\right)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(c*e + d*e*x)^(7/2)*(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] (2*(e*(c + d*x))^(9/2)*(a + b*ArcSinh[c + d*x])^3)/(9*d*e) - (2*b*Defer[Sub
st][Defer[Int][((e*x)^(9/2)*(a + b*ArcSinh[x])^2)/Sqrt[1 + x^2], x], x, c +
 d*x])/(3*d*e)
```

Rubi steps

$$\int (ce+dex)^{7/2}\left(a+b\sinh^{-1}(c+dx)\right)^3 \, dx=\frac{\text{Subst}\left(\int (ex)^{7/2}\left(a+b\sinh^{-1}(x)\right)^3 \, dx,x,c+dx\right)}{d}$$

$$=\frac{2(e(c+dx))^{9/2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{9de}-\frac{(2b)\,\text{Subst}\left(\int \frac{(ex)^{9/2}\left(a+b\sinh\right.}{\sqrt{1+x^2}}\right.}{3de}$$

Mathematica [A] time = 89.80, size = 0, normalized size = 0.00

$$\int (ce+dex)^{7/2}\left(a+b\sinh^{-1}(c+dx)\right)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c*e + d*e*x)^(7/2)*(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] Integrate[(c*e + d*e*x)^(7/2)*(a + b*ArcSinh[c + d*x])^3, x]
```

fricas [A] time = 2.03, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^3d^3e^3x^3+3\,a^3cd^2e^3x^2+3\,a^3c^2de^3x+a^3c^3e^3+\left(b^3d^3e^3x^3+3\,b^3cd^2e^3x^2+3\,b^3c^2de^3x+b^3c^3e^3\right)\text{arsinh}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((a^3*d^3*e^3*x^3 + 3*a^3*c*d^2*e^3*x^2 + 3*a^3*c^2*d*e^3*x + a^3*c
^3*e^3 + (b^3*d^3*e^3*x^3 + 3*b^3*c*d^2*e^3*x^2 + 3*b^3*c^2*d*e^3*x + b^3*c
^3*e^3)*arcsinh(d*x + c)^3 + 3*(a*b^2*d^3*e^3*x^3 + 3*a*b^2*c*d^2*e^3*x^2 +
 3*a*b^2*c^2*d*e^3*x + a*b^2*c^3*e^3)*arcsinh(d*x + c)^2 + 3*(a^2*b*d^3*e^3
```

```
*x^3 + 3*a^2*b*c*d^2*e^3*x^2 + 3*a^2*b*c^2*d*e^3*x + a^2*b*c^3*e^3)*arcsinh
(d*x + c))*sqrt(d*e*x + c*e), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex+ce)^{\frac{7}{2}}(b\operatorname{arsinh}(dx+c)+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^(7/2)*(b*arcsinh(d*x + c) + a)^3, x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int (dex+ce)^{\frac{7}{2}}\left(a+b\operatorname{arcsinh}(dx+c)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^3,x)
```

```
[Out] int((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 2/9*(d*e*x + c*e)^(9/2)*a^3/(d*e) + 2/9*(b^3*d^4*e^(7/2)*x^4 + 4*b^3*c*d^3*
e^(7/2)*x^3 + 6*b^3*c^2*d^2*e^(7/2)*x^2 + 4*b^3*c^3*d*e^(7/2)*x + b^3*c^4*e
^(7/2))*sqrt(d*x + c)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3/d
+ integrate(-1/3*(((2*b^3*c^5*e^(7/2) - (9*a*b^2*d^5*e^(7/2) - 2*b^3*d^5*e^
(7/2))*x^5 - 5*(9*a*b^2*c*d^4*e^(7/2) - 2*b^3*c*d^4*e^(7/2))*x^4 - 9*(c^5*e
^(7/2) + c^3*e^(7/2))*a*b^2 + (20*b^3*c^2*d^3*e^(7/2) - 9*(10*c^2*d^3*e^(7/
2) + d^3*e^(7/2))*a*b^2)*x^3 + (20*b^3*c^3*d^2*e^(7/2) - 9*(10*c^3*d^2*e^(7
/2) + 3*c*d^2*e^(7/2))*a*b^2)*x^2 + (10*b^3*c^4*d*e^(7/2) - 9*(5*c^4*d*e^(7
/2) + 3*c^2*d*e^(7/2))*a*b^2)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*sqrt(d*x
 + c) - ((9*a*b^2*d^6*e^(7/2) - 2*b^3*d^6*e^(7/2))*x^6 + 6*(9*a*b^2*c*d^5*e
^(7/2) - 2*b^3*c*d^5*e^(7/2))*x^5 + (9*(15*c^2*d^4*e^(7/2) + d^4*e^(7/2))*a
*b^2 - 2*(15*c^2*d^4*e^(7/2) + d^4*e^(7/2))*b^3)*x^4 + 9*(c^6*e^(7/2) + c^4
*e^(7/2))*a*b^2 - 2*(c^6*e^(7/2) + c^4*e^(7/2))*b^3 + 4*(9*(5*c^3*d^3*e^(7/
2) + c*d^3*e^(7/2))*a*b^2 - 2*(5*c^3*d^3*e^(7/2) + c*d^3*e^(7/2))*b^3)*x^3
+ 3*(9*(5*c^4*d^2*e^(7/2) + 2*c^2*d^2*e^(7/2))*a*b^2 - 2*(5*c^4*d^2*e^(7/2)
 + 2*c^2*d^2*e^(7/2))*b^3)*x^2 + 2*(9*(3*c^5*d*e^(7/2) + 2*c^3*d*e^(7/2))*a
*b^2 - 2*(3*c^5*d*e^(7/2) + 2*c^3*d*e^(7/2))*b^3)*x)*sqrt(d*x + c))*log(d*x
 + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 - 9*((a^2*b*d^5*e^(7/2)*x^5 + 5
*a^2*b*c*d^4*e^(7/2)*x^4 + (10*c^2*d^3*e^(7/2) + d^3*e^(7/2))*a^2*b*x^3 + (
10*c^3*d^2*e^(7/2) + 3*c*d^2*e^(7/2))*a^2*b*x^2 + (5*c^4*d*e^(7/2) + 3*c^2*
d*e^(7/2))*a^2*b*x + (c^5*e^(7/2) + c^3*e^(7/2))*a^2*b)*sqrt(d^2*x^2 + 2*c*
d*x + c^2 + 1)*sqrt(d*x + c) + (a^2*b*d^6*e^(7/2)*x^6 + 6*a^2*b*c*d^5*e^(7/
2)*x^5 + (15*c^2*d^4*e^(7/2) + d^4*e^(7/2))*a^2*b*x^4 + 4*(5*c^3*d^3*e^(7/2
) + c*d^3*e^(7/2))*a^2*b*x^3 + 3*(5*c^4*d^2*e^(7/2) + 2*c^2*d^2*e^(7/2))*a^
2*b*x^2 + 2*(3*c^5*d*e^(7/2) + 2*c^3*d*e^(7/2))*a^2*b*x + (c^6*e^(7/2) + c^
4*e^(7/2))*a^2*b)*sqrt(d*x + c))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2
 + 1)))/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (3*c^2*d + d)*x + (d^2*x^2 + 2*c*d*x
 + c^2 + 1)^(3/2) + c), x)
```

mupad [**A**] time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e+d\,e\,x)^{7/2}\,(a+b\,\mathrm{asinh}\,(c+d\,x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^(7/2)*(a + b*asinh(c + d*x))^3,x)
[Out] int((c*e + d*e*x)^(7/2)*(a + b*asinh(c + d*x))^3, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(7/2)*(a+b*asinh(d*x+c))**3,x)
[Out] Timed out
```

3.245 $\int (ce+dex)^{5/2}\left(a+b\sinh^{-1}(c+dx)\right)^3\,dx$

Optimal. Leaf size=82

$$\frac{2(e(c+dx))^{7/2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{7de}-\frac{6b\text{Int}\left(\frac{(e(c+dx))^{7/2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{\sqrt{(c+dx)^2+1}},x\right)}{7e}$$

```
[Out] 2/7*(e*(d*x+c))^(7/2)*(a+b*arcsinh(d*x+c))^3/d/e-6/7*b*Unintegrable((e*(d*x
+c))^(7/2)*(a+b*arcsinh(d*x+c))^2/(1+(d*x+c)^2)^(1/2),x)/e
```

Rubi [A] time = 0.20, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (ce+dex)^{5/2}\left(a+b\sinh^{-1}(c+dx)\right)^3\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c*e + d*e*x)^(5/2)*(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] (2*(e*(c + d*x))^(7/2)*(a + b*ArcSinh[c + d*x])^3)/(7*d*e) - (6*b*Defer[Sub
st][Defer[Int][((e*x)^(7/2)*(a + b*ArcSinh[x])^2)/Sqrt[1 + x^2], x], x, c +
 d*x])/(7*d*e)
```

Rubi steps

$$\int (ce+dex)^{5/2}\left(a+b\sinh^{-1}(c+dx)\right)^3\,dx=\frac{\text{Subst}\left(\int (ex)^{5/2}\left(a+b\sinh^{-1}(x)\right)^3\,dx,x,c+dx\right)}{d}$$

$$=\frac{2(e(c+dx))^{7/2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{7de}-\frac{(6b)\,\text{Subst}\left(\int \frac{(ex)^{7/2}\left(a+b\,\text{s}\right.}{\sqrt{1+}}\right.}{7d}$$

Mathematica [A] time = 108.23, size = 0, normalized size = 0.00

$$\int (ce+dex)^{5/2}\left(a+b\sinh^{-1}(c+dx)\right)^3\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c*e + d*e*x)^(5/2)*(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] Integrate[(c*e + d*e*x)^(5/2)*(a + b*ArcSinh[c + d*x])^3, x]
```

fricas [A] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^3d^2e^2x^2+2\,a^3cde^2x+a^3c^2e^2+\left(b^3d^2e^2x^2+2\,b^3cde^2x+b^3c^2e^2\right)\text{arsinh}\left(dx+c\right)^3+3\left(ab^2d^2e^2x^2+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((a^3*d^2*e^2*x^2 + 2*a^3*c*d*e^2*x + a^3*c^2*e^2 + (b^3*d^2*e^2*x^
2 + 2*b^3*c*d*e^2*x + b^3*c^2*e^2)*arcsinh(d*x + c)^3 + 3*(a*b^2*d^2*e^2*x^
2 + 2*a*b^2*c*d*e^2*x + a*b^2*c^2*e^2)*arcsinh(d*x + c)^2 + 3*(a^2*b*d^2*e^
```

```
2*x^2 + 2*a^2*b*c*d*e^2*x + a^2*b*c^2*e^2)*arcsinh(d*x + c))*sqrt(d*e*x + c
*e), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex + ce)^{\frac{5}{2}} (b \operatorname{arsinh}(dx + c) + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^(5/2)*(b*arcsinh(d*x + c) + a)^3, x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int (dex + ce)^{\frac{5}{2}} (a + b \operatorname{arcsinh}(dx + c))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^3,x)
```

```
[Out] int((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 2/7*(d*e*x + c*e)^(7/2)*a^3/(d*e) + 2/7*(b^3*d^3*e^(5/2)*x^3 + 3*b^3*c*d^2*
e^(5/2)*x^2 + 3*b^3*c^2*d*e^(5/2)*x + b^3*c^3*e^(5/2))*sqrt(d*x + c)*log(d*
x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3/d + integrate(-3/7*(((2*b^3*c^
4*e^(5/2) - (7*a*b^2*d^4*e^(5/2) - 2*b^3*d^4*e^(5/2))*x^4 - 7*(c^4*e^(5/2)
+ c^2*e^(5/2))*a*b^2 - 4*(7*a*b^2*c*d^3*e^(5/2) - 2*b^3*c*d^3*e^(5/2))*x^3
+ (12*b^3*c^2*d^2*e^(5/2) - 7*(6*c^2*d^2*e^(5/2) + d^2*e^(5/2))*a*b^2)*x^2
+ 2*(4*b^3*c^3*d*e^(5/2) - 7*(2*c^3*d*e^(5/2) + c*d*e^(5/2))*a*b^2)*x)*sqrt
(d^2*x^2 + 2*c*d*x + c^2 + 1)*sqrt(d*x + c) - ((7*a*b^2*d^5*e^(5/2) - 2*b^3
*d^5*e^(5/2))*x^5 + 5*(7*a*b^2*c*d^4*e^(5/2) - 2*b^3*c*d^4*e^(5/2))*x^4 + 7
*(c^5*e^(5/2) + c^3*e^(5/2))*a*b^2 - 2*(c^5*e^(5/2) + c^3*e^(5/2))*b^3 + (7
*(10*c^2*d^3*e^(5/2) + d^3*e^(5/2))*a*b^2 - 2*(10*c^2*d^3*e^(5/2) + d^3*e^(
5/2))*b^3)*x^3 + (7*(10*c^3*d^2*e^(5/2) + 3*c*d^2*e^(5/2))*a*b^2 - 2*(10*c^
3*d^2*e^(5/2) + 3*c*d^2*e^(5/2))*b^3)*x^2 + (7*(5*c^4*d*e^(5/2) + 3*c^2*d*e
^(5/2))*a*b^2 - 2*(5*c^4*d*e^(5/2) + 3*c^2*d*e^(5/2))*b^3)*x)*sqrt(d*x + c)
)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 - 7*((a^2*b*d^4*e^(5/2
)*x^4 + 4*a^2*b*c*d^3*e^(5/2)*x^3 + (6*c^2*d^2*e^(5/2) + d^2*e^(5/2))*a^2*b
*x^2 + 2*(2*c^3*d*e^(5/2) + c*d*e^(5/2))*a^2*b*x + (c^4*e^(5/2) + c^2*e^(5/
2))*a^2*b)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*sqrt(d*x + c) + (a^2*b*d^5*e^(
5/2)*x^5 + 5*a^2*b*c*d^4*e^(5/2)*x^4 + (10*c^2*d^3*e^(5/2) + d^3*e^(5/2))*a
^2*b*x^3 + (10*c^3*d^2*e^(5/2) + 3*c*d^2*e^(5/2))*a^2*b*x^2 + (5*c^4*d*e^(5
/2) + 3*c^2*d*e^(5/2))*a^2*b*x + (c^5*e^(5/2) + c^3*e^(5/2))*a^2*b)*sqrt(d*
x + c))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)))/(d^3*x^3 + 3*c*d^
2*x^2 + c^3 + (3*c^2*d + d)*x + (d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + c), x
)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(c\,e+d\,e\,x\right)}^{5/2}\,{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^(5/2)*(a + b*asinh(c + d*x))^3,x)
[Out] int((c*e + d*e*x)^(5/2)*(a + b*asinh(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(5/2)*(a+b*asinh(d*x+c))**3,x)
[Out] Timed out
```

3.246 $\int (ce+dex)^{3/2} \left(a+b\sinh^{-1}(c+dx)\right)^3 \, dx$

Optimal. Leaf size=82

$$\frac{2(e(c+dx))^{5/2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{5de}-\frac{6b\text{Int}\left(\frac{(e(c+dx))^{5/2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{\sqrt{(c+dx)^2+1}},x\right)}{5e}$$

```
[Out] 2/5*(e*(d*x+c))^(5/2)*(a+b*arcsinh(d*x+c))^3/d/e-6/5*b*Unintegrable((e*(d*x
+c))^(5/2)*(a+b*arcsinh(d*x+c))^2/(1+(d*x+c)^2)^(1/2),x)/e
```

Rubi [A] time = 0.21, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (ce+dex)^{3/2} \left(a+b\sinh^{-1}(c+dx)\right)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(c*e + d*e*x)^(3/2)*(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] (2*(e*(c + d*x))^(5/2)*(a + b*ArcSinh[c + d*x])^3)/(5*d*e) - (6*b*Defer[Sub
st][Defer[Int][((e*x)^(5/2)*(a + b*ArcSinh[x])^2)/Sqrt[1 + x^2], x], x, c +
 d*x])/(5*d*e)
```

Rubi steps

$$\int (ce+dex)^{3/2} \left(a+b\sinh^{-1}(c+dx)\right)^3 \, dx = \frac{\text{Subst}\left(\int (ex)^{3/2}\left(a+b\sinh^{-1}(x)\right)^3 \, dx,x,c+dx\right)}{d}$$

$$= \frac{2(e(c+dx))^{5/2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{5de}-\frac{(6b)\text{Subst}\left(\int \frac{(ex)^{5/2}\left(a+b\sinh\right.}{\sqrt{1+x^2}}\right.}{5de}$$

Mathematica [A] time = 73.21, size = 0, normalized size = 0.00

$$\int (ce+dex)^{3/2} \left(a+b\sinh^{-1}(c+dx)\right)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c*e + d*e*x)^(3/2)*(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] Integrate[(c*e + d*e*x)^(3/2)*(a + b*ArcSinh[c + d*x])^3, x]
```

fricas [A] time = 1.06, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^3dex+a^3ce+\left(b^3dex+b^3ce\right)\text{arsinh}\left(dx+c\right)^3+3\left(ab^2dex+ab^2ce\right)\text{arsinh}\left(dx+c\right)^2+3\left(a^2bdex+a^2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((a^3*d*e*x + a^3*c*e + (b^3*d*e*x + b^3*c*e)*arcsinh(d*x + c)^3 +
3*(a*b^2*d*e*x + a*b^2*c*e)*arcsinh(d*x + c)^2 + 3*(a^2*b*d*e*x + a^2*b*c*e
)*arcsinh(d*x + c))*sqrt(d*e*x + c*e), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex+ce)^{\frac{3}{2}}(b \operatorname{arsinh}(dx+c)+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^(3/2)*(b*arcsinh(d*x + c) + a)^3, x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int (dex+ce)^{\frac{3}{2}}(a+b \operatorname{arcsinh}(dx+c))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^3,x)
[Out] int((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2(dex+ce)^{\frac{5}{2}}a^3}{5de}+\frac{2\left(b^3d^2e^{\frac{3}{2}}x^2+2b^3cde^{\frac{3}{2}}x+b^3c^2e^{\frac{3}{2}}\right)\sqrt{dx+c}\log\left(dx+c+\sqrt{d^2x^2+2cdx+c^2+1}\right)^3}{5d}+\int -\frac{3(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
[Out] 2/5*(d*e*x + c*e)^(5/2)*a^3/(d*e) + 2/5*(b^3*d^2*e^(3/2)*x^2 + 2*b^3*c*d*e^
(3/2)*x + b^3*c^2*e^(3/2))*sqrt(d*x + c)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d
*x + c^2 + 1))^3/d + integrate(-3/5*(((2*b^3*c^3*e^(3/2) - 5*(c^3*e^(3/2) +
 c*e^(3/2))*a*b^2 - (5*a*b^2*d^3*e^(3/2) - 2*b^3*d^3*e^(3/2))*x^3 - 3*(5*a*
b^2*c*d^2*e^(3/2) - 2*b^3*c*d^2*e^(3/2))*x^2 + (6*b^3*c^2*d*e^(3/2) - 5*(3*
c^2*d*e^(3/2) + d*e^(3/2))*a*b^2)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*sqrt
(d*x + c) - ((5*a*b^2*d^4*e^(3/2) - 2*b^3*d^4*e^(3/2))*x^4 + 5*(c^4*e^(3/2)
 + c^2*e^(3/2))*a*b^2 - 2*(c^4*e^(3/2) + c^2*e^(3/2))*b^3 + 4*(5*a*b^2*c*d^
3*e^(3/2) - 2*b^3*c*d^3*e^(3/2))*x^3 + (5*(6*c^2*d^2*e^(3/2) + d^2*e^(3/2))
*a*b^2 - 2*(6*c^2*d^2*e^(3/2) + d^2*e^(3/2))*b^3)*x^2 + 2*(5*(2*c^3*d*e^(3/
2) + c*d*e^(3/2))*a*b^2 - 2*(2*c^3*d*e^(3/2) + c*d*e^(3/2))*b^3)*x)*sqrt(d*
x + c))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 - 5*((a^2*b*d^3*
e^(3/2)*x^3 + 3*a^2*b*c*d^2*e^(3/2)*x^2 + (3*c^2*d*e^(3/2) + d*e^(3/2))*a^2
*b*x + (c^3*e^(3/2) + c*e^(3/2))*a^2*b)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*s
qrt(d*x + c) + (a^2*b*d^4*e^(3/2)*x^4 + 4*a^2*b*c*d^3*e^(3/2)*x^3 + (6*c^2*
d^2*e^(3/2) + d^2*e^(3/2))*a^2*b*x^2 + 2*(2*c^3*d*e^(3/2) + c*d*e^(3/2))*a^
2*b*x + (c^4*e^(3/2) + c^2*e^(3/2))*a^2*b)*sqrt(d*x + c))*log(d*x + c + sqr
t(d^2*x^2 + 2*c*d*x + c^2 + 1)))/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (3*c^2*d +
d)*x + (d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + c), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e+d\,e\,x)^{3/2}\,(a+b\,\operatorname{asinh}(c+d\,x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^(3/2)*(a + b*asinh(c + d*x))^3,x)
[Out] int((c*e + d*e*x)^(3/2)*(a + b*asinh(c + d*x))^3, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\,(c+dx))^{\frac{3}{2}}\,(a+b\operatorname{asinh}(c+dx))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(3/2)*(a+b*asinh(d*x+c))**3,x)
```

```
[Out] Integral((e*(c + d*x))**(3/2)*(a + b*asinh(c + d*x))**3, x)
```

3.247 $\int \sqrt{ce+dex}\,\left(a+b\sinh^{-1}(c+dx)\right)^3\,dx$

Optimal. Leaf size=80

$$\frac{2(e(c+dx))^{3/2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{3de}-\frac{2b\text{Int}\left(\frac{(e(c+dx))^{3/2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{\sqrt{(c+dx)^2+1}},x\right)}{e}$$

```
[Out] 2/3*(e*(d*x+c))^(3/2)*(a+b*arcsinh(d*x+c))^3/d/e-2*b*Unintegrable((e*(d*x+c
))^(3/2)*(a+b*arcsinh(d*x+c))^2/(1+(d*x+c)^2)^(1/2),x)/e
```

Rubi [A] time = 0.20, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sqrt{ce+dex}\,\left(a+b\sinh^{-1}(c+dx)\right)^3\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[c*e + d*e*x]*(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] (2*(e*(c + d*x))^(3/2)*(a + b*ArcSinh[c + d*x])^3)/(3*d*e) - (2*b*Defer[Sub
st][Defer[Int][((e*x)^(3/2)*(a + b*ArcSinh[x])^2)/Sqrt[1 + x^2], x], x, c +
 d*x])/(d*e)
```

Rubi steps

$$\int \sqrt{ce+dex}\,\left(a+b\sinh^{-1}(c+dx)\right)^3\,dx=\frac{\text{Subst}\left(\int \sqrt{ex}\,\left(a+b\sinh^{-1}(x)\right)^3\,dx,x,c+dx\right)}{d}$$

$$=\frac{2(e(c+dx))^{3/2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{3de}-\frac{(2b)\,\text{Subst}\left(\int \frac{(ex)^{3/2}\left(a+b\sin\right.}{\sqrt{1+x}}\right.}{de}$$

Mathematica [A] time = 93.83, size = 0, normalized size = 0.00

$$\int \sqrt{ce+dex}\,\left(a+b\sinh^{-1}(c+dx)\right)^3\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[c*e + d*e*x]*(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] Integrate[Sqrt[c*e + d*e*x]*(a + b*ArcSinh[c + d*x])^3, x]
```

fricas [A] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^3\,\text{arsinh}\,(dx+c)^3+3\,ab^2\,\text{arsinh}\,(dx+c)^2+3\,a^2b\,\text{arsinh}\,(dx+c)+a^3\right)\sqrt{dex+ce},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3*(d*e*x+c*e)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*arcsinh(d*x + c)^3 + 3*a*b^2*arcsinh(d*x + c)^2 + 3*a^2*b*arc
sinh(d*x + c) + a^3)*sqrt(d*e*x + c*e), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{dex+ce}\,(b\,\mathrm{arsinh}\,(dx+c)+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3*(d*e*x+c*e)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^3, x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int (a+b\,\mathrm{arcsinh}\,(dx+c))^3\,\sqrt{dex+ce}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^3*(d*e*x+c*e)^(1/2),x)
[Out] int((a+b*arcsinh(d*x+c))^3*(d*e*x+c*e)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\left(b^3d\sqrt{e}\,x+b^3c\sqrt{e}\right)\sqrt{dx+c}\,\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)^3}{3\,d}+\frac{2\,(dex+ce)^{\frac{3}{2}}a^3}{3\,de}+\int-\frac{\left(\left(2\,b^3c^2\sqrt{e}\,-3\,(c^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3*(d*e*x+c*e)^(1/2),x, algorithm="maxima")
[Out] 2/3*(b^3*d*sqrt(e)*x + b^3*c*sqrt(e))*sqrt(d*x + c)*log(d*x + c + sqrt(d^2*
x^2 + 2*c*d*x + c^2 + 1))^3/d + 2/3*(d*e*x + c*e)^(3/2)*a^3/(d*e) + integra
te(-(((2*b^3*c^2*sqrt(e) - 3*(c^2*sqrt(e) + sqrt(e))*a*b^2 - (3*a*b^2*d^2*s
qrt(e) - 2*b^3*d^2*sqrt(e))*x^2 - 2*(3*a*b^2*c*d*sqrt(e) - 2*b^3*c*d*sqrt(e
))*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*sqrt(d*x + c) - (3*(c^3*sqrt(e) + c
*sqrt(e))*a*b^2 - 2*(c^3*sqrt(e) + c*sqrt(e))*b^3 + (3*a*b^2*d^3*sqrt(e) -
2*b^3*d^3*sqrt(e))*x^3 + 3*(3*a*b^2*c*d^2*sqrt(e) - 2*b^3*c*d^2*sqrt(e))*x^
2 + (3*(3*c^2*d*sqrt(e) + d*sqrt(e))*a*b^2 - 2*(3*c^2*d*sqrt(e) + d*sqrt(e)
)*b^3)*x)*sqrt(d*x + c))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2
 - 3*((a^2*b*d^2*sqrt(e)*x^2 + 2*a^2*b*c*d*sqrt(e)*x + (c^2*sqrt(e) + sqrt(
e))*a^2*b)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*sqrt(d*x + c) + (a^2*b*d^3*sqr
t(e)*x^3 + 3*a^2*b*c*d^2*sqrt(e)*x^2 + (3*c^2*d*sqrt(e) + d*sqrt(e))*a^2*b*
x + (c^3*sqrt(e) + c*sqrt(e))*a^2*b)*sqrt(d*x + c))*log(d*x + c + sqrt(d^2*
x^2 + 2*c*d*x + c^2 + 1)))/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (3*c^2*d + d)*x +
 (d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + c), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{c\,e+d\,e\,x}\,\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^(1/2)*(a + b*asinh(c + d*x))^3,x)
[Out] int((c*e + d*e*x)^(1/2)*(a + b*asinh(c + d*x))^3, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{e\,(c+dx)}\,\left(a+b\,\mathrm{asinh}\,(c+dx)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**3*(d*e*x+c*e)**(1/2),x)
```

```
[Out] Integral(sqrt(e*(c + d*x))*(a + b*asinh(c + d*x))**3, x)
```

3.248 $\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{\sqrt{ce+dex}}\,dx$

Optimal. Leaf size=78

$$\frac{2\sqrt{e(c+dx)}\left(a+b\sinh^{-1}(c+dx)\right)^3}{de}-\frac{6b\text{Int}\left(\frac{\sqrt{e(c+dx)}\left(a+b\sinh^{-1}(c+dx)\right)^2}{\sqrt{(c+dx)^2+1}},x\right)}{e}$$

```
[Out] 2*(a+b*arcsinh(d*x+c))^3*(e*(d*x+c))^(1/2)/d/e-6*b*Unintegrable((a+b*arcsin
h(d*x+c))^2*(e*(d*x+c))^(1/2)/(1+(d*x+c)^2)^(1/2),x)/e
```

Rubi [A] time = 0.18, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{\sqrt{ce+dex}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcSinh[c + d*x])^3/Sqrt[c*e + d*e*x],x]
```

```
[Out] (2*Sqrt[e*(c + d*x)]*(a + b*ArcSinh[c + d*x])^3)/(d*e) - (6*b*Defer[Subst][
Defer[Int][(Sqrt[e*x]*(a + b*ArcSinh[x])^2)/Sqrt[1 + x^2], x], x, c + d*x])
/(d*e)
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{\sqrt{ce+dex}}\,dx=\frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^3}{\sqrt{ex}}\,dx,x,c+dx\right)}{d}$$
$$=\frac{2\sqrt{e(c+dx)}\left(a+b\sinh^{-1}(c+dx)\right)^3}{de}-\frac{(6b)\,\text{Subst}\left(\int \frac{\sqrt{ex}\left(a+b\sinh^{-1}(x)\right)^2}{\sqrt{1+x^2}}\,dx,x,c+\right.}{de}$$

Mathematica [A] time = 9.44, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{\sqrt{ce+dex}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^3/Sqrt[c*e + d*e*x],x]
```

```
[Out] Integrate[(a + b*ArcSinh[c + d*x])^3/Sqrt[c*e + d*e*x], x]
```

fricas [A] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3\operatorname{arsinh}(dx+c)^3+3\,ab^2\operatorname{arsinh}(dx+c)^2+3\,a^2b\operatorname{arsinh}(dx+c)+a^3}{\sqrt{dex+ce}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*arcsinh(d*x + c)^3 + 3*a*b^2*arcsinh(d*x + c)^2 + 3*a^2*b*arc
sinh(d*x + c) + a^3)/sqrt(d*e*x + c*e), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arsinh}(dx + c) + a)^3}{\sqrt{dex + ce}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^3/sqrt(d*e*x + c*e), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{arcsinh}(dx + c))^3}{\sqrt{dex + ce}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\sqrt{dex + ce}\,a^3}{de} + \frac{2\left(b^3 d\sqrt{e}\,x + b^3 c\sqrt{e}\right)\log\left(dx + c + \sqrt{d^2x^2 + 2\,cdx + c^2 + 1}\right)^3}{\sqrt{dx + c}\,de} - \int \frac{3\left(\left(2\left(c^3\sqrt{e} + c\sqrt{e}\right)b^3 - (ab\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(d*e*x + c*e)*a^3/(d*e) + 2*(b^3*d*sqrt(e)*x + b^3*c*sqrt(e))*log(d*x
 + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3/(sqrt(d*x + c)*d*e) - integrate
(3*((2*(c^3*sqrt(e) + c*sqrt(e))*b^3 - (a*b^2*d^3*sqrt(e) - 2*b^3*d^3*sqrt(
e))*x^3 - (a*c^3*sqrt(e) + a*c*sqrt(e))*b^2 - 3*(a*b^2*c*d^2*sqrt(e) - 2*b^
3*c*d^2*sqrt(e))*x^2 + (2*(3*c^2*d*sqrt(e) + d*sqrt(e))*b^3 - (3*a*c^2*d*sq
rt(e) + a*d*sqrt(e))*b^2)*x + (2*b^3*c^2*sqrt(e) - (a*c^2*sqrt(e) + a*sqrt(
e))*b^2 - (a*b^2*d^2*sqrt(e) - 2*b^3*d^2*sqrt(e))*x^2 - 2*(a*b^2*c*d*sqrt(e
) - 2*b^3*c*d*sqrt(e))*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c +
sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 - (a^2*b*d^3*sqrt(e)*x^3 + 3*a^2*b*c*d
^2*sqrt(e)*x^2 + (3*a^2*c^2*d*sqrt(e) + a^2*d*sqrt(e))*b*x + (a^2*c^3*sqrt(
e) + a^2*c*sqrt(e))*b + (a^2*b*d^2*sqrt(e)*x^2 + 2*a^2*b*c*d*sqrt(e)*x + (a
^2*c^2*sqrt(e) + a^2*sqrt(e))*b)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x
 + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)))/((d^2*e*x^2 + 2*c*d*e*x + c^2*e
+ e)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*sqrt(d*x + c) + (d^3*e*x^3 + 3*c*d^2
*e*x^2 + c^3*e + c*e + (3*c^2*d*e + d*e)*x)*sqrt(d*x + c)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b \operatorname{asinh}(c + d\,x))^3}{\sqrt{c\,e + d\,e\,x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^3/(c*e + d*e*x)^(1/2),x)
```

```
[Out] int((a + b*asinh(c + d*x))^3/(c*e + d*e*x)^(1/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{asinh}(c + dx))^3}{\sqrt{e(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**3/(d*e*x+c*e)**(1/2),x)
[Out] Integral((a + b*asinh(c + d*x))**3/sqrt(e*(c + d*x)), x)
```

3.249 $\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{(ce+dex)^{3/2}}\,dx$

Optimal. Leaf size=78

$$\frac{6b\mathrm{Int}\left(\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{\sqrt{(c+dx)^2+1}\sqrt{e(c+dx)}},x\right)}{e}-\frac{2\left(a+b\sinh^{-1}(c+dx)\right)^3}{de\sqrt{e(c+dx)}}$$

```
[Out] -2*(a+b*arcsinh(d*x+c))^3/d/e/(e*(d*x+c))^(1/2)+6*b*Unintegrable((a+b*arcsi
nh(d*x+c))^2/(e*(d*x+c))^(1/2)/(1+(d*x+c)^2)^(1/2),x)/e
```

Rubi [A] time = 0.19, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{(ce+dex)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^(3/2),x]
```

```
[Out] (-2*(a + b*ArcSinh[c + d*x])^3)/(d*e*Sqrt[e*(c + d*x)]) + (6*b*Defer[Subst]
[Defer[Int][(a + b*ArcSinh[x])^2/(Sqrt[e*x]*Sqrt[1 + x^2]), x], x, c + d*x]
)/(d*e)
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{(ce+dex)^{3/2}}\,dx &= \frac{\mathrm{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^3}{(ex)^{3/2}}\,dx,x,c+dx\right)}{d}\\ &= -\frac{2\left(a+b\sinh^{-1}(c+dx)\right)^3}{de\sqrt{e(c+dx)}}+\frac{(6b)\,\mathrm{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^2}{\sqrt{ex}\sqrt{1+x^2}}\,dx,x,c+dx\right)}{de}\end{aligned}$$

Mathematica [A] time = 19.93, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{(ce+dex)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^(3/2),x]
```

```
[Out] Integrate[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^(3/2), x]
```

fricas [A] time = 0.72, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(b^3\operatorname{arsinh}(dx+c)^3+3ab^2\operatorname{arsinh}(dx+c)^2+3a^2b\operatorname{arsinh}(dx+c)+a^3\right)\sqrt{dex+ce}}{d^2e^2x^2+2cde^2x+c^2e^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*arcsinh(d*x + c)^3 + 3*a*b^2*arcsinh(d*x + c)^2 + 3*a^2*b*arc
sinh(d*x + c) + a^3)*sqrt(d*e*x + c*e)/(d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2
), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arsinh}(dx + c) + a)^3}{(dex + ce)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^3/(d*e*x + c*e)^(3/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{arcsinh}(dx + c))^3}{(dex + ce)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(3/2),x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\, b^3 \log\left(dx + c + \sqrt{d^2x^2 + 2\, cdx + c^2 + 1}\right)^3}{\sqrt{dx + c}\, de^{\frac{3}{2}}} - \frac{2\, a^3}{\sqrt{dex + ce}\, de} + \int \frac{3\left(\left(2\left(c^3 + c\right)b^3 + \left(ab^2d^3 + 2\, b^3d^3\right)x^3 + \left(ac^3 + c\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(3/2),x, algorithm="maxima")
```

```
[Out] -2*b^3*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3/(sqrt(d*x + c)*d*
e^(3/2)) - 2*a^3/(sqrt(d*e*x + c*e)*d*e) + integrate(3*((2*(c^3 + c)*b^3 +
(a*b^2*d^3 + 2*b^3*d^3)*x^3 + (a*c^3 + a*c)*b^2 + 3*(a*b^2*c*d^2 + 2*b^3*c*
d^2)*x^2 + (2*(3*c^2*d + d)*b^3 + (3*a*c^2*d + a*d)*b^2)*x + (2*b^3*c^2 + (
a*c^2 + a)*b^2 + (a*b^2*d^2 + 2*b^3*d^2)*x^2 + 2*(a*b^2*c*d + 2*b^3*c*d)*x)
*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x +
c^2 + 1))^2 + (a^2*b*d^3*x^3 + 3*a^2*b*c*d^2*x^2 + (3*a^2*c^2*d + a^2*d)*b*
x + (a^2*c^3 + a^2*c)*b + (a^2*b*d^2*x^2 + 2*a^2*b*c*d*x + (a^2*c^2 + a^2)*
b)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x
+ c^2 + 1)))/((d^3*e^(3/2)*x^3 + 3*c*d^2*e^(3/2)*x^2 + c^3*e^(3/2) + c*e^(3
/2) + (3*c^2*d*e^(3/2) + d*e^(3/2))*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*sq
rt(d*x + c) + (d^4*e^(3/2)*x^4 + 4*c*d^3*e^(3/2)*x^3 + c^4*e^(3/2) + c^2*e^
(3/2) + (6*c^2*d^2*e^(3/2) + d^2*e^(3/2))*x^2 + 2*(2*c^3*d*e^(3/2) + c*d*e^
(3/2))*x)*sqrt(d*x + c)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b \operatorname{asinh}(c + d\,x))^3}{(c\,e + d\,e\,x)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^3/(c*e + d*e*x)^(3/2),x)
```

```
[Out] int((a + b*asinh(c + d*x))^3/(c*e + d*e*x)^(3/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{asinh}(c + dx))^3}{(e(c + dx))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**3/(d*e*x+c*e)**(3/2),x)
[Out] Integral((a + b*asinh(c + d*x))**3/(e*(c + d*x))**(3/2), x)
```

3.250 $\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{(ce+dex)^{5/2}}\,dx$

Optimal. Leaf size=80

$$\frac{2b\mathrm{Int}\left(\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{\sqrt{(c+dx)^2+1}\,(e(c+dx))^{3/2}},x\right)}{e}-\frac{2\left(a+b\sinh^{-1}(c+dx)\right)^3}{3de(e(c+dx))^{3/2}}$$

```
[Out] -2/3*(a+b*arcsinh(d*x+c))^3/d/e/(e*(d*x+c))^(3/2)+2*b*Unintegrable((a+b*arc
sinh(d*x+c))^2/(e*(d*x+c))^(3/2)/(1+(d*x+c)^2)^(1/2),x)/e
```

Rubi [A] time = 0.21, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{(ce+dex)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^(5/2),x]
```

```
[Out] (-2*(a + b*ArcSinh[c + d*x])^3)/(3*d*e*(e*(c + d*x))^(3/2)) + (2*b*Defer[Su
bst][Defer[Int][(a + b*ArcSinh[x])^2/((e*x)^(3/2)*Sqrt[1 + x^2]), x], x, c
+ d*x])/(d*e)
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{(ce+dex)^{5/2}}\,dx &= \frac{\mathrm{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^3}{(ex)^{5/2}}\,dx,x,c+dx\right)}{d}\\ &= -\frac{2\left(a+b\sinh^{-1}(c+dx)\right)^3}{3de(e(c+dx))^{3/2}}+\frac{(2b)\,\mathrm{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^2}{(ex)^{3/2}\sqrt{1+x^2}}\,dx,x,c+dx\right)}{de}\end{aligned}$$

Mathematica [A] time = 23.07, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{(ce+dex)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^(5/2),x]
```

```
[Out] Integrate[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^(5/2), x]
```

fricas [A] time = 0.76, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(b^3\,\mathrm{arsinh}\,(dx+c)^3+3\,ab^2\,\mathrm{arsinh}\,(dx+c)^2+3\,a^2b\,\mathrm{arsinh}\,(dx+c)+a^3\right)\sqrt{dex+ce}}{d^3e^3x^3+3\,cd^2e^3x^2+3\,c^2de^3x+c^3e^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*arcsinh(d*x + c)^3 + 3*a*b^2*arcsinh(d*x + c)^2 + 3*a^2*b*arc
sinh(d*x + c) + a^3)*sqrt(d*e*x + c*e)/(d^3*e^3*x^3 + 3*c*d^2*e^3*x^2 + 3*c
^2*d*e^3*x + c^3*e^3), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arsinh}(dx+c)+a)^3}{(dex+ce)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^3/(d*e*x + c*e)^(5/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b \operatorname{arcsinh}(dx+c))^3}{(dex+ce)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(5/2),x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(5/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\,b^3\sqrt{e}\,\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)^3}{3\left(d^2e^3x+cde^3\right)\sqrt{dx+c}}-\frac{2\,a^3}{3\,(dex+ce)^{\frac{3}{2}}de}+\int \frac{\left(2\left(c^3\sqrt{e}+c\sqrt{e}\right)b^3+\left(3\,ab^2d^3\sqrt{e}+2\,b\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*b^3*sqrt(e)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3/((d^2*e
^3*x + c*d*e^3)*sqrt(d*x + c)) - 2/3*a^3/((d*e*x + c*e)^(3/2)*d*e) + integr
ate(((2*(c^3*sqrt(e) + c*sqrt(e))*b^3 + (3*a*b^2*d^3*sqrt(e) + 2*b^3*d^3*sq
rt(e))*x^3 + 3*(a*c^3*sqrt(e) + a*c*sqrt(e))*b^2 + 3*(3*a*b^2*c*d^2*sqrt(e)
 + 2*b^3*c*d^2*sqrt(e))*x^2 + (2*(3*c^2*d*sqrt(e) + d*sqrt(e))*b^3 + 3*(3*a
*c^2*d*sqrt(e) + a*d*sqrt(e))*b^2)*x + (2*b^3*c^2*sqrt(e) + 3*(a*c^2*sqrt(e
) + a*sqrt(e))*b^2 + (3*a*b^2*d^2*sqrt(e) + 2*b^3*d^2*sqrt(e))*x^2 + 2*(3*a
*b^2*c*d*sqrt(e) + 2*b^3*c*d*sqrt(e))*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))
*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 + 3*(a^2*b*d^3*sqrt(e)*
x^3 + 3*a^2*b*c*d^2*sqrt(e)*x^2 + (3*a^2*c^2*d*sqrt(e) + a^2*d*sqrt(e))*b*x
 + (a^2*c^3*sqrt(e) + a^2*c*sqrt(e))*b + (a^2*b*d^2*sqrt(e)*x^2 + 2*a^2*b*c
*d*sqrt(e)*x + (a^2*c^2*sqrt(e) + a^2*sqrt(e))*b)*sqrt(d^2*x^2 + 2*c*d*x +
c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)))/((d^4*e^3*x^4 +
 4*c*d^3*e^3*x^3 + c^4*e^3 + c^2*e^3 + (6*c^2*d^2*e^3 + d^2*e^3)*x^2 + 2*(2
*c^3*d*e^3 + c*d*e^3)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*sqrt(d*x + c) +
(d^5*e^3*x^5 + 5*c*d^4*e^3*x^4 + c^5*e^3 + c^3*e^3 + (10*c^2*d^3*e^3 + d^3*
e^3)*x^3 + (10*c^3*d^2*e^3 + 3*c*d^2*e^3)*x^2 + (5*c^4*d*e^3 + 3*c^2*d*e^3)
*x)*sqrt(d*x + c)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{asinh}(c+d\,x))^3}{(c\,e+d\,e\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^3/(c*e + d*e*x)^(5/2),x)
[Out] int((a + b*asinh(c + d*x))^3/(c*e + d*e*x)^(5/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{asinh}\,(c+dx))^3}{(e\,(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**3/(d*e*x+c*e)**(5/2),x)
[Out] Integral((a + b*asinh(c + d*x))**3/(e*(c + d*x))**(5/2), x)
```

3.251 $\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{(ce+dex)^{7/2}}\,dx$

Optimal. Leaf size=82

$$\frac{6b\text{Int}\left(\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{\sqrt{(c+dx)^2+1}\,(e(c+dx))^{5/2}},x\right)}{5e}-\frac{2\left(a+b\sinh^{-1}(c+dx)\right)^3}{5de(e(c+dx))^{5/2}}$$

```
[Out] -2/5*(a+b*arcsinh(d*x+c))^3/d/e/(e*(d*x+c))^(5/2)+6/5*b*Unintegrable((a+b*a
rcsinh(d*x+c))^2/(e*(d*x+c))^(5/2)/(1+(d*x+c)^2)^(1/2),x)/e
```

Rubi [A] time = 0.22, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{(ce+dex)^{7/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^(7/2),x]
```

```
[Out] (-2*(a + b*ArcSinh[c + d*x])^3)/(5*d*e*(e*(c + d*x))^(5/2)) + (6*b*Defer[Su
bst][Defer[Int][(a + b*ArcSinh[x])^2/((e*x)^(5/2)*Sqrt[1 + x^2]), x], x, c
+ d*x])/(5*d*e)
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{(ce+dex)^{7/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^3}{(ex)^{7/2}}\,dx,x,c+dx\right)}{d}\\ &= -\frac{2\left(a+b\sinh^{-1}(c+dx)\right)^3}{5de(e(c+dx))^{5/2}}+\frac{(6b)\,\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^2}{(ex)^{5/2}\sqrt{1+x^2}}\,dx,x,c+dx\right)}{5de}\end{aligned}$$

Mathematica [A] time = 73.71, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{(ce+dex)^{7/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^(7/2),x]
```

```
[Out] Integrate[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^(7/2), x]
```

fricas [A] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^3\,\text{arsinh}\left(dx+c\right)^3+3\,ab^2\,\text{arsinh}\left(dx+c\right)^2+3\,a^2b\,\text{arsinh}\left(dx+c\right)+a^3\right)\sqrt{dex+ce}}{d^4e^4x^4+4\,cd^3e^4x^3+6\,c^2d^2e^4x^2+4\,c^3de^4x+c^4e^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*arcsinh(d*x + c)^3 + 3*a*b^2*arcsinh(d*x + c)^2 + 3*a^2*b*arc
sinh(d*x + c) + a^3)*sqrt(d*e*x + c*e)/(d^4*e^4*x^4 + 4*c*d^3*e^4*x^3 + 6*c
^2*d^2*e^4*x^2 + 4*c^3*d*e^4*x + c^4*e^4), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\mathrm{arsinh}\,(dx+c)+a)^3}{(dex+ce)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^3/(d*e*x + c*e)^(7/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(dx+c))^3}{(dex+ce)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(7/2),x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(7/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\,b^3\sqrt{e}\,\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)^3}{5\left(d^3e^4x^2+2\,cd^2e^4x+c^2de^4\right)\sqrt{dx+c}}-\frac{2\,a^3}{5\,(dex+ce)^{\frac{5}{2}}de}+\int \frac{3\left(\left(2\left(c^3\sqrt{e}+c\sqrt{e}\right)b^3+\left(5\,ab^2d^3\sqrt{e}+2\,b\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(7/2),x, algorithm="maxima")
```

```
[Out] -2/5*b^3*sqrt(e)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3/((d^3*e
^4*x^2 + 2*c*d^2*e^4*x + c^2*d*e^4)*sqrt(d*x + c)) - 2/5*a^3/((d*e*x + c*e)
^(5/2)*d*e) + integrate(3/5*((2*(c^3*sqrt(e) + c*sqrt(e))*b^3 + (5*a*b^2*d^
3*sqrt(e) + 2*b^3*d^3*sqrt(e))*x^3 + 5*(a*c^3*sqrt(e) + a*c*sqrt(e))*b^2 +
3*(5*a*b^2*c*d^2*sqrt(e) + 2*b^3*c*d^2*sqrt(e))*x^2 + (2*(3*c^2*d*sqrt(e) +
 d*sqrt(e))*b^3 + 5*(3*a*c^2*d*sqrt(e) + a*d*sqrt(e))*b^2)*x + (2*b^3*c^2*s
qrt(e) + 5*(a*c^2*sqrt(e) + a*sqrt(e))*b^2 + (5*a*b^2*d^2*sqrt(e) + 2*b^3*d
^2*sqrt(e))*x^2 + 2*(5*a*b^2*c*d*sqrt(e) + 2*b^3*c*d*sqrt(e))*x)*sqrt(d^2*x
^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2
 + 5*(a^2*b*d^3*sqrt(e)*x^3 + 3*a^2*b*c*d^2*sqrt(e)*x^2 + (3*a^2*c^2*d*sqrt
(e) + a^2*d*sqrt(e))*b*x + (a^2*c^3*sqrt(e) + a^2*c*sqrt(e))*b + (a^2*b*d^2
*sqrt(e)*x^2 + 2*a^2*b*c*d*sqrt(e)*x + (a^2*c^2*sqrt(e) + a^2*sqrt(e))*b)*s
qrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^
2 + 1)))/((d^5*e^4*x^5 + 5*c*d^4*e^4*x^4 + c^5*e^4 + c^3*e^4 + (10*c^2*d^3*
e^4 + d^3*e^4)*x^3 + (10*c^3*d^2*e^4 + 3*c*d^2*e^4)*x^2 + (5*c^4*d*e^4 + 3*
c^2*d*e^4)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*sqrt(d*x + c) + (d^6*e^4*x^
6 + 6*c*d^5*e^4*x^5 + c^6*e^4 + c^4*e^4 + (15*c^2*d^4*e^4 + d^4*e^4)*x^4 +
4*(5*c^3*d^3*e^4 + c*d^3*e^4)*x^3 + 3*(5*c^4*d^2*e^4 + 2*c^2*d^2*e^4)*x^2 +
 2*(3*c^5*d*e^4 + 2*c^3*d*e^4)*x)*sqrt(d*x + c)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{asinh}\,(c+d\,x))^3}{(c\,e+d\,e\,x)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^3/(c*e + d*e*x)^(7/2),x)
[Out] int((a + b*asinh(c + d*x))^3/(c*e + d*e*x)^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**3/(d*e*x+c*e)**(7/2),x)
[Out] Timed out
```

3.252 $\int (ce+dex)^{7/2}\left(a+b\sinh^{-1}(c+dx)\right)^4\,dx$

Optimal. Leaf size=82

$$\frac{2(e(c+dx))^{9/2}\left(a+b\sinh^{-1}(c+dx)\right)^4}{9de}-\frac{8b\text{Int}\left(\frac{(e(c+dx))^{9/2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{\sqrt{(c+dx)^2+1}},x\right)}{9e}$$

```
[Out] 2/9*(e*(d*x+c))^(9/2)*(a+b*arcsinh(d*x+c))^4/d/e-8/9*b*Unintegrable((e*(d*x
+c))^(9/2)*(a+b*arcsinh(d*x+c))^3/(1+(d*x+c)^2)^(1/2),x)/e
```

Rubi [A] time = 0.21, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (ce+dex)^{7/2}\left(a+b\sinh^{-1}(c+dx)\right)^4\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c*e + d*e*x)^(7/2)*(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] (2*(e*(c + d*x))^(9/2)*(a + b*ArcSinh[c + d*x])^4)/(9*d*e) - (8*b*Defer[Sub
st][Defer[Int][((e*x)^(9/2)*(a + b*ArcSinh[x])^3)/Sqrt[1 + x^2], x], x, c +
 d*x])/(9*d*e)
```

Rubi steps

$$\begin{aligned}\int (ce+dex)^{7/2}\left(a+b\sinh^{-1}(c+dx)\right)^4\,dx &= \frac{\text{Subst}\left(\int (ex)^{7/2}\left(a+b\sinh^{-1}(x)\right)^4\,dx,x,c+dx\right)}{d}\\ &= \frac{2(e(c+dx))^{9/2}\left(a+b\sinh^{-1}(c+dx)\right)^4}{9de}-\frac{(8b)\,\text{Subst}\left(\int \frac{(ex)^{9/2}\left(a+b\sinh\right.}{\sqrt{1+x^2}}\right.}{9de}\end{aligned}$$

Mathematica [A] time = 98.23, size = 0, normalized size = 0.00

$$\int (ce+dex)^{7/2}\left(a+b\sinh^{-1}(c+dx)\right)^4\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c*e + d*e*x)^(7/2)*(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] Integrate[(c*e + d*e*x)^(7/2)*(a + b*ArcSinh[c + d*x])^4, x]
```

fricas [A] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^4d^3e^3x^3+3\,a^4cd^2e^3x^2+3\,a^4c^2de^3x+a^4c^3e^3+\left(b^4d^3e^3x^3+3\,b^4cd^2e^3x^2+3\,b^4c^2de^3x+b^4c^3e^3\right)\text{arsinh}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral((a^4*d^3*e^3*x^3 + 3*a^4*c*d^2*e^3*x^2 + 3*a^4*c^2*d*e^3*x + a^4*c
^3*e^3 + (b^4*d^3*e^3*x^3 + 3*b^4*c*d^2*e^3*x^2 + 3*b^4*c^2*d*e^3*x + b^4*c
^3*e^3)*arcsinh(d*x + c)^4 + 4*(a*b^3*d^3*e^3*x^3 + 3*a*b^3*c*d^2*e^3*x^2 +
 3*a*b^3*c^2*d*e^3*x + a*b^3*c^3*e^3)*arcsinh(d*x + c)^3 + 6*(a^2*b^2*d^3*e
```

```
^3*x^3 + 3*a^2*b^2*c*d^2*e^3*x^2 + 3*a^2*b^2*c^2*d*e^3*x + a^2*b^2*c^3*e^3)
*arcsinh(d*x + c)^2 + 4*(a^3*b*d^3*e^3*x^3 + 3*a^3*b*c*d^2*e^3*x^2 + 3*a^3*
b*c^2*d*e^3*x + a^3*b*c^3*e^3)*arcsinh(d*x + c))*sqrt(d*e*x + c*e), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex + ce)^{\frac{7}{2}}(b\operatorname{arsinh}(dx + c) + a)^4\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^(7/2)*(b*arcsinh(d*x + c) + a)^4, x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int (dex + ce)^{\frac{7}{2}} (a + b\operatorname{arcsinh}(dx + c))^4\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^4,x)
```

```
[Out] int((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^4,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 2/9*(d*e*x + c*e)^(9/2)*a^4/(d*e) + 2/9*(b^4*d^4*e^(7/2)*x^4 + 4*b^4*c*d^3*
e^(7/2)*x^3 + 6*b^4*c^2*d^2*e^(7/2)*x^2 + 4*b^4*c^3*d*e^(7/2)*x + b^4*c^4*e
^(7/2))*sqrt(d*x + c)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^4/d
+ integrate(-2/9*(2*((2*b^4*c^5*e^(7/2) - (9*a*b^3*d^5*e^(7/2) - 2*b^4*d^5*
e^(7/2))*x^5 - 9*(c^5*e^(7/2) + c^3*e^(7/2))*a*b^3 - 5*(9*a*b^3*c*d^4*e^(7/
2) - 2*b^4*c*d^4*e^(7/2))*x^4 + (20*b^4*c^2*d^3*e^(7/2) - 9*(10*c^2*d^3*e^(
7/2) + d^3*e^(7/2))*a*b^3)*x^3 + (20*b^4*c^3*d^2*e^(7/2) - 9*(10*c^3*d^2*e^
(7/2) + 3*c*d^2*e^(7/2))*a*b^3)*x^2 + (10*b^4*c^4*d*e^(7/2) - 9*(5*c^4*d*e^
(7/2) + 3*c^2*d*e^(7/2))*a*b^3)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*sqrt(d
*x + c) - ((9*a*b^3*d^6*e^(7/2) - 2*b^4*d^6*e^(7/2))*x^6 + 6*(9*a*b^3*c*d^5
*e^(7/2) - 2*b^4*c*d^5*e^(7/2))*x^5 + 9*(c^6*e^(7/2) + c^4*e^(7/2))*a*b^3 -
 2*(c^6*e^(7/2) + c^4*e^(7/2))*b^4 + (9*(15*c^2*d^4*e^(7/2) + d^4*e^(7/2))*
a*b^3 - 2*(15*c^2*d^4*e^(7/2) + d^4*e^(7/2))*b^4)*x^4 + 4*(9*(5*c^3*d^3*e^(
7/2) + c*d^3*e^(7/2))*a*b^3 - 2*(5*c^3*d^3*e^(7/2) + c*d^3*e^(7/2))*b^4)*x^
3 + 3*(9*(5*c^4*d^2*e^(7/2) + 2*c^2*d^2*e^(7/2))*a*b^3 - 2*(5*c^4*d^2*e^(7/
2) + 2*c^2*d^2*e^(7/2))*b^4)*x^2 + 2*(9*(3*c^5*d*e^(7/2) + 2*c^3*d*e^(7/2))
*a*b^3 - 2*(3*c^5*d*e^(7/2) + 2*c^3*d*e^(7/2))*b^4)*x)*sqrt(d*x + c))*log(d
*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3 - 27*((a^2*b^2*d^5*e^(7/2)*x^
5 + 5*a^2*b^2*c*d^4*e^(7/2)*x^4 + (10*c^2*d^3*e^(7/2) + d^3*e^(7/2))*a^2*b^
2*x^3 + (10*c^3*d^2*e^(7/2) + 3*c*d^2*e^(7/2))*a^2*b^2*x^2 + (5*c^4*d*e^(7/
2) + 3*c^2*d*e^(7/2))*a^2*b^2*x + (c^5*e^(7/2) + c^3*e^(7/2))*a^2*b^2)*sqrt
(d^2*x^2 + 2*c*d*x + c^2 + 1)*sqrt(d*x + c) + (a^2*b^2*d^6*e^(7/2)*x^6 + 6*
a^2*b^2*c*d^5*e^(7/2)*x^5 + (15*c^2*d^4*e^(7/2) + d^4*e^(7/2))*a^2*b^2*x^4
+ 4*(5*c^3*d^3*e^(7/2) + c*d^3*e^(7/2))*a^2*b^2*x^3 + 3*(5*c^4*d^2*e^(7/2)
+ 2*c^2*d^2*e^(7/2))*a^2*b^2*x^2 + 2*(3*c^5*d*e^(7/2) + 2*c^3*d*e^(7/2))*a^
2*b^2*x + (c^6*e^(7/2) + c^4*e^(7/2))*a^2*b^2)*sqrt(d*x + c))*log(d*x + c +
 sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 - 18*((a^3*b*d^5*e^(7/2)*x^5 + 5*a^3*
b*c*d^4*e^(7/2)*x^4 + (10*c^2*d^3*e^(7/2) + d^3*e^(7/2))*a^3*b*x^3 + (10*c^
3*d^2*e^(7/2) + 3*c*d^2*e^(7/2))*a^3*b*x^2 + (5*c^4*d*e^(7/2) + 3*c^2*d*e^(
```

```
7/2))*a^3*b*x + (c^5*e^(7/2) + c^3*e^(7/2))*a^3*b)*sqrt(d^2*x^2 + 2*c*d*x +
 c^2 + 1)*sqrt(d*x + c) + (a^3*b*d^6*e^(7/2)*x^6 + 6*a^3*b*c*d^5*e^(7/2)*x^
5 + (15*c^2*d^4*e^(7/2) + d^4*e^(7/2))*a^3*b*x^4 + 4*(5*c^3*d^3*e^(7/2) + c
*d^3*e^(7/2))*a^3*b*x^3 + 3*(5*c^4*d^2*e^(7/2) + 2*c^2*d^2*e^(7/2))*a^3*b*x
^2 + 2*(3*c^5*d*e^(7/2) + 2*c^3*d*e^(7/2))*a^3*b*x + (c^6*e^(7/2) + c^4*e^(
7/2))*a^3*b)*sqrt(d*x + c))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)
))/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (3*c^2*d + d)*x + (d^2*x^2 + 2*c*d*x + c^
2 + 1)^(3/2) + c), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e+d\,e\,x)^{7/2}\,(a+b\,\mathrm{asinh}\,(c+d\,x))^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^(7/2)*(a + b*asinh(c + d*x))^4,x)
```

```
[Out] int((c*e + d*e*x)^(7/2)*(a + b*asinh(c + d*x))^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(7/2)*(a+b*asinh(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.253 $\int (ce+dex)^{5/2} \left(a+b\sinh^{-1}(c+dx)\right)^4 dx$

Optimal. Leaf size=82

$$\frac{2(e(c+dx))^{7/2}\left(a+b\sinh^{-1}(c+dx)\right)^4}{7de}-\frac{8b\text{Int}\left(\frac{(e(c+dx))^{7/2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{\sqrt{(c+dx)^2+1}},x\right)}{7e}$$

```
[Out] 2/7*(e*(d*x+c))^(7/2)*(a+b*arcsinh(d*x+c))^4/d/e-8/7*b*Unintegrable((e*(d*x
+c))^(7/2)*(a+b*arcsinh(d*x+c))^3/(1+(d*x+c)^2)^(1/2),x)/e
```

Rubi [A] time = 0.21, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (ce+dex)^{5/2} \left(a+b\sinh^{-1}(c+dx)\right)^4 dx$$

Verification is Not applicable to the result.

```
[In] Int[(c*e + d*e*x)^(5/2)*(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] (2*(e*(c + d*x))^(7/2)*(a + b*ArcSinh[c + d*x])^4)/(7*d*e) - (8*b*Defer[Sub
st][Defer[Int][((e*x)^(7/2)*(a + b*ArcSinh[x])^3)/Sqrt[1 + x^2], x], x, c +
 d*x])/(7*d*e)
```

Rubi steps

$$\int (ce+dex)^{5/2} \left(a+b\sinh^{-1}(c+dx)\right)^4 dx = \frac{\text{Subst}\left(\int (ex)^{5/2}\left(a+b\sinh^{-1}(x)\right)^4 dx, x, c+dx\right)}{d}$$

$$= \frac{2(e(c+dx))^{7/2}\left(a+b\sinh^{-1}(c+dx)\right)^4}{7de}-\frac{(8b)\,\text{Subst}\left(\int \frac{(ex)^{7/2}\left(a+b\sinh\right.}{\sqrt{1+}}\right.}{7d}$$

Mathematica [A] time = 128.47, size = 0, normalized size = 0.00

$$\int (ce+dex)^{5/2} \left(a+b\sinh^{-1}(c+dx)\right)^4 dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c*e + d*e*x)^(5/2)*(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] Integrate[(c*e + d*e*x)^(5/2)*(a + b*ArcSinh[c + d*x])^4, x]
```

fricas [A] time = 1.01, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^4d^2e^2x^2+2\,a^4cde^2x+a^4c^2e^2+\left(b^4d^2e^2x^2+2\,b^4cde^2x+b^4c^2e^2\right)\text{arsinh}\,(dx+c)^4+4\left(ab^3d^2e^2x^2+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral((a^4*d^2*e^2*x^2 + 2*a^4*c*d*e^2*x + a^4*c^2*e^2 + (b^4*d^2*e^2*x^
2 + 2*b^4*c*d*e^2*x + b^4*c^2*e^2)*arcsinh(d*x + c)^4 + 4*(a*b^3*d^2*e^2*x^
2 + 2*a*b^3*c*d*e^2*x + a*b^3*c^2*e^2)*arcsinh(d*x + c)^3 + 6*(a^2*b^2*d^2*
e^2*x^2 + 2*a^2*b^2*c*d*e^2*x + a^2*b^2*c^2*e^2)*arcsinh(d*x + c)^2 + 4*(a^
```

```
3*b*d^2*e^2*x^2 + 2*a^3*b*c*d*e^2*x + a^3*b*c^2*e^2)*arcsinh(d*x + c))*sqrt
(d*e*x + c*e), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex+ce)^{\frac{5}{2}}(b\operatorname{arsinh}(dx+c)+a)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^(5/2)*(b*arcsinh(d*x + c) + a)^4, x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int (dex+ce)^{\frac{5}{2}}\left(a+b\operatorname{arcsinh}(dx+c)\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^4,x)
```

```
[Out] int((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^4,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 2/7*(d*e*x + c*e)^(7/2)*a^4/(d*e) + 2/7*(b^4*d^3*e^(5/2)*x^3 + 3*b^4*c*d^2*
e^(5/2)*x^2 + 3*b^4*c^2*d*e^(5/2)*x + b^4*c^3*e^(5/2))*sqrt(d*x + c)*log(d*
x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^4/d + integrate(-2/7*(2*((2*b^4*
c^4*e^(5/2) - 7*(c^4*e^(5/2) + c^2*e^(5/2))*a*b^3 - (7*a*b^3*d^4*e^(5/2) -
2*b^4*d^4*e^(5/2))*x^4 - 4*(7*a*b^3*c*d^3*e^(5/2) - 2*b^4*c*d^3*e^(5/2))*x^
3 + (12*b^4*c^2*d^2*e^(5/2) - 7*(6*c^2*d^2*e^(5/2) + d^2*e^(5/2))*a*b^3)*x^
2 + 2*(4*b^4*c^3*d*e^(5/2) - 7*(2*c^3*d*e^(5/2) + c*d*e^(5/2))*a*b^3)*x)*sq
rt(d^2*x^2 + 2*c*d*x + c^2 + 1)*sqrt(d*x + c) - ((7*a*b^3*d^5*e^(5/2) - 2*b
^4*d^5*e^(5/2))*x^5 + 7*(c^5*e^(5/2) + c^3*e^(5/2))*a*b^3 - 2*(c^5*e^(5/2)
+ c^3*e^(5/2))*b^4 + 5*(7*a*b^3*c*d^4*e^(5/2) - 2*b^4*c*d^4*e^(5/2))*x^4 +
(7*(10*c^2*d^3*e^(5/2) + d^3*e^(5/2))*a*b^3 - 2*(10*c^2*d^3*e^(5/2) + d^3*e
^(5/2))*b^4)*x^3 + (7*(10*c^3*d^2*e^(5/2) + 3*c*d^2*e^(5/2))*a*b^3 - 2*(10*
c^3*d^2*e^(5/2) + 3*c*d^2*e^(5/2))*b^4)*x^2 + (7*(5*c^4*d*e^(5/2) + 3*c^2*d
*e^(5/2))*a*b^3 - 2*(5*c^4*d*e^(5/2) + 3*c^2*d*e^(5/2))*b^4)*x)*sqrt(d*x +
c))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3 - 21*((a^2*b^2*d^4*e
^(5/2)*x^4 + 4*a^2*b^2*c*d^3*e^(5/2)*x^3 + (6*c^2*d^2*e^(5/2) + d^2*e^(5/2)
)*a^2*b^2*x^2 + 2*(2*c^3*d*e^(5/2) + c*d*e^(5/2))*a^2*b^2*x + (c^4*e^(5/2)
+ c^2*e^(5/2))*a^2*b^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*sqrt(d*x + c) + (
a^2*b^2*d^5*e^(5/2)*x^5 + 5*a^2*b^2*c*d^4*e^(5/2)*x^4 + (10*c^2*d^3*e^(5/2)
 + d^3*e^(5/2))*a^2*b^2*x^3 + (10*c^3*d^2*e^(5/2) + 3*c*d^2*e^(5/2))*a^2*b^
2*x^2 + (5*c^4*d*e^(5/2) + 3*c^2*d*e^(5/2))*a^2*b^2*x + (c^5*e^(5/2) + c^3*
e^(5/2))*a^2*b^2)*sqrt(d*x + c))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2
 + 1))^2 - 14*((a^3*b*d^4*e^(5/2)*x^4 + 4*a^3*b*c*d^3*e^(5/2)*x^3 + (6*c^2*
d^2*e^(5/2) + d^2*e^(5/2))*a^3*b*x^2 + 2*(2*c^3*d*e^(5/2) + c*d*e^(5/2))*a^
3*b*x + (c^4*e^(5/2) + c^2*e^(5/2))*a^3*b)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1
)*sqrt(d*x + c) + (a^3*b*d^5*e^(5/2)*x^5 + 5*a^3*b*c*d^4*e^(5/2)*x^4 + (10*
c^2*d^3*e^(5/2) + d^3*e^(5/2))*a^3*b*x^3 + (10*c^3*d^2*e^(5/2) + 3*c*d^2*e^
(5/2))*a^3*b*x^2 + (5*c^4*d*e^(5/2) + 3*c^2*d*e^(5/2))*a^3*b*x + (c^5*e^(5/
2) + c^3*e^(5/2))*a^3*b)*sqrt(d*x + c))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*
```

```
x + c^2 + 1)))/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (3*c^2*d + d)*x + (d^2*x^2 +
2*c*d*x + c^2 + 1)^(3/2) + c), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e+d\,e\,x)^{5/2}\,(a+b\,\mathrm{asinh}\,(c+d\,x))^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^(5/2)*(a + b*asinh(c + d*x))^4,x)
```

```
[Out] int((c*e + d*e*x)^(5/2)*(a + b*asinh(c + d*x))^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(5/2)*(a+b*asinh(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.254 $\int (ce+dex)^{3/2} \left(a+b\sinh^{-1}(c+dx)\right)^4 \, dx$

Optimal. Leaf size=82

$$\frac{2(e(c+dx))^{5/2}\left(a+b\sinh^{-1}(c+dx)\right)^4}{5de}-\frac{8b\text{Int}\left(\frac{(e(c+dx))^{5/2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{\sqrt{(c+dx)^2+1}},x\right)}{5e}$$

```
[Out] 2/5*(e*(d*x+c))^(5/2)*(a+b*arcsinh(d*x+c))^4/d/e-8/5*b*Unintegrable((e*(d*x
+c))^(5/2)*(a+b*arcsinh(d*x+c))^3/(1+(d*x+c)^2)^(1/2),x)/e
```

Rubi [A] time = 0.21, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (ce+dex)^{3/2}\left(a+b\sinh^{-1}(c+dx)\right)^4 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(c*e + d*e*x)^(3/2)*(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] (2*(e*(c + d*x))^(5/2)*(a + b*ArcSinh[c + d*x])^4)/(5*d*e) - (8*b*Defer[Sub
st][Defer[Int][((e*x)^(5/2)*(a + b*ArcSinh[x])^3)/Sqrt[1 + x^2], x], x, c +
 d*x])/(5*d*e)
```

Rubi steps

$$\int (ce+dex)^{3/2}\left(a+b\sinh^{-1}(c+dx)\right)^4 \, dx=\frac{\text{Subst}\left(\int (ex)^{3/2}\left(a+b\sinh^{-1}(x)\right)^4 \, dx,x,c+dx\right)}{d}$$

$$=\frac{2(e(c+dx))^{5/2}\left(a+b\sinh^{-1}(c+dx)\right)^4}{5de}-\frac{(8b)\,\text{Subst}\left(\int \frac{(ex)^{5/2}\left(a+b\sinh\right.}{\sqrt{1+x^2}}\right.}{5de}$$

Mathematica [A] time = 83.67, size = 0, normalized size = 0.00

$$\int (ce+dex)^{3/2}\left(a+b\sinh^{-1}(c+dx)\right)^4 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c*e + d*e*x)^(3/2)*(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] Integrate[(c*e + d*e*x)^(3/2)*(a + b*ArcSinh[c + d*x])^4, x]
```

fricas [A] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^4dex+a^4ce+\left(b^4dex+b^4ce\right)\text{arsinh}(dx+c)^4+4\left(ab^3dex+ab^3ce\right)\text{arsinh}(dx+c)^3+6\left(a^2b^2dex+a\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral((a^4*d*e*x + a^4*c*e + (b^4*d*e*x + b^4*c*e)*arcsinh(d*x + c)^4 +
4*(a*b^3*d*e*x + a*b^3*c*e)*arcsinh(d*x + c)^3 + 6*(a^2*b^2*d*e*x + a^2*b^2
*c*e)*arcsinh(d*x + c)^2 + 4*(a^3*b*d*e*x + a^3*b*c*e)*arcsinh(d*x + c))*sq
rt(d*e*x + c*e), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex+ce)^{\frac{3}{2}}(b\operatorname{arsinh}(dx+c)+a)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^(3/2)*(b*arcsinh(d*x + c) + a)^4, x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int (dex+ce)^{\frac{3}{2}}\,(a+b\operatorname{arcsinh}(dx+c))^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^4,x)
```

```
[Out] int((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^4,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 2/5*(d*e*x + c*e)^(5/2)*a^4/(d*e) + 2/5*(b^4*d^2*e^(3/2)*x^2 + 2*b^4*c*d*e^
(3/2)*x + b^4*c^2*e^(3/2))*sqrt(d*x + c)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d
*x + c^2 + 1))^4/d + integrate(-2/5*(2*((2*b^4*c^3*e^(3/2) - 5*(c^3*e^(3/2)
 + c*e^(3/2))*a*b^3 - (5*a*b^3*d^3*e^(3/2) - 2*b^4*d^3*e^(3/2))*x^3 - 3*(5*
a*b^3*c*d^2*e^(3/2) - 2*b^4*c*d^2*e^(3/2))*x^2 + (6*b^4*c^2*d*e^(3/2) - 5*(
3*c^2*d*e^(3/2) + d*e^(3/2))*a*b^3)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*sq
rt(d*x + c) - (5*(c^4*e^(3/2) + c^2*e^(3/2))*a*b^3 - 2*(c^4*e^(3/2) + c^2*e
^(3/2))*b^4 + (5*a*b^3*d^4*e^(3/2) - 2*b^4*d^4*e^(3/2))*x^4 + 4*(5*a*b^3*c*
d^3*e^(3/2) - 2*b^4*c*d^3*e^(3/2))*x^3 + (5*(6*c^2*d^2*e^(3/2) + d^2*e^(3/2
))*a*b^3 - 2*(6*c^2*d^2*e^(3/2) + d^2*e^(3/2))*b^4)*x^2 + 2*(5*(2*c^3*d*e^(
3/2) + c*d*e^(3/2))*a*b^3 - 2*(2*c^3*d*e^(3/2) + c*d*e^(3/2))*b^4)*x)*sqrt(
d*x + c))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3 - 15*((a^2*b^2
*d^3*e^(3/2)*x^3 + 3*a^2*b^2*c*d^2*e^(3/2)*x^2 + (3*c^2*d*e^(3/2) + d*e^(3/
2))*a^2*b^2*x + (c^3*e^(3/2) + c*e^(3/2))*a^2*b^2)*sqrt(d^2*x^2 + 2*c*d*x +
 c^2 + 1)*sqrt(d*x + c) + (a^2*b^2*d^4*e^(3/2)*x^4 + 4*a^2*b^2*c*d^3*e^(3/2
)*x^3 + (6*c^2*d^2*e^(3/2) + d^2*e^(3/2))*a^2*b^2*x^2 + 2*(2*c^3*d*e^(3/2)
+ c*d*e^(3/2))*a^2*b^2*x + (c^4*e^(3/2) + c^2*e^(3/2))*a^2*b^2)*sqrt(d*x +
c))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 - 10*((a^3*b*d^3*e^(
3/2)*x^3 + 3*a^3*b*c*d^2*e^(3/2)*x^2 + (3*c^2*d*e^(3/2) + d*e^(3/2))*a^3*b*
x + (c^3*e^(3/2) + c*e^(3/2))*a^3*b)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*sqrt
(d*x + c) + (a^3*b*d^4*e^(3/2)*x^4 + 4*a^3*b*c*d^3*e^(3/2)*x^3 + (6*c^2*d^2
*e^(3/2) + d^2*e^(3/2))*a^3*b*x^2 + 2*(2*c^3*d*e^(3/2) + c*d*e^(3/2))*a^3*b
*x + (c^4*e^(3/2) + c^2*e^(3/2))*a^3*b)*sqrt(d*x + c))*log(d*x + c + sqrt(d
^2*x^2 + 2*c*d*x + c^2 + 1)))/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (3*c^2*d + d)*
x + (d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + c), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(c\,e+d\,e\,x\right)}^{3/2}\,{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^4 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^(3/2)*(a + b*asinh(c + d*x))^4,x)
```

```
[Out] int((c*e + d*e*x)^(3/2)*(a + b*asinh(c + d*x))^4, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (e(c+dx))^{\frac{3}{2}}(a+b\operatorname{asinh}(c+dx))^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(3/2)*(a+b*asinh(d*x+c))**4,x)
```

```
[Out] Integral((e*(c + d*x))**(3/2)*(a + b*asinh(c + d*x))**4, x)
```

3.255 $\int \sqrt{ce+dex}\left(a+b\sinh^{-1}(c+dx)\right)^4 dx$

Optimal. Leaf size=82

$$\frac{2(e(c+dx))^{3/2}\left(a+b\sinh^{-1}(c+dx)\right)^4}{3de}-\frac{8b\text{Int}\left(\frac{(e(c+dx))^{3/2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{\sqrt{(c+dx)^2+1}},x\right)}{3e}$$

```
[Out] 2/3*(e*(d*x+c))^(3/2)*(a+b*arcsinh(d*x+c))^4/d/e-8/3*b*Unintegrable((e*(d*x
+c))^(3/2)*(a+b*arcsinh(d*x+c))^3/(1+(d*x+c)^2)^(1/2),x)/e
```

Rubi [A] time = 0.19, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sqrt{ce+dex}\left(a+b\sinh^{-1}(c+dx)\right)^4 dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[c*e + d*e*x]*(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] (2*(e*(c + d*x))^(3/2)*(a + b*ArcSinh[c + d*x])^4)/(3*d*e) - (8*b*Defer[Sub
st][Defer[Int][((e*x)^(3/2)*(a + b*ArcSinh[x])^3)/Sqrt[1 + x^2], x], x, c +
 d*x])/(3*d*e)
```

Rubi steps

$$\int \sqrt{ce+dex}\left(a+b\sinh^{-1}(c+dx)\right)^4 dx=\frac{\text{Subst}\left(\int \sqrt{ex}\left(a+b\sinh^{-1}(x)\right)^4 dx,x,c+dx\right)}{d}$$

$$=\frac{2(e(c+dx))^{3/2}\left(a+b\sinh^{-1}(c+dx)\right)^4}{3de}-\frac{(8b)\,\text{Subst}\left(\int \frac{(ex)^{3/2}\left(a+b\sin\right.}{\sqrt{1+x}}\right.}{3de}$$

Mathematica [A] time = 120.92, size = 0, normalized size = 0.00

$$\int \sqrt{ce+dex}\left(a+b\sinh^{-1}(c+dx)\right)^4 dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[c*e + d*e*x]*(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] Integrate[Sqrt[c*e + d*e*x]*(a + b*ArcSinh[c + d*x])^4, x]
```

fricas [A] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^4\operatorname{arsinh}(dx+c)^4+4ab^3\operatorname{arsinh}(dx+c)^3+6a^2b^2\operatorname{arsinh}(dx+c)^2+4a^3b\operatorname{arsinh}(dx+c)+a^4\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4*(d*e*x+c*e)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^4*arcsinh(d*x + c)^4 + 4*a*b^3*arcsinh(d*x + c)^3 + 6*a^2*b^2*a
rcsinh(d*x + c)^2 + 4*a^3*b*arcsinh(d*x + c) + a^4)*sqrt(d*e*x + c*e), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{dex+ce}\,(b\,\mathrm{arsinh}\,(dx+c)+a)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4*(d*e*x+c*e)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^4, x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int (a+b\,\mathrm{arcsinh}\,(dx+c))^4\,\sqrt{dex+ce}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^4*(d*e*x+c*e)^(1/2),x)
[Out] int((a+b*arcsinh(d*x+c))^4*(d*e*x+c*e)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\left(b^4d\sqrt{e}\,x+b^4c\sqrt{e}\right)\sqrt{dx+c}\,\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)^4}{3\,d}+\frac{2\,(dex+ce)^{\frac{3}{2}}a^4}{3\,de}+\int-\frac{2\left(2\left(\left(2\,b^4c^2\sqrt{e}-3\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4*(d*e*x+c*e)^(1/2),x, algorithm="maxima")
[Out] 2/3*(b^4*d*sqrt(e)*x + b^4*c*sqrt(e))*sqrt(d*x + c)*log(d*x + c + sqrt(d^2*
x^2 + 2*c*d*x + c^2 + 1))^4/d + 2/3*(d*e*x + c*e)^(3/2)*a^4/(d*e) + integra
te(-2/3*(2*((2*b^4*c^2*sqrt(e) - 3*(c^2*sqrt(e) + sqrt(e))*a*b^3 - (3*a*b^3
*d^2*sqrt(e) - 2*b^4*d^2*sqrt(e))*x^2 - 2*(3*a*b^3*c*d*sqrt(e) - 2*b^4*c*d*
sqrt(e))*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*sqrt(d*x + c) - (3*(c^3*sqrt(
e) + c*sqrt(e))*a*b^3 - 2*(c^3*sqrt(e) + c*sqrt(e))*b^4 + (3*a*b^3*d^3*sqrt
(e) - 2*b^4*d^3*sqrt(e))*x^3 + 3*(3*a*b^3*c*d^2*sqrt(e) - 2*b^4*c*d^2*sqrt(
e))*x^2 + (3*(3*c^2*d*sqrt(e) + d*sqrt(e))*a*b^3 - 2*(3*c^2*d*sqrt(e) + d*s
qrt(e))*b^4)*x)*sqrt(d*x + c))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 +
 1))^3 - 9*((a^2*b^2*d^2*sqrt(e)*x^2 + 2*a^2*b^2*c*d*sqrt(e)*x + (c^2*sqrt(
e) + sqrt(e))*a^2*b^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*sqrt(d*x + c) + (a
^2*b^2*d^3*sqrt(e)*x^3 + 3*a^2*b^2*c*d^2*sqrt(e)*x^2 + (3*c^2*d*sqrt(e) + d
*sqrt(e))*a^2*b^2*x + (c^3*sqrt(e) + c*sqrt(e))*a^2*b^2)*sqrt(d*x + c))*log
(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 - 6*((a^3*b*d^2*sqrt(e)*x^2
 + 2*a^3*b*c*d*sqrt(e)*x + (c^2*sqrt(e) + sqrt(e))*a^3*b)*sqrt(d^2*x^2 + 2*
c*d*x + c^2 + 1)*sqrt(d*x + c) + (a^3*b*d^3*sqrt(e)*x^3 + 3*a^3*b*c*d^2*sqr
t(e)*x^2 + (3*c^2*d*sqrt(e) + d*sqrt(e))*a^3*b*x + (c^3*sqrt(e) + c*sqrt(e)
)*a^3*b)*sqrt(d*x + c))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)))/(
d^3*x^3 + 3*c*d^2*x^2 + c^3 + (3*c^2*d + d)*x + (d^2*x^2 + 2*c*d*x + c^2 +
1)^(3/2) + c), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{c\,e+d\,e\,x}\,\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^(1/2)*(a + b*asinh(c + d*x))^4,x)
```

```
[Out] int((c*e + d*e*x)^(1/2)*(a + b*asinh(c + d*x))^4, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{e\left(c+dx\right)}\,\left(a+b\,\mathrm{asinh}\left(c+dx\right)\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**4*(d*e*x+c*e)**(1/2),x)
[Out] Integral(sqrt(e*(c + d*x))*(a + b*asinh(c + d*x))**4, x)
```

3.256 $\int \frac{\left(a+b \sinh^{-1}(c+dx)\right)^4}{\sqrt{ce+dex}} \, dx$

Optimal. Leaf size=78

$$\frac{2\sqrt{e(c+dx)}\left(a+b\sinh^{-1}(c+dx)\right)^4}{de}-\frac{8b\text{Int}\left(\frac{\sqrt{e(c+dx)}\left(a+b\sinh^{-1}(c+dx)\right)^3}{\sqrt{(c+dx)^2+1}},x\right)}{e}$$

```
[Out] 2*(a+b*arcsinh(d*x+c))^4*(e*(d*x+c))^(1/2)/d/e-8*b*Unintegrable((a+b*arcsin
h(d*x+c))^3*(e*(d*x+c))^(1/2)/(1+(d*x+c)^2)^(1/2),x)/e
```

Rubi [A] time = 0.18, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{\sqrt{ce+dex}} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcSinh[c + d*x])^4/Sqrt[c*e + d*e*x],x]
```

```
[Out] (2*Sqrt[e*(c + d*x)]*(a + b*ArcSinh[c + d*x])^4)/(d*e) - (8*b*Defer[Subst][
Defer[Int][(Sqrt[e*x]*(a + b*ArcSinh[x])^3)/Sqrt[1 + x^2], x], x, c + d*x])
/(d*e)
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{\sqrt{ce+dex}} \, dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^4}{\sqrt{ex}} \, dx, x, c+dx\right)}{d}$$
$$= \frac{2\sqrt{e(c+dx)}\left(a+b\sinh^{-1}(c+dx)\right)^4}{de}-\frac{(8b)\text{Subst}\left(\int \frac{\sqrt{ex}\left(a+b\sinh^{-1}(x)\right)^3}{\sqrt{1+x^2}} \, dx, x, c+\right.}{de}$$

Mathematica [A] time = 9.33, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{\sqrt{ce+dex}} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^4/Sqrt[c*e + d*e*x],x]
```

```
[Out] Integrate[(a + b*ArcSinh[c + d*x])^4/Sqrt[c*e + d*e*x], x]
```

fricas [A] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^4\operatorname{arsinh}(dx+c)^4+4ab^3\operatorname{arsinh}(dx+c)^3+6a^2b^2\operatorname{arsinh}(dx+c)^2+4a^3b\operatorname{arsinh}(dx+c)+a^4}{\sqrt{dex+ce}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^4*arcsinh(d*x + c)^4 + 4*a*b^3*arcsinh(d*x + c)^3 + 6*a^2*b^2*a
rcsinh(d*x + c)^2 + 4*a^3*b*arcsinh(d*x + c) + a^4)/sqrt(d*e*x + c*e), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arsinh}(dx + c) + a)^4}{\sqrt{dex + ce}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^4/sqrt(d*e*x + c*e), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{arcsinh}(dx + c))^4}{\sqrt{dex + ce}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\sqrt{dx + c}\, b^4 \log\left(dx + c + \sqrt{d^2x^2 + 2\,cdx + c^2 + 1}\right)^4}{d\sqrt{e}} + \frac{2\sqrt{dex + ce}\, a^4}{de} + \int -\frac{2\left(2\left(\left(2\,b^4c^2 - \left(c^2 + 1\right)ab^3 - \left(ab^3 d\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(d*x + c)*b^4*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^4/(d*s
qrt(e)) + 2*sqrt(d*e*x + c*e)*a^4/(d*e) + integrate(-2*(2*((2*b^4*c^2 - (c^
2 + 1)*a*b^3 - (a*b^3*d^2 - 2*b^4*d^2)*x^2 - 2*(a*b^3*c*d - 2*b^4*c*d)*x)*s
qrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*sqrt(d*x + c) - ((c^3 + c)*a*b^3 - 2*(c^3
+ c)*b^4 + (a*b^3*d^3 - 2*b^4*d^3)*x^3 + 3*(a*b^3*c*d^2 - 2*b^4*c*d^2)*x^2
+ ((3*c^2*d + d)*a*b^3 - 2*(3*c^2*d + d)*b^4)*x)*sqrt(d*x + c))*log(d*x + c
 + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3 - 3*((a^2*b^2*d^2*x^2 + 2*a^2*b^2*c
*d*x + (c^2 + 1)*a^2*b^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*sqrt(d*x + c) +
 (a^2*b^2*d^3*x^3 + 3*a^2*b^2*c*d^2*x^2 + (3*c^2*d + d)*a^2*b^2*x + (c^3 +
c)*a^2*b^2)*sqrt(d*x + c))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))
^2 - 2*((a^3*b*d^2*x^2 + 2*a^3*b*c*d*x + (c^2 + 1)*a^3*b)*sqrt(d^2*x^2 + 2*
c*d*x + c^2 + 1)*sqrt(d*x + c) + (a^3*b*d^3*x^3 + 3*a^3*b*c*d^2*x^2 + (3*c^
2*d + d)*a^3*b*x + (c^3 + c)*a^3*b)*sqrt(d*x + c))*log(d*x + c + sqrt(d^2*x
^2 + 2*c*d*x + c^2 + 1)))/(d^4*sqrt(e)*x^4 + 4*c*d^3*sqrt(e)*x^3 + c^4*sqrt
(e) + (6*c^2*d^2*sqrt(e) + d^2*sqrt(e))*x^2 + c^2*sqrt(e) + 2*(2*c^3*d*sqrt
(e) + c*d*sqrt(e))*x + (d^3*sqrt(e)*x^3 + 3*c*d^2*sqrt(e)*x^2 + c^3*sqrt(e)
 + (3*c^2*d*sqrt(e) + d*sqrt(e))*x + c*sqrt(e))*sqrt(d^2*x^2 + 2*c*d*x + c^
2 + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,\mathrm{asinh}(c + d\,x))^4}{\sqrt{c\,e + d\,e\,x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^4/(c*e + d*e*x)^(1/2),x)
[Out] int((a + b*asinh(c + d*x))^4/(c*e + d*e*x)^(1/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\text{asinh}\,(c + dx))^4}{\sqrt{e\,(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**4/(d*e*x+c*e)**(1/2),x)
[Out] Integral((a + b*asinh(c + d*x))**4/sqrt(e*(c + d*x)), x)
```

3.257 $\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{(ce+dex)^{3/2}}\,dx$

Optimal. Leaf size=78

$$\frac{8b\text{Int}\left(\frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{\sqrt{(c+dx)^2+1}\,\sqrt{e(c+dx)}},x\right)}{e}-\frac{2\left(a+b\sinh^{-1}(c+dx)\right)^4}{de\sqrt{e(c+dx)}}$$

```
[Out] -2*(a+b*arcsinh(d*x+c))^4/d/e/(e*(d*x+c))^(1/2)+8*b*Unintegrable((a+b*arcsi
nh(d*x+c))^3/(e*(d*x+c))^(1/2)/(1+(d*x+c)^2)^(1/2),x)/e
```

Rubi [A] time = 0.19, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{(ce+dex)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^(3/2),x]
```

```
[Out] (-2*(a + b*ArcSinh[c + d*x])^4)/(d*e*Sqrt[e*(c + d*x)]) + (8*b*Defer[Subst]
[Defer[Int][(a + b*ArcSinh[x])^3/(Sqrt[e*x]*Sqrt[1 + x^2]), x], x, c + d*x]
)/(d*e)
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{(ce+dex)^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^4}{(ex)^{3/2}}\,dx,x,c+dx\right)}{d}\\ &= -\frac{2\left(a+b\sinh^{-1}(c+dx)\right)^4}{de\sqrt{e(c+dx)}}+\frac{(8b)\,\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^3}{\sqrt{ex}\,\sqrt{1+x^2}}\,dx,x,c+dx\right)}{de}\end{aligned}$$

Mathematica [A] time = 37.76, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{(ce+dex)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^(3/2),x]
```

```
[Out] Integrate[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^(3/2), x]
```

fricas [A] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^4\,\text{arsinh}\left(dx+c\right)^4+4\,ab^3\,\text{arsinh}\left(dx+c\right)^3+6\,a^2b^2\,\text{arsinh}\left(dx+c\right)^2+4\,a^3b\,\text{arsinh}\left(dx+c\right)+a^4\right)}{d^2e^2x^2+2\,cde^2x+c^2e^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^4*arcsinh(d*x + c)^4 + 4*a*b^3*arcsinh(d*x + c)^3 + 6*a^2*b^2*a
rcsinh(d*x + c)^2 + 4*a^3*b*arcsinh(d*x + c) + a^4)*sqrt(d*e*x + c*e)/(d^2*
e^2*x^2 + 2*c*d*e^2*x + c^2*e^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arsinh}(dx+c)+a)^4}{(dex+ce)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^4/(d*e*x + c*e)^(3/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b \operatorname{arcsinh}(dx+c))^4}{(dex+ce)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(3/2),x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\sqrt{dx+c}\,b^4\sqrt{e}\,\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)^4}{d^2e^2x+cde^2}-\frac{2\,a^4}{\sqrt{dex+ce}\,de}+\int \frac{2\left(2\left(\left(2\,b^4c^2\sqrt{e}+\left(c^2\sqrt{e}+\sqrt{e}\right)ab^3\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(3/2),x, algorithm="maxima")
```

```
[Out] -2*sqrt(d*x + c)*b^4*sqrt(e)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1
))^4/(d^2*e^2*x + c*d*e^2) - 2*a^4/(sqrt(d*e*x + c*e)*d*e) + integrate(2*(2
*((2*b^4*c^2*sqrt(e) + (c^2*sqrt(e) + sqrt(e))*a*b^3 + (a*b^3*d^2*sqrt(e) +
 2*b^4*d^2*sqrt(e))*x^2 + 2*(a*b^3*c*d*sqrt(e) + 2*b^4*c*d*sqrt(e))*x)*sqrt
(d^2*x^2 + 2*c*d*x + c^2 + 1)*sqrt(d*x + c) + ((c^3*sqrt(e) + c*sqrt(e))*a*
b^3 + 2*(c^3*sqrt(e) + c*sqrt(e))*b^4 + (a*b^3*d^3*sqrt(e) + 2*b^4*d^3*sqrt
(e))*x^3 + 3*(a*b^3*c*d^2*sqrt(e) + 2*b^4*c*d^2*sqrt(e))*x^2 + ((3*c^2*d*sq
rt(e) + d*sqrt(e))*a*b^3 + 2*(3*c^2*d*sqrt(e) + d*sqrt(e))*b^4)*x)*sqrt(d*x
 + c))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3 + 3*((a^2*b^2*d^2
*sqrt(e)*x^2 + 2*a^2*b^2*c*d*sqrt(e)*x + (c^2*sqrt(e) + sqrt(e))*a^2*b^2)*s
qrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*sqrt(d*x + c) + (a^2*b^2*d^3*sqrt(e)*x^3 +
 3*a^2*b^2*c*d^2*sqrt(e)*x^2 + (3*c^2*d*sqrt(e) + d*sqrt(e))*a^2*b^2*x + (c
^3*sqrt(e) + c*sqrt(e))*a^2*b^2)*sqrt(d*x + c))*log(d*x + c + sqrt(d^2*x^2
+ 2*c*d*x + c^2 + 1))^2 + 2*((a^3*b*d^2*sqrt(e)*x^2 + 2*a^3*b*c*d*sqrt(e)*x
 + (c^2*sqrt(e) + sqrt(e))*a^3*b)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*sqrt(d*
x + c) + (a^3*b*d^3*sqrt(e)*x^3 + 3*a^3*b*c*d^2*sqrt(e)*x^2 + (3*c^2*d*sqrt
(e) + d*sqrt(e))*a^3*b*x + (c^3*sqrt(e) + c*sqrt(e))*a^3*b)*sqrt(d*x + c))*
log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)))/(d^5*e^2*x^5 + 5*c*d^4*e^
2*x^4 + c^5*e^2 + c^3*e^2 + (10*c^2*d^3*e^2 + d^3*e^2)*x^3 + (10*c^3*d^2*e^
2 + 3*c*d^2*e^2)*x^2 + (5*c^4*d*e^2 + 3*c^2*d*e^2)*x + (d^4*e^2*x^4 + 4*c*d
^3*e^2*x^3 + c^4*e^2 + c^2*e^2 + (6*c^2*d^2*e^2 + d^2*e^2)*x^2 + 2*(2*c^3*d
*e^2 + c*d*e^2)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{asinh}\,(c+d\,x))^4}{(c\,e+d\,e\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^4/(c*e + d*e*x)^(3/2),x)
```

```
[Out] int((a + b*asinh(c + d*x))^4/(c*e + d*e*x)^(3/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{asinh}\,(c+dx))^4}{(e\,(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**4/(d*e*x+c*e)**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))**4/(e*(c + d*x))**(3/2), x)
```

3.258 $\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{(ce+dex)^{5/2}}\,dx$

Optimal. Leaf size=82

$$\frac{8b\text{Int}\left(\frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{\sqrt{(c+dx)^2+1}\,(e(c+dx))^{3/2}},x\right)}{3e}-\frac{2\left(a+b\sinh^{-1}(c+dx)\right)^4}{3de(e(c+dx))^{3/2}}$$

```
[Out] -2/3*(a+b*arcsinh(d*x+c))^4/d/e/(e*(d*x+c))^(3/2)+8/3*b*Unintegrable((a+b*a
rcsinh(d*x+c))^3/(e*(d*x+c))^(3/2)/(1+(d*x+c)^2)^(1/2),x)/e
```

Rubi [A] time = 0.21, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{(ce+dex)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^(5/2),x]
```

```
[Out] (-2*(a + b*ArcSinh[c + d*x])^4)/(3*d*e*(e*(c + d*x))^(3/2)) + (8*b*Defer[Su
bst][Defer[Int][(a + b*ArcSinh[x])^3/((e*x)^(3/2)*Sqrt[1 + x^2]), x], x, c
+ d*x])/(3*d*e)
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{(ce+dex)^{5/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^4}{(ex)^{5/2}}\,dx,x,c+dx\right)}{d}\\ &= -\frac{2\left(a+b\sinh^{-1}(c+dx)\right)^4}{3de(e(c+dx))^{3/2}}+\frac{(8b)\,\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^3}{(ex)^{3/2}\sqrt{1+x^2}}\,dx,x,c+dx\right)}{3de}\end{aligned}$$

Mathematica [A] time = 43.22, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{(ce+dex)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^(5/2),x]
```

```
[Out] Integrate[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^(5/2), x]
```

fricas [A] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^4\operatorname{arsinh}(dx+c)^4+4\,ab^3\operatorname{arsinh}(dx+c)^3+6\,a^2b^2\operatorname{arsinh}(dx+c)^2+4\,a^3b\operatorname{arsinh}(dx+c)+a^4\right)\sqrt{d}}{d^3e^3x^3+3\,cd^2e^3x^2+3\,c^2de^3x+c^3e^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^4*arcsinh(d*x + c)^4 + 4*a*b^3*arcsinh(d*x + c)^3 + 6*a^2*b^2*a
rcsinh(d*x + c)^2 + 4*a^3*b*arcsinh(d*x + c) + a^4)*sqrt(d*e*x + c*e)/(d^3*
e^3*x^3 + 3*c*d^2*e^3*x^2 + 3*c^2*d*e^3*x + c^3*e^3), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\operatorname{arsinh}(dx+c)+a)^4}{(dex+ce)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^4/(d*e*x + c*e)^(5/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\operatorname{arcsinh}(dx+c))^4}{(dex+ce)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(5/2),x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(5/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\sqrt{dx+c}\,b^4\sqrt{e}\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)^4}{3\left(d^3e^3x^2+2\,cd^2e^3x+c^2de^3\right)}-\frac{2\,a^4}{3\,(dex+ce)^{\frac{3}{2}}de}+\int \frac{2\left(2\left(\left(2\,b^4c^2\sqrt{e}+3\left(c^2\sqrt{e}+\sqrt{\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*sqrt(d*x + c)*b^4*sqrt(e)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 +
 1))^4/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3) - 2/3*a^4/((d*e*x + c*e)^(
3/2)*d*e) + integrate(2/3*(2*((2*b^4*c^2*sqrt(e) + 3*(c^2*sqrt(e) + sqrt(e)
)*a*b^3 + (3*a*b^3*d^2*sqrt(e) + 2*b^4*d^2*sqrt(e))*x^2 + 2*(3*a*b^3*c*d*sq
rt(e) + 2*b^4*c*d*sqrt(e))*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*sqrt(d*x +
c) + (3*(c^3*sqrt(e) + c*sqrt(e))*a*b^3 + 2*(c^3*sqrt(e) + c*sqrt(e))*b^4 +
 (3*a*b^3*d^3*sqrt(e) + 2*b^4*d^3*sqrt(e))*x^3 + 3*(3*a*b^3*c*d^2*sqrt(e) +
 2*b^4*c*d^2*sqrt(e))*x^2 + (3*(3*c^2*d*sqrt(e) + d*sqrt(e))*a*b^3 + 2*(3*c
^2*d*sqrt(e) + d*sqrt(e))*b^4)*x)*sqrt(d*x + c))*log(d*x + c + sqrt(d^2*x^2
 + 2*c*d*x + c^2 + 1))^3 + 9*((a^2*b^2*d^2*sqrt(e)*x^2 + 2*a^2*b^2*c*d*sqrt
(e)*x + (c^2*sqrt(e) + sqrt(e))*a^2*b^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*
sqrt(d*x + c) + (a^2*b^2*d^3*sqrt(e)*x^3 + 3*a^2*b^2*c*d^2*sqrt(e)*x^2 + (3
*c^2*d*sqrt(e) + d*sqrt(e))*a^2*b^2*x + (c^3*sqrt(e) + c*sqrt(e))*a^2*b^2)*
sqrt(d*x + c))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 + 6*((a^3
*b*d^2*sqrt(e)*x^2 + 2*a^3*b*c*d*sqrt(e)*x + (c^2*sqrt(e) + sqrt(e))*a^3*b)
*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*sqrt(d*x + c) + (a^3*b*d^3*sqrt(e)*x^3 +
 3*a^3*b*c*d^2*sqrt(e)*x^2 + (3*c^2*d*sqrt(e) + d*sqrt(e))*a^3*b*x + (c^3*s
qrt(e) + c*sqrt(e))*a^3*b)*sqrt(d*x + c))*log(d*x + c + sqrt(d^2*x^2 + 2*c*
d*x + c^2 + 1)))/(d^6*e^3*x^6 + 6*c*d^5*e^3*x^5 + c^6*e^3 + c^4*e^3 + (15*c
^2*d^4*e^3 + d^4*e^3)*x^4 + 4*(5*c^3*d^3*e^3 + c*d^3*e^3)*x^3 + 3*(5*c^4*d^
2*e^3 + 2*c^2*d^2*e^3)*x^2 + 2*(3*c^5*d*e^3 + 2*c^3*d*e^3)*x + (d^5*e^3*x^5
 + 5*c*d^4*e^3*x^4 + c^5*e^3 + c^3*e^3 + (10*c^2*d^3*e^3 + d^3*e^3)*x^3 + (
10*c^3*d^2*e^3 + 3*c*d^2*e^3)*x^2 + (5*c^4*d*e^3 + 3*c^2*d*e^3)*x)*sqrt(d^2
*x^2 + 2*c*d*x + c^2 + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)^4}{\left(c\,e+d\,e\,x\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^4/(c*e + d*e*x)^(5/2),x)
[Out] int((a + b*asinh(c + d*x))^4/(c*e + d*e*x)^(5/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{asinh}\left(c+dx\right)\right)^4}{\left(e\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**4/(d*e*x+c*e)**(5/2),x)
[Out] Integral((a + b*asinh(c + d*x))**4/(e*(c + d*x))**(5/2), x)
```

3.259 $\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{(ce+dex)^{7/2}}\,dx$

Optimal. Leaf size=82

$$\frac{8b\text{Int}\left(\frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{\sqrt{(c+dx)^2+1}\,(e(c+dx))^{5/2}},x\right)}{5e}-\frac{2\left(a+b\sinh^{-1}(c+dx)\right)^4}{5de(e(c+dx))^{5/2}}$$

```
[Out] -2/5*(a+b*arcsinh(d*x+c))^4/d/e/(e*(d*x+c))^(5/2)+8/5*b*Unintegrable((a+b*a
rcsinh(d*x+c))^3/(e*(d*x+c))^(5/2)/(1+(d*x+c)^2)^(1/2),x)/e
```

Rubi [A] time = 0.21, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{(ce+dex)^{7/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^(7/2),x]
```

```
[Out] (-2*(a + b*ArcSinh[c + d*x])^4)/(5*d*e*(e*(c + d*x))^(5/2)) + (8*b*Defer[Su
bst][Defer[Int][(a + b*ArcSinh[x])^3/((e*x)^(5/2)*Sqrt[1 + x^2]), x], x, c
+ d*x])/(5*d*e)
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{(ce+dex)^{7/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^4}{(ex)^{7/2}}\,dx,x,c+dx\right)}{d}\\ &= -\frac{2\left(a+b\sinh^{-1}(c+dx)\right)^4}{5de(e(c+dx))^{5/2}}+\frac{(8b)\,\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^3}{(ex)^{5/2}\sqrt{1+x^2}}\,dx,x,c+dx\right)}{5de}\end{aligned}$$

Mathematica [A] time = 112.48, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{(ce+dex)^{7/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^(7/2),x]
```

```
[Out] Integrate[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^(7/2), x]
```

fricas [A] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^4\,\text{arsinh}\,(dx+c)^4+4\,ab^3\,\text{arsinh}\,(dx+c)^3+6\,a^2b^2\,\text{arsinh}\,(dx+c)^2+4\,a^3b\,\text{arsinh}\,(dx+c)+a^4\right)}{d^4e^4x^4+4\,cd^3e^4x^3+6\,c^2d^2e^4x^2+4\,c^3de^4x+c^4e^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((b^4*arcsinh(d*x + c)^4 + 4*a*b^3*arcsinh(d*x + c)^3 + 6*a^2*b^2*a
rcsinh(d*x + c)^2 + 4*a^3*b*arcsinh(d*x + c) + a^4)*sqrt(d*e*x + c*e)/(d^4*
e^4*x^4 + 4*c*d^3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^3*d*e^4*x + c^4*e^4), x
)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arsinh}(dx+c)+a)^4}{(dex+ce)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^4/(d*e*x + c*e)^(7/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b \operatorname{arcsinh}(dx+c))^4}{(dex+ce)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(7/2),x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(7/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(7/2),x, algorithm="maxima")
```

```
[Out] -2/5*sqrt(d*x + c)*b^4*sqrt(e)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 +
 1))^4/(d^4*e^4*x^3 + 3*c*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^4) - 2/5*
a^4/((d*e*x + c*e)^(5/2)*d*e) + integrate(2/5*(2*((2*b^4*c^2*sqrt(e) + 5*(c
^2*sqrt(e) + sqrt(e))*a*b^3 + (5*a*b^3*d^2*sqrt(e) + 2*b^4*d^2*sqrt(e))*x^2
 + 2*(5*a*b^3*c*d*sqrt(e) + 2*b^4*c*d*sqrt(e))*x)*sqrt(d^2*x^2 + 2*c*d*x +
c^2 + 1)*sqrt(d*x + c) + (5*(c^3*sqrt(e) + c*sqrt(e))*a*b^3 + 2*(c^3*sqrt(e
) + c*sqrt(e))*b^4 + (5*a*b^3*d^3*sqrt(e) + 2*b^4*d^3*sqrt(e))*x^3 + 3*(5*a
*b^3*c*d^2*sqrt(e) + 2*b^4*c*d^2*sqrt(e))*x^2 + (5*(3*c^2*d*sqrt(e) + d*sqr
t(e))*a*b^3 + 2*(3*c^2*d*sqrt(e) + d*sqrt(e))*b^4)*x)*sqrt(d*x + c))*log(d*
x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3 + 15*((a^2*b^2*d^2*sqrt(e)*x^2
 + 2*a^2*b^2*c*d*sqrt(e)*x + (c^2*sqrt(e) + sqrt(e))*a^2*b^2)*sqrt(d^2*x^2
+ 2*c*d*x + c^2 + 1)*sqrt(d*x + c) + (a^2*b^2*d^3*sqrt(e)*x^3 + 3*a^2*b^2*c
*d^2*sqrt(e)*x^2 + (3*c^2*d*sqrt(e) + d*sqrt(e))*a^2*b^2*x + (c^3*sqrt(e) +
 c*sqrt(e))*a^2*b^2)*sqrt(d*x + c))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x +
c^2 + 1))^2 + 10*((a^3*b*d^2*sqrt(e)*x^2 + 2*a^3*b*c*d*sqrt(e)*x + (c^2*sqr
t(e) + sqrt(e))*a^3*b)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*sqrt(d*x + c) + (a
^3*b*d^3*sqrt(e)*x^3 + 3*a^3*b*c*d^2*sqrt(e)*x^2 + (3*c^2*d*sqrt(e) + d*sqr
t(e))*a^3*b*x + (c^3*sqrt(e) + c*sqrt(e))*a^3*b)*sqrt(d*x + c))*log(d*x + c
 + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)))/(d^7*e^4*x^7 + 7*c*d^6*e^4*x^6 + c^7
*e^4 + c^5*e^4 + (21*c^2*d^5*e^4 + d^5*e^4)*x^5 + 5*(7*c^3*d^4*e^4 + c*d^4*
e^4)*x^4 + 5*(7*c^4*d^3*e^4 + 2*c^2*d^3*e^4)*x^3 + (21*c^5*d^2*e^4 + 10*c^3
*d^2*e^4)*x^2 + (7*c^6*d*e^4 + 5*c^4*d*e^4)*x + (d^6*e^4*x^6 + 6*c*d^5*e^4*
x^5 + c^6*e^4 + c^4*e^4 + (15*c^2*d^4*e^4 + d^4*e^4)*x^4 + 4*(5*c^3*d^3*e^4
 + c*d^3*e^4)*x^3 + 3*(5*c^4*d^2*e^4 + 2*c^2*d^2*e^4)*x^2 + 2*(3*c^5*d*e^4
+ 2*c^3*d*e^4)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,\mathrm{asinh}\,(c + d\,x))^4}{(c\,e + d\,e\,x)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^4/(c*e + d*e*x)^(7/2),x)
```

```
[Out] int((a + b*asinh(c + d*x))^4/(c*e + d*e*x)^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**4/(d*e*x+c*e)**(7/2),x)
```

```
[Out] Timed out
```

3.260 $\int \sqrt{1+a^2+2abx+b^2x^2}\, \sinh^{-1}(a+bx)^3\, dx$

Optimal. Leaf size=131

$$-\frac{3(a+bx)^2}{8b}+\frac{\sinh^{-1}(a+bx)^4}{8b}+\frac{(a+bx)\sqrt{(a+bx)^2+1}\,\sinh^{-1}(a+bx)^3}{2b}-\frac{3(a+bx)^2\sinh^{-1}(a+bx)^2}{4b}-\frac{3\sinh^{-1}(a}{8b}$$

```
[Out] -3/8*(b*x+a)^2/b-3/8*arcsinh(b*x+a)^2/b-3/4*(b*x+a)^2*arcsinh(b*x+a)^2/b+1/
8*arcsinh(b*x+a)^4/b+3/4*(b*x+a)*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)/b+1/2*(
b*x+a)*arcsinh(b*x+a)^3*(1+(b*x+a)^2)^(1/2)/b
```

Rubi [A] time = 0.19, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5867, 5682, 5675, 5661, 5758, 30}

$$-\frac{3(a+bx)^2}{8b}+\frac{\sinh^{-1}(a+bx)^4}{8b}+\frac{(a+bx)\sqrt{(a+bx)^2+1}\,\sinh^{-1}(a+bx)^3}{2b}-\frac{3(a+bx)^2\sinh^{-1}(a+bx)^2}{4b}-\frac{3\sinh^{-1}(a}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x]^3,x]
```

```
[Out] (-3*(a + b*x)^2)/(8*b) + (3*(a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x
])/(4*b) - (3*ArcSinh[a + b*x]^2)/(8*b) - (3*(a + b*x)^2*ArcSinh[a + b*x]^2
)/(4*b) + ((a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x]^3)/(2*b) + ArcS
inh[a + b*x]^4/(8*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
```

```
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5867

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C*x^2)/d^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\int \sqrt{1+a^2+2abx+b^2x^2}\,\sinh^{-1}(a+bx)^3\,dx = \frac{\text{Subst}\left(\int \sqrt{1+x^2}\,\sinh^{-1}(x)^3\,dx, x, a+bx\right)}{b}$$

$$= \frac{(a+bx)\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)^3}{2b} + \frac{\text{Subst}\left(\int \frac{\sinh^{-1}(x)^3}{\sqrt{1+x^2}}\right.}{2b}$$

$$= -\frac{3(a+bx)^2\sinh^{-1}(a+bx)^2}{4b} + \frac{(a+bx)\sqrt{1+(a+bx)^2}\,\sinh^{-1}}{2b}$$

$$= \frac{3(a+bx)\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{4b} - \frac{3(a+bx)^2\sinh^{-1}(}{4b}$$

$$= -\frac{3(a+bx)^2}{8b} + \frac{3(a+bx)\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{4b} - \frac{3\,\text{si}}{}$$

Mathematica [A] time = 0.12, size = 127, normalized size = 0.97

$$\frac{4(a+bx)\sqrt{a^2+2abx+b^2x^2+1}\,\sinh^{-1}(a+bx)^3 - 3\left(2a^2+4abx+2b^2x^2+1\right)\sinh^{-1}(a+bx)^2 + 6(a+bx)\sqrt{a}}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x]^3,x]
```

```
[Out] (-3*b*x*(2*a + b*x) + 6*(a + b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh
[a + b*x] - 3*(1 + 2*a^2 + 4*a*b*x + 2*b^2*x^2)*ArcSinh[a + b*x]^2 + 4*(a +
 b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x]^3 + ArcSinh[a + b*
x]^4)/(8*b)
```

fricas [A] time = 0.52, size = 199, normalized size = 1.52

$$\frac{4\sqrt{b^2x^2+2\,abx+a^2+1}\,(bx+a)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^3 - 3\,b^2x^2 + \log\left(bx+a+\sqrt{b^2x^2+2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="fric
as")
```

```
[Out] 1/8*(4*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b*x + a)*log(b*x + a + sqrt(b^2*x
^2 + 2*a*b*x + a^2 + 1))^3 - 3*b^2*x^2 + log(b*x + a + sqrt(b^2*x^2 + 2*a*b
*x + a^2 + 1))^4 - 6*a*b*x - 3*(2*b^2*x^2 + 4*a*b*x + 2*a^2 + 1)*log(b*x +
a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2 + 6*sqrt(b^2*x^2 + 2*a*b*x + a^2 +
 1)*(b*x + a)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b^2x^2 + 2\,abx + a^2 + 1}\,\operatorname{arsinh}\left(bx + a\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*arcsinh(b*x + a)^3, x)
```

maple [A] time = 0.13, size = 204, normalized size = 1.56

$$4\sqrt{b^2x^2 + 2abx + a^2 + 1}\,\operatorname{arcsinh}\left(bx + a\right)^3 xb - 6\operatorname{arcsinh}\left(bx + a\right)^2 x^2b^2 + 4\sqrt{b^2x^2 + 2abx + a^2 + 1}\,\operatorname{arcsinh}(bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x)
```

```
[Out] 1/8*(4*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*arcsinh(b*x+a)^3*x*b-6*arcsinh(b*x+a)^
2*x^2*b^2+4*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*arcsinh(b*x+a)^3*a-12*arcsinh(b*x
+a)^2*x*a*b+6*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*arcsinh(b*x+a)*x*b+arcsinh(b*x+
a)^4-6*arcsinh(b*x+a)^2*a^2-3*b^2*x^2+6*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*arcsi
nh(b*x+a)*a-6*a*b*x-3*arcsinh(b*x+a)^2-3*a^2-3)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b^2x^2 + 2\,abx + a^2 + 1}\,\operatorname{arsinh}\left(bx + a\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*arcsinh(b*x + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{asinh}\left(a + b\,x\right)^3\,\sqrt{a^2 + 2\,a\,b\,x + b^2\,x^2 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a + b*x)^3*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2),x)
```

```
[Out] int(asinh(a + b*x)^3*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a^2 + 2abx + b^2x^2 + 1}\,\operatorname{asinh}^3\left(a + bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)**3*(b**2*x**2+2*a*b*x+a**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)**3, x)
```

3.261 $\int \sqrt{1+a^2+2abx+b^2x^2}\,\sinh^{-1}(a+bx)^2\,dx$

Optimal. Leaf size=107

$$\frac{(a+bx)\sqrt{(a+bx)^2+1}}{4b}+\frac{\sinh^{-1}(a+bx)^3}{6b}+\frac{(a+bx)\sqrt{(a+bx)^2+1}\,\sinh^{-1}(a+bx)^2}{2b}-\frac{(a+bx)^2\sinh^{-1}(a+bx)}{2b}$$

```
[Out] -1/4*arcsinh(b*x+a)/b-1/2*(b*x+a)^2*arcsinh(b*x+a)/b+1/6*arcsinh(b*x+a)^3/b
+1/4*(b*x+a)*(1+(b*x+a)^2)^(1/2)/b+1/2*(b*x+a)*arcsinh(b*x+a)^2*(1+(b*x+a)^
2)^(1/2)/b
```

Rubi [A] time = 0.12, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5867, 5682, 5675, 5661, 321, 215}

$$\frac{(a+bx)\sqrt{(a+bx)^2+1}}{4b}+\frac{\sinh^{-1}(a+bx)^3}{6b}+\frac{(a+bx)\sqrt{(a+bx)^2+1}\,\sinh^{-1}(a+bx)^2}{2b}-\frac{(a+bx)^2\sinh^{-1}(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x]^2,x]
```

```
[Out] ((a + b*x)*Sqrt[1 + (a + b*x)^2])/(4*b) - ArcSinh[a + b*x]/(4*b) - ((a + b*
x)^2*ArcSinh[a + b*x])/(2*b) + ((a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcSinh[a +
 b*x]^2)/(2*b) + ArcSinh[a + b*x]^3/(6*b)
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5867

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C*x^2)/d^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\int \sqrt{1+a^2+2abx+b^2x^2}\,\sinh^{-1}(a+bx)^2\,dx = \frac{\text{Subst}\left(\int \sqrt{1+x^2}\,\sinh^{-1}(x)^2\,dx, x, a+bx\right)}{b}$$

$$= \frac{(a+bx)\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)^2}{2b} + \frac{\text{Subst}\left(\int \frac{\sinh^{-1}(x)^2}{\sqrt{1+x^2}}\,dx,\right.}{2b}$$

$$= -\frac{(a+bx)^2\sinh^{-1}(a+bx)}{2b} + \frac{(a+bx)\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+}{2b}$$

$$= \frac{(a+bx)\sqrt{1+(a+bx)^2}}{4b} - \frac{(a+bx)^2\sinh^{-1}(a+bx)}{2b} + \frac{(a+bx)\sqrt{}}{}$$

$$= \frac{(a+bx)\sqrt{1+(a+bx)^2}}{4b} - \frac{\sinh^{-1}(a+bx)}{4b} - \frac{(a+bx)^2\sinh^{-1}(a+}{2b}$$

Mathematica [A] time = 0.09, size = 110, normalized size = 1.03

$$\frac{3(a+bx)\sqrt{a^2+2abx+b^2x^2+1}+6(a+bx)\sqrt{a^2+2abx+b^2x^2+1}\,\sinh^{-1}(a+bx)^2-3\left(2a^2+4abx+2b^2x^2+1\right.}{12b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x]^2,x]
```

```
[Out] (3*(a + b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] - 3*(1 + 2*a^2 + 4*a*b*x + 2
*b^2*x^2)*ArcSinh[a + b*x] + 6*(a + b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*
ArcSinh[a + b*x]^2 + 2*ArcSinh[a + b*x]^3)/(12*b)
```

fricas [A] time = 0.71, size = 161, normalized size = 1.50

$$\frac{6\sqrt{b^2x^2+2\,abx+a^2+1}\,(bx+a)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^2+2\,\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2}\right.}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="fric
as")
```

```
[Out] 1/12*(6*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b*x + a)*log(b*x + a + sqrt(b^2*
x^2 + 2*a*b*x + a^2 + 1))^2 + 2*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2
+ 1))^3 - 3*(2*b^2*x^2 + 4*a*b*x + 2*a^2 + 1)*log(b*x + a + sqrt(b^2*x^2 +
2*a*b*x + a^2 + 1)) + 3*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b*x + a))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b^2x^2+2\,abx+a^2+1}\,\operatorname{arsinh}(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*arcsinh(b*x + a)^2, x)
```

maple [A] time = 0.11, size = 167, normalized size = 1.56

$$\frac{6\,\mathrm{arcsinh}\left(bx+a\right)^{2}\sqrt{b^{2}x^{2}+2abx+a^{2}+1}\;xb-6\,\mathrm{arcsinh}\left(bx+a\right)x^{2}b^{2}+6\,\mathrm{arcsinh}\left(bx+a\right)^{2}\sqrt{b^{2}x^{2}+2abx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x)
```

```
[Out] 1/12*(6*arcsinh(b*x+a)^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x*b-6*arcsinh(b*x+a)
*x^2*b^2+6*arcsinh(b*x+a)^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a-12*arcsinh(b*x+
a)*x*a*b+2*arcsinh(b*x+a)^3-6*arcsinh(b*x+a)*a^2+3*(b^2*x^2+2*a*b*x+a^2+1)^
(1/2)*x*b+3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a-3*arcsinh(b*x+a))/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b^{2}x^{2}+2\,abx+a^{2}+1}\;\mathrm{arsinh}\left(bx+a\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*arcsinh(b*x + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{asinh}\left(a+b\,x\right)^{2}\sqrt{a^{2}+2\,a\,b\,x+b^{2}\,x^{2}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a + b*x)^2*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2),x)
```

```
[Out] int(asinh(a + b*x)^2*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a^{2}+2abx+b^{2}x^{2}+1}\;\mathrm{asinh}^{2}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)**2*(b**2*x**2+2*a*b*x+a**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)**2, x)
```

3.262 $\int \sqrt{1+a^2+2abx+b^2x^2}\,\sinh^{-1}(a+bx)\,dx$

Optimal. Leaf size=61

$$-\frac{(a+bx)^2}{4b}+\frac{\sqrt{(a+bx)^2+1}\,(a+bx)\sinh^{-1}(a+bx)}{2b}+\frac{\sinh^{-1}(a+bx)^2}{4b}$$

```
[Out] -1/4*(b*x+a)^2/b+1/4*arcsinh(b*x+a)^2/b+1/2*(b*x+a)*arcsinh(b*x+a)*(1+(b*x+
a)^2)^(1/2)/b
```

Rubi [A] time = 0.07, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{5867, 5682, 5675, 30\}$

$$-\frac{(a+bx)^2}{4b}+\frac{\sqrt{(a+bx)^2+1}\,(a+bx)\sinh^{-1}(a+bx)}{2b}+\frac{\sinh^{-1}(a+bx)^2}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x],x]
```

```
[Out] -(a + b*x)^2/(4*b) + ((a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x])/(2*
b) + ArcSinh[a + b*x]^2/(4*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5867

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C*x^2)/d^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\int \sqrt{1+a^2+2abx+b^2x^2}\,\sinh^{-1}(a+bx)\,dx = \frac{\text{Subst}\left(\int \sqrt{1+x^2}\,\sinh^{-1}(x)\,dx, x, a+bx\right)}{b}$$

$$= \frac{(a+bx)\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{2b} - \frac{\text{Subst}(\int x\,dx, x, a+}{2b}$$

$$= -\frac{(a+bx)^2}{4b} + \frac{(a+bx)\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{2b} + \frac{\sinh^{-1}}{}$$

Mathematica [**A**] time = 0.06, size = 61, normalized size = 1.00

$$\frac{2(a+bx)\sqrt{a^2+2abx+b^2x^2+1}\,\sinh^{-1}(a+bx) - bx(2a+bx) + \sinh^{-1}(a+bx)^2}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x],x]
```

```
[Out] (-(b*x*(2*a + b*x)) + 2*(a + b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh
[a + b*x] + ArcSinh[a + b*x]^2)/(4*b)
```

fricas [**A**] time = 0.60, size = 98, normalized size = 1.61

$$-\frac{b^2x^2 + 2\,abx - 2\,\sqrt{b^2x^2+2\,abx+a^2+1}\,(bx+a)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right) - \log\left(bx+a+\sqrt{b^2}\right.}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="fricas
")
```

```
[Out] -1/4*(b^2*x^2 + 2*a*b*x - 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b*x + a)*log
(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - log(b*x + a + sqrt(b^2*x^2
+ 2*a*b*x + a^2 + 1))^2)/b
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b^2x^2+2\,abx+a^2+1}\ \text{arsinh}\,(bx+a)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*arcsinh(b*x + a), x)
```

maple [**A**] time = 0.10, size = 91, normalized size = 1.49

$$\frac{2\sqrt{b^2x^2+2abx+a^2+1}\ \text{arcsinh}\,(bx+a)\,xb - b^2x^2 + 2\sqrt{b^2x^2+2abx+a^2+1}\ \text{arcsinh}\,(bx+a)\,a - 2abx + \text{ar}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x)
```

```
[Out] 1/4*(2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*arcsinh(b*x+a)*x*b-b^2*x^2+2*(b^2*x^2+
2*a*b*x+a^2+1)^(1/2)*arcsinh(b*x+a)*a-2*a*b*x+arcsinh(b*x+a)^2-a^2-1)/b
```

maxima [B] time = 0.71, size = 238, normalized size = 3.90

$$-\frac{1}{4}\left(x^2+\frac{2\,ax}{b}+\frac{2\,\operatorname{arsinh}\left(bx+a\right)\operatorname{arsinh}\left(\frac{2\left(b^2x+ab\right)}{\sqrt{-4\,a^2b^2+4\left(a^2+1\right)b^2}}\right)}{b^2}-\frac{\operatorname{arsinh}\left(\frac{2\left(b^2x+ab\right)}{\sqrt{-4\,a^2b^2+4\left(a^2+1\right)b^2}}\right)^2}{b^2}\right)b-\frac{1}{2}\left(\frac{a^2\operatorname{arsinh}\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="maxima
")
```

```
[Out] -1/4*(x^2 + 2*a*x/b + 2*arcsinh(b*x + a)*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^
2*b^2 + 4*(a^2 + 1)*b^2))/b^2 - arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4
*(a^2 + 1)*b^2))^2/b^2)*b - 1/2*(a^2*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^
2 + 4*(a^2 + 1)*b^2))/b - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*x - (a^2 + 1)*a
rcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b - sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1)*a/b)*arcsinh(b*x + a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \operatorname{asinh}\left(a+b\,x\right)\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a + b*x)*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2),x)
```

```
[Out] int(asinh(a + b*x)*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a^2+2abx+b^2x^2+1}\,\operatorname{asinh}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)*(b**2*x**2+2*a*b*x+a**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x), x)
```

3.263 $\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\sinh^{-1}(a+bx)}\,dx$

Optimal. Leaf size=31

$$\frac{\mathrm{Chi}\left(2\sinh^{-1}(a+bx)\right)}{2b}+\frac{\log\left(\sinh^{-1}(a+bx)\right)}{2b}$$

```
[Out] 1/2*Chi(2*arcsinh(b*x+a))/b+1/2*ln(arcsinh(b*x+a))/b
```

Rubi [A] time = 0.12, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5867, 5699, 3312, 3301}

$$\frac{\mathrm{Chi}\left(2\sinh^{-1}(a+bx)\right)}{2b}+\frac{\log\left(\sinh^{-1}(a+bx)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]/ArcSinh[a + b*x],x]
```

```
[Out] CoshIntegral[2*ArcSinh[a + b*x]]/(2*b) + Log[ArcSinh[a + b*x]]/(2*b)
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5699

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[d^p/c, Subst[Int[(a + b*x)^n*Cosh[x]^(2*p + 1), x], x, Ar
cSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*
p, 0] && (IntegerQ[p] || GtQ[d, 0])
```

Rule 5867

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C*x^2)/d^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\sinh^{-1}(a+bx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{1+x^2}}{\sinh^{-1}(x)}\,dx, x, a+bx\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{\cosh^2(x)}{x}\,dx, x, \sinh^{-1}(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{2x}+\frac{\cosh(2x)}{2x}\right)dx, x, \sinh^{-1}(a+bx)\right)}{b}$$
$$= \frac{\log\left(\sinh^{-1}(a+bx)\right)}{2b} + \frac{\text{Subst}\left(\int \frac{\cosh(2x)}{x}\,dx, x, \sinh^{-1}(a+bx)\right)}{2b}$$
$$= \frac{\text{Chi}\left(2\sinh^{-1}(a+bx)\right)}{2b} + \frac{\log\left(\sinh^{-1}(a+bx)\right)}{2b}$$

Mathematica [A] time = 0.07, size = 24, normalized size = 0.77

$$\frac{\text{Chi}\left(2\sinh^{-1}(a+bx)\right)+\log\left(\sinh^{-1}(a+bx)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]/ArcSinh[a + b*x],x]
[Out] (CoshIntegral[2*ArcSinh[a + b*x]] + Log[ArcSinh[a + b*x]])/(2*b)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b^2x^2+2\,abx+a^2+1}}{\text{arsinh}\,(bx+a)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(1/2)/arcsinh(b*x+a),x, algorithm="fricas
")
[Out] integral(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/arcsinh(b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b^2x^2+2\,abx+a^2+1}}{\text{arsinh}\,(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(1/2)/arcsinh(b*x+a),x, algorithm="giac")
[Out] integrate(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/arcsinh(b*x + a), x)
```

maple [A] time = 0.13, size = 28, normalized size = 0.90

$$\frac{\text{X}\,(2\,\text{arcsinh}\,(bx+a))}{2b} + \frac{\ln\,(\text{arcsinh}\,(bx+a))}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2+1)^(1/2)/arcsinh(b*x+a),x)
```

[Out] 1/2*Chi(2*arcsinh(b*x+a))/b+1/2*ln(arcsinh(b*x+a))/b

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b^2x^2 + 2\,abx + a^2 + 1}}{\operatorname{arsinh}\left(bx + a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(1/2)/arcsinh(b*x+a),x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/arcsinh(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{a^2 + 2\,a\,b\,x + b^2\,x^2 + 1}}{\operatorname{asinh}\left(a + b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2)/asinh(a + b*x),x)
```

```
[Out] int((a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2)/asinh(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2 + 1}}{\operatorname{asinh}\left(a + bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2+1)**(1/2)/asinh(b*x+a),x)
```

```
[Out] Integral(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/asinh(a + b*x), x)
```

3.264 $\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\sinh^{-1}(a+bx)^2}\,dx$

Optimal. Leaf size=36

$$\frac{\text{Shi}\left(2\sinh^{-1}(a+bx)\right)}{b}-\frac{(a+bx)^2+1}{b\sinh^{-1}(a+bx)}$$

```
[Out] (-1-(b*x+a)^2)/b/arcsinh(b*x+a)+Shi(2*arcsinh(b*x+a))/b
```

Rubi [A] time = 0.12, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5867, 5696, 5669, 5448, 12, 3298}

$$\frac{\text{Shi}\left(2\sinh^{-1}(a+bx)\right)}{b}-\frac{(a+bx)^2+1}{b\sinh^{-1}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]/ArcSinh[a + b*x]^2,x]
```

```
[Out] -((1 + (a + b*x)^2)/(b*ArcSinh[a + b*x])) + SinhIntegral[2*ArcSinh[a + b*x]
]/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5696

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(Sqrt[1 + c^2*x^2]*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^(n +
 1))/(b*c*(n + 1)), x] - Dist[(c*(2*p + 1)*d^IntPart[p]*(d + e*x^2)^FracPar
t[p])/(b*(n + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*
(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ
[e, c^2*d] && LtQ[n, -1]
```

Rule 5867

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C*x^2)/d^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\sinh^{-1}(a+bx)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{\sqrt{1+x^2}}{\sinh^{-1}(x)^2}\,dx, x, a+bx\right)}{b}\\
&= -\frac{1+(a+bx)^2}{b\sinh^{-1}(a+bx)}+\frac{2\,\text{Subst}\left(\int \frac{x}{\sinh^{-1}(x)}\,dx, x, a+bx\right)}{b}\\
&= -\frac{1+(a+bx)^2}{b\sinh^{-1}(a+bx)}+\frac{2\,\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{x}\,dx, x, \sinh^{-1}(a+bx)\right)}{b}\\
&= -\frac{1+(a+bx)^2}{b\sinh^{-1}(a+bx)}+\frac{2\,\text{Subst}\left(\int \frac{\sinh(2x)}{2x}\,dx, x, \sinh^{-1}(a+bx)\right)}{b}\\
&= -\frac{1+(a+bx)^2}{b\sinh^{-1}(a+bx)}+\frac{\text{Subst}\left(\int \frac{\sinh(2x)}{x}\,dx, x, \sinh^{-1}(a+bx)\right)}{b}\\
&= -\frac{1+(a+bx)^2}{b\sinh^{-1}(a+bx)}+\frac{\text{Shi}\left(2\sinh^{-1}(a+bx)\right)}{b}
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 47, normalized size = 1.31

$$
-\frac{a^2-\sinh^{-1}(a+bx)\text{Shi}\left(2\sinh^{-1}(a+bx)\right)+2abx+b^2x^2+1}{b\sinh^{-1}(a+bx)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]/ArcSinh[a + b*x]^2,x]
```

```
[Out] -((1 + a^2 + 2*a*b*x + b^2*x^2 - ArcSinh[a + b*x]*SinhIntegral[2*ArcSinh[a
+ b*x]])/(b*ArcSinh[a + b*x]))
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$
\text{integral}\left(\frac{\sqrt{b^2x^2+2\,abx+a^2+1}}{\text{arsinh}\,(bx+a)^2}, x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(1/2)/arcsinh(b*x+a)^2,x, algorithm="fric
as")
```

```
[Out] integral(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/arcsinh(b*x + a)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{\sqrt{b^2x^2+2\,abx+a^2+1}}{\text{arsinh}\,(bx+a)^2}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(1/2)/arcsinh(b*x+a)^2,x, algorithm="giac")

[Out] integrate(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/arcsinh(b*x + a)^2, x)

maple [A] time = 0.12, size = 44, normalized size = 1.22

$$\frac{2\,\mathrm{Shi}\left(2\,\mathrm{arcsinh}\left(bx+a\right)\right)\mathrm{arcsinh}\left(bx+a\right)-\cosh\left(2\,\mathrm{arcsinh}\left(bx+a\right)\right)-1}{2b\,\mathrm{arcsinh}\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*x^2+2*a*b*x+a^2+1)^(1/2)/arcsinh(b*x+a)^2,x)

[Out] 1/2/b*(2*Shi(2*arcsinh(b*x+a))*arcsinh(b*x+a)-cosh(2*arcsinh(b*x+a))-1)/arcsinh(b*x+a)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(b^2x^2+2\,abx+a^2+1\right)^2+\left(b^3x^3+3\,ab^2x^2+a^3+\left(3\,a^2b+b\right)x+a\right)\sqrt{b^2x^2+2\,abx+a^2+1}}{\left(b^3x^2+2\,ab^2x+a^2b+\sqrt{b^2x^2+2\,abx+a^2+1}\left(b^2x+ab\right)+b\right)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)}+\int\frac{\left(2\,b^2\right.}{\left(b^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(1/2)/arcsinh(b*x+a)^2,x, algorithm="maxima")

[Out] -((b^2*x^2 + 2*a*b*x + a^2 + 1)^2 + (b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b + b)*x + a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/((b^3*x^2 + 2*a*b^2*x + a^2*b + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b^2*x + a*b) + b)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))) + integrate(((2*b^2*x^2 + 4*a*b*x + 2*a^2 - 1)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + 2*(2*b^3*x^3 + 6*a*b^2*x^2 + 2*a^3 + (6*a^2*b + b)*x + a)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + (2*b^4*x^4 + 8*a*b^3*x^3 + 2*a^4 + 3*(4*a^2*b^2 + b^2)*x^2 + 3*a^2 + 2*(4*a^3*b + 3*a*b)*x + 1)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/((b^4*x^4 + 4*a*b^3*x^3 + a^4 + 2*(3*a^2*b^2 + b^2)*x^2 + (b^2*x^2 + 2*a*b*x + a^2 + 1)*(b^2*x^2 + 2*a*b*x + a^2) + 2*a^2 + 4*(a^3*b + a*b)*x + 2*(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b + b)*x + a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}}{{\mathrm{asinh}\left(a+b\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2)/asinh(a + b*x)^2,x)

[Out] int((a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2)/asinh(a + b*x)^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2+2abx+b^2x^2+1}}{\operatorname{asinh}^2\left(a+bx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b**2*x**2+2*a*b*x+a**2+1)**(1/2)/asinh(b*x+a)**2,x)

[Out] Integral(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/asinh(a + b*x)**2, x)

3.265 $\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\sinh^{-1}(a+bx)^3} dx$

Optimal. Leaf size=71

$$\frac{\text{Chi}\left(2\sinh^{-1}(a+bx)\right)}{b}-\frac{\sqrt{(a+bx)^2+1}\,(a+bx)}{b\sinh^{-1}(a+bx)}+\frac{-(a+bx)^2-1}{2b\sinh^{-1}(a+bx)^2}$$

```
[Out] 1/2*(-1-(b*x+a)^2)/b/arcsinh(b*x+a)^2+Chi(2*arcsinh(b*x+a))/b-(b*x+a)*(1+(b
*x+a)^2)^(1/2)/b/arcsinh(b*x+a)
```

Rubi [A] time = 0.11, antiderivative size = 69, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5867, 5696, 5665, 3301}

$$\frac{\text{Chi}\left(2\sinh^{-1}(a+bx)\right)}{b}-\frac{\sqrt{(a+bx)^2+1}\,(a+bx)}{b\sinh^{-1}(a+bx)}-\frac{(a+bx)^2+1}{2b\sinh^{-1}(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]/ArcSinh[a + b*x]^3,x]
```

```
[Out] -(1 + (a + b*x)^2)/(2*b*ArcSinh[a + b*x]^2) - ((a + b*x)*Sqrt[1 + (a + b*x)
^2])/(b*ArcSinh[a + b*x]) + CoshIntegral[2*ArcSinh[a + b*x]]/b
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 5696

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(Sqrt[1 + c^2*x^2]*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^(n +
 1))/(b*c*(n + 1)), x] - Dist[(c*(2*p + 1)*d^IntPart[p]*(d + e*x^2)^FracPar
t[p])/(b*(n + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*
(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ
[e, c^2*d] && LtQ[n, -1]
```

Rule 5867

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C*x^2)/d^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\sinh^{-1}(a+bx)^3}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{1+x^2}}{\sinh^{-1}(x)^3}\,dx, x, a+bx\right)}{b}$$

$$= -\frac{1+(a+bx)^2}{2b\sinh^{-1}(a+bx)^2} + \frac{\text{Subst}\left(\int \frac{x}{\sinh^{-1}(x)^2}\,dx, x, a+bx\right)}{b}$$

$$= -\frac{1+(a+bx)^2}{2b\sinh^{-1}(a+bx)^2} - \frac{(a+bx)\sqrt{1+(a+bx)^2}}{b\sinh^{-1}(a+bx)} + \frac{\text{Subst}\left(\int \frac{\cosh(2x)}{x}\,dx, x, \sinh^{-1}\right.}{b}$$

$$= -\frac{1+(a+bx)^2}{2b\sinh^{-1}(a+bx)^2} - \frac{(a+bx)\sqrt{1+(a+bx)^2}}{b\sinh^{-1}(a+bx)} + \frac{\text{Chi}\left(2\sinh^{-1}(a+bx)\right)}{b}$$

Mathematica [A] time = 0.29, size = 85, normalized size = 1.20

$$-\frac{2(a+bx)\sqrt{a^2+2abx+b^2x^2+1}\,\sinh^{-1}(a+bx)+a^2-2\sinh^{-1}(a+bx)^2\text{Chi}\left(2\sinh^{-1}(a+bx)\right)+2abx+b^2x^2}{2b\sinh^{-1}(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]/ArcSinh[a + b*x]^3,x]
```

```
[Out] -1/2*(1 + a^2 + 2*a*b*x + b^2*x^2 + 2*(a + b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^
2*x^2]*ArcSinh[a + b*x] - 2*ArcSinh[a + b*x]^2*CoshIntegral[2*ArcSinh[a + b
*x]])/(b*ArcSinh[a + b*x]^2)
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b^2x^2+2\,abx+a^2+1}}{\text{arsinh}\,(bx+a)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(1/2)/arcsinh(b*x+a)^3,x, algorithm="fric
as")
```

```
[Out] integral(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/arcsinh(b*x + a)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b^2x^2+2\,abx+a^2+1}}{\text{arsinh}\,(bx+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(1/2)/arcsinh(b*x+a)^3,x, algorithm="giac
")
```

```
[Out] integrate(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/arcsinh(b*x + a)^3, x)
```

maple [A] time = 0.13, size = 63, normalized size = 0.89

$$\frac{4\text{X}\,(2\,\text{arcsinh}\,(bx+a))\,\text{arcsinh}\,(bx+a)^2-2\sinh\,(2\,\text{arcsinh}\,(bx+a))\,\text{arcsinh}\,(bx+a)-\cosh\,(2\,\text{arcsinh}\,(bx+a}{4b\,\text{arcsinh}\,(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2+1)^(1/2)/arcsinh(b*x+a)^3,x)
```

```
[Out] 1/4/b*(4*Chi(2*arcsinh(b*x+a))*arcsinh(b*x+a)^2-2*sinh(2*arcsinh(b*x+a))*ar
csinh(b*x+a)-cosh(2*arcsinh(b*x+a))-1)/arcsinh(b*x+a)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(1/2)/arcsinh(b*x+a)^3,x, algorithm="maxi
ma")
```

```
[Out] -1/2*((b^4*x^4 + 4*a*b^3*x^3 + a^4 + (6*a^2*b^2 + b^2)*x^2 + a^2 + 2*(2*a^3
*b + a*b)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^2 + (3*b^5*x^5 + 15*a*b^4*x^4 +
3*a^5 + 5*(6*a^2*b^3 + b^3)*x^3 + 5*a^3 + 15*(2*a^3*b^2 + a*b^2)*x^2 + (15*
a^4*b + 15*a^2*b + 2*b)*x + 2*a)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + (3*b
^6*x^6 + 18*a*b^5*x^5 + 3*a^6 + (45*a^2*b^4 + 7*b^4)*x^4 + 7*a^4 + 4*(15*a^
3*b^3 + 7*a*b^3)*x^3 + (45*a^4*b^2 + 42*a^2*b^2 + 5*b^2)*x^2 + 5*a^2 + 2*(9
*a^5*b + 14*a^3*b + 5*a*b)*x + 1)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + ((2*b^4*x
^4 + 8*a*b^3*x^3 + 2*a^4 + (12*a^2*b^2 + b^2)*x^2 + a^2 + 2*(4*a^3*b + a*b)
*x - 1)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^2 + (6*b^5*x^5 + 30*a*b^4*x^4 + 6*a^5
 + (60*a^2*b^3 + 7*b^3)*x^3 + 7*a^3 + 3*(20*a^3*b^2 + 7*a*b^2)*x^2 + (30*a^
4*b + 21*a^2*b + b)*x + a)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + (6*b^6*x^6
 + 36*a*b^5*x^5 + 6*a^6 + (90*a^2*b^4 + 11*b^4)*x^4 + 11*a^4 + 4*(30*a^3*b^
3 + 11*a*b^3)*x^3 + 6*(15*a^4*b^2 + 11*a^2*b^2 + b^2)*x^2 + 6*a^2 + 4*(9*a^
5*b + 11*a^3*b + 3*a*b)*x + 1)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + (2*b^7*x^7 +
 14*a*b^6*x^6 + 2*a^7 + (42*a^2*b^5 + 5*b^5)*x^5 + 5*a^5 + 5*(14*a^3*b^4 +
5*a*b^4)*x^4 + 2*(35*a^4*b^3 + 25*a^2*b^3 + 2*b^3)*x^3 + 4*a^3 + 2*(21*a^5*
b^2 + 25*a^3*b^2 + 6*a*b^2)*x^2 + (14*a^6*b + 25*a^4*b + 12*a^2*b + b)*x +
a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x
+ a^2 + 1)) + (b^7*x^7 + 7*a*b^6*x^6 + a^7 + 3*(7*a^2*b^5 + b^5)*x^5 + 3*a^
5 + 5*(7*a^3*b^4 + 3*a*b^4)*x^4 + (35*a^4*b^3 + 30*a^2*b^3 + 3*b^3)*x^3 + 3
*a^3 + 3*(7*a^5*b^2 + 10*a^3*b^2 + 3*a*b^2)*x^2 + (7*a^6*b + 15*a^4*b + 9*a
^2*b + b)*x + a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/((b^7*x^6 + 6*a*b^6*x^5
 + a^6*b + 3*a^4*b + 3*(5*a^2*b^5 + b^5)*x^4 + 4*(5*a^3*b^4 + 3*a*b^4)*x^3
+ 3*a^2*b + 3*(5*a^4*b^3 + 6*a^2*b^3 + b^3)*x^2 + (b^4*x^3 + 3*a*b^3*x^2 +
3*a^2*b^2*x + a^3*b)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + 3*(b^5*x^4 + 4*a
*b^4*x^3 + a^4*b + a^2*b + (6*a^2*b^3 + b^3)*x^2 + 2*(2*a^3*b^2 + a*b^2)*x)
*(b^2*x^2 + 2*a*b*x + a^2 + 1) + 6*(a^5*b^2 + 2*a^3*b^2 + a*b^2)*x + 3*(b^6
*x^5 + 5*a*b^5*x^4 + a^5*b + 2*a^3*b + 2*(5*a^2*b^4 + b^4)*x^3 + 2*(5*a^3*b
^3 + 3*a*b^3)*x^2 + a*b + (5*a^4*b^2 + 6*a^2*b^2 + b^2)*x)*sqrt(b^2*x^2 + 2
*a*b*x + a^2 + 1) + b)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2)
+ integrate(1/2*((4*b^4*x^4 + 16*a*b^3*x^3 + 4*a^4 + 2*(12*a^2*b^2 - b^2)*x
^2 - 2*a^2 + 4*(4*a^3*b - a*b)*x + 3)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(5/2) +
 2*(8*b^5*x^5 + 40*a*b^4*x^4 + 8*a^5 + 4*(20*a^2*b^3 + b^3)*x^3 + 4*a^3 + 4
*(20*a^3*b^2 + 3*a*b^2)*x^2 + (40*a^4*b + 12*a^2*b + b)*x + a)*(b^2*x^2 + 2
*a*b*x + a^2 + 1)^2 + 2*(12*b^6*x^6 + 72*a*b^5*x^5 + 12*a^6 + 18*(10*a^2*b^
4 + b^4)*x^4 + 18*a^4 + 24*(10*a^3*b^3 + 3*a*b^3)*x^3 + 6*(30*a^4*b^2 + 18*
a^2*b^2 + b^2)*x^2 + 6*a^2 + 12*(6*a^5*b + 6*a^3*b + a*b)*x - 1)*(b^2*x^2 +
 2*a*b*x + a^2 + 1)^(3/2) + 2*(8*b^7*x^7 + 56*a*b^6*x^6 + 8*a^7 + 4*(42*a^2
*b^5 + 5*b^5)*x^5 + 20*a^5 + 20*(14*a^3*b^4 + 5*a*b^4)*x^4 + 5*(56*a^4*b^3
+ 40*a^2*b^3 + 3*b^3)*x^3 + 15*a^3 + (168*a^5*b^2 + 200*a^3*b^2 + 45*a*b^2)
*x^2 + (56*a^6*b + 100*a^4*b + 45*a^2*b + 3*b)*x + 3*a)*(b^2*x^2 + 2*a*b*x
+ a^2 + 1) + (4*b^8*x^8 + 32*a*b^7*x^7 + 4*a^8 + 14*(8*a^2*b^6 + b^6)*x^6 +
 14*a^6 + 28*(8*a^3*b^5 + 3*a*b^5)*x^5 + (280*a^4*b^4 + 210*a^2*b^4 + 17*b^
4)*x^4 + 17*a^4 + 4*(56*a^5*b^3 + 70*a^3*b^3 + 17*a*b^3)*x^3 + 2*(56*a^6*b^
2 + 105*a^4*b^2 + 51*a^2*b^2 + 4*b^2)*x^2 + 8*a^2 + 4*(8*a^7*b + 21*a^5*b +
 17*a^3*b + 4*a*b)*x + 1)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/((b^8*x^8 + 8*
a*b^7*x^7 + a^8 + 4*(7*a^2*b^6 + b^6)*x^6 + 4*a^6 + 8*(7*a^3*b^5 + 3*a*b^5)
```

```
*x^5 + 2*(35*a^4*b^4 + 30*a^2*b^4 + 3*b^4)*x^4 + 6*a^4 + 8*(7*a^5*b^3 + 10*
a^3*b^3 + 3*a*b^3)*x^3 + (b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x
 + a^4)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^2 + 4*(7*a^6*b^2 + 15*a^4*b^2 + 9*a^2
*b^2 + b^2)*x^2 + 4*(b^5*x^5 + 5*a*b^4*x^4 + a^5 + (10*a^2*b^3 + b^3)*x^3 +
 a^3 + (10*a^3*b^2 + 3*a*b^2)*x^2 + (5*a^4*b + 3*a^2*b)*x)*(b^2*x^2 + 2*a*b
*x + a^2 + 1)^(3/2) + 6*(b^6*x^6 + 6*a*b^5*x^5 + a^6 + (15*a^2*b^4 + 2*b^4)
*x^4 + 2*a^4 + 4*(5*a^3*b^3 + 2*a*b^3)*x^3 + (15*a^4*b^2 + 12*a^2*b^2 + b^2
)*x^2 + a^2 + 2*(3*a^5*b + 4*a^3*b + a*b)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1)
+ 4*a^2 + 8*(a^7*b + 3*a^5*b + 3*a^3*b + a*b)*x + 4*(b^7*x^7 + 7*a*b^6*x^6
+ a^7 + 3*(7*a^2*b^5 + b^5)*x^5 + 3*a^5 + 5*(7*a^3*b^4 + 3*a*b^4)*x^4 + (35
*a^4*b^3 + 30*a^2*b^3 + 3*b^3)*x^3 + 3*a^3 + 3*(7*a^5*b^2 + 10*a^3*b^2 + 3*
a*b^2)*x^2 + (7*a^6*b + 15*a^4*b + 9*a^2*b + b)*x + a)*sqrt(b^2*x^2 + 2*a*b
*x + a^2 + 1) + 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}}{{\mathrm{asinh}\left(a+b\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2)/asinh(a + b*x)^3,x)
```

```
[Out] int((a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2)/asinh(a + b*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2+2abx+b^2x^2+1}}{\operatorname{asinh}^3(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2+1)**(1/2)/asinh(b*x+a)**3,x)
```

```
[Out] Integral(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/asinh(a + b*x)**3, x)
```

3.266 $\int \left(1+a^2+2abx+b^2x^2\right)^{3/2} \sinh^{-1}(a+bx)^3 \, dx$

Optimal. Leaf size=235

$$-\frac{3(a+bx)^4}{128b}-\frac{51(a+bx)^2}{128b}-\frac{9(a+bx)^2\sinh^{-1}(a+bx)^2}{16b}+\frac{\left((a+bx)^2+1\right)^{3/2}(a+bx)\sinh^{-1}(a+bx)^3}{4b}+\frac{3\sqrt{(a+b}}{}$$

```
[Out] -51/128*(b*x+a)^2/b-3/128*(b*x+a)^4/b+3/32*(b*x+a)*(1+(b*x+a)^2)^(3/2)*arcs
inh(b*x+a)/b-27/128*arcsinh(b*x+a)^2/b-9/16*(b*x+a)^2*arcsinh(b*x+a)^2/b-3/
16*(1+(b*x+a)^2)^2*arcsinh(b*x+a)^2/b+1/4*(b*x+a)*(1+(b*x+a)^2)^(3/2)*arcsi
nh(b*x+a)^3/b+3/32*arcsinh(b*x+a)^4/b+45/64*(b*x+a)*arcsinh(b*x+a)*(1+(b*x+
a)^2)^(1/2)/b+3/8*(b*x+a)*arcsinh(b*x+a)^3*(1+(b*x+a)^2)^(1/2)/b
```

Rubi [A] time = 0.31, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5867, 5684, 5682, 5675, 5661, 5758, 30, 5717, 14}

$$-\frac{3(a+bx)^4}{128b}-\frac{51(a+bx)^2}{128b}-\frac{9(a+bx)^2\sinh^{-1}(a+bx)^2}{16b}+\frac{\left((a+bx)^2+1\right)^{3/2}(a+bx)\sinh^{-1}(a+bx)^3}{4b}+\frac{3\sqrt{(a+b}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)*ArcSinh[a + b*x]^3,x]
```

```
[Out] (-51*(a + b*x)^2)/(128*b) - (3*(a + b*x)^4)/(128*b) + (45*(a + b*x)*Sqrt[1
+ (a + b*x)^2]*ArcSinh[a + b*x])/(64*b) + (3*(a + b*x)*(1 + (a + b*x)^2)^(3
/2)*ArcSinh[a + b*x])/(32*b) - (27*ArcSinh[a + b*x]^2)/(128*b) - (9*(a + b*
x)^2*ArcSinh[a + b*x]^2)/(16*b) - (3*(1 + (a + b*x)^2)^2*ArcSinh[a + b*x]^2
)/(16*b) + (3*(a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x]^3)/(8*b) + (
(a + b*x)*(1 + (a + b*x)^2)^(3/2)*ArcSinh[a + b*x]^3)/(4*b) + (3*ArcSinh[a
+ b*x]^4)/(32*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
```

```
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5867

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C*x^2)/d^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\int \left(1+a^2+2abx+b^2x^2\right)^{3/2}\sinh^{-1}(a+bx)^3\,dx = \frac{\text{Subst}\left(\int \left(1+x^2\right)^{3/2}\sinh^{-1}(x)^3\,dx,x,a+bx\right)}{b}$$

$$= \frac{(a+bx)\left(1+(a+bx)^2\right)^{3/2}\sinh^{-1}(a+bx)^3}{4b} - \frac{3\,\text{Subst}\left(\int x\left(\right.\right.}{}$$

$$= -\frac{3\left(1+(a+bx)^2\right)^2\sinh^{-1}(a+bx)^2}{16b} + \frac{3(a+bx)\sqrt{1+(a+b}}{8}$$

$$= \frac{3(a+bx)\left(1+(a+bx)^2\right)^{3/2}\sinh^{-1}(a+bx)}{32b} - \frac{9(a+bx)^2\sin}{16}$$

$$= \frac{45(a+bx)\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)}{64b} + \frac{3(a+bx)\left(1+\right.}{}$$

$$= -\frac{51(a+bx)^2}{128b} - \frac{3(a+bx)^4}{128b} + \frac{45(a+bx)\sqrt{1+(a+bx)^2}\sin}{64b}$$

Mathematica [A] time = 0.24, size = 266, normalized size = 1.13

$$-\frac{3\left(6a^2+17\right)b^2x^2+6a\left(2a^2+17\right)bx-16\sqrt{a^2+2abx+b^2x^2+1}\left(2a^3+6a^2bx+6ab^2x^2+5a+2b^3x^3+5bx\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)*ArcSinh[a + b*x]^3,x]
```

```
[Out] -1/128*(6*a*(17 + 2*a^2)*b*x + 3*(17 + 6*a^2)*b^2*x^2 + 12*a*b^3*x^3 + 3*b^
4*x^4 - 6*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(17*a + 2*a^3 + 17*b*x + 6*a^2*
b*x + 6*a*b^2*x^2 + 2*b^3*x^3)*ArcSinh[a + b*x] + 3*(17 + 8*a^4 + 32*a^3*b*
x + 40*b^2*x^2 + 8*b^4*x^4 + 16*a*b*x*(5 + 2*b^2*x^2) + 8*a^2*(5 + 6*b^2*x^
2))*ArcSinh[a + b*x]^2 - 16*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(5*a + 2*a^3
+ 5*b*x + 6*a^2*b*x + 6*a*b^2*x^2 + 2*b^3*x^3)*ArcSinh[a + b*x]^3 - 12*ArcS
inh[a + b*x]^4)/b
```

fricas [A] time = 0.46, size = 332, normalized size = 1.41

$$-\frac{3\,b^4x^4+12\,ab^3x^3+3\left(6\,a^2+17\right)b^2x^2-16\left(2\,b^3x^3+6\,ab^2x^2+2\,a^3+\left(6\,a^2+5\right)bx+5\,a\right)\sqrt{b^2x^2+2\,abx+a^2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)*arcsinh(b*x+a)^3,x, algorithm="fric
as")
```

```
[Out] -1/128*(3*b^4*x^4 + 12*a*b^3*x^3 + 3*(6*a^2 + 17)*b^2*x^2 - 16*(2*b^3*x^3 +
 6*a*b^2*x^2 + 2*a^3 + (6*a^2 + 5)*b*x + 5*a)*sqrt(b^2*x^2 + 2*a*b*x + a^2
+ 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^3 - 12*log(b*x + a +
sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^4 + 6*(2*a^3 + 17*a)*b*x + 3*(8*b^4*x^4
+ 32*a*b^3*x^3 + 8*(6*a^2 + 5)*b^2*x^2 + 8*a^4 + 16*(2*a^3 + 5*a)*b*x + 40*
a^2 + 17)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2 - 6*(2*b^3*x^3
 + 6*a*b^2*x^2 + 2*a^3 + (6*a^2 + 17)*b*x + 17*a)*sqrt(b^2*x^2 + 2*a*b*x +
a^2 + 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b^2x^2+2\,abx+a^2+1\right)^{\frac{3}{2}}\operatorname{arsinh}\left(bx+a\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)*arcsinh(b*x+a)^3,x, algorithm="giac
")
```

```
[Out] integrate((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*arcsinh(b*x + a)^3, x)
```

maple [B] time = 0.14, size = 592, normalized size = 2.52

$$\frac{-48-102abx+80\sqrt{b^2x^2+2abx+a^2+1}\ \operatorname{arcsinh}(bx+a)^3 xb-240\operatorname{arcsinh}(bx+a)^2 xab+102\sqrt{b^2x^2+2abx}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2+1)^(3/2)*arcsinh(b*x+a)^3,x)
```

```
[Out] 1/128*(-48-102*a*b*x+80*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*arcsinh(b*x+a)^3*x*b-
240*arcsinh(b*x+a)^2*x*a*b+102*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*arcsinh(b*x+a)
*x*b-51*arcsinh(b*x+a)^2+36*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*arcsinh(b*x+a)*x*
a^2*b-51*b^2*x^2-51*a^2-120*arcsinh(b*x+a)^2*x^2*b^2-24*arcsinh(b*x+a)^2*a^
4-3*x^4*b^4+32*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*arcsinh(b*x+a)^3*x^3*b^3+12*(b
^2*x^2+2*a*b*x+a^2+1)^(1/2)*arcsinh(b*x+a)*x^3*b^3-144*arcsinh(b*x+a)^2*x^2
*a^2*b^2-96*arcsinh(b*x+a)^2*x*a^3*b-96*arcsinh(b*x+a)^2*x^3*a*b^3-3*a^4+12
*arcsinh(b*x+a)^4-120*arcsinh(b*x+a)^2*a^2-12*x^3*a*b^3-18*x^2*a^2*b^2-12*x
*a^3*b+32*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*arcsinh(b*x+a)^3*a^3+12*(b^2*x^2+2*
a*b*x+a^2+1)^(1/2)*arcsinh(b*x+a)*a^3-24*arcsinh(b*x+a)^2*x^4*b^4+80*(b^2*x
^2+2*a*b*x+a^2+1)^(1/2)*arcsinh(b*x+a)^3*a+102*(b^2*x^2+2*a*b*x+a^2+1)^(1/2
)*arcsinh(b*x+a)*a+96*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*arcsinh(b*x+a)^3*x^2*a*
b^2+96*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*arcsinh(b*x+a)^3*x*a^2*b+36*(b^2*x^2+2
*a*b*x+a^2+1)^(1/2)*arcsinh(b*x+a)*x^2*a*b^2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b^2x^2+2\,abx+a^2+1\right)^{\frac{3}{2}} \operatorname{arsinh}(bx+a)^3\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)*arcsinh(b*x+a)^3,x, algorithm="maxi
ma")
```

```
[Out] integrate((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*arcsinh(b*x + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{asinh}(a+b\,x)^3 \left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a + b*x)^3*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2),x)
```

```
[Out] int(asinh(a + b*x)^3*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2), x)
```

sympy [A] time = 17.37, size = 694, normalized size = 2.95

$$\begin{cases} -\frac{3a^4\operatorname{asinh}^2(a+bx)}{16b}-\frac{3a^3x\operatorname{asinh}^2(a+bx)}{4}-\frac{3a^3x}{32}+\frac{a^3\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}^3(a+bx)}{4b}+\frac{3a^3\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}(a+bx)}{32b}-\frac{9a^2bx^2\operatorname{as}}{} \\ x\left(a^2+1\right)^{\frac{3}{2}}\operatorname{asinh}^3(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2+1)**(3/2)*asinh(b*x+a)**3,x)
```

```
[Out] Piecewise((-3*a**4*asinh(a + b*x)**2/(16*b) - 3*a**3*x*asinh(a + b*x)**2/4
- 3*a**3*x/32 + a**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)**3
/(4*b) + 3*a**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/(32*b)
- 9*a**2*b*x**2*asinh(a + b*x)**2/8 - 9*a**2*b*x**2/64 + 3*a**2*x*sqrt(a**2
 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)**3/4 + 9*a**2*x*sqrt(a**2 + 2*a*
b*x + b**2*x**2 + 1)*asinh(a + b*x)/32 - 15*a**2*asinh(a + b*x)**2/(16*b) -
 3*a*b**2*x**3*asinh(a + b*x)**2/4 - 3*a*b**2*x**3/32 + 3*a*b*x**2*sqrt(a**
2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)**3/4 + 9*a*b*x**2*sqrt(a**2 + 2
*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/32 - 15*a*x*asinh(a + b*x)**2/8 - 51
*a*x/64 + 5*a*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)**3/(8*b)
+ 51*a*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/(64*b) - 3*b**3*
x**4*asinh(a + b*x)**2/16 - 3*b**3*x**4/128 + b**2*x**3*sqrt(a**2 + 2*a*b*x
 + b**2*x**2 + 1)*asinh(a + b*x)**3/4 + 3*b**2*x**3*sqrt(a**2 + 2*a*b*x + b
**2*x**2 + 1)*asinh(a + b*x)/32 - 15*b*x**2*asinh(a + b*x)**2/16 - 51*b*x**
2/128 + 5*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)**3/8 + 51*x
*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/64 + 3*asinh(a + b*x)*
*4/(32*b) - 51*asinh(a + b*x)**2/(128*b), Ne(b, 0)), (x*(a**2 + 1)**(3/2)*a
sinh(a)**3, True))
```

3.267 $\int \left(1+a^2+2abx+b^2x^2\right)^{3/2} \sinh^{-1}(a+bx)^2\,dx$

Optimal. Leaf size=189

$$\frac{(a+bx)\left((a+bx)^2+1\right)^{3/2}}{32b}+\frac{15(a+bx)\sqrt{(a+bx)^2+1}}{64b}+\frac{\sinh^{-1}(a+bx)^3}{8b}+\frac{(a+bx)\left((a+bx)^2+1\right)^{3/2}\sinh^{-1}(a+b}{4b}$$

```
[Out] 1/32*(b*x+a)*(1+(b*x+a)^2)^(3/2)/b-9/64*arcsinh(b*x+a)/b-3/8*(b*x+a)^2*arcs
inh(b*x+a)/b-1/8*(1+(b*x+a)^2)^2*arcsinh(b*x+a)/b+1/4*(b*x+a)*(1+(b*x+a)^2)
^(3/2)*arcsinh(b*x+a)^2/b+1/8*arcsinh(b*x+a)^3/b+15/64*(b*x+a)*(1+(b*x+a)^2
)^(1/2)/b+3/8*(b*x+a)*arcsinh(b*x+a)^2*(1+(b*x+a)^2)^(1/2)/b
```

Rubi [A] time = 0.19, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5867, 5684, 5682, 5675, 5661, 321, 215, 5717, 195}

$$\frac{(a+bx)\left((a+bx)^2+1\right)^{3/2}}{32b}+\frac{15(a+bx)\sqrt{(a+bx)^2+1}}{64b}+\frac{\sinh^{-1}(a+bx)^3}{8b}+\frac{(a+bx)\left((a+bx)^2+1\right)^{3/2}\sinh^{-1}(a+b}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)*ArcSinh[a + b*x]^2,x]
```

```
[Out] (15*(a + b*x)*Sqrt[1 + (a + b*x)^2])/(64*b) + ((a + b*x)*(1 + (a + b*x)^2)^
(3/2))/(32*b) - (9*ArcSinh[a + b*x])/(64*b) - (3*(a + b*x)^2*ArcSinh[a + b*
x])/(8*b) - ((1 + (a + b*x)^2)^2*ArcSinh[a + b*x])/(8*b) + (3*(a + b*x)*Sqr
t[1 + (a + b*x)^2]*ArcSinh[a + b*x]^2)/(8*b) + ((a + b*x)*(1 + (a + b*x)^2)
^(3/2)*ArcSinh[a + b*x]^2)/(4*b) + ArcSinh[a + b*x]^3/(8*b)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5867

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C*x^2)/d^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\int \left(1+a^2+2abx+b^2x^2\right)^{3/2} \sinh^{-1}(a+bx)^2\,dx = \frac{\text{Subst}\left(\int \left(1+x^2\right)^{3/2} \sinh^{-1}(x)^2\,dx, x, a+bx\right)}{b}$$
$$= \frac{(a+bx)\left(1+(a+bx)^2\right)^{3/2} \sinh^{-1}(a+bx)^2}{4b} - \frac{\text{Subst}\left(\int x\left(1+x\right.\right.}{}$$
$$= -\frac{\left(1+(a+bx)^2\right)^2 \sinh^{-1}(a+bx)}{8b} + \frac{3(a+bx)\sqrt{1+(a+bx)^2}\, \text{s}}{8b}$$
$$= \frac{(a+bx)\left(1+(a+bx)^2\right)^{3/2}}{32b} - \frac{3(a+bx)^2 \sinh^{-1}(a+bx)}{8b} - \frac{\left(1+\right.}{}$$
$$= \frac{15(a+bx)\sqrt{1+(a+bx)^2}}{64b} + \frac{(a+bx)\left(1+(a+bx)^2\right)^{3/2}}{32b} - \frac{3(a}{}$$
$$= \frac{15(a+bx)\sqrt{1+(a+bx)^2}}{64b} + \frac{(a+bx)\left(1+(a+bx)^2\right)^{3/2}}{32b} - \frac{9\,\text{sir}}{}$$

Mathematica [A] time = 0.17, size = 211, normalized size = 1.12

$$\frac{-\left(8a^4+40a^2+17\right)\sinh^{-1}(a+bx)+\sqrt{a^2+2abx+b^2x^2+1}\left(2a^3+6a^2bx+6ab^2x^2+17a+2b^3x^3+17bx\right)+8}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)*ArcSinh[a + b*x]^2,x]
```

```
[Out] (Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(17*a + 2*a^3 + 17*b*x + 6*a^2*b*x + 6*a
*b^2*x^2 + 2*b^3*x^3) - (17 + 40*a^2 + 8*a^4)*ArcSinh[a + b*x] - 8*b*x*(10*
a + 4*a^3 + 5*b*x + 6*a^2*b*x + 4*a*b^2*x^2 + b^3*x^3)*ArcSinh[a + b*x] + 8
*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(5*a + 2*a^3 + 5*b*x + 6*a^2*b*x + 6*a*b
^2*x^2 + 2*b^3*x^3)*ArcSinh[a + b*x]^2 + 8*ArcSinh[a + b*x]^3)/(64*b)
```

fricas [A] time = 0.46, size = 259, normalized size = 1.37

$$\frac{8\left(2\,b^3x^3+6\,ab^2x^2+2\,a^3+\left(6\,a^2+5\right)bx+5\,a\right)\sqrt{b^2x^2+2\,abx+a^2+1}\,\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^2+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)*arcsinh(b*x+a)^2,x, algorithm="fric
as")
```

```
[Out] 1/64*(8*(2*b^3*x^3 + 6*a*b^2*x^2 + 2*a^3 + (6*a^2 + 5)*b*x + 5*a)*sqrt(b^2*
x^2 + 2*a*b*x + a^2 + 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2
 + 8*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^3 - (8*b^4*x^4 + 32*a
*b^3*x^3 + 8*(6*a^2 + 5)*b^2*x^2 + 8*a^4 + 16*(2*a^3 + 5*a)*b*x + 40*a^2 +
17)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + (2*b^3*x^3 + 6*a*b^2
*x^2 + 2*a^3 + (6*a^2 + 17)*b*x + 17*a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/
b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b^2x^2+2\,abx+a^2+1\right)^{\frac{3}{2}} \operatorname{arsinh}\left(bx+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)*arcsinh(b*x+a)^2,x, algorithm="giac
")
```

```
[Out] integrate((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*arcsinh(b*x + a)^2, x)
```

maple [B] time = 0.13, size = 479, normalized size = 2.53

$$\frac{16 \operatorname{arcsinh}(bx+a)^2 \sqrt{b^2x^2+2abx+a^2+1}\, x^3b^3 - 8 \operatorname{arcsinh}(bx+a)\, x^4b^4 + 48 \operatorname{arcsinh}(bx+a)^2 \sqrt{b^2x^2+2a}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2+1)^(3/2)*arcsinh(b*x+a)^2,x)
```

```
[Out] 1/64*(16*arcsinh(b*x+a)^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x^3*b^3-8*arcsinh(b
*x+a)*x^4*b^4+48*arcsinh(b*x+a)^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x^2*a*b^2-3
2*arcsinh(b*x+a)*x^3*a*b^3+48*arcsinh(b*x+a)^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2
)*x*a^2*b+2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x^3*b^3-48*arcsinh(b*x+a)*x^2*a^2
*b^2+16*arcsinh(b*x+a)^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a^3+6*(b^2*x^2+2*a*b
*x+a^2+1)^(1/2)*x^2*a*b^2-32*arcsinh(b*x+a)*x*a^3*b+40*arcsinh(b*x+a)^2*(b^
2*x^2+2*a*b*x+a^2+1)^(1/2)*x*b+6*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x*a^2*b-40*a
rcsinh(b*x+a)*x^2*b^2-8*arcsinh(b*x+a)*a^4+40*arcsinh(b*x+a)^2*(b^2*x^2+2*a
*b*x+a^2+1)^(1/2)*a+2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a^3-80*arcsinh(b*x+a)*x
*a*b+17*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x*b+8*arcsinh(b*x+a)^3-40*arcsinh(b*x
+a)*a^2+17*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a-17*arcsinh(b*x+a))/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b^2x^2 + 2\,abx + a^2 + 1\right)^{\frac{3}{2}} \operatorname{arsinh}(bx+a)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)*arcsinh(b*x+a)^2,x, algorithm="maxi
ma")
```

```
[Out] integrate((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*arcsinh(b*x + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{asinh}(a+b\,x)^2\,\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a + b*x)^2*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2),x)
```

```
[Out] int(asinh(a + b*x)^2*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2), x)
```

sympy [A] time = 10.35, size = 568, normalized size = 3.01

$$\begin{cases} -\frac{a^4 \operatorname{asinh}(a+bx)}{8b} - \frac{a^3 x \operatorname{asinh}(a+bx)}{2} + \frac{a^3\sqrt{a^2+2abx+b^2x^2+1}\,\operatorname{asinh}^2(a+bx)}{4b} + \frac{a^3\sqrt{a^2+2abx+b^2x^2+1}}{32b} - \frac{3a^2bx^2 \operatorname{asinh}(a+bx)}{4} + \frac{3a^2x\sqrt{a^2\cdots}}{} \\ x\left(a^2+1\right)^{\frac{3}{2}} \operatorname{asinh}^2(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2+1)**(3/2)*asinh(b*x+a)**2,x)
```

```
[Out] Piecewise((-a**4*asinh(a + b*x)/(8*b) - a**3*x*asinh(a + b*x)/2 + a**3*sqrt
(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)**2/(4*b) + a**3*sqrt(a**2 +
 2*a*b*x + b**2*x**2 + 1)/(32*b) - 3*a**2*b*x**2*asinh(a + b*x)/4 + 3*a**2*
```

```
x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)**2/4 + 3*a**2*x*sqrt(
a**2 + 2*a*b*x + b**2*x**2 + 1)/32 - 5*a**2*asinh(a + b*x)/(8*b) - a*b**2*x
**3*asinh(a + b*x)/2 + 3*a*b*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asin
h(a + b*x)**2/4 + 3*a*b*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/32 - 5*a*
x*asinh(a + b*x)/4 + 5*a*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x
)**2/(8*b) + 17*a*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(64*b) - b**3*x**4*a
sinh(a + b*x)/8 + b**2*x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a +
b*x)**2/4 + b**2*x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/32 - 5*b*x**2*as
inh(a + b*x)/8 + 5*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)**2
/8 + 17*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/64 + asinh(a + b*x)**3/(8*b)
 - 17*asinh(a + b*x)/(64*b), Ne(b, 0)), (x*(a**2 + 1)**(3/2)*asinh(a)**2, T
rue))
```

3.268 $\int \left(1+a^2+2abx+b^2x^2\right)^{3/2} \sinh^{-1}(a+bx)\,dx$

Optimal. Leaf size=106

$$-\frac{(a+bx)^4}{16b}-\frac{5(a+bx)^2}{16b}+\frac{\left((a+bx)^2+1\right)^{3/2}(a+bx)\sinh^{-1}(a+bx)}{4b}+\frac{3\sqrt{(a+bx)^2+1}\,(a+bx)\sinh^{-1}(a+bx)}{8b}+$$

```
[Out] -5/16*(b*x+a)^2/b-1/16*(b*x+a)^4/b+1/4*(b*x+a)*(1+(b*x+a)^2)^(3/2)*arcsinh(
b*x+a)/b+3/16*arcsinh(b*x+a)^2/b+3/8*(b*x+a)*arcsinh(b*x+a)*(1+(b*x+a)^2)^(
1/2)/b
```

Rubi [A] time = 0.10, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {5867, 5684, 5682, 5675, 30, 14}

$$-\frac{(a+bx)^4}{16b}-\frac{5(a+bx)^2}{16b}+\frac{\left((a+bx)^2+1\right)^{3/2}(a+bx)\sinh^{-1}(a+bx)}{4b}+\frac{3\sqrt{(a+bx)^2+1}\,(a+bx)\sinh^{-1}(a+bx)}{8b}+$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)*ArcSinh[a + b*x],x]
```

```
[Out] (-5*(a + b*x)^2)/(16*b) - (a + b*x)^4/(16*b) + (3*(a + b*x)*Sqrt[1 + (a + b
*x)^2]*ArcSinh[a + b*x])/(8*b) + ((a + b*x)*(1 + (a + b*x)^2)^(3/2)*ArcSinh
[a + b*x])/(4*b) + (3*ArcSinh[a + b*x]^2)/(16*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
```

```
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5867

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C*x^2)/d^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\int \left(1+a^2+2abx+b^2x^2\right)^{3/2}\sinh^{-1}(a+bx)\,dx = \frac{\text{Subst}\left(\int \left(1+x^2\right)^{3/2}\sinh^{-1}(x)\,dx, x, a+bx\right)}{b}$$
$$= \frac{(a+bx)\left(1+(a+bx)^2\right)^{3/2}\sinh^{-1}(a+bx)}{4b} - \frac{\text{Subst}\left(\int x\left(1+x^2\right.\right.}{4b}$$
$$= \frac{3(a+bx)\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{8b} + \frac{(a+bx)\left(1+(a+b\right.}{4}$$
$$= -\frac{5(a+bx)^2}{16b} - \frac{(a+bx)^4}{16b} + \frac{3(a+bx)\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+}{8b}$$

Mathematica [A] time = 0.10, size = 124, normalized size = 1.17

$$\frac{-bx\left(4a^3+6a^2bx+4ab^2x^2+10a+b^3x^3+5bx\right)+2\sqrt{a^2+2abx+b^2x^2+1}\,\left(2a^3+6a^2bx+6ab^2x^2+5a+2b^3x^3\right.}{16b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)*ArcSinh[a + b*x],x]
```

```
[Out] (-(b*x*(10*a + 4*a^3 + 5*b*x + 6*a^2*b*x + 4*a*b^2*x^2 + b^3*x^3)) + 2*Sqrt
[1 + a^2 + 2*a*b*x + b^2*x^2]*(5*a + 2*a^3 + 5*b*x + 6*a^2*b*x + 6*a*b^2*x^
2 + 2*b^3*x^3)*ArcSinh[a + b*x] + 3*ArcSinh[a + b*x]^2)/(16*b)
```

fricas [A] time = 0.43, size = 160, normalized size = 1.51

$$-\frac{b^4x^4+4\,ab^3x^3+\left(6\,a^2+5\right)b^2x^2+2\left(2\,a^3+5\,a\right)bx-2\left(2\,b^3x^3+6\,ab^2x^2+2\,a^3+\left(6\,a^2+5\right)bx+5\,a\right)\sqrt{b^2x^2+2\,a}}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)*arcsinh(b*x+a),x, algorithm="fricas
")
```

```
[Out] -1/16*(b^4*x^4 + 4*a*b^3*x^3 + (6*a^2 + 5)*b^2*x^2 + 2*(2*a^3 + 5*a)*b*x -
2*(2*b^3*x^3 + 6*a*b^2*x^2 + 2*a^3 + (6*a^2 + 5)*b*x + 5*a)*sqrt(b^2*x^2 +
2*a*b*x + a^2 + 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - 3*log
(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2)/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b^2x^2+2\,abx+a^2+1\right)^{\frac{3}{2}}\operatorname{arsinh}\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)*arcsinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*arcsinh(b*x + a), x)
```

maple [B] time = 0.11, size = 262, normalized size = 2.47

$$\frac{4\sqrt{b^2x^2+2abx+a^2+1}\ \operatorname{arcsinh}(bx+a)\,x^3b^3-x^4b^4+12\sqrt{b^2x^2+2abx+a^2+1}\ \operatorname{arcsinh}(bx+a)\,x^2a\,b^2-4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2+1)^(3/2)*arcsinh(b*x+a),x)
```

```
[Out] 1/16*(4*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*arcsinh(b*x+a)*x^3*b^3-x^4*b^4+12*(b^
2*x^2+2*a*b*x+a^2+1)^(1/2)*arcsinh(b*x+a)*x^2*a*b^2-4*x^3*a*b^3+12*(b^2*x^2
+2*a*b*x+a^2+1)^(1/2)*arcsinh(b*x+a)*x*a^2*b-6*x^2*a^2*b^2+4*(b^2*x^2+2*a*b
*x+a^2+1)^(1/2)*arcsinh(b*x+a)*a^3-4*x*a^3*b+10*(b^2*x^2+2*a*b*x+a^2+1)^(1/
2)*arcsinh(b*x+a)*x*b-5*b^2*x^2-a^4+10*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*arcsin
h(b*x+a)*a-10*a*b*x+3*arcsinh(b*x+a)^2-5*a^2-4)/b
```

maxima [B] time = 0.79, size = 394, normalized size = 3.72

$$-\frac{1}{16}\left(b^2x^4+4\,abx^3+6\,a^2x^2+\frac{4\,a^3x}{b}+5\,x^2+\frac{10\,ax}{b}+\frac{6\ \operatorname{arsinh}(bx+a)\operatorname{arsinh}\left(\frac{2\left(b^2x+ab\right)}{\sqrt{-4\,a^2b^2+4\left(a^2+1\right)b^2}}\right)}{b^2}-\frac{3\ \operatorname{arsir}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)*arcsinh(b*x+a),x, algorithm="maxima
")
```

```
[Out] -1/16*(b^2*x^4 + 4*a*b*x^3 + 6*a^2*x^2 + 4*a^3*x/b + 5*x^2 + 10*a*x/b + 6*a
rcsinh(b*x + a)*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))
/b^2 - 3*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))^2/b^2)
*b + 1/8*(2*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*x + 2*(b^2*x^2 + 2*a*b*x +
a^2 + 1)^(3/2)*a/b + 3*(a^2*b^2 - (a^2 + 1)*b^2)*a^2*arcsinh(2*(b^2*x + a*b
)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^3 - 3*(a^2*b^2 - (a^2 + 1)*b^2)*sqr
t(b^2*x^2 + 2*a*b*x + a^2 + 1)*x/b^2 - 3*(a^2*b^2 - (a^2 + 1)*b^2)*(a^2 + 1
)*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^3 - 3*(a^2*
b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a/b^3)*arcsinh(b*x +
 a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{asinh}(a+b\,x)\,\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a + b*x)*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2),x)
```

```
[Out] int(asinh(a + b*x)*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2), x)
```

sympy [A] time = 5.28, size = 298, normalized size = 2.81

$$\begin{cases}-\frac{a^3x}{4}+\frac{a^3\sqrt{a^2+2abx+b^2x^2+1}\ \operatorname{asinh}(a+bx)}{4b}-\frac{3a^2bx^2}{8}+\frac{3a^2x\sqrt{a^2+2abx+b^2x^2+1}\ \operatorname{asinh}(a+bx)}{4}-\frac{ab^2x^3}{4}+\frac{3abx^2\sqrt{a^2+2abx+b^2x^2+1}\ \operatorname{asinl}}{4}\\ x\left(a^2+1\right)^{\frac{3}{2}}\operatorname{asinh}(a)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2+1)**(3/2)*asinh(b*x+a),x)
```

```
[Out] Piecewise((-a**3*x/4 + a**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a +
b*x)/(4*b) - 3*a**2*b*x**2/8 + 3*a**2*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1
)*asinh(a + b*x)/4 - a*b**2*x**3/4 + 3*a*b*x**2*sqrt(a**2 + 2*a*b*x + b**2*
x**2 + 1)*asinh(a + b*x)/4 - 5*a*x/8 + 5*a*sqrt(a**2 + 2*a*b*x + b**2*x**2
+ 1)*asinh(a + b*x)/(8*b) - b**3*x**4/16 + b**2*x**3*sqrt(a**2 + 2*a*b*x +
b**2*x**2 + 1)*asinh(a + b*x)/4 - 5*b*x**2/16 + 5*x*sqrt(a**2 + 2*a*b*x + b
**2*x**2 + 1)*asinh(a + b*x)/8 + 3*asinh(a + b*x)**2/(16*b), Ne(b, 0)), (x*
(a**2 + 1)**(3/2)*asinh(a), True))
```

3.269 $\int \frac{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}{\sinh^{-1}(a+bx)}\,dx$

Optimal. Leaf size=47

$$\frac{\text{Chi}\left(2\sinh^{-1}(a+bx)\right)}{2b}+\frac{\text{Chi}\left(4\sinh^{-1}(a+bx)\right)}{8b}+\frac{3\log\left(\sinh^{-1}(a+bx)\right)}{8b}$$

```
[Out] 1/2*Chi(2*arcsinh(b*x+a))/b+1/8*Chi(4*arcsinh(b*x+a))/b+3/8*ln(arcsinh(b*x+
a))/b
```

Rubi [A] time = 0.14, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5867, 5699, 3312, 3301}

$$\frac{\text{Chi}\left(2\sinh^{-1}(a+bx)\right)}{2b}+\frac{\text{Chi}\left(4\sinh^{-1}(a+bx)\right)}{8b}+\frac{3\log\left(\sinh^{-1}(a+bx)\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)/ArcSinh[a + b*x],x]
```

```
[Out] CoshIntegral[2*ArcSinh[a + b*x]]/(2*b) + CoshIntegral[4*ArcSinh[a + b*x]]/(
8*b) + (3*Log[ArcSinh[a + b*x]])/(8*b)
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5699

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[d^p/c, Subst[Int[(a + b*x)^n*Cosh[x]^(2*p + 1), x], x, Ar
cSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*
p, 0] && (IntegerQ[p] || GtQ[d, 0])
```

Rule 5867

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C*x^2)/d^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\int \frac{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}{\sinh^{-1}(a+bx)}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^{3/2}}{\sinh^{-1}(x)}\,dx, x, a+bx\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{\cosh^4(x)}{x}\,dx, x, \sinh^{-1}(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{3}{8x}+\frac{\cosh(2x)}{2x}+\frac{\cosh(4x)}{8x}\right)dx, x, \sinh^{-1}(a+bx)\right)}{b}$$
$$= \frac{3\log\left(\sinh^{-1}(a+bx)\right)}{8b}+\frac{\text{Subst}\left(\int \frac{\cosh(4x)}{x}\,dx, x, \sinh^{-1}(a+bx)\right)}{8b}+\frac{\text{Subst}\left(\int\right.}{}$$
$$= \frac{\text{Chi}\left(2\sinh^{-1}(a+bx)\right)}{2b}+\frac{\text{Chi}\left(4\sinh^{-1}(a+bx)\right)}{8b}+\frac{3\log\left(\sinh^{-1}(a+bx)\right)}{8b}$$

Mathematica [**A**] time = 0.32, size = 37, normalized size = 0.79

$$\frac{4\text{Chi}\left(2\sinh^{-1}(a+bx)\right)+\text{Chi}\left(4\sinh^{-1}(a+bx)\right)+3\log\left(\sinh^{-1}(a+bx)\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)/ArcSinh[a + b*x],x]
```

```
[Out] (4*CoshIntegral[2*ArcSinh[a + b*x]] + CoshIntegral[4*ArcSinh[a + b*x]] + 3*
Log[ArcSinh[a + b*x]])/(8*b)
```

fricas [**F**] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2x^2+2\,abx+a^2+1\right)^{\frac{3}{2}}}{\text{arsinh}\,(bx+a)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a),x, algorithm="fricas
")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/arcsinh(b*x + a), x)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b^2x^2+2\,abx+a^2+1\right)^{\frac{3}{2}}}{\text{arsinh}\,(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/arcsinh(b*x + a), x)
```

maple [**A**] time = 0.13, size = 42, normalized size = 0.89

$$\frac{\text{X}\,(2\,\text{arcsinh}\,(bx+a))}{2b}+\frac{\text{X}\,(4\,\text{arcsinh}\,(bx+a))}{8b}+\frac{3\ln\,(\text{arcsinh}\,(bx+a))}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a),x)
```

```
[Out] 1/2*Chi(2*arcsinh(b*x+a))/b+1/8*Chi(4*arcsinh(b*x+a))/b+3/8*ln(arcsinh(b*x+
a))/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b^2x^2 + 2\,abx + a^2 + 1\right)^{\frac{3}{2}}}{\operatorname{arsinh}\left(bx + a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a),x, algorithm="maxima
")
```

```
[Out] integrate((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/arcsinh(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(a^2 + 2\,a\,b\,x + b^2\,x^2 + 1\right)^{3/2}}{\operatorname{asinh}\left(a + b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2)/asinh(a + b*x),x)
```

```
[Out] int((a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2)/asinh(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2 + 2abx + b^2x^2 + 1\right)^{\frac{3}{2}}}{\operatorname{asinh}\left(a + bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2+1)**(3/2)/asinh(b*x+a),x)
```

```
[Out] Integral((a**2 + 2*a*b*x + b**2*x**2 + 1)**(3/2)/asinh(a + b*x), x)
```

3.270 $\int \frac{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}{\sinh^{-1}(a+bx)^2}\,dx$

Optimal. Leaf size=54

$$\frac{\text{Shi}\left(2\sinh^{-1}(a+bx)\right)}{b}+\frac{\text{Shi}\left(4\sinh^{-1}(a+bx)\right)}{2b}-\frac{\left((a+bx)^2+1\right)^2}{b\sinh^{-1}(a+bx)}$$

```
[Out] -(1+(b*x+a)^2)^2/b/arcsinh(b*x+a)+Shi(2*arcsinh(b*x+a))/b+1/2*Shi(4*arcsinh
(b*x+a))/b
```

Rubi [A] time = 0.16, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5867, 5696, 5779, 5448, 3298}

$$\frac{\text{Shi}\left(2\sinh^{-1}(a+bx)\right)}{b}+\frac{\text{Shi}\left(4\sinh^{-1}(a+bx)\right)}{2b}-\frac{\left((a+bx)^2+1\right)^2}{b\sinh^{-1}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)/ArcSinh[a + b*x]^2,x]
```

```
[Out] -((1 + (a + b*x)^2)^2/(b*ArcSinh[a + b*x])) + SinhIntegral[2*ArcSinh[a + b*
x]]/b + SinhIntegral[4*ArcSinh[a + b*x]]/(2*b)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5696

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(Sqrt[1 + c^2*x^2]*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^(n +
 1))/(b*c*(n + 1)), x] - Dist[(c*(2*p + 1)*d^IntPart[p]*(d + e*x^2)^FracPar
t[p])/(b*(n + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*
(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ
[e, c^2*d] && LtQ[n, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 5867

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C*x^2)/d^2)
```

```
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\int \frac{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}{\sinh^{-1}(a+bx)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^{3/2}}{\sinh^{-1}(x)^2}\,dx, x, a+bx\right)}{b}$$
$$= -\frac{\left(1+(a+bx)^2\right)^2}{b\sinh^{-1}(a+bx)} + \frac{4\,\text{Subst}\left(\int \frac{x\left(1+x^2\right)}{\sinh^{-1}(x)}\,dx, x, a+bx\right)}{b}$$
$$= -\frac{\left(1+(a+bx)^2\right)^2}{b\sinh^{-1}(a+bx)} + \frac{4\,\text{Subst}\left(\int \frac{\cosh^3(x)\sinh(x)}{x}\,dx, x, \sinh^{-1}(a+bx)\right)}{b}$$
$$= -\frac{\left(1+(a+bx)^2\right)^2}{b\sinh^{-1}(a+bx)} + \frac{4\,\text{Subst}\left(\int \left(\frac{\sinh(2x)}{4x}+\frac{\sinh(4x)}{8x}\right)dx, x, \sinh^{-1}(a+bx)\right)}{b}$$
$$= -\frac{\left(1+(a+bx)^2\right)^2}{b\sinh^{-1}(a+bx)} + \frac{\text{Subst}\left(\int \frac{\sinh(4x)}{x}\,dx, x, \sinh^{-1}(a+bx)\right)}{2b} + \frac{\text{Subst}\left(\int \frac{\text{si}}{}\right.}{}$$
$$= -\frac{\left(1+(a+bx)^2\right)^2}{b\sinh^{-1}(a+bx)} + \frac{\text{Shi}\left(2\sinh^{-1}(a+bx)\right)}{b} + \frac{\text{Shi}\left(4\sinh^{-1}(a+bx)\right)}{2b}$$

Mathematica [A] time = 0.29, size = 70, normalized size = 1.30

$$\frac{-2\left(a^2+2abx+b^2x^2+1\right)^2+2\sinh^{-1}(a+bx)\text{Shi}\left(2\sinh^{-1}(a+bx)\right)+\sinh^{-1}(a+bx)\text{Shi}\left(4\sinh^{-1}(a+bx)\right)}{2b\sinh^{-1}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)/ArcSinh[a + b*x]^2,x]
```

```
[Out] (-2*(1 + a^2 + 2*a*b*x + b^2*x^2)^2 + 2*ArcSinh[a + b*x]*SinhIntegral[2*Arc
Sinh[a + b*x]] + ArcSinh[a + b*x]*SinhIntegral[4*ArcSinh[a + b*x]])/(2*b*Ar
cSinh[a + b*x])
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2x^2+2\,abx+a^2+1\right)^{\frac{3}{2}}}{\text{arsinh}\left(bx+a\right)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a)^2,x, algorithm="fric
as")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/arcsinh(b*x + a)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b^2x^2+2\,abx+a^2+1\right)^{\frac{3}{2}}}{\text{arsinh}\left(bx+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a)^2,x, algorithm="giac
")
```

```
[Out] integrate((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/arcsinh(b*x + a)^2, x)
```

maple [A] time = 0.12, size = 72, normalized size = 1.33

$$\frac{4\,\mathrm{Shi}\left(4\operatorname{arcsinh}\left(bx+a\right)\right)\operatorname{arcsinh}\left(bx+a\right)+8\,\mathrm{Shi}\left(2\operatorname{arcsinh}\left(bx+a\right)\right)\operatorname{arcsinh}\left(bx+a\right)-\cosh\left(4\operatorname{arcsinh}\left(bx+a\right)\right.}{8b\operatorname{arcsinh}\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a)^2,x)
```

```
[Out] 1/8/b*(4*Shi(4*arcsinh(b*x+a))*arcsinh(b*x+a)+8*Shi(2*arcsinh(b*x+a))*arcsi
nh(b*x+a)-cosh(4*arcsinh(b*x+a))-4*cosh(2*arcsinh(b*x+a))-3)/arcsinh(b*x+a)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(b^4x^4+4\,ab^3x^3+a^4+2\left(3\,a^2b^2+b^2\right)x^2+2\,a^2+4\left(a^3b+ab\right)x+1\right)\left(b^2x^2+2\,abx+a^2+1\right)+\left(b^5x^5+5\,ab^4x^4+\right.}{\left(b^3x^2+2\,ab^2x+a^2b+\sqrt{b^2x^2+2\,abx+a^2+1}\left(b^2x+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a)^2,x, algorithm="maxi
ma")
```

```
[Out] -((b^4*x^4 + 4*a*b^3*x^3 + a^4 + 2*(3*a^2*b^2 + b^2)*x^2 + 2*a^2 + 4*(a^3*b
 + a*b)*x + 1)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + (b^5*x^5 + 5*a*b^4*x^4 + a^5
 + 2*(5*a^2*b^3 + b^3)*x^3 + 2*a^3 + 2*(5*a^3*b^2 + 3*a*b^2)*x^2 + (5*a^4*b
 + 6*a^2*b + b)*x + a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/((b^3*x^2 + 2*a*b
^2*x + a^2*b + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b^2*x + a*b) + b)*log(b*x
 + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))) + integrate(((4*b^4*x^4 + 16*a*b
^3*x^3 + 4*a^4 + 3*(8*a^2*b^2 + b^2)*x^2 + 3*a^2 + 2*(8*a^3*b + 3*a*b)*x -
1)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + 4*(2*b^5*x^5 + 10*a*b^4*x^4 + 2*a^
5 + (20*a^2*b^3 + 3*b^3)*x^3 + 3*a^3 + (20*a^3*b^2 + 9*a*b^2)*x^2 + (10*a^4
*b + 9*a^2*b + b)*x + a)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + (4*b^6*x^6 + 24*a*
b^5*x^5 + 4*a^6 + 3*(20*a^2*b^4 + 3*b^4)*x^4 + 9*a^4 + 4*(20*a^3*b^3 + 9*a*
b^3)*x^3 + 6*(10*a^4*b^2 + 9*a^2*b^2 + b^2)*x^2 + 6*a^2 + 12*(2*a^5*b + 3*a
^3*b + a*b)*x + 1)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/((b^4*x^4 + 4*a*b^3*x
^3 + a^4 + 2*(3*a^2*b^2 + b^2)*x^2 + (b^2*x^2 + 2*a*b*x + a^2 + 1)*(b^2*x^2
 + 2*a*b*x + a^2) + 2*a^2 + 4*(a^3*b + a*b)*x + 2*(b^3*x^3 + 3*a*b^2*x^2 +
a^3 + (3*a^2*b + b)*x + a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 1)*log(b*x +
 a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)}^{3/2}}{{\mathrm{asinh}\left(a+b\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2)/asinh(a + b*x)^2,x)
```

```
[Out] int((a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2)/asinh(a + b*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^{2} + 2abx + b^{2}x^{2} + 1\right)^{\frac{3}{2}}}{\operatorname{asinh}^{2}{\left(a + bx \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2+1)**(3/2)/asinh(b*x+a)**2,x)
[Out] Integral((a**2 + 2*a*b*x + b**2*x**2 + 1)**(3/2)/asinh(a + b*x)**2, x)
```

3.271 $\int \frac{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}{\sinh^{-1}(a+bx)^3}\,dx$

Optimal. Leaf size=84

$$\frac{\text{Chi}\left(2\sinh^{-1}(a+bx)\right)}{b}+\frac{\text{Chi}\left(4\sinh^{-1}(a+bx)\right)}{b}-\frac{\left((a+bx)^2+1\right)^2}{2b\sinh^{-1}(a+bx)^2}-\frac{2(a+bx)\left((a+bx)^2+1\right)^{3/2}}{b\sinh^{-1}(a+bx)}$$

```
[Out] -1/2*(1+(b*x+a)^2)^2/b/arcsinh(b*x+a)^2-2*(b*x+a)*(1+(b*x+a)^2)^(3/2)/b/arc
sinh(b*x+a)+Chi(2*arcsinh(b*x+a))/b+Chi(4*arcsinh(b*x+a))/b
```

Rubi [A] time = 0.28, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {5867, 5696, 5777, 5699, 3312, 3301, 5779, 5448}

$$\frac{\text{Chi}\left(2\sinh^{-1}(a+bx)\right)}{b}+\frac{\text{Chi}\left(4\sinh^{-1}(a+bx)\right)}{b}-\frac{\left((a+bx)^2+1\right)^2}{2b\sinh^{-1}(a+bx)^2}-\frac{2(a+bx)\left((a+bx)^2+1\right)^{3/2}}{b\sinh^{-1}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)/ArcSinh[a + b*x]^3,x]
```

```
[Out] -(1 + (a + b*x)^2)^2/(2*b*ArcSinh[a + b*x]^2) - (2*(a + b*x)*(1 + (a + b*x)
^2)^(3/2))/(b*ArcSinh[a + b*x]) + CoshIntegral[2*ArcSinh[a + b*x]]/b + Cosh
Integral[4*ArcSinh[a + b*x]]/b
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5696

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(Sqrt[1 + c^2*x^2]*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^(n +
 1))/(b*c*(n + 1)), x] - Dist[(c*(2*p + 1)*d^IntPart[p]*(d + e*x^2)^FracPar
t[p])/(b*(n + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*
(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ
[e, c^2*d] && LtQ[n, -1]
```

Rule 5699

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[d^p/c, Subst[Int[(a + b*x)^n*Cosh[x]^(2*p + 1), x], x, Ar
cSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*
```

```
p, 0] && (IntegerQ[p] || GtQ[d, 0])
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^
p*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-Dist[(f*m*d^IntPart[p
]*(d + e*x^2)^FracPart[p])/(b*c*(n + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*
x)^(m - 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - D
ist[(c*(m + 2*p + 1)*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(b*f*(n + 1)*(1
+ c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*A
rcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2
*d] && LtQ[n, -1] && IGtQ[m, -3] && IGtQ[2*p, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 5867

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C*x^2)/d^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\int \frac{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}{\sinh^{-1}(a+bx)^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^{3/2}}{\sinh^{-1}(x)^3}\,dx, x, a+bx\right)}{b}$$

$$= -\frac{\left(1+(a+bx)^2\right)^2}{2b\sinh^{-1}(a+bx)^2} + \frac{2\,\text{Subst}\left(\int \frac{x\left(1+x^2\right)}{\sinh^{-1}(x)^2}\,dx, x, a+bx\right)}{b}$$

$$= -\frac{\left(1+(a+bx)^2\right)^2}{2b\sinh^{-1}(a+bx)^2} - \frac{2(a+bx)\left(1+(a+bx)^2\right)^{3/2}}{b\sinh^{-1}(a+bx)} + \frac{2\,\text{Subst}\left(\int \frac{\sqrt{1+x^2}}{\sinh^{-1}(x)}\,dx \ldots\right.}{b}$$

$$= -\frac{\left(1+(a+bx)^2\right)^2}{2b\sinh^{-1}(a+bx)^2} - \frac{2(a+bx)\left(1+(a+bx)^2\right)^{3/2}}{b\sinh^{-1}(a+bx)} + \frac{2\,\text{Subst}\left(\int \frac{\cosh^2(x)}{x}\,dx \ldots\right.}{b}$$

$$= -\frac{\left(1+(a+bx)^2\right)^2}{2b\sinh^{-1}(a+bx)^2} - \frac{2(a+bx)\left(1+(a+bx)^2\right)^{3/2}}{b\sinh^{-1}(a+bx)} + \frac{2\,\text{Subst}\left(\int \left(\frac{1}{2x} + \frac{\cosh \ldots}{2 \ldots}\right.\right.}{\ldots}$$

$$= -\frac{\left(1+(a+bx)^2\right)^2}{2b\sinh^{-1}(a+bx)^2} - \frac{2(a+bx)\left(1+(a+bx)^2\right)^{3/2}}{b\sinh^{-1}(a+bx)} + \frac{\text{Subst}\left(\int \frac{\cosh(2x)}{x}\,dx, \ldots\right.}{b}$$

$$= -\frac{\left(1+(a+bx)^2\right)^2}{2b\sinh^{-1}(a+bx)^2} - \frac{2(a+bx)\left(1+(a+bx)^2\right)^{3/2}}{b\sinh^{-1}(a+bx)} + \frac{\text{Chi}\left(2\sinh^{-1}(a+bx \ldots\right.}{b}$$

Mathematica [A] time = 0.40, size = 108, normalized size = 1.29

$$\frac{-\frac{(a^2+2abx+b^2x^2+1)\left(4(a+bx)\sqrt{a^2+2abx+b^2x^2+1}\sinh^{-1}(a+bx)+a^2+2abx+b^2x^2+1\right)}{\sinh^{-1}(a+bx)^2}+2\text{Chi}\left(2\sinh^{-1}(a+bx)\right)+2\text{Chi}\left(4\sinh^{-1}(a+\right.}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)/ArcSinh[a + b*x]^3,x]
```

```
[Out] (-(((1 + a^2 + 2*a*b*x + b^2*x^2)*(1 + a^2 + 2*a*b*x + b^2*x^2 + 4*(a + b*x
)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x]))/ArcSinh[a + b*x]^2)
+ 2*CoshIntegral[2*ArcSinh[a + b*x]] + 2*CoshIntegral[4*ArcSinh[a + b*x]])/
(2*b)
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}}{\operatorname{arsinh}(bx+a)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a)^3,x, algorithm="fric
as")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/arcsinh(b*x + a)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}}{\operatorname{arsinh}(bx+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a)^3,x, algorithm="giac
")
```

```
[Out] integrate((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/arcsinh(b*x + a)^3, x)
```

maple [A] time = 0.12, size = 110, normalized size = 1.31

$$\frac{16X(2\operatorname{arcsinh}(bx+a))\operatorname{arcsinh}(bx+a)^2+16X(4\operatorname{arcsinh}(bx+a))\operatorname{arcsinh}(bx+a)^2-8\sinh(2\operatorname{arcsinh}(bx+}{16b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a)^3,x)
```

```
[Out] 1/16/b*(16*Chi(2*arcsinh(b*x+a))*arcsinh(b*x+a)^2+16*Chi(4*arcsinh(b*x+a))*
arcsinh(b*x+a)^2-8*sinh(2*arcsinh(b*x+a))*arcsinh(b*x+a)-4*sinh(4*arcsinh(b
*x+a))*arcsinh(b*x+a)-4*cosh(2*arcsinh(b*x+a))-cosh(4*arcsinh(b*x+a))-3)/ar
csinh(b*x+a)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a)^3,x, algorithm="maxi
ma")
```

```
[Out] -1/2*((b^6*x^6 + 6*a*b^5*x^5 + a^6 + (15*a^2*b^4 + 2*b^4)*x^4 + 2*a^4 + 4*(
5*a^3*b^3 + 2*a*b^3)*x^3 + (15*a^4*b^2 + 12*a^2*b^2 + b^2)*x^2 + a^2 + 2*(3
*a^5*b + 4*a^3*b + a*b)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^2 + (3*b^7*x^7 + 2
1*a*b^6*x^6 + 3*a^7 + (63*a^2*b^5 + 8*b^5)*x^5 + 8*a^5 + 5*(21*a^3*b^4 + 8*
a*b^4)*x^4 + (105*a^4*b^3 + 80*a^2*b^3 + 7*b^3)*x^3 + 7*a^3 + (63*a^5*b^2 +
 80*a^3*b^2 + 21*a*b^2)*x^2 + (21*a^6*b + 40*a^4*b + 21*a^2*b + 2*b)*x + 2*
a)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + (3*b^8*x^8 + 24*a*b^7*x^7 + 3*a^8
+ 2*(42*a^2*b^6 + 5*b^6)*x^6 + 10*a^6 + 12*(14*a^3*b^5 + 5*a*b^5)*x^5 + 6*(
35*a^4*b^4 + 25*a^2*b^4 + 2*b^4)*x^4 + 12*a^4 + 8*(21*a^5*b^3 + 25*a^3*b^3
+ 6*a*b^3)*x^3 + 6*(14*a^6*b^2 + 25*a^4*b^2 + 12*a^2*b^2 + b^2)*x^2 + 6*a^2
 + 12*(2*a^7*b + 5*a^5*b + 4*a^3*b + a*b)*x + 1)*(b^2*x^2 + 2*a*b*x + a^2 +
 1) + ((4*b^6*x^6 + 24*a*b^5*x^5 + 4*a^6 + (60*a^2*b^4 + 7*b^4)*x^4 + 7*a^4
 + 4*(20*a^3*b^3 + 7*a*b^3)*x^3 + 2*(30*a^4*b^2 + 21*a^2*b^2 + b^2)*x^2 + 2
*a^2 + 4*(6*a^5*b + 7*a^3*b + a*b)*x - 1)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^2 +
 3*(4*b^7*x^7 + 28*a*b^6*x^6 + 4*a^7 + 3*(28*a^2*b^5 + 3*b^5)*x^5 + 9*a^5 +
 5*(28*a^3*b^4 + 9*a*b^4)*x^4 + 2*(70*a^4*b^3 + 45*a^2*b^3 + 3*b^3)*x^3 + 6
*a^3 + 6*(14*a^5*b^2 + 15*a^3*b^2 + 3*a*b^2)*x^2 + (28*a^6*b + 45*a^4*b + 1
8*a^2*b + b)*x + a)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + (12*b^8*x^8 + 96*
a*b^7*x^7 + 12*a^8 + 3*(112*a^2*b^6 + 11*b^6)*x^6 + 33*a^6 + 6*(112*a^3*b^5
 + 33*a*b^5)*x^5 + (840*a^4*b^4 + 495*a^2*b^4 + 31*b^4)*x^4 + 31*a^4 + 4*(1
68*a^5*b^3 + 165*a^3*b^3 + 31*a*b^3)*x^3 + (336*a^6*b^2 + 495*a^4*b^2 + 186
*a^2*b^2 + 11*b^2)*x^2 + 11*a^2 + 2*(48*a^7*b + 99*a^5*b + 62*a^3*b + 11*a*
b)*x + 1)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + (4*b^9*x^9 + 36*a*b^8*x^8 + 4*a^9
 + (144*a^2*b^7 + 13*b^7)*x^7 + 13*a^7 + 7*(48*a^3*b^6 + 13*a*b^6)*x^6 + 3*
(168*a^4*b^5 + 91*a^2*b^5 + 5*b^5)*x^5 + 15*a^5 + (504*a^5*b^4 + 455*a^3*b^
4 + 75*a*b^4)*x^4 + (336*a^6*b^3 + 455*a^4*b^3 + 150*a^2*b^3 + 7*b^3)*x^3 +
 7*a^3 + 3*(48*a^7*b^2 + 91*a^5*b^2 + 50*a^3*b^2 + 7*a*b^2)*x^2 + (36*a^8*b
 + 91*a^6*b + 75*a^4*b + 21*a^2*b + b)*x + a)*sqrt(b^2*x^2 + 2*a*b*x + a^2
+ 1))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + (b^9*x^9 + 9*a*b^8
*x^8 + a^9 + 4*(9*a^2*b^7 + b^7)*x^7 + 4*a^7 + 28*(3*a^3*b^6 + a*b^6)*x^6 +
 6*(21*a^4*b^5 + 14*a^2*b^5 + b^5)*x^5 + 6*a^5 + 2*(63*a^5*b^4 + 70*a^3*b^4
 + 15*a*b^4)*x^4 + 4*(21*a^6*b^3 + 35*a^4*b^3 + 15*a^2*b^3 + b^3)*x^3 + 4*a
^3 + 12*(3*a^7*b^2 + 7*a^5*b^2 + 5*a^3*b^2 + a*b^2)*x^2 + (9*a^8*b + 28*a^6
*b + 30*a^4*b + 12*a^2*b + b)*x + a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/((b
^7*x^6 + 6*a*b^6*x^5 + a^6*b + 3*a^4*b + 3*(5*a^2*b^5 + b^5)*x^4 + 4*(5*a^3
*b^4 + 3*a*b^4)*x^3 + 3*a^2*b + 3*(5*a^4*b^3 + 6*a^2*b^3 + b^3)*x^2 + (b^4*
x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*x + a^3*b)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2
) + 3*(b^5*x^4 + 4*a*b^4*x^3 + a^4*b + a^2*b + (6*a^2*b^3 + b^3)*x^2 + 2*(2
*a^3*b^2 + a*b^2)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + 6*(a^5*b^2 + 2*a^3*b^2
 + a*b^2)*x + 3*(b^6*x^5 + 5*a*b^5*x^4 + a^5*b + 2*a^3*b + 2*(5*a^2*b^4 + b
^4)*x^3 + 2*(5*a^3*b^3 + 3*a*b^3)*x^2 + a*b + (5*a^4*b^2 + 6*a^2*b^2 + b^2)
*x)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + b)*log(b*x + a + sqrt(b^2*x^2 + 2*a
*b*x + a^2 + 1))^2) + integrate(1/2*((16*b^6*x^6 + 96*a*b^5*x^5 + 16*a^6 +
10*(24*a^2*b^4 + b^4)*x^4 + 10*a^4 + 40*(8*a^3*b^3 + a*b^3)*x^3 + 3*(80*a^4
*b^2 + 20*a^2*b^2 - b^2)*x^2 - 3*a^2 + 2*(48*a^5*b + 20*a^3*b - 3*a*b)*x +
3)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(5/2) + 4*(16*b^7*x^7 + 112*a*b^6*x^6 + 16
*a^7 + (336*a^2*b^5 + 23*b^5)*x^5 + 23*a^5 + 5*(112*a^3*b^4 + 23*a*b^4)*x^4
 + (560*a^4*b^3 + 230*a^2*b^3 + 7*b^3)*x^3 + 7*a^3 + (336*a^5*b^2 + 230*a^3
*b^2 + 21*a*b^2)*x^2 + (112*a^6*b + 115*a^4*b + 21*a^2*b)*x)*(b^2*x^2 + 2*a
*b*x + a^2 + 1)^2 + 12*(8*b^8*x^8 + 64*a*b^7*x^7 + 8*a^8 + 2*(112*a^2*b^6 +
 9*b^6)*x^6 + 18*a^6 + 4*(112*a^3*b^5 + 27*a*b^5)*x^5 + (560*a^4*b^4 + 270*
a^2*b^4 + 13*b^4)*x^4 + 13*a^4 + 4*(112*a^5*b^3 + 90*a^3*b^3 + 13*a*b^3)*x^
3 + (224*a^6*b^2 + 270*a^4*b^2 + 78*a^2*b^2 + 3*b^2)*x^2 + 3*a^2 + 2*(32*a^
7*b + 54*a^5*b + 26*a^3*b + 3*a*b)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) +
 4*(16*b^9*x^9 + 144*a*b^8*x^8 + 16*a^9 + (576*a^2*b^7 + 49*b^7)*x^7 + 49*a
^7 + 7*(192*a^3*b^6 + 49*a*b^6)*x^6 + 3*(672*a^4*b^5 + 343*a^2*b^5 + 18*b^5
)*x^5 + 54*a^5 + (2016*a^5*b^4 + 1715*a^3*b^4 + 270*a*b^4)*x^4 + (1344*a^6*
b^3 + 1715*a^4*b^3 + 540*a^2*b^3 + 25*b^3)*x^3 + 25*a^3 + 3*(192*a^7*b^2 +
343*a^5*b^2 + 180*a^3*b^2 + 25*a*b^2)*x^2 + (144*a^8*b + 343*a^6*b + 270*a^
```

```
4*b + 75*a^2*b + 4*b)*x + 4*a)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + (16*b^10*x^1
0 + 160*a*b^9*x^9 + 16*a^10 + 2*(360*a^2*b^8 + 31*b^8)*x^8 + 62*a^8 + 16*(1
20*a^3*b^7 + 31*a*b^7)*x^7 + 7*(480*a^4*b^6 + 248*a^2*b^6 + 13*b^6)*x^6 + 9
1*a^6 + 14*(288*a^5*b^5 + 248*a^3*b^5 + 39*a*b^5)*x^5 + (3360*a^6*b^4 + 434
0*a^4*b^4 + 1365*a^2*b^4 + 61*b^4)*x^4 + 61*a^4 + 4*(480*a^7*b^3 + 868*a^5*
b^3 + 455*a^3*b^3 + 61*a*b^3)*x^3 + (720*a^8*b^2 + 1736*a^6*b^2 + 1365*a^4*
b^2 + 366*a^2*b^2 + 17*b^2)*x^2 + 17*a^2 + 2*(80*a^9*b + 248*a^7*b + 273*a^
5*b + 122*a^3*b + 17*a*b)*x + 1)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/((b^8*x
^8 + 8*a*b^7*x^7 + a^8 + 4*(7*a^2*b^6 + b^6)*x^6 + 4*a^6 + 8*(7*a^3*b^5 + 3
*a*b^5)*x^5 + 2*(35*a^4*b^4 + 30*a^2*b^4 + 3*b^4)*x^4 + 6*a^4 + 8*(7*a^5*b^
3 + 10*a^3*b^3 + 3*a*b^3)*x^3 + (b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*
a^3*b*x + a^4)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^2 + 4*(7*a^6*b^2 + 15*a^4*b^2
+ 9*a^2*b^2 + b^2)*x^2 + 4*(b^5*x^5 + 5*a*b^4*x^4 + a^5 + (10*a^2*b^3 + b^3
)*x^3 + a^3 + (10*a^3*b^2 + 3*a*b^2)*x^2 + (5*a^4*b + 3*a^2*b)*x)*(b^2*x^2
+ 2*a*b*x + a^2 + 1)^(3/2) + 6*(b^6*x^6 + 6*a*b^5*x^5 + a^6 + (15*a^2*b^4 +
 2*b^4)*x^4 + 2*a^4 + 4*(5*a^3*b^3 + 2*a*b^3)*x^3 + (15*a^4*b^2 + 12*a^2*b^
2 + b^2)*x^2 + a^2 + 2*(3*a^5*b + 4*a^3*b + a*b)*x)*(b^2*x^2 + 2*a*b*x + a^
2 + 1) + 4*a^2 + 8*(a^7*b + 3*a^5*b + 3*a^3*b + a*b)*x + 4*(b^7*x^7 + 7*a*b
^6*x^6 + a^7 + 3*(7*a^2*b^5 + b^5)*x^5 + 3*a^5 + 5*(7*a^3*b^4 + 3*a*b^4)*x^
4 + (35*a^4*b^3 + 30*a^2*b^3 + 3*b^3)*x^3 + 3*a^3 + 3*(7*a^5*b^2 + 10*a^3*b
^2 + 3*a*b^2)*x^2 + (7*a^6*b + 15*a^4*b + 9*a^2*b + b)*x + a)*sqrt(b^2*x^2
+ 2*a*b*x + a^2 + 1) + 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)))
, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)^{3/2}}{\mathrm{asinh}\left(a+b\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2)/asinh(a + b*x)^3,x)
[Out] int((a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2)/asinh(a + b*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2+2abx+b^2x^2+1\right)^{\frac{3}{2}}}{\operatorname{asinh}^3\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2+1)**(3/2)/asinh(b*x+a)**3,x)
[Out] Integral((a**2 + 2*a*b*x + b**2*x**2 + 1)**(3/2)/asinh(a + b*x)**3, x)
```

3.272 $\int \frac{\sinh^{-1}(a+bx)^3}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx$

Optimal. Leaf size=15

$$\frac{\sinh^{-1}(a+bx)^4}{4b}$$

```
[Out] 1/4*arcsinh(b*x+a)^4/b
```

Rubi [A] time = 0.07, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {5867, 5675}

$$\frac{\sinh^{-1}(a+bx)^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a + b*x]^3/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] ArcSinh[a + b*x]^4/(4*b)
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5867

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C*x^2)/d^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^{-1}(a+bx)^3}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx &= \frac{\text{Subst}\left(\int \frac{\sinh^{-1}(x)^3}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{b}\\ &= \frac{\sinh^{-1}(a+bx)^4}{4b}\end{aligned}$$

Mathematica [A] time = 0.02, size = 15, normalized size = 1.00

$$\frac{\sinh^{-1}(a+bx)^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a + b*x]^3/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] ArcSinh[a + b*x]^4/(4*b)
```

fricas [B] time = 0.49, size = 32, normalized size = 2.13

$$\frac{\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^4}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="fric
as")
```

```
[Out] 1/4*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^4/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}(bx+a)^3}{\sqrt{b^2x^2+2\,abx+a^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(arcsinh(b*x + a)^3/sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1), x)
```

maple [A] time = 0.07, size = 14, normalized size = 0.93

$$\frac{\operatorname{arcsinh}(bx+a)^4}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x)
```

```
[Out] 1/4*arcsinh(b*x+a)^4/b
```

maxima [B] time = 0.77, size = 179, normalized size = 11.93

$$\frac{\operatorname{arsinh}(bx+a)^3\operatorname{arsinh}\left(\frac{2\left(b^2x+ab\right)}{\sqrt{-4\,a^2b^2+4\left(a^2+1\right)b^2}}\right)}{b}-\frac{3\,\operatorname{arsinh}(bx+a)^2\operatorname{arsinh}\left(\frac{2\left(b^2x+ab\right)}{\sqrt{-4\,a^2b^2+4\left(a^2+1\right)b^2}}\right)^2}{2\,b}+\frac{\operatorname{arsinh}(bx+a)\,\mathrm{ar}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] arcsinh(b*x + a)^3*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^
2))/b - 3/2*arcsinh(b*x + a)^2*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*
(a^2 + 1)*b^2))^2/b + arcsinh(b*x + a)*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*
b^2 + 4*(a^2 + 1)*b^2))^3/b - 1/4*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 +
 4*(a^2 + 1)*b^2))^4/b
```

mupad [B] time = 0.23, size = 13, normalized size = 0.87

$$\frac{\operatorname{asinh}(a+b\,x)^4}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a + b*x)^3/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2),x)
```

```
[Out] asinh(a + b*x)^4/(4*b)
```

sympy [A] time = 1.20, size = 26, normalized size = 1.73

$$\begin{cases} \frac{\operatorname{asinh}^4(a+bx)}{4b} & \text{for}\ b \neq 0 \\ \frac{x\operatorname{asinh}^3(a)}{\sqrt{a^2+1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)**3/(b**2*x**2+2*a*b*x+a**2+1)**(1/2),x)
```

```
[Out] Piecewise((asinh(a + b*x)**4/(4*b), Ne(b, 0)), (x*asinh(a)**3/sqrt(a**2 + 1
), True))
```

3.273 $\int \frac{\sinh^{-1}(a+bx)^2}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx$

Optimal. Leaf size=15

$$\frac{\sinh^{-1}(a+bx)^3}{3b}$$

[Out] 1/3*arcsinh(b*x+a)^3/b

Rubi [A] time = 0.07, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {5867, 5675}

$$\frac{\sinh^{-1}(a+bx)^3}{3b}$$

Antiderivative was successfully verified.

[In] Int[ArcSinh[a + b*x]^2/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2],x]

[Out] ArcSinh[a + b*x]^3/(3*b)

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5867

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C*x^2)/d^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^{-1}(a+bx)^2}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx &= \frac{\text{Subst}\left(\int \frac{\sinh^{-1}(x)^2}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{b}\\ &= \frac{\sinh^{-1}(a+bx)^3}{3b}\end{aligned}$$

Mathematica [A] time = 0.02, size = 15, normalized size = 1.00

$$\frac{\sinh^{-1}(a+bx)^3}{3b}$$

Antiderivative was successfully verified.

[In] Integrate[ArcSinh[a + b*x]^2/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2],x]

[Out] ArcSinh[a + b*x]^3/(3*b)

fricas [B] time = 0.83, size = 32, normalized size = 2.13

$$\frac{\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^3}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="fric
as")
```

```
[Out] 1/3*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^3/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}(bx+a)^2}{\sqrt{b^2x^2+2\,abx+a^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(arcsinh(b*x + a)^2/sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1), x)
```

maple [A] time = 0.07, size = 14, normalized size = 0.93

$$\frac{\operatorname{arcsinh}(bx+a)^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x)
```

```
[Out] 1/3*arcsinh(b*x+a)^3/b
```

maxima [B] time = 0.78, size = 132, normalized size = 8.80

$$\frac{\operatorname{arsinh}(bx+a)^2 \operatorname{arsinh}\left(\frac{2\left(b^2x+ab\right)}{\sqrt{-4\,a^2b^2+4\left(a^2+1\right)b^2}}\right)}{b} - \frac{\operatorname{arsinh}(bx+a)\operatorname{arsinh}\left(\frac{2\left(b^2x+ab\right)}{\sqrt{-4\,a^2b^2+4\left(a^2+1\right)b^2}}\right)^2}{b} + \frac{\operatorname{arsinh}\left(\frac{2\left(b^2x\cdot\right.}{\sqrt{-4\,a^2b^2+4}}\right.}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] arcsinh(b*x + a)^2*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^
2))/b - arcsinh(b*x + a)*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 +
 1)*b^2))^2/b + 1/3*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b
^2))^3/b
```

mupad [B] time = 0.19, size = 13, normalized size = 0.87

$$\frac{\operatorname{asinh}(a+b\,x)^3}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a + b*x)^2/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2),x)
```

```
[Out] asinh(a + b*x)^3/(3*b)
```

sympy [A] time = 0.84, size = 26, normalized size = 1.73

$$\begin{cases} \frac{\operatorname{asinh}^{3}(a+bx)}{3b} & \text{for}\ b \neq 0 \\ \frac{x\operatorname{asinh}^{2}(a)}{\sqrt{a^2+1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)**2/(b**2*x**2+2*a*b*x+a**2+1)**(1/2),x)
```

```
[Out] Piecewise((asinh(a + b*x)**3/(3*b), Ne(b, 0)), (x*asinh(a)**2/sqrt(a**2 + 1
), True))
```

3.274 $\int \frac{\sinh^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx$

Optimal. Leaf size=15

$$\frac{\sinh^{-1}(a+bx)^2}{2b}$$

[Out] 1/2*arcsinh(b*x+a)^2/b

Rubi [A] time = 0.04, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {5867, 5675}

$$\frac{\sinh^{-1}(a+bx)^2}{2b}$$

Antiderivative was successfully verified.

[In] Int[ArcSinh[a + b*x]/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2],x]

[Out] ArcSinh[a + b*x]^2/(2*b)

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5867

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C*x^2)/d^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx &= \frac{\text{Subst}\left(\int \frac{\sinh^{-1}(x)}{\sqrt{1+x^2}}\,dx,x,a+bx\right)}{b}\\ &= \frac{\sinh^{-1}(a+bx)^2}{2b}\end{aligned}$$

Mathematica [A] time = 0.02, size = 15, normalized size = 1.00

$$\frac{\sinh^{-1}(a+bx)^2}{2b}$$

Antiderivative was successfully verified.

[In] Integrate[ArcSinh[a + b*x]/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2],x]

[Out] ArcSinh[a + b*x]^2/(2*b)

fricas [B] time = 0.55, size = 32, normalized size = 2.13

$$\frac{\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^2}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/2*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}(bx+a)}{\sqrt{b^2x^2+2\,abx+a^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(b*x + a)/sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1), x)
```

maple [A] time = 0.07, size = 14, normalized size = 0.93

$$\frac{\operatorname{arcsinh}(bx+a)^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x)
```

```
[Out] 1/2*arcsinh(b*x+a)^2/b
```

maxima [B] time = 0.52, size = 84, normalized size = 5.60

$$\frac{\operatorname{arsinh}(bx+a)\operatorname{arsinh}\left(\frac{2\left(b^2x+ab\right)}{\sqrt{-4\,a^2b^2+4\left(a^2+1\right)b^2}}\right)}{b}-\frac{\operatorname{arsinh}\left(\frac{2\left(b^2x+ab\right)}{\sqrt{-4\,a^2b^2+4\left(a^2+1\right)b^2}}\right)^2}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="maxima
")
```

```
[Out] arcsinh(b*x + a)*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)
)/b - 1/2*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))^2/b
```

mupad [B] time = 0.20, size = 13, normalized size = 0.87

$$\frac{\operatorname{asinh}(a+b\,x)^2}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a + b*x)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2),x)
```

```
[Out] asinh(a + b*x)^2/(2*b)
```

sympy [A] time = 0.74, size = 24, normalized size = 1.60

$$\begin{cases} \frac{\operatorname{asinh}^2(a+bx)}{2b} & \text{for}\ b \neq 0 \\ \frac{x\operatorname{asinh}(a)}{\sqrt{a^2+1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)/(b**2*x**2+2*a*b*x+a**2+1)**(1/2),x)
```

```
[Out] Piecewise((asinh(a + b*x)**2/(2*b), Ne(b, 0)), (x*asinh(a)/sqrt(a**2 + 1),
True))
```

3.275 $\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}\,\sinh^{-1}(a+bx)}\,dx$

Optimal. Leaf size=11

$$\frac{\log\left(\sinh^{-1}(a+bx)\right)}{b}$$

```
[Out] ln(arcsinh(b*x+a))/b
```

Rubi [A] time = 0.08, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {5867, 5673}

$$\frac{\log\left(\sinh^{-1}(a+bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x]),x]
```

```
[Out] Log[ArcSinh[a + b*x]]/b
```

Rule 5673

```
Int[1/(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*Sqrt[(d_) + (e_.)*(x_)^2]), x_Sy
mbol] :> Simp[Log[a + b*ArcSinh[c*x]]/(b*c*Sqrt[d]), x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

Rule 5867

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C*x^2)/d^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}\,\sinh^{-1}(a+bx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}\,\sinh^{-1}(x)}\,dx, x, a+bx\right)}{b}\\ &= \frac{\log\left(\sinh^{-1}(a+bx)\right)}{b} \end{aligned}$$

Mathematica [A] time = 0.03, size = 11, normalized size = 1.00

$$\frac{\log\left(\sinh^{-1}(a+bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x]),x]
```

```
[Out] Log[ArcSinh[a + b*x]]/b
```

fricas [B] time = 0.41, size = 30, normalized size = 2.73

$$\frac{\log\left(\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="fric
as")
```

```
[Out] log(log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b^2x^2 + 2\,abx + a^2 + 1}\,\operatorname{arsinh}\left(bx + a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(1/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*arcsinh(b*x + a)), x)
```

maple [A] time = 0.08, size = 12, normalized size = 1.09

$$\frac{\ln\left(\operatorname{arcsinh}\left(bx + a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arcsinh(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x)
```

```
[Out] ln(arcsinh(b*x+a))/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b^2x^2 + 2\,abx + a^2 + 1}\,\operatorname{arsinh}\left(bx + a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(1/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*arcsinh(b*x + a)), x)
```

mupad [B] time = 0.22, size = 11, normalized size = 1.00

$$\frac{\ln\left(\operatorname{asinh}\left(a + b\,x\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(asinh(a + b*x)*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2)),x)
```

```
[Out] log(asinh(a + b*x))/b
```

sympy [A] time = 1.22, size = 22, normalized size = 2.00

$$\begin{cases} \frac{\log\left(\operatorname{asinh}\left(a+bx\right)\right)}{b} & \text{for}\ b \neq 0 \\ \frac{x}{\sqrt{a^2+1}\,\operatorname{asinh}\left(a\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/asinh(b*x+a)/(b**2*x**2+2*a*b*x+a**2+1)**(1/2),x)
```

```
[Out] Piecewise((log(asinh(a + b*x))/b, Ne(b, 0)), (x/(sqrt(a**2 + 1)*asinh(a)),
True))
```

3.276 $\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}\ \sinh^{-1}(a+bx)^2}\,dx$

Optimal. Leaf size=13

$$-\frac{1}{b\sinh^{-1}(a+bx)}$$

```
[Out] -1/b/arcsinh(b*x+a)
```

Rubi [A] time = 0.07, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {5867, 5675}

$$-\frac{1}{b\sinh^{-1}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x]^2),x]
```

```
[Out] -(1/(b*ArcSinh[a + b*x]))
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5867

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C*x^2)/d^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}\ \sinh^{-1}(a+bx)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}\ \sinh^{-1}(x)^2}\,dx, x, a+bx\right)}{b}\\ &= -\frac{1}{b\sinh^{-1}(a+bx)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 13, normalized size = 1.00

$$-\frac{1}{b\sinh^{-1}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x]^2),x]
```

```
[Out] -(1/(b*ArcSinh[a + b*x]))
```

fricas [B] time = 0.47, size = 32, normalized size = 2.46

$$-\frac{1}{b\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(b*x+a)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="fr
icas")
```

```
[Out] -1/(b*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b^2x^2 + 2\,abx + a^2 + 1}\ \operatorname{arsinh}(bx + a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(b*x+a)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate(1/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*arcsinh(b*x + a)^2), x)
```

maple [A] time = 0.08, size = 14, normalized size = 1.08

$$-\frac{1}{b\,\operatorname{arcsinh}(bx + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arcsinh(b*x+a)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x)
```

```
[Out] -1/b/arcsinh(b*x+a)
```

maxima [B] time = 0.77, size = 150, normalized size = 11.54

$$-\frac{b^3x^3 + 3\,ab^2x^2 + a^3 + \left(3\,a^2b + b\right)x + \left(b^2x^2 + 2\,abx + a^2 + 1\right)^{\frac{3}{2}} + a}{\left(\left(b^2x^2 + 2\,abx + a^2 + 1\right)\left(b^2x + ab\right) + \left(b^3x^2 + 2\,ab^2x + a^2b + b\right)\sqrt{b^2x^2 + 2\,abx + a^2 + 1}\right)\log\left(bx + a + \sqrt{b^2x^2 +}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(b*x+a)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="ma
xima")
```

```
[Out] -(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b + b)*x + (b^2*x^2 + 2*a*b*x + a^2
+ 1)^(3/2) + a)/(((b^2*x^2 + 2*a*b*x + a^2 + 1)*(b^2*x + a*b) + (b^3*x^2 +
2*a*b^2*x + a^2*b + b)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*log(b*x + a + sqr
t(b^2*x^2 + 2*a*b*x + a^2 + 1)))
```

mupad [B] time = 0.21, size = 13, normalized size = 1.00

$$-\frac{1}{b\,\operatorname{asinh}(a + b\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(asinh(a + b*x)^2*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2)),x)
```

```
[Out] -1/(b*asinh(a + b*x))
```

sympy [A] time = 1.88, size = 26, normalized size = 2.00

$$\begin{cases} -\frac{1}{b\,\operatorname{asinh}(a+bx)} & \text{for } b \neq 0 \\ \frac{x}{\sqrt{a^2+1}\ \operatorname{asinh}^2(a)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/asinh(b*x+a)**2/(b**2*x**2+2*a*b*x+a**2+1)**(1/2),x)
```

```
[Out] Piecewise((-1/(b*asinh(a + b*x)), Ne(b, 0)), (x/(sqrt(a**2 + 1)*asinh(a)**2
), True))
```

3.277 $\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2} \sinh^{-1}(a+bx)^3} dx$

Optimal. Leaf size=15

$$-\frac{1}{2b\sinh^{-1}(a+bx)^2}$$

[Out] -1/2/b/arcsinh(b*x+a)^2

Rubi [A] time = 0.07, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {5867, 5675}

$$-\frac{1}{2b\sinh^{-1}(a+bx)^2}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x]^3),x]

[Out] -1/(2*b*ArcSinh[a + b*x]^2)

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5867

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C*x^2)/d^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}\sinh^{-1}(a+bx)^3} dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}\sinh^{-1}(x)^3} dx, x, a+bx\right)}{b} \\ &= -\frac{1}{2b\sinh^{-1}(a+bx)^2} \end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$-\frac{1}{2b\sinh^{-1}(a+bx)^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x]^3),x]

[Out] -1/2*1/(b*ArcSinh[a + b*x]^2)

fricas [B] time = 0.89, size = 32, normalized size = 2.13

$$-\frac{1}{2\,b\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(b*x+a)^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="fr
icas")
```

```
[Out] -1/2/(b*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b^2x^2 + 2\,abx + a^2 + 1}\,\operatorname{arsinh}\left(bx + a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(b*x+a)^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate(1/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*arcsinh(b*x + a)^3), x)
```

maple [A] time = 0.08, size = 14, normalized size = 0.93

$$-\frac{1}{2b\,\operatorname{arcsinh}\left(bx + a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arcsinh(b*x+a)^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x)
```

```
[Out] -1/2/b/arcsinh(b*x+a)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(b*x+a)^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="ma
xima")
```

```
[Out] -1/2*(b^7*x^7 + 7*a*b^6*x^6 + a^7 + 3*(7*a^2*b^5 + b^5)*x^5 + 3*a^5 + 5*(7*
a^3*b^4 + 3*a*b^4)*x^4 + (35*a^4*b^3 + 30*a^2*b^3 + 3*b^3)*x^3 + 3*a^3 + 3*
(7*a^5*b^2 + 10*a^3*b^2 + 3*a*b^2)*x^2 + (b^4*x^4 + 4*a*b^3*x^3 + a^4 + (6*
a^2*b^2 + b^2)*x^2 + a^2 + 2*(2*a^3*b + a*b)*x)*(b^2*x^2 + 2*a*b*x + a^2 +
1)^(3/2) + (3*b^5*x^5 + 15*a*b^4*x^4 + 3*a^5 + 5*(6*a^2*b^3 + b^3)*x^3 + 5*
a^3 + 15*(2*a^3*b^2 + a*b^2)*x^2 + (15*a^4*b + 15*a^2*b + 2*b)*x + 2*a)*(b^
2*x^2 + 2*a*b*x + a^2 + 1) + (7*a^6*b + 15*a^4*b + 9*a^2*b + b)*x + (b^5*x^
5 + 5*a*b^4*x^4 + a^5 + 2*(5*a^2*b^3 + b^3)*x^3 + 2*a^3 + 2*(5*a^3*b^2 + 3*
a*b^2)*x^2 - (b^2*x^2 + 2*a*b*x + a^2 + 1)^(5/2) - (b^3*x^3 + 3*a*b^2*x^2 +
 a^3 + (3*a^2*b + b)*x + a)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + (5*a^4*b + 6*a^
2*b + b)*x + (b^4*x^4 + 4*a*b^3*x^3 + a^4 + 2*(3*a^2*b^2 + b^2)*x^2 + 2*a^2
 + 4*(a^3*b + a*b)*x + 1)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + a)*log(b*x +
a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + (3*b^6*x^6 + 18*a*b^5*x^5 + 3*a^6
+ (45*a^2*b^4 + 7*b^4)*x^4 + 7*a^4 + 4*(15*a^3*b^3 + 7*a*b^3)*x^3 + (45*a^4
*b^2 + 42*a^2*b^2 + 5*b^2)*x^2 + 5*a^2 + 2*(9*a^5*b + 14*a^3*b + 5*a*b)*x +
 1)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + a)/(((b^4*x^3 + 3*a*b^3*x^2 + 3*a^2
*b^2*x + a^3*b)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^2 + 3*(b^5*x^4 + 4*a*b^4*x^3
+ a^4*b + a^2*b + (6*a^2*b^3 + b^3)*x^2 + 2*(2*a^3*b^2 + a*b^2)*x)*(b^2*x^2
 + 2*a*b*x + a^2 + 1)^(3/2) + 3*(b^6*x^5 + 5*a*b^5*x^4 + a^5*b + 2*a^3*b +
2*(5*a^2*b^4 + b^4)*x^3 + 2*(5*a^3*b^3 + 3*a*b^3)*x^2 + a*b + (5*a^4*b^2 +
6*a^2*b^2 + b^2)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + (b^7*x^6 + 6*a*b^6*x^5
+ a^6*b + 3*a^4*b + 3*(5*a^2*b^5 + b^5)*x^4 + 4*(5*a^3*b^4 + 3*a*b^4)*x^3 +
 3*a^2*b + 3*(5*a^4*b^3 + 6*a^2*b^3 + b^3)*x^2 + 6*(a^5*b^2 + 2*a^3*b^2 + a
```

```
*b^2)*x + b)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*log(b*x + a + sqrt(b^2*x^2
+ 2*a*b*x + a^2 + 1))^2) + integrate(-1/2*(2*b^6*x^6 + 12*a*b^5*x^5 + 2*a^6
 + 3*(10*a^2*b^4 + b^4)*x^4 + 3*a^4 + 4*(10*a^3*b^3 + 3*a*b^3)*x^3 - (2*b^2
*x^2 + 4*a*b*x + 2*a^2 + 3)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^2 + 6*(5*a^4*b^2
+ 3*a^2*b^2)*x^2 - 4*(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b + b)*x + a)*(b
^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + 4*(b^2*x^2 + 2*a*b*x + a^2 + 1)^2 + 12*
(a^5*b + a^3*b)*x + 4*(b^5*x^5 + 5*a*b^4*x^4 + a^5 + 2*(5*a^2*b^3 + b^3)*x^
3 + 2*a^3 + 2*(5*a^3*b^2 + 3*a*b^2)*x^2 + (5*a^4*b + 6*a^2*b + b)*x + a)*sq
rt(b^2*x^2 + 2*a*b*x + a^2 + 1) - 1)/(((b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x
^2 + 4*a^3*b*x + a^4)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(5/2) + 4*(b^5*x^5 + 5*
a*b^4*x^4 + a^5 + (10*a^2*b^3 + b^3)*x^3 + a^3 + (10*a^3*b^2 + 3*a*b^2)*x^2
 + (5*a^4*b + 3*a^2*b)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^2 + 6*(b^6*x^6 + 6*
a*b^5*x^5 + a^6 + (15*a^2*b^4 + 2*b^4)*x^4 + 2*a^4 + 4*(5*a^3*b^3 + 2*a*b^3
)*x^3 + (15*a^4*b^2 + 12*a^2*b^2 + b^2)*x^2 + a^2 + 2*(3*a^5*b + 4*a^3*b +
a*b)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + 4*(b^7*x^7 + 7*a*b^6*x^6 + a^
7 + 3*(7*a^2*b^5 + b^5)*x^5 + 3*a^5 + 5*(7*a^3*b^4 + 3*a*b^4)*x^4 + (35*a^4
*b^3 + 30*a^2*b^3 + 3*b^3)*x^3 + 3*a^3 + 3*(7*a^5*b^2 + 10*a^3*b^2 + 3*a*b^
2)*x^2 + (7*a^6*b + 15*a^4*b + 9*a^2*b + b)*x + a)*(b^2*x^2 + 2*a*b*x + a^2
 + 1) + (b^8*x^8 + 8*a*b^7*x^7 + a^8 + 4*(7*a^2*b^6 + b^6)*x^6 + 4*a^6 + 8*
(7*a^3*b^5 + 3*a*b^5)*x^5 + 2*(35*a^4*b^4 + 30*a^2*b^4 + 3*b^4)*x^4 + 6*a^4
 + 8*(7*a^5*b^3 + 10*a^3*b^3 + 3*a*b^3)*x^3 + 4*(7*a^6*b^2 + 15*a^4*b^2 + 9
*a^2*b^2 + b^2)*x^2 + 4*a^2 + 8*(a^7*b + 3*a^5*b + 3*a^3*b + a*b)*x + 1)*sq
rt(b^2*x^2 + 2*a*b*x + a^2 + 1))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2
 + 1))), x)
```

mupad [B] time = 0.20, size = 13, normalized size = 0.87

$$-\frac{1}{2\,b\,\mathrm{asinh}\left(a+b\,x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(asinh(a + b*x)^3*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2)),x)
```

```
[Out] -1/(2*b*asinh(a + b*x)^2)
```

sympy [A] time = 2.34, size = 29, normalized size = 1.93

$$\begin{cases} -\frac{1}{2b\,\mathrm{asinh}^2\,(a+bx)} & \text{for}\ b\neq 0\\ \frac{x}{\sqrt{a^2+1}\ \mathrm{asinh}^3\,(a)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/asinh(b*x+a)**3/(b**2*x**2+2*a*b*x+a**2+1)**(1/2),x)
```

```
[Out] Piecewise((-1/(2*b*asinh(a + b*x)**2), Ne(b, 0)), (x/(sqrt(a**2 + 1)*asinh(
a)**3), True))
```

3.278 $\int \frac{\sinh^{-1}(a+bx)^3}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=115

$$-\frac{3\sinh^{-1}(a+bx)\text{Li}_2\left(-e^{2\sinh^{-1}(a+bx)}\right)}{b}+\frac{3\text{Li}_3\left(-e^{2\sinh^{-1}(a+bx)}\right)}{2b}+\frac{(a+bx)\sinh^{-1}(a+bx)^3}{b\sqrt{(a+bx)^2+1}}+\frac{\sinh^{-1}(a+bx)^3}{b}-\frac{3\text{s}}{}$$

```
[Out] arcsinh(b*x+a)^3/b-3*arcsinh(b*x+a)^2*ln(1+(b*x+a+(1+(b*x+a)^2)^(1/2))^2)/b
-3*arcsinh(b*x+a)*polylog(2,-(b*x+a+(1+(b*x+a)^2)^(1/2))^2)/b+3/2*polylog(3
,-(b*x+a+(1+(b*x+a)^2)^(1/2))^2)/b+(b*x+a)*arcsinh(b*x+a)^3/b/(1+(b*x+a)^2)
^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {5867, 5687, 5714, 3718, 2190, 2531, 2282, 6589}

$$-\frac{3\sinh^{-1}(a+bx)\text{PolyLog}\left(2,-e^{2\sinh^{-1}(a+bx)}\right)}{b}+\frac{3\text{PolyLog}\left(3,-e^{2\sinh^{-1}(a+bx)}\right)}{2b}+\frac{(a+bx)\sinh^{-1}(a+bx)^3}{b\sqrt{(a+bx)^2+1}}+\frac{\text{sinh}}{}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a + b*x]^3/(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

```
[Out] ArcSinh[a + b*x]^3/b + ((a + b*x)*ArcSinh[a + b*x]^3)/(b*Sqrt[1 + (a + b*x)
^2]) - (3*ArcSinh[a + b*x]^2*Log[1 + E^(2*ArcSinh[a + b*x])])/b - (3*ArcSin
h[a + b*x]*PolyLog[2, -E^(2*ArcSinh[a + b*x])])/b + (3*PolyLog[3, -E^(2*Arc
Sinh[a + b*x])])/(2*b)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
```

```
FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5867

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C*x^2)/d^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\sinh^{-1}(a+bx)^3}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{\sinh^{-1}(x)^3}{\left(1+x^2\right)^{3/2}}\,dx, x, a+bx\right)}{b}$$

$$= \frac{(a+bx)\sinh^{-1}(a+bx)^3}{b\sqrt{1+(a+bx)^2}} - \frac{3\,\text{Subst}\left(\int \frac{x\sinh^{-1}(x)^2}{1+x^2}\,dx, x, a+bx\right)}{b}$$

$$= \frac{(a+bx)\sinh^{-1}(a+bx)^3}{b\sqrt{1+(a+bx)^2}} - \frac{3\,\text{Subst}\left(\int x^2\tanh(x)\,dx, x, \sinh^{-1}(a+bx)\right)}{b}$$

$$= \frac{\sinh^{-1}(a+bx)^3}{b} + \frac{(a+bx)\sinh^{-1}(a+bx)^3}{b\sqrt{1+(a+bx)^2}} - \frac{6\,\text{Subst}\left(\int \frac{e^{2x}x^2}{1+e^{2x}}\,dx, x, \sinh^{-}\right.}{b}$$

$$= \frac{\sinh^{-1}(a+bx)^3}{b} + \frac{(a+bx)\sinh^{-1}(a+bx)^3}{b\sqrt{1+(a+bx)^2}} - \frac{3\sinh^{-1}(a+bx)^2\log\left(1+e^{2}\right.}{b}$$

$$= \frac{\sinh^{-1}(a+bx)^3}{b} + \frac{(a+bx)\sinh^{-1}(a+bx)^3}{b\sqrt{1+(a+bx)^2}} - \frac{3\sinh^{-1}(a+bx)^2\log\left(1+e^{2}\right.}{b}$$

$$= \frac{\sinh^{-1}(a+bx)^3}{b} + \frac{(a+bx)\sinh^{-1}(a+bx)^3}{b\sqrt{1+(a+bx)^2}} - \frac{3\sinh^{-1}(a+bx)^2\log\left(1+e^{2}\right.}{b}$$

$$= \frac{\sinh^{-1}(a+bx)^3}{b} + \frac{(a+bx)\sinh^{-1}(a+bx)^3}{b\sqrt{1+(a+bx)^2}} - \frac{3\sinh^{-1}(a+bx)^2\log\left(1+e^{2}\right.}{b}$$

Mathematica [A] time = 0.63, size = 128, normalized size = 1.11

$$\frac{2\sinh^{-1}(a+bx)^2\left(\frac{\left(-\sqrt{a^2+2abx+b^2x^2+1}+a+bx\right)\sinh^{-1}(a+bx)}{\sqrt{a^2+2abx+b^2x^2+1}} - 3\log\left(e^{-2\sinh^{-1}(a+bx)}+1\right)\right) + 6\sinh^{-1}(a+bx)\text{Li}_2\left(-e^{-2}\right.}{2b}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSinh[a + b*x]^3/(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

```
[Out] (2*ArcSinh[a + b*x]^2*(((a + b*x - Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2])*ArcSi
nh[a + b*x])/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] - 3*Log[1 + E^(-2*ArcSinh[a
+ b*x])]) + 6*ArcSinh[a + b*x]*PolyLog[2, -E^(-2*ArcSinh[a + b*x])] + 3*Pol
yLog[3, -E^(-2*ArcSinh[a + b*x])])/(2*b)
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b^2x^2+2\,abx+a^2+1}\,\operatorname{arsinh}(bx+a)^3}{b^4x^4+4\,ab^3x^3+2\left(3\,a^2+1\right)b^2x^2+a^4+4\left(a^3+a\right)bx+2\,a^2+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^3/(b^2*x^2+2*a*b*x+a^2+1)^(3/2),x, algorithm="fric
as")
```

```
[Out] integral(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*arcsinh(b*x + a)^3/(b^4*x^4 + 4*
a*b^3*x^3 + 2*(3*a^2 + 1)*b^2*x^2 + a^4 + 4*(a^3 + a)*b*x + 2*a^2 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}(bx+a)^3}{\left(b^2x^2+2\,abx+a^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^3/(b^2*x^2+2*a*b*x+a^2+1)^(3/2),x, algorithm="giac
")
```

```
[Out] integrate(arcsinh(b*x + a)^3/(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2), x)
```

maple [A] time = 0.26, size = 203, normalized size = 1.77

$$-\frac{\left(b^2x^2-\sqrt{b^2x^2+2abx+a^2+1}\;xb+2abx-\sqrt{b^2x^2+2abx+a^2+1}\;a+a^2+1\right)\operatorname{arcsinh}(bx+a)^3}{b\left(b^2x^2+2abx+a^2+1\right)}+\frac{2\operatorname{arcsinh}(b}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)^3/(b^2*x^2+2*a*b*x+a^2+1)^(3/2),x)
```

```
[Out] -(b^2*x^2-(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x*b+2*a*b*x-(b^2*x^2+2*a*b*x+a^2+1)
^(1/2)*a+a^2+1)/b/(b^2*x^2+2*a*b*x+a^2+1)*arcsinh(b*x+a)^3+2*arcsinh(b*x+a)
^3/b-3*arcsinh(b*x+a)^2*ln(1+(b*x+a+(1+(b*x+a)^2)^(1/2))^2)/b-3*arcsinh(b*x
+a)*polylog(2,-(b*x+a+(1+(b*x+a)^2)^(1/2))^2)/b+3/2*polylog(3,-(b*x+a+(1+(b
*x+a)^2)^(1/2))^2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}(bx+a)^3}{\left(b^2x^2+2\,abx+a^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^3/(b^2*x^2+2*a*b*x+a^2+1)^(3/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(arcsinh(b*x + a)^3/(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{asinh}(a+b\,x)^3}{\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a + b*x)^3/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2),x)
```

```
[Out] int(asinh(a + b*x)^3/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}^3(a+bx)}{\left(a^2+2abx+b^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)**3/(b**2*x**2+2*a*b*x+a**2+1)**(3/2),x)
```

```
[Out] Integral(asinh(a + b*x)**3/(a**2 + 2*a*b*x + b**2*x**2 + 1)**(3/2), x)
```

3.279 $\int \frac{\sinh^{-1}(a+bx)^2}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=86

$$-\frac{\mathrm{Li}_2\left(-e^{2\sinh^{-1}(a+bx)}\right)}{b}+\frac{(a+bx)\sinh^{-1}(a+bx)^2}{b\sqrt{(a+bx)^2+1}}+\frac{\sinh^{-1}(a+bx)^2}{b}-\frac{2\sinh^{-1}(a+bx)\log\left(e^{2\sinh^{-1}(a+bx)}+1\right)}{b}$$

```
[Out] arcsinh(b*x+a)^2/b-2*arcsinh(b*x+a)*ln(1+(b*x+a+(1+(b*x+a)^2)^(1/2))^2)/b-p
olylog(2,-(b*x+a+(1+(b*x+a)^2)^(1/2))^2)/b+(b*x+a)*arcsinh(b*x+a)^2/b/(1+(b
*x+a)^2)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {5867, 5687, 5714, 3718, 2190, 2279, 2391}

$$-\frac{\mathrm{PolyLog}\left(2,-e^{2\sinh^{-1}(a+bx)}\right)}{b}+\frac{(a+bx)\sinh^{-1}(a+bx)^2}{b\sqrt{(a+bx)^2+1}}+\frac{\sinh^{-1}(a+bx)^2}{b}-\frac{2\sinh^{-1}(a+bx)\log\left(e^{2\sinh^{-1}(a+bx}\right.}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a + b*x]^2/(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

```
[Out] ArcSinh[a + b*x]^2/b + ((a + b*x)*ArcSinh[a + b*x]^2)/(b*Sqrt[1 + (a + b*x)
^2]) - (2*ArcSinh[a + b*x]*Log[1 + E^(2*ArcSinh[a + b*x])])/b - PolyLog[2,
-E^(2*ArcSinh[a + b*x])]/b
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
```

```
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5867

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C*x^2)/d^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\int \frac{\sinh^{-1}(a+bx)^2}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{\sinh^{-1}(x)^2}{\left(1+x^2\right)^{3/2}}\,dx, x, a+bx\right)}{b}$$

$$= \frac{(a+bx)\sinh^{-1}(a+bx)^2}{b\sqrt{1+(a+bx)^2}} - \frac{2\,\text{Subst}\left(\int \frac{x\sinh^{-1}(x)}{1+x^2}\,dx, x, a+bx\right)}{b}$$

$$= \frac{(a+bx)\sinh^{-1}(a+bx)^2}{b\sqrt{1+(a+bx)^2}} - \frac{2\,\text{Subst}\left(\int x\tanh(x)\,dx, x, \sinh^{-1}(a+bx)\right)}{b}$$

$$= \frac{\sinh^{-1}(a+bx)^2}{b} + \frac{(a+bx)\sinh^{-1}(a+bx)^2}{b\sqrt{1+(a+bx)^2}} - \frac{4\,\text{Subst}\left(\int \frac{e^{2x}x}{1+e^{2x}}\,dx, x, \sinh^{-1}(a\right.}{b}$$

$$= \frac{\sinh^{-1}(a+bx)^2}{b} + \frac{(a+bx)\sinh^{-1}(a+bx)^2}{b\sqrt{1+(a+bx)^2}} - \frac{2\sinh^{-1}(a+bx)\log\left(1+e^{2\sinh}\right.}{b}$$

$$= \frac{\sinh^{-1}(a+bx)^2}{b} + \frac{(a+bx)\sinh^{-1}(a+bx)^2}{b\sqrt{1+(a+bx)^2}} - \frac{2\sinh^{-1}(a+bx)\log\left(1+e^{2\sinh}\right.}{b}$$

$$= \frac{\sinh^{-1}(a+bx)^2}{b} + \frac{(a+bx)\sinh^{-1}(a+bx)^2}{b\sqrt{1+(a+bx)^2}} - \frac{2\sinh^{-1}(a+bx)\log\left(1+e^{2\sinh}\right.}{b}$$

Mathematica [A] time = 0.40, size = 98, normalized size = 1.14

$$\frac{\sinh^{-1}(a+bx)\left(\frac{\left(-\sqrt{a^2+2abx+b^2x^2+1}+a+bx\right)\sinh^{-1}(a+bx)}{\sqrt{a^2+2abx+b^2x^2+1}} - 2\log\left(e^{-2\sinh^{-1}(a+bx)}+1\right)\right) + \text{Li}_2\left(-e^{-2\sinh^{-1}(a+bx)}\right)}{b}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSinh[a + b*x]^2/(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

```
[Out] (ArcSinh[a + b*x]*(((a + b*x - Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2])*ArcSinh[a
 + b*x])/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] - 2*Log[1 + E^(-2*ArcSinh[a + b*
x])]) + PolyLog[2, -E^(-2*ArcSinh[a + b*x])])/b
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b^2x^2+2\,abx+a^2+1}\,\operatorname{arsinh}\left(bx+a\right)^2}{b^4x^4+4\,ab^3x^3+2\left(3\,a^2+1\right)b^2x^2+a^4+4\left(a^3+a\right)bx+2\,a^2+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2/(b^2*x^2+2*a*b*x+a^2+1)^(3/2),x, algorithm="fric
as")
```

```
[Out] integral(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*arcsinh(b*x + a)^2/(b^4*x^4 + 4*
a*b^3*x^3 + 2*(3*a^2 + 1)*b^2*x^2 + a^4 + 4*(a^3 + a)*b*x + 2*a^2 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}\left(bx+a\right)^2}{\left(b^2x^2+2\,abx+a^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2/(b^2*x^2+2*a*b*x+a^2+1)^(3/2),x, algorithm="giac
")
```

```
[Out] integrate(arcsinh(b*x + a)^2/(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2), x)
```

maple [A] time = 0.25, size = 168, normalized size = 1.95

$$-\frac{\left(b^2x^2-\sqrt{b^2x^2+2abx+a^2+1}\;xb+2abx-\sqrt{b^2x^2+2abx+a^2+1}\;a+a^2+1\right)\operatorname{arcsinh}\left(bx+a\right)^2}{b\left(b^2x^2+2abx+a^2+1\right)}+\frac{2\operatorname{arcsinl}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)^2/(b^2*x^2+2*a*b*x+a^2+1)^(3/2),x)
```

```
[Out] -(b^2*x^2-(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x*b+2*a*b*x-(b^2*x^2+2*a*b*x+a^2+1)
^(1/2)*a+a^2+1)/b/(b^2*x^2+2*a*b*x+a^2+1)*arcsinh(b*x+a)^2+2*arcsinh(b*x+a)
^2/b-2*arcsinh(b*x+a)*ln(1+(b*x+a+(1+(b*x+a)^2)^(1/2))^2)/b-polylog(2,-(b*x
+a+(1+(b*x+a)^2)^(1/2))^2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}\left(bx+a\right)^2}{\left(b^2x^2+2\,abx+a^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2/(b^2*x^2+2*a*b*x+a^2+1)^(3/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(arcsinh(b*x + a)^2/(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\mathrm{asinh}\left(a+b\,x\right)}^2}{{\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a + b*x)^2/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2),x)
```

```
[Out] int(asinh(a + b*x)^2/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}^2(a+bx)}{\left(a^2+2abx+b^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)**2/(b**2*x**2+2*a*b*x+a**2+1)**(3/2),x)
```

```
[Out] Integral(asinh(a + b*x)**2/(a**2 + 2*a*b*x + b**2*x**2 + 1)**(3/2), x)
```

3.280 $\int \frac{\sinh^{-1}(a+bx)}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}} \, dx$

Optimal. Leaf size=46

$$\frac{(a+bx)\sinh^{-1}(a+bx)}{b\sqrt{(a+bx)^2+1}}-\frac{\log\left((a+bx)^2+1\right)}{2b}$$

```
[Out] -1/2*ln(1+(b*x+a)^2)/b+(b*x+a)*arcsinh(b*x+a)/b/(1+(b*x+a)^2)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {5867, 5687, 260}

$$\frac{(a+bx)\sinh^{-1}(a+bx)}{b\sqrt{(a+bx)^2+1}}-\frac{\log\left((a+bx)^2+1\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a + b*x]/(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

```
[Out] ((a + b*x)*ArcSinh[a + b*x])/(b*Sqrt[1 + (a + b*x)^2]) - Log[1 + (a + b*x)^
2]/(2*b)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5867

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C*x^2)/d^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^{-1}(a+bx)}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}} \, dx &= \frac{\text{Subst}\left(\int \frac{\sinh^{-1}(x)}{\left(1+x^2\right)^{3/2}} \, dx, x, a+bx\right)}{b}\\
&= \frac{(a+bx)\sinh^{-1}(a+bx)}{b\sqrt{1+(a+bx)^2}}-\frac{\text{Subst}\left(\int \frac{x}{1+x^2} \, dx, x, a+bx\right)}{b}\\
&= \frac{(a+bx)\sinh^{-1}(a+bx)}{b\sqrt{1+(a+bx)^2}}-\frac{\log\left(1+(a+bx)^2\right)}{2b}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 62, normalized size = 1.35

$$\frac{(a+bx)\sinh^{-1}(a+bx)}{b\sqrt{a^2+2abx+b^2x^2+1}}-\frac{\log\left(a^2+2abx+b^2x^2+1\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a + b*x]/(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

```
[Out] ((a + b*x)*ArcSinh[a + b*x])/(b*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]) - Log[1
+ a^2 + 2*a*b*x + b^2*x^2]/(2*b)
```

fricas [B] time = 0.63, size = 115, normalized size = 2.50

$$\frac{2\sqrt{b^2x^2+2\,abx+a^2+1}\,(bx+a)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)-\left(b^2x^2+2\,abx+a^2+1\right)\log\left(b^2x^2+2\right.}{2\left(b^3x^2+2\,ab^2x+\left(a^2+1\right)b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(3/2),x, algorithm="fricas
")
```

```
[Out] 1/2*(2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b*x + a)*log(b*x + a + sqrt(b^2*x
^2 + 2*a*b*x + a^2 + 1)) - (b^2*x^2 + 2*a*b*x + a^2 + 1)*log(b^2*x^2 + 2*a*
b*x + a^2 + 1))/(b^3*x^2 + 2*a*b^2*x + (a^2 + 1)*b)
```

giac [A] time = 0.55, size = 76, normalized size = 1.65

$$\frac{\left(x+\frac{a}{b}\right)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)}{\sqrt{b^2x^2+2\,abx+a^2+1}}-\frac{\log\left(b^2x^2+2\,abx+a^2+1\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] (x + a/b)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/sqrt(b^2*x^2 + 2
*a*b*x + a^2 + 1) - 1/2*log(b^2*x^2 + 2*a*b*x + a^2 + 1)/b
```

maple [B] time = 0.23, size = 131, normalized size = 2.85

$$\frac{2\,\mathrm{arcsinh}\,(bx+a)}{b}-\frac{\left(b^2x^2-\sqrt{b^2x^2+2abx+a^2+1}\;xb+2abx-\sqrt{b^2x^2+2abx+a^2+1}\;a+a^2+1\right)\mathrm{arcsinh}\,(bx}{b\left(b^2x^2+2abx+a^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(3/2),x)
```

```
[Out] 2*arcsinh(b*x+a)/b-(b^2*x^2-(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x*b+2*a*b*x-(b^2*
x^2+2*a*b*x+a^2+1)^(1/2)*a+a^2+1)/b/(b^2*x^2+2*a*b*x+a^2+1)*arcsinh(b*x+a)-
1/b*ln(1+(b*x+a+(1+(b*x+a)^2)^(1/2))^2)
```

maxima [B] time = 0.60, size = 119, normalized size = 2.59

$$-\left(\frac{b^2x}{\left(a^2b^2-\left(a^2+1\right)b^2\right)\sqrt{b^2x^2+2\,abx+a^2+1}}+\frac{ab}{\left(a^2b^2-\left(a^2+1\right)b^2\right)\sqrt{b^2x^2+2\,abx+a^2+1}}\right)\mathrm{arsinh}\,(bx+a)-\frac{\mathrm{lo}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(3/2),x, algorithm="maxima
")
```

```
[Out] -(b^2*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + a*b
/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)))*arcsinh(b*x
 + a) - 1/2*log(b^2*x^2 + 2*a*b*x + a^2 + 1)/b
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{asinh}(a+b\,x)}{\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a + b*x)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2),x)
```

```
[Out] int(asinh(a + b*x)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}(a+bx)}{\left(a^2+2abx+b^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)/(b**2*x**2+2*a*b*x+a**2+1)**(3/2),x)
```

```
[Out] Integral(asinh(a + b*x)/(a**2 + 2*a*b*x + b**2*x**2 + 1)**(3/2), x)
```

3.281 $\int \frac{1}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}\sinh^{-1}(a+bx)}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{1}{\left((a+bx)^2+1\right)^{3/2}\sinh^{-1}(a+bx)},x\right)$$

```
[Out] Unintegrable(1/(1+(b*x+a)^2)^(3/2)/arcsinh(b*x+a),x)
```

Rubi [A] time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}\sinh^{-1}(a+bx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)*ArcSinh[a + b*x]),x]
[Out] Defer[Subst][Defer[Int][1/((1 + x^2)^(3/2)*ArcSinh[x]), x], x, a + b*x]/b
```

Rubi steps

$$\int \frac{1}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}\sinh^{-1}(a+bx)}\,dx=\frac{\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^{3/2}\sinh^{-1}(x)}\,dx,x,a+bx\right)}{b}$$

Mathematica [A] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}\sinh^{-1}(a+bx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)*ArcSinh[a + b*x]),x]
[Out] Integrate[1/((1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)*ArcSinh[a + b*x]), x]
```

fricas [A] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b^2x^2+2\,abx+a^2+1}}{\left(b^4x^4+4\,ab^3x^3+2\left(3\,a^2+1\right)b^2x^2+a^4+4\left(a^3+a\right)bx+2\,a^2+1\right)\text{arsinh}\left(bx+a\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a),x, algorithm="fric
as")
[Out] integral(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/((b^4*x^4 + 4*a*b^3*x^3 + 2*(3*a
^2 + 1)*b^2*x^2 + a^4 + 4*(a^3 + a)*b*x + 2*a^2 + 1)*arcsinh(b*x + a)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b^2x^2+2\,abx+a^2+1\right)^{\frac{3}{2}}\text{arsinh}\left(bx+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a),x, algorithm="giac
")
```

```
[Out] integrate(1/((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*arcsinh(b*x + a)), x)
```

maple [A] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b^{2} x^{2}+2 a b x+a^{2}+1\right)^{\frac{3}{2}} \operatorname{arcsinh}\left(b x+a\right)}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a),x)
```

```
[Out] int(1/(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b^{2} x^{2}+2 \, a b x+a^{2}+1\right)^{\frac{3}{2}} \operatorname{arsinh}\left(b x+a\right)}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a),x, algorithm="maxi
ma")
```

```
[Out] integrate(1/((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*arcsinh(b*x + a)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\mathrm{asinh}\left(a+b\,x\right)\,{\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(asinh(a + b*x)*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2)),x)
```

```
[Out] int(1/(asinh(a + b*x)*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^{2} + 2 a b x + b^{2} x^{2} + 1\right)^{\frac{3}{2}} \operatorname{asinh}{\left(a + b x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**2+2*a*b*x+a**2+1)**(3/2)/asinh(b*x+a),x)
```

```
[Out] Integral(1/((a**2 + 2*a*b*x + b**2*x**2 + 1)**(3/2)*asinh(a + b*x)), x)
```

3.282 $\int \frac{1}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}\sinh^{-1}(a+bx)^2}\,dx$

Optimal. Leaf size=55

$$-2\mathrm{Int}\left(\frac{a+bx}{\left((a+bx)^2+1\right)^2\sinh^{-1}(a+bx)},x\right)-\frac{1}{b\left((a+bx)^2+1\right)\sinh^{-1}(a+bx)}$$

```
[Out] -1/b/(1+(b*x+a)^2)/arcsinh(b*x+a)-2*Unintegrable((b*x+a)/(1+(b*x+a)^2)^2/ar
csinh(b*x+a),x)
```

Rubi [A] time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}\sinh^{-1}(a+bx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)*ArcSinh[a + b*x]^2),x]
```

```
[Out] -(1/(b*(1 + (a + b*x)^2)*ArcSinh[a + b*x])) - (2*Defer[Subst][Defer[Int][x/
((1 + x^2)^2*ArcSinh[x]), x], x, a + b*x])/b
```

Rubi steps

$$\int \frac{1}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}\sinh^{-1}(a+bx)^2}\,dx=\frac{\mathrm{Subst}\left(\int \frac{1}{\left(1+x^2\right)^{3/2}\sinh^{-1}(x)^2}\,dx,x,a+bx\right)}{b}$$
$$=-\frac{1}{b\left(1+(a+bx)^2\right)\sinh^{-1}(a+bx)}-\frac{2\,\mathrm{Subst}\left(\int \frac{x}{\left(1+x^2\right)^2\sinh^{-1}(x)}$$

Mathematica [A] time = 2.92, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}\sinh^{-1}(a+bx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)*ArcSinh[a + b*x]^2),x]
```

```
[Out] Integrate[1/((1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)*ArcSinh[a + b*x]^2), x]
```

fricas [A] time = 0.62, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{b^2x^2+2\,abx+a^2+1}}{\left(b^4x^4+4\,ab^3x^3+2\left(3\,a^2+1\right)b^2x^2+a^4+4\left(a^3+a\right)bx+2\,a^2+1\right)\mathrm{arsinh}\left(bx+a\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a)^2,x, algorithm="fr
icas")
```

```
[Out] integral(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/((b^4*x^4 + 4*a*b^3*x^3 + 2*(3*a
^2 + 1)*b^2*x^2 + a^4 + 4*(a^3 + a)*b*x + 2*a^2 + 1)*arcsinh(b*x + a)^2), x
)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b^2x^2 + 2\,abx + a^2 + 1\right)^{\frac{3}{2}} \operatorname{arsinh}\left(bx + a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a)^2,x, algorithm="gi
ac")
```

```
[Out] integrate(1/((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*arcsinh(b*x + a)^2), x)
```

maple [A] time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b^2x^2 + 2abx + a^2 + 1\right)^{\frac{3}{2}} \operatorname{arcsinh}\left(bx + a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a)^2,x)
```

```
[Out] int(1/(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{bx + a + \sqrt{b^2x^2 + 2\,abx + a^2 + 1}}{\left(\left(b^2x^2 + 2\,abx + a^2 + 1\right)\left(b^2x + ab\right) + \left(b^3x^2 + 2\,ab^2x + a^2b + b\right)\sqrt{b^2x^2 + 2\,abx + a^2 + 1}\right)\log\left(bx + a + \sqrt{b^2x^2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a)^2,x, algorithm="ma
xima")
```

```
[Out] -(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/(((b^2*x^2 + 2*a*b*x + a^2 +
 1)*(b^2*x + a*b) + (b^3*x^2 + 2*a*b^2*x + a^2*b + b)*sqrt(b^2*x^2 + 2*a*b*
x + a^2 + 1))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))) - integrate
((2*b^4*x^4 + 8*a*b^3*x^3 + 2*a^4 + (12*a^2*b^2 + b^2)*x^2 + (2*b^2*x^2 + 4
*a*b*x + 2*a^2 + 1)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + a^2 + 2*(4*a^3*b + a*b)
*x + 2*(2*b^3*x^3 + 6*a*b^2*x^2 + 2*a^3 + (6*a^2*b + b)*x + a)*sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1) - 1)/(((b^4*x^4 + 4*a*b^3*x^3 + a^4 + (6*a^2*b^2 + b^
2)*x^2 + a^2 + 2*(2*a^3*b + a*b)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + 2
*(b^5*x^5 + 5*a*b^4*x^4 + a^5 + 2*(5*a^2*b^3 + b^3)*x^3 + 2*a^3 + 2*(5*a^3*
b^2 + 3*a*b^2)*x^2 + (5*a^4*b + 6*a^2*b + b)*x + a)*(b^2*x^2 + 2*a*b*x + a^
2 + 1) + (b^6*x^6 + 6*a*b^5*x^5 + a^6 + 3*(5*a^2*b^4 + b^4)*x^4 + 3*a^4 + 4
*(5*a^3*b^3 + 3*a*b^3)*x^3 + 3*(5*a^4*b^2 + 6*a^2*b^2 + b^2)*x^2 + 3*a^2 +
6*(a^5*b + 2*a^3*b + a*b)*x + 1)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*log(b*x
 + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\operatorname{asinh}\left(a + b\,x\right)^2\,{\left(a^2 + 2\,a\,b\,x + b^2\,x^2 + 1\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(asinh(a + b*x)^2*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2)),x)
```

```
[Out] int(1/(asinh(a + b*x)^2*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2)), x)
```

sympy [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2 + 2abx + b^2x^2 + 1\right)^{\frac{3}{2}} \operatorname{asinh}^2\left(a + bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**2+2*a*b*x+a**2+1)**(3/2)/asinh(b*x+a)**2,x)
```

```
[Out] Integral(1/((a**2 + 2*a*b*x + b**2*x**2 + 1)**(3/2)*asinh(a + b*x)**2), x)
```

3.283 $\int x^3 \sinh^{-1}\left(ax^2\right) dx$

Optimal. Leaf size=50

$$\frac{\sinh^{-1}\left(ax^2\right)}{8a^2} - \frac{x^2\sqrt{a^2x^4+1}}{8a} + \frac{1}{4}x^4\sinh^{-1}\left(ax^2\right)$$

```
[Out] 1/8*arcsinh(a*x^2)/a^2+1/4*x^4*arcsinh(a*x^2)-1/8*x^2*(a^2*x^4+1)^(1/2)/a
```

Rubi [A] time = 0.04, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5902, 12, 275, 321, 215}

$$-\frac{x^2\sqrt{a^2x^4+1}}{8a} + \frac{\sinh^{-1}\left(ax^2\right)}{8a^2} + \frac{1}{4}x^4\sinh^{-1}\left(ax^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcSinh[a*x^2],x]
```

```
[Out] -(x^2*Sqrt[1 + a^2*x^4])/(8*a) + ArcSinh[a*x^2]/(8*a^2) + (x^4*ArcSinh[a*x^
2])/4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5902

```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcSinh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/Sqrt[1 + u^2], x], x
], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
 x] && !FunctionOfQ[(c + d*x)^(m + 1), u, x] && !FunctionOfExponentialQ[u
, x]
```

Rubi steps

$$\int x^3 \sinh^{-1}\left(ax^2\right) dx = \frac{1}{4}x^4 \sinh^{-1}\left(ax^2\right) - \frac{1}{4}\int \frac{2ax^5}{\sqrt{1+a^2x^4}} dx$$
$$= \frac{1}{4}x^4 \sinh^{-1}\left(ax^2\right) - \frac{1}{2}a\int \frac{x^5}{\sqrt{1+a^2x^4}} dx$$
$$= \frac{1}{4}x^4 \sinh^{-1}\left(ax^2\right) - \frac{1}{4}a\,\text{Subst}\left(\int \frac{x^2}{\sqrt{1+a^2x^2}} dx, x, x^2\right)$$
$$= -\frac{x^2\sqrt{1+a^2x^4}}{8a} + \frac{1}{4}x^4 \sinh^{-1}\left(ax^2\right) + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+a^2x^2}} dx, x, x^2\right)}{8a}$$
$$= -\frac{x^2\sqrt{1+a^2x^4}}{8a} + \frac{\sinh^{-1}\left(ax^2\right)}{8a^2} + \frac{1}{4}x^4 \sinh^{-1}\left(ax^2\right)$$

Mathematica [A] time = 0.02, size = 44, normalized size = 0.88

$$\frac{\left(2a^2x^4+1\right)\sinh^{-1}\left(ax^2\right) - ax^2\sqrt{a^2x^4+1}}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcSinh[a*x^2],x]
[Out] (-(a*x^2*Sqrt[1 + a^2*x^4]) + (1 + 2*a^2*x^4)*ArcSinh[a*x^2])/(8*a^2)
```

fricas [A] time = 0.76, size = 52, normalized size = 1.04

$$-\frac{\sqrt{a^2x^4+1}\,ax^2 - \left(2\,a^2x^4+1\right)\log\left(ax^2+\sqrt{a^2x^4+1}\right)}{8\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x^2),x, algorithm="fricas")
[Out] -1/8*(sqrt(a^2*x^4 + 1)*a*x^2 - (2*a^2*x^4 + 1)*log(a*x^2 + sqrt(a^2*x^4 +
1)))/a^2
```

giac [A] time = 0.46, size = 74, normalized size = 1.48

$$\frac{1}{4}x^4\log\left(ax^2+\sqrt{a^2x^4+1}\right) - \frac{1}{8}a\left(\frac{\sqrt{a^2x^4+1}\,x^2}{a^2} + \frac{\log\left(-x^2|a|+\sqrt{a^2x^4+1}\right)}{a^2|a|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x^2),x, algorithm="giac")
[Out] 1/4*x^4*log(a*x^2 + sqrt(a^2*x^4 + 1)) - 1/8*a*(sqrt(a^2*x^4 + 1)*x^2/a^2 +
 log(-x^2*abs(a) + sqrt(a^2*x^4 + 1))/(a^2*abs(a)))
```

maple [A] time = 0.03, size = 67, normalized size = 1.34

$$\frac{x^4\,\text{arcsinh}\left(a\,x^2\right)}{4} - \frac{x^2\sqrt{a^2x^4+1}}{8a} + \frac{\ln\left(\frac{a^2x^2}{\sqrt{a^2}}+\sqrt{a^2x^4+1}\right)}{8a\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arcsinh(a*x^2),x)
```

```
[Out] 1/4*x^4*arcsinh(a*x^2)-1/8*x^2*(a^2*x^4+1)^(1/2)/a+1/8/a*ln(a^2*x^2/(a^2)^(
1/2)+(a^2*x^4+1)^(1/2))/(a^2)^(1/2)
```

maxima [B] time = 0.65, size = 102, normalized size = 2.04

$$\frac{1}{4}\,x^{4}\operatorname{arsinh}\left(ax^{2}\right)+\frac{1}{16}\,a\left(\frac{\log\left(a+\frac{\sqrt{a^{2}x^{4}+1}}{x^{2}}\right)}{a^{3}}-\frac{\log\left(-a+\frac{\sqrt{a^{2}x^{4}+1}}{x^{2}}\right)}{a^{3}}+\frac{2\,\sqrt{a^{2}x^{4}+1}}{\left(a^{4}-\frac{\left(a^{2}x^{4}+1\right)a^{2}}{x^{4}}\right)x^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x^2),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arcsinh(a*x^2) + 1/16*a*(log(a + sqrt(a^2*x^4 + 1)/x^2)/a^3 - log(-
a + sqrt(a^2*x^4 + 1)/x^2)/a^3 + 2*sqrt(a^2*x^4 + 1)/((a^4 - (a^2*x^4 + 1)*
a^2/x^4)*x^2))
```

mupad [B] time = 0.25, size = 45, normalized size = 0.90

$$\frac{x^{2}\operatorname{asinh}\left(a\,x^{2}\right)\left(\frac{x^{2}}{2}+\frac{1}{4\,a^{2}\,x^{2}}\right)}{2}-\frac{x^{2}\,\sqrt{a^{2}\,x^{4}+1}}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*asinh(a*x^2),x)
```

```
[Out] (x^2*asinh(a*x^2)*(x^2/2 + 1/(4*a^2*x^2)))/2 - (x^2*(a^2*x^4 + 1)^(1/2))/(8
*a)
```

sympy [A] time = 0.85, size = 42, normalized size = 0.84

$$\begin{cases}\frac{x^{4}\operatorname{asinh}\left(ax^{2}\right)}{4}-\frac{x^{2}\sqrt{a^{2}x^{4}+1}}{8a}+\frac{\operatorname{asinh}\left(ax^{2}\right)}{8a^{2}} & \text{for}\ a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*asinh(a*x**2),x)
```

```
[Out] Piecewise((x**4*asinh(a*x**2)/4 - x**2*sqrt(a**2*x**4 + 1)/(8*a) + asinh(a*
x**2)/(8*a**2), Ne(a, 0)), (0, True))
```

3.284 $\int x^2 \sinh^{-1}(ax^2)\,dx$

Optimal. Leaf size=101

$$-\frac{2x\sqrt{a^2x^4+1}}{9a}+\frac{(ax^2+1)\sqrt{\frac{a^2x^4+1}{(ax^2+1)^2}}F\left(2\tan^{-1}(\sqrt{a}\,x)\middle|\frac{1}{2}\right)}{9a^{3/2}\sqrt{a^2x^4+1}}+\frac{1}{3}x^3\sinh^{-1}(ax^2)$$

```
[Out] 1/3*x^3*arcsinh(a*x^2)-2/9*x*(a^2*x^4+1)^(1/2)/a+1/9*(a*x^2+1)*(cos(2*arcta
n(x*a^(1/2)))^2)^(1/2)/cos(2*arctan(x*a^(1/2)))*EllipticF(sin(2*arctan(x*a^
(1/2))),1/2*2^(1/2))*((a^2*x^4+1)/(a*x^2+1)^2)^(1/2)/a^(3/2)/(a^2*x^4+1)^(1
/2)
```

Rubi [A] time = 0.04, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5902, 12, 321, 220}

$$-\frac{2x\sqrt{a^2x^4+1}}{9a}+\frac{(ax^2+1)\sqrt{\frac{a^2x^4+1}{(ax^2+1)^2}}F\left(2\tan^{-1}(\sqrt{a}\,x)\middle|\frac{1}{2}\right)}{9a^{3/2}\sqrt{a^2x^4+1}}+\frac{1}{3}x^3\sinh^{-1}(ax^2)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcSinh[a*x^2],x]
```

```
[Out] (-2*x*Sqrt[1 + a^2*x^4])/(9*a) + (x^3*ArcSinh[a*x^2])/3 + ((1 + a*x^2)*Sqrt
[(1 + a^2*x^4)/(1 + a*x^2)^2]*EllipticF[2*ArcTan[Sqrt[a]*x], 1/2])/(9*a^(3/
2)*Sqrt[1 + a^2*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5902

```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcSinh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/Sqrt[1 + u^2], x], x
], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
 x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u
, x]
```

Rubi steps

$$\begin{aligned}\int x^2\sinh^{-1}\left(ax^2\right)dx &= \frac{1}{3}x^3\sinh^{-1}\left(ax^2\right)-\frac{1}{3}\int\frac{2ax^4}{\sqrt{1+a^2x^4}}\,dx\\ &= \frac{1}{3}x^3\sinh^{-1}\left(ax^2\right)-\frac{1}{3}(2a)\int\frac{x^4}{\sqrt{1+a^2x^4}}\,dx\\ &= -\frac{2x\sqrt{1+a^2x^4}}{9a}+\frac{1}{3}x^3\sinh^{-1}\left(ax^2\right)+\frac{2\int\frac{1}{\sqrt{1+a^2x^4}}\,dx}{9a}\\ &= -\frac{2x\sqrt{1+a^2x^4}}{9a}+\frac{1}{3}x^3\sinh^{-1}\left(ax^2\right)+\frac{\left(1+ax^2\right)\sqrt{\frac{1+a^2x^4}{\left(1+ax^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt{a}\,x\right)|\frac{1}{2}\right)}{9a^{3/2}\sqrt{1+a^2x^4}}\end{aligned}$$

Mathematica [C] time = 0.13, size = 75, normalized size = 0.74

$$\frac{1}{9}\left(-\frac{2\left(a^2x^5+x\right)}{a\sqrt{a^2x^4+1}}-\frac{2\sqrt{ia}\,F\left(i\sinh^{-1}\left(\sqrt{ia}\,x\right)\middle|-1\right)}{a^2}+3x^3\sinh^{-1}\left(ax^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcSinh[a*x^2],x]
[Out] ((-2*(x + a^2*x^5))/(a*Sqrt[1 + a^2*x^4]) + 3*x^3*ArcSinh[a*x^2] - (2*Sqrt[
I*a]*EllipticF[I*ArcSinh[Sqrt[I*a]*x], -1])/a^2)/9
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(x^2\operatorname{arsinh}\left(ax^2\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x^2),x, algorithm="fricas")
[Out] integral(x^2*arcsinh(a*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\operatorname{arsinh}\left(ax^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x^2),x, algorithm="giac")
[Out] integrate(x^2*arcsinh(a*x^2), x)
```

maple [C] time = 0.01, size = 89, normalized size = 0.88

$$\frac{x^3\operatorname{arcsinh}\left(a\,x^2\right)}{3}-\frac{2a\left(\frac{x\sqrt{a^2x^4+1}}{3a^2}-\frac{\sqrt{-ia\,x^2+1}\,\sqrt{ia\,x^2+1}\,\operatorname{EllipticF}\left(x\sqrt{ia},i\right)}{3a^2\sqrt{ia}\,\sqrt{a^2x^4+1}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsinh(a*x^2),x)
[Out] 1/3*x^3*arcsinh(a*x^2)-2/3*a*(1/3/a^2*x*(a^2*x^4+1)^(1/2)-1/3/a^2/(I*a)^(1/
2)*(1-I*a*x^2)^(1/2)*(1+I*a*x^2)^(1/2)/(a^2*x^4+1)^(1/2)*EllipticF(x*(I*a)^
(1/2),I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}x^3\log\left(ax^2+\sqrt{a^2x^4+1}\right)-\frac{2}{9}x^3-2a\int\frac{x^4}{3\left(a^3x^6+ax^2+\left(a^2x^4+1\right)^{\frac{3}{2}}\right)}\,dx-\frac{i\sqrt{2}\left(\log\left(\frac{i\sqrt{2}\left(2ax+\sqrt{2}\sqrt{a}\right)}{2\sqrt{a}}+1\right)-\log\right.}{12a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x^2),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*log(a*x^2 + sqrt(a^2*x^4 + 1)) - 2/9*x^3 - 2*a*integrate(1/3*x^4/(a
^3*x^6 + a*x^2 + (a^2*x^4 + 1)^(3/2)), x) - 1/12*I*sqrt(2)*(log(1/2*I*sqrt(
2)*(2*a*x + sqrt(2)*sqrt(a))/sqrt(a) + 1) - log(-1/2*I*sqrt(2)*(2*a*x + sqr
t(2)*sqrt(a))/sqrt(a) + 1))/a^(3/2) - 1/12*I*sqrt(2)*(log(1/2*I*sqrt(2)*(2*
a*x - sqrt(2)*sqrt(a))/sqrt(a) + 1) - log(-1/2*I*sqrt(2)*(2*a*x - sqrt(2)*s
qrt(a))/sqrt(a) + 1))/a^(3/2) - 1/12*sqrt(2)*log(a*x^2 + sqrt(2)*sqrt(a)*x
+ 1)/a^(3/2) + 1/12*sqrt(2)*log(a*x^2 - sqrt(2)*sqrt(a)*x + 1)/a^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{asinh}\left(a\,x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*asinh(a*x^2),x)
```

```
[Out] int(x^2*asinh(a*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\,\mathrm{asinh}\left(ax^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asinh(a*x**2),x)
```

```
[Out] Integral(x**2*asinh(a*x**2), x)
```

3.285 $\int x \sinh^{-1}\left(ax^2\right) dx$

Optimal. Leaf size=34

$$\frac{1}{2}x^2\sinh^{-1}\left(ax^2\right)-\frac{\sqrt{a^2x^4+1}}{2a}$$

```
[Out] 1/2*x^2*arcsinh(a*x^2)-1/2*(a^2*x^4+1)^(1/2)/a
```

Rubi [A] time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6715, 5653, 261}

$$\frac{1}{2}x^2\sinh^{-1}\left(ax^2\right)-\frac{\sqrt{a^2x^4+1}}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcSinh[a*x^2],x]
```

```
[Out] -Sqrt[1 + a^2*x^4]/(2*a) + (x^2*ArcSinh[a*x^2])/2
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\begin{aligned}\int x\sinh^{-1}\left(ax^2\right)dx &= \frac{1}{2}\operatorname{Subst}\left(\int \sinh^{-1}(ax)\,dx,x,x^2\right)\\ &= \frac{1}{2}x^2\sinh^{-1}\left(ax^2\right)-\frac{1}{2}a\operatorname{Subst}\left(\int \frac{x}{\sqrt{1+a^2x^2}}\,dx,x,x^2\right)\\ &= -\frac{\sqrt{1+a^2x^4}}{2a}+\frac{1}{2}x^2\sinh^{-1}\left(ax^2\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 34, normalized size = 1.00

$$\frac{1}{2}x^2\sinh^{-1}\left(ax^2\right)-\frac{\sqrt{a^2x^4+1}}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcSinh[a*x^2],x]
```

```
[Out] -1/2*Sqrt[1 + a^2*x^4]/a + (x^2*ArcSinh[a*x^2])/2
```

fricas [A] time = 0.48, size = 42, normalized size = 1.24

$$\frac{ax^2 \log\left(ax^2 + \sqrt{a^2x^4 + 1}\right) - \sqrt{a^2x^4 + 1}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x^2),x, algorithm="fricas")
[Out] 1/2*(a*x^2*log(a*x^2 + sqrt(a^2*x^4 + 1)) - sqrt(a^2*x^4 + 1))/a
```

giac [A] time = 0.33, size = 40, normalized size = 1.18

$$\frac{1}{2}\,x^2 \log\left(ax^2 + \sqrt{a^2x^4 + 1}\right) - \frac{\sqrt{a^2x^4 + 1}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x^2),x, algorithm="giac")
[Out] 1/2*x^2*log(a*x^2 + sqrt(a^2*x^4 + 1)) - 1/2*sqrt(a^2*x^4 + 1)/a
```

maple [A] time = 0.00, size = 31, normalized size = 0.91

$$\frac{x^2 a \operatorname{arcsinh}\left(a\,x^2\right) - \sqrt{a^2x^4 + 1}}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsinh(a*x^2),x)
[Out] 1/2/a*(x^2*a*arcsinh(a*x^2)-(a^2*x^4+1)^(1/2))
```

maxima [A] time = 0.31, size = 30, normalized size = 0.88

$$\frac{ax^2 \operatorname{arsinh}\left(ax^2\right) - \sqrt{a^2x^4 + 1}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x^2),x, algorithm="maxima")
[Out] 1/2*(a*x^2*arcsinh(a*x^2) - sqrt(a^2*x^4 + 1))/a
```

mupad [B] time = 0.25, size = 28, normalized size = 0.82

$$\frac{x^2 \operatorname{asinh}\left(a\,x^2\right)}{2} - \frac{\sqrt{a^2\,x^4 + 1}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*asinh(a*x^2),x)
[Out] (x^2*asinh(a*x^2))/2 - (a^2*x^4 + 1)^(1/2)/(2*a)
```

sympy [A] time = 0.22, size = 27, normalized size = 0.79

$$\begin{cases} \frac{x^2 \operatorname{asinh}\left(ax^2\right)}{2} - \frac{\sqrt{a^2x^4+1}}{2a} & \text{for}\ a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asinh(a*x**2),x)
```

```
[Out] Piecewise((x**2*asinh(a*x**2)/2 - sqrt(a**2*x**4 + 1)/(2*a), Ne(a, 0)), (0,
 True))
```

3.286 $\int \sinh^{-1}\left(ax^2\right) dx$

Optimal. Leaf size=162

$$-\frac{2x\sqrt{a^2x^4+1}}{ax^2+1}-\frac{\left(ax^2+1\right)\sqrt{\frac{a^2x^4+1}{\left(ax^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt{a}\,x\right)|\frac{1}{2}\right)}{\sqrt{a}\,\sqrt{a^2x^4+1}}+\frac{2\left(ax^2+1\right)\sqrt{\frac{a^2x^4+1}{\left(ax^2+1\right)^2}}E\left(2\tan^{-1}\left(\sqrt{a}\,x\right)|\frac{1}{2}\right)}{\sqrt{a}\,\sqrt{a^2x^4+1}}+x\sinh^{-1}$$

```
[Out] x*arcsinh(a*x^2)-2*x*(a^2*x^4+1)^(1/2)/(a*x^2+1)+2*(a*x^2+1)*(cos(2*arctan(
x*a^(1/2)))^2)^(1/2)/cos(2*arctan(x*a^(1/2)))*EllipticE(sin(2*arctan(x*a^(1
/2))),1/2*2^(1/2))*((a^2*x^4+1)/(a*x^2+1)^2)^(1/2)/a^(1/2)/(a^2*x^4+1)^(1/2
)-(a*x^2+1)*(cos(2*arctan(x*a^(1/2)))^2)^(1/2)/cos(2*arctan(x*a^(1/2)))*Ell
ipticF(sin(2*arctan(x*a^(1/2))),1/2*2^(1/2))*((a^2*x^4+1)/(a*x^2+1)^2)^(1/2
)/a^(1/2)/(a^2*x^4+1)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {5900, 12, 305, 220, 1196}

$$-\frac{2x\sqrt{a^2x^4+1}}{ax^2+1}-\frac{\left(ax^2+1\right)\sqrt{\frac{a^2x^4+1}{\left(ax^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt{a}\,x\right)|\frac{1}{2}\right)}{\sqrt{a}\,\sqrt{a^2x^4+1}}+\frac{2\left(ax^2+1\right)\sqrt{\frac{a^2x^4+1}{\left(ax^2+1\right)^2}}E\left(2\tan^{-1}\left(\sqrt{a}\,x\right)|\frac{1}{2}\right)}{\sqrt{a}\,\sqrt{a^2x^4+1}}+x\sinh^{-1}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x^2],x]
```

```
[Out] (-2*x*Sqrt[1 + a^2*x^4])/(1 + a*x^2) + x*ArcSinh[a*x^2] + (2*(1 + a*x^2)*Sq
rt[(1 + a^2*x^4)/(1 + a*x^2)^2]*EllipticE[2*ArcTan[Sqrt[a]*x], 1/2])/(Sqrt[
a]*Sqrt[1 + a^2*x^4]) - ((1 + a*x^2)*Sqrt[(1 + a^2*x^4)/(1 + a*x^2)^2]*Elli
pticF[2*ArcTan[Sqrt[a]*x], 1/2])/(Sqrt[a]*Sqrt[1 + a^2*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 5900

```
Int[ArcSinh[u_], x_Symbol] :> Simp[x*ArcSinh[u], x] - Int[SimplifyIntegrand
[(x*D[u, x])/Sqrt[1 + u^2], x], x] /; InverseFunctionFreeQ[u, x] && !Funct
ionOfExponentialQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \sinh^{-1}\left(ax^2\right)\,dx &= x\sinh^{-1}\left(ax^2\right)-\int \frac{2ax^2}{\sqrt{1+a^2x^4}}\,dx\\ &= x\sinh^{-1}\left(ax^2\right)-(2a)\int \frac{x^2}{\sqrt{1+a^2x^4}}\,dx\\ &= x\sinh^{-1}\left(ax^2\right)-2\int \frac{1}{\sqrt{1+a^2x^4}}\,dx+2\int \frac{1-ax^2}{\sqrt{1+a^2x^4}}\,dx\\ &= -\frac{2x\sqrt{1+a^2x^4}}{1+ax^2}+x\sinh^{-1}\left(ax^2\right)+\frac{2\left(1+ax^2\right)\sqrt{\frac{1+a^2x^4}{\left(1+ax^2\right)^2}}\,E\left(2\tan^{-1}\left(\sqrt{a}\,x\right)|\frac{1}{2}\right)}{\sqrt{a}\sqrt{1+a^2x^4}}-\frac{\left(1+ax\right.}{}\end{aligned}$$

Mathematica [C] time = 0.01, size = 35, normalized size = 0.22

$$x\sinh^{-1}\left(ax^2\right)-\frac{2}{3}ax^3\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-a^2x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x^2],x]
[Out] x*ArcSinh[a*x^2] - (2*a*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, -(a^2*x^4)])/3
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\text{arsinh}\left(ax^2\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^2),x, algorithm="fricas")
[Out] integral(arcsinh(a*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \text{arsinh}\left(ax^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^2),x, algorithm="giac")
[Out] integrate(arcsinh(a*x^2), x)
```

maple [C] time = 0.01, size = 77, normalized size = 0.48

$$x\,\text{arcsinh}\left(a\,x^2\right)-\frac{2i\sqrt{-ia\,x^2+1}\,\sqrt{ia\,x^2+1}\,\left(\text{EllipticF}\left(x\sqrt{ia}\,,i\right)-\text{EllipticE}\left(x\sqrt{ia}\,,i\right)\right)}{\sqrt{ia}\,\sqrt{a^2x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x^2),x)
```

[Out] x*arcsinh(a*x^2)-2*I/(I*a)^(1/2)*(1-I*a*x^2)^(1/2)*(1+I*a*x^2)^(1/2)/(a^2*x^4+1)^(1/2)*(EllipticF(x*(I*a)^(1/2),I)-EllipticE(x*(I*a)^(1/2),I))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-2\,a\int\frac{x^2}{a^3x^6+ax^2+\left(a^2x^4+1\right)^{\frac{3}{2}}}\,dx+x\log\left(ax^2+\sqrt{a^2x^4+1}\right)-2\,x-\frac{i\,\sqrt{2}\left(\log\left(\frac{i\,\sqrt{2}\left(2\,ax+\sqrt{2}\,\sqrt{a}\right)}{2\,\sqrt{a}}+1\right)-\log\left(-\frac{i\,\sqrt{2}\left(\right.}{}\right.\right.}{4\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^2),x, algorithm="maxima")
```

```
[Out] -2*a*integrate(x^2/(a^3*x^6 + a*x^2 + (a^2*x^4 + 1)^(3/2)), x) + x*log(a*x^
2 + sqrt(a^2*x^4 + 1)) - 2*x - 1/4*I*sqrt(2)*(log(1/2*I*sqrt(2)*(2*a*x + sq
rt(2)*sqrt(a))/sqrt(a) + 1) - log(-1/2*I*sqrt(2)*(2*a*x + sqrt(2)*sqrt(a))/
sqrt(a) + 1))/sqrt(a) - 1/4*I*sqrt(2)*(log(1/2*I*sqrt(2)*(2*a*x - sqrt(2)*s
qrt(a))/sqrt(a) + 1) - log(-1/2*I*sqrt(2)*(2*a*x - sqrt(2)*sqrt(a))/sqrt(a)
 + 1))/sqrt(a) + 1/4*sqrt(2)*log(a*x^2 + sqrt(2)*sqrt(a)*x + 1)/sqrt(a) - 1
/4*sqrt(2)*log(a*x^2 - sqrt(2)*sqrt(a)*x + 1)/sqrt(a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\mathrm{asinh}\left(a\,x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x^2),x)
```

```
[Out] int(asinh(a*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\mathrm{asinh}\left(ax^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x**2),x)
```

```
[Out] Integral(asinh(a*x**2), x)
```

3.287 $\int \frac{\sinh^{-1}(ax^2)}{x}\,dx$

Optimal. Leaf size=54

$$\frac{1}{4}\text{Li}_2\left(e^{2\sinh^{-1}(ax^2)}\right)-\frac{1}{4}\sinh^{-1}\left(ax^2\right)^2+\frac{1}{2}\sinh^{-1}\left(ax^2\right)\log\left(1-e^{2\sinh^{-1}(ax^2)}\right)$$

```
[Out] -1/4*arcsinh(a*x^2)^2+1/2*arcsinh(a*x^2)*ln(1-(a*x^2+(a^2*x^4+1)^(1/2))^2)+
1/4*polylog(2,(a*x^2+(a^2*x^4+1)^(1/2))^2)
```

Rubi [A] time = 0.06, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5890, 3716, 2190, 2279, 2391}

$$\frac{1}{4}\text{PolyLog}\left(2,e^{2\sinh^{-1}(ax^2)}\right)-\frac{1}{4}\sinh^{-1}\left(ax^2\right)^2+\frac{1}{2}\sinh^{-1}\left(ax^2\right)\log\left(1-e^{2\sinh^{-1}(ax^2)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x^2]/x,x]
```

```
[Out] -ArcSinh[a*x^2]^2/4 + (ArcSinh[a*x^2]*Log[1 - E^(2*ArcSinh[a*x^2])])/2 + Po
lyLog[2, E^(2*ArcSinh[a*x^2])]/4
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 5890

```
Int[ArcSinh[(a_.)*(x_)^(p_)]^(n_.)/(x_), x_Symbol] :> Dist[1/p, Subst[Int[x
^n*Coth[x], x], x, ArcSinh[a*x^p]], x] /; FreeQ[{a, p}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax^2)}{x}\,dx = \frac{1}{2}\text{Subst}\left(\int x\coth(x)\,dx, x, \sinh^{-1}(ax^2)\right)$$
$$= -\frac{1}{4}\sinh^{-1}(ax^2)^2 - \text{Subst}\left(\int \frac{e^{2x}x}{1-e^{2x}}\,dx, x, \sinh^{-1}(ax^2)\right)$$
$$= -\frac{1}{4}\sinh^{-1}(ax^2)^2 + \frac{1}{2}\sinh^{-1}(ax^2)\log\left(1-e^{2\sinh^{-1}(ax^2)}\right) - \frac{1}{2}\text{Subst}\left(\int \log\left(1-e^{2x}\right)\,dx, x,\right.$$
$$= -\frac{1}{4}\sinh^{-1}(ax^2)^2 + \frac{1}{2}\sinh^{-1}(ax^2)\log\left(1-e^{2\sinh^{-1}(ax^2)}\right) - \frac{1}{4}\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^2\right.$$
$$= -\frac{1}{4}\sinh^{-1}(ax^2)^2 + \frac{1}{2}\sinh^{-1}(ax^2)\log\left(1-e^{2\sinh^{-1}(ax^2)}\right) + \frac{1}{4}\text{Li}_2\left(e^{2\sinh^{-1}(ax^2)}\right)$$

Mathematica [A] time = 0.01, size = 54, normalized size = 1.00

$$\frac{1}{4}\text{Li}_2\left(e^{2\sinh^{-1}(ax^2)}\right) - \frac{1}{4}\sinh^{-1}(ax^2)^2 + \frac{1}{2}\sinh^{-1}(ax^2)\log\left(1-e^{2\sinh^{-1}(ax^2)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x^2]/x,x]
```

```
[Out] -1/4*ArcSinh[a*x^2]^2 + (ArcSinh[a*x^2]*Log[1 - E^(2*ArcSinh[a*x^2])])/2 +
PolyLog[2, E^(2*ArcSinh[a*x^2])]/4
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{arsinh}(ax^2)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^2)/x,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a*x^2)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{arsinh}(ax^2)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^2)/x,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x^2)/x, x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\text{arcsinh}(a\,x^2)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x^2)/x,x)
```

```
[Out] int(arcsinh(a*x^2)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{arsinh}(ax^2)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^2)/x,x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x^2)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{asinh}\left(a\,x^2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x^2)/x,x)
```

```
[Out] int(asinh(a*x^2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}\left(ax^2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x**2)/x,x)
```

```
[Out] Integral(asinh(a*x**2)/x, x)
```

3.288 $\int \frac{\sinh^{-1}(ax^2)}{x^2}\,dx$

Optimal. Leaf size=75

$$\frac{\sqrt{a}\,(ax^2+1)\sqrt{\frac{a^2x^4+1}{(ax^2+1)^2}}\,F\left(2\tan^{-1}\left(\sqrt{a}\,x\right)\middle|\frac{1}{2}\right)}{\sqrt{a^2x^4+1}}-\frac{\sinh^{-1}(ax^2)}{x}$$

```
[Out] -arcsinh(a*x^2)/x+(a*x^2+1)*(cos(2*arctan(x*a^(1/2)))^2)^(1/2)/cos(2*arctan
(x*a^(1/2)))*EllipticF(sin(2*arctan(x*a^(1/2))),1/2*2^(1/2))*a^(1/2)*((a^2*
x^4+1)/(a*x^2+1)^2)^(1/2)/(a^2*x^4+1)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5902, 12, 220}

$$\frac{\sqrt{a}\,(ax^2+1)\sqrt{\frac{a^2x^4+1}{(ax^2+1)^2}}\,F\left(2\tan^{-1}\left(\sqrt{a}\,x\right)\middle|\frac{1}{2}\right)}{\sqrt{a^2x^4+1}}-\frac{\sinh^{-1}(ax^2)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x^2]/x^2,x]
```

```
[Out] -(ArcSinh[a*x^2]/x) + (Sqrt[a]*(1 + a*x^2)*Sqrt[(1 + a^2*x^4)/(1 + a*x^2)^2
]*EllipticF[2*ArcTan[Sqrt[a]*x], 1/2])/Sqrt[1 + a^2*x^4]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 5902

```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcSinh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/Sqrt[1 + u^2], x], x
], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
 x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u
, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh^{-1}\left(ax^2\right)}{x^2}\,dx &= -\frac{\sinh^{-1}\left(ax^2\right)}{x} + \int \frac{2a}{\sqrt{1+a^2x^4}}\,dx \\
&= -\frac{\sinh^{-1}\left(ax^2\right)}{x} + (2a)\int \frac{1}{\sqrt{1+a^2x^4}}\,dx \\
&= -\frac{\sinh^{-1}\left(ax^2\right)}{x} + \frac{\sqrt{a}\left(1+ax^2\right)\sqrt{\frac{1+a^2x^4}{\left(1+ax^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt{a}\,x\right)|\frac{1}{2}\right)}{\sqrt{1+a^2x^4}}
\end{aligned}
$$

Mathematica [C] time = 0.04, size = 42, normalized size = 0.56

$$
-\frac{\sinh^{-1}\left(ax^2\right)+2\sqrt{ia}\,xF\left(i\sinh^{-1}\left(\sqrt{ia}\,x\right)\middle|-1\right)}{x}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x^2]/x^2,x]
[Out] -((ArcSinh[a*x^2] + 2*Sqrt[I*a]*x*EllipticF[I*ArcSinh[Sqrt[I*a]*x], -1])/x)
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$
\text{integral}\left(\frac{\text{arsinh}\left(ax^2\right)}{x^2},x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^2)/x^2,x, algorithm="fricas")
[Out] integral(arcsinh(a*x^2)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{\text{arsinh}\left(ax^2\right)}{x^2}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^2)/x^2,x, algorithm="giac")
[Out] integrate(arcsinh(a*x^2)/x^2, x)
```

maple [C] time = 0.01, size = 66, normalized size = 0.88

$$
-\frac{\text{arcsinh}\left(a\,x^2\right)}{x}+\frac{2a\sqrt{-ia\,x^2+1}\,\sqrt{ia\,x^2+1}\,\;\text{EllipticF}\left(x\sqrt{ia}\,,i\right)}{\sqrt{ia}\,\sqrt{a^2x^4+1}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x^2)/x^2,x)
[Out] -arcsinh(a*x^2)/x+2*a/(I*a)^(1/2)*(1-I*a*x^2)^(1/2)*(1+I*a*x^2)^(1/2)/(a^2*
x^4+1)^(1/2)*EllipticF(x*(I*a)^(1/2),I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$
-\frac{1}{4}a^2\left(\frac{i\sqrt{2}\left(\log\left(\frac{i\sqrt{2}\left(2ax+\sqrt{2}\sqrt{a}\right)}{2\sqrt{a}}+1\right)-\log\left(-\frac{i\sqrt{2}\left(2ax+\sqrt{2}\sqrt{a}\right)}{2\sqrt{a}}+1\right)\right)}{a^{\frac{3}{2}}}+\frac{i\sqrt{2}\left(\log\left(\frac{i\sqrt{2}\left(2ax-\sqrt{2}\sqrt{a}\right)}{2\sqrt{a}}+1\right)-\log\left(\right.\right.}{a^{\frac{3}{2}}}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^2)/x^2,x, algorithm="maxima")
```

```
[Out] -1/4*a^2*(I*sqrt(2)*(log(1/2*I*sqrt(2)*(2*a*x + sqrt(2)*sqrt(a))/sqrt(a) +
1) - log(-1/2*I*sqrt(2)*(2*a*x + sqrt(2)*sqrt(a))/sqrt(a) + 1))/a^(3/2) + I
*sqrt(2)*(log(1/2*I*sqrt(2)*(2*a*x - sqrt(2)*sqrt(a))/sqrt(a) + 1) - log(-1
/2*I*sqrt(2)*(2*a*x - sqrt(2)*sqrt(a))/sqrt(a) + 1))/a^(3/2) + sqrt(2)*log(
a*x^2 + sqrt(2)*sqrt(a)*x + 1)/a^(3/2) - sqrt(2)*log(a*x^2 - sqrt(2)*sqrt(a
)*x + 1)/a^(3/2)) + 2*a*integrate(1/(a^3*x^6 + a*x^2 + (a^2*x^4 + 1)^(3/2))
, x) - log(a*x^2 + sqrt(a^2*x^4 + 1))/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{asinh}\left(a\,x^2\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x^2)/x^2,x)
```

```
[Out] int(asinh(a*x^2)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}\left(ax^2\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x**2)/x**2,x)
```

```
[Out] Integral(asinh(a*x**2)/x**2, x)
```

3.289 $\int \frac{\sinh^{-1}(ax^2)}{x^3} dx$

Optimal. Leaf size=33

$$-\frac{1}{2}a\tanh^{-1}\left(\sqrt{a^2x^4+1}\right)-\frac{\sinh^{-1}\left(ax^2\right)}{2x^2}$$

```
[Out] -1/2*arcsinh(a*x^2)/x^2-1/2*a*arctanh((a^2*x^4+1)^(1/2))
```

Rubi [A] time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5902, 12, 266, 63, 208}

$$-\frac{1}{2}a\tanh^{-1}\left(\sqrt{a^2x^4+1}\right)-\frac{\sinh^{-1}\left(ax^2\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x^2]/x^3,x]
```

```
[Out] -ArcSinh[a*x^2]/(2*x^2) - (a*ArcTanh[Sqrt[1 + a^2*x^4]])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5902

```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcSinh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/Sqrt[1 + u^2], x], x
], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
 x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u
, x]
```

Rubi steps

$$\int \frac{\sinh^{-1}\left(ax^2\right)}{x^3}\,dx = -\frac{\sinh^{-1}\left(ax^2\right)}{2x^2} + \frac{1}{2}\int \frac{2a}{x\sqrt{1+a^2x^4}}\,dx$$
$$= -\frac{\sinh^{-1}\left(ax^2\right)}{2x^2} + a\int \frac{1}{x\sqrt{1+a^2x^4}}\,dx$$
$$= -\frac{\sinh^{-1}\left(ax^2\right)}{2x^2} + \frac{1}{4}a\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}}\,dx, x, x^4\right)$$
$$= -\frac{\sinh^{-1}\left(ax^2\right)}{2x^2} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx, x, \sqrt{1+a^2x^4}\right)}{2a}$$
$$= -\frac{\sinh^{-1}\left(ax^2\right)}{2x^2} - \frac{1}{2}a\tanh^{-1}\left(\sqrt{1+a^2x^4}\right)$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.00

$$-\frac{1}{2}a\tanh^{-1}\left(\sqrt{a^2x^4+1}\right) - \frac{\sinh^{-1}\left(ax^2\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x^2]/x^3,x]
```

```
[Out] -1/2*ArcSinh[a*x^2]/x^2 - (a*ArcTanh[Sqrt[1 + a^2*x^4]])/2
```

fricas [B] time = 1.05, size = 106, normalized size = 3.21

$$-\frac{ax^2\log\left(-ax^2+\sqrt{a^2x^4+1}+1\right) - ax^2\log\left(-ax^2+\sqrt{a^2x^4+1}-1\right) - x^2\log\left(-ax^2+\sqrt{a^2x^4+1}\right) - \left(x^2-1\right)\log}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^2)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(a*x^2*log(-a*x^2 + sqrt(a^2*x^4 + 1) + 1) - a*x^2*log(-a*x^2 + sqrt(a
^2*x^4 + 1) - 1) - x^2*log(-a*x^2 + sqrt(a^2*x^4 + 1)) - (x^2 - 1)*log(a*x^
2 + sqrt(a^2*x^4 + 1)))/x^2
```

giac [B] time = 0.30, size = 58, normalized size = 1.76

$$-\frac{1}{4}a\left(\log\left(\sqrt{a^2x^4+1}+1\right) - \log\left(\sqrt{a^2x^4+1}-1\right)\right) - \frac{\log\left(ax^2+\sqrt{a^2x^4+1}\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^2)/x^3,x, algorithm="giac")
```

```
[Out] -1/4*a*(log(sqrt(a^2*x^4 + 1) + 1) - log(sqrt(a^2*x^4 + 1) - 1)) - 1/2*log(
a*x^2 + sqrt(a^2*x^4 + 1))/x^2
```

maple [A] time = 0.01, size = 28, normalized size = 0.85

$$-\frac{\operatorname{arcsinh}\left(a\,x^2\right)}{2x^2} - \frac{a\,\operatorname{arctanh}\left(\frac{1}{\sqrt{a^2x^4+1}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x^2)/x^3,x)
```

```
[Out] -1/2*arcsinh(a*x^2)/x^2-1/2*a*arctanh(1/(a^2*x^4+1)^(1/2))
```

maxima [A] time = 0.37, size = 46, normalized size = 1.39

$$-\frac{1}{4}\,a\left(\log\left(\sqrt{a^2x^4+1}+1\right)-\log\left(\sqrt{a^2x^4+1}-1\right)\right)-\frac{\operatorname{arsinh}\left(ax^2\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^2)/x^3,x, algorithm="maxima")
```

```
[Out] -1/4*a*(log(sqrt(a^2*x^4 + 1) + 1) - log(sqrt(a^2*x^4 + 1) - 1)) - 1/2*arcs
inh(a*x^2)/x^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\operatorname{asinh}\left(a\,x^2\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x^2)/x^3,x)
```

```
[Out] int(asinh(a*x^2)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}\left(ax^2\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x**2)/x**3,x)
```

```
[Out] Integral(asinh(a*x**2)/x**3, x)
```

3.290 $\int \frac{\sinh^{-1}(ax^2)}{x^4}\,dx$

Optimal. Leaf size=197

$$-\frac{2a\sqrt{a^2x^4+1}}{3x}+\frac{2a^2x\sqrt{a^2x^4+1}}{3\left(ax^2+1\right)}+\frac{a^{3/2}\left(ax^2+1\right)\sqrt{\frac{a^2x^4+1}{\left(ax^2+1\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt{a}\,x\right)|\frac{1}{2}\right)}{3\sqrt{a^2x^4+1}}-\frac{2a^{3/2}\left(ax^2+1\right)\sqrt{\frac{a^2x^4+1}{\left(ax^2+1\right)^2}}\,E\left(2\tan\ldots\right)}{3\sqrt{a^2x^4+1}}$$

```
[Out] -1/3*arcsinh(a*x^2)/x^3-2/3*a*(a^2*x^4+1)^(1/2)/x+2/3*a^2*x*(a^2*x^4+1)^(1/
2)/(a*x^2+1)-2/3*a^(3/2)*(a*x^2+1)*(cos(2*arctan(x*a^(1/2)))^2)^(1/2)/cos(2
*arctan(x*a^(1/2)))*EllipticE(sin(2*arctan(x*a^(1/2))),1/2*2^(1/2))*((a^2*x
^4+1)/(a*x^2+1)^2)^(1/2)/(a^2*x^4+1)^(1/2)+1/3*a^(3/2)*(a*x^2+1)*(cos(2*arc
tan(x*a^(1/2)))^2)^(1/2)/cos(2*arctan(x*a^(1/2)))*EllipticF(sin(2*arctan(x*
a^(1/2))),1/2*2^(1/2))*((a^2*x^4+1)/(a*x^2+1)^2)^(1/2)/(a^2*x^4+1)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5902, 12, 325, 305, 220, 1196}

$$\frac{2a^2x\sqrt{a^2x^4+1}}{3\left(ax^2+1\right)}-\frac{2a\sqrt{a^2x^4+1}}{3x}+\frac{a^{3/2}\left(ax^2+1\right)\sqrt{\frac{a^2x^4+1}{\left(ax^2+1\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt{a}\,x\right)|\frac{1}{2}\right)}{3\sqrt{a^2x^4+1}}-\frac{2a^{3/2}\left(ax^2+1\right)\sqrt{\frac{a^2x^4+1}{\left(ax^2+1\right)^2}}\,E\left(2\tan\ldots\right)}{3\sqrt{a^2x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x^2]/x^4,x]
```

```
[Out] (-2*a*Sqrt[1 + a^2*x^4])/(3*x) + (2*a^2*x*Sqrt[1 + a^2*x^4])/(3*(1 + a*x^2)
) - ArcSinh[a*x^2]/(3*x^3) - (2*a^(3/2)*(1 + a*x^2)*Sqrt[(1 + a^2*x^4)/(1 +
 a*x^2)^2]*EllipticE[2*ArcTan[Sqrt[a]*x], 1/2])/(3*Sqrt[1 + a^2*x^4]) + (a^
(3/2)*(1 + a*x^2)*Sqrt[(1 + a^2*x^4)/(1 + a*x^2)^2]*EllipticF[2*ArcTan[Sqrt
[a]*x], 1/2])/(3*Sqrt[1 + a^2*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 5902

```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcSinh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/Sqrt[1 + u^2], x], x
], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
 x] && !FunctionOfQ[(c + d*x)^(m + 1), u, x] && !FunctionOfExponentialQ[u
, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^{-1}\left(ax^2\right)}{x^4}\,dx &= -\frac{\sinh^{-1}\left(ax^2\right)}{3x^3}+\frac{1}{3}\int \frac{2a}{x^2\sqrt{1+a^2x^4}}\,dx\\
&= -\frac{\sinh^{-1}\left(ax^2\right)}{3x^3}+\frac{1}{3}(2a)\int \frac{1}{x^2\sqrt{1+a^2x^4}}\,dx\\
&= -\frac{2a\sqrt{1+a^2x^4}}{3x}-\frac{\sinh^{-1}\left(ax^2\right)}{3x^3}+\frac{1}{3}\left(2a^3\right)\int \frac{x^2}{\sqrt{1+a^2x^4}}\,dx\\
&= -\frac{2a\sqrt{1+a^2x^4}}{3x}-\frac{\sinh^{-1}\left(ax^2\right)}{3x^3}+\frac{1}{3}\left(2a^2\right)\int \frac{1}{\sqrt{1+a^2x^4}}\,dx-\frac{1}{3}\left(2a^2\right)\int \frac{1-ax^2}{\sqrt{1+a^2x^4}}\,dx\\
&= -\frac{2a\sqrt{1+a^2x^4}}{3x}+\frac{2a^2x\sqrt{1+a^2x^4}}{3\left(1+ax^2\right)}-\frac{\sinh^{-1}\left(ax^2\right)}{3x^3}-\frac{2a^{3/2}\left(1+ax^2\right)\sqrt{\frac{1+a^2x^4}{\left(1+ax^2\right)^2}}\,E\left(2\tan^{-1}\right.}{3\sqrt{1+a^2x^4}}
\end{aligned}$$

Mathematica [C] time = 0.17, size = 88, normalized size = 0.45

$$\frac{1}{3}\left(-\frac{2a\sqrt{a^2x^4+1}}{x}+\frac{2a^2\left(E\left(i\sinh^{-1}\left(\sqrt{ia}\,x\right)\middle|-1\right)-F\left(i\sinh^{-1}\left(\sqrt{ia}\,x\right)\middle|-1\right)\right)}{\sqrt{ia}}-\frac{\sinh^{-1}\left(ax^2\right)}{x^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x^2]/x^4,x]
```

```
[Out] ((-2*a*Sqrt[1 + a^2*x^4])/x - ArcSinh[a*x^2]/x^3 + (2*a^2*(EllipticE[I*ArcS
inh[Sqrt[I*a]*x], -1] - EllipticF[I*ArcSinh[Sqrt[I*a]*x], -1]))/Sqrt[I*a])/
3
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{arsinh}\left(ax^2\right)}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^2)/x^4,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a*x^2)/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}\left(ax^2\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x^2)/x^4, x)
```

maple [C] time = 0.01, size = 101, normalized size = 0.51

$$-\frac{\operatorname{arcsinh}\left(a\,x^2\right)}{3x^3}+\frac{2a\left(-\frac{\sqrt{a^2x^4+1}}{x}+\frac{ia\sqrt{-ia\,x^2+1}\,\sqrt{ia\,x^2+1}\,\left(\operatorname{EllipticF}\left(x\sqrt{ia},i\right)-\operatorname{EllipticE}\left(x\sqrt{ia},i\right)\right)}{\sqrt{ia}\,\sqrt{a^2x^4+1}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x^2)/x^4,x)
```

```
[Out] -1/3*arcsinh(a*x^2)/x^3+2/3*a*(-(a^2*x^4+1)^(1/2)/x+I*a/(I*a)^(1/2)*(1-I*a*
x^2)^(1/2)*(1+I*a*x^2)^(1/2)/(a^2*x^4+1)^(1/2)*(EllipticF(x*(I*a)^(1/2),I)-
EllipticE(x*(I*a)^(1/2),I)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{12}i\sqrt{2}\,a^{\frac{3}{2}}\left(\log\left(\frac{i\sqrt{2}\left(2\,ax+\sqrt{2}\sqrt{a}\right)}{2\sqrt{a}}+1\right)-\log\left(-\frac{i\sqrt{2}\left(2\,ax+\sqrt{2}\sqrt{a}\right)}{2\sqrt{a}}+1\right)\right)-\frac{1}{12}i\sqrt{2}\,a^{\frac{3}{2}}\left(\log\left(\frac{i\sqrt{2}\left(2\,ax-\sqrt{2}\right.}{2\sqrt{a}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^2)/x^4,x, algorithm="maxima")
```

```
[Out] -1/12*I*sqrt(2)*a^(3/2)*(log(1/2*I*sqrt(2)*(2*a*x + sqrt(2)*sqrt(a))/sqrt(a
) + 1) - log(-1/2*I*sqrt(2)*(2*a*x + sqrt(2)*sqrt(a))/sqrt(a) + 1)) - 1/12*
I*sqrt(2)*a^(3/2)*(log(1/2*I*sqrt(2)*(2*a*x - sqrt(2)*sqrt(a))/sqrt(a) + 1)
 - log(-1/2*I*sqrt(2)*(2*a*x - sqrt(2)*sqrt(a))/sqrt(a) + 1)) + 1/12*sqrt(2
)*a^(3/2)*log(a*x^2 + sqrt(2)*sqrt(a)*x + 1) - 1/12*sqrt(2)*a^(3/2)*log(a*x
^2 - sqrt(2)*sqrt(a)*x + 1) + 2*a*integrate(1/3/(a^3*x^8 + a*x^4 + (a^2*x^6
 + x^2)*sqrt(a^2*x^4 + 1)), x) - 1/3*log(a*x^2 + sqrt(a^2*x^4 + 1))/x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{asinh}\left(a\,x^2\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x^2)/x^4,x)
```

```
[Out] int(asinh(a*x^2)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}\left(ax^2\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x**2)/x**4,x)
```

```
[Out] Integral(asinh(a*x**2)/x**4, x)
```

3.291 $\int \frac{\sinh^{-1}(ax^5)}{x} dx$

Optimal. Leaf size=54

$$\frac{1}{10}\text{Li}_2\left(e^{2\sinh^{-1}(ax^5)}\right)-\frac{1}{10}\sinh^{-1}\left(ax^5\right)^2+\frac{1}{5}\sinh^{-1}\left(ax^5\right)\log\left(1-e^{2\sinh^{-1}(ax^5)}\right)$$

```
[Out] -1/10*arcsinh(a*x^5)^2+1/5*arcsinh(a*x^5)*ln(1-(a*x^5+(a^2*x^10+1)^(1/2))^2
)+1/10*polylog(2,(a*x^5+(a^2*x^10+1)^(1/2))^2)
```

Rubi [A] time = 0.06, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5890, 3716, 2190, 2279, 2391}

$$\frac{1}{10}\text{PolyLog}\left(2,e^{2\sinh^{-1}(ax^5)}\right)-\frac{1}{10}\sinh^{-1}\left(ax^5\right)^2+\frac{1}{5}\sinh^{-1}\left(ax^5\right)\log\left(1-e^{2\sinh^{-1}(ax^5)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x^5]/x,x]
```

```
[Out] -ArcSinh[a*x^5]^2/10 + (ArcSinh[a*x^5]*Log[1 - E^(2*ArcSinh[a*x^5])])/5 + P
olyLog[2, E^(2*ArcSinh[a*x^5])]/10
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 5890

```
Int[ArcSinh[(a_.)*(x_)^(p_)]^(n_.)/(x_), x_Symbol] :> Dist[1/p, Subst[Int[x
^n*Coth[x], x], x, ArcSinh[a*x^p]], x] /; FreeQ[{a, p}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax^5)}{x}\,dx = \frac{1}{5}\operatorname{Subst}\left(\int x\coth(x)\,dx, x, \sinh^{-1}(ax^5)\right)$$
$$= -\frac{1}{10}\sinh^{-1}(ax^5)^2 - \frac{2}{5}\operatorname{Subst}\left(\int \frac{e^{2x}x}{1-e^{2x}}\,dx, x, \sinh^{-1}(ax^5)\right)$$
$$= -\frac{1}{10}\sinh^{-1}(ax^5)^2 + \frac{1}{5}\sinh^{-1}(ax^5)\log\left(1-e^{2\sinh^{-1}(ax^5)}\right) - \frac{1}{5}\operatorname{Subst}\left(\int \log\left(1-e^{2x}\right)\,dx, x\right.$$
$$= -\frac{1}{10}\sinh^{-1}(ax^5)^2 + \frac{1}{5}\sinh^{-1}(ax^5)\log\left(1-e^{2\sinh^{-1}(ax^5)}\right) - \frac{1}{10}\operatorname{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x,\right.$$
$$= -\frac{1}{10}\sinh^{-1}(ax^5)^2 + \frac{1}{5}\sinh^{-1}(ax^5)\log\left(1-e^{2\sinh^{-1}(ax^5)}\right) + \frac{1}{10}\operatorname{Li}_2\left(e^{2\sinh^{-1}(ax^5)}\right)$$

Mathematica [A] time = 0.01, size = 54, normalized size = 1.00

$$\frac{1}{10}\operatorname{Li}_2\left(e^{2\sinh^{-1}(ax^5)}\right) - \frac{1}{10}\sinh^{-1}(ax^5)^2 + \frac{1}{5}\sinh^{-1}(ax^5)\log\left(1-e^{2\sinh^{-1}(ax^5)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x^5]/x,x]
[Out] -1/10*ArcSinh[a*x^5]^2 + (ArcSinh[a*x^5]*Log[1 - E^(2*ArcSinh[a*x^5])])/5 +
 PolyLog[2, E^(2*ArcSinh[a*x^5])]/10
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\operatorname{arsinh}(ax^5)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^5)/x,x, algorithm="fricas")
[Out] integral(arcsinh(a*x^5)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}(ax^5)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^5)/x,x, algorithm="giac")
[Out] integrate(arcsinh(a*x^5)/x, x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsinh}(a\,x^5)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x^5)/x,x)
[Out] int(arcsinh(a*x^5)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}(ax^5)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^5)/x,x, algorithm="maxima")
[Out] integrate(arcsinh(a*x^5)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{asinh}\left(a\,x^5\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x^5)/x,x)
[Out] int(asinh(a*x^5)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}\left(ax^{5}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x**5)/x,x)
[Out] Integral(asinh(a*x**5)/x, x)
```

3.292 $\int x^2 \sinh^{-1}\left(\sqrt{x}\right)\,dx$

Optimal. Leaf size=72

$$-\frac{1}{18}\sqrt{x+1}\,x^{5/2}+\frac{5}{72}\sqrt{x+1}\,x^{3/2}+\frac{1}{3}x^3\sinh^{-1}\left(\sqrt{x}\right)-\frac{5}{48}\sqrt{x+1}\,\sqrt{x}+\frac{5}{48}\sinh^{-1}\left(\sqrt{x}\right)$$

```
[Out] 5/48*arcsinh(x^(1/2))+1/3*x^3*arcsinh(x^(1/2))+5/72*x^(3/2)*(1+x)^(1/2)-1/1
8*x^(5/2)*(1+x)^(1/2)-5/48*x^(1/2)*(1+x)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5902, 12, 50, 54, 215}

$$-\frac{1}{18}\sqrt{x+1}\,x^{5/2}+\frac{5}{72}\sqrt{x+1}\,x^{3/2}+\frac{1}{3}x^3\sinh^{-1}\left(\sqrt{x}\right)-\frac{5}{48}\sqrt{x+1}\,\sqrt{x}+\frac{5}{48}\sinh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcSinh[Sqrt[x]],x]
```

```
[Out] (-5*Sqrt[x]*Sqrt[1 + x])/48 + (5*x^(3/2)*Sqrt[1 + x])/72 - (x^(5/2)*Sqrt[1
+ x])/18 + (5*ArcSinh[Sqrt[x]])/48 + (x^3*ArcSinh[Sqrt[x]])/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 5902

```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcSinh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/Sqrt[1 + u^2], x], x
], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
 x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u
, x]
```

Rubi steps

$$\int x^2 \sinh^{-1}\left(\sqrt{x}\right) dx = \frac{1}{3}x^3 \sinh^{-1}\left(\sqrt{x}\right) - \frac{1}{3}\int \frac{x^{5/2}}{2\sqrt{1+x}} dx$$
$$= \frac{1}{3}x^3 \sinh^{-1}\left(\sqrt{x}\right) - \frac{1}{6}\int \frac{x^{5/2}}{\sqrt{1+x}} dx$$
$$= -\frac{1}{18}x^{5/2}\sqrt{1+x} + \frac{1}{3}x^3 \sinh^{-1}\left(\sqrt{x}\right) + \frac{5}{36}\int \frac{x^{3/2}}{\sqrt{1+x}} dx$$
$$= \frac{5}{72}x^{3/2}\sqrt{1+x} - \frac{1}{18}x^{5/2}\sqrt{1+x} + \frac{1}{3}x^3 \sinh^{-1}\left(\sqrt{x}\right) - \frac{5}{48}\int \frac{\sqrt{x}}{\sqrt{1+x}} dx$$
$$= -\frac{5}{48}\sqrt{x}\sqrt{1+x} + \frac{5}{72}x^{3/2}\sqrt{1+x} - \frac{1}{18}x^{5/2}\sqrt{1+x} + \frac{1}{3}x^3 \sinh^{-1}\left(\sqrt{x}\right) + \frac{5}{96}\int \frac{1}{\sqrt{x}\sqrt{1}}$$
$$= -\frac{5}{48}\sqrt{x}\sqrt{1+x} + \frac{5}{72}x^{3/2}\sqrt{1+x} - \frac{1}{18}x^{5/2}\sqrt{1+x} + \frac{1}{3}x^3 \sinh^{-1}\left(\sqrt{x}\right) + \frac{5}{48}\text{Subst}\Big(\int$$
$$= -\frac{5}{48}\sqrt{x}\sqrt{1+x} + \frac{5}{72}x^{3/2}\sqrt{1+x} - \frac{1}{18}x^{5/2}\sqrt{1+x} + \frac{5}{48}\sinh^{-1}\left(\sqrt{x}\right) + \frac{1}{3}x^3 \sinh^{-1}\big(\sqrt{}$$

Mathematica [A] time = 0.02, size = 43, normalized size = 0.60

$$\frac{1}{144}\left(3\left(16x^3+5\right)\sinh^{-1}\left(\sqrt{x}\right)+\sqrt{x}\sqrt{x+1}\left(-8x^2+10x-15\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcSinh[Sqrt[x]],x]
```

```
[Out] (Sqrt[x]*Sqrt[1 + x]*(-15 + 10*x - 8*x^2) + 3*(5 + 16*x^3)*ArcSinh[Sqrt[x]]
)/144
```

fricas [A] time = 0.78, size = 40, normalized size = 0.56

$$-\frac{1}{144}\left(8x^2-10x+15\right)\sqrt{x+1}\sqrt{x}+\frac{1}{48}\left(16x^3+5\right)\log\left(\sqrt{x+1}+\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(x^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/144*(8*x^2 - 10*x + 15)*sqrt(x + 1)*sqrt(x) + 1/48*(16*x^3 + 5)*log(sqrt
(x + 1) + sqrt(x))
```

giac [A] time = 0.34, size = 50, normalized size = 0.69

$$\frac{1}{3}x^3\log\left(\sqrt{x+1}+\sqrt{x}\right)-\frac{1}{144}\left(2\left(4x-5\right)x+15\right)\sqrt{x+1}\sqrt{x}-\frac{5}{48}\log\left(\sqrt{x+1}-\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(x^(1/2)),x, algorithm="giac")
```

```
[Out] 1/3*x^3*log(sqrt(x + 1) + sqrt(x)) - 1/144*(2*(4*x - 5)*x + 15)*sqrt(x + 1)
*sqrt(x) - 5/48*log(sqrt(x + 1) - sqrt(x))
```

maple [A] time = 0.01, size = 47, normalized size = 0.65

$$\frac{5\,\text{arcsinh}\left(\sqrt{x}\right)}{48}+\frac{x^3\,\text{arcsinh}\left(\sqrt{x}\right)}{3}+\frac{5x^{\frac{3}{2}}\sqrt{1+x}}{72}-\frac{x^{\frac{5}{2}}\sqrt{1+x}}{18}-\frac{5\sqrt{x}\,\sqrt{1+x}}{48}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsinh(x^(1/2)),x)
```

```
[Out] 5/48*arcsinh(x^(1/2))+1/3*x^3*arcsinh(x^(1/2))+5/72*x^(3/2)*(1+x)^(1/2)-1/1
8*x^(5/2)*(1+x)^(1/2)-5/48*x^(1/2)*(1+x)^(1/2)
```

maxima [A] time = 0.86, size = 46, normalized size = 0.64

$$\frac{1}{3}\,x^3\,\operatorname{arsinh}\left(\sqrt{x}\right)-\frac{1}{18}\,\sqrt{x+1}\,x^{\frac{5}{2}}+\frac{5}{72}\,\sqrt{x+1}\,x^{\frac{3}{2}}-\frac{5}{48}\,\sqrt{x+1}\,\sqrt{x}+\frac{5}{48}\,\operatorname{arsinh}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(x^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arcsinh(sqrt(x)) - 1/18*sqrt(x + 1)*x^(5/2) + 5/72*sqrt(x + 1)*x^(3
/2) - 5/48*sqrt(x + 1)*sqrt(x) + 5/48*arcsinh(sqrt(x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\operatorname{asinh}\left(\sqrt{x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*asinh(x^(1/2)),x)
```

```
[Out] int(x^2*asinh(x^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2}\operatorname{asinh}{\left(\sqrt{x}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asinh(x**(1/2)),x)
```

```
[Out] Integral(x**2*asinh(sqrt(x)), x)
```

3.293 $\int x \sinh^{-1}\left(\sqrt{x}\right) dx$

Optimal. Leaf size=56

$$-\frac{1}{8}\sqrt{x+1}\,x^{3/2}+\frac{1}{2}x^2\sinh^{-1}\left(\sqrt{x}\right)+\frac{3}{16}\sqrt{x+1}\,\sqrt{x}-\frac{3}{16}\sinh^{-1}\left(\sqrt{x}\right)$$

```
[Out] -3/16*arcsinh(x^(1/2))+1/2*x^2*arcsinh(x^(1/2))-1/8*x^(3/2)*(1+x)^(1/2)+3/1
6*x^(1/2)*(1+x)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5902, 12, 50, 54, 215}

$$-\frac{1}{8}\sqrt{x+1}\,x^{3/2}+\frac{1}{2}x^2\sinh^{-1}\left(\sqrt{x}\right)+\frac{3}{16}\sqrt{x+1}\,\sqrt{x}-\frac{3}{16}\sinh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcSinh[Sqrt[x]],x]
```

```
[Out] (3*Sqrt[x]*Sqrt[1 + x])/16 - (x^(3/2)*Sqrt[1 + x])/8 - (3*ArcSinh[Sqrt[x]])
/16 + (x^2*ArcSinh[Sqrt[x]])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 5902

```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcSinh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/Sqrt[1 + u^2], x], x
], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
 x] && !FunctionOfQ[(c + d*x)^(m + 1), u, x] && !FunctionOfExponentialQ[u
, x]
```

Rubi steps

$$
\begin{aligned}
\int x\sinh^{-1}\left(\sqrt{x}\right)dx &= \frac{1}{2}x^2\sinh^{-1}\left(\sqrt{x}\right)-\frac{1}{2}\int\frac{x^{3/2}}{2\sqrt{1+x}}dx\\
&= \frac{1}{2}x^2\sinh^{-1}\left(\sqrt{x}\right)-\frac{1}{4}\int\frac{x^{3/2}}{\sqrt{1+x}}dx\\
&= -\frac{1}{8}x^{3/2}\sqrt{1+x}+\frac{1}{2}x^2\sinh^{-1}\left(\sqrt{x}\right)+\frac{3}{16}\int\frac{\sqrt{x}}{\sqrt{1+x}}dx\\
&= \frac{3}{16}\sqrt{x}\sqrt{1+x}-\frac{1}{8}x^{3/2}\sqrt{1+x}+\frac{1}{2}x^2\sinh^{-1}\left(\sqrt{x}\right)-\frac{3}{32}\int\frac{1}{\sqrt{x}\sqrt{1+x}}dx\\
&= \frac{3}{16}\sqrt{x}\sqrt{1+x}-\frac{1}{8}x^{3/2}\sqrt{1+x}+\frac{1}{2}x^2\sinh^{-1}\left(\sqrt{x}\right)-\frac{3}{16}\text{Subst}\left(\int\frac{1}{\sqrt{1+x^2}}dx,x,\sqrt{x}\right)\\
&= \frac{3}{16}\sqrt{x}\sqrt{1+x}-\frac{1}{8}x^{3/2}\sqrt{1+x}-\frac{3}{16}\sinh^{-1}\left(\sqrt{x}\right)+\frac{1}{2}x^2\sinh^{-1}\left(\sqrt{x}\right)
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 37, normalized size = 0.66

$$\frac{1}{16}\left(\left(8x^2-3\right)\sinh^{-1}\left(\sqrt{x}\right)+\sqrt{x}\sqrt{x+1}(3-2x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcSinh[Sqrt[x]],x]
[Out] ((3 - 2*x)*Sqrt[x]*Sqrt[1 + x] + (-3 + 8*x^2)*ArcSinh[Sqrt[x]])/16
```

fricas [A] time = 0.72, size = 35, normalized size = 0.62

$$-\frac{1}{16}(2x-3)\sqrt{x+1}\sqrt{x}+\frac{1}{16}\left(8x^2-3\right)\log\left(\sqrt{x+1}+\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(x^(1/2)),x, algorithm="fricas")
[Out] -1/16*(2*x - 3)*sqrt(x + 1)*sqrt(x) + 1/16*(8*x^2 - 3)*log(sqrt(x + 1) + sq
rt(x))
```

giac [A] time = 0.48, size = 48, normalized size = 0.86

$$\frac{1}{2}x^2\log\left(\sqrt{x+1}+\sqrt{x}\right)-\frac{1}{16}\sqrt{x^2+x}(2x-3)+\frac{3}{32}\log\left(\left|-2x+2\sqrt{x^2+x}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(x^(1/2)),x, algorithm="giac")
[Out] 1/2*x^2*log(sqrt(x + 1) + sqrt(x)) - 1/16*sqrt(x^2 + x)*(2*x - 3) + 3/32*lo
g(abs(-2*x + 2*sqrt(x^2 + x) - 1))
```

maple [A] time = 0.00, size = 37, normalized size = 0.66

$$-\frac{3\,\text{arcsinh}\left(\sqrt{x}\right)}{16}+\frac{x^2\,\text{arcsinh}\left(\sqrt{x}\right)}{2}-\frac{x^{\frac{3}{2}}\sqrt{1+x}}{8}+\frac{3\sqrt{x}\,\sqrt{1+x}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsinh(x^(1/2)),x)
```

```
[Out] -3/16*arcsinh(x^(1/2))+1/2*x^2*arcsinh(x^(1/2))-1/8*x^(3/2)*(1+x)^(1/2)+3/1
6*x^(1/2)*(1+x)^(1/2)
```

maxima [A] time = 0.79, size = 36, normalized size = 0.64

$$\frac{1}{2}\,x^2\operatorname{arsinh}\left(\sqrt{x}\right) - \frac{1}{8}\,\sqrt{x+1}\,x^{\frac{3}{2}} + \frac{3}{16}\,\sqrt{x+1}\,\sqrt{x} - \frac{3}{16}\,\operatorname{arsinh}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(x^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arcsinh(sqrt(x)) - 1/8*sqrt(x + 1)*x^(3/2) + 3/16*sqrt(x + 1)*sqrt(
x) - 3/16*arcsinh(sqrt(x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\operatorname{asinh}\left(\sqrt{x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*asinh(x^(1/2)),x)
```

```
[Out] int(x*asinh(x^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\operatorname{asinh}\left(\sqrt{x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asinh(x**(1/2)),x)
```

```
[Out] Integral(x*asinh(sqrt(x)), x)
```

3.294 $\int \sinh^{-1}\left(\sqrt{x}\right) dx$

Optimal. Leaf size=35

$$-\frac{1}{2}\sqrt{x}\sqrt{x+1} + x\sinh^{-1}\left(\sqrt{x}\right) + \frac{1}{2}\sinh^{-1}\left(\sqrt{x}\right)$$

```
[Out] 1/2*arcsinh(x^(1/2))+x*arcsinh(x^(1/2))-1/2*x^(1/2)*(1+x)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5900, 12, 1958, 50, 54, 215}

$$-\frac{1}{2}\sqrt{x}\sqrt{x+1} + x\sinh^{-1}\left(\sqrt{x}\right) + \frac{1}{2}\sinh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[Sqrt[x]],x]
```

```
[Out] -(Sqrt[x]*Sqrt[1 + x])/2 + ArcSinh[Sqrt[x]]/2 + x*ArcSinh[Sqrt[x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 1958

```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[(u*(e*(a + b*x^n))^p)/(c + d*x^n)^p, x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - (a*d)/b, 0]
```

Rule 5900

```
Int[ArcSinh[u_], x_Symbol] :> Simp[x*ArcSinh[u], x] - Int[SimplifyIntegrand
[(x*D[u, x])/Sqrt[1 + u^2], x], x] /; InverseFunctionFreeQ[u, x] &&  !Funct
ionOfExponentialQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \sinh^{-1}\left(\sqrt{x}\right)dx &= x\sinh^{-1}\left(\sqrt{x}\right) - \int \frac{1}{2}\sqrt{\frac{x}{1+x}}\,dx\\ &= x\sinh^{-1}\left(\sqrt{x}\right) - \frac{1}{2}\int \sqrt{\frac{x}{1+x}}\,dx\\ &= x\sinh^{-1}\left(\sqrt{x}\right) - \frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{1+x}}\,dx\\ &= -\frac{1}{2}\sqrt{x}\sqrt{1+x} + x\sinh^{-1}\left(\sqrt{x}\right) + \frac{1}{4}\int \frac{1}{\sqrt{x}\sqrt{1+x}}\,dx\\ &= -\frac{1}{2}\sqrt{x}\sqrt{1+x} + x\sinh^{-1}\left(\sqrt{x}\right) + \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \sqrt{x}\right)\\ &= -\frac{1}{2}\sqrt{x}\sqrt{1+x} + \frac{1}{2}\sinh^{-1}\left(\sqrt{x}\right) + x\sinh^{-1}\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A] time = 0.04, size = 33, normalized size = 0.94

$$\frac{1}{2}\left((2x+1)\sinh^{-1}\left(\sqrt{x}\right) - \sqrt{\frac{x}{x+1}}\,(x+1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[Sqrt[x]],x]
[Out] (-(Sqrt[x/(1 + x)]*(1 + x)) + (1 + 2*x)*ArcSinh[Sqrt[x]])/2
```

fricas [A] time = 0.77, size = 28, normalized size = 0.80

$$\frac{1}{2}\,(2\,x+1)\log\left(\sqrt{x+1} + \sqrt{x}\right) - \frac{1}{2}\,\sqrt{x+1}\,\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x^(1/2)),x, algorithm="fricas")
[Out] 1/2*(2*x + 1)*log(sqrt(x + 1) + sqrt(x)) - 1/2*sqrt(x + 1)*sqrt(x)
```

giac [A] time = 0.52, size = 40, normalized size = 1.14

$$x\log\left(\sqrt{x+1} + \sqrt{x}\right) - \frac{1}{2}\,\sqrt{x^2+x} - \frac{1}{4}\,\log\left(\left|-2\,x + 2\,\sqrt{x^2+x} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x^(1/2)),x, algorithm="giac")
[Out] x*log(sqrt(x + 1) + sqrt(x)) - 1/2*sqrt(x^2 + x) - 1/4*log(abs(-2*x + 2*sqr
t(x^2 + x) - 1))
```

maple [A] time = 0.00, size = 24, normalized size = 0.69

$$\frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{2} + x\operatorname{arcsinh}\left(\sqrt{x}\right) - \frac{\sqrt{x}\,\sqrt{1+x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(x^(1/2)),x)
[Out] 1/2*arcsinh(x^(1/2))+x*arcsinh(x^(1/2))-1/2*x^(1/2)*(1+x)^(1/2)
```

maxima [A] time = 0.85, size = 23, normalized size = 0.66

$$x\,\mathrm{arsinh}\left(\sqrt{x}\right)-\frac{1}{2}\,\sqrt{x+1}\,\sqrt{x}+\frac{1}{2}\,\mathrm{arsinh}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x^(1/2)),x, algorithm="maxima")
```

```
[Out] x*arcsinh(sqrt(x)) - 1/2*sqrt(x + 1)*sqrt(x) + 1/2*arcsinh(sqrt(x))
```

mupad [B] time = 0.92, size = 31, normalized size = 0.89

$$\mathrm{atanh}\left(\frac{\sqrt{x}}{\sqrt{x+1}-1}\right)+x\,\mathrm{asinh}\left(\sqrt{x}\right)-\frac{\sqrt{x}\,\sqrt{x+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(x^(1/2)),x)
```

```
[Out] atanh(x^(1/2)/((x + 1)^(1/2) - 1)) + x*asinh(x^(1/2)) - (x^(1/2)*(x + 1)^(1
/2))/2
```

sympy [A] time = 0.30, size = 29, normalized size = 0.83

$$-\frac{\sqrt{x}\,\sqrt{x+1}}{2}+x\,\mathrm{asinh}\left(\sqrt{x}\right)+\frac{\mathrm{asinh}\left(\sqrt{x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(x**(1/2)),x)
```

```
[Out] -sqrt(x)*sqrt(x + 1)/2 + x*asinh(sqrt(x)) + asinh(sqrt(x))/2
```

3.295 $\int \frac{\sinh^{-1}\left(\sqrt{x}\right)}{x}\,dx$

Optimal. Leaf size=46

$$\text{Li}_2\left(e^{2\sinh^{-1}\left(\sqrt{x}\right)}\right)-\sinh^{-1}\left(\sqrt{x}\right)^2+2\sinh^{-1}\left(\sqrt{x}\right)\log\left(1-e^{2\sinh^{-1}\left(\sqrt{x}\right)}\right)$$

```
[Out] -arcsinh(x^(1/2))^2+2*arcsinh(x^(1/2))*ln(1-(x^(1/2)+(1+x)^(1/2))^2)+polylo
g(2,(x^(1/2)+(1+x)^(1/2))^2)
```

Rubi [A] time = 0.06, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5890, 3716, 2190, 2279, 2391}

$$\text{PolyLog}\left(2,e^{2\sinh^{-1}\left(\sqrt{x}\right)}\right)-\sinh^{-1}\left(\sqrt{x}\right)^2+2\sinh^{-1}\left(\sqrt{x}\right)\log\left(1-e^{2\sinh^{-1}\left(\sqrt{x}\right)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[Sqrt[x]]/x,x]
```

```
[Out] -ArcSinh[Sqrt[x]]^2 + 2*ArcSinh[Sqrt[x]]*Log[1 - E^(2*ArcSinh[Sqrt[x]])] +
PolyLog[2, E^(2*ArcSinh[Sqrt[x]])]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 5890

```
Int[ArcSinh[(a_.)*(x_)^(p_)]^(n_.)/(x_), x_Symbol] :> Dist[1/p, Subst[Int[x
^n*Coth[x], x], x, ArcSinh[a*x^p]], x] /; FreeQ[{a, p}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sinh^{-1}\left(\sqrt{x}\right)}{x}\,dx = 2\,\text{Subst}\left(\int x\coth(x)\,dx, x, \sinh^{-1}\left(\sqrt{x}\right)\right)$$
$$= -\sinh^{-1}\left(\sqrt{x}\right)^2 - 4\,\text{Subst}\left(\int \frac{e^{2x}x}{1-e^{2x}}\,dx, x, \sinh^{-1}\left(\sqrt{x}\right)\right)$$
$$= -\sinh^{-1}\left(\sqrt{x}\right)^2 + 2\sinh^{-1}\left(\sqrt{x}\right)\log\left(1-e^{2\sinh^{-1}\left(\sqrt{x}\right)}\right) - 2\,\text{Subst}\left(\int \log\left(1-e^{2x}\right)\,dx, x, \text{si}$$
$$= -\sinh^{-1}\left(\sqrt{x}\right)^2 + 2\sinh^{-1}\left(\sqrt{x}\right)\log\left(1-e^{2\sinh^{-1}\left(\sqrt{x}\right)}\right) - \text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2\sinh^{-1}}$$
$$= -\sinh^{-1}\left(\sqrt{x}\right)^2 + 2\sinh^{-1}\left(\sqrt{x}\right)\log\left(1-e^{2\sinh^{-1}\left(\sqrt{x}\right)}\right) + \text{Li}_2\left(e^{2\sinh^{-1}\left(\sqrt{x}\right)}\right)$$

Mathematica [A] time = 0.01, size = 46, normalized size = 1.00

$$\text{Li}_2\left(e^{2\sinh^{-1}\left(\sqrt{x}\right)}\right) - \sinh^{-1}\left(\sqrt{x}\right)^2 + 2\sinh^{-1}\left(\sqrt{x}\right)\log\left(1-e^{2\sinh^{-1}\left(\sqrt{x}\right)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[Sqrt[x]]/x,x]
[Out] -ArcSinh[Sqrt[x]]^2 + 2*ArcSinh[Sqrt[x]]*Log[1 - E^(2*ArcSinh[Sqrt[x]])] +
PolyLog[2, E^(2*ArcSinh[Sqrt[x]])]
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{arsinh}\left(\sqrt{x}\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x^(1/2))/x,x, algorithm="fricas")
[Out] integral(arcsinh(sqrt(x))/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{arsinh}\left(\sqrt{x}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x^(1/2))/x,x, algorithm="giac")
[Out] integrate(arcsinh(sqrt(x))/x, x)
```

maple [A] time = 0.07, size = 78, normalized size = 1.70

$$-\text{arcsinh}\left(\sqrt{x}\right)^2 + 2\,\text{arcsinh}\left(\sqrt{x}\right)\ln\left(1+\sqrt{x}+\sqrt{1+x}\right) + 2\,\text{polylog}\left(2, -\sqrt{x}-\sqrt{1+x}\right) + 2\,\text{arcsinh}\left(\sqrt{x}\right)\ln\left(1-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(x^(1/2))/x,x)
[Out] -arcsinh(x^(1/2))^2+2*arcsinh(x^(1/2))*ln(1+x^(1/2)+(1+x)^(1/2))+2*polylog(
2,-x^(1/2)-(1+x)^(1/2))+2*arcsinh(x^(1/2))*ln(1-x^(1/2)-(1+x)^(1/2))+2*poly
log(2,x^(1/2)+(1+x)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}\left(\sqrt{x}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x^(1/2))/x,x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(sqrt(x))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{asinh}\left(\sqrt{x}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(x^(1/2))/x,x)
```

```
[Out] int(asinh(x^(1/2))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}\left(\sqrt{x}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(x**(1/2))/x,x)
```

```
[Out] Integral(asinh(sqrt(x))/x, x)
```

3.296 $\int \frac{\sinh^{-1}(\sqrt{x})}{x^2}\,dx$

Optimal. Leaf size=26

$$-\frac{\sqrt{x+1}}{\sqrt{x}}-\frac{\sinh^{-1}\left(\sqrt{x}\right)}{x}$$

```
[Out] -arcsinh(x^(1/2))/x-(1+x)^(1/2)/x^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5902, 12, 37}

$$-\frac{\sqrt{x+1}}{\sqrt{x}}-\frac{\sinh^{-1}\left(\sqrt{x}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[Sqrt[x]]/x^2,x]
```

```
[Out] -(Sqrt[1 + x]/Sqrt[x]) - ArcSinh[Sqrt[x]]/x
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 5902

```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcSinh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/Sqrt[1 + u^2], x], x
], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
 x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u
, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^{-1}\left(\sqrt{x}\right)}{x^2}\,dx &= -\frac{\sinh^{-1}\left(\sqrt{x}\right)}{x}+\int \frac{1}{2x^{3/2}\sqrt{1+x}}\,dx\\
&= -\frac{\sinh^{-1}\left(\sqrt{x}\right)}{x}+\frac{1}{2}\int \frac{1}{x^{3/2}\sqrt{1+x}}\,dx\\
&= -\frac{\sqrt{1+x}}{\sqrt{x}}-\frac{\sinh^{-1}\left(\sqrt{x}\right)}{x}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 1.00

$$-\frac{\sqrt{x+1}}{\sqrt{x}}-\frac{\sinh^{-1}\left(\sqrt{x}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[Sqrt[x]]/x^2,x]
[Out] -(Sqrt[1 + x]/Sqrt[x]) - ArcSinh[Sqrt[x]]/x
```

fricas [A] time = 0.55, size = 25, normalized size = 0.96

$$-\frac{\sqrt{x+1}\sqrt{x}+\log\left(\sqrt{x+1}+\sqrt{x}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x^(1/2))/x^2,x, algorithm="fricas")
[Out] -(sqrt(x + 1)*sqrt(x) + log(sqrt(x + 1) + sqrt(x)))/x
```

giac [A] time = 0.37, size = 35, normalized size = 1.35

$$-\frac{\log\left(\sqrt{x+1}+\sqrt{x}\right)}{x}+\frac{2}{\left(\sqrt{x+1}-\sqrt{x}\right)^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x^(1/2))/x^2,x, algorithm="giac")
[Out] -log(sqrt(x + 1) + sqrt(x))/x + 2/((sqrt(x + 1) - sqrt(x))^2 - 1)
```

maple [A] time = 0.00, size = 21, normalized size = 0.81

$$-\frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{x}-\frac{\sqrt{1+x}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(x^(1/2))/x^2,x)
[Out] -arcsinh(x^(1/2))/x-(1+x)^(1/2)/x^(1/2)
```

maxima [A] time = 0.86, size = 20, normalized size = 0.77

$$-\frac{\sqrt{x+1}}{\sqrt{x}}-\frac{\operatorname{arsinh}\left(\sqrt{x}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x^(1/2))/x^2,x, algorithm="maxima")
[Out] -sqrt(x + 1)/sqrt(x) - arcsinh(sqrt(x))/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\operatorname{asinh}\left(\sqrt{x}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(x^(1/2))/x^2,x)
[Out] int(asinh(x^(1/2))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}\left(\sqrt{x}\right)}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(x**(1/2))/x**2,x)
```

```
[Out] Integral(asinh(sqrt(x))/x**2, x)
```

3.297 $\int \frac{\sinh^{-1}(\sqrt{x})}{x^3} \, dx$

Optimal. Leaf size=46

$$-\frac{\sqrt{x+1}}{6x^{3/2}}-\frac{\sinh^{-1}\left(\sqrt{x}\right)}{2x^2}+\frac{\sqrt{x+1}}{3\sqrt{x}}$$

```
[Out] -1/2*arcsinh(x^(1/2))/x^2-1/6*(1+x)^(1/2)/x^(3/2)+1/3*(1+x)^(1/2)/x^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5902, 12, 45, 37}

$$-\frac{\sqrt{x+1}}{6x^{3/2}}-\frac{\sinh^{-1}\left(\sqrt{x}\right)}{2x^2}+\frac{\sqrt{x+1}}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[Sqrt[x]]/x^3,x]
```

```
[Out] -Sqrt[1 + x]/(6*x^(3/2)) + Sqrt[1 + x]/(3*Sqrt[x]) - ArcSinh[Sqrt[x]]/(2*x^
2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 5902

```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcSinh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/Sqrt[1 + u^2], x], x
], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
 x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u
, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^{-1}\left(\sqrt{x}\right)}{x^3}\,dx &= -\frac{\sinh^{-1}\left(\sqrt{x}\right)}{2x^2}+\frac{1}{2}\int \frac{1}{2x^{5/2}\sqrt{1+x}}\,dx\\ &= -\frac{\sinh^{-1}\left(\sqrt{x}\right)}{2x^2}+\frac{1}{4}\int \frac{1}{x^{5/2}\sqrt{1+x}}\,dx\\ &= -\frac{\sqrt{1+x}}{6x^{3/2}}-\frac{\sinh^{-1}\left(\sqrt{x}\right)}{2x^2}-\frac{1}{6}\int \frac{1}{x^{3/2}\sqrt{1+x}}\,dx\\ &= -\frac{\sqrt{1+x}}{6x^{3/2}}+\frac{\sqrt{1+x}}{3\sqrt{x}}-\frac{\sinh^{-1}\left(\sqrt{x}\right)}{2x^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 34, normalized size = 0.74

$$\frac{\sqrt{x}\sqrt{x+1}(2x-1)-3\sinh^{-1}\left(\sqrt{x}\right)}{6x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[Sqrt[x]]/x^3,x]
[Out] (Sqrt[x]*Sqrt[1 + x]*(-1 + 2*x) - 3*ArcSinh[Sqrt[x]])/(6*x^2)
```

fricas [A] time = 0.57, size = 32, normalized size = 0.70

$$\frac{(2\,x-1)\sqrt{x+1}\,\sqrt{x}-3\,\log\left(\sqrt{x+1}+\sqrt{x}\right)}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x^(1/2))/x^3,x, algorithm="fricas")
[Out] 1/6*((2*x - 1)*sqrt(x + 1)*sqrt(x) - 3*log(sqrt(x + 1) + sqrt(x)))/x^2
```

giac [A] time = 0.32, size = 52, normalized size = 1.13

$$-\frac{\log\left(\sqrt{x+1}+\sqrt{x}\right)}{2\,x^2}+\frac{2\left(3\left(\sqrt{x+1}-\sqrt{x}\right)^2-1\right)}{3\left(\left(\sqrt{x+1}-\sqrt{x}\right)^2-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x^(1/2))/x^3,x, algorithm="giac")
[Out] -1/2*log(sqrt(x + 1) + sqrt(x))/x^2 + 2/3*(3*(sqrt(x + 1) - sqrt(x))^2 - 1)
/((sqrt(x + 1) - sqrt(x))^2 - 1)^3
```

maple [A] time = 0.00, size = 31, normalized size = 0.67

$$-\frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{2x^2}-\frac{\sqrt{1+x}}{6x^{\frac{3}{2}}}+\frac{\sqrt{1+x}}{3\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(x^(1/2))/x^3,x)
[Out] -1/2*arcsinh(x^(1/2))/x^2-1/6*(1+x)^(1/2)/x^(3/2)+1/3*(1+x)^(1/2)/x^(1/2)
```

maxima [A] time = 0.84, size = 30, normalized size = 0.65

$$\frac{\sqrt{x+1}}{3\sqrt{x}} - \frac{\sqrt{x+1}}{6x^{\frac{3}{2}}} - \frac{\operatorname{arsinh}\left(\sqrt{x}\right)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x^(1/2))/x^3,x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(x + 1)/sqrt(x) - 1/6*sqrt(x + 1)/x^(3/2) - 1/2*arcsinh(sqrt(x))/x^
2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{asinh}\left(\sqrt{x}\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(x^(1/2))/x^3,x)
```

```
[Out] int(asinh(x^(1/2))/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}\left(\sqrt{x}\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(x**(1/2))/x**3,x)
```

```
[Out] Integral(asinh(sqrt(x))/x**3, x)
```

3.298 $\int \frac{\sinh^{-1}(\sqrt{x})}{x^4} dx$

Optimal. Leaf size=62

$$\frac{4\sqrt{x+1}}{45x^{3/2}}-\frac{\sqrt{x+1}}{15x^{5/2}}-\frac{\sinh^{-1}(\sqrt{x})}{3x^3}-\frac{8\sqrt{x+1}}{45\sqrt{x}}$$

```
[Out] -1/3*arcsinh(x^(1/2))/x^3-1/15*(1+x)^(1/2)/x^(5/2)+4/45*(1+x)^(1/2)/x^(3/2)
-8/45*(1+x)^(1/2)/x^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5902, 12, 45, 37}

$$\frac{4\sqrt{x+1}}{45x^{3/2}}-\frac{\sqrt{x+1}}{15x^{5/2}}-\frac{\sinh^{-1}(\sqrt{x})}{3x^3}-\frac{8\sqrt{x+1}}{45\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[Sqrt[x]]/x^4,x]
```

```
[Out] -Sqrt[1 + x]/(15*x^(5/2)) + (4*Sqrt[1 + x])/(45*x^(3/2)) - (8*Sqrt[1 + x])/
(45*Sqrt[x]) - ArcSinh[Sqrt[x]]/(3*x^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] && !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] || !SumSimplerQ[n, 1])
```

Rule 5902

```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcSinh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/Sqrt[1 + u^2], x], x
], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
 x] && !FunctionOfQ[(c + d*x)^(m + 1), u, x] && !FunctionOfExponentialQ[u
, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh^{-1}\left(\sqrt{x}\right)}{x^4}\,dx &= -\frac{\sinh^{-1}\left(\sqrt{x}\right)}{3x^3} + \frac{1}{3}\int \frac{1}{2x^{7/2}\sqrt{1+x}}\,dx \\
&= -\frac{\sinh^{-1}\left(\sqrt{x}\right)}{3x^3} + \frac{1}{6}\int \frac{1}{x^{7/2}\sqrt{1+x}}\,dx \\
&= -\frac{\sqrt{1+x}}{15x^{5/2}} - \frac{\sinh^{-1}\left(\sqrt{x}\right)}{3x^3} - \frac{2}{15}\int \frac{1}{x^{5/2}\sqrt{1+x}}\,dx \\
&= -\frac{\sqrt{1+x}}{15x^{5/2}} + \frac{4\sqrt{1+x}}{45x^{3/2}} - \frac{\sinh^{-1}\left(\sqrt{x}\right)}{3x^3} + \frac{4}{45}\int \frac{1}{x^{3/2}\sqrt{1+x}}\,dx \\
&= -\frac{\sqrt{1+x}}{15x^{5/2}} + \frac{4\sqrt{1+x}}{45x^{3/2}} - \frac{8\sqrt{1+x}}{45\sqrt{x}} - \frac{\sinh^{-1}\left(\sqrt{x}\right)}{3x^3}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 39, normalized size = 0.63

$$\frac{\sqrt{x}\sqrt{x+1}\left(-8x^2+4x-3\right)-15\sinh^{-1}\left(\sqrt{x}\right)}{45x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[Sqrt[x]]/x^4,x]
[Out] (Sqrt[x]*Sqrt[1 + x]*(-3 + 4*x - 8*x^2) - 15*ArcSinh[Sqrt[x]])/(45*x^3)
```

fricas [A] time = 0.61, size = 37, normalized size = 0.60

$$-\frac{\left(8\,x^2-4\,x+3\right)\sqrt{x+1}\,\sqrt{x}\,+15\,\log\left(\sqrt{x+1}\,+\sqrt{x}\right)}{45\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x^(1/2))/x^4,x, algorithm="fricas")
[Out] -1/45*((8*x^2 - 4*x + 3)*sqrt(x + 1)*sqrt(x) + 15*log(sqrt(x + 1) + sqrt(x)
))/x^3
```

giac [A] time = 0.37, size = 67, normalized size = 1.08

$$-\frac{\log\left(\sqrt{x+1}\,+\sqrt{x}\right)}{3\,x^3}+\frac{16\left(10\left(\sqrt{x+1}\,-\sqrt{x}\right)^4-5\left(\sqrt{x+1}\,-\sqrt{x}\right)^2+1\right)}{45\left(\left(\sqrt{x+1}\,-\sqrt{x}\right)^2-1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x^(1/2))/x^4,x, algorithm="giac")
[Out] -1/3*log(sqrt(x + 1) + sqrt(x))/x^3 + 16/45*(10*(sqrt(x + 1) - sqrt(x))^4 -
 5*(sqrt(x + 1) - sqrt(x))^2 + 1)/((sqrt(x + 1) - sqrt(x))^2 - 1)^5
```

maple [A] time = 0.00, size = 41, normalized size = 0.66

$$-\frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{3x^3}-\frac{\sqrt{1+x}}{15x^{\frac{5}{2}}}+\frac{4\sqrt{1+x}}{45x^{\frac{3}{2}}}-\frac{8\sqrt{1+x}}{45\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(x^(1/2))/x^4,x)
```

```
[Out] -1/3*arcsinh(x^(1/2))/x^3-1/15*(1+x)^(1/2)/x^(5/2)+4/45*(1+x)^(1/2)/x^(3/2)
-8/45*(1+x)^(1/2)/x^(1/2)
```

maxima [A] time = 0.89, size = 40, normalized size = 0.65

$$-\frac{8\sqrt{x+1}}{45\sqrt{x}}+\frac{4\sqrt{x+1}}{45\,x^{\frac{3}{2}}}-\frac{\sqrt{x+1}}{15\,x^{\frac{5}{2}}}-\frac{\operatorname{arsinh}\left(\sqrt{x}\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x^(1/2))/x^4,x, algorithm="maxima")
```

```
[Out] -8/45*sqrt(x + 1)/sqrt(x) + 4/45*sqrt(x + 1)/x^(3/2) - 1/15*sqrt(x + 1)/x^(
5/2) - 1/3*arcsinh(sqrt(x))/x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{asinh}\left(\sqrt{x}\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(x^(1/2))/x^4,x)
```

```
[Out] int(asinh(x^(1/2))/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}\left(\sqrt{x}\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(x**(1/2))/x**4,x)
```

```
[Out] Integral(asinh(sqrt(x))/x**4, x)
```

3.299 $\int \frac{\sinh^{-1}(\sqrt{x})}{x^5} dx$

Optimal. Leaf size=78

$$-\frac{2\sqrt{x+1}}{35x^{3/2}}+\frac{3\sqrt{x+1}}{70x^{5/2}}-\frac{\sqrt{x+1}}{28x^{7/2}}-\frac{\sinh^{-1}(\sqrt{x})}{4x^4}+\frac{4\sqrt{x+1}}{35\sqrt{x}}$$

```
[Out] -1/4*arcsinh(x^(1/2))/x^4-1/28*(1+x)^(1/2)/x^(7/2)+3/70*(1+x)^(1/2)/x^(5/2)
-2/35*(1+x)^(1/2)/x^(3/2)+4/35*(1+x)^(1/2)/x^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5902, 12, 45, 37}

$$-\frac{2\sqrt{x+1}}{35x^{3/2}}+\frac{3\sqrt{x+1}}{70x^{5/2}}-\frac{\sqrt{x+1}}{28x^{7/2}}-\frac{\sinh^{-1}(\sqrt{x})}{4x^4}+\frac{4\sqrt{x+1}}{35\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[Sqrt[x]]/x^5,x]
```

```
[Out] -Sqrt[1 + x]/(28*x^(7/2)) + (3*Sqrt[1 + x])/(70*x^(5/2)) - (2*Sqrt[1 + x])/
(35*x^(3/2)) + (4*Sqrt[1 + x])/(35*Sqrt[x]) - ArcSinh[Sqrt[x]]/(4*x^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 5902

```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcSinh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/Sqrt[1 + u^2], x], x
], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
 x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u
, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh^{-1}\left(\sqrt{x}\right)}{x^5}\,dx &= -\frac{\sinh^{-1}\left(\sqrt{x}\right)}{4x^4} + \frac{1}{4}\int \frac{1}{2x^{9/2}\sqrt{1+x}}\,dx \\
&= -\frac{\sinh^{-1}\left(\sqrt{x}\right)}{4x^4} + \frac{1}{8}\int \frac{1}{x^{9/2}\sqrt{1+x}}\,dx \\
&= -\frac{\sqrt{1+x}}{28x^{7/2}} - \frac{\sinh^{-1}\left(\sqrt{x}\right)}{4x^4} - \frac{3}{28}\int \frac{1}{x^{7/2}\sqrt{1+x}}\,dx \\
&= -\frac{\sqrt{1+x}}{28x^{7/2}} + \frac{3\sqrt{1+x}}{70x^{5/2}} - \frac{\sinh^{-1}\left(\sqrt{x}\right)}{4x^4} + \frac{3}{35}\int \frac{1}{x^{5/2}\sqrt{1+x}}\,dx \\
&= -\frac{\sqrt{1+x}}{28x^{7/2}} + \frac{3\sqrt{1+x}}{70x^{5/2}} - \frac{2\sqrt{1+x}}{35x^{3/2}} - \frac{\sinh^{-1}\left(\sqrt{x}\right)}{4x^4} - \frac{2}{35}\int \frac{1}{x^{3/2}\sqrt{1+x}}\,dx \\
&= -\frac{\sqrt{1+x}}{28x^{7/2}} + \frac{3\sqrt{1+x}}{70x^{5/2}} - \frac{2\sqrt{1+x}}{35x^{3/2}} + \frac{4\sqrt{1+x}}{35\sqrt{x}} - \frac{\sinh^{-1}\left(\sqrt{x}\right)}{4x^4}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 44, normalized size = 0.56

$$
\frac{\sqrt{x}\sqrt{x+1}\left(16x^3-8x^2+6x-5\right)-35\sinh^{-1}\left(\sqrt{x}\right)}{140x^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[Sqrt[x]]/x^5,x]
```

```
[Out] (Sqrt[x]*Sqrt[1 + x]*(-5 + 6*x - 8*x^2 + 16*x^3) - 35*ArcSinh[Sqrt[x]])/(14
0*x^4)
```

fricas [A] time = 0.59, size = 42, normalized size = 0.54

$$
\frac{\left(16\,x^3-8\,x^2+6\,x-5\right)\sqrt{x+1}\,\sqrt{x}-35\,\log\left(\sqrt{x+1}+\sqrt{x}\right)}{140\,x^4}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x^(1/2))/x^5,x, algorithm="fricas")
```

```
[Out] 1/140*((16*x^3 - 8*x^2 + 6*x - 5)*sqrt(x + 1)*sqrt(x) - 35*log(sqrt(x + 1)
+ sqrt(x)))/x^4
```

giac [A] time = 0.76, size = 82, normalized size = 1.05

$$
-\frac{\log\left(\sqrt{x+1}+\sqrt{x}\right)}{4\,x^4}+\frac{8\left(35\left(\sqrt{x+1}-\sqrt{x}\right)^6-21\left(\sqrt{x+1}-\sqrt{x}\right)^4+7\left(\sqrt{x+1}-\sqrt{x}\right)^2-1\right)}{35\left(\left(\sqrt{x+1}-\sqrt{x}\right)^2-1\right)^7}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x^(1/2))/x^5,x, algorithm="giac")
```

```
[Out] -1/4*log(sqrt(x + 1) + sqrt(x))/x^4 + 8/35*(35*(sqrt(x + 1) - sqrt(x))^6 -
21*(sqrt(x + 1) - sqrt(x))^4 + 7*(sqrt(x + 1) - sqrt(x))^2 - 1)/((sqrt(x +
1) - sqrt(x))^2 - 1)^7
```

maple [A] time = 0.01, size = 51, normalized size = 0.65

$$
-\frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{4x^4}-\frac{\sqrt{1+x}}{28x^{\frac{7}{2}}}+\frac{3\sqrt{1+x}}{70x^{\frac{5}{2}}}-\frac{2\sqrt{1+x}}{35x^{\frac{3}{2}}}+\frac{4\sqrt{1+x}}{35\sqrt{x}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(x^(1/2))/x^5,x)
```

```
[Out] -1/4*arcsinh(x^(1/2))/x^4-1/28*(1+x)^(1/2)/x^(7/2)+3/70*(1+x)^(1/2)/x^(5/2)
-2/35*(1+x)^(1/2)/x^(3/2)+4/35*(1+x)^(1/2)/x^(1/2)
```

maxima [A] time = 0.86, size = 50, normalized size = 0.64

$$\frac{4\sqrt{x+1}}{35\sqrt{x}} - \frac{2\sqrt{x+1}}{35\,x^{\frac{3}{2}}} + \frac{3\sqrt{x+1}}{70\,x^{\frac{5}{2}}} - \frac{\sqrt{x+1}}{28\,x^{\frac{7}{2}}} - \frac{\operatorname{arsinh}\left(\sqrt{x}\right)}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x^(1/2))/x^5,x, algorithm="maxima")
```

```
[Out] 4/35*sqrt(x + 1)/sqrt(x) - 2/35*sqrt(x + 1)/x^(3/2) + 3/70*sqrt(x + 1)/x^(5
/2) - 1/28*sqrt(x + 1)/x^(7/2) - 1/4*arcsinh(sqrt(x))/x^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{asinh}\left(\sqrt{x}\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(x^(1/2))/x^5,x)
```

```
[Out] int(asinh(x^(1/2))/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}\left(\sqrt{x}\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(x**(1/2))/x**5,x)
```

```
[Out] Integral(asinh(sqrt(x))/x**5, x)
```

3.300 $\int x^2 \sinh^{-1}\left(\frac{a}{x}\right) dx$

Optimal. Leaf size=56

$$\frac{1}{6}ax^2\sqrt{\frac{a^2}{x^2}+1}-\frac{1}{6}a^3\tanh^{-1}\left(\sqrt{\frac{a^2}{x^2}+1}\right)+\frac{1}{3}x^3\text{csch}^{-1}\left(\frac{x}{a}\right)$$

```
[Out] 1/3*x^3*arccsch(x/a)-1/6*a^3*arctanh((1+a^2/x^2)^(1/2))+1/6*a*x^2*(1+a^2/x^
2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5892, 6284, 266, 51, 63, 208}

$$\frac{1}{6}ax^2\sqrt{\frac{a^2}{x^2}+1}-\frac{1}{6}a^3\tanh^{-1}\left(\sqrt{\frac{a^2}{x^2}+1}\right)+\frac{1}{3}x^3\text{csch}^{-1}\left(\frac{x}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcSinh[a/x],x]
```

```
[Out] (a*Sqrt[1 + a^2/x^2]*x^2)/6 + (x^3*ArcCsch[x/a])/3 - (a^3*ArcTanh[Sqrt[1 +
a^2/x^2]])/6
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5892

```
Int[ArcSinh[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int
[u*ArcCsch[a/c + (b*x^n)/c]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rule 6284

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((d*x)^(m + 1)*(a + b*ArcCsch[c*x]))/(d*(m + 1)), x] + Dist[(b*d)/(c*(m
+ 1)), Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int x^2\sinh^{-1}\left(\frac{a}{x}\right)dx &= \int x^2\operatorname{csch}^{-1}\left(\frac{x}{a}\right)dx\\
&= \frac{1}{3}x^3\operatorname{csch}^{-1}\left(\frac{x}{a}\right)+\frac{1}{3}a\int\frac{x}{\sqrt{1+\frac{a^2}{x^2}}}\,dx\\
&= \frac{1}{3}x^3\operatorname{csch}^{-1}\left(\frac{x}{a}\right)-\frac{1}{6}a\,\mathrm{Subst}\left(\int\frac{1}{x^2\sqrt{1+a^2x}}\,dx,x,\frac{1}{x^2}\right)\\
&= \frac{1}{6}a\sqrt{1+\frac{a^2}{x^2}}\,x^2+\frac{1}{3}x^3\operatorname{csch}^{-1}\left(\frac{x}{a}\right)+\frac{1}{12}a^3\,\mathrm{Subst}\left(\int\frac{1}{x\sqrt{1+a^2x}}\,dx,x,\frac{1}{x^2}\right)\\
&= \frac{1}{6}a\sqrt{1+\frac{a^2}{x^2}}\,x^2+\frac{1}{3}x^3\operatorname{csch}^{-1}\left(\frac{x}{a}\right)+\frac{1}{6}a\,\mathrm{Subst}\left(\int\frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx,x,\sqrt{1+\frac{a^2}{x^2}}\right)\\
&= \frac{1}{6}a\sqrt{1+\frac{a^2}{x^2}}\,x^2+\frac{1}{3}x^3\operatorname{csch}^{-1}\left(\frac{x}{a}\right)-\frac{1}{6}a^3\tanh^{-1}\left(\sqrt{1+\frac{a^2}{x^2}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 57, normalized size = 1.02

$$\frac{1}{6}\left(ax^2\sqrt{\frac{a^2}{x^2}+1}+a^3\left(-\log\left(x\left(\sqrt{\frac{a^2}{x^2}+1}+1\right)\right)\right)+2x^3\sinh^{-1}\left(\frac{a}{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcSinh[a/x],x]
```

```
[Out] (a*Sqrt[1 + a^2/x^2]*x^2 + 2*x^3*ArcSinh[a/x] - a^3*Log[(1 + Sqrt[1 + a^2/x
^2])*x])/6
```

fricas [B] time = 0.68, size = 122, normalized size = 2.18

$$\frac{1}{6}a^3\log\left(x\sqrt{\frac{a^2+x^2}{x^2}}-x\right)+\frac{1}{6}ax^2\sqrt{\frac{a^2+x^2}{x^2}}+\frac{1}{3}\left(x^3-1\right)\log\left(\frac{x\sqrt{\frac{a^2+x^2}{x^2}}+a}{x}\right)+\frac{1}{3}\log\left(x\sqrt{\frac{a^2+x^2}{x^2}}+a-x\right)-\frac{1}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a/x),x, algorithm="fricas")
```

```
[Out] 1/6*a^3*log(x*sqrt((a^2 + x^2)/x^2) - x) + 1/6*a*x^2*sqrt((a^2 + x^2)/x^2)
+ 1/3*(x^3 - 1)*log((x*sqrt((a^2 + x^2)/x^2) + a)/x) + 1/3*log(x*sqrt((a^2
+ x^2)/x^2) + a - x) - 1/3*log(x*sqrt((a^2 + x^2)/x^2) - a - x)
```

giac [A] time = 0.55, size = 74, normalized size = 1.32

$$-\frac{1}{6}a^3\log\left(|a|\right)\operatorname{sgn}(x)+\frac{1}{3}x^3\log\left(\sqrt{\frac{a^2}{x^2}+1}+\frac{a}{x}\right)+\frac{a^3\log\left(-x+\sqrt{a^2+x^2}\right)}{6\operatorname{sgn}(x)}+\frac{\sqrt{a^2+x^2}\,ax}{6\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a/x),x, algorithm="giac")
```

```
[Out] -1/6*a^3*log(abs(a))*sgn(x) + 1/3*x^3*log(sqrt(a^2/x^2 + 1) + a/x) + 1/6*a^
3*log(-x + sqrt(a^2 + x^2))/sgn(x) + 1/6*sqrt(a^2 + x^2)*a*x/sgn(x)
```

maple [A] time = 0.03, size = 54, normalized size = 0.96

$$-a^{3}\left(-\frac{x^{3}\operatorname{arcsinh}\left(\frac{a}{x}\right)}{3a^{3}}-\frac{x^{2}\sqrt{1+\frac{a^{2}}{x^{2}}}}{6a^{2}}+\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{1+\frac{a^{2}}{x^{2}}}}\right)}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsinh(a/x),x)
```

```
[Out] -a^3*(-1/3/a^3*x^3*arcsinh(a/x)-1/6/a^2*x^2*(1+a^2/x^2)^(1/2)+1/6*arctanh(1
/(1+a^2/x^2)^(1/2)))
```

maxima [A] time = 0.46, size = 69, normalized size = 1.23

$$\frac{1}{3}x^{3}\operatorname{arsinh}\left(\frac{a}{x}\right)-\frac{1}{12}\left(a^{2}\log\left(\sqrt{\frac{a^{2}}{x^{2}}+1}+1\right)-a^{2}\log\left(\sqrt{\frac{a^{2}}{x^{2}}+1}-1\right)-2x^{2}\sqrt{\frac{a^{2}}{x^{2}}+1}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a/x),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arcsinh(a/x) - 1/12*(a^2*log(sqrt(a^2/x^2 + 1) + 1) - a^2*log(sqrt(
a^2/x^2 + 1) - 1) - 2*x^2*sqrt(a^2/x^2 + 1))*a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^{2}\operatorname{asinh}\left(\frac{a}{x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*asinh(a/x),x)
```

```
[Out] int(x^2*asinh(a/x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2}\operatorname{asinh}\left(\frac{a}{x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asinh(a/x),x)
```

```
[Out] Integral(x**2*asinh(a/x), x)
```

3.301 $\int x \sinh^{-1}\left(\frac{a}{x}\right) dx$

Optimal. Leaf size=33

$$\frac{1}{2}ax\sqrt{\frac{a^2}{x^2}+1}+\frac{1}{2}x^2\text{csch}^{-1}\left(\frac{x}{a}\right)$$

```
[Out] 1/2*x^2*arccsch(x/a)+1/2*a*x*(1+a^2/x^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5892, 6284, 191}

$$\frac{1}{2}ax\sqrt{\frac{a^2}{x^2}+1}+\frac{1}{2}x^2\text{csch}^{-1}\left(\frac{x}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcSinh[a/x],x]
```

```
[Out] (a*Sqrt[1 + a^2/x^2]*x)/2 + (x^2*ArcCsch[x/a])/2
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 5892

```
Int[ArcSinh[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int
[u*ArcCsch[a/c + (b*x^n)/c]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rule 6284

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((d*x)^(m + 1)*(a + b*ArcCsch[c*x]))/(d*(m + 1)), x] + Dist[(b*d)/(c*(m
+ 1)), Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x \sinh^{-1}\left(\frac{a}{x}\right) dx &= \int x\text{csch}^{-1}\left(\frac{x}{a}\right) dx\\
&= \frac{1}{2}x^2\text{csch}^{-1}\left(\frac{x}{a}\right)+\frac{1}{2}a\int \frac{1}{\sqrt{1+\frac{a^2}{x^2}}}\,dx\\
&= \frac{1}{2}a\sqrt{1+\frac{a^2}{x^2}}\,x+\frac{1}{2}x^2\text{csch}^{-1}\left(\frac{x}{a}\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 29, normalized size = 0.88

$$\frac{1}{2}x\left(a\sqrt{\frac{a^2}{x^2}+1}+x\sinh^{-1}\left(\frac{a}{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcSinh[a/x],x]
```

```
[Out] (x*(a*Sqrt[1 + a^2/x^2] + x*ArcSinh[a/x]))/2
```

fricas [A] time = 0.87, size = 45, normalized size = 1.36

$$\frac{1}{2}\,x^{2}\log\left(\frac{x\sqrt{\frac{a^{2}+x^{2}}{x^{2}}}+a}{x}\right)+\frac{1}{2}\,ax\sqrt{\frac{a^{2}+x^{2}}{x^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a/x),x, algorithm="fricas")
```

```
[Out] 1/2*x^2*log((x*sqrt((a^2 + x^2)/x^2) + a)/x) + 1/2*a*x*sqrt((a^2 + x^2)/x^2
)
```

giac [A] time = 0.49, size = 47, normalized size = 1.42

$$\frac{1}{2}\,x^{2}\log\left(\sqrt{\frac{a^{2}}{x^{2}}+1}+\frac{a}{x}\right)-\frac{1}{2}\,a|a|\mathrm{sgn}(x)+\frac{\sqrt{a^{2}+x^{2}}\,a}{2\,\mathrm{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a/x),x, algorithm="giac")
```

```
[Out] 1/2*x^2*log(sqrt(a^2/x^2 + 1) + a/x) - 1/2*a*abs(a)*sgn(x) + 1/2*sqrt(a^2 +
 x^2)*a/sgn(x)
```

maple [A] time = 0.00, size = 38, normalized size = 1.15

$$-a^{2}\left(-\frac{x^{2}\,\mathrm{arcsinh}\left(\frac{a}{x}\right)}{2a^{2}}-\frac{x\sqrt{1+\frac{a^{2}}{x^{2}}}}{2a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsinh(a/x),x)
```

```
[Out] -a^2*(-1/2/a^2*x^2*arcsinh(a/x)-1/2/a*x*(1+a^2/x^2)^(1/2))
```

maxima [A] time = 0.32, size = 27, normalized size = 0.82

$$\frac{1}{2}\,x^{2}\,\mathrm{arsinh}\left(\frac{a}{x}\right)+\frac{1}{2}\,ax\sqrt{\frac{a^{2}}{x^{2}}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a/x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arcsinh(a/x) + 1/2*a*x*sqrt(a^2/x^2 + 1)
```

mupad [B] time = 0.03, size = 27, normalized size = 0.82

$$\frac{x^{2}\,\mathrm{asinh}\left(\frac{a}{x}\right)}{2}+\frac{a\,x\,\sqrt{\frac{a^{2}}{x^{2}}+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*asinh(a/x),x)
```

```
[Out] (x^2*asinh(a/x))/2 + (a*x*(a^2/x^2 + 1)^(1/2))/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{asinh}\left(\frac{a}{x}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asinh(a/x),x)
```

```
[Out] Integral(x*asinh(a/x), x)
```

3.302 $\int \sinh^{-1}\left(\frac{a}{x}\right) dx$

Optimal. Leaf size=25

$$a \tanh^{-1}\left(\sqrt{\frac{a^2}{x^2}+1}\right)+x\text{csch}^{-1}\left(\frac{x}{a}\right)$$

[Out] x*arccsch(x/a)+a*arctanh((1+a^2/x^2)^(1/2))

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {5892, 6278, 266, 63, 208}

$$a \tanh^{-1}\left(\sqrt{\frac{a^2}{x^2}+1}\right)+x\text{csch}^{-1}\left(\frac{x}{a}\right)$$

Antiderivative was successfully verified.

[In] Int[ArcSinh[a/x],x]

[Out] x*ArcCsch[x/a] + a*ArcTanh[Sqrt[1 + a^2/x^2]]

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5892

```
Int[ArcSinh[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int
[u*ArcCsch[a/c + (b*x^n)/c]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rule 6278

```
Int[ArcCsch[(c_.)*(x_)], x_Symbol] :> Simp[x*ArcCsch[c*x], x] + Dist[1/c, I
nt[1/(x*Sqrt[1 + 1/(c^2*x^2)]), x], x] /; FreeQ[c, x]
```

Rubi steps

$$
\begin{aligned}
\int \sinh^{-1}\left(\frac{a}{x}\right)dx &= \int \operatorname{csch}^{-1}\left(\frac{x}{a}\right)dx \\
&= x\operatorname{csch}^{-1}\left(\frac{x}{a}\right) + a\int \frac{1}{\sqrt{1+\frac{a^2}{x^2}}\,x}\,dx \\
&= x\operatorname{csch}^{-1}\left(\frac{x}{a}\right) - \frac{1}{2}a\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}}\,dx, x, \frac{1}{x^2}\right) \\
&= x\operatorname{csch}^{-1}\left(\frac{x}{a}\right) - \frac{\mathrm{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx, x, \sqrt{1+\frac{a^2}{x^2}}\right)}{a} \\
&= x\operatorname{csch}^{-1}\left(\frac{x}{a}\right) + a\tanh^{-1}\left(\sqrt{1+\frac{a^2}{x^2}}\right)
\end{aligned}
$$

Mathematica [B] time = 0.10, size = 77, normalized size = 3.08

$$
\frac{a\sqrt{a^2+x^2}\left(\log\left(\frac{x}{\sqrt{a^2+x^2}}+1\right)-\log\left(1-\frac{x}{\sqrt{a^2+x^2}}\right)\right)}{2x\sqrt{\frac{a^2}{x^2}+1}} + x\sinh^{-1}\left(\frac{a}{x}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a/x],x]
```

```
[Out] x*ArcSinh[a/x] + (a*Sqrt[a^2 + x^2]*(-Log[1 - x/Sqrt[a^2 + x^2]] + Log[1 +
x/Sqrt[a^2 + x^2]]))/(2*Sqrt[1 + a^2/x^2]*x)
```

fricas [B] time = 0.50, size = 96, normalized size = 3.84

$$
-a\log\left(x\sqrt{\frac{a^2+x^2}{x^2}}-x\right)+(x-1)\log\left(\frac{x\sqrt{\frac{a^2+x^2}{x^2}}+a}{x}\right)+\log\left(x\sqrt{\frac{a^2+x^2}{x^2}}+a-x\right)-\log\left(x\sqrt{\frac{a^2+x^2}{x^2}}-a-x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a/x),x, algorithm="fricas")
```

```
[Out] -a*log(x*sqrt((a^2 + x^2)/x^2) - x) + (x - 1)*log((x*sqrt((a^2 + x^2)/x^2)
+ a)/x) + log(x*sqrt((a^2 + x^2)/x^2) + a - x) - log(x*sqrt((a^2 + x^2)/x^2
) - a - x)
```

giac [B] time = 0.32, size = 49, normalized size = 1.96

$$
a\log\left(|a|\right)\operatorname{sgn}(x) + x\log\left(\sqrt{\frac{a^2}{x^2}+1}+\frac{a}{x}\right) - \frac{a\log\left(-x+\sqrt{a^2+x^2}\right)}{\operatorname{sgn}(x)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a/x),x, algorithm="giac")
```

```
[Out] a*log(abs(a))*sgn(x) + x*log(sqrt(a^2/x^2 + 1) + a/x) - a*log(-x + sqrt(a^2
 + x^2))/sgn(x)
```

maple [A] time = 0.01, size = 31, normalized size = 1.24

$$
-a\left(-\frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)x}{a} - \operatorname{arctanh}\left(\frac{1}{\sqrt{1+\frac{a^2}{x^2}}}\right)\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a/x),x)
```

```
[Out] -a*(-arcsinh(a/x)/a*x-arctanh(1/(1+a^2/x^2)^(1/2)))
```

maxima [A] time = 0.63, size = 43, normalized size = 1.72

$$\frac{1}{2}a\left(\log\left(\sqrt{\frac{a^2}{x^2}+1}+1\right)-\log\left(\sqrt{\frac{a^2}{x^2}+1}-1\right)\right)+x\operatorname{arsinh}\left(\frac{a}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a/x),x, algorithm="maxima")
```

```
[Out] 1/2*a*(log(sqrt(a^2/x^2 + 1) + 1) - log(sqrt(a^2/x^2 + 1) - 1)) + x*arcsinh
(a/x)
```

mupad [B] time = 0.55, size = 25, normalized size = 1.00

$$x\operatorname{asinh}\left(\frac{a}{x}\right)+a\ \ln\left(x+\sqrt{a^2+x^2}\right)\ \operatorname{sign}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a/x),x)
```

```
[Out] x*asinh(a/x) + a*log(x + (a^2 + x^2)^(1/2))*sign(x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\operatorname{asinh}\left(\frac{a}{x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a/x),x)
```

```
[Out] Integral(asinh(a/x), x)
```

3.303 $\int \frac{\sinh^{-1}\left(\frac{a}{x}\right)}{x}\,dx$

Optimal. Leaf size=52

$$-\frac{1}{2}\text{Li}_2\left(e^{2\sinh^{-1}\left(\frac{a}{x}\right)}\right)+\frac{1}{2}\sinh^{-1}\left(\frac{a}{x}\right)^2-\sinh^{-1}\left(\frac{a}{x}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{a}{x}\right)}\right)$$

```
[Out] 1/2*arcsinh(a/x)^2-arcsinh(a/x)*ln(1-(a/x+(1+a^2/x^2)^(1/2))^2)-1/2*polylog
(2,(a/x+(1+a^2/x^2)^(1/2))^2)
```

Rubi [A] time = 0.06, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5890, 3716, 2190, 2279, 2391}

$$-\frac{1}{2}\text{PolyLog}\left(2,e^{2\sinh^{-1}\left(\frac{a}{x}\right)}\right)+\frac{1}{2}\sinh^{-1}\left(\frac{a}{x}\right)^2-\sinh^{-1}\left(\frac{a}{x}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{a}{x}\right)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a/x]/x,x]
```

```
[Out] ArcSinh[a/x]^2/2 - ArcSinh[a/x]*Log[1 - E^(2*ArcSinh[a/x])] - PolyLog[2, E^
(2*ArcSinh[a/x])]/2
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 5890

```
Int[ArcSinh[(a_.)*(x_)^(p_)]^(n_.)/(x_), x_Symbol] :> Dist[1/p, Subst[Int[x
^n*Coth[x], x], x, ArcSinh[a*x^p]], x] /; FreeQ[{a, p}, x] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh^{-1}\left(\frac{a}{x}\right)}{x}\,dx &= -\operatorname{Subst}\left(\int x\coth(x)\,dx, x, \sinh^{-1}\left(\frac{a}{x}\right)\right)\\
&= \frac{1}{2}\sinh^{-1}\left(\frac{a}{x}\right)^2 + 2\operatorname{Subst}\left(\int \frac{e^{2x}x}{1-e^{2x}}\,dx, x, \sinh^{-1}\left(\frac{a}{x}\right)\right)\\
&= \frac{1}{2}\sinh^{-1}\left(\frac{a}{x}\right)^2 - \sinh^{-1}\left(\frac{a}{x}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{a}{x}\right)}\right) + \operatorname{Subst}\left(\int \log\left(1-e^{2x}\right)\,dx, x, \sinh^{-1}\left(\frac{a}{x}\right)\right)\\
&= \frac{1}{2}\sinh^{-1}\left(\frac{a}{x}\right)^2 - \sinh^{-1}\left(\frac{a}{x}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{a}{x}\right)}\right) + \frac{1}{2}\operatorname{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2\sinh^{-1}\left(\frac{a}{x}\right)}\right)\\
&= \frac{1}{2}\sinh^{-1}\left(\frac{a}{x}\right)^2 - \sinh^{-1}\left(\frac{a}{x}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{a}{x}\right)}\right) - \frac{1}{2}\operatorname{Li}_2\left(e^{2\sinh^{-1}\left(\frac{a}{x}\right)}\right)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 52, normalized size = 1.00

$$
-\frac{1}{2}\operatorname{Li}_2\left(e^{2\sinh^{-1}\left(\frac{a}{x}\right)}\right) + \frac{1}{2}\sinh^{-1}\left(\frac{a}{x}\right)^2 - \sinh^{-1}\left(\frac{a}{x}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{a}{x}\right)}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a/x]/x,x]
[Out] ArcSinh[a/x]^2/2 - ArcSinh[a/x]*Log[1 - E^(2*ArcSinh[a/x])] - PolyLog[2, E^
(2*ArcSinh[a/x])]/2
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$
\operatorname{integral}\left(\frac{\operatorname{arsinh}\left(\frac{a}{x}\right)}{x}, x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a/x)/x,x, algorithm="fricas")
[Out] integral(arcsinh(a/x)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{\operatorname{arsinh}\left(\frac{a}{x}\right)}{x}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a/x)/x,x, algorithm="giac")
[Out] integrate(arcsinh(a/x)/x, x)
```

maple [A] time = 0.01, size = 114, normalized size = 2.19

$$
\frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)^2}{2} - \operatorname{arcsinh}\left(\frac{a}{x}\right)\ln\left(1+\frac{a}{x}+\sqrt{1+\frac{a^2}{x^2}}\right) - \operatorname{polylog}\left(2, -\frac{a}{x}-\sqrt{1+\frac{a^2}{x^2}}\right) - \operatorname{arcsinh}\left(\frac{a}{x}\right)\ln\left(1-\frac{a}{x}-\sqrt{1+\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a/x)/x,x)
[Out] 1/2*arcsinh(a/x)^2-arcsinh(a/x)*ln(1+a/x+(1+a^2/x^2)^(1/2))-polylog(2,-a/x-
(1+a^2/x^2)^(1/2))-arcsinh(a/x)*ln(1-a/x-(1+a^2/x^2)^(1/2))-polylog(2,a/x+(
1+a^2/x^2)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$a\int \frac{x\log(x)}{a^3 + ax^2 + \left(a^2 + x^2\right)^{\frac{3}{2}}}\,dx + \log\left(a + \sqrt{a^2 + x^2}\right)\log(x) - \frac{1}{2}\log(x)^2 - \frac{1}{2}\log(x)\log\left(\frac{x^2}{a^2} + 1\right) - \frac{1}{4}\,\mathrm{Li}_2\left(-\frac{x^2}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a/x)/x,x, algorithm="maxima")
```

```
[Out] a*integrate(x*log(x)/(a^3 + a*x^2 + (a^2 + x^2)^(3/2)), x) + log(a + sqrt(a
^2 + x^2))*log(x) - 1/2*log(x)^2 - 1/2*log(x)*log(x^2/a^2 + 1) - 1/4*dilog(
-x^2/a^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{asinh}\left(\frac{a}{x}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a/x)/x,x)
```

```
[Out] int(asinh(a/x)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{asinh}\left(\frac{a}{x}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a/x)/x,x)
```

```
[Out] Integral(asinh(a/x)/x, x)
```

3.304 $\int \frac{\sinh^{-1}\left(\frac{a}{x}\right)}{x^2}\,dx$

Optimal. Leaf size=29

$$\frac{\sqrt{\frac{a^2}{x^2}+1}}{a}-\frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{x}$$

```
[Out] -arccsch(x/a)/x+(1+a^2/x^2)^(1/2)/a
```

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5892, 6284, 261}

$$\frac{\sqrt{\frac{a^2}{x^2}+1}}{a}-\frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a/x]/x^2,x]
```

```
[Out] Sqrt[1 + a^2/x^2]/a - ArcCsch[x/a]/x
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5892

```
Int[ArcSinh[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int
[u*ArcCsch[a/c + (b*x^n)/c]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rule 6284

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((d*x)^(m + 1)*(a + b*ArcCsch[c*x]))/(d*(m + 1)), x] + Dist[(b*d)/(c*(m
+ 1)), Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^{-1}\left(\frac{a}{x}\right)}{x^2}\,dx &= \int \frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{x^2}\,dx\\ &= -\frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{x}-a\int \frac{1}{\sqrt{1+\frac{a^2}{x^2}}\,x^3}\,dx\\ &= \frac{\sqrt{1+\frac{a^2}{x^2}}}{a}-\frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{x}\end{aligned}$$

Mathematica [A] time = 0.02, size = 29, normalized size = 1.00

$$\frac{\sqrt{\frac{a^2}{x^2}+1}}{a}-\frac{\sinh^{-1}\left(\frac{a}{x}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a/x]/x^2,x]
[Out] Sqrt[1 + a^2/x^2]/a - ArcSinh[a/x]/x
```

fricas [A] time = 0.68, size = 49, normalized size = 1.69

$$-\frac{a \log\left(\frac{x\sqrt{\frac{a^2+x^2}{x^2}}+a}{x}\right) - x\sqrt{\frac{a^2+x^2}{x^2}}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a/x)/x^2,x, algorithm="fricas")
[Out] -(a*log((x*sqrt((a^2 + x^2)/x^2) + a)/x) - x*sqrt((a^2 + x^2)/x^2))/(a*x)
```

giac [A] time = 0.42, size = 39, normalized size = 1.34

$$-\frac{\log\left(\sqrt{\frac{a^2}{x^2}+1}+\frac{a}{x}\right)}{x} + \frac{\sqrt{\frac{a^2}{x^2}+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a/x)/x^2,x, algorithm="giac")
[Out] -log(sqrt(a^2/x^2 + 1) + a/x)/x + sqrt(a^2/x^2 + 1)/a
```

maple [A] time = 0.00, size = 31, normalized size = 1.07

$$-\frac{\frac{a \operatorname{arcsinh}\left(\frac{a}{x}\right)}{x} - \sqrt{1+\frac{a^2}{x^2}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a/x)/x^2,x)
[Out] -1/a*(a/x*arcsinh(a/x)-(1+a^2/x^2)^(1/2))
```

maxima [A] time = 0.67, size = 30, normalized size = 1.03

$$-\frac{\frac{a \operatorname{arsinh}\left(\frac{a}{x}\right)}{x} - \sqrt{\frac{a^2}{x^2}+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a/x)/x^2,x, algorithm="maxima")
[Out] -(a*arcsinh(a/x)/x - sqrt(a^2/x^2 + 1))/a
```

mupad [B] time = 0.23, size = 27, normalized size = 0.93

$$\frac{\sqrt{\frac{a^2}{x^2}+1}}{a} - \frac{\operatorname{asinh}\left(\frac{a}{x}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a/x)/x^2,x)
```

```
[Out] (a^2/x^2 + 1)^(1/2)/a - asinh(a/x)/x
```

sympy [A] time = 1.93, size = 20, normalized size = 0.69

$$\begin{cases} -\frac{\operatorname{asinh}\left(\frac{a}{x}\right)}{x} + \frac{\sqrt{\frac{a^2}{x^2}+1}}{a} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a/x)/x**2,x)
```

```
[Out] Piecewise((-asinh(a/x)/x + sqrt(a**2/x**2 + 1)/a, Ne(a, 0)), (0, True))
```

3.305 $\int \frac{\sinh^{-1}\left(\frac{a}{x}\right)}{x^3}\,dx$

Optimal. Leaf size=50

$$\frac{\sqrt{\frac{a^2}{x^2}+1}}{4ax}-\frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{4a^2}-\frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{2x^2}$$

```
[Out] -1/4*arccsch(x/a)/a^2-1/2*arccsch(x/a)/x^2+1/4*(1+a^2/x^2)^(1/2)/a/x
```

Rubi [A] time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5892, 6284, 335, 321, 215}

$$\frac{\sqrt{\frac{a^2}{x^2}+1}}{4ax}-\frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{4a^2}-\frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a/x]/x^3,x]
```

```
[Out] Sqrt[1 + a^2/x^2]/(4*a*x) - ArcCsch[x/a]/(4*a^2) - ArcCsch[x/a]/(2*x^2)
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5892

```
Int[ArcSinh[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int
[u*ArcCsch[a/c + (b*x^n)/c]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rule 6284

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((d*x)^(m + 1)*(a + b*ArcCsch[c*x]))/(d*(m + 1)), x] + Dist[(b*d)/(c*(m
+ 1)), Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh^{-1}\left(\frac{a}{x}\right)}{x^3}\,dx &= \int \frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{x^3}\,dx \\
&= -\frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{2x^2} - \frac{1}{2}a\int \frac{1}{\sqrt{1+\frac{a^2}{x^2}}\,x^4}\,dx \\
&= -\frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{2x^2} + \frac{1}{2}a\,\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{1+a^2x^2}}\,dx, x, \frac{1}{x}\right) \\
&= \frac{\sqrt{1+\frac{a^2}{x^2}}}{4ax} - \frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{2x^2} - \frac{\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+a^2x^2}}\,dx, x, \frac{1}{x}\right)}{4a} \\
&= \frac{\sqrt{1+\frac{a^2}{x^2}}}{4ax} - \frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{4a^2} - \frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{2x^2}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 44, normalized size = 0.88

$$\frac{ax\sqrt{\frac{a^2}{x^2}+1} - \left(2a^2+x^2\right)\sinh^{-1}\left(\frac{a}{x}\right)}{4a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a/x]/x^3,x]
```

```
[Out] (a*Sqrt[1 + a^2/x^2]*x - (2*a^2 + x^2)*ArcSinh[a/x])/(4*a^2*x^2)
```

fricas [A] time = 0.70, size = 58, normalized size = 1.16

$$\frac{ax\sqrt{\frac{a^2+x^2}{x^2}} - \left(2\,a^2+x^2\right)\log\left(\frac{x\sqrt{\frac{a^2+x^2}{x^2}}+a}{x}\right)}{4\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a/x)/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(a*x*sqrt((a^2 + x^2)/x^2) - (2*a^2 + x^2)*log((x*sqrt((a^2 + x^2)/x^2)
 + a)/x))/(a^2*x^2)
```

giac [A] time = 0.42, size = 84, normalized size = 1.68

$$-\frac{a\left(\frac{\log\left(a+\sqrt{a^2+x^2}\right)}{a^3} - \frac{\log\left(-a+\sqrt{a^2+x^2}\right)}{a^3} - \frac{2\sqrt{a^2+x^2}}{a^2x^2}\right)}{8\,\mathrm{sgn}(x)} - \frac{\log\left(\sqrt{\frac{a^2}{x^2}+1}+\frac{a}{x}\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a/x)/x^3,x, algorithm="giac")
```

```
[Out] -1/8*a*(log(a + sqrt(a^2 + x^2))/a^3 - log(-a + sqrt(a^2 + x^2))/a^3 - 2*sq
rt(a^2 + x^2)/(a^2*x^2))/sgn(x) - 1/2*log(sqrt(a^2/x^2 + 1) + a/x)/x^2
```

maple [A] time = 0.00, size = 46, normalized size = 0.92

$$-\frac{\frac{a^2\,\mathrm{arcsinh}\left(\frac{a}{x}\right)}{2x^2} - \frac{a\sqrt{1+\frac{a^2}{x^2}}}{4x} + \frac{\mathrm{arcsinh}\left(\frac{a}{x}\right)}{4}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a/x)/x^3,x)
```

```
[Out] -1/a^2*(1/2*a^2/x^2*arcsinh(a/x)-1/4*a/x*(1+a^2/x^2)^(1/2)+1/4*arcsinh(a/x)
)
```

maxima [B] time = 0.63, size = 97, normalized size = 1.94

$$\frac{1}{8}a\left(\frac{2x\sqrt{\frac{a^2}{x^2}+1}}{a^2x^2\left(\frac{a^2}{x^2}+1\right)-a^4}-\frac{\log\left(x\sqrt{\frac{a^2}{x^2}+1}+a\right)}{a^3}+\frac{\log\left(x\sqrt{\frac{a^2}{x^2}+1}-a\right)}{a^3}\right)-\frac{\operatorname{arsinh}\left(\frac{a}{x}\right)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a/x)/x^3,x, algorithm="maxima")
```

```
[Out] 1/8*a*(2*x*sqrt(a^2/x^2 + 1)/(a^2*x^2*(a^2/x^2 + 1) - a^4) - log(x*sqrt(a^2
/x^2 + 1) + a)/a^3 + log(x*sqrt(a^2/x^2 + 1) - a)/a^3) - 1/2*arcsinh(a/x)/x
^2
```

mupad [B] time = 0.24, size = 43, normalized size = 0.86

$$\frac{\sqrt{\frac{a^2}{x^2}+1}}{4ax}-\frac{\operatorname{asinh}\left(\frac{a}{x}\right)\left(\frac{x}{4a^2}+\frac{1}{2x}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a/x)/x^3,x)
```

```
[Out] (a^2/x^2 + 1)^(1/2)/(4*a*x) - (asinh(a/x)*(x/(4*a^2) + 1/(2*x)))/x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{asinh}\left(\frac{a}{x}\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a/x)/x**3,x)
```

```
[Out] Integral(asinh(a/x)/x**3, x)
```

3.306 $\int \frac{\sinh^{-1}\left(\frac{a}{x}\right)}{x^4} dx$

Optimal. Leaf size=54

$$\frac{\left(\frac{a^2}{x^2}+1\right)^{3/2}}{9a^3}-\frac{\sqrt{\frac{a^2}{x^2}+1}}{3a^3}-\frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{3x^3}$$

```
[Out] 1/9*(1+a^2/x^2)^(3/2)/a^3-1/3*arccsch(x/a)/x^3-1/3*(1+a^2/x^2)^(1/2)/a^3
```

Rubi [A] time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5892, 6284, 266, 43}

$$\frac{\left(\frac{a^2}{x^2}+1\right)^{3/2}}{9a^3}-\frac{\sqrt{\frac{a^2}{x^2}+1}}{3a^3}-\frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a/x]/x^4,x]
```

```
[Out] -Sqrt[1 + a^2/x^2]/(3*a^3) + (1 + a^2/x^2)^(3/2)/(9*a^3) - ArcCsch[x/a]/(3*
x^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5892

```
Int[ArcSinh[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int
[u*ArcCsch[a/c + (b*x^n)/c]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rule 6284

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((d*x)^(m + 1)*(a + b*ArcCsch[c*x]))/(d*(m + 1)), x] + Dist[(b*d)/(c*(m
+ 1)), Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sinh^{-1}\left(\frac{a}{x}\right)}{x^4}\,dx = \int \frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{x^4}\,dx$$
$$= -\frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{3x^3} - \frac{1}{3}a\int \frac{1}{\sqrt{1+\frac{a^2}{x^2}}\,x^5}\,dx$$
$$= -\frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{3x^3} + \frac{1}{6}a\,\text{Subst}\left(\int \frac{x}{\sqrt{1+a^2x}}\,dx, x, \frac{1}{x^2}\right)$$
$$= -\frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{3x^3} + \frac{1}{6}a\,\text{Subst}\left(\int \left(-\frac{1}{a^2\sqrt{1+a^2x}} + \frac{\sqrt{1+a^2x}}{a^2}\right)dx, x, \frac{1}{x^2}\right)$$
$$= -\frac{\sqrt{1+\frac{a^2}{x^2}}}{3a^3} + \frac{\left(1+\frac{a^2}{x^2}\right)^{3/2}}{9a^3} - \frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{3x^3}$$

Mathematica [A] time = 0.03, size = 48, normalized size = 0.89

$$\left(\frac{1}{9ax^2} - \frac{2}{9a^3}\right)\sqrt{\frac{a^2+x^2}{x^2}} - \frac{\sinh^{-1}\left(\frac{a}{x}\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a/x]/x^4,x]
[Out] (-2/(9*a^3) + 1/(9*a*x^2))*Sqrt[(a^2 + x^2)/x^2] - ArcSinh[a/x]/(3*x^3)
```

fricas [A] time = 0.81, size = 62, normalized size = 1.15

$$-\frac{3\,a^3\log\left(\frac{x\sqrt{\frac{a^2+x^2}{x^2}}+a}{x}\right) - \left(a^2x - 2\,x^3\right)\sqrt{\frac{a^2+x^2}{x^2}}}{9\,a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a/x)/x^4,x, algorithm="fricas")
[Out] -1/9*(3*a^3*log((x*sqrt((a^2 + x^2)/x^2) + a)/x) - (a^2*x - 2*x^3)*sqrt((a^
2 + x^2)/x^2))/(a^3*x^3)
```

giac [A] time = 0.67, size = 75, normalized size = 1.39

$$-\frac{\log\left(\sqrt{\frac{a^2}{x^2}+1}+\frac{a}{x}\right)}{3\,x^3} - \frac{4\left(a^2 - 3\left(x-\sqrt{a^2+x^2}\right)^2\right)a}{9\left(a^2-\left(x-\sqrt{a^2+x^2}\right)^2\right)^3\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a/x)/x^4,x, algorithm="giac")
[Out] -1/3*log(sqrt(a^2/x^2 + 1) + a/x)/x^3 - 4/9*(a^2 - 3*(x - sqrt(a^2 + x^2))^
2)*a/((a^2 - (x - sqrt(a^2 + x^2))^2)^3*sgn(x))
```

maple [A] time = 0.01, size = 53, normalized size = 0.98

$$-\frac{\frac{a^3\operatorname{arcsinh}\left(\frac{a}{x}\right)}{3x^3} - \frac{a^2\sqrt{1+\frac{a^2}{x^2}}}{9x^2} + \frac{2\sqrt{1+\frac{a^2}{x^2}}}{9}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a/x)/x^4,x)
```

```
[Out] -1/a^3*(1/3*a^3/x^3*arcsinh(a/x)-1/9*a^2/x^2*(1+a^2/x^2)^(1/2)+2/9*(1+a^2/x
^2)^(1/2))
```

maxima [A] time = 0.46, size = 47, normalized size = 0.87

$$\frac{1}{9}a\left(\frac{\left(\frac{a^2}{x^2}+1\right)^{\frac{3}{2}}}{a^4}-\frac{3\sqrt{\frac{a^2}{x^2}+1}}{a^4}\right)-\frac{\operatorname{arsinh}\left(\frac{a}{x}\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a/x)/x^4,x, algorithm="maxima")
```

```
[Out] 1/9*a*((a^2/x^2 + 1)^(3/2)/a^4 - 3*sqrt(a^2/x^2 + 1)/a^4) - 1/3*arcsinh(a/x
)/x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{asinh}\left(\frac{a}{x}\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a/x)/x^4,x)
```

```
[Out] int(asinh(a/x)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}\left(\frac{a}{x}\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a/x)/x**4,x)
```

```
[Out] Integral(asinh(a/x)/x**4, x)
```

3.307 $\int x^m \sinh^{-1}(ax^n)\,dx$

Optimal. Leaf size=77

$$\frac{x^{m+1}\sinh^{-1}(ax^n)}{m+1}-\frac{anx^{m+n+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+n+1}{2n};\frac{m+3n+1}{2n};-a^2x^{2n}\right)}{(m+1)(m+n+1)}$$

```
[Out] x^(1+m)*arcsinh(a*x^n)/(1+m)-a*n*x^(1+m+n)*hypergeom([1/2, 1/2*(1+m+n)/n],[
1/2*(1+m+3*n)/n],-a^2*x^(2*n))/(1+m)/(1+m+n)
```

Rubi [A] time = 0.04, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5902, 12, 364}

$$\frac{x^{m+1}\sinh^{-1}(ax^n)}{m+1}-\frac{anx^{m+n+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+n+1}{2n};\frac{m+3n+1}{2n};-a^2x^{2n}\right)}{(m+1)(m+n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*ArcSinh[a*x^n],x]
```

```
[Out] (x^(1 + m)*ArcSinh[a*x^n])/(1 + m) - (a*n*x^(1 + m + n)*Hypergeometric2F1[1
/2, (1 + m + n)/(2*n), (1 + m + 3*n)/(2*n), -(a^2*x^(2*n))])/((1 + m)*(1 +
m + n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5902

```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcSinh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/Sqrt[1 + u^2], x], x
], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
 x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u
, x]
```

Rubi steps

$$\begin{aligned}\int x^m\sinh^{-1}(ax^n)\,dx&=\frac{x^{1+m}\sinh^{-1}(ax^n)}{1+m}-\frac{\int\frac{anx^{m+n}}{\sqrt{1+a^2x^{2n}}}\,dx}{1+m}\\&=\frac{x^{1+m}\sinh^{-1}(ax^n)}{1+m}-\frac{(an)\int\frac{x^{m+n}}{\sqrt{1+a^2x^{2n}}}\,dx}{1+m}\\&=\frac{x^{1+m}\sinh^{-1}(ax^n)}{1+m}-\frac{anx^{1+m+n}\,{}_2F_1\left(\frac{1}{2},\frac{1+m+n}{2n};\frac{1+m+3n}{2n};-a^2x^{2n}\right)}{(1+m)(1+m+n)}\end{aligned}$$

Mathematica [A] time = 0.08, size = 74, normalized size = 0.96

$$\frac{x^{m+1}\left((m+n+1)\sinh^{-1}(ax^n)-anx^n\,{}_2F_1\left(\frac{1}{2},\frac{m+n+1}{2n};\frac{m+3n+1}{2n};-a^2x^{2n}\right)\right)}{(m+1)(m+n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*ArcSinh[a*x^n],x]
```

```
[Out] (x^(1 + m)*((1 + m + n)*ArcSinh[a*x^n] - a*n*x^n*Hypergeometric2F1[1/2, (1
+ m + n)/(2*n), (1 + m + 3*n)/(2*n), -(a^2*x^(2*n))]))/((1 + m)*(1 + m + n)
)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x^n),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \operatorname{arsinh}(ax^n)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^m*arcsinh(a*x^n), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int x^m \operatorname{arcsinh}(a\,x^n)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arcsinh(a*x^n),x)
```

```
[Out] int(x^m*arcsinh(a*x^n),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-an\int \frac{e^{\left(m\log(x)+n\log(x)\right)}}{a^3(m+1)x^{3n}+a(m+1)x^n+\left(a^2(m+1)x^{2n}+m+1\right)\sqrt{a^2x^{2n}+1}}\,dx+n\int \frac{x^m}{a^2(m+1)x^{2n}+m+1}\,dx+\frac{(m}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x^n),x, algorithm="maxima")
```

```
[Out] -a*n*integrate(e^(m*log(x) + n*log(x))/(a^3*(m + 1)*x^(3*n) + a*(m + 1)*x^n
 + (a^2*(m + 1)*x^(2*n) + m + 1)*sqrt(a^2*x^(2*n) + 1)), x) + n*integrate(x
^m/(a^2*(m + 1)*x^(2*n) + m + 1), x) + ((m + 1)*x*x^m*log(a*x^n + sqrt(a^2*
x^(2*n) + 1)) - n*x*x^m)/(m^2 + 2*m + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^m\,\mathrm{asinh}(a\,x^n)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*asinh(a*x^n),x)
[Out] int(x^m*asinh(a*x^n), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \operatorname{asinh}(ax^n)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*asinh(a*x**n),x)
[Out] Integral(x**m*asinh(a*x**n), x)
```

3.308 $\int x^2 \sinh^{-1}(ax^n)\,dx$

Optimal. Leaf size=64

$$\frac{1}{3}x^3\sinh^{-1}(ax^n) - \frac{anx^{n+3}\,{}_2F_1\left(\frac{1}{2},\frac{n+3}{2n};\frac{3(n+1)}{2n};-a^2x^{2n}\right)}{3(n+3)}$$

```
[Out] 1/3*x^3*arcsinh(a*x^n)-1/3*a*n*x^(3+n)*hypergeom([1/2, 1/2*(3+n)/n],[3/2*(1
+n)/n],-a^2*x^(2*n))/(3+n)
```

Rubi [A] time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5902, 12, 364}

$$\frac{1}{3}x^3\sinh^{-1}(ax^n) - \frac{anx^{n+3}\,{}_2F_1\left(\frac{1}{2},\frac{n+3}{2n};\frac{3(n+1)}{2n};-a^2x^{2n}\right)}{3(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcSinh[a*x^n],x]
```

```
[Out] (x^3*ArcSinh[a*x^n])/3 - (a*n*x^(3 + n)*Hypergeometric2F1[1/2, (3 + n)/(2*n
), (3*(1 + n))/(2*n), -(a^2*x^(2*n))])/(3*(3 + n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5902

```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcSinh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/Sqrt[1 + u^2], x], x
], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
 x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u
, x]
```

Rubi steps

$$\begin{aligned}
\int x^2\sinh^{-1}(ax^n)\,dx &= \frac{1}{3}x^3\sinh^{-1}(ax^n) - \frac{1}{3}\int \frac{anx^{2+n}}{\sqrt{1+a^2x^{2n}}}\,dx \\
&= \frac{1}{3}x^3\sinh^{-1}(ax^n) - \frac{1}{3}(an)\int \frac{x^{2+n}}{\sqrt{1+a^2x^{2n}}}\,dx \\
&= \frac{1}{3}x^3\sinh^{-1}(ax^n) - \frac{anx^{3+n}\,{}_2F_1\left(\frac{1}{2},\frac{3+n}{2n};\frac{3(1+n)}{2n};-a^2x^{2n}\right)}{3(3+n)}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 66, normalized size = 1.03

$$\frac{1}{3}x^3\sinh^{-1}(ax^n) - \frac{anx^{n+3}\,{}_2F_1\left(\frac{1}{2},\frac{n+3}{2n};\frac{n+3}{2n}+1;-a^2x^{2n}\right)}{3(n+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcSinh[a*x^n],x]
```

```
[Out] (x^3*ArcSinh[a*x^n])/3 - (a*n*x^(3 + n)*Hypergeometric2F1[1/2, (3 + n)/(2*n
), 1 + (3 + n)/(2*n), -(a^2*x^(2*n))])/(3*(3 + n))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x^n),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\,\mathrm{arsinh}\,(ax^n)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^2*arcsinh(a*x^n), x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int x^2\,\mathrm{arcsinh}\,(a\,x^n)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsinh(a*x^n),x)
```

```
[Out] int(x^2*arcsinh(a*x^n),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{9}\,nx^3+\frac{1}{3}\,x^3\log\left(ax^n+\sqrt{a^2x^{2\,n}+1}\right)-an\int\frac{x^2x^n}{3\left(a^3x^{3\,n}+ax^n+\left(a^2x^{2\,n}+1\right)^{\frac{3}{2}}\right)}\,dx+n\int\frac{x^2}{3\left(a^2x^{2\,n}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x^n),x, algorithm="maxima")
```

```
[Out] -1/9*n*x^3 + 1/3*x^3*log(a*x^n + sqrt(a^2*x^(2*n) + 1)) - a*n*integrate(1/3
*x^2*x^n/(a^3*x^(3*n) + a*x^n + (a^2*x^(2*n) + 1)^(3/2)), x) + n*integrate(
1/3*x^2/(a^2*x^(2*n) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^2\,\mathrm{asinh}\,(a\,x^n)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*asinh(a*x^n),x)
```

```
[Out] int(x^2*asinh(a*x^n), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{asinh}(ax^n)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asinh(a*x**n),x)
```

```
[Out] Integral(x**2*asinh(a*x**n), x)
```

3.309 $\int x \sinh^{-1}(ax^n) \, dx$

Optimal. Leaf size=65

$$\frac{1}{2}x^2 \sinh^{-1}(ax^n) - \frac{anx^{n+2} {}_2F_1\left(\frac{1}{2}, \frac{n+2}{2n}; \frac{1}{2}\left(3+\frac{2}{n}\right); -a^2x^{2n}\right)}{2(n+2)}$$

```
[Out] 1/2*x^2*arcsinh(a*x^n)-1/2*a*n*x^(2+n)*hypergeom([1/2, 1/2*(2+n)/n],[3/2+1/
n],-a^2*x^(2*n))/(2+n)
```

Rubi [A] time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5902, 12, 364}

$$\frac{1}{2}x^2 \sinh^{-1}(ax^n) - \frac{anx^{n+2} {}_2F_1\left(\frac{1}{2}, \frac{n+2}{2n}; \frac{1}{2}\left(3+\frac{2}{n}\right); -a^2x^{2n}\right)}{2(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcSinh[a*x^n],x]
```

```
[Out] (x^2*ArcSinh[a*x^n])/2 - (a*n*x^(2 + n)*Hypergeometric2F1[1/2, (2 + n)/(2*n
), (3 + 2/n)/2, -(a^2*x^(2*n))])/(2*(2 + n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5902

```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcSinh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/Sqrt[1 + u^2], x], x
], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
 x] && !FunctionOfQ[(c + d*x)^(m + 1), u, x] && !FunctionOfExponentialQ[u
, x]
```

Rubi steps

$$\begin{aligned}\int x \sinh^{-1}(ax^n) \, dx &= \frac{1}{2}x^2 \sinh^{-1}(ax^n) - \frac{1}{2}\int \frac{anx^{1+n}}{\sqrt{1+a^2x^{2n}}} \, dx\\ &= \frac{1}{2}x^2 \sinh^{-1}(ax^n) - \frac{1}{2}(an)\int \frac{x^{1+n}}{\sqrt{1+a^2x^{2n}}} \, dx\\ &= \frac{1}{2}x^2 \sinh^{-1}(ax^n) - \frac{anx^{2+n} {}_2F_1\left(\frac{1}{2}, \frac{2+n}{2n}; \frac{1}{2}\left(3+\frac{2}{n}\right); -a^2x^{2n}\right)}{2(2+n)}\end{aligned}$$

Mathematica [A] time = 0.05, size = 58, normalized size = 0.89

$$\frac{x^2\left((n+2)\sinh^{-1}(ax^n)-anx^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}+\frac{1}{n};\frac{3}{2}+\frac{1}{n};-a^2x^{2n}\right)\right)}{2(n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcSinh[a*x^n],x]
[Out] (x^2*((2 + n)*ArcSinh[a*x^n] - a*n*x^n*Hypergeometric2F1[1/2, 1/2 + n^(-1),
 3/2 + n^(-1), -(a^2*x^(2*n))]))/(2*(2 + n))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x^n),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\,\mathrm{arsinh}\left(ax^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x^n),x, algorithm="giac")
[Out] integrate(x*arcsinh(a*x^n), x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int x\,\mathrm{arcsinh}\left(a\,x^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsinh(a*x^n),x)
[Out] int(x*arcsinh(a*x^n),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{4}\,nx^2-an\int\frac{xx^n}{2\left(a^3x^{3\,n}+ax^n+\left(a^2x^{2\,n}+1\right)^{\frac{3}{2}}\right)}\,dx+\frac{1}{2}\,x^2\log\left(ax^n+\sqrt{a^2x^{2\,n}+1}\right)+n\int\frac{x}{2\left(a^2x^{2\,n}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x^n),x, algorithm="maxima")
[Out] -1/4*n*x^2 - a*n*integrate(1/2*x*x^n/(a^3*x^(3*n) + a*x^n + (a^2*x^(2*n) +
1)^(3/2)), x) + 1/2*x^2*log(a*x^n + sqrt(a^2*x^(2*n) + 1)) + n*integrate(1/
2*x/(a^2*x^(2*n) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\mathrm{asinh}\left(a\,x^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*asinh(a*x^n),x)
```

```
[Out] int(x*asinh(a*x^n), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{asinh}(ax^n)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asinh(a*x**n),x)
```

```
[Out] Integral(x*asinh(a*x**n), x)
```

3.310 $\int \sinh^{-1}(ax^n)\,dx$

Optimal. Leaf size=56

$$x\sinh^{-1}(ax^n)-\frac{anx^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-a^2x^{2n}\right)}{n+1}$$

```
[Out] x*arcsinh(a*x^n)-a*n*x^(1+n)*hypergeom([1/2, 1/2*(1+n)/n],[3/2+1/2/n],-a^2*
x^(2*n))/(1+n)
```

Rubi [A] time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5900, 12, 364}

$$x\sinh^{-1}(ax^n)-\frac{anx^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-a^2x^{2n}\right)}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x^n],x]
```

```
[Out] x*ArcSinh[a*x^n] - (a*n*x^(1 + n)*Hypergeometric2F1[1/2, (1 + n)/(2*n), (3
+ n^(-1))/2, -(a^2*x^(2*n))])/(1 + n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5900

```
Int[ArcSinh[u_], x_Symbol] :> Simp[x*ArcSinh[u], x] - Int[SimplifyIntegrand
[(x*D[u, x])/Sqrt[1 + u^2], x], x] /; InverseFunctionFreeQ[u, x] &&  !Funct
ionOfExponentialQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \sinh^{-1}(ax^n)\,dx &= x\sinh^{-1}(ax^n)-\int \frac{anx^n}{\sqrt{1+a^2x^{2n}}}\,dx\\ &= x\sinh^{-1}(ax^n)-(an)\int \frac{x^n}{\sqrt{1+a^2x^{2n}}}\,dx\\ &= x\sinh^{-1}(ax^n)-\frac{anx^{1+n}\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-a^2x^{2n}\right)}{1+n}\end{aligned}$$

Mathematica [A] time = 0.03, size = 56, normalized size = 1.00

$$x\sinh^{-1}(ax^n)-\frac{anx^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-a^2x^{2n}\right)}{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x^n],x]
```

```
[Out] x*ArcSinh[a*x^n] - (a*n*x^(1 + n)*Hypergeometric2F1[1/2, (1 + n)/(2*n), (3
+ n^(-1))/2, -(a^2*x^(2*n))])/(1 + n)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^n),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{arsinh}\left(ax^{n}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^n),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x^n), x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int \operatorname{arcsinh}\left(a\,x^{n}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x^n),x)
```

```
[Out] int(arcsinh(a*x^n),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-an\int \frac{x^{n}}{a^{3}x^{3\,n} + ax^{n} + \left(a^{2}x^{2\,n} + 1\right)^{\frac{3}{2}}}\,dx - nx + n\int \frac{1}{a^{2}x^{2\,n} + 1}\,dx + x\log\left(ax^{n} + \sqrt{a^{2}x^{2\,n} + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^n),x, algorithm="maxima")
```

```
[Out] -a*n*integrate(x^n/(a^3*x^(3*n) + a*x^n + (a^2*x^(2*n) + 1)^(3/2)), x) - n*
x + n*integrate(1/(a^2*x^(2*n) + 1), x) + x*log(a*x^n + sqrt(a^2*x^(2*n) +
1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \operatorname{asinh}\left(a\,x^{n}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x^n),x)
```

```
[Out] int(asinh(a*x^n), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{asinh}(ax^n)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x**n),x)
```

```
[Out] Integral(asinh(a*x**n), x)
```

3.311 $\int \frac{\sinh^{-1}(ax^n)}{x} \, dx$

Optimal. Leaf size=60

$$\frac{\text{Li}_2\left(e^{2\sinh^{-1}(ax^n)}\right)}{2n}-\frac{\sinh^{-1}(ax^n)^2}{2n}+\frac{\sinh^{-1}(ax^n)\log\left(1-e^{2\sinh^{-1}(ax^n)}\right)}{n}$$

```
[Out] -1/2*arcsinh(a*x^n)^2/n+arcsinh(a*x^n)*ln(1-(a*x^n+(1+a^2*(x^n)^2)^(1/2))^2
)/n+1/2*polylog(2,(a*x^n+(1+a^2*(x^n)^2)^(1/2))^2)/n
```

Rubi [A] time = 0.07, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5890, 3716, 2190, 2279, 2391}

$$\frac{\text{PolyLog}\left(2,e^{2\sinh^{-1}(ax^n)}\right)}{2n}-\frac{\sinh^{-1}(ax^n)^2}{2n}+\frac{\sinh^{-1}(ax^n)\log\left(1-e^{2\sinh^{-1}(ax^n)}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x^n]/x,x]
```

```
[Out] -ArcSinh[a*x^n]^2/(2*n) + (ArcSinh[a*x^n]*Log[1 - E^(2*ArcSinh[a*x^n])])/n
+ PolyLog[2, E^(2*ArcSinh[a*x^n])]/(2*n)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 5890

```
Int[ArcSinh[(a_.)*(x_)^(p_)]^(n_.)/(x_), x_Symbol] :> Dist[1/p, Subst[Int[x
^n*Coth[x], x], x, ArcSinh[a*x^p]], x] /; FreeQ[{a, p}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax^n)}{x}\,dx = \frac{\text{Subst}\left(\int x\coth(x)\,dx, x, \sinh^{-1}(ax^n)\right)}{n}$$
$$= -\frac{\sinh^{-1}(ax^n)^2}{2n} - \frac{2\,\text{Subst}\left(\int \frac{e^{2x}x}{1-e^{2x}}\,dx, x, \sinh^{-1}(ax^n)\right)}{n}$$
$$= -\frac{\sinh^{-1}(ax^n)^2}{2n} + \frac{\sinh^{-1}(ax^n)\log\left(1-e^{2\sinh^{-1}(ax^n)}\right)}{n} - \frac{\text{Subst}\left(\int \log\left(1-e^{2x}\right)\,dx, x, \sinh^{-1}\right.}{n}$$
$$= -\frac{\sinh^{-1}(ax^n)^2}{2n} + \frac{\sinh^{-1}(ax^n)\log\left(1-e^{2\sinh^{-1}(ax^n)}\right)}{n} - \frac{\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2\sinh^{-1}(ax^n)}\right)}{2n}$$
$$= -\frac{\sinh^{-1}(ax^n)^2}{2n} + \frac{\sinh^{-1}(ax^n)\log\left(1-e^{2\sinh^{-1}(ax^n)}\right)}{n} + \frac{\text{Li}_2\left(e^{2\sinh^{-1}(ax^n)}\right)}{2n}$$

Mathematica [A] time = 0.01, size = 60, normalized size = 1.00

$$\frac{\text{Li}_2\left(e^{2\sinh^{-1}(ax^n)}\right)}{2n} - \frac{\sinh^{-1}(ax^n)^2}{2n} + \frac{\sinh^{-1}(ax^n)\log\left(1-e^{2\sinh^{-1}(ax^n)}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x^n]/x,x]
```

```
[Out] -1/2*ArcSinh[a*x^n]^2/n + (ArcSinh[a*x^n]*Log[1 - E^(2*ArcSinh[a*x^n])])/n
+ PolyLog[2, E^(2*ArcSinh[a*x^n])]/(2*n)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^n)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}(ax^n)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^n)/x,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x^n)/x, x)
```

maple [A] time = 0.00, size = 133, normalized size = 2.22

$$-\frac{\operatorname{arcsinh}(a\,x^n)^2}{2n} + \frac{\operatorname{arcsinh}(a\,x^n)\ln\left(1+a\,x^n+\sqrt{1+a^2x^{2n}}\right)}{n} + \frac{\operatorname{polylog}\left(2,-a\,x^n-\sqrt{1+a^2x^{2n}}\right)}{n} + \frac{\operatorname{arcsinh}(a\,x^n)\,\mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x^n)/x,x)
```

```
[Out] -1/2*arcsinh(a*x^n)^2/n+1/n*arcsinh(a*x^n)*ln(1+a*x^n+(1+a^2*(x^n)^2)^(1/2)
)+1/n*polylog(2,-a*x^n-(1+a^2*(x^n)^2)^(1/2))+1/n*arcsinh(a*x^n)*ln(1-a*x^n
-(1+a^2*(x^n)^2)^(1/2))+1/n*polylog(2,a*x^n+(1+a^2*(x^n)^2)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-an\int \frac{x^n \log(x)}{a^3xx^{3n} + axx^n + \left(a^2xx^{2n} + x\right)\sqrt{a^2x^{2n} + 1}}\,dx - \frac{1}{2}\,n\log(x)^2 + n\int \frac{\log(x)}{a^2xx^{2n} + x}\,dx + \log\left(ax^n + \sqrt{a^2x^{2n} +\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^n)/x,x, algorithm="maxima")
```

```
[Out] -a*n*integrate(x^n*log(x)/(a^3*x*x^(3*n) + a*x*x^n + (a^2*x*x^(2*n) + x)*sq
rt(a^2*x^(2*n) + 1)), x) - 1/2*n*log(x)^2 + n*integrate(log(x)/(a^2*x*x^(2*
n) + x), x) + log(a*x^n + sqrt(a^2*x^(2*n) + 1))*log(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{asinh}\left(a\,x^n\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x^n)/x,x)
```

```
[Out] int(asinh(a*x^n)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}\left(ax^n\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x**n)/x,x)
```

```
[Out] Integral(asinh(a*x**n)/x, x)
```

3.312 $\int \frac{\sinh^{-1}(ax^n)}{x^2}\,dx$

Optimal. Leaf size=65

$$-\frac{anx^{n-1}\,{}_2F_1\left(\frac{1}{2},-\frac{1-n}{2n};\frac{1}{2}\left(3-\frac{1}{n}\right);-a^2x^{2n}\right)}{1-n}-\frac{\sinh^{-1}(ax^n)}{x}$$

```
[Out] -arcsinh(a*x^n)/x-a*n*x^(-1+n)*hypergeom([1/2, 1/2*(-1+n)/n],[3/2-1/2/n],-a
^2*x^(2*n))/(1-n)
```

Rubi [A] time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5902, 12, 364}

$$-\frac{anx^{n-1}\,{}_2F_1\left(\frac{1}{2},-\frac{1-n}{2n};\frac{1}{2}\left(3-\frac{1}{n}\right);-a^2x^{2n}\right)}{1-n}-\frac{\sinh^{-1}(ax^n)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x^n]/x^2,x]
```

```
[Out] -(ArcSinh[a*x^n]/x) - (a*n*x^(-1 + n)*Hypergeometric2F1[1/2, -(1 - n)/(2*n)
, (3 - n^(-1))/2, -(a^2*x^(2*n))])/(1 - n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5902

```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcSinh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/Sqrt[1 + u^2], x], x
], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
 x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u
, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^{-1}(ax^n)}{x^2}\,dx &= -\frac{\sinh^{-1}(ax^n)}{x}+\int \frac{anx^{-2+n}}{\sqrt{1+a^2x^{2n}}}\,dx\\
&= -\frac{\sinh^{-1}(ax^n)}{x}+(an)\int \frac{x^{-2+n}}{\sqrt{1+a^2x^{2n}}}\,dx\\
&= -\frac{\sinh^{-1}(ax^n)}{x}-\frac{anx^{-1+n}\,{}_2F_1\left(\frac{1}{2},-\frac{1-n}{2n};\frac{1}{2}\left(3-\frac{1}{n}\right);-a^2x^{2n}\right)}{1-n}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 61, normalized size = 0.94

$$\frac{anx^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{n-1}{2n};\frac{n-1}{2n}+1;-a^2x^{2n}\right)}{n-1}-\frac{\sinh^{-1}(ax^n)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x^n]/x^2,x]
```

```
[Out] -(ArcSinh[a*x^n]/x) + (a*n*x^(-1 + n)*Hypergeometric2F1[1/2, (-1 + n)/(2*n)
, 1 + (-1 + n)/(2*n), -(a^2*x^(2*n))])/(-1 + n)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^n)/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}(ax^n)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^n)/x^2,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x^n)/x^2, x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsinh}(a\,x^n)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x^n)/x^2,x)
```

```
[Out] int(arcsinh(a*x^n)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$an\int \frac{x^n}{a^3x^2x^{3\,n}+ax^2x^n+\left(a^2x^2x^{2\,n}+x^2\right)\sqrt{a^2x^{2\,n}+1}}\,dx-n\int \frac{1}{a^2x^2x^{2\,n}+x^2}\,dx-\frac{n+\log\left(ax^n+\sqrt{a^2x^{2\,n}+1}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^n)/x^2,x, algorithm="maxima")
```

```
[Out] a*n*integrate(x^n/(a^3*x^2*x^(3*n) + a*x^2*x^n + (a^2*x^2*x^(2*n) + x^2)*sq
rt(a^2*x^(2*n) + 1)), x) - n*integrate(1/(a^2*x^2*x^(2*n) + x^2), x) - (n +
 log(a*x^n + sqrt(a^2*x^(2*n) + 1)))/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{asinh}(a\,x^n)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x^n)/x^2,x)
```

```
[Out] int(asinh(a*x^n)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}(ax^n)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x**n)/x**2,x)
```

```
[Out] Integral(asinh(a*x**n)/x**2, x)
```

3.313 $\int \frac{\sinh^{-1}(ax^n)}{x^3} dx$

Optimal. Leaf size=68

$$-\frac{anx^{n-2}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}\left(1-\frac{2}{n}\right);\frac{1}{2}\left(3-\frac{2}{n}\right);-a^2x^{2n}\right)}{2(2-n)}-\frac{\sinh^{-1}(ax^n)}{2x^2}$$

```
[Out] -1/2*arcsinh(a*x^n)/x^2-1/2*a*n*x^(-2+n)*hypergeom([1/2, 1/2-1/n],[3/2-1/n]
,-a^2*x^(2*n))/(2-n)
```

Rubi [A] time = 0.04, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5902, 12, 364}

$$-\frac{anx^{n-2}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}\left(1-\frac{2}{n}\right);\frac{1}{2}\left(3-\frac{2}{n}\right);-a^2x^{2n}\right)}{2(2-n)}-\frac{\sinh^{-1}(ax^n)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x^n]/x^3,x]
```

```
[Out] -ArcSinh[a*x^n]/(2*x^2) - (a*n*x^(-2 + n)*Hypergeometric2F1[1/2, (1 - 2/n)/
2, (3 - 2/n)/2, -(a^2*x^(2*n))])/(2*(2 - n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5902

```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcSinh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/Sqrt[1 + u^2], x], x
], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
 x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u
, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^{-1}(ax^n)}{x^3}\,dx &= -\frac{\sinh^{-1}(ax^n)}{2x^2}+\frac{1}{2}\int \frac{anx^{-3+n}}{\sqrt{1+a^2x^{2n}}}\,dx\\ &= -\frac{\sinh^{-1}(ax^n)}{2x^2}+\frac{1}{2}(an)\int \frac{x^{-3+n}}{\sqrt{1+a^2x^{2n}}}\,dx\\ &= -\frac{\sinh^{-1}(ax^n)}{2x^2}-\frac{anx^{-2+n}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}\left(1-\frac{2}{n}\right);\frac{1}{2}\left(3-\frac{2}{n}\right);-a^2x^{2n}\right)}{2(2-n)}\end{aligned}$$

Mathematica [A] time = 0.05, size = 62, normalized size = 0.91

$$\frac{anx^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-\frac{1}{n};\frac{3}{2}-\frac{1}{n};-a^2x^{2n}\right)-(n-2)\sinh^{-1}(ax^n)}{2(n-2)x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x^n]/x^3,x]
[Out] (-((-2 + n)*ArcSinh[a*x^n]) + a*n*x^n*Hypergeometric2F1[1/2, 1/2 - n^(-1),
3/2 - n^(-1), -(a^2*x^(2*n))])/(2*(-2 + n)*x^2)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^n)/x^3,x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}(ax^n)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^n)/x^3,x, algorithm="giac")
[Out] integrate(arcsinh(a*x^n)/x^3, x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsinh}(a\,x^n)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x^n)/x^3,x)
[Out] int(arcsinh(a*x^n)/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$an\int \frac{x^n}{2\left(a^3x^3x^{3\,n}+ax^3x^n+\left(a^2x^3x^{2\,n}+x^3\right)\sqrt{a^2x^{2\,n}+1}\right)}\,dx-n\int \frac{1}{2\left(a^2x^3x^{2\,n}+x^3\right)}\,dx-\frac{n+2\log\left(ax^n+\sqrt{a^2x^{2\,n}\dots}\right.}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^n)/x^3,x, algorithm="maxima")
[Out] a*n*integrate(1/2*x^n/(a^3*x^3*x^(3*n) + a*x^3*x^n + (a^2*x^3*x^(2*n) + x^3
)*sqrt(a^2*x^(2*n) + 1)), x) - n*integrate(1/2/(a^2*x^3*x^(2*n) + x^3), x)
- 1/4*(n + 2*log(a*x^n + sqrt(a^2*x^(2*n) + 1)))/x^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{asinh}(a\,x^n)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x^n)/x^3,x)
```

```
[Out] int(asinh(a*x^n)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}(ax^n)}{x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x**n)/x**3,x)
```

```
[Out] Integral(asinh(a*x**n)/x**3, x)
```

3.314 $\int \left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^4\,dx$

Optimal. Leaf size=153

$$-\frac{192b^3\sqrt{d^2x^4+2idx^2}\,\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)}{dx}+48b^2x\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^2-\frac{8b\sqrt{d^2x^4+2idx^2}\,\left(a+ib\sin^{-1}\right.}{dx}$$

```
[Out] 384*b^4*x+48*b^2*x*(a-I*b*arcsin(-1+I*d*x^2))^2+x*(a-I*b*arcsin(-1+I*d*x^2)
)^4-192*b^3*(a-I*b*arcsin(-1+I*d*x^2))*(2*I*d*x^2+d^2*x^4)^(1/2)/d/x-8*b*(a
-I*b*arcsin(-1+I*d*x^2))^3*(2*I*d*x^2+d^2*x^4)^(1/2)/d/x
```

Rubi [A] time = 0.04, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {4814, 8}

$$-\frac{192b^3\sqrt{d^2x^4+2idx^2}\,\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)}{dx}+48b^2x\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^2-\frac{8b\sqrt{d^2x^4+2idx^2}\,\left(a+ib\sin^{-1}\right.}{dx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*b*ArcSin[1 - I*d*x^2])^4,x]
```

```
[Out] 384*b^4*x - (192*b^3*Sqrt[(2*I)*d*x^2 + d^2*x^4]*(a + I*b*ArcSin[1 - I*d*x^
2]))/(d*x) + 48*b^2*x*(a + I*b*ArcSin[1 - I*d*x^2])^2 - (8*b*Sqrt[(2*I)*d*x
^2 + d^2*x^4]*(a + I*b*ArcSin[1 - I*d*x^2])^3)/(d*x) + x*(a + I*b*ArcSin[1
- I*d*x^2])^4
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4814

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcSin[c + d*x^2])^n, x] + (-Dist[4*b^2*n*(n - 1), Int[(a + b*ArcSin[
c + d*x^2])^(n - 2), x], x] + Simp[(2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*(a + b
*ArcSin[c + d*x^2])^(n - 1))/(d*x), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^
2, 1] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}
\int \left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^4\,dx &= -\frac{8b\sqrt{2idx^2+d^2x^4}\,\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^3}{dx}+x\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^4+(4\\
&= -\frac{192b^3\sqrt{2idx^2+d^2x^4}\,\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)}{dx}+48b^2x\left(a+ib\sin^{-1}\left(1-idx^2\right)\right.\\
&= 384b^4x-\frac{192b^3\sqrt{2idx^2+d^2x^4}\,\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)}{dx}+48b^2x\left(a+ib\sin^{-1}\left(\right.\right.
\end{aligned}$$

Mathematica [A] time = 0.13, size = 149, normalized size = 0.97

$$48b^2\left(-\frac{4b\sqrt{dx^2\left(dx^2+2i\right)}\,\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)}{dx}+x\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^2+8b^2x\right)+x\left(a+ib\sin^{-1}\left(1-id\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*b*ArcSin[1 - I*d*x^2])^4,x]
```

```
[Out] (-8*b*Sqrt[d*x^2*(2*I + d*x^2)]*(a + I*b*ArcSin[1 - I*d*x^2])^3)/(d*x) + x*
(a + I*b*ArcSin[1 - I*d*x^2])^4 + 48*b^2*(8*b^2*x - (4*b*Sqrt[d*x^2*(2*I +
d*x^2)]*(a + I*b*ArcSin[1 - I*d*x^2]))/(d*x) + x*(a + I*b*ArcSin[1 - I*d*x^
2])^2)
```

fricas [B] time = 0.52, size = 269, normalized size = 1.76

$$\frac{b^4dx\log\left(dx^2+\sqrt{d^2x^2+2id}\,x+i\right)^4+4\left(ab^3dx-2\sqrt{d^2x^2+2id}\,b^4\right)\log\left(dx^2+\sqrt{d^2x^2+2id}\,x+i\right)^3+\left(a^4+4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(I+d*x^2))^4,x, algorithm="fricas")
```

```
[Out] (b^4*d*x*log(d*x^2 + sqrt(d^2*x^2 + 2*I*d)*x + I)^4 + 4*(a*b^3*d*x - 2*sqrt
(d^2*x^2 + 2*I*d)*b^4)*log(d*x^2 + sqrt(d^2*x^2 + 2*I*d)*x + I)^3 + (a^4 +
48*a^2*b^2 + 384*b^4)*d*x - 6*(4*sqrt(d^2*x^2 + 2*I*d)*a*b^3 - (a^2*b^2 + 8
*b^4)*d*x)*log(d*x^2 + sqrt(d^2*x^2 + 2*I*d)*x + I)^2 + 4*((a^3*b + 24*a*b^
3)*d*x - 6*(a^2*b^2 + 8*b^4)*sqrt(d^2*x^2 + 2*I*d))*log(d*x^2 + sqrt(d^2*x^
2 + 2*I*d)*x + I) - 8*(a^3*b + 24*a*b^3)*sqrt(d^2*x^2 + 2*I*d))/d
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(I+d*x^2))^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [d,x]=[45,-28]Bad conditionned root j= 1 value -35280.3655931 rati
o 0.379661795171 mindist 0.443514529616Bad conditionned root j= 1 value -51
05.29327315 ratio 1.07361778233 mindist 2.29350729132Warning, choosing root
 of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-8,[3,6]%%%}+%%%{-32,[1,2]%%
%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{16,[0,0]%%%}] at parameters value
s [7,-27]Warning, need to choose a branch for the root of a polynomial with
 parameters. This might be wrong.The choice was done assuming [d,t_nostep]=
[79,3]schur row 3 8.74347e-08Bad conditionned root j= 2 value -151313.41286
2 ratio 11.2206791301 mindist 48.7986537395Warning, choosing root of [1,0,%
%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-8,[3,6]%%%}+%%%{-32,[1,2]%%%},%%%{-3,
[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{16,[0,0]%%%}] at parameters values [63,-49]
Warning, need to choose a branch for the root of a polynomial with paramete
rs. This might be wrong.The choice was done assuming [d,t_nostep]=[-27,9]Ba
d conditionned root j= 2 value 147025.62453 ratio 5.74493624992 mindist 24.
9427695529Warning, choosing root of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},
%%%{-8,[3,6]%%%}+%%%{-32,[1,2]%%%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{1
6,[0,0]%%%}] at parameters values [-30,70]Evaluation time: 29.4sym2poly/r2s
ym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument V
alue
```

maple [F] time = 0.23, size = 0, normalized size = 0.00

$$\int \left(a+b\,\mathrm{arcsinh}\left(d\,x^2+i\right)\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(I+d*x^2))^4,x)
```

```
[Out] int((a+b*arcsinh(I+d*x^2))^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b^4 x \log\left(dx^2 + \sqrt{dx^2 + 2i}\,\sqrt{d}\,x + i\right)^4 + 4\left(x \operatorname{arsinh}\left(dx^2 + i\right) - \frac{2\left(d^{\frac{3}{2}}x^2 + 2i\sqrt{d}\right)}{\sqrt{dx^2 + 2i}\,d}\right)a^3 b + a^4 x + \int \frac{\left(4\left(ab^3d^2 - 2\,b^4d^2\right)x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(I+d*x^2))^4,x, algorithm="maxima")
```

```
[Out] b^4*x*log(d*x^2 + sqrt(d*x^2 + 2*I)*sqrt(d)*x + I)^4 + 4*(x*arcsinh(d*x^2 +
 I) - 2*(d^(3/2)*x^2 + 2*I*sqrt(d))/(sqrt(d*x^2 + 2*I)*d))*a^3*b + a^4*x +
integrate(((4*(a*b^3*d^2 - 2*b^4*d^2)*x^4 - 8*a*b^3 + (12*I*a*b^3*d - 16*I*
b^4*d)*x^2 + (4*(a*b^3*d^(3/2) - 2*b^4*d^(3/2))*x^3 + (8*I*a*b^3*sqrt(d) -
8*I*b^4*sqrt(d))*x)*sqrt(d*x^2 + 2*I))*log(d*x^2 + sqrt(d*x^2 + 2*I)*sqrt(d
)*x + I)^3 + (6*a^2*b^2*d^2*x^4 + 18*I*a^2*b^2*d*x^2 - 12*a^2*b^2 + (6*a^2*
b^2*d^(3/2)*x^3 + 12*I*a^2*b^2*sqrt(d)*x)*sqrt(d*x^2 + 2*I))*log(d*x^2 + sq
rt(d*x^2 + 2*I)*sqrt(d)*x + I)^2)/(d^2*x^4 + 3*I*d*x^2 + (d^(3/2)*x^3 + 2*I
*sqrt(d)*x)*sqrt(d*x^2 + 2*I) - 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a + b\operatorname{asinh}\left(d\,x^2 + 1\mathrm{i}\right)\right)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(d*x^2 + 1i))^4,x)
```

```
[Out] int((a + b*asinh(d*x^2 + 1i))^4, x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(I+d*x**2))**4,x)
```

```
[Out] Exception raised: TypeError
```

3.315 $\int \left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^3\,dx$

Optimal. Leaf size=129

$$24ab^2x-\frac{6b\sqrt{d^2x^4+2idx^2}\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^2}{dx}+x\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^3-\frac{48b^3\sqrt{d^2x^4+2idx^2}}{dx}+24ib^3x$$

```
[Out] 24*a*b^2*x-24*I*b^3*x*arcsin(-1+I*d*x^2)+x*(a-I*b*arcsin(-1+I*d*x^2))^3-48*
b^3*(2*I*d*x^2+d^2*x^4)^(1/2)/d/x-6*b*(a-I*b*arcsin(-1+I*d*x^2))^2*(2*I*d*x
^2+d^2*x^4)^(1/2)/d/x
```

Rubi [A] time = 0.06, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4814, 4840, 12, 1588}

$$24ab^2x-\frac{6b\sqrt{d^2x^4+2idx^2}\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^2}{dx}+x\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^3-\frac{48b^3\sqrt{d^2x^4+2idx^2}}{dx}+24ib^3x$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*b*ArcSin[1 - I*d*x^2])^3,x]
```

```
[Out] 24*a*b^2*x - (48*b^3*Sqrt[(2*I)*d*x^2 + d^2*x^4])/(d*x) + (24*I)*b^3*x*ArcS
in[1 - I*d*x^2] - (6*b*Sqrt[(2*I)*d*x^2 + d^2*x^4]*(a + I*b*ArcSin[1 - I*d*
x^2])^2)/(d*x) + x*(a + I*b*ArcSin[1 - I*d*x^2])^3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 4814

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcSin[c + d*x^2])^n, x] + (-Dist[4*b^2*n*(n - 1), Int[(a + b*ArcSin[
c + d*x^2])^(n - 2), x], x] + Simp[(2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*(a + b
*ArcSin[c + d*x^2])^(n - 1))/(d*x), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^
2, 1] && GtQ[n, 1]
```

Rule 4840

```
Int[ArcSin[u_], x_Symbol] :> Simp[x*ArcSin[u], x] - Int[SimplifyIntegrand[(
x*D[u, x])/Sqrt[1 - u^2], x], x] /; InverseFunctionFreeQ[u, x] &&  !Functio
nOfExponentialQ[u, x]
```

Rubi steps

$$\int \left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^3 dx = -\frac{6b\sqrt{2idx^2+d^2x^4}\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^2}{dx} + x\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^3 + (2$$

$$= 24ab^2x - \frac{6b\sqrt{2idx^2+d^2x^4}\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^2}{dx} + x\left(a+ib\sin^{-1}\left(1-idx\right.\right.$$

$$= 24ab^2x + 24ib^3x\sin^{-1}\left(1-idx^2\right) - \frac{6b\sqrt{2idx^2+d^2x^4}\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^2}{dx}$$

$$= 24ab^2x + 24ib^3x\sin^{-1}\left(1-idx^2\right) - \frac{6b\sqrt{2idx^2+d^2x^4}\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^2}{dx}$$

$$= 24ab^2x - \frac{48b^3\sqrt{2idx^2+d^2x^4}}{dx} + 24ib^3x\sin^{-1}\left(1-idx^2\right) - \frac{6b\sqrt{2idx^2+d^2x^4}\left(a\right.}{}$$

Mathematica [A] time = 0.15, size = 180, normalized size = 1.40

$$\frac{adx^2\left(a^2+24b^2\right)-6b\left(a^2+8b^2\right)\sqrt{dx^2\left(dx^2+2i\right)}+3ib\sin^{-1}\left(1-idx^2\right)\left(a^2dx^2-4ab\sqrt{dx^2\left(dx^2+2i\right)}+8b^2dx^2\right)}{dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*b*ArcSin[1 - I*d*x^2])^3,x]
```

```
[Out] (a*(a^2 + 24*b^2)*d*x^2 - 6*b*(a^2 + 8*b^2)*Sqrt[d*x^2*(2*I + d*x^2)] + (3*
I)*b*(a^2*d*x^2 + 8*b^2*d*x^2 - 4*a*b*Sqrt[d*x^2*(2*I + d*x^2)])*ArcSin[1 -
 I*d*x^2] + 3*b^2*(-(a*d*x^2) + 2*b*Sqrt[d*x^2*(2*I + d*x^2)])*ArcSin[1 - I
*d*x^2]^2 - I*b^3*d*x^2*ArcSin[1 - I*d*x^2]^3)/(d*x)
```

fricas [A] time = 0.48, size = 188, normalized size = 1.46

$$\frac{b^3dx\log\left(dx^2+\sqrt{d^2x^2+2i\,d}\,x+i\right)^3+\left(a^3+24\,ab^2\right)dx+3\left(ab^2dx-2\sqrt{d^2x^2+2i\,d}\,b^3\right)\log\left(dx^2+\sqrt{d^2x^2+2i\,d}\,x\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(I+d*x^2))^3,x, algorithm="fricas")
```

```
[Out] (b^3*d*x*log(d*x^2 + sqrt(d^2*x^2 + 2*I*d)*x + I)^3 + (a^3 + 24*a*b^2)*d*x
+ 3*(a*b^2*d*x - 2*sqrt(d^2*x^2 + 2*I*d)*b^3)*log(d*x^2 + sqrt(d^2*x^2 + 2*
I*d)*x + I)^2 - 3*(4*sqrt(d^2*x^2 + 2*I*d)*a*b^2 - (a^2*b + 8*b^3)*d*x)*log
(d*x^2 + sqrt(d^2*x^2 + 2*I*d)*x + I) - 6*sqrt(d^2*x^2 + 2*I*d)*(a^2*b + 8*
b^3))/d
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(I+d*x^2))^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [d,x]=[45,-28]Bad conditionned root j= 1 value -35280.3655931 rati
o 0.379661795171 mindist 0.443514529616Bad conditionned root j= 1 value -51
05.29327315 ratio 1.07361778233 mindist 2.29350729132Warning, choosing root
```

```
of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-8,[3,6]%%%}+%%%{-32,[1,2]%%
%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{16,[0,0]%%%}] at parameters value
s [7,-27]Warning, need to choose a branch for the root of a polynomial with
 parameters. This might be wrong.The choice was done assuming [d,t_nostep]=
[97,-52]Francis algorithm failure for[1.0,0.0,-137282971022,-2.76877787308e
+16,-1.57055117809e+21]Warning, choosing root of [1,0,%%%{-6,[2,4]%%%}+%%%{
-8,[0,0]%%%},%%%{-8,[3,6]%%%}+%%%{-32,[1,2]%%%},%%%{-3,[4,8]%%%}+%%%{-24,[2
,4]%%%}+%%%{16,[0,0]%%%}] at parameters values [63,-49]Warning, need to cho
ose a branch for the root of a polynomial with parameters. This might be wr
ong.The choice was done assuming [d,t_nostep]=[-9,-4]Francis algorithm fail
ure for[1.0,0.0,-129654000008,2.54121840047e+16,-1.40084664352e+21]Warning,
 choosing root of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-8,[3,6]%%%}+%
%%{-32,[1,2]%%%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{16,[0,0]%%%}] at pa
rameters values [-30,70]Evaluation time: 15.95sym2poly/r2sym(const gen & e,
const index_m & i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.21, size = 0, normalized size = 0.00

$$\int \left(a+b\,\mathrm{arcsinh}\left(d\,x^{2}+i\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(I+d*x^2))^3,x)
```

```
[Out] int((a+b*arcsinh(I+d*x^2))^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b^3x\log\left(dx^2+\sqrt{dx^2+2i}\,\sqrt{d}\,x+i\right)^3+3\left(x\,\mathrm{arsinh}\left(dx^2+i\right)-\frac{2\left(d^{\frac{3}{2}}x^2+2i\,\sqrt{d}\right)}{\sqrt{dx^2+2i}\,d}\right)a^2b+a^3x+\int\frac{\left(3\left(ab^2d^2-2\,b^3d\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(I+d*x^2))^3,x, algorithm="maxima")
```

```
[Out] b^3*x*log(d*x^2 + sqrt(d*x^2 + 2*I)*sqrt(d)*x + I)^3 + 3*(x*arcsinh(d*x^2 +
 I) - 2*(d^(3/2)*x^2 + 2*I*sqrt(d))/(sqrt(d*x^2 + 2*I)*d))*a^2*b + a^3*x +
integrate((3*(a*b^2*d^2 - 2*b^3*d^2)*x^4 - 6*a*b^2 + (9*I*a*b^2*d - 12*I*b^
3*d)*x^2 + (3*(a*b^2*d^(3/2) - 2*b^3*d^(3/2))*x^3 + (6*I*a*b^2*sqrt(d) - 6*
I*b^3*sqrt(d))*x)*sqrt(d*x^2 + 2*I))*log(d*x^2 + sqrt(d*x^2 + 2*I)*sqrt(d)*
x + I)^2/(d^2*x^4 + 3*I*d*x^2 + (d^(3/2)*x^3 + 2*I*sqrt(d)*x)*sqrt(d*x^2 +
2*I) - 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+b\,\mathrm{asinh}\left(d\,x^2+1{}\mathrm{i}\right)\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(d*x^2 + 1i))^3,x)
```

```
[Out] int((a + b*asinh(d*x^2 + 1i))^3, x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(I+d*x**2))**3,x)
```

```
[Out] Exception raised: TypeError
```

3.316 $\int \left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^2\,dx$

Optimal. Leaf size=76

$$-\frac{4b\sqrt{d^2x^4+2idx^2}\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)}{dx}+x\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^2+8b^2x$$

```
[Out] 8*b^2*x+x*(a-I*b*arcsin(-1+I*d*x^2))^2-4*b*(a-I*b*arcsin(-1+I*d*x^2))*(2*I*
d*x^2+d^2*x^4)^(1/2)/d/x
```

Rubi [A] time = 0.02, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {4814, 8}

$$-\frac{4b\sqrt{d^2x^4+2idx^2}\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)}{dx}+x\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^2+8b^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*b*ArcSin[1 - I*d*x^2])^2,x]
```

```
[Out] 8*b^2*x - (4*b*Sqrt[(2*I)*d*x^2 + d^2*x^4]*(a + I*b*ArcSin[1 - I*d*x^2]))/(
d*x) + x*(a + I*b*ArcSin[1 - I*d*x^2])^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4814

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcSin[c + d*x^2])^n, x] + (-Dist[4*b^2*n*(n - 1), Int[(a + b*ArcSin[
c + d*x^2])^(n - 2), x], x] + Simp[(2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*(a + b
*ArcSin[c + d*x^2])^(n - 1))/(d*x), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^
2, 1] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^2\,dx &= -\frac{4b\sqrt{2idx^2+d^2x^4}\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)}{dx}+x\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^2+(8b\\ &= 8b^2x-\frac{4b\sqrt{2idx^2+d^2x^4}\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)}{dx}+x\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^2\end{aligned}$$

Mathematica [A] time = 0.03, size = 76, normalized size = 1.00

$$-\frac{4b\sqrt{d^2x^4+2idx^2}\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)}{dx}+x\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^2+8b^2x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*b*ArcSin[1 - I*d*x^2])^2,x]
```

```
[Out] 8*b^2*x - (4*b*Sqrt[(2*I)*d*x^2 + d^2*x^4]*(a + I*b*ArcSin[1 - I*d*x^2]))/(
d*x) + x*(a + I*b*ArcSin[1 - I*d*x^2])^2
```

fricas [A] time = 0.54, size = 114, normalized size = 1.50

$$\frac{b^2dx\log\left(dx^2+\sqrt{d^2x^2+2i\,d}\,x+i\right)^2+\left(a^2+8\,b^2\right)dx-4\,\sqrt{d^2x^2+2i\,d}\,ab+2\left(abdx-2\,\sqrt{d^2x^2+2i\,d}\,b^2\right)\log\left(d\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(I+d*x^2))^2,x, algorithm="fricas")
```

```
[Out] (b^2*d*x*log(d*x^2 + sqrt(d^2*x^2 + 2*I*d)*x + I)^2 + (a^2 + 8*b^2)*d*x - 4
*sqrt(d^2*x^2 + 2*I*d)*a*b + 2*(a*b*d*x - 2*sqrt(d^2*x^2 + 2*I*d)*b^2)*log(
d*x^2 + sqrt(d^2*x^2 + 2*I*d)*x + I))/d
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(I+d*x^2))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [d,x]=[45,-28]Bad conditionned root j= 1 value -35280.3655931 rati
o 0.379661795171 mindist 0.443514529616Bad conditionned root j= 1 value -51
05.29327315 ratio 1.07361778233 mindist 2.29350729132Warning, choosing root
 of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-8,[3,6]%%%}+%%%{-32,[1,2]%%
%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{16,[0,0]%%%}] at parameters value
s [7,-27]Warning, need to choose a branch for the root of a polynomial with
 parameters. This might be wrong.The choice was done assuming [d,t_nostep]=
[60,97]schur row 1 7.48504e-11Francis algorithm not precise enough for[1.0,
0.0,-1.91223246961e+12,-1.43937562454e+18,-3.04719418157e+23]Bad conditionn
ed root j= 2 value -564549.069246 ratio 4.13534933689 mindist 8.26009499958
schur row 3 8.32254e-09schur row 3 8.32254e-09Francis algorithm not precise
 enough for[1.0,0.0,-137282971022,-2.76877787308e+16,-1.57055117809e+21]Bad
 conditionned root j= 3 value -151279.357647 ratio 3.67253338015 mindist 17
.3373140811Warning, choosing root of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%}
,%%%{-8,[3,6]%%%}+%%%{-32,[1,2]%%%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{
16,[0,0]%%%}] at parameters values [63,-49]Warning, need to choose a branch
 for the root of a polynomial with parameters. This might be wrong.The choi
ce was done assuming [d,t_nostep]=[-20,14]schur row 1 6.86402e-10Francis al
gorithm not precise enough for[1.0,0.0,-129654000008,2.54121840047e+16,-1.4
0084664352e+21]Bad conditionned root j= 2 value 146992.858887 ratio 1.59770
7895 mindist 8.30647902455Warning, choosing root of [1,0,%%%{-6,[2,4]%%%}+%
%%{-8,[0,0]%%%},%%%{-8,[3,6]%%%}+%%%{-32,[1,2]%%%},%%%{-3,[4,8]%%%}+%%%{-24
,[2,4]%%%}+%%%{16,[0,0]%%%}] at parameters values [-30,70]Evaluation time:
6.99sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error
: Bad Argument Value
```

maple [F] time = 0.20, size = 0, normalized size = 0.00

$$\int \left(a+b\,\mathrm{arcsinh}\left(d\,x^2+i\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(I+d*x^2))^2,x)
```

```
[Out] int((a+b*arcsinh(I+d*x^2))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$2\left(x\operatorname{arsinh}\left(dx^2+i\right)-\frac{2\left(d^{\frac{3}{2}}x^2+2i\sqrt{d}\right)}{\sqrt{dx^2+2i}\,d}\right)ab+\left(x\log\left(dx^2+\sqrt{dx^2+2i}\,\sqrt{d}\,x+i\right)^2-\int\frac{\left(4\,d^2x^4+8i\,dx^2+\left(4\,d^{\frac{3}{2}}x^3\right.\right.}{d^2x^4+3i}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(I+d*x^2))^2,x, algorithm="maxima")
```

```
[Out] 2*(x*arcsinh(d*x^2 + I) - 2*(d^(3/2)*x^2 + 2*I*sqrt(d))/(sqrt(d*x^2 + 2*I)*
d))*a*b + (x*log(d*x^2 + sqrt(d*x^2 + 2*I)*sqrt(d)*x + I)^2 - integrate((4*
d^2*x^4 + 8*I*d*x^2 + (4*d^(3/2)*x^3 + 4*I*sqrt(d)*x)*sqrt(d*x^2 + 2*I))*lo
g(d*x^2 + sqrt(d*x^2 + 2*I)*sqrt(d)*x + I)/(d^2*x^4 + 3*I*d*x^2 + (d^(3/2)*
x^3 + 2*I*sqrt(d)*x)*sqrt(d*x^2 + 2*I) - 2), x))*b^2 + a^2*x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\left(a+b\,\operatorname{asinh}\left(d\,x^2+1\mathrm{i}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(d*x^2 + 1i))^2,x)
```

```
[Out] int((a + b*asinh(d*x^2 + 1i))^2, x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(I+d*x**2))**2,x)
```

```
[Out] Exception raised: TypeError
```

3.317 $\int \left(a+ib\sin^{-1}\left(1-idx^2\right)\right) dx$

Optimal. Leaf size=50

$$ax-\frac{2b\sqrt{d^2x^4+2idx^2}}{dx}+ibx\sin^{-1}\left(1-idx^2\right)$$

```
[Out] a*x-I*b*x*arcsin(-1+I*d*x^2)-2*b*(2*I*d*x^2+d^2*x^4)^(1/2)/d/x
```

Rubi [A] time = 0.04, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4840, 12, 1588}

$$ax-\frac{2b\sqrt{d^2x^4+2idx^2}}{dx}+ibx\sin^{-1}\left(1-idx^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[a + I*b*ArcSin[1 - I*d*x^2],x]
```

```
[Out] a*x - (2*b*Sqrt[(2*I)*d*x^2 + d^2*x^4])/(d*x) + I*b*x*ArcSin[1 - I*d*x^2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 4840

```
Int[ArcSin[u_], x_Symbol] :> Simp[x*ArcSin[u], x] - Int[SimplifyIntegrand[(
x*D[u, x])/Sqrt[1 - u^2], x], x] /; InverseFunctionFreeQ[u, x] &&  !Functio
nOfExponentialQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \left(a+ib\sin^{-1}\left(1-idx^2\right)\right) dx &= ax+(ib)\int \sin^{-1}\left(1-idx^2\right) dx\\
&= ax+ibx\sin^{-1}\left(1-idx^2\right)-(ib)\int -\frac{2idx^2}{\sqrt{2idx^2+d^2x^4}} dx\\
&= ax+ibx\sin^{-1}\left(1-idx^2\right)-(2bd)\int \frac{x^2}{\sqrt{2idx^2+d^2x^4}} dx\\
&= ax-\frac{2b\sqrt{2idx^2+d^2x^4}}{dx}+ibx\sin^{-1}\left(1-idx^2\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 48, normalized size = 0.96

$$ax-\frac{2b\sqrt{dx^2\left(dx^2+2i\right)}}{dx}+ibx\sin^{-1}\left(1-idx^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + I*b*ArcSin[1 - I*d*x^2],x]
```

```
[Out] a*x - (2*b*Sqrt[d*x^2*(2*I + d*x^2)])/(d*x) + I*b*x*ArcSin[1 - I*d*x^2]
```

fricas [A] time = 0.45, size = 52, normalized size = 1.04

$$\frac{bdx\log\left(dx^2+\sqrt{d^2x^2+2id}\,x+i\right)+adx-2\sqrt{d^2x^2+2id}\,b}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arcsinh(I+d*x^2),x, algorithm="fricas")
```

```
[Out] (b*d*x*log(d*x^2 + sqrt(d^2*x^2 + 2*I*d)*x + I) + a*d*x - 2*sqrt(d^2*x^2 +
2*I*d)*b)/d
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arcsinh(I+d*x^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [d,x]=[45,-28]Bad conditionned root j= 1 value -35280.3655931 rati
o 0.379661795171 mindist 0.443514529616Bad conditionned root j= 1 value -51
05.29327315 ratio 1.07361778233 mindist 2.29350729132Warning, choosing root
 of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-8,[3,6]%%%}+%%%{-32,[1,2]%%
%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{16,[0,0]%%%}] at parameters value
s [7,-27]Warning, need to choose a branch for the root of a polynomial with
 parameters. This might be wrong.The choice was done assuming [d,x]=[29,3]s
chur row 1 3.19975e-11schur row 2 -2.1116e-07Francis algorithm not precise
enough for[1.0,0.0,-137282971022,-2.76877787308e+16,-1.57055117809e+21]Bad
conditionned root j= 1 value -151344.383904 ratio 18.0968847681 mindist 79.
7696956145Warning, choosing root of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},
%%%{-8,[3,6]%%%}+%%%{-32,[1,2]%%%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{1
6,[0,0]%%%}] at parameters values [63,-49]Evaluation time: 1.26sym2poly/r2s
ym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument V
alue
```

maple [A] time = 0.02, size = 47, normalized size = 0.94

$$ax+b\left(x\,\mathrm{arcsinh}\left(d\,x^2+i\right)-\frac{2x\left(d\,x^2+2i\right)}{\sqrt{d^2x^4+2id\,x^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arcsinh(I+d*x^2),x)
```

```
[Out] a*x+b*(x*arcsinh(I+d*x^2)-2/(2*I*d*x^2+d^2*x^4)^(1/2)*x*(d*x^2+2*I))
```

maxima [A] time = 0.73, size = 44, normalized size = 0.88

$$\left(x\,\mathrm{arsinh}\left(dx^2+i\right)-\frac{2\left(d^{\frac{3}{2}}x^2+2i\sqrt{d}\right)}{\sqrt{dx^2+2i}\,d}\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arcsinh(I+d*x^2),x, algorithm="maxima")
```

```
[Out] (x*arcsinh(d*x^2 + I) - 2*(d^(3/2)*x^2 + 2*I*sqrt(d))/(sqrt(d*x^2 + 2*I)*d)
)*b + a*x
```

mupad [B] time = 0.53, size = 39, normalized size = 0.78

$$a\,x+b\,x\,\mathrm{asinh}\left(d\,x^2+1{}\mathrm{i}\right)-\frac{2\,b\,\sqrt{{\left(d\,x^2+1{}\mathrm{i}\right)}^2+1}}{d\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*asinh(d*x^2 + 1i),x)
```

```
[Out] a*x + b*x*asinh(d*x^2 + 1i) - (2*b*((d*x^2 + 1i)^2 + 1)^(1/2))/(d*x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*asinh(I+d*x**2),x)
```

```
[Out] Exception raised: TypeError
```

3.318 $\int \frac{1}{a+ib\sin^{-1}(1-idx^2)}\,dx$

Optimal. Leaf size=194

$$\frac{x\left(-\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)\mathrm{Ci}\left(-\frac{i(a+ib\sin^{-1}(1-idx^2))}{2b}\right)}{2b\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)\right)}-\frac{x\left(\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)\mathrm{Si}\left(\frac{ia}{2b}-\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)}{2b\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)\right)}$$

```
[Out] 1/2*x*Ci(-1/2*I*(a-I*b*arcsin(-1+I*d*x^2))/b)*(I*cosh(1/2*a/b)-sinh(1/2*a/b
))/b/(cos(1/2*arcsin(-1+I*d*x^2))+sin(1/2*arcsin(-1+I*d*x^2)))-1/2*x*Si(1/2
*I*a/b+1/2*arcsin(-1+I*d*x^2))*(I*cosh(1/2*a/b)+sinh(1/2*a/b))/b/(cos(1/2*a
rcsin(-1+I*d*x^2))+sin(1/2*arcsin(-1+I*d*x^2)))
```

Rubi [A] time = 0.05, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {4816}

$$\frac{x\left(-\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)\text{CosIntegral}\left(-\frac{i(a+ib\sin^{-1}(1-idx^2))}{2b}\right)}{2b\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)\right)}-\frac{x\left(\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)\mathrm{Si}\left(\frac{ia}{2b}-\frac{1}{2}\sin^{-1}\left(1-\right.\right.}{2b\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-i\right.\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*b*ArcSin[1 - I*d*x^2])^(-1),x]
```

```
[Out] (x*CosIntegral[((-I/2)*(a + I*b*ArcSin[1 - I*d*x^2]))/b]*(I*Cosh[a/(2*b)] -
 Sinh[a/(2*b)]))/(2*b*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]
/2])) - (x*(I*Cosh[a/(2*b)] + Sinh[a/(2*b)])*SinIntegral[((I/2)*a)/b - ArcS
in[1 - I*d*x^2]/2])/(2*b*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x
^2]/2]))
```

Rule 4816

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(-1), x_Symbol] :> -Simp[(x
*(c*Cos[a/(2*b)] - Sin[a/(2*b)])*CosIntegral[(c/(2*b))*(a + b*ArcSin[c + d*
x^2])])/(2*b*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2])), x] -
 Simp[(x*(c*Cos[a/(2*b)] + Sin[a/(2*b)])*SinIntegral[(c/(2*b))*(a + b*ArcSi
n[c + d*x^2])])/(2*b*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2]
)), x] /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

Rubi steps

$$\int \frac{1}{a+ib\sin^{-1}\left(1-idx^2\right)}\,dx=\frac{x\mathrm{Ci}\left(-\frac{i(a+ib\sin^{-1}(1-idx^2))}{2b}\right)\left(i\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)}{2b\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)\right)}-\frac{x\left(i\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{\right.\right.}{2b\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx\right.\right.\right.}$$

Mathematica [A] time = 0.79, size = 150, normalized size = 0.77

$$\frac{x\left(\left(-\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)\mathrm{Ci}\left(\frac{1}{2}\left(\sin^{-1}\left(1-idx^2\right)-\frac{ia}{b}\right)\right)+\left(-\sinh\left(\frac{a}{2b}\right)-i\cosh\left(\frac{a}{2b}\right)\right)\mathrm{Si}\left(\frac{ia}{2b}-\frac{1}{2}\sin^{-1}\left(1-idx\right.\right.\right.}{2b\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*b*ArcSin[1 - I*d*x^2])^(-1),x]
```

```
[Out] (x*(CosIntegral[(((-I)*a)/b + ArcSin[1 - I*d*x^2])/2]*(I*Cosh[a/(2*b)] - Si
nh[a/(2*b)]) + ((-I)*Cosh[a/(2*b)] - Sinh[a/(2*b)])*SinIntegral[((I/2)*a)/b
 - ArcSin[1 - I*d*x^2]/2]))/(2*b*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1
 - I*d*x^2]/2]))
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{b\log\left(dx^2+\sqrt{d^2x^2+2i\,d}\,x+i\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2)),x, algorithm="fricas")
```

```
[Out] integral(1/(b*log(d*x^2 + sqrt(d^2*x^2 + 2*I*d)*x + I) + a), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [d,x]=[45,-28]Bad conditionned root j= 1 value -35280.3655931 rati
o 0.379661795171 mindist 0.443514529616Bad conditionned root j= 1 value -51
05.29327315 ratio 1.07361778233 mindist 2.29350729132Warning, choosing root
 of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-8,[3,6]%%%}+%%%{-32,[1,2]%%
%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{16,[0,0]%%%}] at parameters value
s [7,-27]Warning, need to choose a branch for the root of a polynomial with
 parameters. This might be wrong.The choice was done assuming [d,t_nostep]=
[60,97]schur row 1 7.48504e-11Francis algorithm not precise enough for[1.0,
0.0,-1.91223246961e+12,-1.43937562454e+18,-3.04719418157e+23]Bad conditionn
ed root j= 2 value -564549.069246 ratio 4.13534933689 mindist 8.26009499958
schur row 3 8.32254e-09schur row 3 8.32254e-09Francis algorithm not precise
 enough for[1.0,0.0,-137282971022,-2.76877787308e+16,-1.57055117809e+21]Bad
 conditionned root j= 3 value -151279.357647 ratio 3.67253338015 mindist 17
.3373140811Warning, choosing root of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%}
,%%%{-8,[3,6]%%%}+%%%{-32,[1,2]%%%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{
16,[0,0]%%%}] at parameters values [63,-49]Warning, need to choose a branch
 for the root of a polynomial with parameters. This might be wrong.The choi
ce was done assuming [d,t_nostep]=[-20,14]schur row 1 6.86402e-10Francis al
gorithm not precise enough for[1.0,0.0,-129654000008,2.54121840047e+16,-1.4
0084664352e+21]Bad conditionned root j= 2 value 146992.858887 ratio 1.59770
7895 mindist 8.30647902455Warning, choosing root of [1,0,%%%{-6,[2,4]%%%}+%
%%{-8,[0,0]%%%},%%%{-8,[3,6]%%%}+%%%{-32,[1,2]%%%},%%%{-3,[4,8]%%%}+%%%{-24
,[2,4]%%%}+%%%{16,[0,0]%%%}] at parameters values [-30,70]Evaluation time:
3.81sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error
: Bad Argument Value
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{1}{a+b\,\mathrm{arcsinh}\left(d\,x^2+i\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(I+d*x^2)),x)
[Out] int(1/(a+b*arcsinh(I+d*x^2)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{b\,\mathrm{arsinh}\left(dx^2+i\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2)),x, algorithm="maxima")
[Out] integrate(1/(b*arcsinh(d*x^2 + I) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{a+b\,\mathrm{asinh}\left(d\,x^2+1\mathrm{i}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*asinh(d*x^2 + 1i)),x)
[Out] int(1/(a + b*asinh(d*x^2 + 1i)), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(I+d*x**2)),x)
[Out] Exception raised: TypeError
```

3.319 $\int \frac{1}{\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^2}\,dx$

Optimal. Leaf size=245

$$\frac{x\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)\mathrm{Ci}\left(-\frac{i\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)}{2b}\right)}{4b^2\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)\right)}+\frac{x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\mathrm{Si}\left(\frac{ia}{2b}-\frac{1}{2}\sin^{-1}\left(1-idx^2\right.\right.}{4b^2\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right.\right.\right.}$$

```
[Out] 1/4*x*Ci(-1/2*I*(a-I*b*arcsin(-1+I*d*x^2))/b)*(cosh(1/2*a/b)-I*sinh(1/2*a/b
))/b^2/(cos(1/2*arcsin(-1+I*d*x^2))+sin(1/2*arcsin(-1+I*d*x^2)))+1/4*x*Si(1
/2*I*a/b+1/2*arcsin(-1+I*d*x^2))*(cosh(1/2*a/b)+I*sinh(1/2*a/b))/b^2/(cos(1
/2*arcsin(-1+I*d*x^2))+sin(1/2*arcsin(-1+I*d*x^2)))-1/2*(2*I*d*x^2+d^2*x^4)
^(1/2)/b/d/x/(a-I*b*arcsin(-1+I*d*x^2))
```

Rubi [A] time = 0.04, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {4825}

$$\frac{x\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)\mathrm{CosIntegral}\left(-\frac{i\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)}{2b}\right)}{4b^2\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)\right)}+\frac{x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\mathrm{Si}\left(\frac{ia}{2b}-\frac{1}{2}\sin^{-1}\left(1\right.\right.}{4b^2\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1\right.\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*b*ArcSin[1 - I*d*x^2])^(-2),x]
```

```
[Out] -Sqrt[(2*I)*d*x^2 + d^2*x^4]/(2*b*d*x*(a + I*b*ArcSin[1 - I*d*x^2])) + (x*C
osIntegral[((-I/2)*(a + I*b*ArcSin[1 - I*d*x^2]))/b]*(Cosh[a/(2*b)] - I*Sin
h[a/(2*b)]))/(4*b^2*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2
])) + (x*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)])*SinIntegral[((I/2)*a)/b - ArcSin
[1 - I*d*x^2]/2])/(4*b^2*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x
^2]/2]))
```

Rule 4825

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(-2), x_Symbol] :> -Simp[Sq
rt[-2*c*d*x^2 - d^2*x^4]/(2*b*d*x*(a + b*ArcSin[c + d*x^2])), x] + (-Simp[(
x*(Cos[a/(2*b)] + c*Sin[a/(2*b)])*CosIntegral[(c/(2*b))*(a + b*ArcSin[c + d
*x^2])])/(4*b^2*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2])), x
] + Simp[(x*(Cos[a/(2*b)] - c*Sin[a/(2*b)])*SinIntegral[(c/(2*b))*(a + b*Ar
cSin[c + d*x^2])])/(4*b^2*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^
2]/2])), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

Rubi steps

$$\int \frac{1}{\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^2}\,dx=-\frac{\sqrt{2idx^2+d^2x^4}}{2bdx\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)}+\frac{x\mathrm{Ci}\left(-\frac{i\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)}{2b}\right)\left(\cosh\left(\frac{a}{2b}\right)-\right.}{4b^2\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\right.\right.}$$

Mathematica [A] time = 1.45, size = 197, normalized size = 0.80

$$\frac{\frac{x^2\left(\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)\mathrm{Ci}\left(\frac{1}{2}\left(\sin^{-1}\left(1-idx^2\right)-\frac{ia}{b}\right)\right)+\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\mathrm{Si}\left(\frac{ia}{2b}-\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)\right)}{\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)}-\frac{2b\sqrt{dx^2\left(dx^2+2i\right)}}{d\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)}}{4b^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*b*ArcSin[1 - I*d*x^2])^(-2),x]
```

```
[Out] ((-2*b*Sqrt[d*x^2*(2*I + d*x^2)])/(d*(a + I*b*ArcSin[1 - I*d*x^2])) + (x^2*
(CosIntegral[(((-I)*a)/b + ArcSin[1 - I*d*x^2])/2]*(Cosh[a/(2*b)] - I*Sinh[
a/(2*b)]) + (Cosh[a/(2*b)] + I*Sinh[a/(2*b)])*SinIntegral[((I/2)*a)/b - Arc
Sin[1 - I*d*x^2]/2]))/(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]
/2]))/(4*b^2*x)
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\frac{2\left(b^2 d\log\left(dx^2+\sqrt{d^2x^2+2i\,d}\,x+i\right)+abd\right)\text{integral}\left(\frac{\sqrt{d^2x^2+2i\,d}\,x}{2\,abdx^2+4i\,ab+\left(2\,b^2dx^2+4i\,b^2\right)\log\left(dx^2+\sqrt{d^2x^2+2i\,d}\,x+i\right)},x\right)-\sqrt{d^2x^2+}}{2\left(b^2d\log\left(dx^2+\sqrt{d^2x^2+2i\,d}\,x+i\right)+abd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*(b^2*d*log(d*x^2 + sqrt(d^2*x^2 + 2*I*d)*x + I) + a*b*d)*integral(sq
rt(d^2*x^2 + 2*I*d)*x/(2*a*b*d*x^2 + 4*I*a*b + (2*b^2*d*x^2 + 4*I*b^2)*log(
d*x^2 + sqrt(d^2*x^2 + 2*I*d)*x + I)), x) - sqrt(d^2*x^2 + 2*I*d))/(b^2*d*l
og(d*x^2 + sqrt(d^2*x^2 + 2*I*d)*x + I) + a*b*d)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [d,x]=[45,-28]Bad conditionned root j= 1 value -35280.3655931 rati
o 0.379661795171 mindist 0.443514529616Bad conditionned root j= 1 value -51
05.29327315 ratio 1.07361778233 mindist 2.29350729132Warning, choosing root
 of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-8,[3,6]%%%}+%%%{-32,[1,2]%%
%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{16,[0,0]%%%}] at parameters value
s [7,-27]Warning, need to choose a branch for the root of a polynomial with
 parameters. This might be wrong.The choice was done assuming [d,t_nostep]=
[79,3]schur row 3 8.74347e-08Bad conditionned root j= 2 value -151313.41286
2 ratio 11.2206791301 mindist 48.7986537395Warning, choosing root of [1,0,%
%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-8,[3,6]%%%}+%%%{-32,[1,2]%%%},%%%{-3,
[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{16,[0,0]%%%}] at parameters values [63,-49]
Warning, need to choose a branch for the root of a polynomial with paramete
rs. This might be wrong.The choice was done assuming [d,t_nostep]=[-27,9]Ba
d conditionned root j= 2 value 147025.62453 ratio 5.74493624992 mindist 24.
9427695529Warning, choosing root of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},
%%%{-8,[3,6]%%%}+%%%{-32,[1,2]%%%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{1
6,[0,0]%%%}] at parameters values [-30,70]Evaluation time: 12.09sym2poly/r2
sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument
Value
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\,\mathrm{arcsinh}\left(d\,x^2+i\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(I+d*x^2))^2,x)
```

```
[Out] int(1/(a+b*arcsinh(I+d*x^2))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{d^2x^4 + 3i\,dx^2 + \left(d^{\frac{3}{2}}x^3 + 2i\,\sqrt{d}\,x\right)\sqrt{dx^2 + 2i} - 2}{2\,abd^2x^3 + 4i\,abdx + \left(2\,b^2d^2x^3 + 4i\,b^2dx + \left(2\,b^2d^{\frac{3}{2}}x^2 + 2i\,b^2\sqrt{d}\right)\sqrt{dx^2 + 2i}\right)\log\left(dx^2 + \sqrt{dx^2 + 2i}\,\sqrt{d}\,x + i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2))^2,x, algorithm="maxima")
```

```
[Out] -(d^2*x^4 + 3*I*d*x^2 + (d^(3/2)*x^3 + 2*I*sqrt(d)*x)*sqrt(d*x^2 + 2*I) - 2
)/(2*a*b*d^2*x^3 + 4*I*a*b*d*x + (2*b^2*d^2*x^3 + 4*I*b^2*d*x + (2*b^2*d^(3
/2)*x^2 + 2*I*b^2*sqrt(d))*sqrt(d*x^2 + 2*I))*log(d*x^2 + sqrt(d*x^2 + 2*I)
*sqrt(d)*x + I) + 2*(a*b*d^(3/2)*x^2 + I*a*b*sqrt(d))*sqrt(d*x^2 + 2*I)) +
integrate((2*d^3*x^6 + 6*I*d^2*x^4 + (2*d^2*x^4 + 2*I*d*x^2 - 4)*(d*x^2 + 2
*I) + 2*(2*d^(5/2)*x^5 + 4*I*d^(3/2)*x^3 - sqrt(d)*x)*sqrt(d*x^2 + 2*I) + 8
*I)/(4*a*b*d^3*x^6 + 16*I*a*b*d^2*x^4 - 16*a*b*d*x^2 + (4*a*b*d^2*x^4 + 8*I
*a*b*d*x^2 - 4*a*b)*(d*x^2 + 2*I) + (4*b^2*d^3*x^6 + 16*I*b^2*d^2*x^4 - 16*
b^2*d*x^2 + 4*(b^2*d^2*x^4 + 2*I*b^2*d*x^2 - b^2)*(d*x^2 + 2*I) + 8*(b^2*d^
(5/2)*x^5 + 3*I*b^2*d^(3/2)*x^3 - 2*b^2*sqrt(d)*x)*sqrt(d*x^2 + 2*I))*log(d
*x^2 + sqrt(d*x^2 + 2*I)*sqrt(d)*x + I) + (8*a*b*d^(5/2)*x^5 + 24*I*a*b*d^(
3/2)*x^3 - 16*a*b*sqrt(d)*x)*sqrt(d*x^2 + 2*I)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a + b\,\mathrm{asinh}\left(d\,x^2 + 1\mathrm{i}\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*asinh(d*x^2 + 1i))^2,x)
```

```
[Out] int(1/(a + b*asinh(d*x^2 + 1i))^2, x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(I+d*x**2))**2,x)
```

```
[Out] Exception raised: TypeError
```

3.320 $\int \frac{1}{\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^3}\,dx$

Optimal. Leaf size=275

$$\frac{x\left(-\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)\mathrm{Ci}\left(-\frac{i\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)}{2b}\right)}{16b^3\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)\right)}-\frac{x\left(\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)\mathrm{Si}\left(\frac{ia}{2b}-\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right.}{16b^3\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right.\right.\right.}$$

```
[Out] -1/8*x/b^2/(a-I*b*arcsin(-1+I*d*x^2))+1/16*x*Ci(-1/2*I*(a-I*b*arcsin(-1+I*d
*x^2))/b)*(I*cosh(1/2*a/b)-sinh(1/2*a/b))/b^3/(cos(1/2*arcsin(-1+I*d*x^2))+
sin(1/2*arcsin(-1+I*d*x^2)))-1/16*x*Si(1/2*I*a/b+1/2*arcsin(-1+I*d*x^2))*(I
*cosh(1/2*a/b)+sinh(1/2*a/b))/b^3/(cos(1/2*arcsin(-1+I*d*x^2))+sin(1/2*arcs
in(-1+I*d*x^2)))-1/4*(2*I*d*x^2+d^2*x^4)^(1/2)/b/d/x/(a-I*b*arcsin(-1+I*d*x
^2))^2
```

Rubi [A] time = 0.05, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {4828, 4816}

$$\frac{x\left(-\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)\text{CosIntegral}\left(-\frac{i\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)}{2b}\right)}{16b^3\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)\right)}-\frac{x\left(\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)\mathrm{Si}\left(\frac{ia}{2b}-\frac{1}{2}\sin^{-1}\left(1-\right.\right.}{16b^3\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-\right.\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*b*ArcSin[1 - I*d*x^2])^(-3),x]
```

```
[Out] -Sqrt[(2*I)*d*x^2 + d^2*x^4]/(4*b*d*x*(a + I*b*ArcSin[1 - I*d*x^2])^2) - x/
(8*b^2*(a + I*b*ArcSin[1 - I*d*x^2])) + (x*CosIntegral[((-I/2)*(a + I*b*Arc
Sin[1 - I*d*x^2]))/b]*(I*Cosh[a/(2*b)] - Sinh[a/(2*b)]))/(16*b^3*(Cos[ArcSi
n[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2])) - (x*(I*Cosh[a/(2*b)] + Si
nh[a/(2*b)])*SinIntegral[((I/2)*a)/b - ArcSin[1 - I*d*x^2]/2])/(16*b^3*(Cos
[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2]))
```

Rule 4816

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(-1), x_Symbol] :> -Simp[(x
*(c*Cos[a/(2*b)] - Sin[a/(2*b)])*CosIntegral[(c/(2*b))*(a + b*ArcSin[c + d*
x^2])])/(2*b*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2])), x] -
 Simp[(x*(c*Cos[a/(2*b)] + Sin[a/(2*b)])*SinIntegral[(c/(2*b))*(a + b*ArcSi
n[c + d*x^2])])/(2*b*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2]
)), x] /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

Rule 4828

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[(x*
(a + b*ArcSin[c + d*x^2])^(n + 2))/(4*b^2*(n + 1)*(n + 2)), x] + (-Dist[1/(
4*b^2*(n + 1)*(n + 2)), Int[(a + b*ArcSin[c + d*x^2])^(n + 2), x], x] + Sim
p[(Sqrt[-2*c*d*x^2 - d^2*x^4]*(a + b*ArcSin[c + d*x^2])^(n + 1))/(2*b*d*(n
+ 1)*x), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && LtQ[n, -1] && NeQ[
n, -2]
```

Rubi steps

$$\int \frac{1}{\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^3}\,dx = -\frac{\sqrt{2idx^2+d^2x^4}}{4bdx\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^2} - \frac{x}{8b^2\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)} + \frac{\int \frac{}{a+ib\sin^{-}}}{8}$$

$$= -\frac{\sqrt{2idx^2+d^2x^4}}{4bdx\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^2} - \frac{x}{8b^2\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)} + \frac{x\mathrm{Ci}\left(\right.}{16b^3\left(\cos\right.}$$

Mathematica [A] time = 0.58, size = 229, normalized size = 0.83

$$\frac{-\frac{8b^2\sqrt{dx^2(dx^2+2i)}}{d\left(a+ib\sin^{-1}(1-idx^2)\right)^2} + \frac{2ix^2\left(\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\mathrm{Ci}\left(\frac{1}{2}\left(\sin^{-1}(1-idx^2)-\frac{ia}{b}\right)\right)-\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)\mathrm{Si}\left(\frac{ia}{2b}-\frac{1}{2}\sin^{-1}(1-idx^2)\right)\right)}{\cos\left(\frac{1}{2}\sin^{-1}(1-idx^2)\right)-\sin\left(\frac{1}{2}\sin^{-1}(1-idx^2)\right)} - \frac{}{a+ib\sin}}{32b^3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*b*ArcSin[1 - I*d*x^2])^(-3),x]
```

```
[Out] ((-8*b^2*Sqrt[d*x^2*(2*I + d*x^2)])/(d*(a + I*b*ArcSin[1 - I*d*x^2])^2) - (
4*b*x^2)/(a + I*b*ArcSin[1 - I*d*x^2]) + ((2*I)*x^2*(CosIntegral[(((-I)*a)/
b + ArcSin[1 - I*d*x^2])/2]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)]) - (Cosh[a/(2*
b)] - I*Sinh[a/(2*b)])*SinIntegral[((I/2)*a)/b - ArcSin[1 - I*d*x^2]/2]))/(
Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2]))/(32*b^3*x)
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$-\frac{bdx\log\left(dx^2+\sqrt{d^2x^2+2i\,d}\,x+i\right)+adx-8\left(b^4d\log\left(dx^2+\sqrt{d^2x^2+2i\,d}\,x+i\right)^2+2\,ab^3d\log\left(dx^2+\sqrt{d^2x^2}\right.\right.}{8\left(b^4d\log\left(dx^2+\sqrt{d^2x^2+2i\,d}\,x+i\right)^2+2\,ab^3d\log\left(dx^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2))^3,x, algorithm="fricas")
```

```
[Out] -1/8*(b*d*x*log(d*x^2 + sqrt(d^2*x^2 + 2*I*d)*x + I) + a*d*x - 8*(b^4*d*log
(d*x^2 + sqrt(d^2*x^2 + 2*I*d)*x + I)^2 + 2*a*b^3*d*log(d*x^2 + sqrt(d^2*x^
2 + 2*I*d)*x + I) + a^2*b^2*d)*integral(1/8/(b^3*log(d*x^2 + sqrt(d^2*x^2 +
 2*I*d)*x + I) + a*b^2), x) + 2*sqrt(d^2*x^2 + 2*I*d)*b)/(b^4*d*log(d*x^2 +
 sqrt(d^2*x^2 + 2*I*d)*x + I)^2 + 2*a*b^3*d*log(d*x^2 + sqrt(d^2*x^2 + 2*I*
d)*x + I) + a^2*b^2*d)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2))^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [d,x]=[45,-28]Bad conditionned root j= 1 value -35280.3655931 rati
o 0.379661795171 mindist 0.443514529616Bad conditionned root j= 1 value -51
05.29327315 ratio 1.07361778233 mindist 2.29350729132Warning, choosing root
```

```
 of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-8,[3,6]%%%}+%%%{-32,[1,2]%%
%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{16,[0,0]%%%}] at parameters value
s [7,-27]Warning, need to choose a branch for the root of a polynomial with
 parameters. This might be wrong.The choice was done assuming [d,t_nostep]=
[65,-98]schur row 1 5.57208e-12schur row 2 -6.7303e-09Francis algorithm not
 precise enough for[1.0,0.0,-2.33820328561e+12,-1.94619571078e+18,-4.555995
50409e+23]Bad conditionned root j= 1 value -624323.751966 ratio 14.00780622
15 mindist 69.4834240274Bad conditionned root j= 1 value -151261.012618 rat
io 2.78093290945 mindist 7.18852886464Bad conditionned root j= 2 value -151
274.163312 ratio 2.51776304852 mindist 13.1506943274Warning, choosing root
of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-8,[3,6]%%%}+%%%{-32,[1,2]%%%
},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{16,[0,0]%%%}] at parameters values
 [63,-49]Warning, need to choose a branch for the root of a polynomial with
 parameters. This might be wrong.The choice was done assuming [d,t_nostep]=
[-86,2]Bad conditionned root j= 2 value 147044.272515 ratio 9.86512495427 m
indist 45.8740775194Warning, choosing root of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,
[0,0]%%%},%%%{-8,[3,6]%%%}+%%%{-32,[1,2]%%%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]
%%%}+%%%{16,[0,0]%%%}] at parameters values [-30,70]Evaluation time: 29.04s
ym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad
 Argument Value
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\,\mathrm{arcsinh}\left(d\,x^{2}+i\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(I+d*x^2))^3,x)
```

```
[Out] int(1/(a+b*arcsinh(I+d*x^2))^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2))^3,x, algorithm="maxima")
```

```
[Out] -(4*(a*d^(11/2) + 2*b*d^(11/2))*x^10 + (24*I*a*d^(9/2) + 56*I*b*d^(9/2))*x^
8 - 4*(11*a*d^(7/2) + 36*b*d^(7/2))*x^6 + (-8*I*a*d^(5/2) - 160*I*b*d^(5/2)
)*x^4 - 16*(3*a*d^(3/2) - 4*b*d^(3/2))*x^2 + (4*(a*d^4 + 2*b*d^4)*x^7 + (12
*I*a*d^3 + 32*I*b*d^3)*x^5 - 8*(2*a*d^2 + 5*b*d^2)*x^3 + (-16*I*a*d - 16*I*
b*d)*x)*(d*x^2 + 2*I)^(3/2) + (12*(a*d^(9/2) + 2*b*d^(9/2))*x^8 + (48*I*a*d
^(7/2) + 120*I*b*d^(7/2))*x^6 - 8*(8*a*d^(5/2) + 25*b*d^(5/2))*x^4 + (-40*I
*a*d^(3/2) - 120*I*b*d^(3/2))*x^2 + 16*a*sqrt(d) + 16*b*sqrt(d))*(d*x^2 + 2
*I) + (4*b*d^(11/2)*x^10 + 24*I*b*d^(9/2)*x^8 - 44*b*d^(7/2)*x^6 - 8*I*b*d^
(5/2)*x^4 - 48*b*d^(3/2)*x^2 + (4*b*d^4*x^7 + 12*I*b*d^3*x^5 - 16*b*d^2*x^3
 - 16*I*b*d*x)*(d*x^2 + 2*I)^(3/2) + (12*b*d^(9/2)*x^8 + 48*I*b*d^(7/2)*x^6
 - 64*b*d^(5/2)*x^4 - 40*I*b*d^(3/2)*x^2 + 16*b*sqrt(d))*(d*x^2 + 2*I) + (1
2*b*d^5*x^9 + 60*I*b*d^4*x^7 - 92*b*d^3*x^5 - 28*I*b*d^2*x^3 - 24*b*d*x)*sq
rt(d*x^2 + 2*I) - 32*I*b*sqrt(d))*log(d*x^2 + sqrt(d*x^2 + 2*I)*sqrt(d)*x +
 I) + (12*(a*d^5 + 2*b*d^5)*x^9 + (60*I*a*d^4 + 144*I*b*d^4)*x^7 - 4*(23*a*
d^3 + 76*b*d^3)*x^5 + (-28*I*a*d^2 - 256*I*b*d^2)*x^3 - 8*(3*a*d - 8*b*d)*x
)*sqrt(d*x^2 + 2*I) - 32*I*a*sqrt(d))/(32*a^2*b^2*d^(11/2)*x^9 + 192*I*a^2*
b^2*d^(9/2)*x^7 - 384*a^2*b^2*d^(7/2)*x^5 - 256*I*a^2*b^2*d^(5/2)*x^3 + (32
*b^4*d^(11/2)*x^9 + 192*I*b^4*d^(9/2)*x^7 - 384*b^4*d^(7/2)*x^5 - 256*I*b^4
*d^(5/2)*x^3 + (32*b^4*d^4*x^6 + 96*I*b^4*d^3*x^4 - 96*b^4*d^2*x^2 - 32*I*b
^4*d)*(d*x^2 + 2*I)^(3/2) + (96*b^4*d^(9/2)*x^7 + 384*I*b^4*d^(7/2)*x^5 - 4
80*b^4*d^(5/2)*x^3 - 192*I*b^4*d^(3/2)*x)*(d*x^2 + 2*I) + (96*b^4*d^5*x^8 +
```

```
 480*I*b^4*d^4*x^6 - 768*b^4*d^3*x^4 - 384*I*b^4*d^2*x^2)*sqrt(d*x^2 + 2*I)
)*log(d*x^2 + sqrt(d*x^2 + 2*I)*sqrt(d)*x + I)^2 + (32*a^2*b^2*d^4*x^6 + 96
*I*a^2*b^2*d^3*x^4 - 96*a^2*b^2*d^2*x^2 - 32*I*a^2*b^2*d)*(d*x^2 + 2*I)^(3/
2) + (96*a^2*b^2*d^(9/2)*x^7 + 384*I*a^2*b^2*d^(7/2)*x^5 - 480*a^2*b^2*d^(5
/2)*x^3 - 192*I*a^2*b^2*d^(3/2)*x)*(d*x^2 + 2*I) + (64*a*b^3*d^(11/2)*x^9 +
 384*I*a*b^3*d^(9/2)*x^7 - 768*a*b^3*d^(7/2)*x^5 - 512*I*a*b^3*d^(5/2)*x^3
+ (64*a*b^3*d^4*x^6 + 192*I*a*b^3*d^3*x^4 - 192*a*b^3*d^2*x^2 - 64*I*a*b^3*
d)*(d*x^2 + 2*I)^(3/2) + (192*a*b^3*d^(9/2)*x^7 + 768*I*a*b^3*d^(7/2)*x^5 -
 960*a*b^3*d^(5/2)*x^3 - 384*I*a*b^3*d^(3/2)*x)*(d*x^2 + 2*I) + (192*a*b^3*
d^5*x^8 + 960*I*a*b^3*d^4*x^6 - 1536*a*b^3*d^3*x^4 - 768*I*a*b^3*d^2*x^2)*s
qrt(d*x^2 + 2*I))*log(d*x^2 + sqrt(d*x^2 + 2*I)*sqrt(d)*x + I) + (96*a^2*b^
2*d^5*x^8 + 480*I*a^2*b^2*d^4*x^6 - 768*a^2*b^2*d^3*x^4 - 384*I*a^2*b^2*d^2
*x^2)*sqrt(d*x^2 + 2*I)) + integrate((d^6*x^12 + 8*I*d^5*x^10 - 27*d^4*x^8
- 56*I*d^3*x^6 + 88*d^2*x^4 + (d^4*x^8 + 4*I*d^3*x^6 - 3*d^2*x^4 + 8*I*d*x^
2 + 4)*(d*x^2 + 2*I)^2 + 96*I*d*x^2 + (4*d^(9/2)*x^9 + 20*I*d^(7/2)*x^7 - 3
0*d^(5/2)*x^5 + 2*I*d^(3/2)*x^3 - 22*sqrt(d)*x)*(d*x^2 + 2*I)^(3/2) + (6*d^
5*x^10 + 36*I*d^4*x^8 - 78*d^3*x^6 - 72*I*d^2*x^4 + 9*d*x^2 - 30*I)*(d*x^2
+ 2*I) + (4*d^(11/2)*x^11 + 28*I*d^(9/2)*x^9 - 78*d^(7/2)*x^7 - 122*I*d^(5/
2)*x^5 + 122*d^(3/2)*x^3 + 60*I*sqrt(d)*x)*sqrt(d*x^2 + 2*I) - 48)/(8*a*b^2
*d^6*x^12 + 64*I*a*b^2*d^5*x^10 - 192*a*b^2*d^4*x^8 - 256*I*a*b^2*d^3*x^6 +
 128*a*b^2*d^2*x^4 + (8*a*b^2*d^4*x^8 + 32*I*a*b^2*d^3*x^6 - 48*a*b^2*d^2*x
^4 - 32*I*a*b^2*d*x^2 + 8*a*b^2)*(d*x^2 + 2*I)^2 + (32*a*b^2*d^(9/2)*x^9 +
160*I*a*b^2*d^(7/2)*x^7 - 288*a*b^2*d^(5/2)*x^5 - 224*I*a*b^2*d^(3/2)*x^3 +
 64*a*b^2*sqrt(d)*x)*(d*x^2 + 2*I)^(3/2) + (48*a*b^2*d^5*x^10 + 288*I*a*b^2
*d^4*x^8 - 624*a*b^2*d^3*x^6 - 576*I*a*b^2*d^2*x^4 + 192*a*b^2*d*x^2)*(d*x^
2 + 2*I) + (8*b^3*d^6*x^12 + 64*I*b^3*d^5*x^10 - 192*b^3*d^4*x^8 - 256*I*b^
3*d^3*x^6 + 128*b^3*d^2*x^4 + (8*b^3*d^4*x^8 + 32*I*b^3*d^3*x^6 - 48*b^3*d^
2*x^4 - 32*I*b^3*d*x^2 + 8*b^3)*(d*x^2 + 2*I)^2 + (32*b^3*d^(9/2)*x^9 + 160
*I*b^3*d^(7/2)*x^7 - 288*b^3*d^(5/2)*x^5 - 224*I*b^3*d^(3/2)*x^3 + 64*b^3*s
qrt(d)*x)*(d*x^2 + 2*I)^(3/2) + (48*b^3*d^5*x^10 + 288*I*b^3*d^4*x^8 - 624*
b^3*d^3*x^6 - 576*I*b^3*d^2*x^4 + 192*b^3*d*x^2)*(d*x^2 + 2*I) + (32*b^3*d^
(11/2)*x^11 + 224*I*b^3*d^(9/2)*x^9 - 576*b^3*d^(7/2)*x^7 - 640*I*b^3*d^(5/
2)*x^5 + 256*b^3*d^(3/2)*x^3)*sqrt(d*x^2 + 2*I))*log(d*x^2 + sqrt(d*x^2 + 2
*I)*sqrt(d)*x + I) + (32*a*b^2*d^(11/2)*x^11 + 224*I*a*b^2*d^(9/2)*x^9 - 57
6*a*b^2*d^(7/2)*x^7 - 640*I*a*b^2*d^(5/2)*x^5 + 256*a*b^2*d^(3/2)*x^3)*sqrt
(d*x^2 + 2*I)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+b\,\mathrm{asinh}\left(d\,x^2+1{}\mathrm{i}\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*asinh(d*x^2 + 1i))^3,x)
[Out] int(1/(a + b*asinh(d*x^2 + 1i))^3, x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(I+d*x**2))**3,x)
[Out] Exception raised: TypeError
```

3.321 $\int \left(a - ib\sin^{-1}\left(1 + idx^2\right)\right)^4 dx$

Optimal. Leaf size=153

$$-\frac{192b^3\sqrt{d^2x^4 - 2idx^2}\left(a - ib\sin^{-1}\left(1 + idx^2\right)\right)}{dx} + 48b^2x\left(a - ib\sin^{-1}\left(1 + idx^2\right)\right)^2 - \frac{8b\sqrt{d^2x^4 - 2idx^2}\left(a - ib\sin^{-1}\left(\right.\right.}{dx}$$

```
[Out] 384*b^4*x+48*b^2*x*(a-I*b*arcsin(1+I*d*x^2))^2+x*(a-I*b*arcsin(1+I*d*x^2))^
4-192*b^3*(a-I*b*arcsin(1+I*d*x^2))*(-2*I*d*x^2+d^2*x^4)^(1/2)/d/x-8*b*(a-I
*b*arcsin(1+I*d*x^2))^3*(-2*I*d*x^2+d^2*x^4)^(1/2)/d/x
```

Rubi [A] time = 0.03, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {4814, 8}

$$-\frac{192b^3\sqrt{d^2x^4 - 2idx^2}\left(a - ib\sin^{-1}\left(1 + idx^2\right)\right)}{dx} + 48b^2x\left(a - ib\sin^{-1}\left(1 + idx^2\right)\right)^2 - \frac{8b\sqrt{d^2x^4 - 2idx^2}\left(a - ib\sin^{-1}\left(\right.\right.}{dx}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*b*ArcSin[1 + I*d*x^2])^4,x]
```

```
[Out] 384*b^4*x - (192*b^3*Sqrt[(-2*I)*d*x^2 + d^2*x^4]*(a - I*b*ArcSin[1 + I*d*x
^2]))/(d*x) + 48*b^2*x*(a - I*b*ArcSin[1 + I*d*x^2])^2 - (8*b*Sqrt[(-2*I)*d
*x^2 + d^2*x^4]*(a - I*b*ArcSin[1 + I*d*x^2])^3)/(d*x) + x*(a - I*b*ArcSin[
1 + I*d*x^2])^4
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4814

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcSin[c + d*x^2])^n, x] + (-Dist[4*b^2*n*(n - 1), Int[(a + b*ArcSin[
c + d*x^2])^(n - 2), x], x] + Simp[(2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*(a + b
*ArcSin[c + d*x^2])^(n - 1))/(d*x), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^
2, 1] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \left(a - ib\sin^{-1}\left(1 + idx^2\right)\right)^4 dx &= -\frac{8b\sqrt{-2idx^2 + d^2x^4}\left(a - ib\sin^{-1}\left(1 + idx^2\right)\right)^3}{dx} + x\left(a - ib\sin^{-1}\left(1 + idx^2\right)\right)^4 + \\ &= -\frac{192b^3\sqrt{-2idx^2 + d^2x^4}\left(a - ib\sin^{-1}\left(1 + idx^2\right)\right)}{dx} + 48b^2x\left(a - ib\sin^{-1}\left(1 + idx\right.\right. \\ &= 384b^4x - \frac{192b^3\sqrt{-2idx^2 + d^2x^4}\left(a - ib\sin^{-1}\left(1 + idx^2\right)\right)}{dx} + 48b^2x\left(a - ib\sin^{-1}\right.\end{aligned}$$

Mathematica [A] time = 0.12, size = 149, normalized size = 0.97

$$48b^2\left(-\frac{4b\sqrt{dx^2\left(dx^2 - 2i\right)}\left(a - ib\sin^{-1}\left(1 + idx^2\right)\right)}{dx} + x\left(a - ib\sin^{-1}\left(1 + idx^2\right)\right)^2 + 8b^2x\right) + x\left(a - ib\sin^{-1}\left(1 + idx\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*b*ArcSin[1 + I*d*x^2])^4,x]
```

```
[Out] (-8*b*Sqrt[d*x^2*(-2*I + d*x^2)]*(a - I*b*ArcSin[1 + I*d*x^2])^3)/(d*x) + x
*(a - I*b*ArcSin[1 + I*d*x^2])^4 + 48*b^2*(8*b^2*x - (4*b*Sqrt[d*x^2*(-2*I
+ d*x^2)]*(a - I*b*ArcSin[1 + I*d*x^2]))/(d*x) + x*(a - I*b*ArcSin[1 + I*d*
x^2])^2)
```

fricas [B] time = 0.57, size = 269, normalized size = 1.76

$$b^4dx\log\left(dx^2+\sqrt{d^2x^2-2id}\,x-i\right)^4+4\left(ab^3dx-2\sqrt{d^2x^2-2id}\,b^4\right)\log\left(dx^2+\sqrt{d^2x^2-2id}\,x-i\right)^3+\left(a^4+4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^4,x, algorithm="fricas")
```

```
[Out] (b^4*d*x*log(d*x^2 + sqrt(d^2*x^2 - 2*I*d)*x - I)^4 + 4*(a*b^3*d*x - 2*sqrt
(d^2*x^2 - 2*I*d)*b^4)*log(d*x^2 + sqrt(d^2*x^2 - 2*I*d)*x - I)^3 + (a^4 +
48*a^2*b^2 + 384*b^4)*d*x - 6*(4*sqrt(d^2*x^2 - 2*I*d)*a*b^3 - (a^2*b^2 + 8
*b^4)*d*x)*log(d*x^2 + sqrt(d^2*x^2 - 2*I*d)*x - I)^2 + 4*((a^3*b + 24*a*b^
3)*d*x - 6*(a^2*b^2 + 8*b^4)*sqrt(d^2*x^2 - 2*I*d))*log(d*x^2 + sqrt(d^2*x^
2 - 2*I*d)*x - I) - 8*(a^3*b + 24*a*b^3)*sqrt(d^2*x^2 - 2*I*d))/d
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [d,x]=[45,-28]Bad conditionned root j= 1 value -35280.3655931 rati
o 0.379661795171 mindist 0.443514529616Bad conditionned root j= 1 value -51
05.29327315 ratio 1.07361778233 mindist 2.29350729132Warning, choosing root
 of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-8,[3,6]%%%}+%%%{-32,[1,2]%%
%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{16,[0,0]%%%}] at parameters value
s [7,-27]Warning, need to choose a branch for the root of a polynomial with
 parameters. This might be wrong.The choice was done assuming [d,t_nostep]=
[79,3]schur row 3 8.74347e-08Bad conditionned root j= 2 value -151313.41286
2 ratio 11.2206791301 mindist 48.7986537395Warning, choosing root of [1,0,%
%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-8,[3,6]%%%}+%%%{-32,[1,2]%%%},%%%{-3,
[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{16,[0,0]%%%}] at parameters values [63,-49]
Warning, need to choose a branch for the root of a polynomial with paramete
rs. This might be wrong.The choice was done assuming [d,t_nostep]=[-27,9]Ba
d conditionned root j= 2 value 147025.62453 ratio 5.74493624992 mindist 24.
9427695529Warning, choosing root of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},
%%%{-8,[3,6]%%%}+%%%{-32,[1,2]%%%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{1
6,[0,0]%%%}] at parameters values [-30,70]Evaluation time: 30.29sym2poly/r2
sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument
Value
```

maple [F] time = 0.19, size = 0, normalized size = 0.00

$$\int \left(a+b\,\mathrm{arcsinh}\left(d\,x^2-i\right)\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(-I+d*x^2))^4,x)
```

[Out] int((a+b*arcsinh(-I+d*x^2))^4,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b^4x\log\left(dx^2+\sqrt{dx^2-2i}\sqrt{d}\,x-i\right)^4+4\left(x\,\mathrm{arsinh}\left(dx^2-i\right)-\frac{2\left(d^{\frac{3}{2}}x^2-2i\sqrt{d}\right)}{\sqrt{dx^2-2i}\,d}\right)a^3b+a^4x+\int\frac{\left(4\left(ab^3d^2-2\,b^4d^2\right)x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^4,x, algorithm="maxima")
```

```
[Out] b^4*x*log(d*x^2 + sqrt(d*x^2 - 2*I)*sqrt(d)*x - I)^4 + 4*(x*arcsinh(d*x^2 -
 I) - 2*(d^(3/2)*x^2 - 2*I*sqrt(d))/(sqrt(d*x^2 - 2*I)*d))*a^3*b + a^4*x +
integrate(((4*(a*b^3*d^2 - 2*b^4*d^2)*x^4 - 8*a*b^3 + (-12*I*a*b^3*d + 16*I
*b^4*d)*x^2 + (4*(a*b^3*d^(3/2) - 2*b^4*d^(3/2))*x^3 + (-8*I*a*b^3*sqrt(d)
+ 8*I*b^4*sqrt(d))*x)*sqrt(d*x^2 - 2*I))*log(d*x^2 + sqrt(d*x^2 - 2*I)*sqrt
(d)*x - I)^3 + (6*a^2*b^2*d^2*x^4 - 18*I*a^2*b^2*d*x^2 - 12*a^2*b^2 + (6*a^
2*b^2*d^(3/2)*x^3 - 12*I*a^2*b^2*sqrt(d)*x)*sqrt(d*x^2 - 2*I))*log(d*x^2 +
sqrt(d*x^2 - 2*I)*sqrt(d)*x - I)^2)/(d^2*x^4 - 3*I*d*x^2 + (d^(3/2)*x^3 - 2
*I*sqrt(d)*x)*sqrt(d*x^2 - 2*I) - 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\left(a+b\,\mathrm{asinh}\left(d\,x^2-\mathrm{i}\right)\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(d*x^2 - 1i))^4,x)
```

```
[Out] int((a + b*asinh(d*x^2 - 1i))^4, x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(-I+d*x**2))**4,x)
```

```
[Out] Exception raised: TypeError
```

3.322 $\int \left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^3\,dx$

Optimal. Leaf size=129

$$24ab^2x-\frac{6b\sqrt{d^2x^4-2idx^2}\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^2}{dx}+x\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^3-\frac{48b^3\sqrt{d^2x^4-2idx^2}}{dx}-24ib^3x$$

```
[Out] 24*a*b^2*x-24*I*b^3*x*arcsin(1+I*d*x^2)+x*(a-I*b*arcsin(1+I*d*x^2))^3-48*b^
3*(-2*I*d*x^2+d^2*x^4)^(1/2)/d/x-6*b*(a-I*b*arcsin(1+I*d*x^2))^2*(-2*I*d*x^
2+d^2*x^4)^(1/2)/d/x
```

Rubi [A] time = 0.06, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4814, 4840, 12, 1588}

$$24ab^2x-\frac{6b\sqrt{d^2x^4-2idx^2}\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^2}{dx}+x\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^3-\frac{48b^3\sqrt{d^2x^4-2idx^2}}{dx}-24ib^3x$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*b*ArcSin[1 + I*d*x^2])^3,x]
```

```
[Out] 24*a*b^2*x - (48*b^3*Sqrt[(-2*I)*d*x^2 + d^2*x^4])/(d*x) - (24*I)*b^3*x*Arc
Sin[1 + I*d*x^2] - (6*b*Sqrt[(-2*I)*d*x^2 + d^2*x^4]*(a - I*b*ArcSin[1 + I*
d*x^2])^2)/(d*x) + x*(a - I*b*ArcSin[1 + I*d*x^2])^3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 4814

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcSin[c + d*x^2])^n, x] + (-Dist[4*b^2*n*(n - 1), Int[(a + b*ArcSin[
c + d*x^2])^(n - 2), x], x] + Simp[(2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*(a + b
*ArcSin[c + d*x^2])^(n - 1))/(d*x), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^
2, 1] && GtQ[n, 1]
```

Rule 4840

```
Int[ArcSin[u_], x_Symbol] :> Simp[x*ArcSin[u], x] - Int[SimplifyIntegrand[(
x*D[u, x])/Sqrt[1 - u^2], x], x] /; InverseFunctionFreeQ[u, x] &&  !Functio
nOfExponentialQ[u, x]
```

Rubi steps

$$\int \left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^3 dx = -\frac{6b\sqrt{-2idx^2+d^2x^4}\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^2}{dx}+x\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^3+$$

$$=24ab^2x-\frac{6b\sqrt{-2idx^2+d^2x^4}\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^2}{dx}+x\left(a-ib\sin^{-1}\left(1+id\right.\right.$$

$$=24ab^2x-24ib^3x\sin^{-1}\left(1+idx^2\right)-\frac{6b\sqrt{-2idx^2+d^2x^4}\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)}{dx}$$

$$=24ab^2x-24ib^3x\sin^{-1}\left(1+idx^2\right)-\frac{6b\sqrt{-2idx^2+d^2x^4}\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)}{dx}$$

$$=24ab^2x-\frac{48b^3\sqrt{-2idx^2+d^2x^4}}{dx}-24ib^3x\sin^{-1}\left(1+idx^2\right)-\frac{6b\sqrt{-2idx^2+d^2x^4}}{}$$

Mathematica [A] time = 0.15, size = 180, normalized size = 1.40

$$\frac{adx^2\left(a^2+24b^2\right)-6b\left(a^2+8b^2\right)\sqrt{dx^2\left(dx^2-2i\right)}-3ib\sin^{-1}\left(1+idx^2\right)\left(a^2dx^2-4ab\sqrt{dx^2\left(dx^2-2i\right)}+8b^2dx^2\right)}{dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*b*ArcSin[1 + I*d*x^2])^3,x]
```

```
[Out] (a*(a^2 + 24*b^2)*d*x^2 - 6*b*(a^2 + 8*b^2)*Sqrt[d*x^2*(-2*I + d*x^2)] - (3
*I)*b*(a^2*d*x^2 + 8*b^2*d*x^2 - 4*a*b*Sqrt[d*x^2*(-2*I + d*x^2)])*ArcSin[1
 + I*d*x^2] + 3*b^2*(-(a*d*x^2) + 2*b*Sqrt[d*x^2*(-2*I + d*x^2)])*ArcSin[1
+ I*d*x^2]^2 + I*b^3*d*x^2*ArcSin[1 + I*d*x^2]^3)/(d*x)
```

fricas [A] time = 0.59, size = 188, normalized size = 1.46

$$\frac{b^3dx\log\left(dx^2+\sqrt{d^2x^2-2id}\,x-i\right)^3+\left(a^3+24\,ab^2\right)dx+3\left(ab^2dx-2\sqrt{d^2x^2-2id}\,b^3\right)\log\left(dx^2+\sqrt{d^2x^2-2id}\,x\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^3,x, algorithm="fricas")
```

```
[Out] (b^3*d*x*log(d*x^2 + sqrt(d^2*x^2 - 2*I*d)*x - I)^3 + (a^3 + 24*a*b^2)*d*x
+ 3*(a*b^2*d*x - 2*sqrt(d^2*x^2 - 2*I*d)*b^3)*log(d*x^2 + sqrt(d^2*x^2 - 2*
I*d)*x - I)^2 - 3*(4*sqrt(d^2*x^2 - 2*I*d)*a*b^2 - (a^2*b + 8*b^3)*d*x)*log
(d*x^2 + sqrt(d^2*x^2 - 2*I*d)*x - I) - 6*sqrt(d^2*x^2 - 2*I*d)*(a^2*b + 8*
b^3))/d
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [d,x]=[45,-28]Bad conditionned root j= 1 value -35280.3655931 rati
o 0.379661795171 mindist 0.443514529616Bad conditionned root j= 1 value -51
05.29327315 ratio 1.07361778233 mindist 2.29350729132Warning, choosing root
```

```
of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-8,[3,6]%%%}+%%%{-32,[1,2]%%
%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{16,[0,0]%%%}] at parameters value
s [7,-27]Warning, need to choose a branch for the root of a polynomial with
 parameters. This might be wrong.The choice was done assuming [d,t_nostep]=
[97,-52]Francis algorithm failure for[1.0,0.0,-137282971022,-2.76877787308e
+16,-1.57055117809e+21]Warning, choosing root of [1,0,%%%{-6,[2,4]%%%}+%%%{
-8,[0,0]%%%},%%%{-8,[3,6]%%%}+%%%{-32,[1,2]%%%},%%%{-3,[4,8]%%%}+%%%{-24,[2
,4]%%%}+%%%{16,[0,0]%%%}] at parameters values [63,-49]Warning, need to cho
ose a branch for the root of a polynomial with parameters. This might be wr
ong.The choice was done assuming [d,t_nostep]=[-9,-4]Francis algorithm fail
ure for[1.0,0.0,-129654000008,2.54121840047e+16,-1.40084664352e+21]Warning,
 choosing root of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-8,[3,6]%%%}+%
%%{-32,[1,2]%%%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{16,[0,0]%%%}] at pa
rameters values [-30,70]Evaluation time: 15.74sym2poly/r2sym(const gen & e,
const index_m & i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.19, size = 0, normalized size = 0.00

$$\int \left(a + b\,\mathrm{arcsinh}\left(d\,x^2 - i\right)\right)^3 \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(-I+d*x^2))^3,x)
```

```
[Out] int((a+b*arcsinh(-I+d*x^2))^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b^3 x \log\left(dx^2 + \sqrt{dx^2 - 2i}\,\sqrt{d}\,x - i\right)^3 + 3\left(x\,\mathrm{arsinh}\left(dx^2 - i\right) - \frac{2\left(d^{\frac{3}{2}}x^2 - 2i\sqrt{d}\right)}{\sqrt{dx^2 - 2i}\,d}\right)a^2 b + a^3 x + \int \frac{\left(3\left(ab^2d^2 - 2\,b^3d^{\cdot}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^3,x, algorithm="maxima")
```

```
[Out] b^3*x*log(d*x^2 + sqrt(d*x^2 - 2*I)*sqrt(d)*x - I)^3 + 3*(x*arcsinh(d*x^2 -
 I) - 2*(d^(3/2)*x^2 - 2*I*sqrt(d))/(sqrt(d*x^2 - 2*I)*d))*a^2*b + a^3*x +
integrate((3*(a*b^2*d^2 - 2*b^3*d^2)*x^4 - 6*a*b^2 + (-9*I*a*b^2*d + 12*I*b
^3*d)*x^2 + (3*(a*b^2*d^(3/2) - 2*b^3*d^(3/2))*x^3 + (-6*I*a*b^2*sqrt(d) +
6*I*b^3*sqrt(d))*x)*sqrt(d*x^2 - 2*I))*log(d*x^2 + sqrt(d*x^2 - 2*I)*sqrt(d
)*x - I)^2/(d^2*x^4 - 3*I*d*x^2 + (d^(3/2)*x^3 - 2*I*sqrt(d)*x)*sqrt(d*x^2
- 2*I) - 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a + b\,\mathrm{asinh}\left(d\,x^2 - \mathrm{i}\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(d*x^2 - 1i))^3,x)
```

```
[Out] int((a + b*asinh(d*x^2 - 1i))^3, x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(-I+d*x**2))**3,x)
```

```
[Out] Exception raised: TypeError
```

3.323 $\int \left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^2 \, dx$

Optimal. Leaf size=76

$$-\frac{4b\sqrt{d^2x^4-2idx^2}\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)}{dx}+x\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^2+8b^2x$$

```
[Out] 8*b^2*x+x*(a-I*b*arcsin(1+I*d*x^2))^2-4*b*(a-I*b*arcsin(1+I*d*x^2))*(-2*I*d
*x^2+d^2*x^4)^(1/2)/d/x
```

Rubi [A] time = 0.01, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {4814, 8}

$$-\frac{4b\sqrt{d^2x^4-2idx^2}\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)}{dx}+x\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^2+8b^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*b*ArcSin[1 + I*d*x^2])^2,x]
```

```
[Out] 8*b^2*x - (4*b*Sqrt[(-2*I)*d*x^2 + d^2*x^4]*(a - I*b*ArcSin[1 + I*d*x^2]))/
(d*x) + x*(a - I*b*ArcSin[1 + I*d*x^2])^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4814

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcSin[c + d*x^2])^n, x] + (-Dist[4*b^2*n*(n - 1), Int[(a + b*ArcSin[
c + d*x^2])^(n - 2), x], x] + Simp[(2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*(a + b
*ArcSin[c + d*x^2])^(n - 1))/(d*x), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^
2, 1] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^2 \, dx &= -\frac{4b\sqrt{-2idx^2+d^2x^4}\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)}{dx}+x\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^2+\left(\right.\\ &= 8b^2x-\frac{4b\sqrt{-2idx^2+d^2x^4}\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)}{dx}+x\left(a-ib\sin^{-1}\left(1+idx^2\right)\right.\end{aligned}$$

Mathematica [A] time = 0.02, size = 76, normalized size = 1.00

$$-\frac{4b\sqrt{d^2x^4-2idx^2}\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)}{dx}+x\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^2+8b^2x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*b*ArcSin[1 + I*d*x^2])^2,x]
```

```
[Out] 8*b^2*x - (4*b*Sqrt[(-2*I)*d*x^2 + d^2*x^4]*(a - I*b*ArcSin[1 + I*d*x^2]))/
(d*x) + x*(a - I*b*ArcSin[1 + I*d*x^2])^2
```

fricas [A] time = 0.51, size = 114, normalized size = 1.50

$$\frac{b^2dx\log\left(dx^2+\sqrt{d^2x^2-2i\,d}\,x-i\right)^2+\left(a^2+8\,b^2\right)dx-4\,\sqrt{d^2x^2-2i\,d}\,ab+2\left(abdx-2\,\sqrt{d^2x^2-2i\,d}\,b^2\right)\log\left(d\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^2,x, algorithm="fricas")
```

```
[Out] (b^2*d*x*log(d*x^2 + sqrt(d^2*x^2 - 2*I*d)*x - I)^2 + (a^2 + 8*b^2)*d*x - 4
*sqrt(d^2*x^2 - 2*I*d)*a*b + 2*(a*b*d*x - 2*sqrt(d^2*x^2 - 2*I*d)*b^2)*log(
d*x^2 + sqrt(d^2*x^2 - 2*I*d)*x - I))/d
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [d,x]=[45,-28]Bad conditionned root j= 1 value -35280.3655931 rati
o 0.379661795171 mindist 0.443514529616Bad conditionned root j= 1 value -51
05.29327315 ratio 1.07361778233 mindist 2.29350729132Warning, choosing root
 of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-8,[3,6]%%%}+%%%{-32,[1,2]%%
%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{16,[0,0]%%%}] at parameters value
s [7,-27]Warning, need to choose a branch for the root of a polynomial with
 parameters. This might be wrong.The choice was done assuming [d,t_nostep]=
[60,97]schur row 1 7.48504e-11Francis algorithm not precise enough for[1.0,
0.0,-1.91223246961e+12,-1.43937562454e+18,-3.04719418157e+23]Bad conditionn
ed root j= 2 value -564549.069246 ratio 4.13534933689 mindist 8.26009499958
schur row 3 8.32254e-09schur row 3 8.32254e-09Francis algorithm not precise
 enough for[1.0,0.0,-137282971022,-2.76877787308e+16,-1.57055117809e+21]Bad
 conditionned root j= 3 value -151279.357647 ratio 3.67253338015 mindist 17
.3373140811Warning, choosing root of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%}
,%%%{-8,[3,6]%%%}+%%%{-32,[1,2]%%%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{
16,[0,0]%%%}] at parameters values [63,-49]Warning, need to choose a branch
 for the root of a polynomial with parameters. This might be wrong.The choi
ce was done assuming [d,t_nostep]=[-20,14]schur row 1 6.86402e-10Francis al
gorithm not precise enough for[1.0,0.0,-129654000008,2.54121840047e+16,-1.4
0084664352e+21]Bad conditionned root j= 2 value 146992.858887 ratio 1.59770
7895 mindist 8.30647902455Warning, choosing root of [1,0,%%%{-6,[2,4]%%%}+%
%%{-8,[0,0]%%%},%%%{-8,[3,6]%%%}+%%%{-32,[1,2]%%%},%%%{-3,[4,8]%%%}+%%%{-24
,[2,4]%%%}+%%%{16,[0,0]%%%}] at parameters values [-30,70]Evaluation time:
6.85sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error
: Bad Argument Value
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \left(a+b\,\mathrm{arcsinh}\left(d\,x^2-i\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(-I+d*x^2))^2,x)
```

```
[Out] int((a+b*arcsinh(-I+d*x^2))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$2\left(x\operatorname{arsinh}\left(dx^2-i\right)-\frac{2\left(d^{\frac{3}{2}}x^2-2i\sqrt{d}\right)}{\sqrt{dx^2-2i}\,d}\right)ab+\left(x\log\left(dx^2+\sqrt{dx^2-2i}\sqrt{d}\,x-i\right)^2-\int\frac{\left(4\,d^2x^4-8i\,dx^2+\left(4\,d^{\frac{3}{2}}x^3\right.\right.}{d^2x^4-3i\,c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^2,x, algorithm="maxima")
```

```
[Out] 2*(x*arcsinh(d*x^2 - I) - 2*(d^(3/2)*x^2 - 2*I*sqrt(d))/(sqrt(d*x^2 - 2*I)*
d))*a*b + (x*log(d*x^2 + sqrt(d*x^2 - 2*I)*sqrt(d)*x - I)^2 - integrate((4*
d^2*x^4 - 8*I*d*x^2 + (4*d^(3/2)*x^3 - 4*I*sqrt(d)*x)*sqrt(d*x^2 - 2*I))*lo
g(d*x^2 + sqrt(d*x^2 - 2*I)*sqrt(d)*x - I)/(d^2*x^4 - 3*I*d*x^2 + (d^(3/2)*
x^3 - 2*I*sqrt(d)*x)*sqrt(d*x^2 - 2*I) - 2), x))*b^2 + a^2*x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\left(a+b\operatorname{asinh}\left(d\,x^2-\mathrm{i}\right)\right)^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(d*x^2 - 1i))^2,x)
```

```
[Out] int((a + b*asinh(d*x^2 - 1i))^2, x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(-I+d*x**2))**2,x)
```

```
[Out] Exception raised: TypeError
```

3.324 $\int \left(a-ib\sin^{-1}\left(1+idx^2\right)\right)dx$

Optimal. Leaf size=50

$$ax-\frac{2b\sqrt{d^2x^4-2idx^2}}{dx}-ibx\sin^{-1}\left(1+idx^2\right)$$

```
[Out] a*x-I*b*x*arcsin(1+I*d*x^2)-2*b*(-2*I*d*x^2+d^2*x^4)^(1/2)/d/x
```

Rubi [A] time = 0.04, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4840, 12, 1588}

$$ax-\frac{2b\sqrt{d^2x^4-2idx^2}}{dx}-ibx\sin^{-1}\left(1+idx^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[a - I*b*ArcSin[1 + I*d*x^2],x]
```

```
[Out] a*x - (2*b*Sqrt[(-2*I)*d*x^2 + d^2*x^4])/(d*x) - I*b*x*ArcSin[1 + I*d*x^2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 4840

```
Int[ArcSin[u_], x_Symbol] :> Simp[x*ArcSin[u], x] - Int[SimplifyIntegrand[(
x*D[u, x])/Sqrt[1 - u^2], x], x] /; InverseFunctionFreeQ[u, x] && !Functio
nOfExponentialQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \left(a-ib\sin^{-1}\left(1+idx^2\right)\right)dx &= ax-(ib)\int \sin^{-1}\left(1+idx^2\right)dx\\ &= ax-ibx\sin^{-1}\left(1+idx^2\right)+(ib)\int \frac{2idx^2}{\sqrt{-2idx^2+d^2x^4}}dx\\ &= ax-ibx\sin^{-1}\left(1+idx^2\right)-(2bd)\int \frac{x^2}{\sqrt{-2idx^2+d^2x^4}}dx\\ &= ax-\frac{2b\sqrt{-2idx^2+d^2x^4}}{dx}-ibx\sin^{-1}\left(1+idx^2\right)\end{aligned}$$

Mathematica [A] time = 0.03, size = 48, normalized size = 0.96

$$ax-\frac{2b\sqrt{dx^2\left(dx^2-2i\right)}}{dx}-ibx\sin^{-1}\left(1+idx^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[a - I*b*ArcSin[1 + I*d*x^2],x]
```

```
[Out] a*x - (2*b*Sqrt[d*x^2*(-2*I + d*x^2)])/(d*x) - I*b*x*ArcSin[1 + I*d*x^2]
```

fricas [A] time = 0.48, size = 52, normalized size = 1.04

$$\frac{bdx\log\left(dx^2+\sqrt{d^2x^2-2i\,d}\,x-i\right)+adx-2\sqrt{d^2x^2-2i\,d}\,b}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arcsinh(-I+d*x^2),x, algorithm="fricas")
```

```
[Out] (b*d*x*log(d*x^2 + sqrt(d^2*x^2 - 2*I*d)*x - I) + a*d*x - 2*sqrt(d^2*x^2 -
2*I*d)*b)/d
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arcsinh(-I+d*x^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [d,x]=[45,-28]Bad conditionned root j= 1 value -35280.3655931 rati
o 0.379661795171 mindist 0.443514529616Bad conditionned root j= 1 value -51
05.29327315 ratio 1.07361778233 mindist 2.29350729132Warning, choosing root
 of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-8,[3,6]%%%}+%%%{-32,[1,2]%%
%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{16,[0,0]%%%}] at parameters value
s [7,-27]Warning, need to choose a branch for the root of a polynomial with
 parameters. This might be wrong.The choice was done assuming [d,x]=[29,3]s
chur row 1 3.19975e-11schur row 2 -2.1116e-07Francis algorithm not precise
enough for[1.0,0.0,-137282971022,-2.76877787308e+16,-1.57055117809e+21]Bad
conditionned root j= 1 value -151344.383904 ratio 18.0968847681 mindist 79.
7696956145Warning, choosing root of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},
%%%{-8,[3,6]%%%}+%%%{-32,[1,2]%%%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{1
6,[0,0]%%%}] at parameters values [63,-49]Evaluation time: 1.23sym2poly/r2s
ym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument V
alue
```

maple [A] time = 0.02, size = 48, normalized size = 0.96

$$ax+b\left(x\,\mathrm{arcsinh}\left(d\,x^2-i\right)+\frac{2x\left(-d\,x^2+2i\right)}{\sqrt{d^2x^4-2id\,x^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arcsinh(-I+d*x^2),x)
```

```
[Out] a*x+b*(x*arcsinh(-I+d*x^2)+2/(-2*I*d*x^2+d^2*x^4)^(1/2)*x*(-d*x^2+2*I))
```

maxima [A] time = 0.70, size = 44, normalized size = 0.88

$$\left(x\,\mathrm{arsinh}\left(dx^2-i\right)-\frac{2\left(d^{\frac{3}{2}}x^2-2i\sqrt{d}\right)}{\sqrt{dx^2-2i\,d}}\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arcsinh(-I+d*x^2),x, algorithm="maxima")
```

```
[Out] (x*arcsinh(d*x^2 - I) - 2*(d^(3/2)*x^2 - 2*I*sqrt(d))/(sqrt(d*x^2 - 2*I)*d)
)*b + a*x
```

mupad [B] time = 0.45, size = 39, normalized size = 0.78

$$ax + bx\operatorname{asinh}\left(dx^2 - \mathrm{i}\right) - \frac{2b\sqrt{\left(dx^2 - \mathrm{i}\right)^2 + 1}}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*asinh(d*x^2 - 1i),x)
```

```
[Out] a*x + b*x*asinh(d*x^2 - 1i) - (2*b*((d*x^2 - 1i)^2 + 1)^(1/2))/(d*x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*asinh(-I+d*x**2),x)
```

```
[Out] Exception raised: TypeError
```

3.325 $\int \frac{1}{a-ib\sin^{-1}(1+idx^2)}\,dx$

Optimal. Leaf size=191

$$\frac{x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{Shi}\left(\frac{a-ib\sin^{-1}(idx^2+1)}{2b}\right)}{2b\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)\right)}-\frac{x\left(\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)\text{Ci}\left(\frac{i\left(a-ib\sin^{-1}(idx^2+1)\right)}{2b}\right)}{2b\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)\right)}$$

```
[Out] 1/2*x*Shi(1/2*(a-I*b*arcsin(1+I*d*x^2))/b)*(cosh(1/2*a/b)+I*sinh(1/2*a/b))/
b/(cos(1/2*arcsin(1+I*d*x^2))-sin(1/2*arcsin(1+I*d*x^2)))-1/2*x*Ci(1/2*I*(a
-I*b*arcsin(1+I*d*x^2))/b)*(I*cosh(1/2*a/b)+sinh(1/2*a/b))/b/(cos(1/2*arcsi
n(1+I*d*x^2))-sin(1/2*arcsin(1+I*d*x^2)))
```

Rubi [A] time = 0.02, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {4816}

$$\frac{x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{Shi}\left(\frac{a-ib\sin^{-1}(idx^2+1)}{2b}\right)}{2b\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)\right)}-\frac{x\left(\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)\text{CosIntegral}\left(\frac{i\left(a-ib\sin^{-1}(1+idx^2)\right)}{2b}\right)}{2b\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*b*ArcSin[1 + I*d*x^2])^(-1),x]
```

```
[Out] -(x*CosIntegral[((I/2)*(a - I*b*ArcSin[1 + I*d*x^2]))/b]*(I*Cosh[a/(2*b)] +
 Sinh[a/(2*b)]))/(2*b*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]
/2])) + (x*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)])*SinhIntegral[(a - I*b*ArcSin[1
 + I*d*x^2])/(2*b)])/(2*b*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*
x^2]/2]))
```

Rule 4816

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(-1), x_Symbol] :> -Simp[(x
*(c*Cos[a/(2*b)] - Sin[a/(2*b)])*CosIntegral[(c/(2*b))*(a + b*ArcSin[c + d*
x^2])])/(2*b*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2])), x] -
 Simp[(x*(c*Cos[a/(2*b)] + Sin[a/(2*b)])*SinIntegral[(c/(2*b))*(a + b*ArcSi
n[c + d*x^2])])/(2*b*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2]
)), x] /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

Rubi steps

$$\int \frac{1}{a-ib\sin^{-1}\left(1+idx^2\right)}\,dx=-\frac{x\text{Ci}\left(\frac{i\left(a-ib\sin^{-1}(1+idx^2)\right)}{2b}\right)\left(i\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)}{2b\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)\right)}+\frac{x\left(\cosh\left(\frac{a}{2b}\right)+i\sin\ldots\right.}{2b\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+\ldots\right.\right.\right.}$$

Mathematica [A] time = 0.69, size = 146, normalized size = 0.76

$$\frac{x\left(\left(-\sinh\left(\frac{a}{2b}\right)-i\cosh\left(\frac{a}{2b}\right)\right)\text{Ci}\left(\frac{1}{2}\left(\frac{ia}{b}+\sin^{-1}\left(idx^2+1\right)\right)\right)+\left(\sinh\left(\frac{a}{2b}\right)-i\cosh\left(\frac{a}{2b}\right)\right)\text{Si}\left(\frac{1}{2}\left(\frac{ia}{b}+\sin^{-1}\left(idx^2+1\ldots\right.\right.\right.}{2b\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a - I*b*ArcSin[1 + I*d*x^2])^(-1),x]
```

```
[Out] (x*(CosIntegral[((I*a)/b + ArcSin[1 + I*d*x^2])/2]*((-I)*Cosh[a/(2*b)] - Si
nh[a/(2*b)]) + ((-I)*Cosh[a/(2*b)] + Sinh[a/(2*b)])*SinIntegral[((I*a)/b +
ArcSin[1 + I*d*x^2])/2]))/(2*b*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 +
 I*d*x^2]/2]))
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{1}{b\log\left(dx^2+\sqrt{d^2x^2-2i\,d}\,x-i\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2)),x, algorithm="fricas")
```

```
[Out] integral(1/(b*log(d*x^2 + sqrt(d^2*x^2 - 2*I*d)*x - I) + a), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [d,x]=[45,-28]Bad conditionned root j= 1 value -35280.3655931 rati
o 0.379661795171 mindist 0.443514529616Bad conditionned root j= 1 value -51
05.29327315 ratio 1.07361778233 mindist 2.29350729132Warning, choosing root
 of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-8,[3,6]%%%}+%%%{-32,[1,2]%%
%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{16,[0,0]%%%}] at parameters value
s [7,-27]Warning, need to choose a branch for the root of a polynomial with
 parameters. This might be wrong.The choice was done assuming [d,t_nostep]=
[60,97]schur row 1 7.48504e-11Francis algorithm not precise enough for[1.0,
0.0,-1.91223246961e+12,-1.43937562454e+18,-3.04719418157e+23]Bad conditionn
ed root j= 2 value -564549.069246 ratio 4.13534933689 mindist 8.26009499958
schur row 3 8.32254e-09schur row 3 8.32254e-09Francis algorithm not precise
 enough for[1.0,0.0,-137282971022,-2.76877787308e+16,-1.57055117809e+21]Bad
 conditionned root j= 3 value -151279.357647 ratio 3.67253338015 mindist 17
.3373140811Warning, choosing root of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%}
,%%%{-8,[3,6]%%%}+%%%{-32,[1,2]%%%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{
16,[0,0]%%%}] at parameters values [63,-49]Warning, need to choose a branch
 for the root of a polynomial with parameters. This might be wrong.The choi
ce was done assuming [d,t_nostep]=[-20,14]schur row 1 6.86402e-10Francis al
gorithm not precise enough for[1.0,0.0,-129654000008,2.54121840047e+16,-1.4
0084664352e+21]Bad conditionned root j= 2 value 146992.858887 ratio 1.59770
7895 mindist 8.30647902455Warning, choosing root of [1,0,%%%{-6,[2,4]%%%}+%
%%{-8,[0,0]%%%},%%%{-8,[3,6]%%%}+%%%{-32,[1,2]%%%},%%%{-3,[4,8]%%%}+%%%{-24
,[2,4]%%%}+%%%{16,[0,0]%%%}] at parameters values [-30,70]Evaluation time:
3.85sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error
: Bad Argument Value
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{1}{a+b\,\mathrm{arcsinh}\left(d\,x^2-i\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(-I+d*x^2)),x)
[Out] int(1/(a+b*arcsinh(-I+d*x^2)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{b\,\mathrm{arsinh}\left(dx^2-i\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2)),x, algorithm="maxima")
[Out] integrate(1/(b*arcsinh(d*x^2 - I) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{a+b\,\mathrm{asinh}\left(d\,x^2-\mathrm{i}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*asinh(d*x^2 - 1i)),x)
[Out] int(1/(a + b*asinh(d*x^2 - 1i)), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(-I+d*x**2)),x)
[Out] Exception raised: TypeError
```

3.326 $\int \frac{1}{\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^2}\,dx$

Optimal. Leaf size=244

$$\frac{x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{Ci}\left(\frac{i\left(a-ib\sin^{-1}\left(idx^2+1\right)\right)}{2b}\right)}{4b^2\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)\right)}-\frac{x\left(\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)\text{Shi}\left(\frac{a-ib\sin^{-1}\left(idx^2+1\right)}{2b}\right)}{4b^2\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)\right)}$$

```
[Out] 1/4*x*Ci(1/2*I*(a-I*b*arcsin(1+I*d*x^2))/b)*(cosh(1/2*a/b)+I*sinh(1/2*a/b))
/b^2/(cos(1/2*arcsin(1+I*d*x^2))-sin(1/2*arcsin(1+I*d*x^2)))-1/4*x*Shi(1/2*
(a-I*b*arcsin(1+I*d*x^2))/b)*(I*cosh(1/2*a/b)+sinh(1/2*a/b))/b^2/(cos(1/2*a
rcsin(1+I*d*x^2))-sin(1/2*arcsin(1+I*d*x^2)))-1/2*(-2*I*d*x^2+d^2*x^4)^(1/2
)/b/d/x/(a-I*b*arcsin(1+I*d*x^2))
```

Rubi [A] time = 0.03, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {4825}

$$\frac{x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{CosIntegral}\left(\frac{i\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)}{2b}\right)}{4b^2\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)\right)}-\frac{x\left(\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)\text{Shi}\left(\frac{a-ib\sin^{-1}\left(idx^2+1\right)}{2b}\right)}{4b^2\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*b*ArcSin[1 + I*d*x^2])^(-2),x]
```

```
[Out] -Sqrt[(-2*I)*d*x^2 + d^2*x^4]/(2*b*d*x*(a - I*b*ArcSin[1 + I*d*x^2])) + (x*
CosIntegral[((I/2)*(a - I*b*ArcSin[1 + I*d*x^2]))/b]*(Cosh[a/(2*b)] + I*Sin
h[a/(2*b)]))/(4*b^2*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2
])) - (x*(I*Cosh[a/(2*b)] + Sinh[a/(2*b)])*SinhIntegral[(a - I*b*ArcSin[1 +
 I*d*x^2])/(2*b)])/(4*b^2*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*
x^2]/2]))
```

Rule 4825

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(-2), x_Symbol] :> -Simp[Sq
rt[-2*c*d*x^2 - d^2*x^4]/(2*b*d*x*(a + b*ArcSin[c + d*x^2])), x] + (-Simp[(
x*(Cos[a/(2*b)] + c*Sin[a/(2*b)])*CosIntegral[(c/(2*b))*(a + b*ArcSin[c + d
*x^2])])/(4*b^2*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2])), x
] + Simp[(x*(Cos[a/(2*b)] - c*Sin[a/(2*b)])*SinIntegral[(c/(2*b))*(a + b*Ar
cSin[c + d*x^2])])/(4*b^2*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^
2]/2])), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

Rubi steps

$$\int \frac{1}{\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^2}\,dx=-\frac{\sqrt{-2idx^2+d^2x^4}}{2bdx\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)}+\frac{x\text{Ci}\left(\frac{i\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)}{2b}\right)\left(\cosh\left(\frac{a}{2b}\right)+i\ldots\right.}{4b^2\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\ldots\right.\right.}$$

Mathematica [A] time = 1.49, size = 196, normalized size = 0.80

$$\frac{\frac{x^2\left(\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{Ci}\left(\frac{1}{2}\left(\frac{ia}{b}+\sin^{-1}\left(idx^2+1\right)\right)\right)-\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)\text{Si}\left(\frac{1}{2}\left(\frac{ia}{b}+\sin^{-1}\left(idx^2+1\right)\right)\right)\right)}{\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)}-\frac{2b\sqrt{dx^2\left(dx^2-2i\right)}}{d\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)}}{4b^2x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a - I*b*ArcSin[1 + I*d*x^2])^(-2),x]
```

```
[Out] ((-2*b*Sqrt[d*x^2*(-2*I + d*x^2)])/(d*(a - I*b*ArcSin[1 + I*d*x^2])) + (x^2
*(CosIntegral[((I*a)/b + ArcSin[1 + I*d*x^2])/2]*(Cosh[a/(2*b)] + I*Sinh[a/
(2*b)]) - (Cosh[a/(2*b)] - I*Sinh[a/(2*b)])*SinIntegral[((I*a)/b + ArcSin[1
 + I*d*x^2])/2]))/(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2])
)/(4*b^2*x)
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\frac{2\left(b^2 d\log\left(dx^2+\sqrt{d^2x^2-2i\,d}\,x-i\right)+abd\right)\mathrm{integral}\left(\frac{\sqrt{d^2x^2-2i\,d}\,x}{2\,abdx^2-4i\,ab+\left(2\,b^2dx^2-4i\,b^2\right)\log\left(dx^2+\sqrt{d^2x^2-2i\,d}\,x-i\right)},x\right)-\sqrt{d^2x^2-2}}{2\left(b^2d\log\left(dx^2+\sqrt{d^2x^2-2i\,d}\,x-i\right)+abd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*(b^2*d*log(d*x^2 + sqrt(d^2*x^2 - 2*I*d)*x - I) + a*b*d)*integral(sq
rt(d^2*x^2 - 2*I*d)*x/(2*a*b*d*x^2 - 4*I*a*b + (2*b^2*d*x^2 - 4*I*b^2)*log(
d*x^2 + sqrt(d^2*x^2 - 2*I*d)*x - I)), x) - sqrt(d^2*x^2 - 2*I*d))/(b^2*d*l
og(d*x^2 + sqrt(d^2*x^2 - 2*I*d)*x - I) + a*b*d)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [d,x]=[45,-28]Bad conditionned root j= 1 value -35280.3655931 rati
o 0.379661795171 mindist 0.443514529616Bad conditionned root j= 1 value -51
05.29327315 ratio 1.07361778233 mindist 2.29350729132Warning, choosing root
 of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-8,[3,6]%%%}+%%%{-32,[1,2]%%
%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{16,[0,0]%%%}] at parameters value
s [7,-27]Warning, need to choose a branch for the root of a polynomial with
 parameters. This might be wrong.The choice was done assuming [d,t_nostep]=
[79,3]schur row 3 8.74347e-08Bad conditionned root j= 2 value -151313.41286
2 ratio 11.2206791301 mindist 48.7986537395Warning, choosing root of [1,0,%
%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-8,[3,6]%%%}+%%%{-32,[1,2]%%%},%%%{-3,
[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{16,[0,0]%%%}] at parameters values [63,-49]
Warning, need to choose a branch for the root of a polynomial with paramete
rs. This might be wrong.The choice was done assuming [d,t_nostep]=[-27,9]Ba
d conditionned root j= 2 value 147025.62453 ratio 5.74493624992 mindist 24.
9427695529Warning, choosing root of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},
%%%{-8,[3,6]%%%}+%%%{-32,[1,2]%%%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{1
6,[0,0]%%%}] at parameters values [-30,70]Evaluation time: 12.5sym2poly/r2s
ym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument V
alue
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\,\mathrm{arcsinh}\left(d\,x^2-i\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(-I+d*x^2))^2,x)
```

```
[Out] int(1/(a+b*arcsinh(-I+d*x^2))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{d^2x^4 - 3i\,dx^2 + \left(d^{\frac{3}{2}}x^3 - 2i\,\sqrt{d}\,x\right)\sqrt{dx^2 - 2i} - 2}{2\,abd^2x^3 - 4i\,abdx + \left(2\,b^2d^2x^3 - 4i\,b^2dx + \left(2\,b^2d^{\frac{3}{2}}x^2 - 2i\,b^2\sqrt{d}\right)\sqrt{dx^2 - 2i}\right)\log\left(dx^2 + \sqrt{dx^2 - 2i}\,\sqrt{d}\,x - i\right) \cdot}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^2,x, algorithm="maxima")
```

```
[Out] -(d^2*x^4 - 3*I*d*x^2 + (d^(3/2)*x^3 - 2*I*sqrt(d)*x)*sqrt(d*x^2 - 2*I) - 2
)/(2*a*b*d^2*x^3 - 4*I*a*b*d*x + (2*b^2*d^2*x^3 - 4*I*b^2*d*x + (2*b^2*d^(3
/2)*x^2 - 2*I*b^2*sqrt(d))*sqrt(d*x^2 - 2*I))*log(d*x^2 + sqrt(d*x^2 - 2*I)
*sqrt(d)*x - I) + 2*(a*b*d^(3/2)*x^2 - I*a*b*sqrt(d))*sqrt(d*x^2 - 2*I)) +
integrate((2*d^3*x^6 - 6*I*d^2*x^4 + (2*d^2*x^4 - 2*I*d*x^2 - 4)*(d*x^2 - 2
*I) + 2*(2*d^(5/2)*x^5 - 4*I*d^(3/2)*x^3 - sqrt(d)*x)*sqrt(d*x^2 - 2*I) - 8
*I)/(4*a*b*d^3*x^6 - 16*I*a*b*d^2*x^4 - 16*a*b*d*x^2 + (4*a*b*d^2*x^4 - 8*I
*a*b*d*x^2 - 4*a*b)*(d*x^2 - 2*I) + (4*b^2*d^3*x^6 - 16*I*b^2*d^2*x^4 - 16*
b^2*d*x^2 + 4*(b^2*d^2*x^4 - 2*I*b^2*d*x^2 - b^2)*(d*x^2 - 2*I) + 8*(b^2*d^
(5/2)*x^5 - 3*I*b^2*d^(3/2)*x^3 - 2*b^2*sqrt(d)*x)*sqrt(d*x^2 - 2*I))*log(d
*x^2 + sqrt(d*x^2 - 2*I)*sqrt(d)*x - I) + (8*a*b*d^(5/2)*x^5 - 24*I*a*b*d^(
3/2)*x^3 - 16*a*b*sqrt(d)*x)*sqrt(d*x^2 - 2*I)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a + b\,\mathrm{asinh}\left(d\,x^2 - \mathrm{i}\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*asinh(d*x^2 - 1i))^2,x)
```

```
[Out] int(1/(a + b*asinh(d*x^2 - 1i))^2, x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(-I+d*x**2))**2,x)
```

```
[Out] Exception raised: TypeError
```

3.327 $\int \frac{1}{\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^3}\,dx$

Optimal. Leaf size=272

$$-\frac{x\left(\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)\mathrm{Ci}\left(\frac{i\left(a-ib\sin^{-1}\left(idx^2+1\right)\right)}{2b}\right)}{16b^3\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)\right)}+\frac{x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\mathrm{Shi}\left(\frac{a-ib\sin^{-1}\left(idx^2+1\right)}{2b}\right)}{16b^3\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+id\right.\right.\right.}$$

```
[Out] -1/8*x/b^2/(a-I*b*arcsin(1+I*d*x^2))+1/16*x*Shi(1/2*(a-I*b*arcsin(1+I*d*x^2
))/b)*(cosh(1/2*a/b)+I*sinh(1/2*a/b))/b^3/(cos(1/2*arcsin(1+I*d*x^2))-sin(1
/2*arcsin(1+I*d*x^2)))-1/16*x*Ci(1/2*I*(a-I*b*arcsin(1+I*d*x^2))/b)*(I*cosh
(1/2*a/b)+sinh(1/2*a/b))/b^3/(cos(1/2*arcsin(1+I*d*x^2))-sin(1/2*arcsin(1+I
*d*x^2)))-1/4*(-2*I*d*x^2+d^2*x^4)^(1/2)/b/d/x/(a-I*b*arcsin(1+I*d*x^2))^2
```

Rubi [A] time = 0.05, antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {4828, 4816}

$$-\frac{x\left(\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)\mathrm{CosIntegral}\left(\frac{i\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)}{2b}\right)}{16b^3\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)\right)}+\frac{x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\mathrm{Shi}\left(\frac{a-ib\sin^{-1}\left(idx^2+\right.}{2b}\right.}{16b^3\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+\right.\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*b*ArcSin[1 + I*d*x^2])^(-3),x]
```

```
[Out] -Sqrt[(-2*I)*d*x^2 + d^2*x^4]/(4*b*d*x*(a - I*b*ArcSin[1 + I*d*x^2])^2) - x
/(8*b^2*(a - I*b*ArcSin[1 + I*d*x^2])) - (x*CosIntegral[((I/2)*(a - I*b*Arc
Sin[1 + I*d*x^2]))/b]*(I*Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(16*b^3*(Cos[ArcSi
n[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2])) + (x*(Cosh[a/(2*b)] + I*Si
nh[a/(2*b)])*SinhIntegral[(a - I*b*ArcSin[1 + I*d*x^2])/(2*b)])/(16*b^3*(Co
s[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2]))
```

Rule 4816

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(-1), x_Symbol] :> -Simp[(x
*(c*Cos[a/(2*b)] - Sin[a/(2*b)])*CosIntegral[(c/(2*b))*(a + b*ArcSin[c + d*
x^2])])/(2*b*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2])), x] -
 Simp[(x*(c*Cos[a/(2*b)] + Sin[a/(2*b)])*SinIntegral[(c/(2*b))*(a + b*ArcSi
n[c + d*x^2])])/(2*b*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2]
)), x] /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

Rule 4828

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[(x*
(a + b*ArcSin[c + d*x^2])^(n + 2))/(4*b^2*(n + 1)*(n + 2)), x] + (-Dist[1/(
4*b^2*(n + 1)*(n + 2)), Int[(a + b*ArcSin[c + d*x^2])^(n + 2), x], x] + Sim
p[(Sqrt[-2*c*d*x^2 - d^2*x^4]*(a + b*ArcSin[c + d*x^2])^(n + 1))/(2*b*d*(n
+ 1)*x), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && LtQ[n, -1] && NeQ[
n, -2]
```

Rubi steps

$$\int \frac{1}{\left(a - ib\sin^{-1}\left(1 + idx^2\right)\right)^3}\,dx = -\frac{\sqrt{-2idx^2 + d^2x^4}}{4bdx\left(a - ib\sin^{-1}\left(1 + idx^2\right)\right)^2} - \frac{x}{8b^2\left(a - ib\sin^{-1}\left(1 + idx^2\right)\right)} + \frac{\int \frac{}{a-ib\sin^{-}}}{8}$$

$$= -\frac{\sqrt{-2idx^2 + d^2x^4}}{4bdx\left(a - ib\sin^{-1}\left(1 + idx^2\right)\right)^2} - \frac{x}{8b^2\left(a - ib\sin^{-1}\left(1 + idx^2\right)\right)} - \frac{x\mathrm{Ci}\left(\right.}{16b^3\left(\cos\right.}$$

Mathematica [A] time = 0.77, size = 227, normalized size = 0.83

$$\frac{\frac{8b^2\sqrt{dx^2(dx^2-2i)}}{d\left(ia+b\sin^{-1}\left(1+idx^2\right)\right)^2} - \frac{2ix^2\left(\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)\mathrm{Ci}\left(\frac{1}{2}\left(\frac{ia}{b}+\sin^{-1}\left(idx^2+1\right)\right)\right)+\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\mathrm{Si}\left(\frac{1}{2}\left(\frac{ia}{b}+\sin^{-1}\left(idx^2+1\right)\right)\right)\right)}{\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)} - \frac{}{a-ib\sin}}{32b^3x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a - I*b*ArcSin[1 + I*d*x^2])^(-3),x]
```

```
[Out] ((-4*b*x^2)/(a - I*b*ArcSin[1 + I*d*x^2]) + (8*b^2*Sqrt[d*x^2*(-2*I + d*x^2
)])/(d*(I*a + b*ArcSin[1 + I*d*x^2])^2) - ((2*I)*x^2*(CosIntegral[((I*a)/b
+ ArcSin[1 + I*d*x^2])/2]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]) + (Cosh[a/(2*b)
] + I*Sinh[a/(2*b)])*SinIntegral[((I*a)/b + ArcSin[1 + I*d*x^2])/2]))/(Cos[
ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2]))/(32*b^3*x)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$-\frac{bdx\log\left(dx^2 + \sqrt{d^2x^2 - 2i\,d}\,x - i\right) + adx - 8\left(b^4d\log\left(dx^2 + \sqrt{d^2x^2 - 2i\,d}\,x - i\right)^2 + 2\,ab^3d\log\left(dx^2 + \sqrt{d^2x^2}\right.\right.}{8\left(b^4d\log\left(dx^2 + \sqrt{d^2x^2 - 2i\,d}\,x - i\right)^2 + 2\,ab^3d\log\left(dx^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^3,x, algorithm="fricas")
```

```
[Out] -1/8*(b*d*x*log(d*x^2 + sqrt(d^2*x^2 - 2*I*d)*x - I) + a*d*x - 8*(b^4*d*log
(d*x^2 + sqrt(d^2*x^2 - 2*I*d)*x - I)^2 + 2*a*b^3*d*log(d*x^2 + sqrt(d^2*x^
2 - 2*I*d)*x - I) + a^2*b^2*d)*integral(1/8/(b^3*log(d*x^2 + sqrt(d^2*x^2 -
 2*I*d)*x - I) + a*b^2), x) + 2*sqrt(d^2*x^2 - 2*I*d)*b)/(b^4*d*log(d*x^2 +
 sqrt(d^2*x^2 - 2*I*d)*x - I)^2 + 2*a*b^3*d*log(d*x^2 + sqrt(d^2*x^2 - 2*I*
d)*x - I) + a^2*b^2*d)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [d,x]=[45,-28]Bad conditionned root j= 1 value -35280.3655931 rati
o 0.379661795171 mindist 0.443514529616Bad conditionned root j= 1 value -51
05.29327315 ratio 1.07361778233 mindist 2.29350729132Warning, choosing root
```

```
 of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-8,[3,6]%%%}+%%%{-32,[1,2]%%
%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{16,[0,0]%%%}] at parameters value
s [7,-27]Warning, need to choose a branch for the root of a polynomial with
 parameters. This might be wrong.The choice was done assuming [d,t_nostep]=
[65,-98]schur row 1 5.57208e-12schur row 2 -6.7303e-09Francis algorithm not
 precise enough for[1.0,0.0,-2.33820328561e+12,-1.94619571078e+18,-4.555995
50409e+23]Bad conditionned root j= 1 value -624323.751966 ratio 14.00780622
15 mindist 69.4834240274Bad conditionned root j= 1 value -151261.012618 rat
io 2.78093290945 mindist 7.18852886464Bad conditionned root j= 2 value -151
274.163312 ratio 2.51776304852 mindist 13.1506943274Warning, choosing root
of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-8,[3,6]%%%}+%%%{-32,[1,2]%%%
},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{16,[0,0]%%%}] at parameters values
 [63,-49]Warning, need to choose a branch for the root of a polynomial with
 parameters. This might be wrong.The choice was done assuming [d,t_nostep]=
[-86,2]Bad conditionned root j= 2 value 147044.272515 ratio 9.86512495427 m
indist 45.8740775194Warning, choosing root of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,
[0,0]%%%},%%%{-8,[3,6]%%%}+%%%{-32,[1,2]%%%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]
%%%}+%%%{16,[0,0]%%%}] at parameters values [-30,70]Evaluation time: 29.51s
ym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad
 Argument Value
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\,\mathrm{arcsinh}\left(d\,x^{2}-i\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(-I+d*x^2))^3,x)
```

```
[Out] int(1/(a+b*arcsinh(-I+d*x^2))^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^3,x, algorithm="maxima")
```

```
[Out] -(4*(a*d^(11/2) + 2*b*d^(11/2))*x^10 + (-24*I*a*d^(9/2) - 56*I*b*d^(9/2))*x
^8 - 4*(11*a*d^(7/2) + 36*b*d^(7/2))*x^6 + (8*I*a*d^(5/2) + 160*I*b*d^(5/2)
)*x^4 - 16*(3*a*d^(3/2) - 4*b*d^(3/2))*x^2 + (4*(a*d^4 + 2*b*d^4)*x^7 + (-1
2*I*a*d^3 - 32*I*b*d^3)*x^5 - 8*(2*a*d^2 + 5*b*d^2)*x^3 + (16*I*a*d + 16*I*
b*d)*x)*(d*x^2 - 2*I)^(3/2) + (12*(a*d^(9/2) + 2*b*d^(9/2))*x^8 + (-48*I*a*
d^(7/2) - 120*I*b*d^(7/2))*x^6 - 8*(8*a*d^(5/2) + 25*b*d^(5/2))*x^4 + (40*I
*a*d^(3/2) + 120*I*b*d^(3/2))*x^2 + 16*a*sqrt(d) + 16*b*sqrt(d))*(d*x^2 - 2
*I) + (4*b*d^(11/2)*x^10 - 24*I*b*d^(9/2)*x^8 - 44*b*d^(7/2)*x^6 + 8*I*b*d^
(5/2)*x^4 - 48*b*d^(3/2)*x^2 + (4*b*d^4*x^7 - 12*I*b*d^3*x^5 - 16*b*d^2*x^3
 + 16*I*b*d*x)*(d*x^2 - 2*I)^(3/2) + (12*b*d^(9/2)*x^8 - 48*I*b*d^(7/2)*x^6
 - 64*b*d^(5/2)*x^4 + 40*I*b*d^(3/2)*x^2 + 16*b*sqrt(d))*(d*x^2 - 2*I) + (1
2*b*d^5*x^9 - 60*I*b*d^4*x^7 - 92*b*d^3*x^5 + 28*I*b*d^2*x^3 - 24*b*d*x)*sq
rt(d*x^2 - 2*I) + 32*I*b*sqrt(d))*log(d*x^2 + sqrt(d*x^2 - 2*I)*sqrt(d)*x -
 I) + (12*(a*d^5 + 2*b*d^5)*x^9 + (-60*I*a*d^4 - 144*I*b*d^4)*x^7 - 4*(23*a
*d^3 + 76*b*d^3)*x^5 + (28*I*a*d^2 + 256*I*b*d^2)*x^3 - 8*(3*a*d - 8*b*d)*x
)*sqrt(d*x^2 - 2*I) + 32*I*a*sqrt(d))/(32*a^2*b^2*d^(11/2)*x^9 - 192*I*a^2*
b^2*d^(9/2)*x^7 - 384*a^2*b^2*d^(7/2)*x^5 + 256*I*a^2*b^2*d^(5/2)*x^3 + (32
*b^4*d^(11/2)*x^9 - 192*I*b^4*d^(9/2)*x^7 - 384*b^4*d^(7/2)*x^5 + 256*I*b^4
*d^(5/2)*x^3 + (32*b^4*d^4*x^6 - 96*I*b^4*d^3*x^4 - 96*b^4*d^2*x^2 + 32*I*b
^4*d)*(d*x^2 - 2*I)^(3/2) + (96*b^4*d^(9/2)*x^7 - 384*I*b^4*d^(7/2)*x^5 - 4
80*b^4*d^(5/2)*x^3 + 192*I*b^4*d^(3/2)*x)*(d*x^2 - 2*I) + (96*b^4*d^5*x^8 -
```

```
480*I*b^4*d^4*x^6 - 768*b^4*d^3*x^4 + 384*I*b^4*d^2*x^2)*sqrt(d*x^2 - 2*I)
)*log(d*x^2 + sqrt(d*x^2 - 2*I)*sqrt(d)*x - I)^2 + (32*a^2*b^2*d^4*x^6 - 96
*I*a^2*b^2*d^3*x^4 - 96*a^2*b^2*d^2*x^2 + 32*I*a^2*b^2*d)*(d*x^2 - 2*I)^(3/
2) + (96*a^2*b^2*d^(9/2)*x^7 - 384*I*a^2*b^2*d^(7/2)*x^5 - 480*a^2*b^2*d^(5
/2)*x^3 + 192*I*a^2*b^2*d^(3/2)*x)*(d*x^2 - 2*I) + (64*a*b^3*d^(11/2)*x^9 -
 384*I*a*b^3*d^(9/2)*x^7 - 768*a*b^3*d^(7/2)*x^5 + 512*I*a*b^3*d^(5/2)*x^3
+ (64*a*b^3*d^4*x^6 - 192*I*a*b^3*d^3*x^4 - 192*a*b^3*d^2*x^2 + 64*I*a*b^3*
d)*(d*x^2 - 2*I)^(3/2) + (192*a*b^3*d^(9/2)*x^7 - 768*I*a*b^3*d^(7/2)*x^5 -
 960*a*b^3*d^(5/2)*x^3 + 384*I*a*b^3*d^(3/2)*x)*(d*x^2 - 2*I) + (192*a*b^3*
d^5*x^8 - 960*I*a*b^3*d^4*x^6 - 1536*a*b^3*d^3*x^4 + 768*I*a*b^3*d^2*x^2)*s
qrt(d*x^2 - 2*I))*log(d*x^2 + sqrt(d*x^2 - 2*I)*sqrt(d)*x - I) + (96*a^2*b^
2*d^5*x^8 - 480*I*a^2*b^2*d^4*x^6 - 768*a^2*b^2*d^3*x^4 + 384*I*a^2*b^2*d^2
*x^2)*sqrt(d*x^2 - 2*I)) + integrate((d^6*x^12 - 8*I*d^5*x^10 - 27*d^4*x^8
+ 56*I*d^3*x^6 + 88*d^2*x^4 + (d^4*x^8 - 4*I*d^3*x^6 - 3*d^2*x^4 - 8*I*d*x^
2 + 4)*(d*x^2 - 2*I)^2 - 96*I*d*x^2 + (4*d^(9/2)*x^9 - 20*I*d^(7/2)*x^7 - 3
0*d^(5/2)*x^5 - 2*I*d^(3/2)*x^3 - 22*sqrt(d)*x)*(d*x^2 - 2*I)^(3/2) + (6*d^
5*x^10 - 36*I*d^4*x^8 - 78*d^3*x^6 + 72*I*d^2*x^4 + 9*d*x^2 + 30*I)*(d*x^2
- 2*I) + (4*d^(11/2)*x^11 - 28*I*d^(9/2)*x^9 - 78*d^(7/2)*x^7 + 122*I*d^(5/
2)*x^5 + 122*d^(3/2)*x^3 - 60*I*sqrt(d)*x)*sqrt(d*x^2 - 2*I) - 48)/(8*a*b^2
*d^6*x^12 - 64*I*a*b^2*d^5*x^10 - 192*a*b^2*d^4*x^8 + 256*I*a*b^2*d^3*x^6 +
 128*a*b^2*d^2*x^4 + (8*a*b^2*d^4*x^8 - 32*I*a*b^2*d^3*x^6 - 48*a*b^2*d^2*x
^4 + 32*I*a*b^2*d*x^2 + 8*a*b^2)*(d*x^2 - 2*I)^2 + (32*a*b^2*d^(9/2)*x^9 -
160*I*a*b^2*d^(7/2)*x^7 - 288*a*b^2*d^(5/2)*x^5 + 224*I*a*b^2*d^(3/2)*x^3 +
 64*a*b^2*sqrt(d)*x)*(d*x^2 - 2*I)^(3/2) + (48*a*b^2*d^5*x^10 - 288*I*a*b^2
*d^4*x^8 - 624*a*b^2*d^3*x^6 + 576*I*a*b^2*d^2*x^4 + 192*a*b^2*d*x^2)*(d*x^
2 - 2*I) + (8*b^3*d^6*x^12 - 64*I*b^3*d^5*x^10 - 192*b^3*d^4*x^8 + 256*I*b^
3*d^3*x^6 + 128*b^3*d^2*x^4 + (8*b^3*d^4*x^8 - 32*I*b^3*d^3*x^6 - 48*b^3*d^
2*x^4 + 32*I*b^3*d*x^2 + 8*b^3)*(d*x^2 - 2*I)^2 + (32*b^3*d^(9/2)*x^9 - 160
*I*b^3*d^(7/2)*x^7 - 288*b^3*d^(5/2)*x^5 + 224*I*b^3*d^(3/2)*x^3 + 64*b^3*s
qrt(d)*x)*(d*x^2 - 2*I)^(3/2) + (48*b^3*d^5*x^10 - 288*I*b^3*d^4*x^8 - 624*
b^3*d^3*x^6 + 576*I*b^3*d^2*x^4 + 192*b^3*d*x^2)*(d*x^2 - 2*I) + (32*b^3*d^
(11/2)*x^11 - 224*I*b^3*d^(9/2)*x^9 - 576*b^3*d^(7/2)*x^7 + 640*I*b^3*d^(5/
2)*x^5 + 256*b^3*d^(3/2)*x^3)*sqrt(d*x^2 - 2*I))*log(d*x^2 + sqrt(d*x^2 - 2
*I)*sqrt(d)*x - I) + (32*a*b^2*d^(11/2)*x^11 - 224*I*a*b^2*d^(9/2)*x^9 - 57
6*a*b^2*d^(7/2)*x^7 + 640*I*a*b^2*d^(5/2)*x^5 + 256*a*b^2*d^(3/2)*x^3)*sqrt
(d*x^2 - 2*I)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+b\,\mathrm{asinh}\left(d\,x^2-\mathrm{i}\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*asinh(d*x^2 - 1i))^3,x)
[Out] int(1/(a + b*asinh(d*x^2 - 1i))^3, x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(-I+d*x**2))**3,x)
[Out] Exception raised: TypeError
```

3.328 $\int \left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^{5/2}\,dx$

Optimal. Leaf size=348

$$-\frac{15\sqrt{\pi}\sqrt{-\frac{i}{b}}\,b^3x\left(\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)C\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\sin^{-1}\left(1-idx^2\right)}}{\sqrt{\pi}}\right)}{\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)}+\frac{15\sqrt{\pi}\,b^2x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)S\left(\frac{\sqrt{-\frac{i}{b}}\cdots}{\cdots}\right.}{\sqrt{-\frac{i}{b}}\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\mathrm{si}\cdots\right.\right.}$$

```
[Out] x*(a-I*b*arcsin(-1+I*d*x^2))^(5/2)+15*b^2*x*FresnelS((-I/b)^(1/2)*(a-I*b*ar
csin(-1+I*d*x^2))^(1/2)/Pi^(1/2))*(cosh(1/2*a/b)+I*sinh(1/2*a/b))*Pi^(1/2)/
(cos(1/2*arcsin(-1+I*d*x^2))+sin(1/2*arcsin(-1+I*d*x^2)))/(-I/b)^(1/2)-15*b
^3*x*FresnelC((-I/b)^(1/2)*(a-I*b*arcsin(-1+I*d*x^2))^(1/2)/Pi^(1/2))*(I*co
sh(1/2*a/b)+sinh(1/2*a/b))*(-I/b)^(1/2)*Pi^(1/2)/(cos(1/2*arcsin(-1+I*d*x^2
))+sin(1/2*arcsin(-1+I*d*x^2)))-5*b*(a-I*b*arcsin(-1+I*d*x^2))^(3/2)*(2*I*d
*x^2+d^2*x^4)^(1/2)/d/x+15*b^2*x*(a-I*b*arcsin(-1+I*d*x^2))^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 348, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4814, 4811}

$$-\frac{15\sqrt{\pi}\sqrt{-\frac{i}{b}}\,b^3x\left(\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\sin^{-1}\left(1-idx^2\right)}}{\sqrt{\pi}}\right)}{\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)}+\frac{15\sqrt{\pi}\,b^2x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\cdots}{\sqrt{-\frac{i}{b}}\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*b*ArcSin[1 - I*d*x^2])^(5/2),x]
```

```
[Out] 15*b^2*x*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]] - (5*b*Sqrt[(2*I)*d*x^2 + d^2*x^
4]*(a + I*b*ArcSin[1 - I*d*x^2])^(3/2))/(d*x) + x*(a + I*b*ArcSin[1 - I*d*x
^2])^(5/2) + (15*b^2*Sqrt[Pi]*x*FresnelS[(Sqrt[(-I)/b]*Sqrt[a + I*b*ArcSin[
1 - I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)]))/(Sqrt[(-I)/b]*(
Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2])) - (15*Sqrt[(-I)/b
]*b^3*Sqrt[Pi]*x*FresnelC[(Sqrt[(-I)/b]*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]])/
Sqrt[Pi]]*(I*Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(Cos[ArcSin[1 - I*d*x^2]/2] -
Sin[ArcSin[1 - I*d*x^2]/2])
```

Rule 4811

```
Int[Sqrt[(a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[x*Sq
rt[a + b*ArcSin[c + d*x^2]], x] + (-Simp[(Sqrt[Pi]*x*(Cos[a/(2*b)] + c*Sin[
a/(2*b)])*FresnelC[Sqrt[c/(Pi*b)]*Sqrt[a + b*ArcSin[c + d*x^2]]])/(Sqrt[c/b
]*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2])), x] + Simp[(Sqrt
[Pi]*x*(Cos[a/(2*b)] - c*Sin[a/(2*b)])*FresnelS[Sqrt[c/(Pi*b)]*Sqrt[a + b*A
rcSin[c + d*x^2]]])/(Sqrt[c/b]*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c +
 d*x^2]/2])), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

Rule 4814

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcSin[c + d*x^2])^n, x] + (-Dist[4*b^2*n*(n - 1), Int[(a + b*ArcSin[
c + d*x^2])^(n - 2), x], x] + Simp[(2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*(a + b
*ArcSin[c + d*x^2])^(n - 1))/(d*x), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^
2, 1] && GtQ[n, 1]
```

Rubi steps

$$\int \left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^{5/2}dx = -\frac{5b\sqrt{2idx^2+d^2x^4}\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^{3/2}}{dx}+x\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)$$

$$= 15b^2x\sqrt{a+ib\sin^{-1}\left(1-idx^2\right)}-\frac{5b\sqrt{2idx^2+d^2x^4}\left(a+ib\sin^{-1}\left(1-idx^2\right)\right.}{dx}$$

Mathematica [A] time = 0.29, size = 337, normalized size = 0.97

$$\frac{15b^2x\left(-\sqrt{\pi}\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)C\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\sin^{-1}\left(1-idx^2\right)}}{\sqrt{\pi}}\right)+\sqrt{\pi}\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)S\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\mathrm{s}}}{\sqrt{}}\right.\right.}{\sqrt{-\frac{i}{b}}\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\mathrm{si}\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*b*ArcSin[1 - I*d*x^2])^(5/2),x]
```

```
[Out] (-5*b*Sqrt[d*x^2*(2*I + d*x^2)]*(a + I*b*ArcSin[1 - I*d*x^2])^(3/2))/(d*x)
+ x*(a + I*b*ArcSin[1 - I*d*x^2])^(5/2) + (15*b^2*x*(Sqrt[(-I)/b]*Sqrt[a +
I*b*ArcSin[1 - I*d*x^2]]*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x
^2]/2]) - Sqrt[Pi]*FresnelC[(Sqrt[(-I)/b]*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]]
)/Sqrt[Pi]]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]) + Sqrt[Pi]*FresnelS[(Sqrt[(-I
)/b]*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] + I*Sinh[a
/(2*b)])))/(Sqrt[(-I)/b]*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x
^2]/2]))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(I+d*x^2))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(I+d*x^2))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [d,t_nostep]=[33,44]Bad conditionned root j= 2 value -63897.972732
 ratio 2.32090080903 mindist 10.1217065137Warning, choosing root of [1,0,%%
%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-8,[3,6]%%%}+%%%{-32,[1,2]%%%},%%%{-3,[
4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{16,[0,0]%%%}] at parameters values [7,-27]Wa
rning, need to choose a branch for the root of a polynomial with parameters
. This might be wrong.The choice was done assuming [d,t_nostep]=[50,45]Fran
cis algorithm failure for[1.0,0.0,-61509375008,-8.30376562824e+15,-3.152836
0132e+20]schur row 1 1.29492e-09Francis algorithm not precise enough for[1.
```

```
0,0.0,-61509375008,-8.30376562824e+15,-3.1528360132e+20]Bad conditionned ro
ot j= 2 value -101243.096423 ratio 1.67716116885 mindist 7.19967768138Franc
is algorithm failure for[1.0,0.0,-137282971022,-2.76877787308e+16,-1.570551
17809e+21]Warning, choosing root of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},
%%%{-8,[3,6]%%%}+%%%{-32,[1,2]%%%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{1
6,[0,0]%%%}] at parameters values [63,-49]Evaluation time: 1.58sym2poly/r2s
ym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument V
alue
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \left(a + b \operatorname{arcsinh}\left(d x^2 + i\right)\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(I+d*x^2))^(5/2),x)
[Out] int((a+b*arcsinh(I+d*x^2))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \operatorname{arsinh}\left(dx^2 + i\right) + a\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(I+d*x^2))^(5/2),x, algorithm="maxima")
[Out] integrate((b*arcsinh(d*x^2 + I) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a + b \operatorname{asinh}\left(d x^2 + 1\mathrm{i}\right)\right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(d*x^2 + 1i))^(5/2),x)
[Out] int((a + b*asinh(d*x^2 + 1i))^(5/2), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(I+d*x**2))**(5/2),x)
[Out] Exception raised: TypeError
```

3.329 $\int \left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^{3/2}\,dx$

Optimal. Leaf size=312

$$-\frac{3\sqrt{\pi}\,b^2x\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)S\left(\frac{\sqrt{a+ib\sin^{-1}\left(1-idx^2\right)}}{\sqrt{ib}\,\sqrt{\pi}}\right)}{\sqrt{ib}\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)\right)}-\frac{3b\sqrt{d^2x^4+2idx^2}\,\sqrt{a+ib\sin^{-1}\left(1-idx^2\right)}}{dx}+\frac{3\sqrt{\pi}\cdots}{\cdots}$$

```
[Out] x*(a-I*b*arcsin(-1+I*d*x^2))^(3/2)-3*b^2*x*FresnelS((a-I*b*arcsin(-1+I*d*x^
2))^(1/2)/(I*b)^(1/2)/Pi^(1/2))*(cosh(1/2*a/b)-I*sinh(1/2*a/b))*Pi^(1/2)/(c
os(1/2*arcsin(-1+I*d*x^2))+sin(1/2*arcsin(-1+I*d*x^2)))/(I*b)^(1/2)+3*b*x*F
resnelC((a-I*b*arcsin(-1+I*d*x^2))^(1/2)/(I*b)^(1/2)/Pi^(1/2))*(I*cosh(1/2*
a/b)-sinh(1/2*a/b))*(I*b)^(1/2)*Pi^(1/2)/(cos(1/2*arcsin(-1+I*d*x^2))+sin(1
/2*arcsin(-1+I*d*x^2)))-3*b*(2*I*d*x^2+d^2*x^4)^(1/2)*(a-I*b*arcsin(-1+I*d*
x^2))^(1/2)/d/x
```

Rubi [A] time = 0.11, antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4814, 4819}

$$-\frac{3\sqrt{\pi}\,b^2x\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)S\left(\frac{\sqrt{a+ib\sin^{-1}\left(1-idx^2\right)}}{\sqrt{ib}\,\sqrt{\pi}}\right)}{\sqrt{ib}\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)\right)}-\frac{3b\sqrt{d^2x^4+2idx^2}\,\sqrt{a+ib\sin^{-1}\left(1-idx^2\right)}}{dx}+\frac{3\sqrt{\pi}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*b*ArcSin[1 - I*d*x^2])^(3/2),x]
```

```
[Out] (-3*b*Sqrt[(2*I)*d*x^2 + d^2*x^4]*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]])/(d*x)
+ x*(a + I*b*ArcSin[1 - I*d*x^2])^(3/2) + (3*Sqrt[I*b]*b*Sqrt[Pi]*x*Fresnel
C[Sqrt[a + I*b*ArcSin[1 - I*d*x^2]]/(Sqrt[I*b]*Sqrt[Pi])]*(I*Cosh[a/(2*b)]
- Sinh[a/(2*b)]))/(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2])
 - (3*b^2*Sqrt[Pi]*x*FresnelS[Sqrt[a + I*b*ArcSin[1 - I*d*x^2]]/(Sqrt[I*b]*
Sqrt[Pi])]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]))/(Sqrt[I*b]*(Cos[ArcSin[1 - I*
d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2]))
```

Rule 4814

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcSin[c + d*x^2])^n, x] + (-Dist[4*b^2*n*(n - 1), Int[(a + b*ArcSin[
c + d*x^2])^(n - 2), x], x] + Simp[(2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*(a + b
*ArcSin[c + d*x^2])^(n - 1))/(d*x), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^
2, 1] && GtQ[n, 1]
```

Rule 4819

```
Int[1/Sqrt[(a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> -Simp[(
Sqrt[Pi]*x*(Cos[a/(2*b)] - c*Sin[a/(2*b)])*FresnelC[(1*Sqrt[a + b*ArcSin[c
+ d*x^2]])/(Sqrt[b*c]*Sqrt[Pi])])/(Sqrt[b*c]*(Cos[ArcSin[c + d*x^2]/2] - c*
Sin[ArcSin[c + d*x^2]/2])), x] - Simp[(Sqrt[Pi]*x*(Cos[a/(2*b)] + c*Sin[a/(
2*b)])*FresnelS[(1/(Sqrt[b*c]*Sqrt[Pi]))*Sqrt[a + b*ArcSin[c + d*x^2]]])/(S
qrt[b*c]*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2])), x] /; Fr
eeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

Rubi steps

$$\int \left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^{3/2}\,dx = -\frac{3b\sqrt{2idx^2+d^2x^4}\,\sqrt{a+ib\sin^{-1}\left(1-idx^2\right)}}{dx}+x\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^{3/2}+$$

$$= -\frac{3b\sqrt{2idx^2+d^2x^4}\,\sqrt{a+ib\sin^{-1}\left(1-idx^2\right)}}{dx}+x\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^{3/2}+$$

Mathematica [A] time = 0.23, size = 258, normalized size = 0.83

$$\frac{3\sqrt{\pi}\,b^2x\left(\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)\left(-S\left(\frac{\sqrt{a+ib\sin^{-1}\left(1-idx^2\right)}}{\sqrt{ib}\,\sqrt{\pi}}\right)\right)-\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)C\left(\frac{\sqrt{a+ib\sin^{-1}\left(1-idx^2\right)}}{\sqrt{ib}\,\sqrt{\pi}}\right)\right)}{\sqrt{ib}\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)\right)}-3$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*b*ArcSin[1 - I*d*x^2])^(3/2),x]
```

```
[Out] (-3*b*Sqrt[d*x^2*(2*I + d*x^2)]*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]])/(d*x) +
x*(a + I*b*ArcSin[1 - I*d*x^2])^(3/2) + (3*b^2*Sqrt[Pi]*x*(-(FresnelS[Sqrt[
a + I*b*ArcSin[1 - I*d*x^2]]/(Sqrt[I*b]*Sqrt[Pi])]*(Cosh[a/(2*b)] - I*Sinh[
a/(2*b)])) - FresnelC[Sqrt[a + I*b*ArcSin[1 - I*d*x^2]]/(Sqrt[I*b]*Sqrt[Pi]
)]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)])))/(Sqrt[I*b]*(Cos[ArcSin[1 - I*d*x^2]/
2] - Sin[ArcSin[1 - I*d*x^2]/2]))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(I+d*x^2))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(I+d*x^2))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [d,t_nostep]=[33,44]Bad conditionned root j= 2 value -63897.972732
 ratio 2.32090080903 mindist 10.1217065137Warning, choosing root of [1,0,%%
%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-8,[3,6]%%%}+%%%{-32,[1,2]%%%},%%%{-3,[
4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{16,[0,0]%%%}] at parameters values [7,-27]Wa
rning, need to choose a branch for the root of a polynomial with parameters
. This might be wrong.The choice was done assuming [d,t_nostep]=[50,45]Fran
cis algorithm failure for[1.0,0.0,-61509375008,-8.30376562824e+15,-3.152836
0132e+20]schur row 1 1.29492e-09Francis algorithm not precise enough for[1.
0,0.0,-61509375008,-8.30376562824e+15,-3.1528360132e+20]Bad conditionned ro
```

```
ot j= 2 value -101243.096423 ratio 1.67716116885 mindist 7.19967768138Franc
is algorithm failure for[1.0,0.0,-137282971022,-2.76877787308e+16,-1.570551
17809e+21]Warning, choosing root of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},
%%%{-8,[3,6]%%%}+%%%{-32,[1,2]%%%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{1
6,[0,0]%%%}] at parameters values [63,-49]Evaluation time: 1.36sym2poly/r2s
ym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument V
alue
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \left(a + b \operatorname{arcsinh}\left(d x^2 + i\right)\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(I+d*x^2))^(3/2),x)
[Out] int((a+b*arcsinh(I+d*x^2))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \operatorname{arsinh}\left(dx^2 + i\right) + a\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(I+d*x^2))^(3/2),x, algorithm="maxima")
[Out] integrate((b*arcsinh(d*x^2 + I) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a + b \operatorname{asinh}\left(d x^2 + 1\mathrm{i}\right)\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(d*x^2 + 1i))^(3/2),x)
[Out] int((a + b*asinh(d*x^2 + 1i))^(3/2), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(I+d*x**2))**(3/2),x)
[Out] Exception raised: TypeError
```

3.330 $\int \sqrt{a+ib\sin^{-1}(1-idx^2)}\,dx$

Optimal. Leaf size=263

$$-\frac{\sqrt{\pi}\sqrt{-\frac{i}{b}}\,bx\left(\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)C\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\sin^{-1}(1-idx^2)}}{\sqrt{\pi}}\right)}{\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)}+\frac{\sqrt{\pi}\,x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)S\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\sin^{-1}}}{\sqrt{\pi}}\right.}{\sqrt{-\frac{i}{b}}\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\right.\right.}$$

```
[Out] x*FresnelS((-I/b)^(1/2)*(a-I*b*arcsin(-1+I*d*x^2))^(1/2)/Pi^(1/2))*(cosh(1/
2*a/b)+I*sinh(1/2*a/b))*Pi^(1/2)/(cos(1/2*arcsin(-1+I*d*x^2))+sin(1/2*arcsi
n(-1+I*d*x^2)))/(-I/b)^(1/2)-b*x*FresnelC((-I/b)^(1/2)*(a-I*b*arcsin(-1+I*d
*x^2))^(1/2)/Pi^(1/2))*(I*cosh(1/2*a/b)+sinh(1/2*a/b))*(-I/b)^(1/2)*Pi^(1/2
)/(cos(1/2*arcsin(-1+I*d*x^2))+sin(1/2*arcsin(-1+I*d*x^2)))+x*(a-I*b*arcsin
(-1+I*d*x^2))^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {4811}

$$-\frac{\sqrt{\pi}\sqrt{-\frac{i}{b}}\,bx\left(\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\sin^{-1}(1-idx^2)}}{\sqrt{\pi}}\right)}{\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)}+\frac{\sqrt{\pi}\,x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)S\left(\sqrt{-\right.}{\sqrt{-\frac{i}{b}}\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + I*b*ArcSin[1 - I*d*x^2]],x]
```

```
[Out] x*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]] + (Sqrt[Pi]*x*FresnelS[(Sqrt[(-I)/b]*Sq
rt[a + I*b*ArcSin[1 - I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)]
))/(Sqrt[(-I)/b]*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2]))
 - (Sqrt[(-I)/b]*b*Sqrt[Pi]*x*FresnelC[(Sqrt[(-I)/b]*Sqrt[a + I*b*ArcSin[1
- I*d*x^2]])/Sqrt[Pi]]*(I*Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(Cos[ArcSin[1 - I
*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2])
```

Rule 4811

```
Int[Sqrt[(a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[x*Sq
rt[a + b*ArcSin[c + d*x^2]], x] + (-Simp[(Sqrt[Pi]*x*(Cos[a/(2*b)] + c*Sin[
a/(2*b)])*FresnelC[Sqrt[c/(Pi*b)]*Sqrt[a + b*ArcSin[c + d*x^2]]])/(Sqrt[c/b
]*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2])), x] + Simp[(Sqrt
[Pi]*x*(Cos[a/(2*b)] - c*Sin[a/(2*b)])*FresnelS[Sqrt[c/(Pi*b)]*Sqrt[a + b*A
rcSin[c + d*x^2]]])/(Sqrt[c/b]*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c +
 d*x^2]/2])), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

Rubi steps

$$\int\sqrt{a+ib\sin^{-1}\left(1-idx^2\right)}\,dx=x\sqrt{a+ib\sin^{-1}\left(1-idx^2\right)}+\frac{\sqrt{\pi}\,xS\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\sin^{-1}(1-idx^2)}}{\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\right.}{\sqrt{-\frac{i}{b}}\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-i\right.\right.\right.}$$

Mathematica [A] time = 0.06, size = 259, normalized size = 0.98

$$\frac{x\left(-\sqrt{\pi}\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)C\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\sin^{-1}\left(1-idx^2\right)}}{\sqrt{\pi}}\right)+\sqrt{\pi}\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)S\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\sin^{-1}(}}{\sqrt{\pi}}\right.\right.}{\sqrt{-\frac{i}{b}}\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + I*b*ArcSin[1 - I*d*x^2]],x]
```

```
[Out] (x*(Sqrt[(-I)/b]*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]]*(Cos[ArcSin[1 - I*d*x^2]
/2] - Sin[ArcSin[1 - I*d*x^2]/2]) - Sqrt[Pi]*FresnelC[(Sqrt[(-I)/b]*Sqrt[a
+ I*b*ArcSin[1 - I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]) + S
qrt[Pi]*FresnelS[(Sqrt[(-I)/b]*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]])/Sqrt[Pi]]
*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)])))/(Sqrt[(-I)/b]*(Cos[ArcSin[1 - I*d*x^2]
/2] - Sin[ArcSin[1 - I*d*x^2]/2]))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(I+d*x^2))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(I+d*x^2))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [d,t_nostep]=[84,56]Bad conditionned root j= 0 value -263411.19793
6 ratio 0.852649578726 mindist 22.2638977566Bad conditionned root j= 1 valu
e -263430.401038-11.2659672545*i ratio 0.734861252591 mindist 22.2638977566
Bad conditionned root j= 2 value -263430.401038+11.2659672545*i ratio 0.734
86125259 mindist 22.2638977566schur row 1 7.1988e-07Francis algorithm not p
recise enough for[1.0,0.0,-156243662.0,-1.06308198511e+12,-2.03434057625e+1
5]Bad conditionned root j= 0 value -5110.30395302 ratio 1.76012200073 mindi
st 7.30402974054Warning, choosing root of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,0
]%%%},%%%{-8,[3,6]%%%}+%%%{-32,[1,2]%%%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]%%%}
+%%%{16,[0,0]%%%}] at parameters values [7,-27]Warning, need to choose a br
anch for the root of a polynomial with parameters. This might be wrong.The
choice was done assuming [d,t_nostep]=[2,-51]schur row 1 4.28283e-07Francis
 algorithm not precise enough for[1.0,0.0,-162364832.0,-1.12616258573e+12,-
2.19686198884e+15]Bad conditionned root j= 0 value -5207.61031281 ratio 1.4
2050071801 mindist 5.61479175428Francis algorithm failure for[1.0,0.0,-1372
82971022,-2.76877787308e+16,-1.57055117809e+21]Warning, choosing root of [1
,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-8,[3,6]%%%}+%%%{-32,[1,2]%%%},%%%
{-3,[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{16,[0,0]%%%}] at parameters values [63,
-49]Evaluation time: 1.27sym2poly/r2sym(const gen & e,const index_m & i,con
st vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\,\mathrm{arcsinh}\left(d\,x^{2} + i\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(I+d*x^2))^(1/2),x)
[Out] int((a+b*arcsinh(I+d*x^2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\,\mathrm{arsinh}\left(dx^{2} + i\right) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(I+d*x^2))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*arcsinh(d*x^2 + I) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a + b\,\mathrm{asinh}\left(d\,x^{2} + 1\mathrm{i}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(d*x^2 + 1i))^(1/2),x)
[Out] int((a + b*asinh(d*x^2 + 1i))^(1/2), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(I+d*x**2))**(1/2),x)
[Out] Exception raised: TypeError
```

3.331 $\int \frac{1}{\sqrt{a+ib\sin^{-1}(1-idx^2)}}\,dx$

Optimal. Leaf size=231

$$-\frac{\sqrt{\pi}\,x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)C\left(\frac{\sqrt{a+ib\sin^{-1}(1-idx^2)}}{\sqrt{ib}\,\sqrt{\pi}}\right)}{\sqrt{ib}\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)\right)}-\frac{\sqrt{\pi}\,x\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)S\left(\frac{\sqrt{a+ib\sin^{-1}(1-id}}{\sqrt{ib}\,\sqrt{\pi}}\right.}{\sqrt{ib}\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-i\right.\right.\right.}$$

```
[Out] -x*FresnelS((a-I*b*arcsin(-1+I*d*x^2))^(1/2)/(I*b)^(1/2)/Pi^(1/2))*(cosh(1/
2*a/b)-I*sinh(1/2*a/b))*Pi^(1/2)/(cos(1/2*arcsin(-1+I*d*x^2))+sin(1/2*arcsi
n(-1+I*d*x^2)))/(I*b)^(1/2)-x*FresnelC((a-I*b*arcsin(-1+I*d*x^2))^(1/2)/(I*
b)^(1/2)/Pi^(1/2))*(cosh(1/2*a/b)+I*sinh(1/2*a/b))*Pi^(1/2)/(cos(1/2*arcsin
(-1+I*d*x^2))+sin(1/2*arcsin(-1+I*d*x^2)))/(I*b)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {4819}

$$-\frac{\sqrt{\pi}\,x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{a+ib\sin^{-1}(1-idx^2)}}{\sqrt{\pi}\,\sqrt{ib}}\right)}{\sqrt{ib}\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)\right)}-\frac{\sqrt{\pi}\,x\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)S\left(\frac{\sqrt{a+ib\sin^{-1}(}}{\sqrt{ib}\,\sqrt{}}\right.}{\sqrt{ib}\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1\right.\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + I*b*ArcSin[1 - I*d*x^2]],x]
```

```
[Out] -((Sqrt[Pi]*x*FresnelS[Sqrt[a + I*b*ArcSin[1 - I*d*x^2]]/(Sqrt[I*b]*Sqrt[Pi
])]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]))/(Sqrt[I*b]*(Cos[ArcSin[1 - I*d*x^2]/
2] - Sin[ArcSin[1 - I*d*x^2]/2]))) - (Sqrt[Pi]*x*FresnelC[Sqrt[a + I*b*ArcS
in[1 - I*d*x^2]]/(Sqrt[I*b]*Sqrt[Pi])]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)]))/(
Sqrt[I*b]*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2]))
```

Rule 4819

```
Int[1/Sqrt[(a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> -Simp[(
Sqrt[Pi]*x*(Cos[a/(2*b)] - c*Sin[a/(2*b)])*FresnelC[(1*Sqrt[a + b*ArcSin[c
+ d*x^2]])/(Sqrt[b*c]*Sqrt[Pi])])/(Sqrt[b*c]*(Cos[ArcSin[c + d*x^2]/2] - c*
Sin[ArcSin[c + d*x^2]/2])), x] - Simp[(Sqrt[Pi]*x*(Cos[a/(2*b)] + c*Sin[a/(
2*b)])*FresnelS[(1/(Sqrt[b*c]*Sqrt[Pi]))*Sqrt[a + b*ArcSin[c + d*x^2]]])/(S
qrt[b*c]*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2])), x] /; Fr
eeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+ib\sin^{-1}\left(1-idx^2\right)}}\,dx=-\frac{\sqrt{\pi}\,xS\left(\frac{\sqrt{a+ib\sin^{-1}(1-idx^2)}}{\sqrt{ib}\,\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)}{\sqrt{ib}\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)\right)}-\frac{\sqrt{\pi}\,xC\left(\sqrt{a+}\right.}{\sqrt{ib}\left(\cos\left(\frac{1}{2}\sin\right.\right.}$$

Mathematica [A] time = 0.00, size = 180, normalized size = 0.78

$$\frac{\sqrt{\pi}\,x\left(\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)\left(-S\left(\frac{\sqrt{a+ib\sin^{-1}(1-idx^2)}}{\sqrt{ib}\,\sqrt{\pi}}\right)\right)-\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)C\left(\frac{\sqrt{a+ib\sin^{-1}(1-idx^2)}}{\sqrt{ib}\,\sqrt{\pi}}\right)\right)}{\sqrt{ib}\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + I*b*ArcSin[1 - I*d*x^2]],x]
```

```
[Out] (Sqrt[Pi]*x*(-(FresnelS[Sqrt[a + I*b*ArcSin[1 - I*d*x^2]]/(Sqrt[I*b]*Sqrt[P
i])]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)])) - FresnelC[Sqrt[a + I*b*ArcSin[1 -
I*d*x^2]]/(Sqrt[I*b]*Sqrt[Pi])]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)])))/(Sqrt[I
*b]*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2]))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [d,t_nostep]=[92,-31]schur row 1 2.27097e-10Francis algorithm not
precise enough for[1.0,0.0,-46900090472,-5.52870773363e+15,-1.83301540649e+
20]Bad conditionned root j= 1 value -88413.1703987 ratio 0.928933994168 min
dist 1.4753899225Warning, choosing root of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,
0]%%%},%%%{-8,[3,6]%%%}+%%%{-32,[1,2]%%%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]%%%
}+%%%{16,[0,0]%%%}] at parameters values [7,-27]Warning, need to choose a b
ranch for the root of a polynomial with parameters. This might be wrong.The
 choice was done assuming [d,t_nostep]=[69,-52]Bad conditionned root j= 3 v
alue -186564.652407 ratio 2.53779195402 mindist 12.5891719132schur row 1 1.
93713e-09Francis algorithm not precise enough for[1.0,0.0,-137282971022,-2.
76877787308e+16,-1.57055117809e+21]Bad conditionned root j= 1 value -151275
.671615 ratio 2.85445476241 mindist 13.6978000362Warning, choosing root of
[1,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-8,[3,6]%%%}+%%%{-32,[1,2]%%%},%
%%{-3,[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{16,[0,0]%%%}] at parameters values [6
3,-49]Evaluation time: 1.31sym2poly/r2sym(const gen & e,const index_m & i,c
onst vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\,\mathrm{arcsinh}\left(d\,x^{2}+i\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(I+d*x^2))^(1/2),x)
```

```
[Out] int(1/(a+b*arcsinh(I+d*x^2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\,\mathrm{arsinh}\left(dx^{2}+i\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*arcsinh(d*x^2 + I) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{a + b\,\mathrm{asinh}\left(d\,x^2 + 1\mathrm{i}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*asinh(d*x^2 + 1i))^(1/2),x)
```

```
[Out] int(1/(a + b*asinh(d*x^2 + 1i))^(1/2), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(I+d*x**2))**(1/2),x)
```

```
[Out] Exception raised: TypeError
```

3.332 $\int \frac{1}{\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^{3/2}}\,dx$

Optimal. Leaf size=291

$$-\frac{\sqrt{d^2x^4+2idx^2}}{bdx\sqrt{a+ib\sin^{-1}\left(1-idx^2\right)}}-\frac{\sqrt{\pi}\left(-\frac{i}{b}\right)^{3/2}x\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)C\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\sin^{-1}\left(1-idx^2\right)}}{\sqrt{\pi}}\right)}{\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)}+\frac{\sqrt{\pi}\left(-\frac{i}{b}\right)^{3/2}x\left(\ldots\right.}{\cos\left(\frac{1}{2}\ldots\right.}$$

```
[Out] -(-I/b)^(3/2)*x*FresnelC((-I/b)^(1/2)*(a-I*b*arcsin(-1+I*d*x^2))^(1/2)/Pi^(
1/2))*(cosh(1/2*a/b)-I*sinh(1/2*a/b))*Pi^(1/2)/(cos(1/2*arcsin(-1+I*d*x^2))
+sin(1/2*arcsin(-1+I*d*x^2)))+(-I/b)^(3/2)*x*FresnelS((-I/b)^(1/2)*(a-I*b*a
rcsin(-1+I*d*x^2))^(1/2)/Pi^(1/2))*(cosh(1/2*a/b)+I*sinh(1/2*a/b))*Pi^(1/2)
/(cos(1/2*arcsin(-1+I*d*x^2))+sin(1/2*arcsin(-1+I*d*x^2)))-(2*I*d*x^2+d^2*x
^4)^(1/2)/b/d/x/(a-I*b*arcsin(-1+I*d*x^2))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {4822}

$$-\frac{\sqrt{d^2x^4+2idx^2}}{bdx\sqrt{a+ib\sin^{-1}\left(1-idx^2\right)}}-\frac{\sqrt{\pi}\left(-\frac{i}{b}\right)^{3/2}x\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\sin^{-1}\left(1-idx^2\right)}}{\sqrt{\pi}}\right)}{\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)}+\frac{\sqrt{\pi}\left(-\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*b*ArcSin[1 - I*d*x^2])^(-3/2),x]
```

```
[Out] -(Sqrt[(2*I)*d*x^2 + d^2*x^4]/(b*d*x*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]])) -
(((-I)/b)^(3/2)*Sqrt[Pi]*x*FresnelC[(Sqrt[(-I)/b]*Sqrt[a + I*b*ArcSin[1 - I
*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]))/(Cos[ArcSin[1 - I*d*
x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2]) + (((-I)/b)^(3/2)*Sqrt[Pi]*x*FresnelS
[(Sqrt[(-I)/b]*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)]
+ I*Sinh[a/(2*b)]))/(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2
])
```

Rule 4822

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(-3/2), x_Symbol] :> -Simp[
Sqrt[-2*c*d*x^2 - d^2*x^4]/(b*d*x*Sqrt[a + b*ArcSin[c + d*x^2]]), x] + (-Si
mp[((c/b)^(3/2)*Sqrt[Pi]*x*(Cos[a/(2*b)] + c*Sin[a/(2*b)])*FresnelC[Sqrt[c/
(Pi*b)]*Sqrt[a + b*ArcSin[c + d*x^2]]])/(Cos[(1/2)*ArcSin[c + d*x^2]] - c*S
in[ArcSin[c + d*x^2]/2]), x] + Simp[((c/b)^(3/2)*Sqrt[Pi]*x*(Cos[a/(2*b)] -
 c*Sin[a/(2*b)])*FresnelS[Sqrt[c/(Pi*b)]*Sqrt[a + b*ArcSin[c + d*x^2]]])/(C
os[(1/2)*ArcSin[c + d*x^2]] - c*Sin[ArcSin[c + d*x^2]/2]), x]) /; FreeQ[{a,
 b, c, d}, x] && EqQ[c^2, 1]
```

Rubi steps

$$\int \frac{1}{\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^{3/2}}\,dx=-\frac{\sqrt{2idx^2+d^2x^4}}{bdx\sqrt{a+ib\sin^{-1}\left(1-idx^2\right)}}-\frac{\left(-\frac{i}{b}\right)^{3/2}\sqrt{\pi}\,xC\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\sin^{-1}\left(1-idx^2\right)}}{\sqrt{\pi}}\right)\left(\cosh\ldots\right.}{\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\ldots\right.}$$

Mathematica [A] time = 0.39, size = 291, normalized size = 1.00

$$-\frac{\sqrt{d^2x^4+2idx^2}}{bdx\sqrt{a+ib\sin^{-1}(1-idx^2)}}-\frac{\sqrt{\pi}\left(-\frac{i}{b}\right)^{3/2}x\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)C\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\sin^{-1}(1-idx^2)}}{\sqrt{\pi}}\right)}{\cos\left(\frac{1}{2}\sin^{-1}(1-idx^2)\right)-\sin\left(\frac{1}{2}\sin^{-1}(1-idx^2)\right)}+\frac{\sqrt{\pi}\left(-\frac{i}{b}\right)^{3/2}}{\cos}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*b*ArcSin[1 - I*d*x^2])^(-3/2),x]
```

```
[Out] -(Sqrt[(2*I)*d*x^2 + d^2*x^4]/(b*d*x*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]])) -
(((-I)/b)^(3/2)*Sqrt[Pi]*x*FresnelC[(Sqrt[(-I)/b]*Sqrt[a + I*b*ArcSin[1 - I
*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]))/(Cos[ArcSin[1 - I*d*
x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2]) + (((-I)/b)^(3/2)*Sqrt[Pi]*x*FresnelS
[(Sqrt[(-I)/b]*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)]
+ I*Sinh[a/(2*b)]))/(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2
])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [d,x]=[89,77]schur row 1 1.22263e-09Francis algorithm not precise
enough for[1.0,0.0,-1.67068342657e+12,-1.17545053497e+18,-2.32598592657e+23
]Bad conditionned root j= 2 value -527644.406418 ratio 8.33415008204 mindis
t 33.7256546889Warning, need to choose a branch for the root of a polynomia
l with parameters. This might be wrong.The choice was done assuming [d,x]=[
-66,8]schur row 1 9.91586e-07Francis algorithm not precise enough for[1.0,0
.0,-107053064.0,602922946560,-9.55030161457e+14]Bad conditionned root j= 0
value 4231.08593187 ratio 1.71555050372 mindist 7.08837726861schur row 1 9.
85057e-10Francis algorithm not precise enough for[1.0,0.0,-137282971022,-2.
76877787308e+16,-1.57055117809e+21]Unable to isolate roots number Vector [0
,1][-0.151267996566355e6,-0.151267183209932e6]Bad conditionned root j= 2 va
lue -151253.820244 ratio 2.23329136282 mindist 13.3629663866Warning, choosi
ng root of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-8,[3,6]%%%}+%%%{-32,
[1,2]%%%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{16,[0,0]%%%}] at parameter
s values [63,-49]Warning, need to choose a branch for the root of a polynom
ial with parameters. This might be wrong.The choice was done assuming [d,x]
=[84,-65]Bad conditionned root j= 1 value -354917.321547 ratio 3.5819922982
8 mindist 17.6876820158Bad conditionned root j= 2 value -354899.633865 rati
o 36.3553454927 mindist 16.5892679524Bad conditionned root j= 1 value -3388
0.7669046 ratio 0.612379387673 mindist 0.825253607938Warning, choosing root
```

```
 of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-8,[3,6]%%%}+%%%{-32,[1,2]%%
%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{16,[0,0]%%%}] at parameters value
s [70,22]Warning, need to choose a branch for the root of a polynomial with
 parameters. This might be wrong.The choice was done assuming [d,x]=[30,-21
]Warning, need to choose a branch for the root of a polynomial with paramet
ers. This might be wrong.The choice was done assuming [d,x]=[-24,-63]Warnin
g, need to choose a branch for the root of a polynomial with parameters. Th
is might be wrong.The choice was done assuming [d,x]=[11,52]schur row 3 -3.
22193e-07Warning, choosing root of [1,0,%%%{-4,[2,4]%%%}+%%%{-8,[0,0]%%%},%
%%{-4,[3,6]%%%}+%%%{-16,[1,2]%%%},%%%{-1,[4,8]%%%}+%%%{-4,[2,4]%%%}+%%%{16,
[0,0]%%%}] at parameters values [18,-49]Warning, choosing root of [1,0,%%%{
-4,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-4,[3,6]%%%}+%%%{-16,[1,2]%%%},%%%{-1,[4,
8]%%%}+%%%{-4,[2,4]%%%}+%%%{16,[0,0]%%%}] at parameters values [-33,-70]sch
ur row 3 1.93169e-07Warning, choosing root of [1,0,%%%{-4,[2,4]%%%}+%%%{-8,
[0,0]%%%},%%%{-4,[3,6]%%%}+%%%{-16,[1,2]%%%},%%%{-1,[4,8]%%%}+%%%{-4,[2,4]%
%%}+%%%{16,[0,0]%%%}] at parameters values [8,63]Warning, need to choose a
branch for the root of a polynomial with parameters. This might be wrong.Th
e choice was done assuming [d,x]=[-8,-94]schur row 1 1.15782e-08Francis alg
orithm not precise enough for[1.0,0.0,-29980760072,2.82570662547e+15,-7.490
38312879e+19]Bad conditionned root j= 0 value 70702.5530055 ratio 3.3049102
8239 mindist 18.5135468412Warning, need to choose a branch for the root of
a polynomial with parameters. This might be wrong.The choice was done assum
ing [d,x]=[-8,-38]schur row 1 3.47275e-08Francis algorithm not precise enou
gh for[1.0,0.0,-800692232.0,1.23327957985e+13,-5.34256730006e+16]Bad condit
ionned root j= 0 value 11555.1783957 ratio 1.30663425153 mindist 3.17957631
341schur row 1 5.69583e-10Francis algorithm not precise enough for[1.0,0.0,
-1.32328584376e+12,-8.28597485843e+17,-1.45923785361e+23]Unable to isolate
 roots number Vector [0,2][-0.469612557480389e6,-0.469615331440520e6]Bad con
ditionned root j= 1 value -469647.111085 ratio 3.72291056632 mindist 31.779
6449475Warning, choosing root of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%
{-8,[3,6]%%%}+%%%{-32,[1,2]%%%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{16,[
0,0]%%%}] at parameters values [65,-85]Warning, need to choose a branch for
 the root of a polynomial with parameters. This might be wrong.The choice w
as done assuming [d,x]=[-86,73]Bad conditionned root j= 0 value 458272.6215
08 ratio 152.169961926 mindist 255.569113305Bad conditionned root j= 2 valu
e 458592.326104 ratio 66.2919566381 mindist 302.967080007Bad conditionned r
oot j= 1 value -770132.243981 ratio 141.661540265 mindist 52.7628472447Bad
conditionned root j= 2 value -770187.27489 ratio 12.2575973716 mindist 55.0
309094898Unable to isolate roots number Vector [1,3][-0.770132243980564e6,-
0.770079481133319e6]Warning, choosing root of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,
[0,0]%%%},%%%{-8,[3,6]%%%}+%%%{-32,[1,2]%%%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]
%%%}+%%%{16,[0,0]%%%}] at parameters values [93,91]Warning, need to choose
a branch for the root of a polynomial with parameters. This might be wrong.
The choice was done assuming [d,x]=[-82,36]Warning, need to choose a branch
 for the root of a polynomial with parameters. This might be wrong.The choi
ce was done assuming [d,x]=[9,15]Warning, need to choose a branch for the r
oot of a polynomial with parameters. This might be wrong.The choice was don
e assuming [d,x]=[-11,52]schur row 3 1.00361e-07Warning, choosing root of [
1,0,%%%{-4,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-4,[3,6]%%%}+%%%{-16,[1,2]%%%},%%
%{-1,[4,8]%%%}+%%%{-4,[2,4]%%%}+%%%{16,[0,0]%%%}] at parameters values [79,
-88]Warning, choosing root of [1,0,%%%{-4,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-4
,[3,6]%%%}+%%%{-16,[1,2]%%%},%%%{-1,[4,8]%%%}+%%%{-4,[2,4]%%%}+%%%{16,[0,0]
%%%}] at parameters values [9,6]Warning, choosing root of [1,0,%%%{-4,[2,4]
%%%}+%%%{-8,[0,0]%%%},%%%{-4,[3,6]%%%}+%%%{-16,[1,2]%%%},%%%{-1,[4,8]%%%}+%
%%{-4,[2,4]%%%}+%%%{16,[0,0]%%%}] at parameters values [-69,-8]Warning, nee
d to choose a branch for the root of a polynomial with parameters. This mig
ht be wrong.The choice was done assuming [d,t_nostep]=[91,52]Bad conditionn
ed root j= 1 value -246064.602658 ratio 4.13844710897 mindist 2.78272759262
schur row 1 9.14435e-09Francis algorithm not precise enough for[1.0,0.0,-21
```

```
24702752,-5.33102089176e+13,-3.76196821029e+17]Bad conditionned root j= 1 v
alue -18820.5074185 ratio 0.838796175516 mindist 3.20710914307Warning, choo
sing root of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-8,[3,6]%%%}+%%%{-3
2,[1,2]%%%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{16,[0,0]%%%}] at paramet
ers values [2,97]Warning, need to choose a branch for the root of a polynom
ial with parameters. This might be wrong.The choice was done assuming [d,t_
nostep]=[-82,75]schur row 1 1.2297e-10Francis algorithm not precise enough
for[1.0,0.0,-1.27650937501e+12,7.8505326564e+17,-1.35789682044e+23]Bad cond
itionned root j= 2 value 461240.783946 ratio 2.37092177178 mindist 11.77450
41251schur row 1 1.96129e-07Francis algorithm not precise enough for[1.0,0.
0,-288648584.0,2.6694222528e+12,-6.94316785683e+15]Bad conditionned root j=
 0 value 6940.98554667 ratio 1.29997372862 mindist 4.99589011288Warning, ch
oosing root of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-8,[3,6]%%%}+%%%{
-32,[1,2]%%%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{16,[0,0]%%%}] at param
eters values [-24,-17]Evaluation time: 1.99sym2poly/r2sym(const gen & e,con
st index_m & i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\,\mathrm{arcsinh}\left(d\,x^{2}+i\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(I+d*x^2))^(3/2),x)
[Out] int(1/(a+b*arcsinh(I+d*x^2))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,\mathrm{arsinh}\left(dx^{2}+i\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2))^(3/2),x, algorithm="maxima")
[Out] integrate((b*arcsinh(d*x^2 + I) + a)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+b\,\mathrm{asinh}\left(d\,x^{2}+1\mathrm{i}\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*asinh(d*x^2 + 1i))^(3/2),x)
[Out] int(1/(a + b*asinh(d*x^2 + 1i))^(3/2), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(I+d*x**2))**(3/2),x)
[Out] Exception raised: TypeError
```

3.333 $\int \frac{1}{\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^{5/2}}\,dx$

Optimal. Leaf size=326

$$-\frac{\sqrt{\pi}\,x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)C\left(\frac{\sqrt{a+ib\sin^{-1}\left(1-idx^2\right)}}{\sqrt{ib}\,\sqrt{\pi}}\right)}{3\sqrt{ib}\,b^2\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)\right)}-\frac{\sqrt{\pi}\,x\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)S\left(\frac{\sqrt{a+ib\sin^{-1}(1\cdots}}{\sqrt{ib}\,\sqrt{\pi}}\cdots\right.}{3\sqrt{ib}\,b^2\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1\cdots\right.\right.\right.}$$

```
[Out] -1/3*x*FresnelS((a-I*b*arcsin(-1+I*d*x^2))^(1/2)/(I*b)^(1/2)/Pi^(1/2))*(cos
h(1/2*a/b)-I*sinh(1/2*a/b))*Pi^(1/2)/b^2/(cos(1/2*arcsin(-1+I*d*x^2))+sin(1
/2*arcsin(-1+I*d*x^2)))/(I*b)^(1/2)-1/3*x*FresnelC((a-I*b*arcsin(-1+I*d*x^2
))^(1/2)/(I*b)^(1/2)/Pi^(1/2))*(cosh(1/2*a/b)+I*sinh(1/2*a/b))*Pi^(1/2)/b^2
/(cos(1/2*arcsin(-1+I*d*x^2))+sin(1/2*arcsin(-1+I*d*x^2)))/(I*b)^(1/2)-1/3*
(2*I*d*x^2+d^2*x^4)^(1/2)/b/d/x/(a-I*b*arcsin(-1+I*d*x^2))^(3/2)-1/3*x/b^2/
(a-I*b*arcsin(-1+I*d*x^2))^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4828, 4819}

$$-\frac{\sqrt{\pi}\,x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{a+ib\sin^{-1}\left(1-idx^2\right)}}{\sqrt{\pi}\,\sqrt{ib}}\right)}{3\sqrt{ib}\,b^2\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)\right)}-\frac{\sqrt{\pi}\,x\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)S\left(\frac{\sqrt{a+ib\sin^{-1}(1\cdots}}{\sqrt{ib}\,\sqrt{\pi}}\cdots\right.}{3\sqrt{ib}\,b^2\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(\cdots\right.\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*b*ArcSin[1 - I*d*x^2])^(-5/2),x]
```

```
[Out] -Sqrt[(2*I)*d*x^2 + d^2*x^4]/(3*b*d*x*(a + I*b*ArcSin[1 - I*d*x^2])^(3/2))
- x/(3*b^2*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]]) - (Sqrt[Pi]*x*FresnelS[Sqrt[a
 + I*b*ArcSin[1 - I*d*x^2]]/(Sqrt[I*b]*Sqrt[Pi])]*(Cosh[a/(2*b)] - I*Sinh[a
/(2*b)]))/(3*Sqrt[I*b]*b^2*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d
*x^2]/2])) - (Sqrt[Pi]*x*FresnelC[Sqrt[a + I*b*ArcSin[1 - I*d*x^2]]/(Sqrt[I
*b]*Sqrt[Pi])]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)]))/(3*Sqrt[I*b]*b^2*(Cos[Arc
Sin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2]))
```

Rule 4819

```
Int[1/Sqrt[(a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> -Simp[(
Sqrt[Pi]*x*(Cos[a/(2*b)] - c*Sin[a/(2*b)])*FresnelC[(1*Sqrt[a + b*ArcSin[c
+ d*x^2]])/(Sqrt[b*c]*Sqrt[Pi])])/(Sqrt[b*c]*(Cos[ArcSin[c + d*x^2]/2] - c*
Sin[ArcSin[c + d*x^2]/2])), x] - Simp[(Sqrt[Pi]*x*(Cos[a/(2*b)] + c*Sin[a/(
2*b)])*FresnelS[(1/(Sqrt[b*c]*Sqrt[Pi]))*Sqrt[a + b*ArcSin[c + d*x^2]]])/(S
qrt[b*c]*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2])), x] /; Fr
eeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

Rule 4828

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[(x*
(a + b*ArcSin[c + d*x^2])^(n + 2))/(4*b^2*(n + 1)*(n + 2)), x] + (-Dist[1/(
4*b^2*(n + 1)*(n + 2)), Int[(a + b*ArcSin[c + d*x^2])^(n + 2), x], x] + Sim
p[(Sqrt[-2*c*d*x^2 - d^2*x^4]*(a + b*ArcSin[c + d*x^2])^(n + 1))/(2*b*d*(n
+ 1)*x), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && LtQ[n, -1] && NeQ[
n, -2]
```

Rubi steps

$$\int \frac{1}{\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^{5/2}}\,dx = -\frac{\sqrt{2idx^2+d^2x^4}}{3bdx\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^{3/2}} - \frac{x}{3b^2\sqrt{a+ib\sin^{-1}\left(1-idx^2\right)}} + \frac{\int \frac{}{\sqrt{a+}}}{}$$

$$= -\frac{\sqrt{2idx^2+d^2x^4}}{3bdx\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^{3/2}} - \frac{x}{3b^2\sqrt{a+ib\sin^{-1}\left(1-idx^2\right)}} - \frac{\sqrt{}}{3\sqrt{ib}\,b}$$

Mathematica [A] time = 0.83, size = 308, normalized size = 0.94

$$-\frac{\frac{\sqrt{\pi}x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)C\left(\frac{\sqrt{a+ib\sin^{-1}(1-idx^2)}}{\sqrt{ib}\sqrt{\pi}}\right)}{\sqrt{ib}\left(\cos\left(\frac{1}{2}\sin^{-1}(1-idx^2)\right)-\sin\left(\frac{1}{2}\sin^{-1}(1-idx^2)\right)\right)} + \frac{\sqrt{\pi}x\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)S\left(\frac{\sqrt{a+ib\sin^{-1}(1-idx^2)}}{\sqrt{ib}\sqrt{\pi}}\right)}{\sqrt{ib}\left(\cos\left(\frac{1}{2}\sin^{-1}(1-idx^2)\right)-\sin\left(\frac{1}{2}\sin^{-1}(1-idx^2)\right)\right)} + \frac{b\sqrt{dx^2(dx^2+2i)}}{dx\left(a+ib\sin^{-1}(1-idx^2)\right)^{3/2}} + \cdots}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*b*ArcSin[1 - I*d*x^2])^(-5/2),x]
```

```
[Out] -1/3*((b*Sqrt[d*x^2*(2*I + d*x^2)])/(d*x*(a + I*b*ArcSin[1 - I*d*x^2])^(3/2
)) + x/Sqrt[a + I*b*ArcSin[1 - I*d*x^2]] + (Sqrt[Pi]*x*FresnelS[Sqrt[a + I*
b*ArcSin[1 - I*d*x^2]]/(Sqrt[I*b]*Sqrt[Pi])]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b
)]))/(Sqrt[I*b]*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2]))
+ (Sqrt[Pi]*x*FresnelC[Sqrt[a + I*b*ArcSin[1 - I*d*x^2]]/(Sqrt[I*b]*Sqrt[Pi
])]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)]))/(Sqrt[I*b]*(Cos[ArcSin[1 - I*d*x^2]/
2] - Sin[ArcSin[1 - I*d*x^2]/2])))/b^2
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [d,x]=[89,77]schur row 1 1.22263e-09Francis algorithm not precise
enough for[1.0,0.0,-1.67068342657e+12,-1.17545053497e+18,-2.32598592657e+23
]Bad conditionned root j= 2 value -527644.406418 ratio 8.33415008204 mindis
t 33.7256546889Warning, need to choose a branch for the root of a polynomia
l with parameters. This might be wrong.The choice was done assuming [d,x]=[
-66,8]schur row 1 9.91586e-07Francis algorithm not precise enough for[1.0,0
.0,-107053064.0,602922946560,-9.55030161457e+14]Bad conditionned root j= 0
value 4231.08593187 ratio 1.71555050372 mindist 7.08837726861schur row 1 9.
```

85057e-10Francis algorithm not precise enough for[1.0,0.0,-137282971022,-2.76877787308e+16,-1.57055117809e+21]Unable to isolate roots number Vector [0,1][-0.151267996566355e6,-0.151267183209932e6]Bad conditionned root j= 2 value -151253.820244 ratio 2.23329136282 mindist 13.3629663866Warning, choosing root of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-8,[3,6]%%%}+%%%{-32,[1,2]%%%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{16,[0,0]%%%}] at parameters values [63,-49]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [d,x]=[84,-65]Bad conditionned root j= 1 value -354917.321547 ratio 3.58199229828 mindist 17.6876820158Bad conditionned root j= 2 value -354899.633865 ratio 36.3553454927 mindist 16.5892679524Bad conditionned root j= 1 value -33880.7669046 ratio 0.612379387673 mindist 0.825253607938Warning, choosing root of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-8,[3,6]%%%}+%%%{-32,[1,2]%%%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{16,[0,0]%%%}] at parameters values [70,22]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [d,x]=[30,-21]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [d,x]=[-24,-63]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [d,x]=[11,52]schur row 3 -3.22193e-07Warning, choosing root of [1,0,%%%{-4,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-4,[3,6]%%%}+%%%{-16,[1,2]%%%},%%%{-1,[4,8]%%%}+%%%{-4,[2,4]%%%}+%%%{16,[0,0]%%%}] at parameters values [18,-49]Warning, choosing root of [1,0,%%%{-4,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-4,[3,6]%%%}+%%%{-16,[1,2]%%%},%%%{-1,[4,8]%%%}+%%%{-4,[2,4]%%%}+%%%{16,[0,0]%%%}] at parameters values [-33,-70]schur row 3 1.93169e-07Warning, choosing root of [1,0,%%%{-4,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-4,[3,6]%%%}+%%%{-16,[1,2]%%%},%%%{-1,[4,8]%%%}+%%%{-4,[2,4]%%%}+%%%{16,[0,0]%%%}] at parameters values [8,63]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [d,x]=[-8,-94]schur row 1 1.15782e-08Francis algorithm not precise enough for[1.0,0.0,-29980760072,2.82570662547e+15,-7.49038312879e+19]Bad conditionned root j= 0 value 70702.5530055 ratio 3.30491028239 mindist 18.5135468412Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [d,x]=[-8,-38]schur row 1 3.47275e-08Francis algorithm not precise enough for[1.0,0.0,-800692232.0,1.23327957985e+13,-5.34256730006e+16]Bad conditionned root j= 0 value 11555.1783957 ratio 1.30663425153 mindist 3.17957631341schur row 1 5.69583e-10Francis algorithm not precise enough for[1.0,0.0,-1.32328584376e+12,-8.28597485843e+17,-1.45923785361e+23]Unable to isolate roots number Vector [0,2][-0.469612557480389e6,-0.469615331440520e6]Bad conditionned root j= 1 value -469647.111085 ratio 3.72291056632 mindist 31.7796449475Warning, choosing root of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-8,[3,6]%%%}+%%%{-32,[1,2]%%%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{16,[0,0]%%%}] at parameters values [65,-85]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [d,x]=[-86,73]Bad conditionned root j= 0 value 458272.621508 ratio 152.169961926 mindist 255.569113305Bad conditionned root j= 2 value 458592.326104 ratio 66.2919566381 mindist 302.967080007Bad conditionned root j= 1 value -770132.243981 ratio 141.661540265 mindist 52.7628472447Bad conditionned root j= 2 value -770187.27489 ratio 12.2575973716 mindist 55.0309094898Unable to isolate roots number Vector [1,3][-0.770132243980564e6,-0.770079481133319e6]Warning, choosing root of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-8,[3,6]%%%}+%%%{-32,[1,2]%%%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{16,[0,0]%%%}] at parameters values [93,91]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [d,x]=[-82,36]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [d,x]=[9,15]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was don

```
e assuming [d,x]=[-11,52]schur row 3 1.00361e-07Warning, choosing root of [
1,0,%%%{-4,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-4,[3,6]%%%}+%%%{-16,[1,2]%%%},%%
%{-1,[4,8]%%%}+%%%{-4,[2,4]%%%}+%%%{16,[0,0]%%%}] at parameters values [79,
-88]Warning, choosing root of [1,0,%%%{-4,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-4
,[3,6]%%%}+%%%{-16,[1,2]%%%},%%%{-1,[4,8]%%%}+%%%{-4,[2,4]%%%}+%%%{16,[0,0]
%%%}] at parameters values [9,6]Warning, choosing root of [1,0,%%%{-4,[2,4]
%%%}+%%%{-8,[0,0]%%%},%%%{-4,[3,6]%%%}+%%%{-16,[1,2]%%%},%%%{-1,[4,8]%%%}+%
%%{-4,[2,4]%%%}+%%%{16,[0,0]%%%}] at parameters values [-69,-8]Warning, nee
d to choose a branch for the root of a polynomial with parameters. This mig
ht be wrong.The choice was done assuming [d,t_nostep]=[91,52]Bad conditionn
ed root j= 1 value -246064.602658 ratio 4.13844710897 mindist 2.78272759262
schur row 1 9.14435e-09Francis algorithm not precise enough for[1.0,0.0,-21
24702752,-5.33102089176e+13,-3.76196821029e+17]Bad conditionned root j= 1 v
alue -18820.5074185 ratio 0.838796175516 mindist 3.20710914307Warning, choo
sing root of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-8,[3,6]%%%}+%%%{-3
2,[1,2]%%%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{16,[0,0]%%%}] at paramet
ers values [2,97]Warning, need to choose a branch for the root of a polynom
ial with parameters. This might be wrong.The choice was done assuming [d,t_
nostep]=[-82,75]schur row 1 1.2297e-10Francis algorithm not precise enough
for[1.0,0.0,-1.27650937501e+12,7.8505326564e+17,-1.35789682044e+23]Bad cond
itionned root j= 2 value 461240.783946 ratio 2.37092177178 mindist 11.77450
41251schur row 1 1.96129e-07Francis algorithm not precise enough for[1.0,0.
0,-288648584.0,2.6694222528e+12,-6.94316785683e+15]Bad conditionned root j=
 0 value 6940.98554667 ratio 1.29997372862 mindist 4.99589011288Warning, ch
oosing root of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-8,[3,6]%%%}+%%%{
-32,[1,2]%%%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{16,[0,0]%%%}] at param
eters values [-24,-17]Evaluation time: 1.97sym2poly/r2sym(const gen & e,con
st index_m & i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b \operatorname{arcsinh}\left(d\,x^{2}+i\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(I+d*x^2))^(5/2),x)
```

```
[Out] int(1/(a+b*arcsinh(I+d*x^2))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b \operatorname{arsinh}\left(dx^{2}+i\right)+a\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(d*x^2 + I) + a)^(-5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+b\,\mathrm{asinh}\left(d\,x^2+1{}\mathrm{i}\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*asinh(d*x^2 + 1i))^(5/2),x)
```

```
[Out] int(1/(a + b*asinh(d*x^2 + 1i))^(5/2), x)
```

sympy [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(I+d*x**2))**(5/2),x)
```

```
[Out] Exception raised: TypeError
```

3.334 $\int \frac{1}{\left(a+ib\sin^{-1}(1-idx^2)\right)^{7/2}}\,dx$

Optimal. Leaf size=389

$$-\frac{\sqrt{d^2x^4+2idx^2}}{15b^3dx\sqrt{a+ib\sin^{-1}\left(1-idx^2\right)}}-\frac{\sqrt{\pi}\left(-\frac{i}{b}\right)^{3/2}x\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)C\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\sin^{-1}(1-idx^2)}}{\sqrt{\pi}}\right)}{15b^2\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)\right)}+\frac{\sqrt{\pi}\left(-\frac{i}{b}\right.}{15b^2}$$

```
[Out] -1/15*x/b^2/(a-I*b*arcsin(-1+I*d*x^2))^(3/2)-1/15*(-I/b)^(3/2)*x*FresnelC((
-I/b)^(1/2)*(a-I*b*arcsin(-1+I*d*x^2))^(1/2)/Pi^(1/2))*(cosh(1/2*a/b)-I*sin
h(1/2*a/b))*Pi^(1/2)/b^2/(cos(1/2*arcsin(-1+I*d*x^2))+sin(1/2*arcsin(-1+I*d
*x^2)))+1/15*(-I/b)^(3/2)*x*FresnelS((-I/b)^(1/2)*(a-I*b*arcsin(-1+I*d*x^2)
)^(1/2)/Pi^(1/2))*(cosh(1/2*a/b)+I*sinh(1/2*a/b))*Pi^(1/2)/b^2/(cos(1/2*arc
sin(-1+I*d*x^2))+sin(1/2*arcsin(-1+I*d*x^2)))-1/5*(2*I*d*x^2+d^2*x^4)^(1/2)
/b/d/x/(a-I*b*arcsin(-1+I*d*x^2))^(5/2)-1/15*(2*I*d*x^2+d^2*x^4)^(1/2)/b^3/
d/x/(a-I*b*arcsin(-1+I*d*x^2))^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 389, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4828, 4822}

$$-\frac{\sqrt{d^2x^4+2idx^2}}{15b^3dx\sqrt{a+ib\sin^{-1}\left(1-idx^2\right)}}-\frac{\sqrt{\pi}\left(-\frac{i}{b}\right)^{3/2}x\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\sin^{-1}(1-idx^2)}}{\sqrt{\pi}}\right)}{15b^2\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)\right)}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*b*ArcSin[1 - I*d*x^2])^(-7/2),x]
```

```
[Out] -Sqrt[(2*I)*d*x^2 + d^2*x^4]/(5*b*d*x*(a + I*b*ArcSin[1 - I*d*x^2])^(5/2))
- x/(15*b^2*(a + I*b*ArcSin[1 - I*d*x^2])^(3/2)) - Sqrt[(2*I)*d*x^2 + d^2*x
^4]/(15*b^3*d*x*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]]) - (((-I)/b)^(3/2)*Sqrt[P
i]*x*FresnelC[(Sqrt[(-I)/b]*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]])/Sqrt[Pi]]*(C
osh[a/(2*b)] - I*Sinh[a/(2*b)]))/(15*b^2*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[
ArcSin[1 - I*d*x^2]/2])) + (((-I)/b)^(3/2)*Sqrt[Pi]*x*FresnelS[(Sqrt[(-I)/b
]*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] + I*Sinh[a/(2
*b)]))/(15*b^2*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2]))
```

Rule 4822

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(-3/2), x_Symbol] :> -Simp[
Sqrt[-2*c*d*x^2 - d^2*x^4]/(b*d*x*Sqrt[a + b*ArcSin[c + d*x^2]]), x] + (-Si
mp[((c/b)^(3/2)*Sqrt[Pi]*x*(Cos[a/(2*b)] + c*Sin[a/(2*b)])*FresnelC[Sqrt[c/
(Pi*b)]*Sqrt[a + b*ArcSin[c + d*x^2]]])/(Cos[(1/2)*ArcSin[c + d*x^2]] - c*S
in[ArcSin[c + d*x^2]/2]), x] + Simp[((c/b)^(3/2)*Sqrt[Pi]*x*(Cos[a/(2*b)] -
 c*Sin[a/(2*b)])*FresnelS[Sqrt[c/(Pi*b)]*Sqrt[a + b*ArcSin[c + d*x^2]]])/(C
os[(1/2)*ArcSin[c + d*x^2]] - c*Sin[ArcSin[c + d*x^2]/2]), x]) /; FreeQ[{a,
 b, c, d}, x] && EqQ[c^2, 1]
```

Rule 4828

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[(x*
(a + b*ArcSin[c + d*x^2])^(n + 2))/(4*b^2*(n + 1)*(n + 2)), x] + (-Dist[1/(
4*b^2*(n + 1)*(n + 2)), Int[(a + b*ArcSin[c + d*x^2])^(n + 2), x], x] + Sim
p[(Sqrt[-2*c*d*x^2 - d^2*x^4]*(a + b*ArcSin[c + d*x^2])^(n + 1))/(2*b*d*(n
+ 1)*x), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && LtQ[n, -1] && NeQ[
```

n, -2]

Rubi steps

$$\int \frac{1}{\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^{7/2}}\,dx = -\frac{\sqrt{2idx^2+d^2x^4}}{5bdx\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^{5/2}} - \frac{x}{15b^2\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^{3/2}} + \frac{\int \frac{}{(a+i}}{}$$

$$= -\frac{\sqrt{2idx^2+d^2x^4}}{5bdx\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^{5/2}} - \frac{x}{15b^2\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^{3/2}} - \frac{}{15b^3d}$$

Mathematica [A] time = 0.99, size = 365, normalized size = 0.94

$$\frac{-\frac{\sqrt{\pi}\left(-\frac{i}{b}\right)^{3/2}x\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)C\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\sin^{-1}\left(1-idx^2\right)}}{\sqrt{\pi}}\right)}{\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)} + \frac{\sqrt{\pi}\left(-\frac{i}{b}\right)^{3/2}x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)S\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\sin^{-1}\left(1-idx^2\right)}}{\sqrt{\pi}}\right)}{\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)} + \frac{-\left(x^2\left(a+ib\right.\right.}{}}{15b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*b*ArcSin[1 - I*d*x^2])^(-7/2),x]
```

```
[Out] (((-3*b*Sqrt[d*x^2*(2*I + d*x^2)])/d - x^2*(a + I*b*ArcSin[1 - I*d*x^2]) +
(Sqrt[d*x^2*(2*I + d*x^2)]*((-I)*a + b*ArcSin[1 - I*d*x^2])^2)/(b*d))/(x*(a
 + I*b*ArcSin[1 - I*d*x^2])^(5/2)) - (((-I)/b)^(3/2)*Sqrt[Pi]*x*FresnelC[(S
qrt[(-I)/b]*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] - I
*Sinh[a/(2*b)]))/(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2])
+ (((-I)/b)^(3/2)*Sqrt[Pi]*x*FresnelS[(Sqrt[(-I)/b]*Sqrt[a + I*b*ArcSin[1 -
 I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)]))/(Cos[ArcSin[1 - I*
d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2]))/(15*b^2)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2))^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [d,x]=[89,77]schur row 1 1.22263e-09Francis algorithm not precise
enough for[1.0,0.0,-1.67068342657e+12,-1.17545053497e+18,-2.32598592657e+23
```

```
]Bad conditionned root j= 2 value -527644.406418 ratio 8.33415008204 mindis
t 33.7256546889Warning, need to choose a branch for the root of a polynomia
l with parameters. This might be wrong.The choice was done assuming [d,x]=[
-66,8]schur row 1 9.91586e-07Francis algorithm not precise enough for[1.0,0
.0,-107053064.0,602922946560,-9.55030161457e+14]Bad conditionned root j= 0
value 4231.08593187 ratio 1.71555050372 mindist 7.08837726861schur row 1 9.
85057e-10Francis algorithm not precise enough for[1.0,0.0,-137282971022,-2.
76877787308e+16,-1.57055117809e+21]Unable to isolate roots number Vector [0
,1][-0.151267996566355e6,-0.151267183209932e6]Bad conditionned root j= 2 va
lue -151253.820244 ratio 2.23329136282 mindist 13.3629663866Warning, choosi
ng root of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-8,[3,6]%%%}+%%%{-32,
[1,2]%%%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{16,[0,0]%%%}] at parameter
s values [63,-49]Warning, need to choose a branch for the root of a polynom
ial with parameters. This might be wrong.The choice was done assuming [d,x]
=[84,-65]Bad conditionned root j= 1 value -354917.321547 ratio 3.5819922982
8 mindist 17.6876820158Bad conditionned root j= 2 value -354899.633865 rati
o 36.3553454927 mindist 16.5892679524Bad conditionned root j= 1 value -3388
0.7669046 ratio 0.612379387673 mindist 0.825253607938Warning, choosing root
 of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-8,[3,6]%%%}+%%%{-32,[1,2]%%
%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{16,[0,0]%%%}] at parameters value
s [70,22]Warning, need to choose a branch for the root of a polynomial with
 parameters. This might be wrong.The choice was done assuming [d,x]=[30,-21
]Warning, need to choose a branch for the root of a polynomial with paramet
ers. This might be wrong.The choice was done assuming [d,x]=[-24,-63]Warnin
g, need to choose a branch for the root of a polynomial with parameters. Th
is might be wrong.The choice was done assuming [d,x]=[11,52]schur row 3 -3.
22193e-07Warning, choosing root of [1,0,%%%{-4,[2,4]%%%}+%%%{-8,[0,0]%%%},%
%%{-4,[3,6]%%%}+%%%{-16,[1,2]%%%},%%%{-1,[4,8]%%%}+%%%{-4,[2,4]%%%}+%%%{16,
[0,0]%%%}] at parameters values [18,-49]Warning, choosing root of [1,0,%%%{
-4,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-4,[3,6]%%%}+%%%{-16,[1,2]%%%},%%%{-1,[4,
8]%%%}+%%%{-4,[2,4]%%%}+%%%{16,[0,0]%%%}] at parameters values [-33,-70]sch
ur row 3 1.93169e-07Warning, choosing root of [1,0,%%%{-4,[2,4]%%%}+%%%{-8,
[0,0]%%%},%%%{-4,[3,6]%%%}+%%%{-16,[1,2]%%%},%%%{-1,[4,8]%%%}+%%%{-4,[2,4]%
%%}+%%%{16,[0,0]%%%}] at parameters values [8,63]Warning, need to choose a
branch for the root of a polynomial with parameters. This might be wrong.Th
e choice was done assuming [d,x]=[-8,-94]schur row 1 1.15782e-08Francis alg
orithm not precise enough for[1.0,0.0,-29980760072,2.82570662547e+15,-7.490
38312879e+19]Bad conditionned root j= 0 value 70702.5530055 ratio 3.3049102
8239 mindist 18.5135468412Warning, need to choose a branch for the root of
a polynomial with parameters. This might be wrong.The choice was done assum
ing [d,x]=[-8,-38]schur row 1 3.47275e-08Francis algorithm not precise enou
gh for[1.0,0.0,-800692232.0,1.23327957985e+13,-5.34256730006e+16]Bad condit
ionned root j= 0 value 11555.1783957 ratio 1.30663425153 mindist 3.17957631
341schur row 1 5.69583e-10Francis algorithm not precise enough for[1.0,0.0,
-1.32328584376e+12,-8.28597485843e+17,-1.45923785361e+23]Unable to isolate
roots number Vector [0,2][-0.469612557480389e6,-0.469615331440520e6]Bad con
ditionned root j= 1 value -469647.111085 ratio 3.72291056632 mindist 31.779
6449475Warning, choosing root of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%
{-8,[3,6]%%%}+%%%{-32,[1,2]%%%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{16,[
0,0]%%%}] at parameters values [65,-85]Warning, need to choose a branch for
 the root of a polynomial with parameters. This might be wrong.The choice w
as done assuming [d,x]=[-86,73]Bad conditionned root j= 0 value 458272.6215
08 ratio 152.169961926 mindist 255.569113305Bad conditionned root j= 2 valu
e 458592.326104 ratio 66.2919566381 mindist 302.967080007Bad conditionned r
oot j= 1 value -770132.243981 ratio 141.661540265 mindist 52.7628472447Bad
conditionned root j= 2 value -770187.27489 ratio 12.2575973716 mindist 55.0
309094898Unable to isolate roots number Vector [1,3][-0.770132243980564e6,-
0.770079481133319e6]Warning, choosing root of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,
[0,0]%%%},%%%{-8,[3,6]%%%}+%%%{-32,[1,2]%%%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]
```

```
%%%}+%%%{16,[0,0]%%%}] at parameters values [93,91]Warning, need to choose
a branch for the root of a polynomial with parameters. This might be wrong.
The choice was done assuming [d,x]=[-82,36]Warning, need to choose a branch
 for the root of a polynomial with parameters. This might be wrong.The choi
ce was done assuming [d,x]=[9,15]Warning, need to choose a branch for the r
oot of a polynomial with parameters. This might be wrong.The choice was don
e assuming [d,x]=[-11,52]schur row 3 1.00361e-07Warning, choosing root of [
1,0,%%%{-4,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-4,[3,6]%%%}+%%%{-16,[1,2]%%%},%%
%{-1,[4,8]%%%}+%%%{-4,[2,4]%%%}+%%%{16,[0,0]%%%}] at parameters values [79,
-88]Warning, choosing root of [1,0,%%%{-4,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-4
,[3,6]%%%}+%%%{-16,[1,2]%%%},%%%{-1,[4,8]%%%}+%%%{-4,[2,4]%%%}+%%%{16,[0,0]
%%%}] at parameters values [9,6]Warning, choosing root of [1,0,%%%{-4,[2,4]
%%%}+%%%{-8,[0,0]%%%},%%%{-4,[3,6]%%%}+%%%{-16,[1,2]%%%},%%%{-1,[4,8]%%%}+%
%%{-4,[2,4]%%%}+%%%{16,[0,0]%%%}] at parameters values [-69,-8]Warning, nee
d to choose a branch for the root of a polynomial with parameters. This mig
ht be wrong.The choice was done assuming [d,t_nostep]=[91,52]Bad conditionn
ed root j= 1 value -246064.602658 ratio 4.13844710897 mindist 2.78272759262
schur row 1 9.14435e-09Francis algorithm not precise enough for[1.0,0.0,-21
24702752,-5.33102089176e+13,-3.76196821029e+17]Bad conditionned root j= 1 v
alue -18820.5074185 ratio 0.838796175516 mindist 3.20710914307Warning, choo
sing root of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-8,[3,6]%%%}+%%%{-3
2,[1,2]%%%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{16,[0,0]%%%}] at paramet
ers values [2,97]Warning, need to choose a branch for the root of a polynom
ial with parameters. This might be wrong.The choice was done assuming [d,t_
nostep]=[-82,75]schur row 1 1.2297e-10Francis algorithm not precise enough
for[1.0,0.0,-1.27650937501e+12,7.8505326564e+17,-1.35789682044e+23]Bad cond
itionned root j= 2 value 461240.783946 ratio 2.37092177178 mindist 11.77450
41251schur row 1 1.96129e-07Francis algorithm not precise enough for[1.0,0.
0,-288648584.0,2.6694222528e+12,-6.94316785683e+15]Bad conditionned root j=
 0 value 6940.98554667 ratio 1.29997372862 mindist 4.99589011288Warning, ch
oosing root of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-8,[3,6]%%%}+%%%{
-32,[1,2]%%%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{16,[0,0]%%%}] at param
eters values [-24,-17]Evaluation time: 1.97sym2poly/r2sym(const gen & e,con
st index_m & i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\,\mathrm{arcsinh}\left(d\,x^{2}+i\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(I+d*x^2))^(7/2),x)
```

```
[Out] int(1/(a+b*arcsinh(I+d*x^2))^(7/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,\mathrm{arsinh}\left(dx^{2}+i\right)+a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(d*x^2 + I) + a)^(-7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+b\,\mathrm{asinh}\left(d\,x^{2}+1{}\mathrm{i}\right)\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*asinh(d*x^2 + 1i))^(7/2),x)
[Out] int(1/(a + b*asinh(d*x^2 + 1i))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(I+d*x**2))**(7/2),x)
[Out] Timed out
```

3.335 $\int \left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^{5/2}\,dx$

Optimal. Leaf size=348

$$-\frac{15\sqrt{\pi}\,b^2x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)C\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\sin^{-1}(idx^2+1)}}{\sqrt{\pi}}\right)}{\sqrt{\frac{i}{b}}\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)\right)}+\frac{15\sqrt{\pi}\,b^2x\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)S\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\sin\ldots}}{\sqrt{\pi}}\right)}{\sqrt{\frac{i}{b}}\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}(1+\ldots\right.\right.}$$

```
[Out] x*(a-I*b*arcsin(1+I*d*x^2))^(5/2)+15*b^2*x*FresnelS((I/b)^(1/2)*(a-I*b*arcs
in(1+I*d*x^2))^(1/2)/Pi^(1/2))*(cosh(1/2*a/b)-I*sinh(1/2*a/b))*Pi^(1/2)/(co
s(1/2*arcsin(1+I*d*x^2))-sin(1/2*arcsin(1+I*d*x^2)))/(I/b)^(1/2)-15*b^2*x*F
resnelC((I/b)^(1/2)*(a-I*b*arcsin(1+I*d*x^2))^(1/2)/Pi^(1/2))*(cosh(1/2*a/b
)+I*sinh(1/2*a/b))*Pi^(1/2)/(cos(1/2*arcsin(1+I*d*x^2))-sin(1/2*arcsin(1+I*
d*x^2)))/(I/b)^(1/2)-5*b*(a-I*b*arcsin(1+I*d*x^2))^(3/2)*(-2*I*d*x^2+d^2*x^
4)^(1/2)/d/x+15*b^2*x*(a-I*b*arcsin(1+I*d*x^2))^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 348, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4814, 4811}

$$-\frac{15\sqrt{\pi}\,b^2x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\sin^{-1}(1+idx^2)}}{\sqrt{\pi}}\right)}{\sqrt{\frac{i}{b}}\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)\right)}+\frac{15\sqrt{\pi}\,b^2x\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)S\left(\ldots\right)}{\sqrt{\frac{i}{b}}\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\ldots\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*b*ArcSin[1 + I*d*x^2])^(5/2),x]
```

```
[Out] 15*b^2*x*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]] - (5*b*Sqrt[(-2*I)*d*x^2 + d^2*x
^4]*(a - I*b*ArcSin[1 + I*d*x^2])^(3/2))/(d*x) + x*(a - I*b*ArcSin[1 + I*d*
x^2])^(5/2) + (15*b^2*Sqrt[Pi]*x*FresnelS[(Sqrt[I/b]*Sqrt[a - I*b*ArcSin[1
+ I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]))/(Sqrt[I/b]*(Cos[A
rcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2])) - (15*b^2*Sqrt[Pi]*x*F
resnelC[(Sqrt[I/b]*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*
b)] + I*Sinh[a/(2*b)]))/(Sqrt[I/b]*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin
[1 + I*d*x^2]/2]))
```

Rule 4811

```
Int[Sqrt[(a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[x*Sq
rt[a + b*ArcSin[c + d*x^2]], x] + (-Simp[(Sqrt[Pi]*x*(Cos[a/(2*b)] + c*Sin[
a/(2*b)])*FresnelC[Sqrt[c/(Pi*b)]*Sqrt[a + b*ArcSin[c + d*x^2]]])/(Sqrt[c/b
]*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2])), x] + Simp[(Sqrt
[Pi]*x*(Cos[a/(2*b)] - c*Sin[a/(2*b)])*FresnelS[Sqrt[c/(Pi*b)]*Sqrt[a + b*A
rcSin[c + d*x^2]]])/(Sqrt[c/b]*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c +
 d*x^2]/2])), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

Rule 4814

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcSin[c + d*x^2])^n, x] + (-Dist[4*b^2*n*(n - 1), Int[(a + b*ArcSin[
c + d*x^2])^(n - 2), x], x] + Simp[(2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*(a + b
*ArcSin[c + d*x^2])^(n - 1))/(d*x), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^
2, 1] && GtQ[n, 1]
```

Rubi steps

$$\int \left(a - ib\sin^{-1}\left(1 + idx^2\right)\right)^{5/2} dx = -\frac{5b\sqrt{-2idx^2 + d^2x^4}\left(a - ib\sin^{-1}\left(1 + idx^2\right)\right)^{3/2}}{dx} + x\left(a - ib\sin^{-1}\left(1 + idx^2\right.\right.$$

$$= 15b^2x\sqrt{a - ib\sin^{-1}\left(1 + idx^2\right)} - \frac{5b\sqrt{-2idx^2 + d^2x^4}\left(a - ib\sin^{-1}\left(1 + idx^2\right.\right.}{dx}$$

Mathematica [A] time = 0.29, size = 337, normalized size = 0.97

$$\frac{15b^2x\left(-\sqrt{\pi}\left(\cosh\left(\frac{a}{2b}\right) + i\sinh\left(\frac{a}{2b}\right)\right)C\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\sin^{-1}\left(idx^2+1\right)}}{\sqrt{\pi}}\right) + \sqrt{\pi}\left(\cosh\left(\frac{a}{2b}\right) - i\sinh\left(\frac{a}{2b}\right)\right)S\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\sin^{-1}}}{\sqrt{\pi}}\right.\right.}{\sqrt{\frac{i}{b}}\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1 + idx^2\right)\right) - \sin\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*b*ArcSin[1 + I*d*x^2])^(5/2),x]
```

```
[Out] (-5*b*Sqrt[d*x^2*(-2*I + d*x^2)]*(a - I*b*ArcSin[1 + I*d*x^2])^(3/2))/(d*x)
 + x*(a - I*b*ArcSin[1 + I*d*x^2])^(5/2) + (15*b^2*x*(Sqrt[I/b]*Sqrt[a - I*
b*ArcSin[1 + I*d*x^2]]*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2
]/2]) + Sqrt[Pi]*FresnelS[(Sqrt[I/b]*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]])/Sqr
t[Pi]]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]) - Sqrt[Pi]*FresnelC[(Sqrt[I/b]*Sqr
t[a - I*b*ArcSin[1 + I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)])
))/(Sqrt[I/b]*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2]))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [d,t_nostep]=[33,44]Bad conditionned root j= 2 value -63897.972732
 ratio 2.32090080903 mindist 10.1217065137Warning, choosing root of [1,0,%%
%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-8,[3,6]%%%}+%%%{-32,[1,2]%%%},%%%{-3,[
4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{16,[0,0]%%%}] at parameters values [7,-27]Wa
rning, need to choose a branch for the root of a polynomial with parameters
. This might be wrong.The choice was done assuming [d,t_nostep]=[50,45]Fran
cis algorithm failure for[1.0,0.0,-61509375008,-8.30376562824e+15,-3.152836
0132e+20]schur row 1 1.29492e-09Francis algorithm not precise enough for[1.
0,0.0,-61509375008,-8.30376562824e+15,-3.1528360132e+20]Bad conditionned ro
```

```
ot j= 2 value -101243.096423 ratio 1.67716116885 mindist 7.19967768138Franc
is algorithm failure for[1.0,0.0,-137282971022,-2.76877787308e+16,-1.570551
17809e+21]Warning, choosing root of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},
%%%{-8,[3,6]%%%}+%%%{-32,[1,2]%%%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{1
6,[0,0]%%%}] at parameters values [63,-49]Evaluation time: 1.57sym2poly/r2s
ym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument V
alue
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \left(a+b\operatorname{arcsinh}\left(d\,x^2-i\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(-I+d*x^2))^(5/2),x)
[Out] int((a+b*arcsinh(-I+d*x^2))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\operatorname{arsinh}\left(dx^2-i\right)+a\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^(5/2),x, algorithm="maxima")
[Out] integrate((b*arcsinh(d*x^2 - I) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+b\operatorname{asinh}\left(d\,x^2-\mathrm{i}\right)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(d*x^2 - 1i))^(5/2),x)
[Out] int((a + b*asinh(d*x^2 - 1i))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(-I+d*x**2))**(5/2),x)
[Out] Timed out
```

3.336 $\int \left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^{3/2}\,dx$

Optimal. Leaf size=310

$$-\frac{3\sqrt{\pi}\,b^2x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)S\left(\frac{\sqrt{a-ib\sin^{-1}\left(idx^2+1\right)}}{\sqrt{-ib}\,\sqrt{\pi}}\right)}{\sqrt{-ib}\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)\right)}-\frac{3b\sqrt{d^2x^4-2idx^2}\sqrt{a-ib\sin^{-1}\left(1+idx^2\right)}}{dx}-\frac{3\sqrt{\pi}}{}$$

```
[Out] x*(a-I*b*arcsin(1+I*d*x^2))^(3/2)-3*b^2*x*FresnelS((a-I*b*arcsin(1+I*d*x^2)
)^(1/2)/(-I*b)^(1/2)/Pi^(1/2))*(cosh(1/2*a/b)+I*sinh(1/2*a/b))*Pi^(1/2)/(co
s(1/2*arcsin(1+I*d*x^2))-sin(1/2*arcsin(1+I*d*x^2)))/(-I*b)^(1/2)-3*b*x*Fre
snelC((a-I*b*arcsin(1+I*d*x^2))^(1/2)/(-I*b)^(1/2)/Pi^(1/2))*(I*cosh(1/2*a/
b)+sinh(1/2*a/b))*(-I*b)^(1/2)*Pi^(1/2)/(cos(1/2*arcsin(1+I*d*x^2))-sin(1/2
*arcsin(1+I*d*x^2)))-3*b*(-2*I*d*x^2+d^2*x^4)^(1/2)*(a-I*b*arcsin(1+I*d*x^2
))^(1/2)/d/x
```

Rubi [A] time = 0.10, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4814, 4819}

$$-\frac{3\sqrt{\pi}\,b^2x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)S\left(\frac{\sqrt{a-ib\sin^{-1}\left(idx^2+1\right)}}{\sqrt{-ib}\,\sqrt{\pi}}\right)}{\sqrt{-ib}\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)\right)}-\frac{3b\sqrt{d^2x^4-2idx^2}\sqrt{a-ib\sin^{-1}\left(1+idx^2\right)}}{dx}-\frac{3\sqrt{\pi}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*b*ArcSin[1 + I*d*x^2])^(3/2),x]
```

```
[Out] (-3*b*Sqrt[(-2*I)*d*x^2 + d^2*x^4]*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]])/(d*x)
 + x*(a - I*b*ArcSin[1 + I*d*x^2])^(3/2) - (3*b^2*Sqrt[Pi]*x*FresnelS[Sqrt[
a - I*b*ArcSin[1 + I*d*x^2]]/(Sqrt[(-I)*b]*Sqrt[Pi])]*(Cosh[a/(2*b)] + I*Si
nh[a/(2*b)]))/(Sqrt[(-I)*b]*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*
d*x^2]/2])) - (3*Sqrt[(-I)*b]*b*Sqrt[Pi]*x*FresnelC[Sqrt[a - I*b*ArcSin[1 +
 I*d*x^2]]/(Sqrt[(-I)*b]*Sqrt[Pi])]*(I*Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(Cos
[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2])
```

Rule 4814

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcSin[c + d*x^2])^n, x] + (-Dist[4*b^2*n*(n - 1), Int[(a + b*ArcSin[
c + d*x^2])^(n - 2), x], x] + Simp[(2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*(a + b
*ArcSin[c + d*x^2])^(n - 1))/(d*x), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^
2, 1] && GtQ[n, 1]
```

Rule 4819

```
Int[1/Sqrt[(a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> -Simp[(
Sqrt[Pi]*x*(Cos[a/(2*b)] - c*Sin[a/(2*b)])*FresnelC[(1*Sqrt[a + b*ArcSin[c
+ d*x^2]])/(Sqrt[b*c]*Sqrt[Pi])])/(Sqrt[b*c]*(Cos[ArcSin[c + d*x^2]/2] - c*
Sin[ArcSin[c + d*x^2]/2])), x] - Simp[(Sqrt[Pi]*x*(Cos[a/(2*b)] + c*Sin[a/(
2*b)])*FresnelS[(1/(Sqrt[b*c]*Sqrt[Pi]))*Sqrt[a + b*ArcSin[c + d*x^2]]])/(S
qrt[b*c]*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2])), x] /; Fr
eeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

Rubi steps

$$\int \left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^{3/2}\,dx = -\frac{3b\sqrt{-2idx^2+d^2x^4}\,\sqrt{a-ib\sin^{-1}\left(1+idx^2\right)}}{dx}+x\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^{3/2}$$

$$= -\frac{3b\sqrt{-2idx^2+d^2x^4}\,\sqrt{a-ib\sin^{-1}\left(1+idx^2\right)}}{dx}+x\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^{3/2}$$

Mathematica [A] time = 0.26, size = 255, normalized size = 0.82

$$-\frac{3\sqrt{\pi}\,(-ib)^{3/2}x\left(\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)\left(-C\left(\frac{\sqrt{a-ib\sin^{-1}\left(idx^2+1\right)}}{\sqrt{-ib}\,\sqrt{\pi}}\right)\right)-\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)S\left(\frac{\sqrt{a-ib\sin^{-1}\left(idx^2+1\right)}}{\sqrt{-ib}\,\sqrt{\pi}}\right.}{\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*b*ArcSin[1 + I*d*x^2])^(3/2),x]
```

```
[Out] (-3*b*Sqrt[d*x^2*(-2*I + d*x^2)]*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]])/(d*x) +
 x*(a - I*b*ArcSin[1 + I*d*x^2])^(3/2) - (3*((-I)*b)^(3/2)*Sqrt[Pi]*x*(-(Fr
esnelC[Sqrt[a - I*b*ArcSin[1 + I*d*x^2]]/(Sqrt[(-I)*b]*Sqrt[Pi])]*(Cosh[a/(
2*b)] - I*Sinh[a/(2*b)])) - FresnelS[Sqrt[a - I*b*ArcSin[1 + I*d*x^2]]/(Sqr
t[(-I)*b]*Sqrt[Pi])]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)])))/(Cos[ArcSin[1 + I*
d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [d,t_nostep]=[33,44]Bad conditionned root j= 2 value -63897.972732
 ratio 2.32090080903 mindist 10.1217065137Warning, choosing root of [1,0,%%
%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-8,[3,6]%%%}+%%%{-32,[1,2]%%%},%%%{-3,[
4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{16,[0,0]%%%}] at parameters values [7,-27]Wa
rning, need to choose a branch for the root of a polynomial with parameters
. This might be wrong.The choice was done assuming [d,t_nostep]=[50,45]Fran
cis algorithm failure for[1.0,0.0,-61509375008,-8.30376562824e+15,-3.152836
0132e+20]schur row 1 1.29492e-09Francis algorithm not precise enough for[1.
0,0.0,-61509375008,-8.30376562824e+15,-3.1528360132e+20]Bad conditionned ro
```

```
ot j= 2 value -101243.096423 ratio 1.67716116885 mindist 7.19967768138Franc
is algorithm failure for[1.0,0.0,-137282971022,-2.76877787308e+16,-1.570551
17809e+21]Warning, choosing root of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},
%%%{-8,[3,6]%%%}+%%%{-32,[1,2]%%%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{1
6,[0,0]%%%}] at parameters values [63,-49]Evaluation time: 1.41sym2poly/r2s
ym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument V
alue
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \left(a+b\,\mathrm{arcsinh}\left(d\,x^{2}-i\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(-I+d*x^2))^(3/2),x)
```

```
[Out] int((a+b*arcsinh(-I+d*x^2))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\,\mathrm{arsinh}\left(dx^{2}-i\right)+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(d*x^2 - I) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+b\,\mathrm{asinh}\left(d\,x^{2}-\mathrm{i}\right)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(d*x^2 - 1i))^(3/2),x)
```

```
[Out] int((a + b*asinh(d*x^2 - 1i))^(3/2), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(-I+d*x**2))**(3/2),x)
```

```
[Out] Exception raised: TypeError
```

3.337 $\int \sqrt{a - ib\sin^{-1}(1 + idx^2)}\, dx$

Optimal. Leaf size=262

$$-\frac{\sqrt{\pi}\, x \left(\cosh\left(\frac{a}{2b}\right) + i\sinh\left(\frac{a}{2b}\right)\right) C\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\sin^{-1}(idx^2+1)}}{\sqrt{\pi}}\right)}{\sqrt{\frac{i}{b}}\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right) - \sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)\right)} + \frac{\sqrt{\pi}\, x \left(\cosh\left(\frac{a}{2b}\right) - i\sinh\left(\frac{a}{2b}\right)\right) S\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\sin^{-1}(idx^2+1)}}{\sqrt{\pi}}\right.}{\sqrt{\frac{i}{b}}\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right) - \sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right.\right.}$$

```
[Out] x*FresnelS((I/b)^(1/2)*(a-I*b*arcsin(1+I*d*x^2))^(1/2)/Pi^(1/2))*(cosh(1/2*
a/b)-I*sinh(1/2*a/b))*Pi^(1/2)/(cos(1/2*arcsin(1+I*d*x^2))-sin(1/2*arcsin(1
+I*d*x^2)))/(I/b)^(1/2)-x*FresnelC((I/b)^(1/2)*(a-I*b*arcsin(1+I*d*x^2))^(1
/2)/Pi^(1/2))*(cosh(1/2*a/b)+I*sinh(1/2*a/b))*Pi^(1/2)/(cos(1/2*arcsin(1+I*
d*x^2))-sin(1/2*arcsin(1+I*d*x^2)))/(I/b)^(1/2)+x*(a-I*b*arcsin(1+I*d*x^2))
^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {4811}

$$-\frac{\sqrt{\pi}\, x \left(\cosh\left(\frac{a}{2b}\right) + i\sinh\left(\frac{a}{2b}\right)\right) \text{FresnelC}\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\sin^{-1}(1+idx^2)}}{\sqrt{\pi}}\right)}{\sqrt{\frac{i}{b}}\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right) - \sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)\right)} + \frac{\sqrt{\pi}\, x \left(\cosh\left(\frac{a}{2b}\right) - i\sinh\left(\frac{a}{2b}\right)\right) S\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\sin}}{\sqrt{\pi}}\right.}{\sqrt{\frac{i}{b}}\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right) - \sin\left(\frac{1}{2}\sin^{-1}\left(1\right.\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - I*b*ArcSin[1 + I*d*x^2]],x]
```

```
[Out] x*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]] + (Sqrt[Pi]*x*FresnelS[(Sqrt[I/b]*Sqrt[
a - I*b*ArcSin[1 + I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]))/
(Sqrt[I/b]*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2])) - (Sq
rt[Pi]*x*FresnelC[(Sqrt[I/b]*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]])/Sqrt[Pi]]*(
Cosh[a/(2*b)] + I*Sinh[a/(2*b)]))/(Sqrt[I/b]*(Cos[ArcSin[1 + I*d*x^2]/2] -
Sin[ArcSin[1 + I*d*x^2]/2]))
```

Rule 4811

```
Int[Sqrt[(a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[x*Sq
rt[a + b*ArcSin[c + d*x^2]], x] + (-Simp[(Sqrt[Pi]*x*(Cos[a/(2*b)] + c*Sin[
a/(2*b)])*FresnelC[Sqrt[c/(Pi*b)]*Sqrt[a + b*ArcSin[c + d*x^2]]])/(Sqrt[c/b
]*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2])), x] + Simp[(Sqrt
[Pi]*x*(Cos[a/(2*b)] - c*Sin[a/(2*b)])*FresnelS[Sqrt[c/(Pi*b)]*Sqrt[a + b*A
rcSin[c + d*x^2]]])/(Sqrt[c/b]*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c +
 d*x^2]/2])), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

Rubi steps

$$\int \sqrt{a - ib\sin^{-1}\left(1+idx^2\right)}\, dx = x\sqrt{a - ib\sin^{-1}\left(1+idx^2\right)} + \frac{\sqrt{\pi}\, xS\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\sin^{-1}(1+idx^2)}}{\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right) - i\sinh\left(\right.\right.}{\sqrt{\frac{i}{b}}\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right) - \sin\left(\frac{1}{2}\sin^{-1}\left(1+id\right.\right.\right.}$$

Mathematica [A] time = 0.05, size = 259, normalized size = 0.99

$$\frac{x\left(-\sqrt{\pi}\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)C\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\sin^{-1}(idx^2+1)}}{\sqrt{\pi}}\right)+\sqrt{\pi}\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)S\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\sin^{-1}(idx^2 \ldots}}{\sqrt{\pi}}\right.\ldots}{\sqrt{\frac{i}{b}}\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\ldots\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - I*b*ArcSin[1 + I*d*x^2]],x]
```

```
[Out] (x*(Sqrt[I/b]*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]]*(Cos[ArcSin[1 + I*d*x^2]/2]
 - Sin[ArcSin[1 + I*d*x^2]/2]) + Sqrt[Pi]*FresnelS[(Sqrt[I/b]*Sqrt[a - I*b*
ArcSin[1 + I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]) - Sqrt[Pi
]*FresnelC[(Sqrt[I/b]*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/
(2*b)] + I*Sinh[a/(2*b)])))/(Sqrt[I/b]*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[Ar
cSin[1 + I*d*x^2]/2]))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [d,t_nostep]=[84,56]Bad conditionned root j= 0 value -263411.19793
6 ratio 0.852649578726 mindist 22.2638977566Bad conditionned root j= 1 valu
e -263430.401038-11.2659672545*i ratio 0.734861252591 mindist 22.2638977566
Bad conditionned root j= 2 value -263430.401038+11.2659672545*i ratio 0.734
86125259 mindist 22.2638977566schur row 1 7.1988e-07Francis algorithm not p
recise enough for[1.0,0.0,-156243662.0,-1.06308198511e+12,-2.03434057625e+1
5]Bad conditionned root j= 0 value -5110.30395302 ratio 1.76012200073 mindi
st 7.30402974054Warning, choosing root of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,0
]%%%},%%%{-8,[3,6]%%%}+%%%{-32,[1,2]%%%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]%%%}
+%%%{16,[0,0]%%%}] at parameters values [7,-27]Warning, need to choose a br
anch for the root of a polynomial with parameters. This might be wrong.The
choice was done assuming [d,t_nostep]=[2,-51]schur row 1 4.28283e-07Francis
 algorithm not precise enough for[1.0,0.0,-162364832.0,-1.12616258573e+12,-
2.19686198884e+15]Bad conditionned root j= 0 value -5207.61031281 ratio 1.4
2050071801 mindist 5.61479175428Francis algorithm failure for[1.0,0.0,-1372
82971022,-2.76877787308e+16,-1.57055117809e+21]Warning, choosing root of [1
,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-8,[3,6]%%%}+%%%{-32,[1,2]%%%},%%%
{-3,[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{16,[0,0]%%%}] at parameters values [63,
-49]Evaluation time: 1.31sym2poly/r2sym(const gen & e,const index_m & i,con
st vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\,\mathrm{arcsinh}\left(d\,x^{2}-i\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(-I+d*x^2))^(1/2),x)
[Out] int((a+b*arcsinh(-I+d*x^2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\,\mathrm{arsinh}\left(dx^{2}-i\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*arcsinh(d*x^2 - I) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a+b\,\mathrm{asinh}\left(d\,x^{2}-\mathrm{i}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(d*x^2 - 1i))^(1/2),x)
[Out] int((a + b*asinh(d*x^2 - 1i))^(1/2), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(-I+d*x**2))**(1/2),x)
[Out] Exception raised: TypeError
```

3.338 $\int \frac{1}{\sqrt{a-ib\sin^{-1}(1+idx^2)}}\,dx$

Optimal. Leaf size=231

$$-\frac{\sqrt{\pi}\,x\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)C\left(\frac{\sqrt{a-ib\sin^{-1}(idx^2+1)}}{\sqrt{-ib}\sqrt{\pi}}\right)}{\sqrt{-ib}\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)\right)}-\frac{\sqrt{\pi}\,x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)S\left(\frac{\sqrt{a-ib\sin^{-1}(i}}{\sqrt{-ib}\sqrt{\pi}}\right.}{\sqrt{-ib}\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1\right.\right.\right.}$$

```
[Out] -x*FresnelC((a-I*b*arcsin(1+I*d*x^2))^(1/2)/(-I*b)^(1/2)/Pi^(1/2))*(cosh(1/
2*a/b)-I*sinh(1/2*a/b))*Pi^(1/2)/(cos(1/2*arcsin(1+I*d*x^2))-sin(1/2*arcsin
(1+I*d*x^2)))/(-I*b)^(1/2)-x*FresnelS((a-I*b*arcsin(1+I*d*x^2))^(1/2)/(-I*b
)^(1/2)/Pi^(1/2))*(cosh(1/2*a/b)+I*sinh(1/2*a/b))*Pi^(1/2)/(cos(1/2*arcsin(
1+I*d*x^2))-sin(1/2*arcsin(1+I*d*x^2)))/(-I*b)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {4819}

$$-\frac{\sqrt{\pi}\,x\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{a-ib\sin^{-1}(1+idx^2)}}{\sqrt{\pi}\sqrt{-ib}}\right)}{\sqrt{-ib}\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)\right)}-\frac{\sqrt{\pi}\,x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)S\left(\frac{\sqrt{a-ib\sin^{-1}}}{\sqrt{-ib}}\right.}{\sqrt{-ib}\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a - I*b*ArcSin[1 + I*d*x^2]],x]
```

```
[Out] -((Sqrt[Pi]*x*FresnelC[Sqrt[a - I*b*ArcSin[1 + I*d*x^2]]/(Sqrt[(-I)*b]*Sqrt
[Pi])]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]))/(Sqrt[(-I)*b]*(Cos[ArcSin[1 + I*d
*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2]))) - (Sqrt[Pi]*x*FresnelS[Sqrt[a - I*
b*ArcSin[1 + I*d*x^2]]/(Sqrt[(-I)*b]*Sqrt[Pi])]*(Cosh[a/(2*b)] + I*Sinh[a/(
2*b)]))/(Sqrt[(-I)*b]*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]
/2]))
```

Rule 4819

```
Int[1/Sqrt[(a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> -Simp[(
Sqrt[Pi]*x*(Cos[a/(2*b)] - c*Sin[a/(2*b)])*FresnelC[(1*Sqrt[a + b*ArcSin[c
+ d*x^2]])/(Sqrt[b*c]*Sqrt[Pi])])/(Sqrt[b*c]*(Cos[ArcSin[c + d*x^2]/2] - c*
Sin[ArcSin[c + d*x^2]/2])), x] - Simp[(Sqrt[Pi]*x*(Cos[a/(2*b)] + c*Sin[a/(
2*b)])*FresnelS[(1/(Sqrt[b*c]*Sqrt[Pi]))*Sqrt[a + b*ArcSin[c + d*x^2]]])/(S
qrt[b*c]*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2])), x] /; Fr
eeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

Rubi steps

$$\int \frac{1}{\sqrt{a-ib\sin^{-1}\left(1+idx^2\right)}}\,dx=-\frac{\sqrt{\pi}\,xC\left(\frac{\sqrt{a-ib\sin^{-1}(1+idx^2)}}{\sqrt{-ib}\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)}{\sqrt{-ib}\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)\right)}-\frac{\sqrt{\pi}\,xS\left(\right.}{\sqrt{-ib}\left(\cos\left(\right.\right.}$$

Mathematica [A] time = 0.00, size = 180, normalized size = 0.78

$$\frac{\sqrt{\pi}\,x\left(\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)\left(-C\left(\frac{\sqrt{a-ib\sin^{-1}\left(idx^2+1\right)}}{\sqrt{-ib}\,\sqrt{\pi}}\right)\right)-\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)S\left(\frac{\sqrt{a-ib\sin^{-1}\left(idx^2+1\right)}}{\sqrt{-ib}\,\sqrt{\pi}}\right)\right)}{\sqrt{-ib}\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a - I*b*ArcSin[1 + I*d*x^2]],x]
```

```
[Out] (Sqrt[Pi]*x*(-(FresnelC[Sqrt[a - I*b*ArcSin[1 + I*d*x^2]]/(Sqrt[(-I)*b]*Sqr
t[Pi])]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)])) - FresnelS[Sqrt[a - I*b*ArcSin[1
 + I*d*x^2]]/(Sqrt[(-I)*b]*Sqrt[Pi])]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)])))/(
Sqrt[(-I)*b]*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2]))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [d,t_nostep]=[92,-31]schur row 1 2.27097e-10Francis algorithm not
precise enough for[1.0,0.0,-46900090472,-5.52870773363e+15,-1.83301540649e+
20]Bad conditionned root j= 1 value -88413.1703987 ratio 0.928933994168 min
dist 1.4753899225Warning, choosing root of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,
0]%%%},%%%{-8,[3,6]%%%}+%%%{-32,[1,2]%%%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]%%%
}+%%%{16,[0,0]%%%}] at parameters values [7,-27]Warning, need to choose a b
ranch for the root of a polynomial with parameters. This might be wrong.The
 choice was done assuming [d,t_nostep]=[69,-52]Bad conditionned root j= 3 v
alue -186564.652407 ratio 2.53779195402 mindist 12.5891719132schur row 1 1.
93713e-09Francis algorithm not precise enough for[1.0,0.0,-137282971022,-2.
76877787308e+16,-1.57055117809e+21]Bad conditionned root j= 1 value -151275
.671615 ratio 2.85445476241 mindist 13.6978000362Warning, choosing root of
[1,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-8,[3,6]%%%}+%%%{-32,[1,2]%%%},%
%%{-3,[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{16,[0,0]%%%}] at parameters values [6
3,-49]Evaluation time: 1.31sym2poly/r2sym(const gen & e,const index_m & i,c
onst vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\,\mathrm{arcsinh}\left(d\,x^2-i\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(-I+d*x^2))^(1/2),x)
[Out] int(1/(a+b*arcsinh(-I+d*x^2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b \operatorname{arsinh}\left(dx^2 - i\right) + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(b*arcsinh(d*x^2 - I) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{a + b \operatorname{asinh}\left(d\, x^2 - \mathrm{i}\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*asinh(d*x^2 - 1i))^(1/2),x)
[Out] int(1/(a + b*asinh(d*x^2 - 1i))^(1/2), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(-I+d*x**2))**(1/2),x)
[Out] Exception raised: TypeError
```

3.339 $\int \frac{1}{\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^{3/2}}\,dx$

Optimal. Leaf size=291

$$-\frac{\sqrt{d^2x^4-2idx^2}}{bdx\sqrt{a-ib\sin^{-1}\left(1+idx^2\right)}}-\frac{\sqrt{\pi}\left(\frac{i}{b}\right)^{3/2}x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)C\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\sin^{-1}(idx^2+1)}}{\sqrt{\pi}}\right)}{\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)}+\frac{\sqrt{\pi}\left(\frac{i}{b}\right)^{3/2}x\left(\cos\right.}{\cos\left(\frac{1}{2}\sin^{-1}\right.}$$

```
[Out] (I/b)^(3/2)*x*FresnelS((I/b)^(1/2)*(a-I*b*arcsin(1+I*d*x^2))^(1/2)/Pi^(1/2)
)*(cosh(1/2*a/b)-I*sinh(1/2*a/b))*Pi^(1/2)/(cos(1/2*arcsin(1+I*d*x^2))-sin(
1/2*arcsin(1+I*d*x^2)))-(I/b)^(3/2)*x*FresnelC((I/b)^(1/2)*(a-I*b*arcsin(1+
I*d*x^2))^(1/2)/Pi^(1/2))*(cosh(1/2*a/b)+I*sinh(1/2*a/b))*Pi^(1/2)/(cos(1/2
*arcsin(1+I*d*x^2))-sin(1/2*arcsin(1+I*d*x^2)))-(-2*I*d*x^2+d^2*x^4)^(1/2)/
b/d/x/(a-I*b*arcsin(1+I*d*x^2))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {4822}

$$-\frac{\sqrt{d^2x^4-2idx^2}}{bdx\sqrt{a-ib\sin^{-1}\left(1+idx^2\right)}}-\frac{\sqrt{\pi}\left(\frac{i}{b}\right)^{3/2}x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\sin^{-1}(1+idx^2)}}{\sqrt{\pi}}\right)}{\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)}+\frac{\sqrt{\pi}\left(\frac{i}{b}\right)^{3}}{\cos}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*b*ArcSin[1 + I*d*x^2])^(-3/2),x]
```

```
[Out] -(Sqrt[(-2*I)*d*x^2 + d^2*x^4]/(b*d*x*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]])) +
 ((I/b)^(3/2)*Sqrt[Pi]*x*FresnelS[(Sqrt[I/b]*Sqrt[a - I*b*ArcSin[1 + I*d*x^
2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]))/(Cos[ArcSin[1 + I*d*x^2]/
2] - Sin[ArcSin[1 + I*d*x^2]/2]) - ((I/b)^(3/2)*Sqrt[Pi]*x*FresnelC[(Sqrt[I
/b]*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] + I*Sinh[a/
(2*b)]))/(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2])
```

Rule 4822

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(-3/2), x_Symbol] :> -Simp[
Sqrt[-2*c*d*x^2 - d^2*x^4]/(b*d*x*Sqrt[a + b*ArcSin[c + d*x^2]]), x] + (-Si
mp[((c/b)^(3/2)*Sqrt[Pi]*x*(Cos[a/(2*b)] + c*Sin[a/(2*b)])*FresnelC[Sqrt[c/
(Pi*b)]*Sqrt[a + b*ArcSin[c + d*x^2]]])/(Cos[(1/2)*ArcSin[c + d*x^2]] - c*S
in[ArcSin[c + d*x^2]/2]), x] + Simp[((c/b)^(3/2)*Sqrt[Pi]*x*(Cos[a/(2*b)] -
 c*Sin[a/(2*b)])*FresnelS[Sqrt[c/(Pi*b)]*Sqrt[a + b*ArcSin[c + d*x^2]]])/(C
os[(1/2)*ArcSin[c + d*x^2]] - c*Sin[ArcSin[c + d*x^2]/2]), x]) /; FreeQ[{a,
 b, c, d}, x] && EqQ[c^2, 1]
```

Rubi steps

$$\int \frac{1}{\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^{3/2}}\,dx=-\frac{\sqrt{-2idx^2+d^2x^4}}{bdx\sqrt{a-ib\sin^{-1}\left(1+idx^2\right)}}+\frac{\left(\frac{i}{b}\right)^{3/2}\sqrt{\pi}\,xS\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\sin^{-1}(1+idx^2)}}{\sqrt{\pi}}\right)\left(\cosh\right.}{\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\right.}$$

Mathematica [A] time = 0.37, size = 291, normalized size = 1.00

$$-\frac{\sqrt{d^2x^4-2idx^2}}{bdx\sqrt{a-ib\sin^{-1}\left(1+idx^2\right)}}-\frac{\sqrt{\pi}\left(\frac{i}{b}\right)^{3/2}x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)C\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\sin^{-1}\left(idx^2+1\right)}}{\sqrt{\pi}}\right)}{\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)}+\frac{\sqrt{\pi}\left(\frac{i}{b}\right)^{3/2}x\left(\cdots\right.}{\cos\left(\frac{1}{2}\text{s}\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*b*ArcSin[1 + I*d*x^2])^(-3/2),x]
```

```
[Out] -(Sqrt[(-2*I)*d*x^2 + d^2*x^4]/(b*d*x*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]])) +
 ((I/b)^(3/2)*Sqrt[Pi]*x*FresnelS[(Sqrt[I/b]*Sqrt[a - I*b*ArcSin[1 + I*d*x^
2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]))/(Cos[ArcSin[1 + I*d*x^2]/
2] - Sin[ArcSin[1 + I*d*x^2]/2]) - ((I/b)^(3/2)*Sqrt[Pi]*x*FresnelC[(Sqrt[I
/b]*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] + I*Sinh[a/
(2*b)]))/(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [d,x]=[89,77]schur row 1 1.22263e-09Francis algorithm not precise
enough for[1.0,0.0,-1.67068342657e+12,-1.17545053497e+18,-2.32598592657e+23
]Bad conditionned root j= 2 value -527644.406418 ratio 8.33415008204 mindis
t 33.7256546889Warning, need to choose a branch for the root of a polynomia
l with parameters. This might be wrong.The choice was done assuming [d,x]=[
-66,8]schur row 1 9.91586e-07Francis algorithm not precise enough for[1.0,0
.0,-107053064.0,602922946560,-9.55030161457e+14]Bad conditionned root j= 0
value 4231.08593187 ratio 1.71555050372 mindist 7.08837726861schur row 1 9.
85057e-10Francis algorithm not precise enough for[1.0,0.0,-137282971022,-2.
76877787308e+16,-1.57055117809e+21]Unable to isolate roots number Vector [0
,1][-0.151267996566355e6,-0.151267183209932e6]Bad conditionned root j= 2 va
lue -151253.820244 ratio 2.23329136282 mindist 13.3629663866Warning, choosi
ng root of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-8,[3,6]%%%}+%%%{-32,
[1,2]%%%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{16,[0,0]%%%}] at parameter
s values [63,-49]Warning, need to choose a branch for the root of a polynom
ial with parameters. This might be wrong.The choice was done assuming [d,x]
=[84,-65]Bad conditionned root j= 1 value -354917.321547 ratio 3.5819922982
8 mindist 17.6876820158Bad conditionned root j= 2 value -354899.633865 rati
o 36.3553454927 mindist 16.5892679524Bad conditionned root j= 1 value -3388
0.7669046 ratio 0.612379387673 mindist 0.825253607938Warning, choosing root
 of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-8,[3,6]%%%}+%%%{-32,[1,2]%%
```

%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{16,[0,0]%%%}] at parameters value s [70,22]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [d,x]=[30,-21]Warning, need to choose a branch for the root of a polynomial with paramet ers. This might be wrong.The choice was done assuming [d,x]=[-24,-63]Warnin g, need to choose a branch for the root of a polynomial with parameters. Th is might be wrong.The choice was done assuming [d,x]=[11,52]schur row 3 -3. 22193e-07Warning, choosing root of [1,0,%%%{-4,[2,4]%%%}+%%%{-8,[0,0]%%%},% %%{-4,[3,6]%%%}+%%%{-16,[1,2]%%%},%%%{-1,[4,8]%%%}+%%%{-4,[2,4]%%%}+%%%{16, [0,0]%%%}] at parameters values [18,-49]Warning, choosing root of [1,0,%%%{ -4,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-4,[3,6]%%%}+%%%{-16,[1,2]%%%},%%%{-1,[4, 8]%%%}+%%%{-4,[2,4]%%%}+%%%{16,[0,0]%%%}] at parameters values [-33,-70]sch ur row 3 1.93169e-07Warning, choosing root of [1,0,%%%{-4,[2,4]%%%}+%%%{-8, [0,0]%%%},%%%{-4,[3,6]%%%}+%%%{-16,[1,2]%%%},%%%{-1,[4,8]%%%}+%%%{-4,[2,4]% %%}+%%%{16,[0,0]%%%}] at parameters values [8,63]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.Th e choice was done assuming [d,x]=[-8,-94]schur row 1 1.15782e-08Francis alg orithm not precise enough for[1.0,0.0,-29980760072,2.82570662547e+15,-7.490 38312879e+19]Bad conditionned root j= 0 value 70702.5530055 ratio 3.3049102 8239 mindist 18.5135468412Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assum ing [d,x]=[-8,-38]schur row 1 3.47275e-08Francis algorithm not precise enou gh for[1.0,0.0,-800692232.0,1.23327957985e+13,-5.34256730006e+16]Bad condit ionned root j= 0 value 11555.1783957 ratio 1.30663425153 mindist 3.17957631 341schur row 1 5.69583e-10Francis algorithm not precise enough for[1.0,0.0, -1.32328584376e+12,-8.28597485843e+17,-1.45923785361e+23]Unable to isolate roots number Vector [0,2][-0.469612557480389e6,-0.469615331440520e6]Bad con ditionned root j= 1 value -469647.111085 ratio 3.72291056632 mindist 31.779 6449475Warning, choosing root of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},%%% {-8,[3,6]%%%}+%%%{-32,[1,2]%%%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{16,[0,0]%%%}] at parameters values [65,-85]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice w as done assuming [d,x]=[-86,73]Bad conditionned root j= 0 value 458272.6215 08 ratio 152.169961926 mindist 255.569113305Bad conditionned root j= 2 valu e 458592.326104 ratio 66.2919566381 mindist 302.967080007Bad conditionned r oot j= 1 value -770132.243981 ratio 141.661540265 mindist 52.7628472447Bad conditionned root j= 2 value -770187.27489 ratio 12.2575973716 mindist 55.0 309094898Unable to isolate roots number Vector [1,3][-0.770132243980564e6,- 0.770079481133319e6]Warning, choosing root of [1,0,%%%{-6,[2,4]%%%}+%%%{-8, [0,0]%%%},%%%{-8,[3,6]%%%}+%%%{-32,[1,2]%%%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4] %%%}+%%%{16,[0,0]%%%}] at parameters values [93,91]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong. The choice was done assuming [d,x]=[-82,36]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choi ce was done assuming [d,x]=[9,15]Warning, need to choose a branch for the r oot of a polynomial with parameters. This might be wrong.The choice was don e assuming [d,x]=[-11,52]schur row 3 1.00361e-07Warning, choosing root of [1,0,%%%{-4,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-4,[3,6]%%%}+%%%{-16,[1,2]%%%},%% %{-1,[4,8]%%%}+%%%{-4,[2,4]%%%}+%%%{16,[0,0]%%%}] at parameters values [79, -88]Warning, choosing root of [1,0,%%%{-4,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-4 ,[3,6]%%%}+%%%{-16,[1,2]%%%},%%%{-1,[4,8]%%%}+%%%{-4,[2,4]%%%}+%%%{16,[0,0] %%%}] at parameters values [9,6]Warning, choosing root of [1,0,%%%{-4,[2,4] %%%}+%%%{-8,[0,0]%%%},%%%{-4,[3,6]%%%}+%%%{-16,[1,2]%%%},%%%{-1,[4,8]%%%}+% %%{-4,[2,4]%%%}+%%%{16,[0,0]%%%}] at parameters values [-69,-8]Warning, nee d to choose a branch for the root of a polynomial with parameters. This mig ht be wrong.The choice was done assuming [d,t_nostep]=[91,52]Bad conditionn ed root j= 1 value -246064.602658 ratio 4.13844710897 mindist 2.78272759262 schur row 1 9.14435e-09Francis algorithm not precise enough for[1.0,0.0,-21 24702752,-5.33102089176e+13,-3.76196821029e+17]Bad conditionned root j= 1 v

```
alue -18820.5074185 ratio 0.838796175516 mindist 3.20710914307Warning, choo
sing root of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-8,[3,6]%%%}+%%%{-3
2,[1,2]%%%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{16,[0,0]%%%}] at paramet
ers values [2,97]Warning, need to choose a branch for the root of a polynom
ial with parameters. This might be wrong.The choice was done assuming [d,t_
nostep]=[-82,75]schur row 1 1.2297e-10Francis algorithm not precise enough
for[1.0,0.0,-1.27650937501e+12,7.8505326564e+17,-1.35789682044e+23]Bad cond
itionned root j= 2 value 461240.783946 ratio 2.37092177178 mindist 11.77450
41251schur row 1 1.96129e-07Francis algorithm not precise enough for[1.0,0.
0,-288648584.0,2.6694222528e+12,-6.94316785683e+15]Bad conditionned root j=
 0 value 6940.98554667 ratio 1.29997372862 mindist 4.99589011288Warning, ch
oosing root of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-8,[3,6]%%%}+%%%{
-32,[1,2]%%%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{16,[0,0]%%%}] at param
eters values [-24,-17]Evaluation time: 1.96sym2poly/r2sym(const gen & e,con
st index_m & i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\,\mathrm{arcsinh}\left(d\,x^{2}-i\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(-I+d*x^2))^(3/2),x)
```

```
[Out] int(1/(a+b*arcsinh(-I+d*x^2))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,\mathrm{arsinh}\left(dx^{2}-i\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(d*x^2 - I) + a)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+b\,\mathrm{asinh}\left(d\,x^2-\mathrm{i}\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*asinh(d*x^2 - 1i))^(3/2),x)
```

```
[Out] int(1/(a + b*asinh(d*x^2 - 1i))^(3/2), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(-I+d*x**2))**(3/2),x)
```

```
[Out] Exception raised: TypeError
```

3.340 $\int \frac{1}{\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^{5/2}}\,dx$

Optimal. Leaf size=326

$$-\frac{\sqrt{\pi}\sqrt{-ib}\,x\left(\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)C\left(\frac{\sqrt{a-ib\sin^{-1}(idx^2+1)}}{\sqrt{-ib}\sqrt{\pi}}\right)}{3b^3\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)\right)}-\frac{\sqrt{\pi}\,x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)S\left(\frac{\sqrt{a-ib\sin^{-1}(idx}}{\sqrt{-ib}\sqrt{\pi}}\right.}{3\sqrt{-ib}\,b^2\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1\right.\right.\right.}$$

```
[Out] -1/3*x*FresnelS((a-I*b*arcsin(1+I*d*x^2))^(1/2)/(-I*b)^(1/2)/Pi^(1/2))*(cos
h(1/2*a/b)+I*sinh(1/2*a/b))*Pi^(1/2)/b^2/(cos(1/2*arcsin(1+I*d*x^2))-sin(1/
2*arcsin(1+I*d*x^2)))/(-I*b)^(1/2)-1/3*x*FresnelC((a-I*b*arcsin(1+I*d*x^2))
^(1/2)/(-I*b)^(1/2)/Pi^(1/2))*(I*cosh(1/2*a/b)+sinh(1/2*a/b))*(-I*b)^(1/2)*
Pi^(1/2)/b^3/(cos(1/2*arcsin(1+I*d*x^2))-sin(1/2*arcsin(1+I*d*x^2)))-1/3*(-
2*I*d*x^2+d^2*x^4)^(1/2)/b/d/x/(a-I*b*arcsin(1+I*d*x^2))^(3/2)-1/3*x/b^2/(a
-I*b*arcsin(1+I*d*x^2))^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4828, 4819}

$$-\frac{\sqrt{\pi}\sqrt{-ib}\,x\left(\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{a-ib\sin^{-1}(1+idx^2)}}{\sqrt{\pi}\sqrt{-ib}}\right)}{3b^3\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)\right)}-\frac{\sqrt{\pi}\,x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)S\left(\sqrt{a-il}\right.}{3\sqrt{-ib}\,b^2\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*b*ArcSin[1 + I*d*x^2])^(-5/2),x]
```

```
[Out] -Sqrt[(-2*I)*d*x^2 + d^2*x^4]/(3*b*d*x*(a - I*b*ArcSin[1 + I*d*x^2])^(3/2))
 - x/(3*b^2*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]]) - (Sqrt[Pi]*x*FresnelS[Sqrt[
a - I*b*ArcSin[1 + I*d*x^2]]/(Sqrt[(-I)*b]*Sqrt[Pi])]*(Cosh[a/(2*b)] + I*Si
nh[a/(2*b)]))/(3*Sqrt[(-I)*b]*b^2*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[
1 + I*d*x^2]/2])) - (Sqrt[(-I)*b]*Sqrt[Pi]*x*FresnelC[Sqrt[a - I*b*ArcSin[1
 + I*d*x^2]]/(Sqrt[(-I)*b]*Sqrt[Pi])]*(I*Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(3
*b^3*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2]))
```

Rule 4819

```
Int[1/Sqrt[(a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> -Simp[(
Sqrt[Pi]*x*(Cos[a/(2*b)] - c*Sin[a/(2*b)])*FresnelC[(1*Sqrt[a + b*ArcSin[c
+ d*x^2]])/(Sqrt[b*c]*Sqrt[Pi])])/(Sqrt[b*c]*(Cos[ArcSin[c + d*x^2]/2] - c*
Sin[ArcSin[c + d*x^2]/2])), x] - Simp[(Sqrt[Pi]*x*(Cos[a/(2*b)] + c*Sin[a/(
2*b)])*FresnelS[(1/(Sqrt[b*c]*Sqrt[Pi]))*Sqrt[a + b*ArcSin[c + d*x^2]]])/(S
qrt[b*c]*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2])), x] /; Fr
eeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

Rule 4828

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[(x*
(a + b*ArcSin[c + d*x^2])^(n + 2))/(4*b^2*(n + 1)*(n + 2)), x] + (-Dist[1/(
4*b^2*(n + 1)*(n + 2)), Int[(a + b*ArcSin[c + d*x^2])^(n + 2), x], x] + Sim
p[(Sqrt[-2*c*d*x^2 - d^2*x^4]*(a + b*ArcSin[c + d*x^2])^(n + 1))/(2*b*d*(n
+ 1)*x), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && LtQ[n, -1] && NeQ[
n, -2]
```

Rubi steps

$$\int \frac{1}{\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^{5/2}}\,dx = -\frac{\sqrt{-2idx^2+d^2x^4}}{3bdx\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^{3/2}} - \frac{x}{3b^2\sqrt{a-ib\sin^{-1}\left(1+idx^2\right)}} + \frac{\int \frac{}{\sqrt{a-}}}{}$$

$$= -\frac{\sqrt{-2idx^2+d^2x^4}}{3bdx\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^{3/2}} - \frac{x}{3b^2\sqrt{a-ib\sin^{-1}\left(1+idx^2\right)}} - \frac{\sqrt{}}{3\sqrt{-ib}}$$

Mathematica [A] time = 0.85, size = 308, normalized size = 0.94

$$-\frac{\frac{\sqrt{\pi}x\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)C\left(\frac{\sqrt{a-ib\sin^{-1}\left(idx^2+1\right)}}{\sqrt{-ib}\sqrt{\pi}}\right)}{\sqrt{-ib}\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)\right)}+\frac{\sqrt{\pi}x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)S\left(\frac{\sqrt{a-ib\sin^{-1}\left(idx^2+1\right)}}{\sqrt{-ib}\sqrt{\pi}}\right)}{\sqrt{-ib}\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)\right)}+\frac{b\sqrt{dx^2\left(dx^2-2i\right)}}{dx\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^{3/2}}+}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*b*ArcSin[1 + I*d*x^2])^(-5/2),x]
```

```
[Out] -1/3*((b*Sqrt[d*x^2*(-2*I + d*x^2)])/(d*x*(a - I*b*ArcSin[1 + I*d*x^2])^(3/
2)) + x/Sqrt[a - I*b*ArcSin[1 + I*d*x^2]] + (Sqrt[Pi]*x*FresnelC[Sqrt[a - I
*b*ArcSin[1 + I*d*x^2]]/(Sqrt[(-I)*b]*Sqrt[Pi])]*(Cosh[a/(2*b)] - I*Sinh[a/
(2*b)]))/(Sqrt[(-I)*b]*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2
]/2])) + (Sqrt[Pi]*x*FresnelS[Sqrt[a - I*b*ArcSin[1 + I*d*x^2]]/(Sqrt[(-I)*
b]*Sqrt[Pi])]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)]))/(Sqrt[(-I)*b]*(Cos[ArcSin[
1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2])))/b^2
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [d,x]=[89,77]schur row 1 1.22263e-09Francis algorithm not precise
enough for[1.0,0.0,-1.67068342657e+12,-1.17545053497e+18,-2.32598592657e+23
]Bad conditionned root j= 2 value -527644.406418 ratio 8.33415008204 mindis
t 33.7256546889Warning, need to choose a branch for the root of a polynomia
l with parameters. This might be wrong.The choice was done assuming [d,x]=[
-66,8]schur row 1 9.91586e-07Francis algorithm not precise enough for[1.0,0
.0,-107053064.0,602922946560,-9.55030161457e+14]Bad conditionned root j= 0
value 4231.08593187 ratio 1.71555050372 mindist 7.08837726861schur row 1 9.
```

85057e-10Francis algorithm not precise enough for[1.0,0.0,-137282971022,-2.76877787308e+16,-1.57055117809e+21]Unable to isolate roots number Vector [0,1][-0.151267996566355e6,-0.151267183209932e6]Bad conditionned root j= 2 value -151253.820244 ratio 2.23329136282 mindist 13.3629663866Warning, choosing root of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-8,[3,6]%%%}+%%%{-32,[1,2]%%%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{16,[0,0]%%%}] at parameters values [63,-49]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [d,x]=[84,-65]Bad conditionned root j= 1 value -354917.321547 ratio 3.58199229828 mindist 17.6876820158Bad conditionned root j= 2 value -354899.633865 ratio 36.3553454927 mindist 16.5892679524Bad conditionned root j= 1 value -33880.7669046 ratio 0.612379387673 mindist 0.825253607938Warning, choosing root of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-8,[3,6]%%%}+%%%{-32,[1,2]%%%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{16,[0,0]%%%}] at parameters values [70,22]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [d,x]=[30,-21]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [d,x]=[-24,-63]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [d,x]=[11,52]schur row 3 -3.22193e-07Warning, choosing root of [1,0,%%%{-4,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-4,[3,6]%%%}+%%%{-16,[1,2]%%%},%%%{-1,[4,8]%%%}+%%%{-4,[2,4]%%%}+%%%{16,[0,0]%%%}] at parameters values [18,-49]Warning, choosing root of [1,0,%%%{-4,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-4,[3,6]%%%}+%%%{-16,[1,2]%%%},%%%{-1,[4,8]%%%}+%%%{-4,[2,4]%%%}+%%%{16,[0,0]%%%}] at parameters values [-33,-70]schur row 3 1.93169e-07Warning, choosing root of [1,0,%%%{-4,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-4,[3,6]%%%}+%%%{-16,[1,2]%%%},%%%{-1,[4,8]%%%}+%%%{-4,[2,4]%%%}+%%%{16,[0,0]%%%}] at parameters values [8,63]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [d,x]=[-8,-94]schur row 1 1.15782e-08Francis algorithm not precise enough for[1.0,0.0,-29980760072,2.82570662547e+15,-7.49038312879e+19]Bad conditionned root j= 0 value 70702.5530055 ratio 3.30491028239 mindist 18.5135468412Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [d,x]=[-8,-38]schur row 1 3.47275e-08Francis algorithm not precise enough for[1.0,0.0,-800692232.0,1.23327957985e+13,-5.34256730006e+16]Bad conditionned root j= 0 value 11555.1783957 ratio 1.30663425153 mindist 3.17957631341schur row 1 5.69583e-10Francis algorithm not precise enough for[1.0,0.0,-1.32328584376e+12,-8.28597485843e+17,-1.45923785361e+23]Unable to isolate roots number Vector [0,2][-0.469612557480389e6,-0.469615331440520e6]Bad conditionned root j= 1 value -469647.111085 ratio 3.72291056632 mindist 31.7796449475Warning, choosing root of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-8,[3,6]%%%}+%%%{-32,[1,2]%%%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{16,[0,0]%%%}] at parameters values [65,-85]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [d,x]=[-86,73]Bad conditionned root j= 0 value 458272.621508 ratio 152.169961926 mindist 255.569113305Bad conditionned root j= 2 value 458592.326104 ratio 66.2919566381 mindist 302.967080007Bad conditionned root j= 1 value -770132.243981 ratio 141.661540265 mindist 52.7628472447Bad conditionned root j= 2 value -770187.27489 ratio 12.2575973716 mindist 55.0309094898Unable to isolate roots number Vector [1,3][-0.770132243980564e6,-0.770079481133319e6]Warning, choosing root of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-8,[3,6]%%%}+%%%{-32,[1,2]%%%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{16,[0,0]%%%}] at parameters values [93,91]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [d,x]=[-82,36]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [d,x]=[9,15]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was don

```
e assuming [d,x]=[-11,52]schur row 3 1.00361e-07Warning, choosing root of [
1,0,%%%{-4,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-4,[3,6]%%%}+%%%{-16,[1,2]%%%},%%
%{-1,[4,8]%%%}+%%%{-4,[2,4]%%%}+%%%{16,[0,0]%%%}] at parameters values [79,
-88]Warning, choosing root of [1,0,%%%{-4,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-4
,[3,6]%%%}+%%%{-16,[1,2]%%%},%%%{-1,[4,8]%%%}+%%%{-4,[2,4]%%%}+%%%{16,[0,0]
%%%}] at parameters values [9,6]Warning, choosing root of [1,0,%%%{-4,[2,4]
%%%}+%%%{-8,[0,0]%%%},%%%{-4,[3,6]%%%}+%%%{-16,[1,2]%%%},%%%{-1,[4,8]%%%}+%
%%{-4,[2,4]%%%}+%%%{16,[0,0]%%%}] at parameters values [-69,-8]Warning, nee
d to choose a branch for the root of a polynomial with parameters. This mig
ht be wrong.The choice was done assuming [d,t_nostep]=[91,52]Bad conditionn
ed root j= 1 value -246064.602658 ratio 4.13844710897 mindist 2.78272759262
schur row 1 9.14435e-09Francis algorithm not precise enough for[1.0,0.0,-21
24702752,-5.33102089176e+13,-3.76196821029e+17]Bad conditionned root j= 1 v
alue -18820.5074185 ratio 0.838796175516 mindist 3.20710914307Warning, choo
sing root of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-8,[3,6]%%%}+%%%{-3
2,[1,2]%%%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{16,[0,0]%%%}] at paramet
ers values [2,97]Warning, need to choose a branch for the root of a polynom
ial with parameters. This might be wrong.The choice was done assuming [d,t_
nostep]=[-82,75]schur row 1 1.2297e-10Francis algorithm not precise enough
for[1.0,0.0,-1.27650937501e+12,7.8505326564e+17,-1.35789682044e+23]Bad cond
itionned root j= 2 value 461240.783946 ratio 2.37092177178 mindist 11.77450
41251schur row 1 1.96129e-07Francis algorithm not precise enough for[1.0,0.
0,-288648584.0,2.6694222528e+12,-6.94316785683e+15]Bad conditionned root j=
 0 value 6940.98554667 ratio 1.29997372862 mindist 4.99589011288Warning, ch
oosing root of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-8,[3,6]%%%}+%%%{
-32,[1,2]%%%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{16,[0,0]%%%}] at param
eters values [-24,-17]Evaluation time: 1.98sym2poly/r2sym(const gen & e,con
st index_m & i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b \operatorname{arcsinh}\left(d x^{2}-i\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(-I+d*x^2))^(5/2),x)
[Out] int(1/(a+b*arcsinh(-I+d*x^2))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b \operatorname{arsinh}\left(dx^{2}-i\right)+a\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^(5/2),x, algorithm="maxima")
[Out] integrate((b*arcsinh(d*x^2 - I) + a)^(-5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+b\,\mathrm{asinh}\left(d\,x^2-\mathrm{i}\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*asinh(d*x^2 - 1i))^(5/2),x)
```

```
[Out] int(1/(a + b*asinh(d*x^2 - 1i))^(5/2), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(-I+d*x**2))**(5/2),x)
```

```
[Out] Exception raised: TypeError
```

3.341 $\int \frac{1}{\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^{7/2}} dx$

Optimal. Leaf size=389

$$-\frac{\sqrt{d^2x^4-2idx^2}}{15b^3dx\sqrt{a-ib\sin^{-1}\left(1+idx^2\right)}}+\frac{\sqrt{\pi}\sqrt{\frac{i}{b}}\,x\left(\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)S\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\sin^{-1}\left(idx^2+1\right)}}{\sqrt{\pi}}\right)}{15b^3\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)\right)}-\frac{\sqrt{\pi}\left(\frac{i}{b}\right)^{3/2}x\,(\cdots}{15b^2\left(\cos\left(\cdots\right.\right.}$$

```
[Out] -1/15*x/b^2/(a-I*b*arcsin(1+I*d*x^2))^(3/2)-1/15*(I/b)^(3/2)*x*FresnelC((I/
b)^(1/2)*(a-I*b*arcsin(1+I*d*x^2))^(1/2)/Pi^(1/2))*(cosh(1/2*a/b)+I*sinh(1/
2*a/b))*Pi^(1/2)/b^2/(cos(1/2*arcsin(1+I*d*x^2))-sin(1/2*arcsin(1+I*d*x^2))
)+1/15*x*FresnelS((I/b)^(1/2)*(a-I*b*arcsin(1+I*d*x^2))^(1/2)/Pi^(1/2))*(I*
cosh(1/2*a/b)+sinh(1/2*a/b))*(I/b)^(1/2)*Pi^(1/2)/b^3/(cos(1/2*arcsin(1+I*d
*x^2))-sin(1/2*arcsin(1+I*d*x^2)))-1/5*(-2*I*d*x^2+d^2*x^4)^(1/2)/b/d/x/(a-
I*b*arcsin(1+I*d*x^2))^(5/2)-1/15*(-2*I*d*x^2+d^2*x^4)^(1/2)/b^3/d/x/(a-I*b
*arcsin(1+I*d*x^2))^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 389, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4828, 4822}

$$-\frac{\sqrt{d^2x^4-2idx^2}}{15b^3dx\sqrt{a-ib\sin^{-1}\left(1+idx^2\right)}}-\frac{\sqrt{\pi}\left(\frac{i}{b}\right)^{3/2}x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\sin^{-1}\left(1+idx^2\right)}}{\sqrt{\pi}}\right)}{15b^2\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)\right)}+\frac{\sqrt{\cdots}}{15\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*b*ArcSin[1 + I*d*x^2])^(-7/2),x]
```

```
[Out] -Sqrt[(-2*I)*d*x^2 + d^2*x^4]/(5*b*d*x*(a - I*b*ArcSin[1 + I*d*x^2])^(5/2))
 - x/(15*b^2*(a - I*b*ArcSin[1 + I*d*x^2])^(3/2)) - Sqrt[(-2*I)*d*x^2 + d^2
*x^4]/(15*b^3*d*x*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]]) - ((I/b)^(3/2)*Sqrt[Pi
]*x*FresnelC[(Sqrt[I/b]*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]])/Sqrt[Pi]]*(Cosh[
a/(2*b)] + I*Sinh[a/(2*b)]))/(15*b^2*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcS
in[1 + I*d*x^2]/2])) + (Sqrt[I/b]*Sqrt[Pi]*x*FresnelS[(Sqrt[I/b]*Sqrt[a - I
*b*ArcSin[1 + I*d*x^2]])/Sqrt[Pi]]*(I*Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(15*b
^3*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2]))
```

Rule 4822

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(-3/2), x_Symbol] :> -Simp[
Sqrt[-2*c*d*x^2 - d^2*x^4]/(b*d*x*Sqrt[a + b*ArcSin[c + d*x^2]]), x] + (-Si
mp[((c/b)^(3/2)*Sqrt[Pi]*x*(Cos[a/(2*b)] + c*Sin[a/(2*b)])*FresnelC[Sqrt[c/
(Pi*b)]*Sqrt[a + b*ArcSin[c + d*x^2]]])/(Cos[(1/2)*ArcSin[c + d*x^2]] - c*S
in[ArcSin[c + d*x^2]/2]), x] + Simp[((c/b)^(3/2)*Sqrt[Pi]*x*(Cos[a/(2*b)] -
 c*Sin[a/(2*b)])*FresnelS[Sqrt[c/(Pi*b)]*Sqrt[a + b*ArcSin[c + d*x^2]]])/(C
os[(1/2)*ArcSin[c + d*x^2]] - c*Sin[ArcSin[c + d*x^2]/2]), x]) /; FreeQ[{a,
 b, c, d}, x] && EqQ[c^2, 1]
```

Rule 4828

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[(x*
(a + b*ArcSin[c + d*x^2])^(n + 2))/(4*b^2*(n + 1)*(n + 2)), x] + (-Dist[1/(
4*b^2*(n + 1)*(n + 2)), Int[(a + b*ArcSin[c + d*x^2])^(n + 2), x], x] + Sim
p[(Sqrt[-2*c*d*x^2 - d^2*x^4]*(a + b*ArcSin[c + d*x^2])^(n + 1))/(2*b*d*(n
+ 1)*x), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && LtQ[n, -1] && NeQ[
```

```
n, -2]
```

Rubi steps

$$\int \frac{1}{\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^{7/2}}\,dx = -\frac{\sqrt{-2idx^2+d^2x^4}}{5bdx\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^{5/2}} - \frac{x}{15b^2\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^{3/2}} + \frac{\int \frac{}{(a-i}}{}$$

$$= -\frac{\sqrt{-2idx^2+d^2x^4}}{5bdx\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^{5/2}} - \frac{x}{15b^2\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^{3/2}} - \frac{}{15b^3d}$$

Mathematica [A] time = 0.98, size = 370, normalized size = 0.95

$$\frac{\frac{\sqrt{\pi}\sqrt{\frac{i}{b}}\,x\left(\sinh\left(\frac{a}{2b}\right)-i\cosh\left(\frac{a}{2b}\right)\right)C\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\sin^{-1}\left(idx^2+1\right)}}{\sqrt{\pi}}\right)}{b\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)\right)}+\frac{\sqrt{\pi}\sqrt{\frac{i}{b}}\,x\left(\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)S\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\sin^{-1}\left(idx^2+1\right)}}{\sqrt{\pi}}\right)}{b\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)\right)}+\frac{-\left(x^2\left(a-ib\sin^{-1}\left(1+id\right.\right.\right.}{}}{15b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*b*ArcSin[1 + I*d*x^2])^(-7/2),x]
```

```
[Out] (((-3*b*Sqrt[d*x^2*(-2*I + d*x^2)])/d - x^2*(a - I*b*ArcSin[1 + I*d*x^2]) +
 (Sqrt[d*x^2*(-2*I + d*x^2)]*(I*a + b*ArcSin[1 + I*d*x^2])^2)/(b*d))/(x*(a
- I*b*ArcSin[1 + I*d*x^2])^(5/2)) + (Sqrt[I/b]*Sqrt[Pi]*x*FresnelC[(Sqrt[I/
b]*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]])/Sqrt[Pi]]*((-I)*Cosh[a/(2*b)] + Sinh[
a/(2*b)]))/(b*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2])) +
(Sqrt[I/b]*Sqrt[Pi]*x*FresnelS[(Sqrt[I/b]*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]]
)/Sqrt[Pi]]*(I*Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(b*(Cos[ArcSin[1 + I*d*x^2]/
2] - Sin[ArcSin[1 + I*d*x^2]/2])))/(15*b^2)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [d,x]=[89,77]schur row 1 1.22263e-09Francis algorithm not precise
enough for[1.0,0.0,-1.67068342657e+12,-1.17545053497e+18,-2.32598592657e+23
```

```
]Bad conditionned root j= 2 value -527644.406418 ratio 8.33415008204 mindis
t 33.7256546889Warning, need to choose a branch for the root of a polynomia
l with parameters. This might be wrong.The choice was done assuming [d,x]=[
-66,8]schur row 1 9.91586e-07Francis algorithm not precise enough for[1.0,0
.0,-107053064.0,602922946560,-9.55030161457e+14]Bad conditionned root j= 0
value 4231.08593187 ratio 1.71555050372 mindist 7.08837726861schur row 1 9.
85057e-10Francis algorithm not precise enough for[1.0,0.0,-137282971022,-2.
76877787308e+16,-1.57055117809e+21]Unable to isolate roots number Vector [0
,1][-0.151267996566355e6,-0.151267183209932e6]Bad conditionned root j= 2 va
lue -151253.820244 ratio 2.23329136282 mindist 13.3629663866Warning, choosi
ng root of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-8,[3,6]%%%}+%%%{-32,
[1,2]%%%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{16,[0,0]%%%}] at parameter
s values [63,-49]Warning, need to choose a branch for the root of a polynom
ial with parameters. This might be wrong.The choice was done assuming [d,x]
=[84,-65]Bad conditionned root j= 1 value -354917.321547 ratio 3.5819922982
8 mindist 17.6876820158Bad conditionned root j= 2 value -354899.633865 rati
o 36.3553454927 mindist 16.5892679524Bad conditionned root j= 1 value -3388
0.7669046 ratio 0.612379387673 mindist 0.825253607938Warning, choosing root
 of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-8,[3,6]%%%}+%%%{-32,[1,2]%%
%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{16,[0,0]%%%}] at parameters value
s [70,22]Warning, need to choose a branch for the root of a polynomial with
 parameters. This might be wrong.The choice was done assuming [d,x]=[30,-21
]Warning, need to choose a branch for the root of a polynomial with paramet
ers. This might be wrong.The choice was done assuming [d,x]=[-24,-63]Warnin
g, need to choose a branch for the root of a polynomial with parameters. Th
is might be wrong.The choice was done assuming [d,x]=[11,52]schur row 3 -3.
22193e-07Warning, choosing root of [1,0,%%%{-4,[2,4]%%%}+%%%{-8,[0,0]%%%},%
%%{-4,[3,6]%%%}+%%%{-16,[1,2]%%%},%%%{-1,[4,8]%%%}+%%%{-4,[2,4]%%%}+%%%{16,
[0,0]%%%}] at parameters values [18,-49]Warning, choosing root of [1,0,%%%{
-4,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-4,[3,6]%%%}+%%%{-16,[1,2]%%%},%%%{-1,[4,
8]%%%}+%%%{-4,[2,4]%%%}+%%%{16,[0,0]%%%}] at parameters values [-33,-70]sch
ur row 3 1.93169e-07Warning, choosing root of [1,0,%%%{-4,[2,4]%%%}+%%%{-8,
[0,0]%%%},%%%{-4,[3,6]%%%}+%%%{-16,[1,2]%%%},%%%{-1,[4,8]%%%}+%%%{-4,[2,4]%
%%}+%%%{16,[0,0]%%%}] at parameters values [8,63]Warning, need to choose a
branch for the root of a polynomial with parameters. This might be wrong.Th
e choice was done assuming [d,x]=[-8,-94]schur row 1 1.15782e-08Francis alg
orithm not precise enough for[1.0,0.0,-29980760072,2.82570662547e+15,-7.490
38312879e+19]Bad conditionned root j= 0 value 70702.5530055 ratio 3.3049102
8239 mindist 18.5135468412Warning, need to choose a branch for the root of
a polynomial with parameters. This might be wrong.The choice was done assum
ing [d,x]=[-8,-38]schur row 1 3.47275e-08Francis algorithm not precise enou
gh for[1.0,0.0,-800692232.0,1.23327957985e+13,-5.34256730006e+16]Bad condit
ionned root j= 0 value 11555.1783957 ratio 1.30663425153 mindist 3.17957631
341schur row 1 5.69583e-10Francis algorithm not precise enough for[1.0,0.0,
-1.32328584376e+12,-8.28597485843e+17,-1.45923785361e+23]Unable to isolate
roots number Vector [0,2][-0.469612557480389e6,-0.469615331440520e6]Bad con
ditionned root j= 1 value -469647.111085 ratio 3.72291056632 mindist 31.779
6449475Warning, choosing root of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%
{-8,[3,6]%%%}+%%%{-32,[1,2]%%%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{16,[
0,0]%%%}] at parameters values [65,-85]Warning, need to choose a branch for
 the root of a polynomial with parameters. This might be wrong.The choice w
as done assuming [d,x]=[-86,73]Bad conditionned root j= 0 value 458272.6215
08 ratio 152.169961926 mindist 255.569113305Bad conditionned root j= 2 valu
e 458592.326104 ratio 66.2919566381 mindist 302.967080007Bad conditionned r
oot j= 1 value -770132.243981 ratio 141.661540265 mindist 52.7628472447Bad
conditionned root j= 2 value -770187.27489 ratio 12.2575973716 mindist 55.0
309094898Unable to isolate roots number Vector [1,3][-0.770132243980564e6,-
0.770079481133319e6]Warning, choosing root of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,
[0,0]%%%},%%%{-8,[3,6]%%%}+%%%{-32,[1,2]%%%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]
```

```
%%%}+%%%{16,[0,0]%%%}] at parameters values [93,91]Warning, need to choose
a branch for the root of a polynomial with parameters. This might be wrong.
The choice was done assuming [d,x]=[-82,36]Warning, need to choose a branch
 for the root of a polynomial with parameters. This might be wrong.The choi
ce was done assuming [d,x]=[9,15]Warning, need to choose a branch for the r
oot of a polynomial with parameters. This might be wrong.The choice was don
e assuming [d,x]=[-11,52]schur row 3 1.00361e-07Warning, choosing root of [
1,0,%%%{-4,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-4,[3,6]%%%}+%%%{-16,[1,2]%%%},%%
%{-1,[4,8]%%%}+%%%{-4,[2,4]%%%}+%%%{16,[0,0]%%%}] at parameters values [79,
-88]Warning, choosing root of [1,0,%%%{-4,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-4
,[3,6]%%%}+%%%{-16,[1,2]%%%},%%%{-1,[4,8]%%%}+%%%{-4,[2,4]%%%}+%%%{16,[0,0]
%%%}] at parameters values [9,6]Warning, choosing root of [1,0,%%%{-4,[2,4]
%%%}+%%%{-8,[0,0]%%%},%%%{-4,[3,6]%%%}+%%%{-16,[1,2]%%%},%%%{-1,[4,8]%%%}+%
%%{-4,[2,4]%%%}+%%%{16,[0,0]%%%}] at parameters values [-69,-8]Warning, nee
d to choose a branch for the root of a polynomial with parameters. This mig
ht be wrong.The choice was done assuming [d,t_nostep]=[91,52]Bad conditionn
ed root j= 1 value -246064.602658 ratio 4.13844710897 mindist 2.78272759262
schur row 1 9.14435e-09Francis algorithm not precise enough for[1.0,0.0,-21
24702752,-5.33102089176e+13,-3.76196821029e+17]Bad conditionned root j= 1 v
alue -18820.5074185 ratio 0.838796175516 mindist 3.20710914307Warning, choo
sing root of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-8,[3,6]%%%}+%%%{-3
2,[1,2]%%%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{16,[0,0]%%%}] at paramet
ers values [2,97]Warning, need to choose a branch for the root of a polynom
ial with parameters. This might be wrong.The choice was done assuming [d,t_
nostep]=[-82,75]schur row 1 1.2297e-10Francis algorithm not precise enough
for[1.0,0.0,-1.27650937501e+12,7.8505326564e+17,-1.35789682044e+23]Bad cond
itionned root j= 2 value 461240.783946 ratio 2.37092177178 mindist 11.77450
41251schur row 1 1.96129e-07Francis algorithm not precise enough for[1.0,0.
0,-288648584.0,2.6694222528e+12,-6.94316785683e+15]Bad conditionned root j=
 0 value 6940.98554667 ratio 1.29997372862 mindist 4.99589011288Warning, ch
oosing root of [1,0,%%%{-6,[2,4]%%%}+%%%{-8,[0,0]%%%},%%%{-8,[3,6]%%%}+%%%{
-32,[1,2]%%%},%%%{-3,[4,8]%%%}+%%%{-24,[2,4]%%%}+%%%{16,[0,0]%%%}] at param
eters values [-24,-17]Evaluation time: 1.98sym2poly/r2sym(const gen & e,con
st index_m & i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b \operatorname{arcsinh}\left(d\,x^{2}-i\right)\right)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(-I+d*x^2))^(7/2),x)
```

```
[Out] int(1/(a+b*arcsinh(-I+d*x^2))^(7/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b \operatorname{arsinh}\left(dx^{2}-i\right)+a\right)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(d*x^2 - I) + a)^(-7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+b\,\mathrm{asinh}\left(d\,x^2-\mathrm{i}\right)\right)}^{7/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*asinh(d*x^2 - 1i))^(7/2),x)
```

```
[Out] int(1/(a + b*asinh(d*x^2 - 1i))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(-I+d*x**2))**(7/2),x)
```

```
[Out] Timed out
```

3.342 $\int \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$

Optimal. Leaf size=43

$$\text{Int}\left(\frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^n}{1-c^2x^2},x\right)$$

```
[Out] Unintegrable((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2),x]
[Out] Defer[Int][(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2), x]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=\int \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$$

Mathematica [A] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2),x]
[Out] Integrate[(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2), x]
```

fricas [A] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(b\,\text{arsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^n}{c^2x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, alg
orithm="fricas")
[Out] integral(-(b*arcsinh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(b\,\text{arsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^n}{c^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, alg
orithm="giac")
```

```
[Out] integrate(-(b*arcsinh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x
)
```

maple [A] time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{arcsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^{n}}{-c^{2}x^{2}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

```
[Out] int((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(b\,\mathrm{arsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^{n}}{c^{2}x^{2}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, alg
orithm="maxima")
```

```
[Out] -integrate((b*arcsinh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x
)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{\left(a+b\,\mathrm{asinh}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^{n}}{c^{2}\,x^{2}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*asinh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^n/(c^2*x^2 - 1),x)
```

```
[Out] -int((a + b*asinh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^n/(c^2*x^2 - 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**n/(-c**2*x**2+1),x)
```

```
[Out] Timed out
```

3.343 $\int \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx$

Optimal. Leaf size=261

$$\frac{3b^2\text{Li}_3\left(e^{-2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{2c}+\frac{3b\text{Li}_2\left(e^{-2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{2c}-\frac{\left(a+b\sinh^{-1}\right.}{4b}$$

```
[Out] -1/4*(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^4/b/c-(a+b*arcsinh((-c*x+1
)^(1/2)/(c*x+1)^(1/2)))^3*ln(1-1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/
(c*x+1))^(1/2))^2)/c+3/2*b*(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2*po
lylog(2,1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/(c*x+1))^(1/2))^2)/c+3/
2*b^2*(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))*polylog(3,1/((-c*x+1)^(1/
2)/(c*x+1)^(1/2)+(1+(-c*x+1)/(c*x+1))^(1/2))^2)/c+3/4*b^3*polylog(4,1/((-c*
x+1)^(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/(c*x+1))^(1/2))^2)/c
```

Rubi [A] time = 0.23, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6681, 5659, 3716, 2190, 2531, 6609, 2282, 6589}

$$\frac{3b^2\text{PolyLog}\left(3,e^{2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{2c}-\frac{3b\text{PolyLog}\left(2,e^{2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{2c}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2),x]
```

```
[Out] (a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^4/(4*b*c) - ((a + b*ArcSinh[Sq
rt[1 - c*x]/Sqrt[1 + c*x]])^3*Log[1 - E^(2*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c
*x]])])/c - (3*b*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*PolyLog[2,
E^(2*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])])/(2*c) + (3*b^2*(a + b*ArcSinh[
Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[3, E^(2*ArcSinh[Sqrt[1 - c*x]/Sqrt[1
+ c*x]])])/(2*c) - (3*b^3*PolyLog[4, E^(2*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*
x]])])/(4*c)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
```

```
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6681

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[(2*e*g)/(C*(e*f -
d*g)), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^3}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}$$
$$= -\frac{\text{Subst}\left(\int (a+bx)^3\coth(x)\,dx, x, \sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{c}$$
$$= \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc} + \frac{2\,\text{Subst}\left(\int \frac{e^{2x}(a+bx)^3}{1-e^{2x}}\,dx, x, \sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{c}$$
$$= \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc} - \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} + \cdots$$
$$= \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc} - \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} - \cdots$$
$$= \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc} - \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} - \cdots$$
$$= \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc} - \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} - \cdots$$
$$= \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc} - \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} - \cdots$$

Mathematica [A] time = 0.06, size = 244, normalized size = 0.93

$$\frac{6b^2\text{Li}_3\left(e^{2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right) - 6b\text{Li}_2\left(e^{2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2 + \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-}}{\sqrt{cx}}\right.\right.}{b}\cdots}{4c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2),x]
```

```
[Out] ((a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^4/b - 4*(a + b*ArcSinh[Sqrt[1
 - c*x]/Sqrt[1 + c*x]])^3*Log[1 - E^(2*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]]
)] - 6*b*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*PolyLog[2, E^(2*Arc
Sinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])] + 6*b^2*(a + b*ArcSinh[Sqrt[1 - c*x]/Sq
rt[1 + c*x]])*PolyLog[3, E^(2*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])] - 3*b^
3*PolyLog[4, E^(2*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])])/(4*c)
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{b^3\operatorname{arsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^3 + 3\,ab^2\operatorname{arsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2 + 3\,a^2b\operatorname{arsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + a^3}{c^2x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, alg
orithm="fricas")
```

```
[Out] integral(-(b^3*arcsinh(sqrt(-c*x + 1)/sqrt(c*x + 1))^3 + 3*a*b^2*arcsinh(sq
rt(-c*x + 1)/sqrt(c*x + 1))^2 + 3*a^2*b*arcsinh(sqrt(-c*x + 1)/sqrt(c*x + 1
)) + a^3)/(c^2*x^2 - 1), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, alg
orithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.71, size = 1175, normalized size = 4.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x)
```

```
[Out] 1/2*a^3/c*ln(c*x+1)-1/2*a^3/c*ln(c*x-1)+1/4*b^3/c*arcsinh((-c*x+1)^(1/2)/(c
*x+1)^(1/2))^4-b^3/c*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^3*ln(1+(-c*x+1)^
(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/(c*x+1))^(1/2))-3*b^3/c*arcsinh((-c*x+1)^(1
/2)/(c*x+1)^(1/2))^2*polylog(2,-(-c*x+1)^(1/2)/(c*x+1)^(1/2)-(1+(-c*x+1)/(c
*x+1))^(1/2))+6*b^3/c*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(3,-(-c*
x+1)^(1/2)/(c*x+1)^(1/2)-(1+(-c*x+1)/(c*x+1))^(1/2))-6*b^3/c*polylog(4,-(-c
*x+1)^(1/2)/(c*x+1)^(1/2)-(1+(-c*x+1)/(c*x+1))^(1/2))-b^3/c*arcsinh((-c*x+1
)^(1/2)/(c*x+1)^(1/2))^3*ln(1-(-c*x+1)^(1/2)/(c*x+1)^(1/2)-(1+(-c*x+1)/(c*x
+1))^(1/2))-3*b^3/c*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*polylog(2,(-c*x
+1)^(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/(c*x+1))^(1/2))+6*b^3/c*arcsinh((-c*x+1
)^(1/2)/(c*x+1)^(1/2))*polylog(3,(-c*x+1)^(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/(
c*x+1))^(1/2))-6*b^3/c*polylog(4,(-c*x+1)^(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/(
c*x+1))^(1/2))+a*b^2/c*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^3-3*a*b^2/c*ar
csinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*ln(1+(-c*x+1)^(1/2)/(c*x+1)^(1/2)+(1+
(-c*x+1)/(c*x+1))^(1/2))-6*a*b^2/c*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))*po
lylog(2,-(-c*x+1)^(1/2)/(c*x+1)^(1/2)-(1+(-c*x+1)/(c*x+1))^(1/2))+6*a*b^2/c
*polylog(3,-(-c*x+1)^(1/2)/(c*x+1)^(1/2)-(1+(-c*x+1)/(c*x+1))^(1/2))-3*a*b^
2/c*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*ln(1-(-c*x+1)^(1/2)/(c*x+1)^(1/
2)-(1+(-c*x+1)/(c*x+1))^(1/2))-6*a*b^2/c*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/
2))*polylog(2,(-c*x+1)^(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/(c*x+1))^(1/2))+6*a*
b^2/c*polylog(3,(-c*x+1)^(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/(c*x+1))^(1/2))+3/
2*a^2*b/c*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2-3*a^2*b/c*arcsinh((-c*x+1
)^(1/2)/(c*x+1)^(1/2))*ln(1+(-c*x+1)^(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/(c*x+1
))^(1/2))-3*a^2*b/c*polylog(2,-(-c*x+1)^(1/2)/(c*x+1)^(1/2)-(1+(-c*x+1)/(c*
x+1))^(1/2))-3*a^2*b/c*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))*ln(1-(-c*x+1)^
(1/2)/(c*x+1)^(1/2)-(1+(-c*x+1)/(c*x+1))^(1/2))-3*a^2*b/c*polylog(2,(-c*x+1
)^(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/(c*x+1))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}a^3\left(\frac{\log(cx+1)}{c} - \frac{\log(cx-1)}{c}\right) + \frac{\left(b^3\log(cx+1) - b^3\log(-cx+1)\right)\log\left(\sqrt{2} + \sqrt{-cx+1}\right)^3}{2c} + \int \frac{\left(\sqrt{2}\,b^3 + \sqrt{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, alg
orithm="maxima")
```

```
[Out] 1/2*a^3*(log(c*x + 1)/c - log(c*x - 1)/c) + 1/2*(b^3*log(c*x + 1) - b^3*log
(-c*x + 1))*log(sqrt(2) + sqrt(-c*x + 1))^3/c + integrate(1/8*((sqrt(2)*b^3
 + sqrt(-c*x + 1)*b^3)*log(c*x + 1)^3 - 6*(sqrt(2)*a*b^2 + sqrt(-c*x + 1)*a
*b^2)*log(c*x + 1)^2 - 6*(4*sqrt(2)*a*b^2 - 2*(sqrt(2)*b^3 + sqrt(-c*x + 1)
*b^3)*log(c*x + 1) + (4*a*b^2 + (b^3*c*x + b^3)*log(c*x + 1) - (b^3*c*x + b
^3)*log(-c*x + 1))*sqrt(-c*x + 1))*log(sqrt(2) + sqrt(-c*x + 1))^2 + 12*(sq
rt(2)*a^2*b + sqrt(-c*x + 1)*a^2*b)*log(c*x + 1) - 6*(4*sqrt(2)*a^2*b + 4*s
qrt(-c*x + 1)*a^2*b + (sqrt(2)*b^3 + sqrt(-c*x + 1)*b^3)*log(c*x + 1)^2 - 4
*(sqrt(2)*a*b^2 + sqrt(-c*x + 1)*a*b^2)*log(c*x + 1))*log(sqrt(2) + sqrt(-c
*x + 1)))/(sqrt(2)*c^2*x^2 + (c^2*x^2 - 1)*sqrt(-c*x + 1) - sqrt(2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{\left(a+b\,\mathrm{asinh}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^3}{c^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*asinh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^3/(c^2*x^2 - 1),x)
```

```
[Out] int(-(a + b*asinh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^3/(c^2*x^2 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a^3}{c^2x^2-1}\,dx-\int \frac{b^3\,\mathrm{asinh}^3\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx-\int \frac{3ab^2\,\mathrm{asinh}^2\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx-\int \frac{3a^2b\,\mathrm{asinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**3/(-c**2*x**2+1),x)
```

```
[Out] -Integral(a**3/(c**2*x**2 - 1), x) - Integral(b**3*asinh(sqrt(-c*x + 1)/sqr
t(c*x + 1))**3/(c**2*x**2 - 1), x) - Integral(3*a*b**2*asinh(sqrt(-c*x + 1)
/sqrt(c*x + 1))**2/(c**2*x**2 - 1), x) - Integral(3*a**2*b*asinh(sqrt(-c*x
+ 1)/sqrt(c*x + 1))/(c**2*x**2 - 1), x)
```

3.344 $\int \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx$

Optimal. Leaf size=194

$$\frac{b\text{Li}_2\left(e^{-2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{c}-\frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3}{3bc}-\frac{\log\left(1-e^{-2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+b\sinh\right.}{c}$$

```
[Out] -1/3*(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/b/c-(a+b*arcsinh((-c*x+1
)^(1/2)/(c*x+1)^(1/2)))^2*ln(1-1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/
(c*x+1))^(1/2))^2)/c+b*(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))*polylog(
2,1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/(c*x+1))^(1/2))^2)/c+1/2*b^2*
polylog(3,1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/(c*x+1))^(1/2))^2)/c
```

Rubi [A] time = 0.22, antiderivative size = 195, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {6681, 5659, 3716, 2190, 2531, 2282, 6589}

$$-\frac{b\text{PolyLog}\left(2,e^{2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{c}+\frac{b^2\text{PolyLog}\left(3,e^{2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)}{2c}+\frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right.\right.}{3bc}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2),x]
```

```
[Out] (a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(3*b*c) - ((a + b*ArcSinh[Sq
rt[1 - c*x]/Sqrt[1 + c*x]])^2*Log[1 - E^(2*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c
*x]])])/c - (b*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[2, E^(2
*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])])/c + (b^2*PolyLog[3, E^(2*ArcSinh[S
qrt[1 - c*x]/Sqrt[1 + c*x]])])/(2*c)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6681

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[(2*e*g)/(C*(e*f -
d*g)), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx &= -\frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^2}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c} \\
&= -\frac{\text{Subst}\left(\int (a+bx)^2\coth(x)\,dx, x, \sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{c} \\
&= \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{3bc} + \frac{2\,\text{Subst}\left(\int \frac{e^{2x}(a+bx)^2}{1-e^{2x}}\,dx, x, \sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{c} \\
&= \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{3bc} - \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} + \cdots \\
&= \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{3bc} - \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} - \cdots \\
&= \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{3bc} - \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} - \cdots \\
&= \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{3bc} - \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} - \cdots
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 187, normalized size = 0.96

$$\frac{-6b^2\mathrm{Li}_2\left(e^{2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)+2\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)-3b\log\left(\right.\right.}{6bc}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2),x]
```

```
[Out] (2*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*(a + b*ArcSinh[Sqrt[1 - c
*x]/Sqrt[1 + c*x]] - 3*b*Log[1 - E^(2*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])
]) - 6*b^2*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[2, E^(2*Arc
Sinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])] + 3*b^3*PolyLog[3, E^(2*ArcSinh[Sqrt[1
- c*x]/Sqrt[1 + c*x]])])/(6*b*c)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{b^2\operatorname{arsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2+2\,ab\operatorname{arsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a^2}{c^2x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, alg
orithm="fricas")
```

```
[Out] integral(-(b^2*arcsinh(sqrt(-c*x + 1)/sqrt(c*x + 1))^2 + 2*a*b*arcsinh(sqrt
(-c*x + 1)/sqrt(c*x + 1)) + a^2)/(c^2*x^2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int-\frac{\left(b\operatorname{arsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}{c^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, alg
orithm="giac")
```

```
[Out] integrate(-(b*arcsinh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^2/(c^2*x^2 - 1), x
)
```

maple [B] time = 0.01, size = 649, normalized size = 3.35

$$\frac{a^2\ln(cx+1)}{2c}-\frac{a^2\ln(cx-1)}{2c}+\frac{b^2\operatorname{arcsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^3}{3c}-\frac{b^2\operatorname{arcsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2\ln\left(1+\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}+\sqrt{1+\frac{-cx+1}{cx+1}}\right)}{c}-\frac{2b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x)
```

```
[Out] 1/2*a^2/c*ln(c*x+1)-1/2*a^2/c*ln(c*x-1)+1/3*b^2/c*arcsinh((-c*x+1)^(1/2)/(c
*x+1)^(1/2))^3-b^2/c*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*ln(1+(-c*x+1)^
(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/(c*x+1))^(1/2))-2*b^2/c*arcsinh((-c*x+1)^(1
/2)/(c*x+1)^(1/2))*polylog(2,-(-c*x+1)^(1/2)/(c*x+1)^(1/2)-(1+(-c*x+1)/(c*x
+1))^(1/2))+2*b^2/c*polylog(3,-(-c*x+1)^(1/2)/(c*x+1)^(1/2)-(1+(-c*x+1)/(c*
x+1))^(1/2))-b^2/c*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*ln(1-(-c*x+1)^(1
/2)/(c*x+1)^(1/2)-(1+(-c*x+1)/(c*x+1))^(1/2))-2*b^2/c*arcsinh((-c*x+1)^(1/2
```

```
)/(c*x+1)^(1/2))*polylog(2,(-c*x+1)^(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/(c*x+1)
)^(1/2))+2*b^2/c*polylog(3,(-c*x+1)^(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/(c*x+1)
)^(1/2))+a*b/c*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2-2*a*b/c*arcsinh((-c*
x+1)^(1/2)/(c*x+1)^(1/2))*ln(1+(-c*x+1)^(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/(c*
x+1))^(1/2))-2*a*b/c*polylog(2,-(-c*x+1)^(1/2)/(c*x+1)^(1/2)-(1+(-c*x+1)/(c
*x+1))^(1/2))-2*a*b/c*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))*ln(1-(-c*x+1)^(
1/2)/(c*x+1)^(1/2)-(1+(-c*x+1)/(c*x+1))^(1/2))-2*a*b/c*polylog(2,(-c*x+1)^(
1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/(c*x+1))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}a^2\left(\frac{\log(cx+1)}{c}-\frac{\log(cx-1)}{c}\right)+\frac{\left(b^2\log(cx+1)-b^2\log(-cx+1)\right)\log\left(\sqrt{2}+\sqrt{-cx+1}\right)^2}{2c}+\int -\frac{\left(\sqrt{2}b^2+\sqrt{-}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, alg
orithm="maxima")
```

```
[Out] 1/2*a^2*(log(c*x + 1)/c - log(c*x - 1)/c) + 1/2*(b^2*log(c*x + 1) - b^2*log
(-c*x + 1))*log(sqrt(2) + sqrt(-c*x + 1))^2/c + integrate(-1/4*((sqrt(2)*b^
2 + sqrt(-c*x + 1)*b^2)*log(c*x + 1)^2 - 4*(sqrt(2)*a*b + sqrt(-c*x + 1)*a*
b)*log(c*x + 1) + 2*(4*sqrt(2)*a*b - 2*(sqrt(2)*b^2 + sqrt(-c*x + 1)*b^2)*l
og(c*x + 1) + (4*a*b + (b^2*c*x + b^2)*log(c*x + 1) - (b^2*c*x + b^2)*log(-
c*x + 1))*sqrt(-c*x + 1))*log(sqrt(2) + sqrt(-c*x + 1)))/(sqrt(2)*c^2*x^2 +
 (c^2*x^2 - 1)*sqrt(-c*x + 1) - sqrt(2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\left(a+b\,\mathrm{asinh}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^2}{c^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*asinh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2/(c^2*x^2 - 1),x)
```

```
[Out] int(-(a + b*asinh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2/(c^2*x^2 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a^2}{c^2x^2-1}\,dx-\int \frac{b^2\,\mathrm{asinh}^2\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx-\int \frac{2ab\,\mathrm{asinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**2/(-c**2*x**2+1),x)
```

```
[Out] -Integral(a**2/(c**2*x**2 - 1), x) - Integral(b**2*asinh(sqrt(-c*x + 1)/sqr
t(c*x + 1))**2/(c**2*x**2 - 1), x) - Integral(2*a*b*asinh(sqrt(-c*x + 1)/sq
rt(c*x + 1))/(c**2*x**2 - 1), x)
```

3.345 $\int \frac{a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx$

Optimal. Leaf size=133

$$-\frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{2bc}-\frac{\log\left(1-e^{-2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{c}+\frac{b\mathrm{Li}_2\left(e^{-2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)}{2c}$$

```
[Out] -1/2*(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/b/c-(a+b*arcsinh((-c*x+1
)^(1/2)/(c*x+1)^(1/2)))*ln(1-1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/(c
*x+1))^(1/2))^2)/c+1/2*b*polylog(2,1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+(1+(-c*x
+1)/(c*x+1))^(1/2))^2)/c
```

Rubi [A] time = 0.12, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {206, 6681, 5659, 3716, 2190, 2279, 2391}

$$-\frac{b\text{PolyLog}\left(2,e^{2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)}{2c}+\frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{2bc}-\frac{\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{c}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2),x]
```

```
[Out] (a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(2*b*c) - ((a + b*ArcSinh[Sq
rt[1 - c*x]/Sqrt[1 + c*x]])*Log[1 - E^(2*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x
]])])/c - (b*PolyLog[2, E^(2*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])])/(2*c)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
```

```
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 6681

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[(2*e*g)/(C*(e*f -
d*g)), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx = -\frac{\mathrm{Subst}\left(\int \frac{a+b\sinh^{-1}(x)}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}$$

$$= -\frac{\mathrm{Subst}\left(\int (a+bx)\coth(x)\,dx, x, \sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{c}$$

$$= \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{2bc} + \frac{2\,\mathrm{Subst}\left(\int \frac{e^{2x}(a+bx)}{1-e^{2x}}\,dx, x, \sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{c}$$

$$= \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{2bc} - \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} + \frac{b\,\mathrm{Su}}{}$$

$$= \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{2bc} - \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} + \frac{b\,\mathrm{Su}}{}$$

$$= \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{2bc} - \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} - \frac{b\mathrm{Li}_2}{}$$

Mathematica [A] time = 0.03, size = 127, normalized size = 0.95

$$\frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)-2b\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\right)-b^2\mathrm{Li}_2\left(e^{2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)}{2bc}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2),x]
```

```
[Out] ((a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*(a + b*ArcSinh[Sqrt[1 - c*x]/
Sqrt[1 + c*x]] - 2*b*Log[1 - E^(2*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])]) -
 b^2*PolyLog[2, E^(2*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])])/(2*b*c)
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{b\,\text{arsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a}{c^2x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algor
ithm="fricas")
```

```
[Out] integral(-(b*arcsinh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)/(c^2*x^2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{b\,\text{arsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a}{c^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algor
ithm="giac")
```

```
[Out] integrate(-(b*arcsinh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)/(c^2*x^2 - 1), x)
```

maple [A] time = 0.01, size = 263, normalized size = 1.98

$$\frac{a\ln(cx+1)}{2c}-\frac{a\ln(cx-1)}{2c}+\frac{b\,\text{arcsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2}{2c}-\frac{b\,\text{arcsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\ln\left(1+\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}+\sqrt{1+\frac{-cx+1}{cx+1}}\right)}{c}-\frac{b\,\text{polyl}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x)
```

```
[Out] 1/2*a/c*ln(c*x+1)-1/2*a/c*ln(c*x-1)+1/2*b/c*arcsinh((-c*x+1)^(1/2)/(c*x+1)^
(1/2))^2-b/c*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))*ln(1+(-c*x+1)^(1/2)/(c*x
+1)^(1/2)+(1+(-c*x+1)/(c*x+1))^(1/2))-b/c*polylog(2,-(-c*x+1)^(1/2)/(c*x+1)
^(1/2)-(1+(-c*x+1)/(c*x+1))^(1/2))-b/c*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)
)*ln(1-(-c*x+1)^(1/2)/(c*x+1)^(1/2)-(1+(-c*x+1)/(c*x+1))^(1/2))-b/c*polylog
(2,(-c*x+1)^(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/(c*x+1))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{8}b\left(\frac{2\left(\log(cx+1)-\log(-cx+1)\right)\log(cx+1)-\log(cx+1)^2+2\log(cx+1)\log(-cx+1)-\log(-cx+1}{c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algor
ithm="maxima")
```

```
[Out] -1/8*b*((2*(log(c*x + 1) - log(-c*x + 1))*log(c*x + 1) - log(c*x + 1)^2 + 2
*log(c*x + 1)*log(-c*x + 1) - log(-c*x + 1)^2 - 4*(log(c*x + 1) - log(-c*x
+ 1))*log(sqrt(2) + sqrt(-c*x + 1)))/c + 8*integrate(-1/4*(sqrt(2)*log(c*x
+ 1) - sqrt(2)*log(-c*x + 1))/(sqrt(2)*c*x + (c*x - 1)*sqrt(-c*x + 1) - sqr
t(2)), x)) + 1/2*a*(log(c*x + 1)/c - log(c*x - 1)/c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{a+b\,\text{asinh}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)}{c^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*asinh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))/(c^2*x^2 - 1),x)
```

```
[Out] int(-(a + b*asinh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))/(c^2*x^2 - 1), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a}{c^2x^2-1}\,dx - \int \frac{b \operatorname{asinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh((-c*x+1)**(1/2)/(c*x+1)**(1/2)))/(-c**2*x**2+1),x)
```

```
[Out] -Integral(a/(c**2*x**2 - 1), x) - Integral(b*asinh(sqrt(-c*x + 1)/sqrt(c*x
+ 1))/(c**2*x**2 - 1), x)
```

3.346 $\int \frac{1}{(1-c^2x^2)\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)} dx$

Optimal. Leaf size=43

$$\text{Int}\left(\frac{1}{(1-c^2x^2)\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}, x\right)$$

[Out] Unintegrable(1/(-c^2*x^2+1)/(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(1-c^2x^2)\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)} dx$$

Verification is Not applicable to the result.

[In] Int[1/((1 - c^2*x^2)*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),x]

[Out] Defer[Int][1/((1 - c^2*x^2)*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])), x]

Rubi steps

$$\int \frac{1}{(1-c^2x^2)\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)} dx = \int \frac{1}{(1-c^2x^2)\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)} dx$$

Mathematica [A] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{1}{(1-c^2x^2)\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((1 - c^2*x^2)*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),x]

[Out] Integrate[1/((1 - c^2*x^2)*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])), x]

fricas [A] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{1}{ac^2x^2 + (bc^2x^2-b)\operatorname{arsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) - a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-c^2*x^2+1)/(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, algorithm="fricas")

[Out] integral(-1/(a*c^2*x^2 + (b*c^2*x^2 - b)*arcsinh(sqrt(-c*x + 1)/sqrt(c*x + 1)) - a), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{\left(c^2x^2-1\right)\left(b\operatorname{arsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, alg
orithm="giac")
```

```
[Out] integrate(-1/((c^2*x^2 - 1)*(b*arcsinh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)),
 x)
```

maple [A] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-c^2x^2+1\right)\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-c^2*x^2+1)/(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

```
[Out] int(1/(-c^2*x^2+1)/(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\left(c^2x^2-1\right)\left(b\operatorname{arsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, alg
orithm="maxima")
```

```
[Out] -integrate(1/((c^2*x^2 - 1)*(b*arcsinh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)),
 x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{1}{\left(a+b\,\operatorname{asinh}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)\,\left(c^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((a + b*asinh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))*(c^2*x^2 - 1)),x)
```

```
[Out] -int(1/((a + b*asinh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))*(c^2*x^2 - 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c**2*x**2+1)/(a+b*asinh((-c*x+1)**(1/2)/(c*x+1)**(1/2))),x)
```

```
[Out] Timed out
```

3.347 $\int \frac{1}{(1-c^2x^2)\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$

Optimal. Leaf size=43

$$\text{Int}\left(\frac{1}{(1-c^2x^2)\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(-c^2*x^2+1)/(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x
)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(1-c^2x^2)\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((1 - c^2*x^2)*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),x]
```

```
[Out] Defer[Int][1/((1 - c^2*x^2)*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2)
, x]
```

Rubi steps

$$\int \frac{1}{(1-c^2x^2)\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx=\int \frac{1}{(1-c^2x^2)\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$$

Mathematica [A] time = 0.85, size = 0, normalized size = 0.00

$$\int \frac{1}{(1-c^2x^2)\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((1 - c^2*x^2)*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),
x]
```

```
[Out] Integrate[1/((1 - c^2*x^2)*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),
 x]
```

fricas [A] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{1}{a^2c^2x^2+(b^2c^2x^2-b^2)\,\text{arsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2-a^2+2\,(abc^2x^2-ab)\,\text{arsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, a
lgorithm="fricas")
```

```
[Out] integral(-1/(a^2*c^2*x^2 + (b^2*c^2*x^2 - b^2)*arcsinh(sqrt(-c*x + 1)/sqrt(
c*x + 1))^2 - a^2 + 2*(a*b*c^2*x^2 - a*b)*arcsinh(sqrt(-c*x + 1)/sqrt(c*x +
 1))), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{\left(c^2x^2-1\right)\left(b\operatorname{arsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, a
lgorithm="giac")
```

```
[Out] integrate(-1/((c^2*x^2 - 1)*(b*arcsinh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^2
), x)
```

maple [A] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-c^2x^2+1\right)\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-c^2*x^2+1)/(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

```
[Out] int(1/(-c^2*x^2+1)/(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{4\left(\sqrt{2}+\sqrt{-cx+1}\right)}{2\sqrt{2}\,abc^2x-2\sqrt{2}\,abc-4\sqrt{-cx+1}\,abc-\left(\sqrt{2}\,b^2c^2x-\sqrt{2}\,b^2c-2\sqrt{-cx+1}\,b^2c\right)\log\left(cx+1\right)+2\left(\sqrt{2}\,b^2c^2x-\sqrt{\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, a
lgorithm="maxima")
```

```
[Out] -4*(sqrt(2) + sqrt(-c*x + 1))/(2*sqrt(2)*a*b*c^2*x - 2*sqrt(2)*a*b*c - 4*sq
rt(-c*x + 1)*a*b*c - (sqrt(2)*b^2*c^2*x - sqrt(2)*b^2*c - 2*sqrt(-c*x + 1)*
b^2*c)*log(c*x + 1) + 2*(sqrt(2)*b^2*c^2*x - sqrt(2)*b^2*c - 2*sqrt(-c*x +
1)*b^2*c)*log(sqrt(2) + sqrt(-c*x + 1))) - integrate((4*c*x + (sqrt(2)*c*x
- 3*sqrt(2))*sqrt(-c*x + 1) - 4)/(2*a*b*c^3*x^3 - 6*a*b*c^2*x^2 + 6*a*b*c*x
 - 4*(a*b*c*x - a*b)*(c*x - 1) - 2*a*b - (b^2*c^3*x^3 - 3*b^2*c^2*x^2 + 3*b
^2*c*x - 2*(b^2*c*x - b^2)*(c*x - 1) - b^2 - 2*(sqrt(2)*b^2*c^2*x^2 - 2*sqr
t(2)*b^2*c*x + sqrt(2)*b^2)*sqrt(-c*x + 1))*log(c*x + 1) + 2*(b^2*c^3*x^3 -
 3*b^2*c^2*x^2 + 3*b^2*c*x - 2*(b^2*c*x - b^2)*(c*x - 1) - b^2 - 2*(sqrt(2)
*b^2*c^2*x^2 - 2*sqrt(2)*b^2*c*x + sqrt(2)*b^2)*sqrt(-c*x + 1))*log(sqrt(2)
 + sqrt(-c*x + 1)) - 4*(sqrt(2)*a*b*c^2*x^2 - 2*sqrt(2)*a*b*c*x + sqrt(2)*a
*b)*sqrt(-c*x + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{1}{{\left(a+b\,\mathrm{asinh}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)}^2\,\left(c^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((a + b*asinh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2*(c^2*x^2 - 1)),x)
```

```
[Out] -int(1/((a + b*asinh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2*(c^2*x^2 - 1)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c**2*x**2+1)/(a+b*asinh((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**2,x
)
```

```
[Out] Timed out
```

3.348 $\int \sinh^{-1}\left(ce^{a+bx}\right) dx$

Optimal. Leaf size=76

$$\frac{\text{Li}_2\left(e^{2\sinh^{-1}\left(ce^{a+bx}\right)}\right)}{2b}-\frac{\sinh^{-1}\left(ce^{a+bx}\right)^2}{2b}+\frac{\sinh^{-1}\left(ce^{a+bx}\right)\log\left(1-e^{2\sinh^{-1}\left(ce^{a+bx}\right)}\right)}{b}$$

```
[Out] -1/2*arcsinh(c*exp(b*x+a))^2/b+arcsinh(c*exp(b*x+a))*ln(1-(c*exp(b*x+a)+(1+
c^2*exp(b*x+a)^2)^(1/2))^2)/b+1/2*polylog(2,(c*exp(b*x+a)+(1+c^2*exp(b*x+a)
^2)^(1/2))^2)/b
```

Rubi [A] time = 0.08, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {2282, 5659, 3716, 2190, 2279, 2391}

$$\frac{\text{PolyLog}\left(2,e^{2\sinh^{-1}\left(ce^{a+bx}\right)}\right)}{2b}-\frac{\sinh^{-1}\left(ce^{a+bx}\right)^2}{2b}+\frac{\sinh^{-1}\left(ce^{a+bx}\right)\log\left(1-e^{2\sinh^{-1}\left(ce^{a+bx}\right)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[c*E^(a + b*x)],x]
```

```
[Out] -ArcSinh[c*E^(a + b*x)]^2/(2*b) + (ArcSinh[c*E^(a + b*x)]*Log[1 - E^(2*ArcS
inh[c*E^(a + b*x)])])/b + PolyLog[2, E^(2*ArcSinh[c*E^(a + b*x)])]/(2*b)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rubi steps

$$
\begin{aligned}
\int \sinh^{-1}\left(ce^{a+bx}\right) dx &= \frac{\text{Subst}\left(\int \frac{\sinh^{-1}(cx)}{x} dx, x, e^{a+bx}\right)}{b} \\
&= \frac{\text{Subst}\left(\int x\coth(x)\, dx, x, \sinh^{-1}\left(ce^{a+bx}\right)\right)}{b} \\
&= -\frac{\sinh^{-1}\left(ce^{a+bx}\right)^2}{2b} - \frac{2\,\text{Subst}\left(\int \frac{e^{2x}x}{1-e^{2x}} dx, x, \sinh^{-1}\left(ce^{a+bx}\right)\right)}{b} \\
&= -\frac{\sinh^{-1}\left(ce^{a+bx}\right)^2}{2b} + \frac{\sinh^{-1}\left(ce^{a+bx}\right)\log\left(1-e^{2\sinh^{-1}\left(ce^{a+bx}\right)}\right)}{b} - \frac{\text{Subst}\left(\int \log\left(1-e^{2x}\right)\right.}{} \\
&= -\frac{\sinh^{-1}\left(ce^{a+bx}\right)^2}{2b} + \frac{\sinh^{-1}\left(ce^{a+bx}\right)\log\left(1-e^{2\sinh^{-1}\left(ce^{a+bx}\right)}\right)}{b} - \frac{\text{Subst}\left(\int \frac{\log(1-x)}{x} dx, x\right.}{2b} \\
&= -\frac{\sinh^{-1}\left(ce^{a+bx}\right)^2}{2b} + \frac{\sinh^{-1}\left(ce^{a+bx}\right)\log\left(1-e^{2\sinh^{-1}\left(ce^{a+bx}\right)}\right)}{b} + \frac{\text{Li}_2\left(e^{2\sinh^{-1}\left(ce^{a+bx}\right)}\right)}{2b}
\end{aligned}
$$

Mathematica [A] time = 0.53, size = 76, normalized size = 1.00

$$
\frac{\text{Li}_2\left(e^{2\sinh^{-1}\left(ce^{a+bx}\right)}\right)}{2b} - \frac{\sinh^{-1}\left(ce^{a+bx}\right)^2}{2b} + \frac{\sinh^{-1}\left(ce^{a+bx}\right)\log\left(1-e^{2\sinh^{-1}\left(ce^{a+bx}\right)}\right)}{b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[c*E^(a + b*x)],x]
```

```
[Out] -1/2*ArcSinh[c*E^(a + b*x)]^2/b + (ArcSinh[c*E^(a + b*x)]*Log[1 - E^(2*ArcS
inh[c*E^(a + b*x)])])/b + PolyLog[2, E^(2*ArcSinh[c*E^(a + b*x)])]/(2*b)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(c*exp(b*x+a)),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \operatorname{arsinh}\left(ce^{(bx+a)}\right) dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(c*exp(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(c*e^(b*x + a)), x)
```

maple [A] time = 0.01, size = 166, normalized size = 2.18

$$-\frac{\operatorname{arcsinh}\left(c\,\mathrm{e}^{bx+a}\right)^{2}}{2b}+\frac{\operatorname{arcsinh}\left(c\,\mathrm{e}^{bx+a}\right)\ln\left(1+c\,\mathrm{e}^{bx+a}+\sqrt{1+c^{2}\mathrm{e}^{2bx+2a}}\right)}{b}+\frac{\operatorname{polylog}\left(2,-c\,\mathrm{e}^{bx+a}-\sqrt{1+c^{2}\mathrm{e}^{2bx+2a}}\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(c*exp(b*x+a)),x)
```

```
[Out] -1/2*arcsinh(c*exp(b*x+a))^2/b+1/b*arcsinh(c*exp(b*x+a))*ln(1+c*exp(b*x+a)+
(1+c^2*exp(b*x+a)^2)^(1/2))+1/b*polylog(2,-c*exp(b*x+a)-(1+c^2*exp(b*x+a)^2
)^(1/2))+1/b*arcsinh(c*exp(b*x+a))*ln(1-c*exp(b*x+a)-(1+c^2*exp(b*x+a)^2)^(
1/2))+1/b*polylog(2,c*exp(b*x+a)+(1+c^2*exp(b*x+a)^2)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-bc\int\frac{xe^{(bx+a)}}{c^{3}e^{(3\,bx+3\,a)}+ce^{(bx+a)}+\left(c^{2}e^{(2\,bx+2\,a)}+1\right)^{\frac{3}{2}}}\,dx+x\log\left(ce^{(bx+a)}+\sqrt{c^{2}e^{(2\,bx+2\,a)}+1}\right)-\frac{2\,bx\log\left(c^{2}e^{(2\,bx+2\,a)}+1\right.}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(c*exp(b*x+a)),x, algorithm="maxima")
```

```
[Out] -b*c*integrate(x*e^(b*x + a)/(c^3*e^(3*b*x + 3*a) + c*e^(b*x + a) + (c^2*e^
(2*b*x + 2*a) + 1)^(3/2)), x) + x*log(c*e^(b*x + a) + sqrt(c^2*e^(2*b*x + 2
*a) + 1)) - 1/4*(2*b*x*log(c^2*e^(2*b*x + 2*a) + 1) + dilog(-c^2*e^(2*b*x +
 2*a)))/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{asinh}\left(c\,\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(c*exp(a + b*x)),x)
```

```
[Out] int(asinh(c*exp(b*x)*exp(a)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{asinh}\left(ce^{a+bx}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(c*exp(b*x+a)),x)
```

```
[Out] Integral(asinh(c*exp(a + b*x)), x)
```

3.349 $\int e^{\sinh^{-1}(a+bx)}x^3\,dx$

Optimal. Leaf size=165

$$\frac{(3-4a^2)\,ae^{2\sinh^{-1}(a+bx)}}{16b^4}+\frac{(3-4a^2)\,a\sinh^{-1}(a+bx)}{8b^4}-\frac{(1-6a^2)\,e^{-\sinh^{-1}(a+bx)}}{8b^4}-\frac{(1-6a^2)\,e^{3\sinh^{-1}(a+bx)}}{24b^4}+\frac{3ae^{-2\sinh^{-1}(a+bx)}}{16b^4}$$

```
[Out] 1/48/b^4/(b*x+a+(1+(b*x+a)^2)^(1/2))^3+3/16*a/b^4/(b*x+a+(1+(b*x+a)^2)^(1/2
))^2+1/8*(6*a^2-1)/b^4/(b*x+a+(1+(b*x+a)^2)^(1/2))+1/16*a*(-4*a^2+3)*(b*x+a
+(1+(b*x+a)^2)^(1/2))^2/b^4-1/24*(-6*a^2+1)*(b*x+a+(1+(b*x+a)^2)^(1/2))^3/b
^4-3/32*a*(b*x+a+(1+(b*x+a)^2)^(1/2))^4/b^4+1/80*(b*x+a+(1+(b*x+a)^2)^(1/2)
)^5/b^4+1/8*a*(-4*a^2+3)*arcsinh(b*x+a)/b^4
```

Rubi [A] time = 0.17, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5898, 2282, 12, 1628}

$$\frac{(3-4a^2)\,ae^{2\sinh^{-1}(a+bx)}}{16b^4}+\frac{(3-4a^2)\,a\sinh^{-1}(a+bx)}{8b^4}-\frac{(1-6a^2)\,e^{-\sinh^{-1}(a+bx)}}{8b^4}-\frac{(1-6a^2)\,e^{3\sinh^{-1}(a+bx)}}{24b^4}+\frac{3ae^{-2\sinh^{-1}(a+bx)}}{16b^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSinh[a + b*x]*x^3,x]
```

```
[Out] 1/(48*b^4*E^(3*ArcSinh[a + b*x])) + (3*a)/(16*b^4*E^(2*ArcSinh[a + b*x])) -
 (1 - 6*a^2)/(8*b^4*E^ArcSinh[a + b*x]) + (a*(3 - 4*a^2)*E^(2*ArcSinh[a + b
*x]))/(16*b^4) - ((1 - 6*a^2)*E^(3*ArcSinh[a + b*x]))/(24*b^4) - (3*a*E^(4*
ArcSinh[a + b*x]))/(32*b^4) + E^(5*ArcSinh[a + b*x])/(80*b^4) + (a*(3 - 4*a
^2)*ArcSinh[a + b*x])/(8*b^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 5898

```
Int[(f_)^(ArcSinh[(a_.) + (b_.)*(x_)]^(n_.)*(c_.))*(x_)^(m_.), x_Symbol] :>
 Dist[1/b, Subst[Int[(-(a/b) + Sinh[x]/b)^m*f^(c*x^n)*Cosh[x], x], x, ArcSi
nh[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int e^{\sinh^{-1}(a+bx)}x^3\,dx = \frac{\text{Subst}\left(\int e^x\cosh(x)\left(-\frac{a}{b}+\frac{\sinh(x)}{b}\right)^3 dx, x, \sinh^{-1}(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{(-1-x^2)(1+2ax-x^2)^3}{16b^3x^4}dx, x, e^{\sinh^{-1}(a+bx)}\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{(-1-x^2)(1+2ax-x^2)^3}{x^4}dx, x, e^{\sinh^{-1}(a+bx)}\right)}{16b^4}$$
$$= \frac{\text{Subst}\left(\int\left(-\frac{1}{x^4}-\frac{6a}{x^3}-\frac{2(-1+6a^2)}{x^2}+\frac{2a(3-4a^2)}{x}+2a\left(3-4a^2\right)x+2\left(-1+6a^2\right)x^2-6ax^3+x^4\right)\right.}{16b^4}$$
$$= \frac{e^{-3\sinh^{-1}(a+bx)}}{48b^4}+\frac{3ae^{-2\sinh^{-1}(a+bx)}}{16b^4}-\frac{\left(1-6a^2\right)e^{-\sinh^{-1}(a+bx)}}{8b^4}+\frac{a\left(3-4a^2\right)e^{2\sinh^{-1}(a+bx)}}{16b^4}-\frac{(1}{}$$

Mathematica [A] time = 0.09, size = 119, normalized size = 0.72

$$\frac{15a\left(3-4a^2\right)\sinh^{-1}(a+bx)-\sqrt{a^2+2abx+b^2x^2+1}\left(6a^4+2\left(3a^2-4\right)b^2x^2+\left(29-6a^2\right)abx-83a^2-6ab^3x^3\right.}{120b^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSinh[a + b*x]*x^3,x]
```

```
[Out] (30*a*b^4*x^4 + 24*b^5*x^5 - Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(16 - 83*a^2
 + 6*a^4 + a*(29 - 6*a^2)*b*x + 2*(-4 + 3*a^2)*b^2*x^2 - 6*a*b^3*x^3 - 24*b
^4*x^4) + 15*a*(3 - 4*a^2)*ArcSinh[a + b*x])/(120*b^4)
```

fricas [A] time = 0.56, size = 138, normalized size = 0.84

$$\frac{24\,b^5x^5+30\,ab^4x^4+15\left(4\,a^3-3\,a\right)\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)+\left(24\,b^4x^4+6\,ab^3x^3-2\left(3\,a^2-4\right)b\right.}{120\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))*x^3,x, algorithm="fricas")
```

```
[Out] 1/120*(24*b^5*x^5 + 30*a*b^4*x^4 + 15*(4*a^3 - 3*a)*log(-b*x - a + sqrt(b^2
*x^2 + 2*a*b*x + a^2 + 1)) + (24*b^4*x^4 + 6*a*b^3*x^3 - 2*(3*a^2 - 4)*b^2*
x^2 - 6*a^4 + (6*a^3 - 29*a)*b*x + 83*a^2 - 16)*sqrt(b^2*x^2 + 2*a*b*x + a^
2 + 1))/b^4
```

giac [A] time = 0.41, size = 173, normalized size = 1.05

$$\frac{1}{5}\,bx^5+\frac{1}{4}\,ax^4+\frac{1}{120}\,\sqrt{b^2x^2+2\,abx+a^2+1}\left(\left(2\left(3\left(4\,x+\frac{a}{b}\right)x-\frac{3\,a^2b^5-4\,b^5}{b^7}\right)x+\frac{6\,a^3b^4-29\,ab^4}{b^7}\right)x-\frac{6\,a^4b^3-83}{b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))*x^3,x, algorithm="giac")
```

```
[Out] 1/5*b*x^5 + 1/4*a*x^4 + 1/120*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*((2*(3*(4*x
 + a/b)*x - (3*a^2*b^5 - 4*b^5)/b^7)*x + (6*a^3*b^4 - 29*a*b^4)/b^7)*x - (6
*a^4*b^3 - 83*a^2*b^3 + 16*b^3)/b^7) + 1/8*(4*a^3 - 3*a)*log(-a*b - (x*abs(
b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*abs(b))/(b^3*abs(b))
```

maple [A] time = 0.01, size = 322, normalized size = 1.95

$$\frac{x^2 \left(b^2x^2+2abx+a^2+1\right)^{\frac{3}{2}}}{5b^2}-\frac{7ax \left(b^2x^2+2abx+a^2+1\right)^{\frac{3}{2}}}{20b^3}+\frac{9a^2 \left(b^2x^2+2abx+a^2+1\right)^{\frac{3}{2}}}{20b^4}-\frac{a^3\sqrt{b^2x^2+2abx+a}}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+(1+(b*x+a)^2)^(1/2))*x^3,x)
```

```
[Out] 1/5*x^2*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/b^2-7/20*a/b^3*x*(b^2*x^2+2*a*b*x+a^2
+1)^(3/2)+9/20*a^2/b^4*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)-1/2*a^3/b^3*(b^2*x^2+2
*a*b*x+a^2+1)^(1/2)*x-1/2*a^4/b^4*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-1/2*a^3/b^3
*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+3/8*
a/b^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x+3/8*a^2/b^4*(b^2*x^2+2*a*b*x+a^2+1)^(
1/2)+3/8*a/b^3*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b
^2)^(1/2)-2/15/b^4*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)+1/5*b*x^5+1/4*x^4*a
```

maxima [B] time = 0.70, size = 491, normalized size = 2.98

$$\frac{1}{5}\,bx^5+\frac{1}{4}\,ax^4+\frac{\left(b^2x^2+2\,abx+a^2+1\right)^{\frac{3}{2}}x^2}{5\,b^2}-\frac{\left(a^2+1\right)a^3 \operatorname{arsinh}\left(\frac{2\left(b^2x+ab\right)}{\sqrt{-4\,a^2b^2+4\left(a^2+1\right)b^2}}\right)}{5\,b^4}-\frac{7\left(b^2x^2+2\,abx+a^2+1\right)^{\frac{3}{2}}}{20\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))*x^3,x, algorithm="maxima")
```

```
[Out] 1/5*b*x^5 + 1/4*a*x^4 + 1/5*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*x^2/b^2 - 1
/5*(a^2 + 1)*a^3*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)
)/b^4 - 7/20*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*a*x/b^3 + 1/5*sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1)*(a^2 + 1)*a*x/b^3 + 1/5*(a^2 + 1)^2*a*arcsinh(2*(b^2*
x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^4 + 7/12*(b^2*x^2 + 2*a*b*x
+ a^2 + 1)^(3/2)*a^2/b^4 + 1/5*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)*
a^2/b^4 + 7/40*(5*a^2*b^2 - (a^2 + 1)*b^2)*a^3*arcsinh(2*(b^2*x + a*b)/sqrt
(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^6 - 2/15*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3
/2)*(a^2 + 1)/b^4 - 7/40*(5*a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x
 + a^2 + 1)*a*x/b^5 - 7/40*(5*a^2*b^2 - (a^2 + 1)*b^2)*(a^2 + 1)*a*arcsinh(
2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^6 - 7/40*(5*a^2*b^2 -
 (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^2/b^6
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\left(a+\sqrt{{\left(a+b\,x\right)}^2+1}+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + ((a + b*x)^2 + 1)^(1/2) + b*x),x)
```

```
[Out] int(x^3*(a + ((a + b*x)^2 + 1)^(1/2) + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3} \left(a + bx + \sqrt{a^{2} + 2abx + b^{2}x^{2} + 1}\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)**2)**(1/2))*x**3,x)
```

```
[Out] Integral(x**3*(a + b*x + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x)
```

3.350 $\int e^{\sinh^{-1}(a+bx)}x^2\,dx$

Optimal. Leaf size=115

$$-\frac{(1-4a^2)\,e^{2\sinh^{-1}(a+bx)}}{16b^3}-\frac{(1-4a^2)\sinh^{-1}(a+bx)}{8b^3}-\frac{ae^{-\sinh^{-1}(a+bx)}}{2b^3}-\frac{ae^{3\sinh^{-1}(a+bx)}}{6b^3}-\frac{e^{-2\sinh^{-1}(a+bx)}}{16b^3}+\frac{e^{4\sinh^{-1}(a+bx)}}{32b^3}$$

```
[Out] -1/16/b^3/(b*x+a+(1+(b*x+a)^2)^(1/2))^2-1/2*a/b^3/(b*x+a+(1+(b*x+a)^2)^(1/2
))-1/16*(-4*a^2+1)*(b*x+a+(1+(b*x+a)^2)^(1/2))^2/b^3-1/6*a*(b*x+a+(1+(b*x+a
)^2)^(1/2))^3/b^3+1/32*(b*x+a+(1+(b*x+a)^2)^(1/2))^4/b^3-1/8*(-4*a^2+1)*arc
sinh(b*x+a)/b^3
```

Rubi [A] time = 0.13, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5898, 2282, 12, 1628}

$$-\frac{(1-4a^2)\,e^{2\sinh^{-1}(a+bx)}}{16b^3}-\frac{(1-4a^2)\sinh^{-1}(a+bx)}{8b^3}-\frac{ae^{-\sinh^{-1}(a+bx)}}{2b^3}-\frac{ae^{3\sinh^{-1}(a+bx)}}{6b^3}-\frac{e^{-2\sinh^{-1}(a+bx)}}{16b^3}+\frac{e^{4\sinh^{-1}(a+bx)}}{32b^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSinh[a + b*x]*x^2,x]
```

```
[Out] -1/(16*b^3*E^(2*ArcSinh[a + b*x])) - a/(2*b^3*E^ArcSinh[a + b*x]) - ((1 - 4
*a^2)*E^(2*ArcSinh[a + b*x]))/(16*b^3) - (a*E^(3*ArcSinh[a + b*x]))/(6*b^3)
 + E^(4*ArcSinh[a + b*x])/(32*b^3) - ((1 - 4*a^2)*ArcSinh[a + b*x])/(8*b^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 5898

```
Int[(f_)^(ArcSinh[(a_.) + (b_.)*(x_)]^(n_.)*(c_.))*(x_)^(m_.), x_Symbol] :>
 Dist[1/b, Subst[Int[(-(a/b) + Sinh[x]/b)^m*f^(c*x^n)*Cosh[x], x], x, ArcSi
nh[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int e^{\sinh^{-1}(a+bx)}x^2\,dx = \frac{\text{Subst}\left(\int e^x\cosh(x)\left(-\frac{a}{b}+\frac{\sinh(x)}{b}\right)^2dx, x, \sinh^{-1}(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{(1+2ax-x^2)^2(1+x^2)}{8b^2x^3}dx, x, e^{\sinh^{-1}(a+bx)}\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{(1+2ax-x^2)^2(1+x^2)}{x^3}dx, x, e^{\sinh^{-1}(a+bx)}\right)}{8b^3}$$
$$= \frac{\text{Subst}\left(\int\left(\frac{1}{x^3}+\frac{4a}{x^2}+\frac{-1+4a^2}{x}+(-1+4a^2)x-4ax^2+x^3\right)dx, x, e^{\sinh^{-1}(a+bx)}\right)}{8b^3}$$
$$= -\frac{e^{-2\sinh^{-1}(a+bx)}}{16b^3}-\frac{ae^{-\sinh^{-1}(a+bx)}}{2b^3}-\frac{(1-4a^2)e^{2\sinh^{-1}(a+bx)}}{16b^3}-\frac{ae^{3\sinh^{-1}(a+bx)}}{6b^3}+\frac{e^{4\sinh^{-1}(a+}}{32b^3}$$

Mathematica [A] time = 0.12, size = 102, normalized size = 0.89

$$\frac{\sqrt{a^2+2abx+b^2x^2+1}\left(2a^3-2a^2bx+a\left(2b^2x^2-13\right)+6b^3x^3+3bx\right)+8ab^3x^3+3(2a-1)(2a+1)\sinh^{-1}(a+}{24b^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSinh[a + b*x]*x^2,x]
```

```
[Out] (8*a*b^3*x^3 + 6*b^4*x^4 + Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(2*a^3 + 3*b*x
 - 2*a^2*b*x + 6*b^3*x^3 + a*(-13 + 2*b^2*x^2)) + 3*(-1 + 2*a)*(1 + 2*a)*Ar
cSinh[a + b*x])/(24*b^3)
```

fricas [A] time = 0.63, size = 117, normalized size = 1.02

$$\frac{6\,b^4x^4+8\,ab^3x^3-3\left(4\,a^2-1\right)\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)+\left(6\,b^3x^3+2\,ab^2x^2+2\,a^3-\left(2\,a^2-3\right)\right.}{24\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))*x^2,x, algorithm="fricas")
```

```
[Out] 1/24*(6*b^4*x^4 + 8*a*b^3*x^3 - 3*(4*a^2 - 1)*log(-b*x - a + sqrt(b^2*x^2 +
 2*a*b*x + a^2 + 1)) + (6*b^3*x^3 + 2*a*b^2*x^2 + 2*a^3 - (2*a^2 - 3)*b*x -
 13*a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/b^3
```

giac [A] time = 0.61, size = 140, normalized size = 1.22

$$\frac{1}{4}bx^4+\frac{1}{3}ax^3+\frac{1}{24}\sqrt{b^2x^2+2\,abx+a^2+1}\left(\left(2\left(3\,x+\frac{a}{b}\right)x-\frac{2\,a^2b^3-3\,b^3}{b^5}\right)x+\frac{2\,a^3b^2-13\,ab^2}{b^5}\right)-\frac{\left(4\,a^2-1\right)\log\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))*x^2,x, algorithm="giac")
```

```
[Out] 1/4*b*x^4 + 1/3*a*x^3 + 1/24*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*((2*(3*x + a
/b)*x - (2*a^2*b^3 - 3*b^3)/b^5)*x + (2*a^3*b^2 - 13*a*b^2)/b^5) - 1/8*(4*a
^2 - 1)*log(-a*b - (x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*abs(b))/(
b^2*abs(b))
```

maple [A] time = 0.00, size = 264, normalized size = 2.30

$$\frac{x\left(b^2x^2+2abx+a^2+1\right)^{\frac{3}{2}}}{4b^2}-\frac{5a\left(b^2x^2+2abx+a^2+1\right)^{\frac{3}{2}}}{12b^3}+\frac{a^2\sqrt{b^2x^2+2abx+a^2+1}\,x}{2b^2}+\frac{a^3\sqrt{b^2x^2+2abx+a^2+1}}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+(1+(b*x+a)^2)^(1/2))*x^2,x)
```

```
[Out] 1/4*x*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/b^2-5/12*a/b^3*(b^2*x^2+2*a*b*x+a^2+1)^
(3/2)+1/2*a^2/b^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x+1/2*a^3/b^3*(b^2*x^2+2*a*
b*x+a^2+1)^(1/2)+1/2*a^2/b^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^
2+1)^(1/2))/(b^2)^(1/2)-1/8/b^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x-1/8/b^3*(b^
2*x^2+2*a*b*x+a^2+1)^(1/2)*a-1/8/b^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*
a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+1/4*b*x^4+1/3*x^3*a
```

maxima [A] time = 0.69, size = 273, normalized size = 2.37

$$\frac{1}{4}\,bx^4+\frac{1}{3}\,ax^3+\frac{\left(b^2x^2+2\,abx+a^2+1\right)^{\frac{3}{2}}x}{4\,b^2}-\frac{5\left(b^2x^2+2\,abx+a^2+1\right)^{\frac{3}{2}}a}{12\,b^3}-\frac{\left(5\,a^2b^2-\left(a^2+1\right)b^2\right)a^2\operatorname{arsinh}\left(\frac{2\left(b^2\right.}{\sqrt{-4\,a^2b^2+}}\right.}{8\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))*x^2,x, algorithm="maxima")
```

```
[Out] 1/4*b*x^4 + 1/3*a*x^3 + 1/4*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*x/b^2 - 5/1
2*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*a/b^3 - 1/8*(5*a^2*b^2 - (a^2 + 1)*b^
2)*a^2*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^5 + 1/
8*(5*a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*x/b^4 + 1/8
*(5*a^2*b^2 - (a^2 + 1)*b^2)*(a^2 + 1)*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*
b^2 + 4*(a^2 + 1)*b^2))/b^5 + 1/8*(5*a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2
+ 2*a*b*x + a^2 + 1)*a/b^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\left(a+\sqrt{(a+b\,x)^2+1}+b\,x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + ((a + b*x)^2 + 1)^(1/2) + b*x),x)
```

```
[Out] int(x^2*(a + ((a + b*x)^2 + 1)^(1/2) + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(a+bx+\sqrt{a^2+2abx+b^2x^2+1}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)**2)**(1/2))*x**2,x)
```

```
[Out] Integral(x**2*(a + b*x + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x)
```

3.351 $\int e^{\sinh^{-1}(a+bx)} x \, dx$

Optimal. Leaf size=67

$$-\frac{ae^{2\sinh^{-1}(a+bx)}}{4b^2}-\frac{a\sinh^{-1}(a+bx)}{2b^2}+\frac{e^{-\sinh^{-1}(a+bx)}}{4b^2}+\frac{e^{3\sinh^{-1}(a+bx)}}{12b^2}$$

```
[Out] 1/4/b^2/(b*x+a+(1+(b*x+a)^2)^(1/2))-1/4*a*(b*x+a+(1+(b*x+a)^2)^(1/2))^2/b^2
+1/12*(b*x+a+(1+(b*x+a)^2)^(1/2))^3/b^2-1/2*a*arcsinh(b*x+a)/b^2
```

Rubi [A] time = 0.07, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5898, 2282, 12, 1628}

$$-\frac{ae^{2\sinh^{-1}(a+bx)}}{4b^2}-\frac{a\sinh^{-1}(a+bx)}{2b^2}+\frac{e^{-\sinh^{-1}(a+bx)}}{4b^2}+\frac{e^{3\sinh^{-1}(a+bx)}}{12b^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSinh[a + b*x]*x,x]
```

```
[Out] 1/(4*b^2*E^ArcSinh[a + b*x]) - (a*E^(2*ArcSinh[a + b*x]))/(4*b^2) + E^(3*Ar
cSinh[a + b*x])/(12*b^2) - (a*ArcSinh[a + b*x])/(2*b^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 5898

```
Int[(f_)^(ArcSinh[(a_.) + (b_.)*(x_)]^(n_.)*(c_.))*(x_)^(m_.), x_Symbol] :>
 Dist[1/b, Subst[Int[(-(a/b) + Sinh[x]/b)^m*f^(c*x^n)*Cosh[x], x], x, ArcSi
nh[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int e^{\sinh^{-1}(a+bx)}x\,dx = \frac{\text{Subst}\left(\int e^x\cosh(x)\left(-\frac{a}{b}+\frac{\sinh(x)}{b}\right)dx,x,\sinh^{-1}(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{(-1-x^2)(1+2ax-x^2)}{4bx^2}dx,x,e^{\sinh^{-1}(a+bx)}\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{(-1-x^2)(1+2ax-x^2)}{x^2}dx,x,e^{\sinh^{-1}(a+bx)}\right)}{4b^2}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{x^2}-\frac{2a}{x}-2ax+x^2\right)dx,x,e^{\sinh^{-1}(a+bx)}\right)}{4b^2}$$
$$= \frac{e^{-\sinh^{-1}(a+bx)}}{4b^2}-\frac{ae^{2\sinh^{-1}(a+bx)}}{4b^2}+\frac{e^{3\sinh^{-1}(a+bx)}}{12b^2}-\frac{a\sinh^{-1}(a+bx)}{2b^2}$$

Mathematica [A] time = 0.11, size = 73, normalized size = 1.09

$$\frac{1}{6}\left(\frac{\sqrt{a^2+2abx+b^2x^2+1}\,(-a^2+abx+2b^2x^2+2)}{b^2}-\frac{3a\sinh^{-1}(a+bx)}{b^2}+3ax^2+2bx^3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSinh[a + b*x]*x,x]
```

```
[Out] (3*a*x^2 + 2*b*x^3 + (Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(2 - a^2 + a*b*x +
2*b^2*x^2))/b^2 - (3*a*ArcSinh[a + b*x])/b^2)/6
```

fricas [A] time = 0.56, size = 93, normalized size = 1.39

$$\frac{2\,b^3x^3+3\,ab^2x^2+3\,a\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)+\left(2\,b^2x^2+abx-a^2+2\right)\sqrt{b^2x^2+2\,abx+a^2+1}}{6\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))*x,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*x^3 + 3*a*b^2*x^2 + 3*a*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x +
a^2 + 1)) + (2*b^2*x^2 + a*b*x - a^2 + 2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)
)/b^2
```

giac [A] time = 0.32, size = 106, normalized size = 1.58

$$\frac{1}{3}\,bx^3+\frac{1}{2}\,ax^2+\frac{1}{6}\,\sqrt{b^2x^2+2\,abx+a^2+1}\left(\left(2\,x+\frac{a}{b}\right)x-\frac{a^2b-2\,b}{b^3}\right)+\frac{a\log\left(-ab-\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))*x,x, algorithm="giac")
```

```
[Out] 1/3*b*x^3 + 1/2*a*x^2 + 1/6*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*((2*x + a/b)*
x - (a^2*b - 2*b)/b^3) + 1/2*a*log(-a*b - (x*abs(b) - sqrt(b^2*x^2 + 2*a*b*
x + a^2 + 1))*abs(b))/(b*abs(b))
```

maple [A] time = 0.00, size = 138, normalized size = 2.06

$$\frac{\left(b^2x^2+2abx+a^2+1\right)^{\frac{3}{2}}}{3b^2}-\frac{ax\sqrt{b^2x^2+2abx+a^2+1}}{2b}-\frac{a^2\sqrt{b^2x^2+2abx+a^2+1}}{2b^2}-\frac{a\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a}\right.}{2b\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+(1+(b*x+a)^2)^(1/2))*x,x)
```

```
[Out] 1/3*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/b^2-1/2*a/b*x*(b^2*x^2+2*a*b*x+a^2+1)^(1/
2)-1/2*a^2/b^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-1/2*a/b*ln((b^2*x+a*b)/(b^2)^(
1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+1/3*b*x^3+1/2*a*x^2
```

maxima [B] time = 0.68, size = 175, normalized size = 2.61

$$\frac{1}{3}\,bx^3+\frac{1}{2}\,ax^2+\frac{a^3\operatorname{arsinh}\left(\frac{2\left(b^2x+ab\right)}{\sqrt{-4\,a^2b^2+4\left(a^2+1\right)b^2}}\right)}{2\,b^2}-\frac{\sqrt{b^2x^2+2\,abx+a^2+1}\,ax}{2\,b}-\frac{\left(a^2+1\right)a\operatorname{arsinh}\left(\frac{2\left(b^2x+ab\right)}{\sqrt{-4\,a^2b^2+4\left(a^2+1\right)b^2}}\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))*x,x, algorithm="maxima")
```

```
[Out] 1/3*b*x^3 + 1/2*a*x^2 + 1/2*a^3*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4
*(a^2 + 1)*b^2))/b^2 - 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a*x/b - 1/2*(a
^2 + 1)*a*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^2 -
 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^2/b^2 + 1/3*(b^2*x^2 + 2*a*b*x + a
^2 + 1)^(3/2)/b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\left(a+\sqrt{{\left(a+b\,x\right)}^2+1}+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + ((a + b*x)^2 + 1)^(1/2) + b*x),x)
```

```
[Out] int(x*(a + ((a + b*x)^2 + 1)^(1/2) + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+bx+\sqrt{a^2+2abx+b^2x^2+1}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)**2)**(1/2))*x,x)
```

```
[Out] Integral(x*(a + b*x + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x)
```

3.352 $\int e^{\sinh^{-1}(a+bx)}\,dx$

Optimal. Leaf size=31

$$\frac{\sinh^{-1}(a+bx)}{2b}+\frac{e^{2\sinh^{-1}(a+bx)}}{4b}$$

```
[Out] 1/4*(b*x+a+(1+(b*x+a)^2)^(1/2))^2/b+1/2*arcsinh(b*x+a)/b
```

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5896, 2282, 12, 14}

$$\frac{\sinh^{-1}(a+bx)}{2b}+\frac{e^{2\sinh^{-1}(a+bx)}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSinh[a + b*x],x]
```

```
[Out] E^(2*ArcSinh[a + b*x])/(4*b) + ArcSinh[a + b*x]/(2*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 5896

```
Int[(f_)^(ArcSinh[(a_.) + (b_.)*(x_)]^(n_.)*(c_.)), x_Symbol] :> Dist[1/b,
Subst[Int[f^(c*x^n)*Cosh[x], x], x, ArcSinh[a + b*x]], x] /; FreeQ[{a, b, c
, f}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int e^{\sinh^{-1}(a+bx)}\,dx = \frac{\text{Subst}\left(\int e^x\cosh(x)\,dx, x, \sinh^{-1}(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{1+x^2}{2x}\,dx, x, e^{\sinh^{-1}(a+bx)}\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{1+x^2}{x}\,dx, x, e^{\sinh^{-1}(a+bx)}\right)}{2b}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{x}+x\right)\,dx, x, e^{\sinh^{-1}(a+bx)}\right)}{2b}$$
$$= \frac{e^{2\sinh^{-1}(a+bx)}}{4b} + \frac{\sinh^{-1}(a+bx)}{2b}$$

Mathematica [A] time = 0.04, size = 46, normalized size = 1.48

$$\frac{(a+bx)\left(\sqrt{a^2+2abx+b^2x^2+1}+a+bx\right)+\sinh^{-1}(a+bx)}{2b}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSinh[a + b*x],x]
```

```
[Out] ((a + b*x)*(a + b*x + Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]) + ArcSinh[a + b*x]
)/(2*b)
```

fricas [B] time = 0.60, size = 73, normalized size = 2.35

$$\frac{b^2x^2+2\,abx+\sqrt{b^2x^2+2\,abx+a^2+1}\,(bx+a)-\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x+a+(1+(b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2 + 2*a*b*x + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b*x + a) - log(
-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)))/b
```

giac [B] time = 0.29, size = 80, normalized size = 2.58

$$\frac{1}{2}\,bx^2+ax+\frac{1}{2}\,\sqrt{b^2x^2+2\,abx+a^2+1}\left(x+\frac{a}{b}\right)-\frac{\log\left(-ab-\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)|b|\right)}{2\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x+a+(1+(b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*b*x^2 + a*x + 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(x + a/b) - 1/2*log
(-a*b - (x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*abs(b))/abs(b)
```

maple [B] time = 0.00, size = 89, normalized size = 2.87

$$ax+\frac{\left(2b^2x+2ab\right)\sqrt{b^2x^2+2abx+a^2+1}}{4b^2}+\frac{\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{2\sqrt{b^2}}+\frac{b\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b*x+a+(1+(b*x+a)^2)^(1/2),x)
```

```
[Out] a*x+1/4*(2*b^2*x+2*a*b)/b^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+1/2*ln((b^2*x+a*b
)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+1/2*b*x^2
```

maxima [B] time = 0.46, size = 141, normalized size = 4.55

$$\frac{1}{2}\,bx^2+ax-\frac{a^2\,\mathrm{arsinh}\left(\frac{2\left(b^2x+ab\right)}{\sqrt{-4\,a^2b^2+4\left(a^2+1\right)b^2}}\right)}{2\,b}+\frac{1}{2}\,\sqrt{b^2x^2+2\,abx+a^2+1}\,x+\frac{\left(a^2+1\right)\mathrm{arsinh}\left(\frac{2\left(b^2x+ab\right)}{\sqrt{-4\,a^2b^2+4\left(a^2+1\right)b^2}}\right)}{2\,b}+\sqrt{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x+a+(1+(b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*b*x^2 + a*x - 1/2*a^2*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2
+ 1)*b^2))/b + 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*x + 1/2*(a^2 + 1)*arcs
inh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b + 1/2*sqrt(b^2*x^
2 + 2*a*b*x + a^2 + 1)*a/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int a+\sqrt{(a+b\,x)^2+1}+b\,x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + ((a + b*x)^2 + 1)^(1/2) + b*x,x)
```

```
[Out] int(a + ((a + b*x)^2 + 1)^(1/2) + b*x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+bx+\sqrt{(a+bx)^2+1}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x+a+(1+(b*x+a)**2)**(1/2),x)
```

```
[Out] Integral(a + b*x + sqrt((a + b*x)**2 + 1), x)
```

3.353 $\int \frac{e^{\sinh^{-1}(a+bx)}}{x} dx$

Optimal. Leaf size=89

$$\sqrt{a^2+2abx+b^2x^2+1}-\sqrt{a^2+1}\tanh^{-1}\left(\frac{a^2+abx+1}{\sqrt{a^2+1}\sqrt{a^2+2abx+b^2x^2+1}}\right)+a\sinh^{-1}(a+bx)+a\log(x)+bx$$

```
[Out] b*x+a*arcsinh(b*x+a)+a*ln(x)-arctanh((a*b*x+a^2+1)/(a^2+1)^(1/2)/(b^2*x^2+2
*a*b*x+a^2+1)^(1/2))*(a^2+1)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {5907, 14, 734, 843, 619, 215, 724, 206}

$$\sqrt{a^2+2abx+b^2x^2+1}-\sqrt{a^2+1}\tanh^{-1}\left(\frac{a^2+abx+1}{\sqrt{a^2+1}\sqrt{a^2+2abx+b^2x^2+1}}\right)+a\sinh^{-1}(a+bx)+a\log(x)+bx$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSinh[a + b*x]/x,x]
```

```
[Out] b*x + Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] + a*ArcSinh[a + b*x] - Sqrt[1 + a^2
]*ArcTanh[(1 + a^2 + a*b*x)/(Sqrt[1 + a^2]*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2
])] + a*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
```

```
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 5907

```
Int[E^(ArcSinh[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(u + Sqrt[1 + u^
2])^n, x] /; RationalQ[m] && IntegerQ[n] && PolynomialQ[u, x]
```

Rubi steps

$$\int \frac{e^{\sinh^{-1}(a+bx)}}{x}\,dx = \int \frac{a+bx+\sqrt{1+(a+bx)^2}}{x}\,dx$$
$$= \int \left(b+\frac{a}{x}+\frac{\sqrt{1+a^2+2abx+b^2x^2}}{x}\right)dx$$
$$= bx + a\log(x) + \int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{x}\,dx$$
$$= bx + \sqrt{1+a^2+2abx+b^2x^2} + a\log(x) - \frac{1}{2}\int \frac{-2\left(1+a^2\right)-2abx}{x\sqrt{1+a^2+2abx+b^2x^2}}\,dx$$
$$= bx + \sqrt{1+a^2+2abx+b^2x^2} + a\log(x) - \left(-1-a^2\right)\int \frac{1}{x\sqrt{1+a^2+2abx+b^2x^2}}\,dx + (ab)\int$$
$$= bx + \sqrt{1+a^2+2abx+b^2x^2} + a\log(x) - \left(2\left(1+a^2\right)\right)\mathrm{Subst}\left(\int \frac{1}{4\left(1+a^2\right)-x^2}\,dx, x, \frac{2(1}{\sqrt{1+a}}\right.$$
$$= bx + \sqrt{1+a^2+2abx+b^2x^2} + a\sinh^{-1}(a+bx) - \sqrt{1+a^2}\tanh^{-1}\left(\frac{1+a^2+abx}{\sqrt{1+a^2}\sqrt{1+a^2+2abx}}\right.$$

Mathematica [A] time = 0.08, size = 99, normalized size = 1.11

$$\sqrt{a^2+2abx+b^2x^2+1}-\sqrt{a^2+1}\log\left(\sqrt{a^2+1}\sqrt{a^2+2abx+b^2x^2+1}+a^2+abx+1\right)+\left(\sqrt{a^2+1}+a\right)\log(x)+a$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSinh[a + b*x]/x,x]
```

```
[Out] b*x + Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] + a*ArcSinh[a + b*x] + (a + Sqrt[1
+ a^2])*Log[x] - Sqrt[1 + a^2]*Log[1 + a^2 + a*b*x + Sqrt[1 + a^2]*Sqrt[1 +
 a^2 + 2*a*b*x + b^2*x^2]]
```

fricas [A] time = 0.49, size = 136, normalized size = 1.53

$$bx-a\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)+a\log(x)+\sqrt{a^2+1}\log\left(-\frac{a^2bx+a^3+\sqrt{b^2x^2+2\,abx+a^2+1}\left(a^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x,x, algorithm="fricas")
```

```
[Out] b*x - a*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + a*log(x) + sqrt
(a^2 + 1)*log(-(a^2*b*x + a^3 + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 - sq
rt(a^2 + 1)*a + 1) - (a*b*x + a^2 + 1)*sqrt(a^2 + 1) + a)/x) + sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1)
```

giac [A] time = 0.42, size = 158, normalized size = 1.78

$$bx - \frac{ab\log\left(-ab - \left(x|b| - \sqrt{b^2x^2 + 2\,abx + a^2 + 1}\right)|b|\right)}{|b|} + a\log\left(|x|\right) + \sqrt{a^2+1}\,\log\left(\frac{\left|-2\,x|b| + 2\,\sqrt{b^2x^2 + 2\,abx + a\right.}{\left|-2\,x|b| + 2\,\sqrt{b^2x^2 + 2\,abx + a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x,x, algorithm="giac")
```

```
[Out] b*x - a*b*log(-a*b - (x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*abs(b))
/abs(b) + a*log(abs(x)) + sqrt(a^2 + 1)*log(abs(-2*x*abs(b) + 2*sqrt(b^2*x^
2 + 2*a*b*x + a^2 + 1) - 2*sqrt(a^2 + 1))/abs(-2*x*abs(b) + 2*sqrt(b^2*x^2
+ 2*a*b*x + a^2 + 1) + 2*sqrt(a^2 + 1))) + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1
)
```

maple [A] time = 0.00, size = 126, normalized size = 1.42

$$\sqrt{b^2x^2 + 2abx + a^2 + 1} + \frac{ab\ln\left(\frac{b^2x+ab}{\sqrt{b^2}} + \sqrt{b^2x^2 + 2abx + a^2 + 1}\right)}{\sqrt{b^2}} - \sqrt{a^2+1}\,\ln\left(\frac{2a^2 + 2 + 2abx + 2\sqrt{a^2+1}\,\sqrt{b}}{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+(1+(b*x+a)^2)^(1/2))/x,x)
```

```
[Out] (b^2*x^2+2*a*b*x+a^2+1)^(1/2)+a*b*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b
*x+a^2+1)^(1/2))/(b^2)^(1/2)-(a^2+1)^(1/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1
/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)+b*x+a*ln(x)
```

maxima [A] time = 0.34, size = 160, normalized size = 1.80

$$bx + a\,\mathrm{arsinh}\left(\frac{2\left(b^2x + ab\right)}{\sqrt{-4\,a^2b^2 + 4\left(a^2+1\right)b^2}}\right) + a\log(x) - \sqrt{a^2+1}\,\mathrm{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^2b^2 + 4\left(a^2+1\right)b^2}\,|x|} + \frac{}{\sqrt{-4\,a^2b^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x,x, algorithm="maxima")
```

```
[Out] b*x + a*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)) + a*log
(x) - sqrt(a^2 + 1)*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs
(x)) + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b
^2 + 4*(a^2 + 1)*b^2)*abs(x))) + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)
```

mupad [B] time = 1.04, size = 180, normalized size = 2.02

$$\sqrt{a^2 + 2\,a\,b\,x + b^2\,x^2 + 1} + b\,x + a\,\ln(x) - \frac{\ln\left(a\,b + \frac{a^2+1}{x} + \frac{\sqrt{a^2+1}\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}}{x}\right)}{\sqrt{a^2+1}} - \frac{a^2\,\ln\left(a\,b + \frac{a^2+1}{x} + \frac{\sqrt{a^2+1}\,\sqrt{a}}{}\right.}{\sqrt{a^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + ((a + b*x)^2 + 1)^(1/2) + b*x)/x,x)
```

```
[Out] (a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2) + b*x + a*log(x) - log(a*b + (a^2 + 1)/
x + ((a^2 + 1)^(1/2)*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2))/x)/(a^2 + 1)^(1/2
) - (a^2*log(a*b + (a^2 + 1)/x + ((a^2 + 1)^(1/2)*(a^2 + b^2*x^2 + 2*a*b*x
+ 1)^(1/2))/x))/(a^2 + 1)^(1/2) + (a*b*log((a^2 + b^2*x^2 + 2*a*b*x + 1)^(1
/2) + (a*b + b^2*x)/(b^2)^(1/2)))/(b^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + bx + \sqrt{a^2 + 2abx + b^2x^2 + 1}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)**2)**(1/2))/x,x)
```

```
[Out] Integral((a + b*x + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1))/x, x)
```

3.354 $\int \frac{e^{\sinh^{-1}(a+bx)}}{x^2}\,dx$

Optimal. Leaf size=99

$$-\frac{\sqrt{a^2+2abx+b^2x^2+1}}{x}-\frac{ab\tanh^{-1}\left(\frac{a^2+abx+1}{\sqrt{a^2+1}\sqrt{a^2+2abx+b^2x^2+1}}\right)}{\sqrt{a^2+1}}+b\sinh^{-1}(a+bx)-\frac{a}{x}+b\log(x)$$

```
[Out] -a/x+b*arcsinh(b*x+a)+b*ln(x)-a*b*arctanh((a*b*x+a^2+1)/(a^2+1)^(1/2)/(b^2*
x^2+2*a*b*x+a^2+1)^(1/2))/(a^2+1)^(1/2)-(b^2*x^2+2*a*b*x+a^2+1)^(1/2)/x
```

Rubi [A] time = 0.11, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5907, 14, 732, 843, 619, 215, 724, 206}

$$-\frac{\sqrt{a^2+2abx+b^2x^2+1}}{x}-\frac{ab\tanh^{-1}\left(\frac{a^2+abx+1}{\sqrt{a^2+1}\sqrt{a^2+2abx+b^2x^2+1}}\right)}{\sqrt{a^2+1}}+b\sinh^{-1}(a+bx)-\frac{a}{x}+b\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSinh[a + b*x]/x^2,x]
```

```
[Out] -(a/x) - Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]/x + b*ArcSinh[a + b*x] - (a*b*Ar
cTanh[(1 + a^2 + a*b*x)/(Sqrt[1 + a^2]*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2])])
/Sqrt[1 + a^2] + b*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 5907

```
Int[E^(ArcSinh[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(u + Sqrt[1 + u^
2])^n, x] /; RationalQ[m] && IntegerQ[n] && PolynomialQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\sinh^{-1}(a+bx)}}{x^2}\,dx &= \int \frac{a+bx+\sqrt{1+(a+bx)^2}}{x^2}\,dx\\ &= \int \left(\frac{a}{x^2}+\frac{b}{x}+\frac{\sqrt{1+a^2+2abx+b^2x^2}}{x^2}\right)dx\\ &= -\frac{a}{x}+b\log(x)+\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{x^2}\,dx\\ &= -\frac{a}{x}-\frac{\sqrt{1+a^2+2abx+b^2x^2}}{x}+b\log(x)+\frac{1}{2}\int \frac{2ab+2b^2x}{x\sqrt{1+a^2+2abx+b^2x^2}}\,dx\\ &= -\frac{a}{x}-\frac{\sqrt{1+a^2+2abx+b^2x^2}}{x}+b\log(x)+(ab)\int \frac{1}{x\sqrt{1+a^2+2abx+b^2x^2}}\,dx+b^2\int \frac{}{\sqrt{1+}}\\ &= -\frac{a}{x}-\frac{\sqrt{1+a^2+2abx+b^2x^2}}{x}+b\log(x)+\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{4b^2}}}\,dx,x,2ab+2b^2x\right)-(2ab)\\ &= -\frac{a}{x}-\frac{\sqrt{1+a^2+2abx+b^2x^2}}{x}+b\sinh^{-1}(a+bx)-\frac{ab\tanh^{-1}\left(\frac{1+a^2+abx}{\sqrt{1+a^2}\sqrt{1+a^2+2abx+b^2x^2}}\right)}{\sqrt{1+a^2}}+b\log\end{aligned}$$

Mathematica [A] time = 0.17, size = 110, normalized size = 1.11

$$b\sinh^{-1}(a+bx)-\frac{\sqrt{a^2+2abx+b^2x^2+1}+\frac{abx\log\left(\sqrt{a^2+1}\sqrt{a^2+2abx+b^2x^2+1}+a^2+abx+1\right)}{\sqrt{a^2+1}}+\left(-\frac{a}{\sqrt{a^2+1}}-1\right)bx\log(x)+a}{x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSinh[a + b*x]/x^2,x]
```

```
[Out] b*ArcSinh[a + b*x] - (a + Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] + (-1 - a/Sqrt[
1 + a^2])*b*x*Log[x] + (a*b*x*Log[1 + a^2 + a*b*x + Sqrt[1 + a^2]*Sqrt[1 +
a^2 + 2*a*b*x + b^2*x^2]])/Sqrt[1 + a^2])/x
```

fricas [B] time = 0.69, size = 183, normalized size = 1.85

$$\frac{\sqrt{a^2+1}\,abx\log\left(-\frac{a^2bx+a^3+\sqrt{b^2x^2+2\,abx+a^2+1}\left(a^2-\sqrt{a^2+1}\,a+1\right)-\left(abx+a^2+1\right)\sqrt{a^2+1}+a}{x}\right)-\left(a^2+1\right)bx\log\left(-bx-a+\sqrt{b^2x^2\cdots}\right.}{\left(a^2+1\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x^2,x, algorithm="fricas")
```

```
[Out] (sqrt(a^2 + 1)*a*b*x*log(-(a^2*b*x + a^3 + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1
)*(a^2 - sqrt(a^2 + 1)*a + 1) - (a*b*x + a^2 + 1)*sqrt(a^2 + 1) + a)/x) - (
a^2 + 1)*b*x*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + (a^2 + 1)*
b*x*log(x) - a^3 - (a^2 + 1)*b*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 +
 1) - a)/((a^2 + 1)*x)
```

giac [B] time = 0.94, size = 234, normalized size = 2.36

$$\frac{ab\log\left(\frac{\left|-2\,x|b|+2\,\sqrt{b^2x^2+2\,abx+a^2+1}-2\,\sqrt{a^2+1}\right|}{\left|-2\,x|b|+2\,\sqrt{b^2x^2+2\,abx+a^2+1}+2\,\sqrt{a^2+1}\right|}\right)}{\sqrt{a^2+1}}-\frac{b^2\log\left(-ab-\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)|b|\right)}{|b|}+b\log\left(|x|\right)-\frac{a}{x}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x^2,x, algorithm="giac")
```

```
[Out] a*b*log(abs(-2*x*abs(b) + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*sqrt(a^2
+ 1))/abs(-2*x*abs(b) + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 2*sqrt(a^2 +
1)))/sqrt(a^2 + 1) - b^2*log(-a*b - (x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^
2 + 1))*abs(b))/abs(b) + b*log(abs(x)) - a/x + 2*((x*abs(b) - sqrt(b^2*x^2
+ 2*a*b*x + a^2 + 1))*a*b^5 + a^2*b^4*abs(b) + b^4*abs(b))/(((x*abs(b) - sq
rt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2 - a^2 - 1)*b^4)
```

maple [B] time = 0.01, size = 267, normalized size = 2.70

$$-\frac{\left(b^2x^2+2abx+a^2+1\right)^{\frac{3}{2}}}{\left(a^2+1\right)x}+\frac{2ab\sqrt{b^2x^2+2abx+a^2+1}}{a^2+1}+\frac{a^2b^2\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{\left(a^2+1\right)\sqrt{b^2}}-\frac{ab\ln\left(\frac{2a^2\cdots}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+(1+(b*x+a)^2)^(1/2))/x^2,x)
```

```
[Out] -1/(a^2+1)/x*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)+2*a*b/(a^2+1)*(b^2*x^2+2*a*b*x+a
^2+1)^(1/2)+a^2*b^2/(a^2+1)*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2
+1)^(1/2))/(b^2)^(1/2)-a*b/(a^2+1)^(1/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2
)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)+b^2/(a^2+1)*(b^2*x^2+2*a*b*x+a^2+1)^(1/
2)*x+b^2/(a^2+1)*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/
(b^2)^(1/2)+b*ln(x)-a/x
```

maxima [A] time = 0.36, size = 170, normalized size = 1.72

$$-\frac{ab\,\mathrm{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^2b^2+4\,\left(a^2+1\right)b^2}\,|x|}+\frac{2\,a^2}{\sqrt{-4\,a^2b^2+4\,\left(a^2+1\right)b^2}\,|x|}+\frac{2}{\sqrt{-4\,a^2b^2+4\,\left(a^2+1\right)b^2}\,|x|}\right)}{\sqrt{a^2+1}}+b\,\mathrm{arsinh}\left(\frac{2\left(b^2x+ab\right)}{\sqrt{-4\,a^2b^2+4\,\left(a^2+\cdots\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x^2,x, algorithm="maxima")
```

```
[Out] -a*b*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2*a^2/(s
qrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1
)*b^2)*abs(x)))/sqrt(a^2 + 1) + b*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 +
 4*(a^2 + 1)*b^2)) + b*log(x) - a/x - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/x
```

mupad [B] time = 1.26, size = 269, normalized size = 2.72

$$b\ln(x) - \frac{a}{x} + \ln\left(\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1} + \frac{x\,b^2+a\,b}{\sqrt{b^2}}\right)\sqrt{b^2} - \frac{\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}}{x\,\left(a^2+1\right)} + \frac{a^3\,b\,\mathrm{atanh}\left(\frac{a^2+b\,x}{\sqrt{a^2+1}\,\sqrt{a^2+2\,a}}\right.}{\left(a^2+1\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + ((a + b*x)^2 + 1)^(1/2) + b*x)/x^2,x)
```

```
[Out] b*log(x) - a/x + log((a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2) + (a*b + b^2*x)/(b
^2)^(1/2))*(b^2)^(1/2) - (a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2)/(x*(a^2 + 1))
+ (a^3*b*atanh((a^2 + a*b*x + 1)/((a^2 + 1)^(1/2)*(a^2 + b^2*x^2 + 2*a*b*x
+ 1)^(1/2))))/(a^2 + 1)^(3/2) - (a^2*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2))/(
x*(a^2 + 1)) + (a*b*atanh((a^2 + a*b*x + 1)/((a^2 + 1)^(1/2)*(a^2 + b^2*x^2
 + 2*a*b*x + 1)^(1/2))))/(a^2 + 1)^(3/2) - (2*a*b*log(a*b + (a^2 + 1)/x + (
(a^2 + 1)^(1/2)*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2))/x))/(a^2 + 1)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + bx + \sqrt{a^2 + 2abx + b^2x^2 + 1}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)**2)**(1/2))/x**2,x)
```

```
[Out] Integral((a + b*x + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1))/x**2, x)
```

3.355 $\int \frac{e^{\sinh^{-1}(a+bx)}}{x^3} dx$

Optimal. Leaf size=116

$$-\frac{\left(a^2+abx+1\right)\sqrt{a^2+2abx+b^2x^2+1}}{2\left(a^2+1\right)x^2}-\frac{b^2\tanh^{-1}\left(\frac{a^2+abx+1}{\sqrt{a^2+1}\sqrt{a^2+2abx+b^2x^2+1}}\right)}{2\left(a^2+1\right)^{3/2}}-\frac{a}{2x^2}-\frac{b}{x}$$

```
[Out] -1/2*a/x^2-b/x-1/2*b^2*arctanh((a*b*x+a^2+1)/(a^2+1)^(1/2)/(b^2*x^2+2*a*b*x
+a^2+1)^(1/2))/(a^2+1)^(3/2)-1/2*(a*b*x+a^2+1)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2
)/(a^2+1)/x^2
```

Rubi [A] time = 0.09, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5907, 14, 720, 724, 206}

$$-\frac{\left(a^2+abx+1\right)\sqrt{a^2+2abx+b^2x^2+1}}{2\left(a^2+1\right)x^2}-\frac{b^2\tanh^{-1}\left(\frac{a^2+abx+1}{\sqrt{a^2+1}\sqrt{a^2+2abx+b^2x^2+1}}\right)}{2\left(a^2+1\right)^{3/2}}-\frac{a}{2x^2}-\frac{b}{x}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSinh[a + b*x]/x^3,x]
```

```
[Out] -a/(2*x^2) - b/x - ((1 + a^2 + a*b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2])/(2
*(1 + a^2)*x^2) - (b^2*ArcTanh[(1 + a^2 + a*b*x)/(Sqrt[1 + a^2]*Sqrt[1 + a^
2 + 2*a*b*x + b^2*x^2])])/(2*(1 + a^2)^(3/2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 5907

```
Int[E^(ArcSinh[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(u + Sqrt[1 + u^
2])^n, x] /; RationalQ[m] && IntegerQ[n] && PolynomialQ[u, x]
```

Rubi steps

$$\int \frac{e^{\sinh^{-1}(a+bx)}}{x^3}\,dx = \int \frac{a+bx+\sqrt{1+(a+bx)^2}}{x^3}\,dx$$

$$= \int \left(\frac{a}{x^3}+\frac{b}{x^2}+\frac{\sqrt{1+a^2+2abx+b^2x^2}}{x^3}\right)dx$$

$$= -\frac{a}{2x^2}-\frac{b}{x}+\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{x^3}\,dx$$

$$= -\frac{a}{2x^2}-\frac{b}{x}-\frac{\left(1+a^2+abx\right)\sqrt{1+a^2+2abx+b^2x^2}}{2\left(1+a^2\right)x^2}+\frac{b^2\int \frac{1}{x\sqrt{1+a^2+2abx+b^2x^2}}\,dx}{2\left(1+a^2\right)}$$

$$= -\frac{a}{2x^2}-\frac{b}{x}-\frac{\left(1+a^2+abx\right)\sqrt{1+a^2+2abx+b^2x^2}}{2\left(1+a^2\right)x^2}-\frac{b^2\,\text{Subst}\left(\int \frac{1}{4\left(1+a^2\right)-x^2}\,dx, x, \frac{2\left(1+a^2\right)+2\ldots}{\sqrt{1+a^2+2abx\ldots}}\right.}{1+a^2}$$

$$= -\frac{a}{2x^2}-\frac{b}{x}-\frac{\left(1+a^2+abx\right)\sqrt{1+a^2+2abx+b^2x^2}}{2\left(1+a^2\right)x^2}-\frac{b^2\tanh^{-1}\left(\frac{1+a^2+abx}{\sqrt{1+a^2}\sqrt{1+a^2+2abx+b^2x^2}}\right)}{2\left(1+a^2\right)^{3/2}}$$

Mathematica [A] time = 0.22, size = 129, normalized size = 1.11

$$\frac{1}{2}\left(-\frac{\left(a^2+abx+1\right)\sqrt{a^2+2abx+b^2x^2+1}}{\left(a^2+1\right)x^2}-\frac{b^2\log\left(\sqrt{a^2+1}\sqrt{a^2+2abx+b^2x^2+1}+a^2+abx+1\right)}{\left(a^2+1\right)^{3/2}}+\frac{b^2\log(\ldots}{\left(a^2+1\right)\ldots}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSinh[a + b*x]/x^3,x]
```

```
[Out] (-(a/x^2) - (2*b)/x - ((1 + a^2 + a*b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2])
/((1 + a^2)*x^2) + (b^2*Log[x])/(1 + a^2)^(3/2) - (b^2*Log[1 + a^2 + a*b*x
+ Sqrt[1 + a^2]*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]])/(1 + a^2)^(3/2))/2
```

fricas [A] time = 0.71, size = 181, normalized size = 1.56

$$\frac{\sqrt{a^2+1}\,b^2x^2\log\left(-\frac{a^2bx+a^3+\sqrt{b^2x^2+2\,abx+a^2+1}\left(a^2-\sqrt{a^2+1}\,a+1\right)-\left(abx+a^2+1\right)\sqrt{a^2+1}+a}{x}\right)-a^5-\left(a^3+a\right)b^2x^2-2\,a^3-2\left(a^4+2\ldots\right.}{2\left(a^4+2\,a^2+1\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(a^2 + 1)*b^2*x^2*log(-(a^2*b*x + a^3 + sqrt(b^2*x^2 + 2*a*b*x + a
^2 + 1)*(a^2 - sqrt(a^2 + 1)*a + 1) - (a*b*x + a^2 + 1)*sqrt(a^2 + 1) + a)/
x) - a^5 - (a^3 + a)*b^2*x^2 - 2*a^3 - 2*(a^4 + 2*a^2 + 1)*b*x - (a^4 + (a^
3 + a)*b*x + 2*a^2 + 1)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) - a)/((a^4 + 2*a^
2 + 1)*x^2)
```

giac [B] time = 0.38, size = 384, normalized size = 3.31

$$\frac{b^2\log\left(\frac{\left|-2\,x|b|+2\,\sqrt{b^2x^2+2\,abx+a^2+1}-2\,\sqrt{a^2+1}\right|}{\left|-2\,x|b|+2\,\sqrt{b^2x^2+2\,abx+a^2+1}+2\,\sqrt{a^2+1}\right|}\right)}{2\,(a^2+1)^{\frac{3}{2}}}-\frac{2\,bx+a}{2\,x^2}+\frac{2\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)^3a^2b^2+2\left(x|b|-\sqrt{b^2x^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x^3,x, algorithm="giac")
```

```
[Out] 1/2*b^2*log(abs(-2*x*abs(b) + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*sqrt(
a^2 + 1))/abs(-2*x*abs(b) + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 2*sqrt(a^
2 + 1)))/(a^2 + 1)^(3/2) - 1/2*(2*b*x + a)/x^2 + (2*(x*abs(b) - sqrt(b^2*x^
2 + 2*a*b*x + a^2 + 1))^3*a^2*b^2 + 2*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x +
a^2 + 1))*a^4*b^2 + 4*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2*a^3*
b*abs(b) + (x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^3*b^2 + 3*(x*abs(
b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*a^2*b^2 + 4*(x*abs(b) - sqrt(b^2*x^
2 + 2*a*b*x + a^2 + 1))^2*a*b*abs(b) + (x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x +
 a^2 + 1))*b^2)/(((x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2 - a^2 -
1)^2*(a^2 + 1))
```

maple [B] time = 0.01, size = 457, normalized size = 3.94

$$-\frac{\left(b^2x^2+2abx+a^2+1\right)^{\frac{3}{2}}}{2\left(a^2+1\right)x^2}+\frac{ab\left(b^2x^2+2abx+a^2+1\right)^{\frac{3}{2}}}{2\left(a^2+1\right)^2x}-\frac{a^2b^2\sqrt{b^2x^2+2abx+a^2+1}}{\left(a^2+1\right)^2}-\frac{a^3b^3\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2}\right.}{2\left(a^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+(1+(b*x+a)^2)^(1/2))/x^3,x)
```

```
[Out] -1/2/(a^2+1)/x^2*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)+1/2*a*b/(a^2+1)^2/x*(b^2*x^2
+2*a*b*x+a^2+1)^(3/2)-a^2*b^2/(a^2+1)^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-1/2*a
^3*b^3/(a^2+1)^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/
(b^2)^(1/2)+1/2*a^2*b^2/(a^2+1)^(3/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(
b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)-1/2*a*b^3/(a^2+1)^2*(b^2*x^2+2*a*b*x+a^2+1
)^(1/2)*x-1/2*a*b^3/(a^2+1)^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a
^2+1)^(1/2))/(b^2)^(1/2)+1/2*b^2/(a^2+1)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+1/2*
b^3/(a^2+1)*a*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^
2)^(1/2)-1/2*b^2/(a^2+1)^(1/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2
+2*a*b*x+a^2+1)^(1/2))/x)-b/x-1/2*a/x^2
```

maxima [B] time = 0.96, size = 313, normalized size = 2.70

$$\frac{a^2b^2\operatorname{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}\,|x|}+\frac{2\,a^2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}\,|x|}+\frac{2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}\,|x|}\right)}{2\,(a^2+1)^{\frac{3}{2}}}-\frac{b^2\operatorname{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}\,|x|}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*b^2*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2
*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(
a^2 + 1)*b^2)*abs(x)))/(a^2 + 1)^(3/2) - 1/2*b^2*arcsinh(2*a*b*x/(sqrt(-4*a
^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^
2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)))/sqrt(a^2 + 1) +
```

```
 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^2/(a^2 + 1) + 1/2*sqrt(b^2*x^2 + 2
*a*b*x + a^2 + 1)*a*b/((a^2 + 1)*x) - b/x - 1/2*a/x^2 - 1/2*(b^2*x^2 + 2*a*
b*x + a^2 + 1)^(3/2)/((a^2 + 1)*x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+\sqrt{(a+b\,x)^2+1}+b\,x}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + ((a + b*x)^2 + 1)^(1/2) + b*x)/x^3,x)
```

```
[Out] int((a + ((a + b*x)^2 + 1)^(1/2) + b*x)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx+\sqrt{a^2+2abx+b^2x^2+1}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)**2)**(1/2))/x**3,x)
```

```
[Out] Integral((a + b*x + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1))/x**3, x)
```

3.356 $\int \frac{e^{\sinh^{-1}(a+bx)}}{x^4} dx$

Optimal. Leaf size=156

$$\frac{ab\left(a^2+abx+1\right)\sqrt{a^2+2abx+b^2x^2+1}}{2\left(a^2+1\right)^2x^2}-\frac{\left(a^2+2abx+b^2x^2+1\right)^{3/2}}{3\left(a^2+1\right)x^3}+\frac{ab^3\tanh^{-1}\left(\frac{a^2+abx+1}{\sqrt{a^2+1}\sqrt{a^2+2abx+b^2x^2+1}}\right)}{2\left(a^2+1\right)^{5/2}}-\frac{a}{3x^3}$$

```
[Out] -1/3*a/x^3-1/2*b/x^2-1/3*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/(a^2+1)/x^3+1/2*a*b^
3*arctanh((a*b*x+a^2+1)/(a^2+1)^(1/2)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(a^2+1
)^(5/2)+1/2*a*b*(a*b*x+a^2+1)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)/(a^2+1)^2/x^2
```

Rubi [A] time = 0.11, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5907, 14, 730, 720, 724, 206}

$$\frac{ab\left(a^2+abx+1\right)\sqrt{a^2+2abx+b^2x^2+1}}{2\left(a^2+1\right)^2x^2}-\frac{\left(a^2+2abx+b^2x^2+1\right)^{3/2}}{3\left(a^2+1\right)x^3}+\frac{ab^3\tanh^{-1}\left(\frac{a^2+abx+1}{\sqrt{a^2+1}\sqrt{a^2+2abx+b^2x^2+1}}\right)}{2\left(a^2+1\right)^{5/2}}-\frac{a}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSinh[a + b*x]/x^4,x]
```

```
[Out] -a/(3*x^3) - b/(2*x^2) + (a*b*(1 + a^2 + a*b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^
2*x^2])/(2*(1 + a^2)^2*x^2) - (1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)/(3*(1 + a
^2)*x^3) + (a*b^3*ArcTanh[(1 + a^2 + a*b*x)/(Sqrt[1 + a^2]*Sqrt[1 + a^2 + 2
*a*b*x + b^2*x^2])])/(2*(1 + a^2)^(5/2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 730

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 5907

```
Int[E^(ArcSinh[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(u + Sqrt[1 + u^
2])^n, x] /; RationalQ[m] && IntegerQ[n] && PolynomialQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\sinh^{-1}(a+bx)}}{x^4}\,dx &= \int \frac{a+bx+\sqrt{1+(a+bx)^2}}{x^4}\,dx\\ &= \int \left(\frac{a}{x^4}+\frac{b}{x^3}+\frac{\sqrt{1+a^2+2abx+b^2x^2}}{x^4}\right)dx\\ &= -\frac{a}{3x^3}-\frac{b}{2x^2}+\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{x^4}\,dx\\ &= -\frac{a}{3x^3}-\frac{b}{2x^2}-\frac{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}{3\left(1+a^2\right)x^3}-\frac{(ab)\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{x^3}\,dx}{1+a^2}\\ &= -\frac{a}{3x^3}-\frac{b}{2x^2}+\frac{ab\left(1+a^2+abx\right)\sqrt{1+a^2+2abx+b^2x^2}}{2\left(1+a^2\right)^2x^2}-\frac{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}{3\left(1+a^2\right)x^3}-\frac{\left(ab^3\right.}{}\\ &= -\frac{a}{3x^3}-\frac{b}{2x^2}+\frac{ab\left(1+a^2+abx\right)\sqrt{1+a^2+2abx+b^2x^2}}{2\left(1+a^2\right)^2x^2}-\frac{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}{3\left(1+a^2\right)x^3}+\frac{\left(ab^3\right.}{}\\ &= -\frac{a}{3x^3}-\frac{b}{2x^2}+\frac{ab\left(1+a^2+abx\right)\sqrt{1+a^2+2abx+b^2x^2}}{2\left(1+a^2\right)^2x^2}-\frac{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}{3\left(1+a^2\right)x^3}+\frac{ab^3\,\mathrm{t}}{}\end{aligned}$$

Mathematica [A] time = 0.12, size = 162, normalized size = 1.04

$$\frac{1}{6}\left(-\frac{3ab^3\log(x)}{\left(a^2+1\right)^{5/2}}+\frac{3ab^3\log\left(\sqrt{a^2+1}\sqrt{a^2+2abx+b^2x^2+1}+a^2+abx+1\right)}{\left(a^2+1\right)^{5/2}}-\frac{\sqrt{a^2+2abx+b^2x^2+1}\left(2a^4+a\right.}{\left(a^2\right.}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSinh[a + b*x]/x^4,x]
```

```
[Out] ((-2*a)/x^3 - (3*b)/x^2 - (Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(2 + 2*a^4 + a
*b*x + a^3*b*x + 2*b^2*x^2 + a^2*(4 - b^2*x^2)))/((1 + a^2)^2*x^3) - (3*a*b
^3*Log[x])/(1 + a^2)^(5/2) + (3*a*b^3*Log[1 + a^2 + a*b*x + Sqrt[1 + a^2]*S
qrt[1 + a^2 + 2*a*b*x + b^2*x^2]])/(1 + a^2)^(5/2))/6
```

fricas [A] time = 0.51, size = 230, normalized size = 1.47

$$\frac{3\sqrt{a^2+1}\,ab^3x^3\log\left(-\frac{a^2bx+a^3+\sqrt{b^2x^2+2\,abx+a^2+1}\left(a^2+\sqrt{a^2+1}\,a+1\right)+\left(abx+a^2+1\right)\sqrt{a^2+1}+a}{x}\right)-2\,a^7+\left(a^4-a^2-2\right)b^3x^3-6\,a^5-}{6\left(a^6\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(3*sqrt(a^2 + 1)*a*b^3*x^3*log(-(a^2*b*x + a^3 + sqrt(b^2*x^2 + 2*a*b*x
 + a^2 + 1)*(a^2 + sqrt(a^2 + 1)*a + 1) + (a*b*x + a^2 + 1)*sqrt(a^2 + 1) +
 a)/x) - 2*a^7 + (a^4 - a^2 - 2)*b^3*x^3 - 6*a^5 - 6*a^3 - 3*(a^6 + 3*a^4 +
 3*a^2 + 1)*b*x - (2*a^6 - (a^4 - a^2 - 2)*b^2*x^2 + 6*a^4 + (a^5 + 2*a^3 +
 a)*b*x + 6*a^2 + 2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*a)/((a^6 + 3*a^4
 + 3*a^2 + 1)*x^3)
```

giac [B] time = 0.53, size = 715, normalized size = 4.58

$$-\frac{ab^3\log\left(\frac{\left|-2x|b|+2\sqrt{b^2x^2+2abx+a^2+1}-2\sqrt{a^2+1}\right|}{\left|-2x|b|+2\sqrt{b^2x^2+2abx+a^2+1}+2\sqrt{a^2+1}\right|}\right)}{2\left(a^4+2a^2+1\right)\sqrt{a^2+1}}-\frac{3bx+2a}{6x^3}+\frac{20\left(x|b|-\sqrt{b^2x^2+2abx+a^2+1}\right)^3a^5b^3+12\left(x|b|-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x^4,x, algorithm="giac")
```

```
[Out] -1/2*a*b^3*log(abs(-2*x*abs(b) + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*sq
rt(a^2 + 1))/abs(-2*x*abs(b) + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 2*sqrt
(a^2 + 1)))/((a^4 + 2*a^2 + 1)*sqrt(a^2 + 1)) - 1/6*(3*b*x + 2*a)/x^3 + 1/3
*(20*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^3*a^5*b^3 + 12*(x*abs(b
) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*a^7*b^3 + 6*(x*abs(b) - sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1))^4*a^4*b^2*abs(b) + 24*(x*abs(b) - sqrt(b^2*x^2 + 2*a
*b*x + a^2 + 1))^2*a^6*b^2*abs(b) + 2*a^8*b^2*abs(b) + 3*(x*abs(b) - sqrt(b
^2*x^2 + 2*a*b*x + a^2 + 1))^5*a*b^3 + 32*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*
x + a^2 + 1))^3*a^3*b^3 + 33*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))
*a^5*b^3 + 12*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^4*a^2*b^2*abs(
b) + 48*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2*a^4*b^2*abs(b) + 8
*a^6*b^2*abs(b) + 12*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^3*a*b^3
 + 30*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*a^3*b^3 + 6*(x*abs(b)
- sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^4*b^2*abs(b) + 24*(x*abs(b) - sqrt(b^2
*x^2 + 2*a*b*x + a^2 + 1))^2*a^2*b^2*abs(b) + 12*a^4*b^2*abs(b) + 9*(x*abs(
b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*a*b^3 + 8*a^2*b^2*abs(b) + 2*b^2*ab
s(b))/((a^4 + 2*a^2 + 1)*((x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2
- a^2 - 1)^3)
```

maple [B] time = 0.01, size = 501, normalized size = 3.21

$$-\frac{\left(b^2x^2+2abx+a^2+1\right)^{\frac{3}{2}}}{3\left(a^2+1\right)x^3}+\frac{ab\left(b^2x^2+2abx+a^2+1\right)^{\frac{3}{2}}}{2\left(a^2+1\right)^2x^2}-\frac{a^2b^2\left(b^2x^2+2abx+a^2+1\right)^{\frac{3}{2}}}{2\left(a^2+1\right)^3x}+\frac{a^3b^3\sqrt{b^2x^2+2abx+\cdots}}{\left(a^2+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+(1+(b*x+a)^2)^(1/2))/x^4,x)
```

```
[Out] -1/3*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/(a^2+1)/x^3+1/2*a*b/(a^2+1)^2/x^2*(b^2*x
^2+2*a*b*x+a^2+1)^(3/2)-1/2*a^2*b^2/(a^2+1)^3/x*(b^2*x^2+2*a*b*x+a^2+1)^(3/
2)+a^3*b^3/(a^2+1)^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+1/2*a^4*b^4/(a^2+1)^3*ln
((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-1/2*a^3
*b^3/(a^2+1)^(5/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2
+1)^(1/2))/x)+1/2*a^2*b^4/(a^2+1)^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x+1/2*a^2
*b^4/(a^2+1)^3*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b
^2)^(1/2)-1/2*a*b^3/(a^2+1)^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-1/2*a^2*b^4/(a^
2+1)^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2
```

```
)+1/2*a*b^3/(a^2+1)^(3/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*
b*x+a^2+1)^(1/2))/x)-1/3*a/x^3-1/2*b/x^2
```

maxima [B] time = 0.56, size = 352, normalized size = 2.26

$$-\frac{a^3b^3 \operatorname{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^2b^2+4\left(a^2+1\right)b^2}\,|x|}+\frac{2\,a^2}{\sqrt{-4\,a^2b^2+4\left(a^2+1\right)b^2}\,|x|}+\frac{2}{\sqrt{-4\,a^2b^2+4\left(a^2+1\right)b^2}\,|x|}\right)}{2\left(a^2+1\right)^{\frac{5}{2}}}+\frac{ab^3 \operatorname{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^2b^2+4\left(a^2+1\right)b^2}\,|x|}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x^4,x, algorithm="maxima")
```

```
[Out] -1/2*a^3*b^3*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) +
2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*
(a^2 + 1)*b^2)*abs(x)))/(a^2 + 1)^(5/2) + 1/2*a*b^3*arcsinh(2*a*b*x/(sqrt(-
4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)
*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)))/(a^2 + 1)^(3
/2) - 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a*b^3/(a^2 + 1)^2 - 1/2*sqrt(b^
2*x^2 + 2*a*b*x + a^2 + 1)*a^2*b^2/((a^2 + 1)^2*x) + 1/2*(b^2*x^2 + 2*a*b*x
 + a^2 + 1)^(3/2)*a*b/((a^2 + 1)^2*x^2) - 1/2*b/x^2 - 1/3*a/x^3 - 1/3*(b^2*
x^2 + 2*a*b*x + a^2 + 1)^(3/2)/((a^2 + 1)*x^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+\sqrt{(a+b\,x)^2+1}+b\,x}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + ((a + b*x)^2 + 1)^(1/2) + b*x)/x^4,x)
```

```
[Out] int((a + ((a + b*x)^2 + 1)^(1/2) + b*x)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx+\sqrt{a^2+2abx+b^2x^2+1}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)**2)**(1/2))/x**4,x)
```

```
[Out] Integral((a + b*x + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1))/x**4, x)
```

3.357 $\int \frac{e^{\sinh^{-1}(a+bx)}}{x^5} dx$

Optimal. Leaf size=207

$$\frac{(1-4a^2) b^2 (a^2+abx+1) \sqrt{a^2+2abx+b^2x^2+1}}{8 (a^2+1)^3 x^2}-\frac{(a^2+2abx+b^2x^2+1)^{3/2}}{4 (a^2+1) x^4}+\frac{5ab (a^2+2abx+b^2x^2+1)^{3/2}}{12 (a^2+1)^2 x^3}$$

```
[Out] -1/4*a/x^4-1/3*b/x^3-1/4*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/(a^2+1)/x^4+5/12*a*b
*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/(a^2+1)^2/x^3+1/8*(-4*a^2+1)*b^4*arctanh((a*
b*x+a^2+1)/(a^2+1)^(1/2)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(a^2+1)^(7/2)+1/8*(
-4*a^2+1)*b^2*(a*b*x+a^2+1)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)/(a^2+1)^3/x^2
```

Rubi [A] time = 0.17, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5907, 14, 744, 806, 720, 724, 206}

$$\frac{(1-4a^2) b^2 (a^2+abx+1) \sqrt{a^2+2abx+b^2x^2+1}}{8 (a^2+1)^3 x^2}+\frac{5ab (a^2+2abx+b^2x^2+1)^{3/2}}{12 (a^2+1)^2 x^3}-\frac{(a^2+2abx+b^2x^2+1)^{3/2}}{4 (a^2+1) x^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSinh[a + b*x]/x^5,x]
```

```
[Out] -a/(4*x^4) - b/(3*x^3) + ((1 - 4*a^2)*b^2*(1 + a^2 + a*b*x)*Sqrt[1 + a^2 +
2*a*b*x + b^2*x^2])/(8*(1 + a^2)^3*x^2) - (1 + a^2 + 2*a*b*x + b^2*x^2)^(3/
2)/(4*(1 + a^2)*x^4) + (5*a*b*(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2))/(12*(1 +
 a^2)^2*x^3) + ((1 - 4*a^2)*b^4*ArcTanh[(1 + a^2 + a*b*x)/(Sqrt[1 + a^2]*Sq
rt[1 + a^2 + 2*a*b*x + b^2*x^2])])/(8*(1 + a^2)^(7/2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 5907

```
Int[E^(ArcSinh[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(u + Sqrt[1 + u^
2])^n, x] /; RationalQ[m] && IntegerQ[n] && PolynomialQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\sinh^{-1}(a+bx)}}{x^5}\,dx &= \int \frac{a+bx+\sqrt{1+(a+bx)^2}}{x^5}\,dx\\
&= \int \left(\frac{a}{x^5}+\frac{b}{x^4}+\frac{\sqrt{1+a^2+2abx+b^2x^2}}{x^5}\right)dx\\
&= -\frac{a}{4x^4}-\frac{b}{3x^3}+\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{x^5}\,dx\\
&= -\frac{a}{4x^4}-\frac{b}{3x^3}-\frac{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}{4\left(1+a^2\right)x^4}-\frac{\int \frac{\left(5ab+b^2x\right)\sqrt{1+a^2+2abx+b^2x^2}}{x^4}\,dx}{4\left(1+a^2\right)}\\
&= -\frac{a}{4x^4}-\frac{b}{3x^3}-\frac{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}{4\left(1+a^2\right)x^4}+\frac{5ab\left(1+a^2+2abx+b^2x^2\right)^{3/2}}{12\left(1+a^2\right)^2x^3}-\frac{\left(\left(1-4a^2\right)b^2\right)\int}{4(1}\\
&= -\frac{a}{4x^4}-\frac{b}{3x^3}+\frac{\left(1-4a^2\right)b^2\left(1+a^2+abx\right)\sqrt{1+a^2+2abx+b^2x^2}}{8\left(1+a^2\right)^3x^2}-\frac{\left(1+a^2+2abx+b^2x^2\right)^3}{4\left(1+a^2\right)x^4}\\
&= -\frac{a}{4x^4}-\frac{b}{3x^3}+\frac{\left(1-4a^2\right)b^2\left(1+a^2+abx\right)\sqrt{1+a^2+2abx+b^2x^2}}{8\left(1+a^2\right)^3x^2}-\frac{\left(1+a^2+2abx+b^2x^2\right)^3}{4\left(1+a^2\right)x^4}\\
&= -\frac{a}{4x^4}-\frac{b}{3x^3}+\frac{\left(1-4a^2\right)b^2\left(1+a^2+abx\right)\sqrt{1+a^2+2abx+b^2x^2}}{8\left(1+a^2\right)^3x^2}-\frac{\left(1+a^2+2abx+b^2x^2\right)^3}{4\left(1+a^2\right)x^4}
\end{aligned}
$$

Mathematica [A] time = 0.81, size = 192, normalized size = 0.93

$$\frac{1}{24}\left(\frac{3(2a-1)(2a+1)b^4\log(x)}{\left(a^2+1\right)^{7/2}}-\frac{3(2a-1)(2a+1)b^4\log\left(\sqrt{a^2+1}\sqrt{a^2+2abx+b^2x^2+1}+a^2+abx+1\right)}{\left(a^2+1\right)^{7/2}}-\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSinh[a + b*x]/x^5,x]
```

```
[Out] ((-6*a)/x^4 - (8*b)/x^3 - (Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(6 + (2*a*b*x)
/(1 + a^2) - ((-3 + 2*a^2)*b^2*x^2)/(1 + a^2)^2 + (a*(-13 + 2*a^2)*b^3*x^3)
/(1 + a^2)^3))/x^4 + (3*(-1 + 2*a)*(1 + 2*a)*b^4*Log[x])/(1 + a^2)^(7/2) -
(3*(-1 + 2*a)*(1 + 2*a)*b^4*Log[1 + a^2 + a*b*x + Sqrt[1 + a^2]*Sqrt[1 + a^
2 + 2*a*b*x + b^2*x^2]])/(1 + a^2)^(7/2))/24
```

fricas [A] time = 0.69, size = 295, normalized size = 1.43

$$\frac{3\left(4a^2-1\right)\sqrt{a^2+1}\,b^4x^4\log\left(-\frac{a^2bx+a^3+\sqrt{b^2x^2+2abx+a^2+1}\left(a^2-\sqrt{a^2+1}\,a+1\right)-\left(abx+a^2+1\right)\sqrt{a^2+1}+a}{x}\right)-6a^9-\left(2a^5-11a^3-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x^5,x, algorithm="fricas")
```

```
[Out] 1/24*(3*(4*a^2 - 1)*sqrt(a^2 + 1)*b^4*x^4*log(-(a^2*b*x + a^3 + sqrt(b^2*x^
2 + 2*a*b*x + a^2 + 1)*(a^2 - sqrt(a^2 + 1)*a + 1) - (a*b*x + a^2 + 1)*sqrt
(a^2 + 1) + a)/x) - 6*a^9 - (2*a^5 - 11*a^3 - 13*a)*b^4*x^4 - 24*a^7 - 36*a
^5 - 24*a^3 - 8*(a^8 + 4*a^6 + 6*a^4 + 4*a^2 + 1)*b*x - (6*a^8 + (2*a^5 - 1
1*a^3 - 13*a)*b^3*x^3 + 24*a^6 - (2*a^6 + a^4 - 4*a^2 - 3)*b^2*x^2 + 36*a^4
 + 2*(a^7 + 3*a^5 + 3*a^3 + a)*b*x + 24*a^2 + 6)*sqrt(b^2*x^2 + 2*a*b*x + a
^2 + 1) - 6*a)/((a^8 + 4*a^6 + 6*a^4 + 4*a^2 + 1)*x^4)
```

giac [B] time = 0.49, size = 1173, normalized size = 5.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x^5,x, algorithm="giac")
```

```
[Out] 1/8*(4*a^2*b^4 - b^4)*log(abs(-2*x*abs(b) + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2
+ 1) - 2*sqrt(a^2 + 1))/abs(-2*x*abs(b) + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 +
1) + 2*sqrt(a^2 + 1)))/((a^6 + 3*a^4 + 3*a^2 + 1)*sqrt(a^2 + 1)) - 1/12*(4*
b*x + 3*a)/x^4 + 1/12*(32*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^5*
a^6*b^4 + 256*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^3*a^8*b^4 + 96
*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*a^10*b^4 + 144*(x*abs(b) -
sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^4*a^7*b^3*abs(b) + 224*(x*abs(b) - sqrt(
b^2*x^2 + 2*a*b*x + a^2 + 1))^2*a^9*b^3*abs(b) + 16*a^11*b^3*abs(b) - 12*(x
*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^7*a^2*b^4 + 140*(x*abs(b) - sq
rt(b^2*x^2 + 2*a*b*x + a^2 + 1))^5*a^4*b^4 + 716*(x*abs(b) - sqrt(b^2*x^2 +
 2*a*b*x + a^2 + 1))^3*a^6*b^4 + 372*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a
^2 + 1))*a^8*b^4 + 432*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^4*a^5
*b^3*abs(b) + 704*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2*a^7*b^3*
abs(b) + 80*a^9*b^3*abs(b) + 3*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1
))^7*b^4 + 129*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^5*a^2*b^4 + 6
85*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^3*a^4*b^4 + 543*(x*abs(b)
 - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*a^6*b^4 + 432*(x*abs(b) - sqrt(b^2*x^
```

```
2 + 2*a*b*x + a^2 + 1))^4*a^3*b^3*abs(b) + 768*(x*abs(b) - sqrt(b^2*x^2 + 2
*a*b*x + a^2 + 1))^2*a^5*b^3*abs(b) + 160*a^7*b^3*abs(b) + 21*(x*abs(b) - s
qrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^5*b^4 + 246*(x*abs(b) - sqrt(b^2*x^2 + 2*
a*b*x + a^2 + 1))^3*a^2*b^4 + 357*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2
+ 1))*a^4*b^4 + 144*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^4*a*b^3*
abs(b) + 320*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2*a^3*b^3*abs(b
) + 160*a^5*b^3*abs(b) + 21*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^
3*b^4 + 93*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*a^2*b^4 + 32*(x*a
bs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2*a*b^3*abs(b) + 80*a^3*b^3*abs(
b) + 3*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*b^4 + 16*a*b^3*abs(b)
)/((a^6 + 3*a^4 + 3*a^2 + 1)*((x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)
)^2 - a^2 - 1)^4)
```

maple [B] time = 0.01, size = 841, normalized size = 4.06

$$-\frac{\left(b^2x^2+2abx+a^2+1\right)^{\frac{3}{2}}}{4\left(a^2+1\right)x^4}+\frac{5ab\left(b^2x^2+2abx+a^2+1\right)^{\frac{3}{2}}}{12\left(a^2+1\right)^2x^3}-\frac{5a^2b^2\left(b^2x^2+2abx+a^2+1\right)^{\frac{3}{2}}}{8\left(a^2+1\right)^3x^2}+\frac{5a^3b^3\left(b^2x^2+2abx+\right.}{8\left(a^2+1\right)^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+(1+(b*x+a)^2)^(1/2))/x^5,x)
```

```
[Out] -1/4*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/(a^2+1)/x^4+5/12*a*b*(b^2*x^2+2*a*b*x+a^
2+1)^(3/2)/(a^2+1)^2/x^3-5/8*a^2*b^2/(a^2+1)^3/x^2*(b^2*x^2+2*a*b*x+a^2+1)^
(3/2)+5/8*a^3*b^3/(a^2+1)^4/x*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)-5/4*a^4*b^4/(a^
2+1)^4*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-5/8*a^5*b^5/(a^2+1)^4*ln((b^2*x+a*b)/(
b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+5/8*a^4*b^4/(a^2+1)^(
7/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)-
5/8*a^3*b^5/(a^2+1)^4*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*x-5/8*a^3*b^5/(a^2+1)^4
*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+7/8*
a^2*b^4/(a^2+1)^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+3/4*a^3*b^5/(a^2+1)^3*ln((b
^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-3/4*a^2*b^
4/(a^2+1)^(5/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)
^(1/2))/x)+1/8*b^2/(a^2+1)^2/x^2*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)-1/8*b^3/(a^2
+1)^3*a/x*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)+1/8*b^5/(a^2+1)^3*a*(b^2*x^2+2*a*b*
x+a^2+1)^(1/2)*x+1/8*b^5/(a^2+1)^3*a*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*
a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-1/8*b^4/(a^2+1)^2*(b^2*x^2+2*a*b*x+a^2+1)^(
1/2)-1/8*b^5/(a^2+1)^2*a*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)
^(1/2))/(b^2)^(1/2)+1/8*b^4/(a^2+1)^(3/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/
2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)-1/3*b/x^3-1/4*a/x^4
```

maxima [B] time = 0.45, size = 594, normalized size = 2.87

$$\frac{5\,a^4b^4\,\mathrm{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^2b^2+4\left(a^2+1\right)b^2}\,|x|}+\frac{2\,a^2}{\sqrt{-4\,a^2b^2+4\left(a^2+1\right)b^2}\,|x|}+\frac{2}{\sqrt{-4\,a^2b^2+4\left(a^2+1\right)b^2}\,|x|}\right)}{8\left(a^2+1\right)^{\frac{7}{2}}}-\frac{3\,a^2b^4\,\mathrm{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^2b^2+4\left(a^2+1\right)b^2}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x^5,x, algorithm="maxima")
```

```
[Out] 5/8*a^4*b^4*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2
*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(
a^2 + 1)*b^2)*abs(x)))/(a^2 + 1)^(7/2) - 3/4*a^2*b^4*arcsinh(2*a*b*x/(sqrt(
-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1
)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)))/(a^2 + 1)^(
5/2) + 5/8*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^2*b^4/(a^2 + 1)^3 + 1/8*b^4*
arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2*a^2/(sqrt(-
```

```
4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2
)*abs(x)))/(a^2 + 1)^(3/2) - 1/8*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^4/(a^2
 + 1)^2 + 5/8*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^3*b^3/((a^2 + 1)^3*x) - 1
/8*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a*b^3/((a^2 + 1)^2*x) - 5/8*(b^2*x^2 +
 2*a*b*x + a^2 + 1)^(3/2)*a^2*b^2/((a^2 + 1)^3*x^2) + 1/8*(b^2*x^2 + 2*a*b*
x + a^2 + 1)^(3/2)*b^2/((a^2 + 1)^2*x^2) + 5/12*(b^2*x^2 + 2*a*b*x + a^2 +
1)^(3/2)*a*b/((a^2 + 1)^2*x^3) - 1/3*b/x^3 - 1/4*a/x^4 - 1/4*(b^2*x^2 + 2*a
*b*x + a^2 + 1)^(3/2)/((a^2 + 1)*x^4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+\sqrt{(a+b\,x)^2+1}+b\,x}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + ((a + b*x)^2 + 1)^(1/2) + b*x)/x^5,x)
```

```
[Out] int((a + ((a + b*x)^2 + 1)^(1/2) + b*x)/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx+\sqrt{a^2+2abx+b^2x^2+1}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)**2)**(1/2))/x**5,x)
```

```
[Out] Integral((a + b*x + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1))/x**5, x)
```

3.358 $\int e^{\sinh^{-1}(a+bx)^2}x^3\,dx$

Optimal. Leaf size=359

$$-\frac{\sqrt{\pi}\,a^3\text{erfi}\left(\frac{1}{2}\left(2\sinh^{-1}(a+bx)-1\right)\right)}{4\sqrt[4]{e}\,b^4}-\frac{\sqrt{\pi}\,a^3\text{erfi}\left(\frac{1}{2}\left(2\sinh^{-1}(a+bx)+1\right)\right)}{4\sqrt[4]{e}\,b^4}+\frac{3\sqrt{\pi}\,a^2\text{erfi}\left(1-\sinh^{-1}(a+bx)\right)}{8eb^4}+$$

```
[Out] -1/32*erfi(-2+arcsinh(b*x+a))*Pi^(1/2)/b^4/exp(4)+1/16*erfi(-1+arcsinh(b*x+
a))*Pi^(1/2)/b^4/E-3/8*a^2*erfi(-1+arcsinh(b*x+a))*Pi^(1/2)/b^4/E-1/16*erfi
(1+arcsinh(b*x+a))*Pi^(1/2)/b^4/E+3/8*a^2*erfi(1+arcsinh(b*x+a))*Pi^(1/2)/b
^4/E+1/32*erfi(2+arcsinh(b*x+a))*Pi^(1/2)/b^4/exp(4)-3/16*a*erfi(-3/2+arcsi
nh(b*x+a))*Pi^(1/2)/b^4/exp(9/4)+3/16*a*erfi(-1/2+arcsinh(b*x+a))*Pi^(1/2)/
b^4/exp(1/4)-1/4*a^3*erfi(-1/2+arcsinh(b*x+a))*Pi^(1/2)/b^4/exp(1/4)+3/16*a
*erfi(1/2+arcsinh(b*x+a))*Pi^(1/2)/b^4/exp(1/4)-1/4*a^3*erfi(1/2+arcsinh(b*
x+a))*Pi^(1/2)/b^4/exp(1/4)-3/16*a*erfi(3/2+arcsinh(b*x+a))*Pi^(1/2)/b^4/ex
p(9/4)
```

Rubi [A] time = 0.74, antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 37, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5898, 6741, 12, 6742, 5513, 2234, 2204, 5514}

$$-\frac{\sqrt{\pi}\,a^3\text{Erfi}\left(\frac{1}{2}\left(2\sinh^{-1}(a+bx)-1\right)\right)}{4\sqrt[4]{e}\,b^4}-\frac{\sqrt{\pi}\,a^3\text{Erfi}\left(\frac{1}{2}\left(2\sinh^{-1}(a+bx)+1\right)\right)}{4\sqrt[4]{e}\,b^4}+\frac{3\sqrt{\pi}\,a^2\text{Erfi}\left(1-\sinh^{-1}(a+bx)\right)}{8eb^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSinh[a + b*x]^2*x^3,x]
```

```
[Out] -(Sqrt[Pi]*Erfi[1 - ArcSinh[a + b*x]])/(16*b^4*E) + (3*a^2*Sqrt[Pi]*Erfi[1
 - ArcSinh[a + b*x]])/(8*b^4*E) + (Sqrt[Pi]*Erfi[2 - ArcSinh[a + b*x]])/(32*
b^4*E^4) - (Sqrt[Pi]*Erfi[1 + ArcSinh[a + b*x]])/(16*b^4*E) + (3*a^2*Sqrt[P
i]*Erfi[1 + ArcSinh[a + b*x]])/(8*b^4*E) + (Sqrt[Pi]*Erfi[2 + ArcSinh[a + b
*x]])/(32*b^4*E^4) - (3*a*Sqrt[Pi]*Erfi[(-3 + 2*ArcSinh[a + b*x])/2])/(16*b
^4*E^(9/4)) + (3*a*Sqrt[Pi]*Erfi[(-1 + 2*ArcSinh[a + b*x])/2])/(16*b^4*E^(1
/4)) - (a^3*Sqrt[Pi]*Erfi[(-1 + 2*ArcSinh[a + b*x])/2])/(4*b^4*E^(1/4)) + (
3*a*Sqrt[Pi]*Erfi[(1 + 2*ArcSinh[a + b*x])/2])/(16*b^4*E^(1/4)) - (a^3*Sqrt
[Pi]*Erfi[(1 + 2*ArcSinh[a + b*x])/2])/(4*b^4*E^(1/4)) - (3*a*Sqrt[Pi]*Erfi
[(3 + 2*ArcSinh[a + b*x])/2])/(16*b^4*E^(9/4))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 5513

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
```

```
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 5514

```
Int[Cosh[v_]^(n_.)*(F_)^(u_)*Sinh[v_]^(m_.), x_Symbol] :> Int[ExpandTrigToE
xp[F^u, Sinh[v]^m*Cosh[v]^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || Pol
yQ[u, x, 2]) && (LinearQ[v, x] || PolyQ[v, x, 2]) && IGtQ[m, 0] && IGtQ[n,
0]
```

Rule 5898

```
Int[(f_)^(ArcSinh[(a_.) + (b_.)*(x_)]^(n_.)*(c_.))*(x_)^(m_.), x_Symbol] :>
 Dist[1/b, Subst[Int[(-(a/b) + Sinh[x]/b)^m*f^(c*x^n)*Cosh[x], x], x, ArcSi
nh[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int e^{\sinh^{-1}(a+bx)^2}x^3\,dx = \frac{\text{Subst}\left(\int e^{x^2}\cosh(x)\left(-\frac{a}{b}+\frac{\sinh(x)}{b}\right)^3 dx, x, \sinh^{-1}(a+bx)\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \frac{e^{x^2}\cosh(x)(-a+\sinh(x))^3}{b^3} dx, x, \sinh^{-1}(a+bx)\right)}{b}$$

$$= \frac{\text{Subst}\left(\int e^{x^2}\cosh(x)(-a+\sinh(x))^3 dx, x, \sinh^{-1}(a+bx)\right)}{b^4}$$

$$= \frac{\text{Subst}\left(\int \left(-a^3e^{x^2}\cosh(x)+3a^2e^{x^2}\cosh(x)\sinh(x)-3ae^{x^2}\cosh(x)\sinh^2(x)+e^{x^2}\cosh\right.\right.}{b^4}$$

$$= \frac{\text{Subst}\left(\int e^{x^2}\cosh(x)\sinh^3(x)\,dx, x, \sinh^{-1}(a+bx)\right)}{b^4} - \frac{(3a)\,\text{Subst}\left(\int e^{x^2}\cosh(x)\sinh^2\right.}{b^4}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{16}e^{-4x+x^2}+\frac{1}{8}e^{-2x+x^2}-\frac{1}{8}e^{2x+x^2}+\frac{1}{16}e^{4x+x^2}\right)dx, x, \sinh^{-1}(a+bx)\right)}{b^4} - \frac{(3a)\,\text{S}}{}$$

$$= -\frac{\text{Subst}\left(\int e^{-4x+x^2}dx, x, \sinh^{-1}(a+bx)\right)}{16b^4} + \frac{\text{Subst}\left(\int e^{4x+x^2}dx, x, \sinh^{-1}(a+bx)\right)}{16b^4} + \frac{\text{Su}}{}$$

$$= -\frac{\text{Subst}\left(\int e^{\frac{1}{4}(-4+2x)^2}dx, x, \sinh^{-1}(a+bx)\right)}{16b^4e^4} + \frac{\text{Subst}\left(\int e^{\frac{1}{4}(4+2x)^2}dx, x, \sinh^{-1}(a+bx)\right)}{16b^4e^4}$$

$$= -\frac{\sqrt{\pi}\,\text{erfi}\left(1-\sinh^{-1}(a+bx)\right)}{16b^4e} + \frac{3a^2\sqrt{\pi}\,\text{erfi}\left(1-\sinh^{-1}(a+bx)\right)}{8b^4e} + \frac{\sqrt{\pi}\,\text{erfi}\left(2-\sinh\right.}{32b^4e^4}$$

Mathematica [A] time = 0.33, size = 198, normalized size = 0.55

$$\frac{\sqrt{\pi}\left(-8e^{15/4}a^3\text{erfi}\left(\sinh^{-1}(a+bx)+\frac{1}{2}\right)+12e^3a^2\text{erfi}\left(\sinh^{-1}(a+bx)+1\right)+2e^{15/4}\left(4a^2-3\right)a\text{erfi}\left(\frac{1}{2}-\sinh^{-1}(a\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcSinh[a + b*x]^2*x^3,x]
```

```
[Out] (Sqrt[Pi]*(2*a*(-3 + 4*a^2)*E^(15/4)*Erfi[1/2 - ArcSinh[a + b*x]] + 2*(-1 +
 6*a^2)*E^3*Erfi[1 - ArcSinh[a + b*x]] + 6*a*E^(7/4)*Erfi[3/2 - ArcSinh[a +
 b*x]] + Erfi[2 - ArcSinh[a + b*x]] + 6*a*E^(15/4)*Erfi[1/2 + ArcSinh[a + b
*x]] - 8*a^3*E^(15/4)*Erfi[1/2 + ArcSinh[a + b*x]] - 2*E^3*Erfi[1 + ArcSinh
[a + b*x]] + 12*a^2*E^3*Erfi[1 + ArcSinh[a + b*x]] - 6*a*E^(7/4)*Erfi[3/2 +
 ArcSinh[a + b*x]] + Erfi[2 + ArcSinh[a + b*x]]))/(32*b^4*E^4)
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(x^3e^{\left(\text{arsinh}(bx+a)^2\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsinh(b*x+a)^2)*x^3,x, algorithm="fricas")
```

```
[Out] integral(x^3*e^(arcsinh(b*x + a)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3e^{\left(\text{arsinh}(bx+a)^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsinh(b*x+a)^2)*x^3,x, algorithm="giac")
```

```
[Out] integrate(x^3*e^(arcsinh(b*x + a)^2), x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int e^{\text{arcsinh}(bx+a)^2}x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arcsinh(b*x+a)^2)*x^3,x)
```

```
[Out] int(exp(arcsinh(b*x+a)^2)*x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3e^{\left(\text{arsinh}(bx+a)^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsinh(b*x+a)^2)*x^3,x, algorithm="maxima")
```

```
[Out] integrate(x^3*e^(arcsinh(b*x + a)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,e^{\text{asinh}(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*exp(asinh(a + b*x)^2),x)
```

```
[Out] int(x^3*exp(asinh(a + b*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 e^{\operatorname{asinh}^2(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(asinh(b*x+a)**2)*x**3,x)
```

```
[Out] Integral(x**3*exp(asinh(a + b*x)**2), x)
```

3.359 $\int e^{\sinh^{-1}(a+bx)^2} x^2\,dx$

Optimal. Leaf size=251

$$\frac{\sqrt{\pi}\,a^2\mathrm{erfi}\left(\frac{1}{2}\left(2\sinh^{-1}(a+bx)-1\right)\right)}{4\sqrt[4]{e}\,b^3}+\frac{\sqrt{\pi}\,a^2\mathrm{erfi}\left(\frac{1}{2}\left(2\sinh^{-1}(a+bx)+1\right)\right)}{4\sqrt[4]{e}\,b^3}-\frac{\sqrt{\pi}\,a\mathrm{erfi}\left(1-\sinh^{-1}(a+bx)\right)}{4eb^3}-\sqrt{\pi}$$

```
[Out] 1/4*a*erfi(-1+arcsinh(b*x+a))*Pi^(1/2)/b^3/E-1/4*a*erfi(1+arcsinh(b*x+a))*P
i^(1/2)/b^3/E+1/16*erfi(-3/2+arcsinh(b*x+a))*Pi^(1/2)/b^3/exp(9/4)-1/16*erf
i(-1/2+arcsinh(b*x+a))*Pi^(1/2)/b^3/exp(1/4)+1/4*a^2*erfi(-1/2+arcsinh(b*x+
a))*Pi^(1/2)/b^3/exp(1/4)-1/16*erfi(1/2+arcsinh(b*x+a))*Pi^(1/2)/b^3/exp(1/
4)+1/4*a^2*erfi(1/2+arcsinh(b*x+a))*Pi^(1/2)/b^3/exp(1/4)+1/16*erfi(3/2+arc
sinh(b*x+a))*Pi^(1/2)/b^3/exp(9/4)
```

Rubi [A] time = 0.52, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5898, 6741, 12, 6742, 5513, 2234, 2204, 5514}

$$\frac{\sqrt{\pi}\,a^2\mathrm{Erfi}\left(\frac{1}{2}\left(2\sinh^{-1}(a+bx)-1\right)\right)}{4\sqrt[4]{e}\,b^3}+\frac{\sqrt{\pi}\,a^2\mathrm{Erfi}\left(\frac{1}{2}\left(2\sinh^{-1}(a+bx)+1\right)\right)}{4\sqrt[4]{e}\,b^3}-\frac{\sqrt{\pi}\,a\mathrm{Erfi}\left(1-\sinh^{-1}(a+bx)\right)}{4eb^3}-\sqrt{}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSinh[a + b*x]^2*x^2,x]
```

```
[Out] -(a*Sqrt[Pi]*Erfi[1 - ArcSinh[a + b*x]])/(4*b^3*E) - (a*Sqrt[Pi]*Erfi[1 + A
rcSinh[a + b*x]])/(4*b^3*E) + (Sqrt[Pi]*Erfi[(-3 + 2*ArcSinh[a + b*x])/2])/
(16*b^3*E^(9/4)) - (Sqrt[Pi]*Erfi[(-1 + 2*ArcSinh[a + b*x])/2])/(16*b^3*E^(
1/4)) + (a^2*Sqrt[Pi]*Erfi[(-1 + 2*ArcSinh[a + b*x])/2])/(4*b^3*E^(1/4)) -
(Sqrt[Pi]*Erfi[(1 + 2*ArcSinh[a + b*x])/2])/(16*b^3*E^(1/4)) + (a^2*Sqrt[Pi
]*Erfi[(1 + 2*ArcSinh[a + b*x])/2])/(4*b^3*E^(1/4)) + (Sqrt[Pi]*Erfi[(3 + 2
*ArcSinh[a + b*x])/2])/(16*b^3*E^(9/4))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 5513

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 5514

```
Int[Cosh[v_]^(n_.)*(F_)^(u_)*Sinh[v_]^(m_.), x_Symbol] :> Int[ExpandTrigToE
xp[F^u, Sinh[v]^m*Cosh[v]^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || Pol
```

```
yQ[u, x, 2]) && (LinearQ[v, x] || PolyQ[v, x, 2]) && IGtQ[m, 0] && IGtQ[n,
0]
```

Rule 5898

```
Int[(f_)^(ArcSinh[(a_.) + (b_.)*(x_)]^(n_.)*(c_.))*(x_)^(m_.), x_Symbol] :>
 Dist[1/b, Subst[Int[(-(a/b) + Sinh[x]/b)^m*f^(c*x^n)*Cosh[x], x], x, ArcSi
nh[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int e^{\sinh^{-1}(a+bx)^2} x^2\,dx = \frac{\text{Subst}\left(\int e^{x^2}\cosh(x)\left(-\frac{a}{b}+\frac{\sinh(x)}{b}\right)^2 dx, x, \sinh^{-1}(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{e^{x^2}\cosh(x)(a-\sinh(x))^2}{b^2} dx, x, \sinh^{-1}(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int e^{x^2}\cosh(x)(a-\sinh(x))^2 dx, x, \sinh^{-1}(a+bx)\right)}{b^3}$$
$$= \frac{\text{Subst}\left(\int \left(a^2e^{x^2}\cosh(x)-2ae^{x^2}\cosh(x)\sinh(x)+e^{x^2}\cosh(x)\sinh^2(x)\right) dx, x, \sinh^{-1}(\ldots}{b^3}$$
$$= \frac{\text{Subst}\left(\int e^{x^2}\cosh(x)\sinh^2(x)\,dx, x, \sinh^{-1}(a+bx)\right)}{b^3} - \frac{(2a)\,\text{Subst}\left(\int e^{x^2}\cosh(x)\sinh(\ldots}{b^3}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{8}e^{-3x+x^2}-\frac{1}{8}e^{-x+x^2}-\frac{e^{x+x^2}}{8}+\frac{1}{8}e^{3x+x^2}\right) dx, x, \sinh^{-1}(a+bx)\right)}{b^3} - \frac{(2a)\,\text{Subst}\left(\int\ldots\right.}{\ldots}$$
$$= \frac{\text{Subst}\left(\int e^{-3x+x^2} dx, x, \sinh^{-1}(a+bx)\right)}{8b^3} - \frac{\text{Subst}\left(\int e^{-x+x^2} dx, x, \sinh^{-1}(a+bx)\right)}{8b^3} - \frac{\text{Sub}\ldots}{\ldots}$$
$$= \frac{\text{Subst}\left(\int e^{\frac{1}{4}(-3+2x)^2} dx, x, \sinh^{-1}(a+bx)\right)}{8b^3e^{9/4}} + \frac{\text{Subst}\left(\int e^{\frac{1}{4}(3+2x)^2} dx, x, \sinh^{-1}(a+bx)\right)}{8b^3e^{9/4}} + \ldots$$
$$= -\frac{a\sqrt{\pi}\,\text{erfi}\left(1-\sinh^{-1}(a+bx)\right)}{4b^3e} - \frac{a\sqrt{\pi}\,\text{erfi}\left(1+\sinh^{-1}(a+bx)\right)}{4b^3e} + \frac{\sqrt{\pi}\,\text{erfi}\left(\frac{1}{2}\left(-3+2\ldots\right.\right.}{16b^3\ldots}$$

Mathematica [A] time = 0.21, size = 138, normalized size = 0.55

$$-\frac{\sqrt{\pi}\left(-4e^2a^2\text{erfi}\left(\sinh^{-1}(a+bx)+\frac{1}{2}\right)+e^2\left(4a^2-1\right)\text{erfi}\left(\frac{1}{2}-\sinh^{-1}(a+bx)\right)+4e^{5/4}a\,\text{erfi}\left(1-\sinh^{-1}(a+b\ldots\right.\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcSinh[a + b*x]^2*x^2,x]
```

```
[Out] -1/16*(Sqrt[Pi]*((-1 + 4*a^2)*E^2*Erfi[1/2 - ArcSinh[a + b*x]] + 4*a*E^(5/4
)*Erfi[1 - ArcSinh[a + b*x]] + Erfi[3/2 - ArcSinh[a + b*x]] + E^2*Erfi[1/2
+ ArcSinh[a + b*x]] - 4*a^2*E^2*Erfi[1/2 + ArcSinh[a + b*x]] + 4*a*E^(5/4)*
Erfi[1 + ArcSinh[a + b*x]] - Erfi[3/2 + ArcSinh[a + b*x]]))/(b^3*E^(9/4))
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(x^2 e^{\left(\mathrm{arsinh}(bx+a)^2\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsinh(b*x+a)^2)*x^2,x, algorithm="fricas")
```

```
[Out] integral(x^2*e^(arcsinh(b*x + a)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 e^{\left(\mathrm{arsinh}(bx+a)^2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsinh(b*x+a)^2)*x^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*e^(arcsinh(b*x + a)^2), x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int e^{\mathrm{arcsinh}(bx+a)^2} x^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arcsinh(b*x+a)^2)*x^2,x)
```

```
[Out] int(exp(arcsinh(b*x+a)^2)*x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 e^{\left(\mathrm{arsinh}(bx+a)^2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsinh(b*x+a)^2)*x^2,x, algorithm="maxima")
```

```
[Out] integrate(x^2*e^(arcsinh(b*x + a)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\, e^{\mathrm{asinh}(a+b\,x)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*exp(asinh(a + b*x)^2),x)
```

```
[Out] int(x^2*exp(asinh(a + b*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 e^{\mathrm{asinh}^2(a+bx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(asinh(b*x+a)**2)*x**2,x)
```

```
[Out] Integral(x**2*exp(asinh(a + b*x)**2), x)
```

3.360 $\int e^{\sinh^{-1}(a+bx)^2} x\,dx$

Optimal. Leaf size=117

$$\frac{\sqrt{\pi}\,\text{erfi}\left(1-\sinh^{-1}(a+bx)\right)}{8eb^2}+\frac{\sqrt{\pi}\,\text{erfi}\left(\sinh^{-1}(a+bx)+1\right)}{8eb^2}-\frac{\sqrt{\pi}\,a\text{erfi}\left(\frac{1}{2}\left(2\sinh^{-1}(a+bx)-1\right)\right)}{4\sqrt[4]{e}\,b^2}-\frac{\sqrt{\pi}\,a\text{erfi}\left(\right.}{}$$

```
[Out] -1/8*erfi(-1+arcsinh(b*x+a))*Pi^(1/2)/b^2/E+1/8*erfi(1+arcsinh(b*x+a))*Pi^(
1/2)/b^2/E-1/4*a*erfi(-1/2+arcsinh(b*x+a))*Pi^(1/2)/b^2/exp(1/4)-1/4*a*erfi
(1/2+arcsinh(b*x+a))*Pi^(1/2)/b^2/exp(1/4)
```

Rubi [A] time = 0.28, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5898, 6741, 12, 6742, 5513, 2234, 2204, 5514}

$$\frac{\sqrt{\pi}\,\text{Erfi}\left(1-\sinh^{-1}(a+bx)\right)}{8eb^2}+\frac{\sqrt{\pi}\,\text{Erfi}\left(\sinh^{-1}(a+bx)+1\right)}{8eb^2}-\frac{\sqrt{\pi}\,a\text{Erfi}\left(\frac{1}{2}\left(2\sinh^{-1}(a+bx)-1\right)\right)}{4\sqrt[4]{e}\,b^2}-\frac{\sqrt{\pi}\,a\text{Erfi}}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSinh[a + b*x]^2*x,x]
```

```
[Out] (Sqrt[Pi]*Erfi[1 - ArcSinh[a + b*x]])/(8*b^2*E) + (Sqrt[Pi]*Erfi[1 + ArcSin
h[a + b*x]])/(8*b^2*E) - (a*Sqrt[Pi]*Erfi[(-1 + 2*ArcSinh[a + b*x])/2])/(4*
b^2*E^(1/4)) - (a*Sqrt[Pi]*Erfi[(1 + 2*ArcSinh[a + b*x])/2])/(4*b^2*E^(1/4)
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 5513

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 5514

```
Int[Cosh[v_]^(n_.)*(F_)^(u_)*Sinh[v_]^(m_.), x_Symbol] :> Int[ExpandTrigToE
xp[F^u, Sinh[v]^m*Cosh[v]^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || Pol
yQ[u, x, 2]) && (LinearQ[v, x] || PolyQ[v, x, 2]) && IGtQ[m, 0] && IGtQ[n,
0]
```

Rule 5898

```
Int[(f_)^(ArcSinh[(a_.) + (b_.)*(x_)]^(n_.)*(c_.))*(x_)^(m_.), x_Symbol] :>
 Dist[1/b, Subst[Int[(-(a/b) + Sinh[x]/b)^m*f^(c*x^n)*Cosh[x], x], x, ArcSi
nh[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int e^{\sinh^{-1}(a+bx)^2}x\,dx = \frac{\text{Subst}\left(\int e^{x^2}\cosh(x)\left(-\frac{a}{b}+\frac{\sinh(x)}{b}\right)dx,x,\sinh^{-1}(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{e^{x^2}\cosh(x)(-a+\sinh(x))}{b}dx,x,\sinh^{-1}(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int e^{x^2}\cosh(x)(-a+\sinh(x))\,dx,x,\sinh^{-1}(a+bx)\right)}{b^2}$$
$$= \frac{\text{Subst}\left(\int \left(-ae^{x^2}\cosh(x)+e^{x^2}\cosh(x)\sinh(x)\right)dx,x,\sinh^{-1}(a+bx)\right)}{b^2}$$
$$= \frac{\text{Subst}\left(\int e^{x^2}\cosh(x)\sinh(x)\,dx,x,\sinh^{-1}(a+bx)\right)}{b^2} - \frac{a\,\text{Subst}\left(\int e^{x^2}\cosh(x)\,dx,x,\sinh^{-1}(\right.}{b^2}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{4}e^{-2x+x^2}+\frac{1}{4}e^{2x+x^2}\right)dx,x,\sinh^{-1}(a+bx)\right)}{b^2} - \frac{a\,\text{Subst}\left(\int \left(\frac{1}{2}e^{-x+x^2}+\frac{e^{x+x^2}}{2}\right)dx,\right.}{b^2}$$
$$= -\frac{\text{Subst}\left(\int e^{-2x+x^2}dx,x,\sinh^{-1}(a+bx)\right)}{4b^2} + \frac{\text{Subst}\left(\int e^{2x+x^2}dx,x,\sinh^{-1}(a+bx)\right)}{4b^2} - \frac{a\,\text{Subs}}{}$$
$$= -\frac{\text{Subst}\left(\int e^{\frac{1}{4}(-2+2x)^2}dx,x,\sinh^{-1}(a+bx)\right)}{4b^2e} + \frac{\text{Subst}\left(\int e^{\frac{1}{4}(2+2x)^2}dx,x,\sinh^{-1}(a+bx)\right)}{4b^2e} - \frac{a}{}$$
$$= \frac{\sqrt{\pi}\,\text{erfi}\left(1-\sinh^{-1}(a+bx)\right)}{8b^2e} + \frac{\sqrt{\pi}\,\text{erfi}\left(1+\sinh^{-1}(a+bx)\right)}{8b^2e} - \frac{a\sqrt{\pi}\,\text{erfi}\left(\frac{1}{2}\left(-1+2\sinh^{-1}\right.\right.}{4b^2\sqrt[4]{e}}$$

Mathematica [A] time = 0.11, size = 76, normalized size = 0.65

$$\frac{\sqrt{\pi}\left(2e^{3/4}a\,\text{erfi}\left(\frac{1}{2}-\sinh^{-1}(a+bx)\right)+\text{erfi}\left(1-\sinh^{-1}(a+bx)\right)-2e^{3/4}a\,\text{erfi}\left(\sinh^{-1}(a+bx)+\frac{1}{2}\right)+\text{erfi}\left(\sinh^{-1}(\right.\right.}{8eb^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcSinh[a + b*x]^2*x,x]
```

```
[Out] (Sqrt[Pi]*(2*a*E^(3/4)*Erfi[1/2 - ArcSinh[a + b*x]] + Erfi[1 - ArcSinh[a +
b*x]] - 2*a*E^(3/4)*Erfi[1/2 + ArcSinh[a + b*x]] + Erfi[1 + ArcSinh[a + b*x
]]))/(8*b^2*E)
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(xe^{\left(\text{arsinh}(bx+a)^2\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsinh(b*x+a)^2)*x,x, algorithm="fricas")
[Out] integral(x*e^(arcsinh(b*x + a)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int xe^{\left(\text{arsinh}(bx+a)^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsinh(b*x+a)^2)*x,x, algorithm="giac")
[Out] integrate(x*e^(arcsinh(b*x + a)^2), x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int e^{\text{arcsinh}(bx+a)^2} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arcsinh(b*x+a)^2)*x,x)
[Out] int(exp(arcsinh(b*x+a)^2)*x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int xe^{\left(\text{arsinh}(bx+a)^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsinh(b*x+a)^2)*x,x, algorithm="maxima")
[Out] integrate(x*e^(arcsinh(b*x + a)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,e^{\text{asinh}(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*exp(asinh(a + b*x)^2),x)
[Out] int(x*exp(asinh(a + b*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int xe^{\text{asinh}^2(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(asinh(b*x+a)**2)*x,x)
[Out] Integral(x*exp(asinh(a + b*x)**2), x)
```

3.361 $\int e^{\sinh^{-1}(a+bx)^2}\,dx$

Optimal. Leaf size=65

$$\frac{\sqrt{\pi}\,\mathrm{erfi}\left(\frac{1}{2}\left(2\sinh^{-1}(a+bx)-1\right)\right)}{4\sqrt[4]{e}\,b}+\frac{\sqrt{\pi}\,\mathrm{erfi}\left(\frac{1}{2}\left(2\sinh^{-1}(a+bx)+1\right)\right)}{4\sqrt[4]{e}\,b}$$

```
[Out] 1/4*erfi(-1/2+arcsinh(b*x+a))*Pi^(1/2)/b/exp(1/4)+1/4*erfi(1/2+arcsinh(b*x+
a))*Pi^(1/2)/b/exp(1/4)
```

Rubi [A] time = 0.06, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5896, 5513, 2234, 2204}

$$\frac{\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{1}{2}\left(2\sinh^{-1}(a+bx)-1\right)\right)}{4\sqrt[4]{e}\,b}+\frac{\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{1}{2}\left(2\sinh^{-1}(a+bx)+1\right)\right)}{4\sqrt[4]{e}\,b}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSinh[a + b*x]^2,x]
```

```
[Out] (Sqrt[Pi]*Erfi[(-1 + 2*ArcSinh[a + b*x])/2])/(4*b*E^(1/4)) + (Sqrt[Pi]*Erfi
[(1 + 2*ArcSinh[a + b*x])/2])/(4*b*E^(1/4))
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 5513

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 5896

```
Int[(f_)^(ArcSinh[(a_.) + (b_.)*(x_)]^(n_.)*(c_.)), x_Symbol] :> Dist[1/b,
Subst[Int[f^(c*x^n)*Cosh[x], x], x, ArcSinh[a + b*x]], x] /; FreeQ[{a, b, c
, f}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int e^{\sinh^{-1}(a+bx)^2}\,dx = \frac{\text{Subst}\left(\int e^{x^2}\cosh(x)\,dx, x, \sinh^{-1}(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{2}e^{-x+x^2}+\frac{e^{x+x^2}}{2}\right)dx, x, \sinh^{-1}(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int e^{-x+x^2}\,dx, x, \sinh^{-1}(a+bx)\right)}{2b} + \frac{\text{Subst}\left(\int e^{x+x^2}\,dx, x, \sinh^{-1}(a+bx)\right)}{2b}$$
$$= \frac{\text{Subst}\left(\int e^{\frac{1}{4}(-1+2x)^2}\,dx, x, \sinh^{-1}(a+bx)\right)}{2b\sqrt[4]{e}} + \frac{\text{Subst}\left(\int e^{\frac{1}{4}(1+2x)^2}\,dx, x, \sinh^{-1}(a+bx)\right)}{2b\sqrt[4]{e}}$$
$$= \frac{\sqrt{\pi}\,\text{erfi}\left(\frac{1}{2}\left(-1+2\sinh^{-1}(a+bx)\right)\right)}{4b\sqrt[4]{e}} + \frac{\sqrt{\pi}\,\text{erfi}\left(\frac{1}{2}\left(1+2\sinh^{-1}(a+bx)\right)\right)}{4b\sqrt[4]{e}}$$

Mathematica [A] time = 0.04, size = 44, normalized size = 0.68

$$\frac{\sqrt{\pi}\left(\text{erfi}\left(\sinh^{-1}(a+bx)+\frac{1}{2}\right)+\text{erfi}\left(\frac{1}{2}\left(2\sinh^{-1}(a+bx)-1\right)\right)\right)}{4\sqrt[4]{e}\,b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcSinh[a + b*x]^2,x]
[Out] (Sqrt[Pi]*(Erfi[1/2 + ArcSinh[a + b*x]] + Erfi[(-1 + 2*ArcSinh[a + b*x])/2]
))/(4*b*E^(1/4))
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(e^{\left(\text{arsinh}(bx+a)^2\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsinh(b*x+a)^2),x, algorithm="fricas")
[Out] integral(e^(arcsinh(b*x + a)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{\left(\text{arsinh}(bx+a)^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsinh(b*x+a)^2),x, algorithm="giac")
[Out] integrate(e^(arcsinh(b*x + a)^2), x)
```

maple [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{\text{arcsinh}(bx+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arcsinh(b*x+a)^2),x)
[Out] int(exp(arcsinh(b*x+a)^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{\left(\operatorname{arsinh}(bx+a)^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsinh(b*x+a)^2),x, algorithm="maxima")
```

```
[Out] integrate(e^(arcsinh(b*x + a)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \mathrm{e}^{\operatorname{asinh}(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(asinh(a + b*x)^2),x)
```

```
[Out] int(exp(asinh(a + b*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{\operatorname{asinh}^2(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(asinh(b*x+a)**2),x)
```

```
[Out] Integral(exp(asinh(a + b*x)**2), x)
```

3.362 $\int \frac{e^{\sinh^{-1}(a+bx)^2}}{x}\,dx$

Optimal. Leaf size=17

$$\text{Int}\left(\frac{e^{\sinh^{-1}(a+bx)^2}}{x},x\right)$$

```
[Out] CannotIntegrate(exp(arcsinh(b*x+a)^2)/x,x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{e^{\sinh^{-1}(a+bx)^2}}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[E^ArcSinh[a + b*x]^2/x,x]
[Out] Defer[Int][E^ArcSinh[a + b*x]^2/x, x]
```

Rubi steps

$$\int \frac{e^{\sinh^{-1}(a+bx)^2}}{x}\,dx = \int \frac{e^{\sinh^{-1}(a+bx)^2}}{x}\,dx$$

Mathematica [A] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{e^{\sinh^{-1}(a+bx)^2}}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^ArcSinh[a + b*x]^2/x,x]
[Out] Integrate[E^ArcSinh[a + b*x]^2/x, x]
```

fricas [A] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{e^{\left(\text{arsinh}(bx+a)^2\right)}}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsinh(b*x+a)^2)/x,x, algorithm="fricas")
[Out] integral(e^(arcsinh(b*x + a)^2)/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{\left(\text{arsinh}(bx+a)^2\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsinh(b*x+a)^2)/x,x, algorithm="giac")
[Out] integrate(e^(arcsinh(b*x + a)^2)/x, x)
```

maple [A] time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{e}^{\operatorname{arcsinh}(bx+a)^2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arcsinh(b*x+a)^2)/x,x)
[Out] int(exp(arcsinh(b*x+a)^2)/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{\left(\operatorname{arsinh}(bx+a)^2\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsinh(b*x+a)^2)/x,x, algorithm="maxima")
[Out] integrate(e^(arcsinh(b*x + a)^2)/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\mathrm{e}^{\operatorname{asinh}(a+b\,x)^2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(asinh(a + b*x)^2)/x,x)
[Out] int(exp(asinh(a + b*x)^2)/x, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{\operatorname{asinh}^{2}(a+bx)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(asinh(b*x+a)**2)/x,x)
[Out] Integral(exp(asinh(a + b*x)**2)/x, x)
```

3.363 $\int \frac{e^{\sinh^{-1}(a+bx)^2}}{x^2}\, dx$

Optimal. Leaf size=17

$$\text{Int}\left(\frac{e^{\sinh^{-1}(a+bx)^2}}{x^2}, x\right)$$

[Out] CannotIntegrate(exp(arcsinh(b*x+a)^2)/x^2,x)

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{e^{\sinh^{-1}(a+bx)^2}}{x^2}\, dx$$

Verification is Not applicable to the result.

[In] Int[E^ArcSinh[a + b*x]^2/x^2,x]

[Out] Defer[Int][E^ArcSinh[a + b*x]^2/x^2, x]

Rubi steps

$$\int \frac{e^{\sinh^{-1}(a+bx)^2}}{x^2}\, dx = \int \frac{e^{\sinh^{-1}(a+bx)^2}}{x^2}\, dx$$

Mathematica [A] time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{e^{\sinh^{-1}(a+bx)^2}}{x^2}\, dx$$

Verification is Not applicable to the result.

[In] Integrate[E^ArcSinh[a + b*x]^2/x^2,x]

[Out] Integrate[E^ArcSinh[a + b*x]^2/x^2, x]

fricas [A] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{e^{\left(\text{arsinh}(bx+a)^2\right)}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(arcsinh(b*x+a)^2)/x^2,x, algorithm="fricas")

[Out] integral(e^(arcsinh(b*x + a)^2)/x^2, x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{\left(\text{arsinh}(bx+a)^2\right)}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(arcsinh(b*x+a)^2)/x^2,x, algorithm="giac")

[Out] integrate(e^(arcsinh(b*x + a)^2)/x^2, x)

maple [A] time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{e}^{\operatorname{arcsinh}(bx+a)^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arcsinh(b*x+a)^2)/x^2,x)
```

```
[Out] int(exp(arcsinh(b*x+a)^2)/x^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{\left(\operatorname{arsinh}(bx+a)^2\right)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsinh(b*x+a)^2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(e^(arcsinh(b*x + a)^2)/x^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\mathrm{e}^{\operatorname{asinh}(a+b\,x)^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(asinh(a + b*x)^2)/x^2,x)
```

```
[Out] int(exp(asinh(a + b*x)^2)/x^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{\operatorname{asinh}^{2}(a+bx)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(asinh(b*x+a)**2)/x**2,x)
```

```
[Out] Integral(exp(asinh(a + b*x)**2)/x**2, x)
```

3.364 $\int \frac{\sinh^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx$

Optimal. Leaf size=60

$$\frac{\text{Li}_2\left(e^{2\sinh^{-1}(a+bx)}\right)}{2d}-\frac{\sinh^{-1}(a+bx)^2}{2d}+\frac{\sinh^{-1}(a+bx)\log\left(1-e^{2\sinh^{-1}(a+bx)}\right)}{d}$$

```
[Out] -1/2*arcsinh(b*x+a)^2/d+arcsinh(b*x+a)*ln(1-(b*x+a+(1+(b*x+a)^2)^(1/2))^2)/
d+1/2*polylog(2,(b*x+a+(1+(b*x+a)^2)^(1/2))^2)/d
```

Rubi [A] time = 0.10, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {5865, 12, 5659, 3716, 2190, 2279, 2391}

$$\frac{\text{PolyLog}\left(2,e^{2\sinh^{-1}(a+bx)}\right)}{2d}-\frac{\sinh^{-1}(a+bx)^2}{2d}+\frac{\sinh^{-1}(a+bx)\log\left(1-e^{2\sinh^{-1}(a+bx)}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a + b*x]/((a*d)/b + d*x),x]
```

```
[Out] -ArcSinh[a + b*x]^2/(2*d) + (ArcSinh[a + b*x]*Log[1 - E^(2*ArcSinh[a + b*x]
)])/d + PolyLog[2, E^(2*ArcSinh[a + b*x])]/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 5865

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx &= \frac{\text{Subst}\left(\int \frac{b\sinh^{-1}(x)}{dx}\,dx, x, a+bx\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{\sinh^{-1}(x)}{x}\,dx, x, a+bx\right)}{d}\\
&= \frac{\text{Subst}\left(\int x\coth(x)\,dx, x, \sinh^{-1}(a+bx)\right)}{d}\\
&= -\frac{\sinh^{-1}(a+bx)^2}{2d} - \frac{2\,\text{Subst}\left(\int \frac{e^{2x}x}{1-e^{2x}}\,dx, x, \sinh^{-1}(a+bx)\right)}{d}\\
&= -\frac{\sinh^{-1}(a+bx)^2}{2d} + \frac{\sinh^{-1}(a+bx)\log\left(1-e^{2\sinh^{-1}(a+bx)}\right)}{d} - \frac{\text{Subst}\left(\int \log\left(1-e^{2x}\right)\,dx, x\right.}{d}\\
&= -\frac{\sinh^{-1}(a+bx)^2}{2d} + \frac{\sinh^{-1}(a+bx)\log\left(1-e^{2\sinh^{-1}(a+bx)}\right)}{d} - \frac{\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2\sin}\right.}{2d}\\
&= -\frac{\sinh^{-1}(a+bx)^2}{2d} + \frac{\sinh^{-1}(a+bx)\log\left(1-e^{2\sinh^{-1}(a+bx)}\right)}{d} + \frac{\text{Li}_2\left(e^{2\sinh^{-1}(a+bx)}\right)}{2d}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 52, normalized size = 0.87

$$\frac{\text{Li}_2\left(e^{2\sinh^{-1}(a+bx)}\right)-\sinh^{-1}(a+bx)\left(\sinh^{-1}(a+bx)-2\log\left(1-e^{2\sinh^{-1}(a+bx)}\right)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a + b*x]/((a*d)/b + d*x),x]
```

```
[Out] (-(ArcSinh[a + b*x]*(ArcSinh[a + b*x] - 2*Log[1 - E^(2*ArcSinh[a + b*x])]))
 + PolyLog[2, E^(2*ArcSinh[a + b*x])])/(2*d)
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\,\text{arsinh}\,(bx+a)}{bdx+ad}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/(a*d/b+d*x),x, algorithm="fricas")
```

```
[Out] integral(b*arcsinh(b*x + a)/(b*d*x + a*d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}(bx+a)}{dx+\frac{ad}{b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/(a*d/b+d*x),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(b*x + a)/(d*x + a*d/b), x)
```

maple [A] time = 0.06, size = 125, normalized size = 2.08

$$-\frac{\operatorname{arcsinh}(bx+a)^2}{2d}+\frac{\operatorname{arcsinh}(bx+a)\ln\left(1+bx+a+\sqrt{1+(bx+a)^2}\right)}{d}+\frac{\operatorname{polylog}\left(2,-bx-a-\sqrt{1+(bx+a)^2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)/(a*d/b+d*x),x)
```

```
[Out] -1/2*arcsinh(b*x+a)^2/d+1/d*arcsinh(b*x+a)*ln(1+b*x+a+(1+(b*x+a)^2)^(1/2))+
1/d*polylog(2,-b*x-a-(1+(b*x+a)^2)^(1/2))+1/d*arcsinh(b*x+a)*ln(1-b*x-a-(1+
(b*x+a)^2)^(1/2))+1/d*polylog(2,b*x+a+(1+(b*x+a)^2)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}(bx+a)}{dx+\frac{ad}{b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/(a*d/b+d*x),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(b*x + a)/(d*x + a*d/b), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{asinh}(a+b\,x)}{d\,x+\frac{a\,d}{b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a + b*x)/(d*x + (a*d)/b),x)
```

```
[Out] int(asinh(a + b*x)/(d*x + (a*d)/b), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{b\int \frac{\operatorname{asinh}(a+bx)}{a+bx}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)/(a*d/b+d*x),x)
```

```
[Out] b*Integral(asinh(a + b*x)/(a + b*x), x)/d
```

3.365 $\int \frac{x}{\sqrt{1+x^2}\,\sinh^{-1}(x)}\,dx$

Optimal. Leaf size=3

$$\text{Shi}\left(\sinh^{-1}(x)\right)$$

```
[Out] Shi(arcsinh(x))
```

Rubi [A] time = 0.06, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5779, 3298}

$$\text{Shi}\left(\sinh^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[1 + x^2]*ArcSinh[x]),x]
```

```
[Out] SinhIntegral[ArcSinh[x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{1+x^2}\,\sinh^{-1}(x)}\,dx &= \text{Subst}\left(\int \frac{\sinh(x)}{x}\,dx, x, \sinh^{-1}(x)\right)\\ &= \text{Shi}\left(\sinh^{-1}(x)\right)\end{aligned}$$

Mathematica [A] time = 0.06, size = 3, normalized size = 1.00

$$\text{Shi}\left(\sinh^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[1 + x^2]*ArcSinh[x]),x]
```

```
[Out] SinhIntegral[ArcSinh[x]]
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x}{\sqrt{x^2+1}\,\text{arsinh}(x)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(x)/(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x/(sqrt(x^2 + 1)*arcsinh(x)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x^2+1}\,\operatorname{arsinh}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(x)/(x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(x/(sqrt(x^2 + 1)*arcsinh(x)), x)
```

maple [A] time = 0.13, size = 4, normalized size = 1.33

$$\operatorname{Shi}(\operatorname{arcsinh}(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arcsinh(x)/(x^2+1)^(1/2),x)
[Out] Shi(arcsinh(x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x^2+1}\,\operatorname{arsinh}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(x)/(x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x/(sqrt(x^2 + 1)*arcsinh(x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.33

$$\int \frac{x}{\operatorname{asinh}(x)\,\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(asinh(x)*(x^2 + 1)^(1/2)),x)
[Out] int(x/(asinh(x)*(x^2 + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x^2+1}\,\operatorname{asinh}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/asinh(x)/(x**2+1)**(1/2),x)
[Out] Integral(x/(sqrt(x**2 + 1)*asinh(x)), x)
```

3.366 $\int x^3 \sinh^{-1}\left(a+bx^4\right)\,dx$

Optimal. Leaf size=45

$$\frac{\left(a+bx^4\right)\sinh^{-1}\left(a+bx^4\right)}{4b}-\frac{\sqrt{\left(a+bx^4\right)^2+1}}{4b}$$

```
[Out] 1/4*(b*x^4+a)*arcsinh(b*x^4+a)/b-1/4*(1+(b*x^4+a)^2)^(1/2)/b
```

Rubi [A] time = 0.05, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6715, 5863, 5653, 261}

$$\frac{\left(a+bx^4\right)\sinh^{-1}\left(a+bx^4\right)}{4b}-\frac{\sqrt{\left(a+bx^4\right)^2+1}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcSinh[a + b*x^4],x]
```

```
[Out] -Sqrt[1 + (a + b*x^4)^2]/(4*b) + ((a + b*x^4)*ArcSinh[a + b*x^4])/(4*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5863

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\begin{aligned}
\int x^3 \sinh^{-1}\left(a+bx^4\right)\,dx &= \frac{1}{4}\text{Subst}\left(\int \sinh^{-1}(a+bx)\,dx,x,x^4\right)\\
&= \frac{\text{Subst}\left(\int \sinh^{-1}(x)\,dx,x,a+bx^4\right)}{4b}\\
&= \frac{\left(a+bx^4\right)\sinh^{-1}\left(a+bx^4\right)}{4b}-\frac{\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}}\,dx,x,a+bx^4\right)}{4b}\\
&= -\frac{\sqrt{1+\left(a+bx^4\right)^2}}{4b}+\frac{\left(a+bx^4\right)\sinh^{-1}\left(a+bx^4\right)}{4b}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 41, normalized size = 0.91

$$\frac{\left(a+bx^4\right)\sinh^{-1}\left(a+bx^4\right)-\sqrt{\left(a+bx^4\right)^2+1}}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcSinh[a + b*x^4],x]
[Out] (-Sqrt[1 + (a + b*x^4)^2] + (a + b*x^4)*ArcSinh[a + b*x^4])/(4*b)
```

fricas [A] time = 0.69, size = 66, normalized size = 1.47

$$\frac{\left(bx^4+a\right)\log\left(bx^4+a+\sqrt{b^2x^8+2\,abx^4+a^2+1}\right)-\sqrt{b^2x^8+2\,abx^4+a^2+1}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(b*x^4+a),x, algorithm="fricas")
[Out] 1/4*((b*x^4 + a)*log(b*x^4 + a + sqrt(b^2*x^8 + 2*a*b*x^4 + a^2 + 1)) - sqr
t(b^2*x^8 + 2*a*b*x^4 + a^2 + 1))/b
```

giac [B] time = 0.32, size = 105, normalized size = 2.33

$$\frac{1}{4}\,x^4\log\left(bx^4+a+\sqrt{\left(bx^4+a\right)^2+1}\right)-\frac{1}{4}\,b\left(\frac{a\log\left(-ab-\left(x^4|b|-\sqrt{b^2x^8+2\,abx^4+a^2+1}\right)|b|\right)}{b|b|}+\frac{\sqrt{b^2x^8+2\,a}}{b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(b*x^4+a),x, algorithm="giac")
[Out] 1/4*x^4*log(b*x^4 + a + sqrt((b*x^4 + a)^2 + 1)) - 1/4*b*(a*log(-a*b - (x^4
*abs(b) - sqrt(b^2*x^8 + 2*a*b*x^4 + a^2 + 1))*abs(b))/(b*abs(b)) + sqrt(b^
2*x^8 + 2*a*b*x^4 + a^2 + 1)/b^2)
```

maple [A] time = 0.00, size = 38, normalized size = 0.84

$$\frac{\left(b\,x^4+a\right)\operatorname{arcsinh}\left(b\,x^4+a\right)-\sqrt{1+\left(b\,x^4+a\right)^2}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arcsinh(b*x^4+a),x)
[Out] 1/4/b*((b*x^4+a)*arcsinh(b*x^4+a)-(1+(b*x^4+a)^2)^(1/2))
```

maxima [A] time = 0.31, size = 37, normalized size = 0.82

$$\frac{\left(bx^4+a\right)\operatorname{arsinh}\left(bx^4+a\right)-\sqrt{\left(bx^4+a\right)^2+1}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(b*x^4+a),x, algorithm="maxima")
[Out] 1/4*((b*x^4 + a)*arcsinh(b*x^4 + a) - sqrt((b*x^4 + a)^2 + 1))/b
```

mupad [B] time = 0.57, size = 88, normalized size = 1.96

$$\frac{x^4\,\mathrm{asinh}\left(b\,x^4+a\right)}{4}-\frac{\sqrt{a^2+2\,a\,b\,x^4+b^2\,x^8+1}}{4\,b}+\frac{a\,\ln\left(\sqrt{a^2+2\,a\,b\,x^4+b^2\,x^8+1}+\frac{b^2\,x^4+a\,b}{\sqrt{b^2}}\right)}{4\,\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*asinh(a + b*x^4),x)
```

```
[Out] (x^4*asinh(a + b*x^4))/4 - (a^2 + b^2*x^8 + 2*a*b*x^4 + 1)^(1/2)/(4*b) + (a
*log((a^2 + b^2*x^8 + 2*a*b*x^4 + 1)^(1/2) + (a*b + b^2*x^4)/(b^2)^(1/2)))/
(4*(b^2)^(1/2))
```

sympy [A] time = 0.85, size = 61, normalized size = 1.36

$$\begin{cases} \frac{a\,\mathrm{asinh}\left(a+bx^4\right)}{4b}+\frac{x^4\,\mathrm{asinh}\left(a+bx^4\right)}{4}-\frac{\sqrt{a^2+2abx^4+b^2x^8+1}}{4b} & \text{for } b\neq 0 \\ \frac{x^4\,\mathrm{asinh}\,(a)}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*asinh(b*x**4+a),x)
```

```
[Out] Piecewise((a*asinh(a + b*x**4)/(4*b) + x**4*asinh(a + b*x**4)/4 - sqrt(a**2
 + 2*a*b*x**4 + b**2*x**8 + 1)/(4*b), Ne(b, 0)), (x**4*asinh(a)/4, True))
```

3.367 $\int x^{-1+n} \sinh^{-1}(a + bx^n)\, dx$

Optimal. Leaf size=46

$$\frac{(a + bx^n)\sinh^{-1}(a + bx^n)}{bn} - \frac{\sqrt{(a + bx^n)^2 + 1}}{bn}$$

```
[Out] (a+b*x^n)*arcsinh(a+b*x^n)/b/n-(1+(a+b*x^n)^2)^(1/2)/b/n
```

Rubi [A] time = 0.05, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6715, 5863, 5653, 261}

$$\frac{(a + bx^n)\sinh^{-1}(a + bx^n)}{bn} - \frac{\sqrt{(a + bx^n)^2 + 1}}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)*ArcSinh[a + b*x^n],x]
```

```
[Out] -(Sqrt[1 + (a + b*x^n)^2]/(b*n)) + ((a + b*x^n)*ArcSinh[a + b*x^n])/(b*n)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5863

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\begin{aligned}
\int x^{-1+n}\sinh^{-1}(a + bx^n)\, dx &= \frac{\text{Subst}\left(\int \sinh^{-1}(a + bx)\, dx, x, x^n\right)}{n}\\
&= \frac{\text{Subst}\left(\int \sinh^{-1}(x)\, dx, x, a + bx^n\right)}{bn}\\
&= \frac{(a + bx^n)\sinh^{-1}(a + bx^n)}{bn} - \frac{\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}}\, dx, x, a + bx^n\right)}{bn}\\
&= -\frac{\sqrt{1 + (a + bx^n)^2}}{bn} + \frac{(a + bx^n)\sinh^{-1}(a + bx^n)}{bn}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 41, normalized size = 0.89

$$\frac{(a+bx^n)\sinh^{-1}(a+bx^n)-\sqrt{(a+bx^n)^2+1}}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)*ArcSinh[a + b*x^n],x]
[Out] (-Sqrt[1 + (a + b*x^n)^2] + (a + b*x^n)*ArcSinh[a + b*x^n])/(b*n)
```

fricas [B] time = 0.64, size = 152, normalized size = 3.30

$$\frac{\left(b\cosh\left(n\log(x)\right)+b\sinh\left(n\log(x)\right)+a\right)\log\left(b\cosh\left(n\log(x)\right)+b\sinh\left(n\log(x)\right)+a+\sqrt{\frac{2ab+\left(a^2+b^2+1\right)\cosh\left(n\right.}{\cosh\left(n\log\right.}}\right.}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*arcsinh(a+b*x^n),x, algorithm="fricas")
[Out] ((b*cosh(n*log(x)) + b*sinh(n*log(x)) + a)*log(b*cosh(n*log(x)) + b*sinh(n*
log(x)) + a + sqrt((2*a*b + (a^2 + b^2 + 1)*cosh(n*log(x)) - (a^2 - b^2 + 1
)*sinh(n*log(x)))/(cosh(n*log(x)) - sinh(n*log(x))))) - sqrt((2*a*b + (a^2
+ b^2 + 1)*cosh(n*log(x)) - (a^2 - b^2 + 1)*sinh(n*log(x)))/(cosh(n*log(x))
 - sinh(n*log(x)))))/(b*n)
```

giac [B] time = 0.36, size = 113, normalized size = 2.46

$$-\frac{b\left(\frac{a\log\left(-ab-\left(x^n|b|-\sqrt{b^2x^{2n}+2abx^n+a^2+1}\right)|b|\right)}{b|b|}+\frac{\sqrt{b^2x^{2n}+2abx^n+a^2+1}}{b^2}\right)-x^n\log\left(bx^n+a+\sqrt{(bx^n+a)^2+1}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*arcsinh(a+b*x^n),x, algorithm="giac")
[Out] -(b*(a*log(-a*b - (x^n*abs(b) - sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2 + 1))*ab
s(b))/(b*abs(b)) + sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2 + 1)/b^2) - x^n*log(b
*x^n + a + sqrt((b*x^n + a)^2 + 1)))/n
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int x^{-1+n}\operatorname{arcsinh}\left(a+b\,x^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+n)*arcsinh(a+b*x^n),x)
[Out] int(x^(-1+n)*arcsinh(a+b*x^n),x)
```

maxima [A] time = 0.35, size = 39, normalized size = 0.85

$$\frac{(bx^n+a)\operatorname{arsinh}(bx^n+a)-\sqrt{(bx^n+a)^2+1}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*arcsinh(a+b*x^n),x, algorithm="maxima")
[Out] ((b*x^n + a)*arcsinh(b*x^n + a) - sqrt((b*x^n + a)^2 + 1))/(b*n)
```

mupad [B] time = 0.36, size = 99, normalized size = 2.15

$$\frac{x^n\,\mathrm{asinh}\left(a+b\,x^n\right)}{n}-\frac{\sqrt{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n+1}}{b\,n}+\frac{a\,\ln\left(\frac{a\,b+b^2\,x^n}{\sqrt{b^2}}+\sqrt{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n+1}\right)}{n\,\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 1)*asinh(a + b*x^n),x)
```

```
[Out] (x^n*asinh(a + b*x^n))/n - (a^2 + b^2*x^(2*n) + 2*a*b*x^n + 1)^(1/2)/(b*n)
+ (a*log((a*b + b^2*x^n)/(b^2)^(1/2) + (a^2 + b^2*x^(2*n) + 2*a*b*x^n + 1)^
(1/2)))/(n*(b^2)^(1/2))
```

sympy [A] time = 71.05, size = 76, normalized size = 1.65

$$\begin{cases} \log{\left(x \right)} \operatorname{asinh}{\left(a \right)} & \text{for}\: b = 0 \wedge n = 0 \\ \frac{x^{n} \operatorname{asinh}{\left(a \right)}}{n} & \text{for}\: b = 0 \\ \log{\left(x \right)} \operatorname{asinh}{\left(a + b \right)} & \text{for}\: n = 0 \\ \frac{a \operatorname{asinh}{\left(a + b x^{n} \right)}}{b n} + \frac{x^{n} \operatorname{asinh}{\left(a + b x^{n} \right)}}{n} - \frac{\sqrt{a^{2} + 2 a b x^{n} + b^{2} x^{2 n} + 1}}{b n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*asinh(a+b*x**n),x)
```

```
[Out] Piecewise((log(x)*asinh(a), Eq(b, 0) & Eq(n, 0)), (x**n*asinh(a)/n, Eq(b, 0
)), (log(x)*asinh(a + b), Eq(n, 0)), (a*asinh(a + b*x**n)/(b*n) + x**n*asin
h(a + b*x**n)/n - sqrt(a**2 + 2*a*b*x**n + b**2*x**(2*n) + 1)/(b*n), True))
```

3.368 $\int \sinh^{-1}\left(\frac{c}{a+bx}\right) dx$

Optimal. Leaf size=49

$$\frac{c\tanh^{-1}\left(\sqrt{\frac{1}{\left(\frac{a}{c}+\frac{bx}{c}\right)^2}+1}\right)}{b}+\frac{(a+bx)\text{csch}^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b}$$

```
[Out] (b*x+a)*arccsch(a/c+b*x/c)/b+c*arctanh((1+1/(a/c+b*x/c)^2)^(1/2))/b
```

Rubi [A] time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5892, 6314, 372, 266, 63, 207}

$$\frac{c\tanh^{-1}\left(\sqrt{\frac{1}{\left(\frac{a}{c}+\frac{bx}{c}\right)^2}+1}\right)}{b}+\frac{(a+bx)\text{csch}^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[c/(a + b*x)],x]
```

```
[Out] ((a + b*x)*ArcCsch[a/c + (b*x)/c])/b + (c*ArcTanh[Sqrt[1 + (a/c + (b*x)/c)^
(-2)]])/b
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rule 5892

```
Int[ArcSinh[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int
[u*ArcCsch[a/c + (b*x^n)/c]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rule 6314

```
Int[ArcCsch[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[((c + d*x)*ArcCsch[c + d*
x])/d, x] + Int[1/((c + d*x)*Sqrt[1 + 1/(c + d*x)^2]), x] /; FreeQ[{c, d},
x]
```

Rubi steps

$$
\begin{aligned}
\int \sinh^{-1}\left(\frac{c}{a+bx}\right)dx &= \int \operatorname{csch}^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)dx\\
&= \frac{(a+bx)\operatorname{csch}^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b}+\int \frac{1}{\left(\frac{a}{c}+\frac{bx}{c}\right)\sqrt{1+\frac{1}{\left(\frac{a}{c}+\frac{bx}{c}\right)^2}}}\,dx\\
&= \frac{(a+bx)\operatorname{csch}^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b}+\frac{c\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{1}{x^2}}\,x}\,dx,x,\frac{a}{c}+\frac{bx}{c}\right)}{b}\\
&= \frac{(a+bx)\operatorname{csch}^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b}-\frac{c\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx,x,\frac{1}{\left(\frac{a}{c}+\frac{bx}{c}\right)^2}\right)}{2b}\\
&= \frac{(a+bx)\operatorname{csch}^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b}-\frac{c\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\sqrt{1+\frac{c^2}{(a+bx)^2}}\right)}{b}\\
&= \frac{(a+bx)\operatorname{csch}^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b}+\frac{c\tanh^{-1}\left(\sqrt{1+\frac{c^2}{(a+bx)^2}}\right)}{b}
\end{aligned}
$$

Mathematica [B] time = 0.13, size = 131, normalized size = 2.67

$$
\frac{(a+bx)\sqrt{\frac{a^2+2abx+b^2x^2+c^2}{(a+bx)^2}}\left(c\tanh^{-1}\left(\frac{a+bx}{\sqrt{a^2+2abx+b^2x^2+c^2}}\right)+a\tanh^{-1}\left(\frac{\sqrt{(a+bx)^2+c^2}}{c}\right)\right)}{b\sqrt{a^2+2abx+b^2x^2+c^2}}+x\sinh^{-1}\left(\frac{c}{a+bx}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[c/(a + b*x)],x]
```

```
[Out] x*ArcSinh[c/(a + b*x)] + ((a + b*x)*Sqrt[(a^2 + c^2 + 2*a*b*x + b^2*x^2)/(a
 + b*x)^2]*(c*ArcTanh[(a + b*x)/Sqrt[a^2 + c^2 + 2*a*b*x + b^2*x^2]] + a*Ar
cTanh[Sqrt[c^2 + (a + b*x)^2]/c]))/(b*Sqrt[a^2 + c^2 + 2*a*b*x + b^2*x^2])
```

fricas [B] time = 0.90, size = 242, normalized size = 4.94

$$
\frac{bx\log\left(\frac{(bx+a)\sqrt{\frac{b^2x^2+2\,abx+a^2+c^2}{b^2x^2+2\,abx+a^2}}+c}{bx+a}\right)+a\log\left(-bx+(bx+a)\sqrt{\frac{b^2x^2+2\,abx+a^2+c^2}{b^2x^2+2\,abx+a^2}}-a+c\right)-a\log\left(-bx+(bx+a)\sqrt{\frac{b^2 \cdots}{b \cdots}}\right.}{b}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(c/(b*x+a)),x, algorithm="fricas")
```

```
[Out] (b*x*log(((b*x + a)*sqrt((b^2*x^2 + 2*a*b*x + a^2 + c^2)/(b^2*x^2 + 2*a*b*x
 + a^2)) + c)/(b*x + a)) + a*log(-b*x + (b*x + a)*sqrt((b^2*x^2 + 2*a*b*x +
 a^2 + c^2)/(b^2*x^2 + 2*a*b*x + a^2)) - a + c) - a*log(-b*x + (b*x + a)*sq
```

```
rt((b^2*x^2 + 2*a*b*x + a^2 + c^2)/(b^2*x^2 + 2*a*b*x + a^2)) - a - c) - c*
log(-b*x + (b*x + a)*sqrt((b^2*x^2 + 2*a*b*x + a^2 + c^2)/(b^2*x^2 + 2*a*b*
x + a^2)) - a))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{arsinh}\left(\frac{c}{bx+a}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(c/(b*x+a)),x, algorithm="giac")
[Out] integrate(arcsinh(c/(b*x + a)), x)
```

maple [A] time = 0.04, size = 46, normalized size = 0.94

$$-\frac{c\left(-\frac{\operatorname{arcsinh}\left(\frac{c}{bx+a}\right)(bx+a)}{c}-\operatorname{arctanh}\left(\frac{1}{\sqrt{\frac{c^2}{(bx+a)^2}+1}}\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(c/(b*x+a)),x)
[Out] -1/b*c*(-arcsinh(c/(b*x+a))/c*(b*x+a)-arctanh(1/(c^2/(b*x+a)^2+1)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{ic\left(\log\left(\frac{i\left(b^2x+ab\right)}{bc}+1\right)-\log\left(-\frac{i\left(b^2x+ab\right)}{bc}+1\right)\right)}{2b}+\frac{2bx\log\left(c+\sqrt{b^2x^2+2abx+a^2+c^2}\right)+a\log\left(b^2x^2+2abx+a\right.}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(c/(b*x+a)),x, algorithm="maxima")
[Out] -1/2*I*c*(log(I*(b^2*x + a*b)/(b*c) + 1) - log(-I*(b^2*x + a*b)/(b*c) + 1))
/b + 1/2*(2*b*x*log(c + sqrt(b^2*x^2 + 2*a*b*x + a^2 + c^2)) + a*log(b^2*x^
2 + 2*a*b*x + a^2 + c^2) - 2*(b*x + a)*log(b*x + a))/b + integrate((b^2*c*x
^2 + a*b*c*x)/(b^2*c*x^2 + 2*a*b*c*x + a^2*c + c^3 + (b^2*x^2 + 2*a*b*x + a
^2 + c^2)^(3/2)), x)
```

mupad [B] time = 1.09, size = 41, normalized size = 0.84

$$\frac{c\,\mathrm{atanh}\left(\sqrt{\frac{c^2}{(a+b\,x)^2}+1}\right)}{b}+\frac{\mathrm{asinh}\left(\frac{c}{a+b\,x}\right)\,(a+b\,x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(c/(a + b*x)),x)
[Out] (c*atanh((c^2/(a + b*x)^2 + 1)^(1/2)))/b + (asinh(c/(a + b*x))*(a + b*x))/b
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{asinh}\left(\frac{c}{a+bx}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(c/(b*x+a)),x)
[Out] Integral(asinh(c/(a + b*x)), x)
```

3.369 $\int \frac{x}{\sinh^{-1}(\sinh(x))} \, dx$

Optimal. Leaf size=27

$$\sinh^{-1}(\sinh(x)) + \log\left(\sinh^{-1}(\sinh(x))\right)\left(x\sqrt{\cosh^2(x)}\,\mathrm{sech}(x) - \sinh^{-1}(\sinh(x))\right)$$

```
[Out] arcsinh(sinh(x))+ln(arcsinh(sinh(x)))*(-arcsinh(sinh(x))+x*sech(x)*(cosh(x)
^2)^(1/2))
```

Rubi [F] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x}{\sinh^{-1}(\sinh(x))} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x/ArcSinh[Sinh[x]],x]
```

```
[Out] Defer[Int][x/ArcSinh[Sinh[x]], x]
```

Rubi steps

$$\int \frac{x}{\sinh^{-1}(\sinh(x))} \, dx = \int \frac{x}{\sinh^{-1}(\sinh(x))} \, dx$$

Mathematica [A] time = 0.56, size = 28, normalized size = 1.04

$$x\sqrt{\cosh^2(x)}\,\mathrm{sech}(x)\log\left(\sinh^{-1}(\sinh(x))\right) - \sinh^{-1}(\sinh(x))\left(\log\left(\sinh^{-1}(\sinh(x))\right) - 1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/ArcSinh[Sinh[x]],x]
```

```
[Out] -(ArcSinh[Sinh[x]]*(-1 + Log[ArcSinh[Sinh[x]]])) + x*Sqrt[Cosh[x]^2]*Log[Ar
cSinh[Sinh[x]]]*Sech[x]
```

fricas [A] time = 0.48, size = 1, normalized size = 0.04

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(sinh(x)),x, algorithm="fricas")
```

```
[Out] x
```

giac [A] time = 0.28, size = 1, normalized size = 0.04

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(sinh(x)),x, algorithm="giac")
```

```
[Out] x
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{x}{\operatorname{arcsinh}\left(\sinh(x)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arcsinh(sinh(x)),x)
```

```
[Out] int(x/arcsinh(sinh(x)),x)
```

maxima [A] time = 0.78, size = 1, normalized size = 0.04

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(sinh(x)),x, algorithm="maxima")
```

```
[Out] x
```

mupad [B] time = 0.22, size = 1, normalized size = 0.04

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/asinh(sinh(x)),x)
```

```
[Out] x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\operatorname{asinh}\left(\sinh\left(x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/asinh(sinh(x)),x)
```

```
[Out] Integral(x/asinh(sinh(x)), x)
```

3.370 $\int \frac{\sinh^{-1}\left(\sqrt{-1+bx^2}\right)^n}{\sqrt{-1+bx^2}} \, dx$

Optimal. Leaf size=37

$$\frac{\sqrt{bx^2}\sinh^{-1}\left(\sqrt{bx^2-1}\right)^{n+1}}{b(n+1)x}$$

```
[Out] arcsinh((b*x^2-1)^(1/2))^(1+n)*(b*x^2)^(1/2)/b/(1+n)/x
```

Rubi [A] time = 0.07, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {5894, 5675}

$$\frac{\sqrt{bx^2}\sinh^{-1}\left(\sqrt{bx^2-1}\right)^{n+1}}{b(n+1)x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[Sqrt[-1 + b*x^2]]^n/Sqrt[-1 + b*x^2],x]
```

```
[Out] (Sqrt[b*x^2]*ArcSinh[Sqrt[-1 + b*x^2]]^(1 + n))/(b*(1 + n)*x)
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5894

```
Int[ArcSinh[Sqrt[-1 + (b_.)*(x_)^2]]^(n_.)/Sqrt[-1 + (b_.)*(x_)^2], x_Symbo
l] :> Dist[Sqrt[b*x^2]/(b*x), Subst[Int[ArcSinh[x]^n/Sqrt[1 + x^2], x], x,
Sqrt[-1 + b*x^2]], x] /; FreeQ[{b, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^{-1}\left(\sqrt{-1+bx^2}\right)^n}{\sqrt{-1+bx^2}} \, dx &= \frac{\sqrt{bx^2}\,\text{Subst}\left(\int \frac{\sinh^{-1}(x)^n}{\sqrt{1+x^2}} \, dx, x, \sqrt{-1+bx^2}\right)}{bx}\\ &= \frac{\sqrt{bx^2}\sinh^{-1}\left(\sqrt{-1+bx^2}\right)^{1+n}}{b(1+n)x}\end{aligned}$$

Mathematica [A] time = 0.04, size = 37, normalized size = 1.00

$$\frac{\sqrt{bx^2}\sinh^{-1}\left(\sqrt{bx^2-1}\right)^{n+1}}{b(n+1)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[Sqrt[-1 + b*x^2]]^n/Sqrt[-1 + b*x^2],x]
```

```
[Out] (Sqrt[b*x^2]*ArcSinh[Sqrt[-1 + b*x^2]]^(1 + n))/(b*(1 + n)*x)
```

fricas [B] time = 0.57, size = 108, normalized size = 2.92

$$\frac{\sqrt{bx^2}\cosh\left(n\log\left(\log\left(\sqrt{bx^2-1}+\sqrt{bx^2}\right)\right)\right)\log\left(\sqrt{bx^2-1}+\sqrt{bx^2}\right)+\sqrt{bx^2}\log\left(\sqrt{bx^2-1}+\sqrt{bx^2}\right)\sinh\left(n\log\left(\log\left(\sqrt{bx^2-1}+\sqrt{bx^2}\right)\right)\right)}{(bn+b)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh((b*x^2-1)^(1/2))^n/(b*x^2-1)^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(b*x^2)*cosh(n*log(log(sqrt(b*x^2 - 1) + sqrt(b*x^2))))*log(sqrt(b*x^2
 - 1) + sqrt(b*x^2)) + sqrt(b*x^2)*log(sqrt(b*x^2 - 1) + sqrt(b*x^2))*sinh(
n*log(log(sqrt(b*x^2 - 1) + sqrt(b*x^2)))))/((b*n + b)*x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh((b*x^2-1)^(1/2))^n/(b*x^2-1)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsinh}\left(\sqrt{b\,x^2-1}\right)^n}{\sqrt{b\,x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh((b*x^2-1)^(1/2))^n/(b*x^2-1)^(1/2),x)
```

```
[Out] int(arcsinh((b*x^2-1)^(1/2))^n/(b*x^2-1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}\left(\sqrt{bx^2-1}\right)^n}{\sqrt{bx^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh((b*x^2-1)^(1/2))^n/(b*x^2-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(sqrt(b*x^2 - 1))^n/sqrt(b*x^2 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\operatorname{asinh}\left(\sqrt{b\,x^2-1}\right)^n}{\sqrt{b\,x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh((b*x^2 - 1)^(1/2))^n/(b*x^2 - 1)^(1/2),x)
```

```
[Out] int(asinh((b*x^2 - 1)^(1/2))^n/(b*x^2 - 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} -\frac{2x}{\pi} & \text{for}\ b = 0 \wedge n = -1 \\ -ix\left(\frac{i\pi}{2}\right)^n & \text{for}\ b = 0 \\ \int \frac{1}{\sqrt{bx^2-1}\ \operatorname{asinh}\left(\sqrt{bx^2-1}\right)}\, dx & \text{for}\ n = -1 \\ \frac{\sqrt{b}\ \sqrt{x^2}\ \operatorname{asinh}\left(\sqrt{bx^2-1}\right)\operatorname{asinh}^n\left(\sqrt{bx^2-1}\right)}{bnx+bx} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh((b*x**2-1)**(1/2))**n/(b*x**2-1)**(1/2),x)
```

```
[Out] Piecewise((-2*x/pi, Eq(b, 0) & Eq(n, -1)), (-I*x*(I*pi/2)**n, Eq(b, 0)), (I
ntegral(1/(sqrt(b*x**2 - 1)*asinh(sqrt(b*x**2 - 1))), x), Eq(n, -1)), (sqrt
(b)*sqrt(x**2)*asinh(sqrt(b*x**2 - 1))*asinh(sqrt(b*x**2 - 1))**n/(b*n*x +
b*x), True))
```

3.371 $\int \frac{1}{\sqrt{-1+bx^2}\,\sinh^{-1}\left(\sqrt{-1+bx^2}\right)}\,dx$

Optimal. Leaf size=29

$$\frac{\sqrt{bx^2}\log\left(\sinh^{-1}\left(\sqrt{bx^2-1}\right)\right)}{bx}$$

```
[Out] ln(arcsinh((b*x^2-1)^(1/2)))*(b*x^2)^(1/2)/b/x
```

Rubi [A] time = 0.06, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {5894, 5673}

$$\frac{\sqrt{bx^2}\log\left(\sinh^{-1}\left(\sqrt{bx^2-1}\right)\right)}{bx}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 + b*x^2]*ArcSinh[Sqrt[-1 + b*x^2]]),x]
```

```
[Out] (Sqrt[b*x^2]*Log[ArcSinh[Sqrt[-1 + b*x^2]]])/(b*x)
```

Rule 5673

```
Int[1/(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*Sqrt[(d_) + (e_.)*(x_)^2]), x_Sy
mbol] :> Simp[Log[a + b*ArcSinh[c*x]]/(b*c*Sqrt[d]), x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

Rule 5894

```
Int[ArcSinh[Sqrt[-1 + (b_.)*(x_)^2]]^(n_.)/Sqrt[-1 + (b_.)*(x_)^2], x_Symbo
l] :> Dist[Sqrt[b*x^2]/(b*x), Subst[Int[ArcSinh[x]^n/Sqrt[1 + x^2], x], x,
Sqrt[-1 + b*x^2]], x] /; FreeQ[{b, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-1+bx^2}\,\sinh^{-1}\left(\sqrt{-1+bx^2}\right)}\,dx &= \frac{\sqrt{bx^2}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}\,\sinh^{-1}(x)}\,dx,x,\sqrt{-1+bx^2}\right)}{bx}\\ &= \frac{\sqrt{bx^2}\log\left(\sinh^{-1}\left(\sqrt{-1+bx^2}\right)\right)}{bx}\end{aligned}$$

Mathematica [A] time = 0.02, size = 24, normalized size = 0.83

$$\frac{x\log\left(\sinh^{-1}\left(\sqrt{bx^2-1}\right)\right)}{\sqrt{bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 + b*x^2]*ArcSinh[Sqrt[-1 + b*x^2]]),x]
```

```
[Out] (x*Log[ArcSinh[Sqrt[-1 + b*x^2]]])/Sqrt[b*x^2]
```

fricas [**A**] time = 0.48, size = 33, normalized size = 1.14

$$\frac{\sqrt{bx^2}\log\left(\log\left(\sqrt{bx^2-1}+\sqrt{bx^2}\right)\right)}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh((b*x^2-1)^(1/2))/(b*x^2-1)^(1/2),x, algorithm="fricas")
[Out] sqrt(b*x^2)*log(log(sqrt(b*x^2 - 1) + sqrt(b*x^2)))/(b*x)
```

giac [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh((b*x^2-1)^(1/2))/(b*x^2-1)^(1/2),x, algorithm="giac")
[Out] Timed out
```

maple [**F**] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{arcsinh}\left(\sqrt{b\,x^2-1}\right)\sqrt{b\,x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arcsinh((b*x^2-1)^(1/2))/(b*x^2-1)^(1/2),x)
[Out] int(1/arcsinh((b*x^2-1)^(1/2))/(b*x^2-1)^(1/2),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^2-1}\,\operatorname{arsinh}\left(\sqrt{bx^2-1}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh((b*x^2-1)^(1/2))/(b*x^2-1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*x^2 - 1)*arcsinh(sqrt(b*x^2 - 1))), x)
```

mupad [**B**] time = 0.25, size = 23, normalized size = 0.79

$$\frac{\ln\left(\operatorname{asinh}\left(\sqrt{b\,x^2-1}\right)\right)\sqrt{x^2}}{\sqrt{b}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(asinh((b*x^2 - 1)^(1/2))*(b*x^2 - 1)^(1/2)),x)
[Out] (log(asinh((b*x^2 - 1)^(1/2)))*(x^2)^(1/2))/(b^(1/2)*x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^2-1}\,\operatorname{asinh}\left(\sqrt{bx^2-1}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/asinh((b*x**2-1)**(1/2))/(b*x**2-1)**(1/2),x)
[Out] Integral(1/(sqrt(b*x**2 - 1)*asinh(sqrt(b*x**2 - 1))), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
          print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9
```

```
#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
```

```
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
   sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
   hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
   sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
   sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
```

```
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```